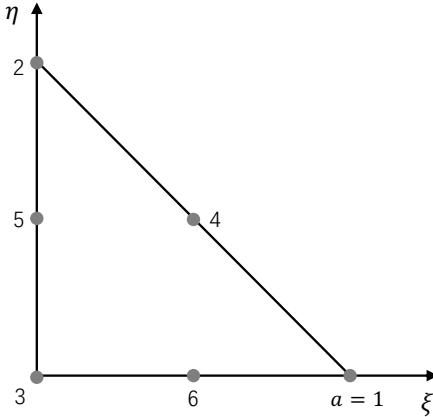


Homework 7

Due: Dec. 15 2025

1. Consider the six-node triangle element shown below. There are three extra nodes on the mid of the three edges. It is possible to define quadratic shape functions on this element, which takes the form $N_a(\xi, \eta) = c_0 + c_1\xi + c_2\eta + c_3\xi^2 + c_4\eta^2 + c_5\xi\eta$.



Determine the expressions for all six shape functions.

2. The integration of monomials $\xi^\alpha\eta^\beta$ over the reference triangle can be given in the exact expression

$$\int_{\bar{\Omega}} \xi^\alpha \eta^\beta d\Omega = \frac{\alpha! \beta!}{(\alpha + \beta + 2)!}.$$

Consider the three-point quadrature rule:

$$w_1 = w_2 = w_3 = 1/6,$$

$$\xi_1 = \frac{2}{3}, \eta_1 = \frac{1}{6},$$

$$\xi_2 = \frac{1}{6}, \eta_2 = \frac{1}{6},$$

$$\xi_3 = \frac{1}{6}, \eta_3 = \frac{2}{3}.$$

Determine the highest order of the monomials (i.e. $\alpha + \beta$) that this quadrature rule can achieve for exact integration.

3. Consider the 2D heat equation solver we developed in class. If we want to switch to using the six-node triangle element, what part of the code needs to be modified? Please discuss the modification in detail. You do **not** have to provide the code.
4. Exercises 2 and 3 on page 82.
5. Exercise 5 on page 83.