Homework 4

Due April 27 2021

1. Show that the stiffness matrix of the linear elastostatic problem is symmetric.

2. (Triangle element) Consider the two-dimensional triangle element. Its parent or reference element is defined in the $r-s$ coordinate as is shown in Figure 1. For convenience, the coordinate $t := 1 - r - s$ is introduced. It can be easily observed that $t = \text{constant}$ represents the lines parallel to the inclined edge of the triangle.

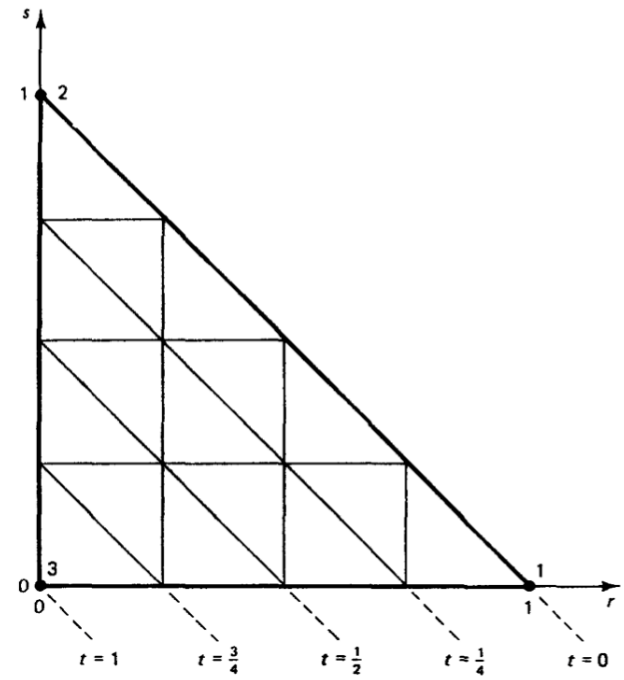

Figure 1: Triangle element in the reference domain.

(a) Consider the three node triangle element, whose shape functions are

$$N_1(r,s) = r, \quad N_2(r,s) = s, \quad N_3(r,s) = t = 1 - r - s.$$

Use the surf function in MATLAB to visualize the shape functions.

(b) The quadratic element defined on the triangle has shape functions

$$N_1(r,s) = r(2r-1), \quad N_2(r,s) = s(2s-1), \quad N_3(r,s) = t(2t-1),$$
$$N_4(r,s) = 4rs, \quad N_5(r,s) = 4st, \quad N_6(r,s) = 4rt.$$

Again, visualize these shape functions.

(c) Consider the 3-point quadrature rule for the triangle shown in Figure 1 as follows.

$$w_1 = \frac{1.0}{3.0}, \quad r_1 = 0.5, \quad s_1 = 0.5;$$
$$w_2 = \frac{1.0}{3.0}, \quad r_2 = 0.5, \quad s_2 = 0.0;$$
$$w_3 = \frac{1.0}{3.0}, \quad r_3 = 0.0, \quad s_3 = 0.5.$$

Determine the algebraic accuracy of this quadrature rule.

3. (convergence rate of the error in the L_2-norm) Revisit the MATLAB code we developed in the second homework and do the following.

   (a) Plot the basis functions for polynomial degree 1, 2, 3, 4, 5, and 6 on the reference or parent domain $\hat{\Omega} = [-1, 1]$.

   (b) Let $e := u^h - u$ be the error in the finite element approximation. Calculate

$$\|e\|_0 := \left(\int_0^1 e^2 dx \right)^{\frac{1}{2}},$$

and

$$\|u\|_0 := \left(\int_0^1 u^2 dx \right)^{\frac{1}{2}},$$

for meshes with 4, 6, 8, 10, 12, and 16 elements. Report the relative errors $\|e\|_0 / \|u\|_0$ for linear, quadratic, cubic, quartic, quintic, and sextic elements in a table. Observe the pattern of the errors and make comments.