**Recent advancements in structure-preserving integration for nonlinear and inelastic problems**

Preserving physical and geometrical structures in dynamic calculations has been demonstrated to be critical in ensuring reliable results. The concept of energy-momentum scheme and the discrete gradient strategy have enabled the design of energy-momentum consistent integrators for shell dynamics, multibody systems, elastoplasticity, viscoelasticity, contact, and impact, to list a few. Abundant numerical evidence has justified its advantage over classical implicit integrators, especially for long-term simulations. In this talk, I will discuss recent advancements in designing structure-preserving integrators.

First, I will focus on the discrete gradient formula initially proposed by O. Gonzalez. Its quotient form renders it to be numerically singular when the adjacent solutions are close. Leveraging a set of specially developed quadrature rules, the potential energy can be split into two parts, each of which is treated separately to guarantee a dissipative nature in the numerical residual. The resulting integrators are non-singular and energy- momentum consistent. Interestingly, they are found to be dissipative for the high-frequency modes without harming the relative equilibria. The technique can be utilized to improve the robustness of existing conserving integrators.

Second, I will focus on incompressible elastodynamics and propose a new Hamiltonian. Based on a recently proposed mixed formulation and the discrete gradient formula, this Hamiltonian and momenta can be conserved in the fully discrete level. The scaled mid-point formula, another popular option for constructing algorithmic stresses, is analyzed and demonstrated to be non-robust numerically. The generalized Taylor-Hood element based on the spline technology offers a higher-order, robust, and inf-sup stable spatial discretization option. This element technology is further enhanced by the grad-div stabilization to improve its discrete mass conservation. I will also discuss and evaluate the spectral decomposition algorithms for stretch-based models.

Third, I will discuss a viscoelastic model that gained popularity over the years but was recently identified to suffer from thermodynamic inconsistency. I will revisit this model by proposing a complete thermomechanical theory of it. The derivation elucidates the origin of the evolution equations of that model, with a few non-negligible differences. Based on the consistent framework, an energy-momentum scheme is constructed using a strain-driven approach and a generalized directionality property for the stress-like variable.

Numerical examples will be provided to justify the effectiveness of the overall methodology. I will draw conclusions and discuss some promising directions at the end of this talk.

Bio: Ju Liu is an assistant professor in the Department of Mechanics and Aerospace Engineering at Southern University of Science and Technology. He received his PhD in Computational and Applied Mathematicss from the University of Texas at Austin. From 2016 to 2020, he was a postdoctoral fellow at Stanford University. His research interests include finite element method, fluid-structure interaction, biomechanics, and high-performance computing.