Homework 2

Due: Oct. 13 2025

1. Consider the boundary-value problem below,

$$u_{,xx} + f = 0,$$
$$u(1) = g,$$
$$-u_{,x}(0) = h.$$

Assume that $f = \sin(x)$.

a. Show the following identities

$$(N_A, N_{A-1}) = \frac{h_{A-1}}{6}, \quad (N_A, N_A) = \frac{h_{A-1} + h_A}{3}, \quad (N_A, N_{A+1}) = \frac{h_A}{6}.$$

b. Employing the linear finite element space with equally spaced nodes, set up the stiffness matrix and load vector using 3 elements. Solve the matrix problem. Use $\hat{f} \approx \sum_{B=1}^{n+1} f_B N_B$ in the assembly of the load vector. Do you get nodally exact solution here?

c. Repeat the calculation without invoking the approximation $\hat{f}$. In other words, calculate (N_A, f) rather than $(N_A, \hat{f})$ in the load vector. What do you observe? Notice that the following identity can be helpful.

$$\int xsin(x)dx = -\int xdcos(x) = -xcos(x) + \int \cos(x)\, dx = -xcos(x) + \sin(x).$$