

Homework 4

Due: Nov. 3 2025

1. Go over the textbook from page 126 to 128 on Lagrange polynomials.
2. Exercise 1 on page 128.
3. A quadrature rule is often used for the calculation of integrals on computers. It involves the quadrature points ξ_l and their weights w_l . It approximates an integral as follows,

$$\int_{-1}^1 g(\xi) d\xi \approx \sum_{l=1}^{n_{\text{int}}} w_l g(\xi_l),$$

where n_{int} is the number of quadrature points. State the trapezoidal rule and the Simpson's rule?

4. For the Gaussian quadrature rule with $n_{\text{int}} = 2$, we have $\xi_1 = -1/\sqrt{3}$, $w_1 = 1$ and $\xi_2 = 1/\sqrt{3}$, $w_2 = 1$. Verify that the two-point Gaussian rule can exactly integrate the monomials 1 , ξ^2 , ξ^3 but not ξ^4 . Compared with the trapezoidal rule, what can you say about the two-point Gaussian quadrature rule?
5. Derive the Gauss quadrature rule for $n_{\text{int}} = 3$. You may assume that $\xi_1 = -\xi_3$, $\xi_2 = 0$, and $w_1 = w_3$.