

Homework 5

Due: Nov. 17 2025

1. Report your github account and the link to your repository. Please make your repository **private** and add me and the course TA to your repository.
2. Work on the 1D Matlab code developed in class. Your code development shall be submitted to your github repository.

- a. Consider the heat conductivity is a function of x , that is, the governing equation now reads

$$\frac{d}{dx} \left(-\kappa(x) \frac{du}{dx} \right) = f.$$

The boundary condition is the same. Modify the code.

- b. Calculate the following two relative errors of the solution using your code,

$$e_{L2} := \frac{\left(\int_0^1 (u^h - u)^2 dx \right)^{1/2}}{\left(\int_0^1 u^2 dx \right)^{1/2}},$$

$$e_{H1} := \frac{\left(\int_0^1 (u_{,x}^h - u_{,x})^2 dx \right)^{1/2}}{\left(\int_0^1 u_{,x}^2 dx \right)^{1/2}},$$

with 2, 4, 6, 8, 10, 12, 14, and 16 elements and uniform mesh size. Plot the relative errors against the mesh size in the log-log plot (i.e., plot $\log(\text{error})$ against $\log(h)$). Determine the slope of the curves.

- c. Enable the code for higher-order elements using quadratic and cubic elements. Modify the code. Repeat the calculation of the errors using the two higher-order elements.
- d. The equivalence between (G) and (M) is contingent upon the accuracy of the quadrature. Run the code with the element of degree 3 (i.e. cubic element). Experiment the code with 1,2,3,4,5,6 quadrature points, respectively. Report your observation and make comments. (The effect of numerical quadrature is discussed in Hughes book p. 191.)