Homework 5

Due: Nov. 17 2025

1. Report your github account and the link to your repository. Please make your repository **private** and add me and the course TA to your repository.

2. Work on the 1D Matlab code developed in class. Your code development shall be submitted to your github repository.

a. Consider the heat conductivity is a function of x, that is, the governing equation now reads

$$\frac{d}{dx}\left(-\kappa(x)\frac{du}{dx}\right) = f.$$

The boundary condition is the same. Modify the code. (Note: to maintain positiveness of the problem, you have to ensure $\kappa(x)$ to be positive over the whole domain. A safe choice is $\kappa(x) = 1 + x^2$, for example.)

b. Calculate the following two relative errors of the solution using your code,

$$e_{L2} := \frac{\left(\int_0^1 (u^h - u)^2 dx\right)^{1/2}}{\left(\int_0^1 u^2 dx\right)^{1/2}},$$

$$e_{H1} := \frac{\left(\int_0^1 (u^h_{,x} - u_{,x})^2 dx\right)^{1/2}}{\left(\int_0^1 u^2_{,x} dx\right)^{1/2}},$$

with 2, 4, 6, 8, 10, 12, 14, and 16 elements and uniform mesh size. Plot the relative errors against the mesh size in the log-log plot (i.e., plot log(error) against log(h)). Determine the slope of the curves.

c. Enable the code for higher-order elements using quadratic and cubic elements. Modify the code. Repeat the calculation of the errors using the two higher-order elements.

d. The equivalence between (G) and (M) is contingent upon the accuracy of the quadrature. Run the code with the element of degree 3 (i.e. cubic element). Experiment the code with 1,2,3,4,5,6 quadrature points, respectively. Report your observation and make comments. (The effect of numerical quadrature is discussed in Hughes book p. 191.)