

1.)

a.)

$$(A \rightarrow C) \vee (B \rightarrow C) \stackrel{?}{\equiv} (A \vee B) \rightarrow C$$

A	B	C	$(A \rightarrow C) \vee (B \rightarrow C)$		$(A \vee B) \rightarrow C$
F	F	F	T		T
F	F	T	T		T
F	T	F	T		F
F	T	T	T	$\neq$	T
T	F	F	T		F
T	F	T	T		T
T	T	F	F		F
T	T	T	T		T

a.) is not equivalent

b.)

$$(A \rightarrow C) \vee (B \rightarrow C) \stackrel{?}{\equiv} (A \wedge B) \rightarrow C$$

A	B	C	$(A \rightarrow C) \vee (B \rightarrow C)$		$(A \wedge B) \rightarrow C$
F	F	F	T		T
F	F	T	T		T
F	T	F	T		T
F	T	T	T	$\equiv$	T
T	F	F	T		T
T	F	T	T		T
T	T	F	F		F
T	T	T	T		T

b.) is equivalent

c.)

$$(A \rightarrow C) \wedge (B \rightarrow C) \stackrel{?}{=} (A \vee B) \rightarrow C$$

A	B	C	$(A \rightarrow C) \wedge (B \rightarrow C)$	$(A \vee B) \rightarrow C$
F	F	F	T	T
F	F	T	T	T
F	T	F	F	F
F	T	T	T	T
T	F	F	F	F
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

c.) is equivalent

d.)

$$(A \rightarrow C) \wedge (B \rightarrow C) \stackrel{?}{=} (A \wedge B) \rightarrow C$$

A	B	C	$(A \rightarrow C) \wedge (B \rightarrow C)$	$(A \wedge B) \rightarrow C$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

d.) is not equivalent

e.)

$$A \rightarrow (B \rightarrow C) \stackrel{?}{=} (A \rightarrow B) \rightarrow C$$

A	B	C	$A \rightarrow (B \rightarrow C)$	$(A \rightarrow B) \rightarrow C$
F	F	F	F	F
F	F	T	T	T
F	T	F	F	F
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

e.) is not equivalent

f.)

$$A \rightarrow (B \rightarrow C) \stackrel{?}{\equiv} A \wedge B \rightarrow C$$

A	B	C	$A \rightarrow (B \rightarrow C)$		$A \wedge B \rightarrow C$
F	F	F	T		T
F	F	T	T		T
F	T	F	T		T
F	T	T	T	$\equiv$	T
T	F	F	T		T
T	F	T	T		T
T	T	F	F		F
T	T	T	T		T

f.) is equivalent

g.)

$$A \rightarrow (B \rightarrow C) \stackrel{?}{=} A \rightarrow B \wedge C$$

A	B	C	$A \rightarrow (B \rightarrow C)$	$A \rightarrow B \wedge C$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	T	F
T	F	T	T	F
T	T	F	F	F
T	T	T	T	T

$\neq$

g.) is not equivalent

2.)

a.)  $\neg (A \wedge \neg A)$

$$\begin{array}{c} \begin{array}{cc} \overset{-1}{A} & \overset{-2}{\neg A} \end{array} \\ \hline A \wedge \neg A \end{array} \qquad \begin{array}{c} \hline \neg (A \wedge \neg A) \end{array} \overset{3}{\phantom{3}} \\ \hline \begin{array}{c} \perp \\ \hline \neg (A \wedge \neg A) \end{array} 1, 2, 3$$



$$b.) \quad \neg A \rightarrow (A \rightarrow B)$$

$$\begin{array}{c}
 \begin{array}{cc}
 \overline{A} & \overline{\neg A} \\
 A & \neg A
 \end{array} \\
 \hline
 \perp \\
 \hline
 B \quad \text{RAA} \\
 \hline
 A \rightarrow B \\
 \hline
 \neg A \rightarrow (A \rightarrow B)
 \end{array}$$

C.)  $A \vee \neg A$

$$\begin{array}{rcl}
 \frac{1}{A} & & \frac{2}{\neg(A \vee \neg A)} \\
 \hline
 A \vee \neg A & & \\
 \hline
 \frac{\perp}{\neg A}
 \end{array}$$

$$\begin{array}{rcl}
 \frac{3}{\neg(A \vee \neg A)} & & \frac{1,2}{A \vee \neg A} \\
 \hline
 \hline
 \end{array}$$

$$\begin{array}{rcl}
 \frac{1}{\neg A} & & 3 \\
 \hline
 A \vee \neg A
 \end{array}$$

d.)

$$(A \rightarrow C \vee D) \rightarrow ((A \rightarrow C) \vee (A \rightarrow D))$$

$\frac{\frac{A \rightarrow C \vee D \quad A}{C \vee D} \quad 1 \quad 2}{(A \rightarrow C) \vee (A \rightarrow D)} \quad 2, 3, 4$	$\frac{\frac{C}{A \rightarrow C} \quad 3}{(A \rightarrow C) \vee (A \rightarrow D)}$	$\frac{\frac{D}{A \rightarrow D} \quad 4}{(A \rightarrow C) \vee (A \rightarrow D)}$
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$$\frac{(A \rightarrow C) \vee (A \rightarrow D)}{(A \rightarrow C \vee D) \rightarrow ((A \rightarrow C) \vee (A \rightarrow D))} \quad 1$$

e.)  $A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$

[illegible]

$$A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$$

$$f.) \quad A \leftrightarrow B \stackrel{?}{=} \neg A \leftrightarrow \neg B$$

$$\begin{array}{c}
 \begin{array}{c}
 \overline{A}^1 \quad A \leftrightarrow B \\
 \hline
 B \quad \overline{\neg B}^2 \\
 \hline
 \perp \\
 \hline
 \neg B^2
 \end{array}
 \quad
 \begin{array}{c}
 \overline{B}^1 \quad A \leftrightarrow B \\
 \hline
 A \quad \overline{\neg A}^3 \\
 \hline
 \perp \\
 \hline
 \neg A^3
 \end{array} \\
 \hline
 \neg A \leftrightarrow \neg B^1
 \end{array}$$

$$9.) \quad \begin{array}{l} A \rightarrow C \\ B \rightarrow D \end{array}$$

$$A \vee B \rightarrow C \vee D$$

$$\begin{array}{c}
 \begin{array}{c} \text{---} 1 \\ A \vee B \end{array} \qquad \begin{array}{c} A \rightarrow C \quad \overline{A}^2 \\ \hline C \\ \hline C \vee D \end{array} \qquad \begin{array}{c} B \rightarrow D \quad \overline{B}^3 \\ \hline D \\ \hline C \vee D \end{array} \\
 \hline
 C \vee D \qquad \qquad \qquad 2,3 \\
 \hline
 A \vee B \rightarrow C \vee D \quad 1
 \end{array}$$