MAE 310 Final Project

Due: Jan. 12 2026

The grading of the final project is based on the code development displayed in your GitHub repository (**30 points**) and your submitted technical report (**70 points**).

Submission Guidelines:

1. Provide the code development history in the GitHub repository with clear file structures.

2. Submit a PDF file of the report on the numerical method and result analysis. Recommended length: 6 pages, excluding figures.

Code development:

Consider a plane elasticity problem. Implement a finite element code using bilinear quadrilateral element or linear triangle element. You may assume that the material is homogeneous isotropic elastic.

- Generate the mesh (including the nodal coordinates, the IEN connectivity, etc.) using Gmsh and load the mesh into your code. (**5 points**)

- Your implementation should allow different boundary segments to carry different prescribed displacement/tractions. (**10 points**)

- Regarding the model, you may allow the users to specify either plane strain or plane stress for the two-dimensional analysis. (**5 points**)

- Provide a visualization routine that can draw the displacement and stress components on the modeled elastic body. (**5 points**)

- Provide an error calculator that can calculate the relative errors in the L_2- and H_1-norms. (**5 points**)

Final project report:

- State the strong-, weak-, Galerkin, and matrix formulations for linear elasticity. Provide the derivation of the formulations. Discuss how different boundary conditions enter your numerical formulation. (**10 points**)

- Elaborate on the implementation of different boundary conditions in Gmsh and your code. (**10 points**)

- Discuss your chosen implementation of the element stiffness matrix. You may want to consult Sec. 3.10 of the textbook for different implementations. (**10 points**)

- Verify your code using a manufactured solution. Report your chosen manufactured solution, the corresponding boundary condition, the corresponding body force, and the resulting errors and convergence rates measured in the L_2- and H_1-norms, respectively. You need to perform at least four levels of mesh refinement to monitor the error reduction pattern. State the theoretical rates of convergence. Compare with your observation and make comments. (**15 points**)

- Consider a hollow thick-walled cylinder. Through a shrink-fit process, a rigid solid cylinder of radius $r_1 + \delta$ is to be inserted into the hollow cylinder of inner radius r_1 and outer radius r_2 (see Figure 1). This process results in a displacement boundary condition $u_r(r_1) = \delta$. The outer surface of the hollow cylinder is to remain stress free. We assume the cylinder is long and the cylinder can be modeled under plane strain conditions. The material is homogeneous and isotropic. We may ignore the body force. Determine the stress field within the cylinder using your finite element code. (**20 points**) Determine the analytical stress field within the cylinder under the above shrink-fit condition. Compare your analytical stress distribution with a numerical simulation. (**Bonus 10 points**)

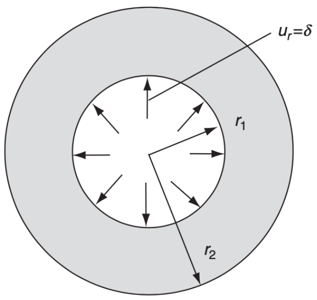

Figure 1 Thick-walled cylinder problem

- Provide conclusive remarks summarizing your findings and insights. (**5 points**) You may also include constructive feedback on the course. (**Bonus 5 points**)