1.) find min of x and y:

```
if x < y:
    z = x
else
    z = y
```

and

- (trivial) precondition: T
- Precondition: $(x \leq y \wedge z = x) \vee (x > y \wedge z = y)$

Given the conditions, there are three possible cases:

- $x = y$, where y is stored in z,

and since $x = y = z$ now, the left side of precondition is satisfied as $x = z$, and $x = y$ thus $x \leq y \wedge z = x$ is true.

The program is true as one side of disjunction $(x \leq y \wedge z = x)$ is satisfied.

- $x > y$, y gets stored in z. Meaning, $y = z$ or $z = y$, and since $x > y$, the right side of precondition is satisfied.

  The program is true as one side of disjunction $(x > y \land z = y)$ is satisfied.

- $x < y$, x gets stored in z. Now, since $z = x$ and $x < y$, the left side of precondition is satisfied.

  The program is true as one side of disjunction $(x \leq y \land z = x)$ is satisfied

Since every possible cases are all proving the program to be correct, the program is correct.

2.) Find quotient / remainder when
a/b

$r = a$
$q = 0$
while $r >= b$;
$r = r - b$
$q += 1$

and

- precondition: $(a > 0) \wedge (b > 0)$
- Postcondition: $(a = bq + r) \wedge (0 \leq r < b)$

- The precondition covers the edge case of division yielding value of 0 or undefined as both numerator and denominator has to be a positive number.

- the postcondition is the definition of quotient - remainder theorem, where given any intiger a and positive integer b, there exist unique integer q and r such that:

$$a = bq + r \quad \wedge \quad 0 \leq r \leq b$$

where q is the quotient and r is the remainder.

Meaning, it means $a \% b = r$

Thus, the program is correct as given input passes both pre/post conditions, we get our desired result (quotient and remainder)

3.) Suppose $m, n \in \mathbb{Z}^+$ are relatively prime. Prove that for all $a, b \in \mathbb{Z}$, $a \equiv b \pmod{mn}$ iff $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$

if we have

$a \equiv b \pmod{mn}$, rewrite as $mn \mid (a-b)$

since both m and n are relatively prime,

$a-b$ is both tee multiple of m and n.

meaning,

$m \mid (a-b) \quad \wedge \quad n \mid (a-b)$

thus,

$a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$

To complete tee proof, if we take tee converse and let

$a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$

which can be re-written as

$m \mid (a-b) \quad \wedge \quad n \mid (a-b)$

since $a-b$ is both tee multiple of m and n,

$mn \mid (a-b)$ which can be rewritten as

$a \equiv b \pmod{mn}$

thus, proof is complete.

4.) H is set of no. of the form $4k+1$ for some $k \in \mathbb{N}$

a. Show $xy \in H$, if $x \in H \wedge y \in H$

if $x = 4k_1 + 1$, $y = 4k_2$

$xy = (4k_1 + 1)(4k_2 + 1)$

$= 16k_1k_2 + 4k_1 + 4k_2 + 1$

$$= \underline{4}\,\underline{(4k_1k_2 + 4k_1 + 4k_2)}\,\underline{+1}$$

4 k + 1 format

$$= 4k + 1$$

thus, $xy \in H$ is proven.

b.

assume $x \in H$, where

x is an H-prime.

then proof is done.

if x is not an H-prime,

do $x = yz$ such that

$x > y$ and $x > z$ such that

both y and z are smaller

H-primes, then proof is done.

if y and z are not H-primes, re-write them as $y = ab$ where $y > a \wedge y > b$ and are smaller H-primes.

continue on and on, which yields

$x = p_1 p_2 \ldots\ldots p_n$

ultimately.

thus, proof is complete.

c. given 441,

$$441 = \underset{4}{4}(\underset{k}{110}) \underset{+}{+} \underset{1}{1}$$

thus,

$\underline{441 \in H}$

so,

$$441 = 49 \cdot 9$$

where

$49 = \underset{4}{4}(\underset{k}{12}) \underset{+}{+} \underset{1}{1}$, exists H

$9 = \underset{4}{4}(\underset{k}{2}) \underset{+}{+} \underset{1}{1}$, exists H

thus 441 is H-prime.

Also,

$441 = 21 \cdot 21$

where

$21 = 4(5) + 1$, exists H

$4k + 1$

Since 21, 9, and 49

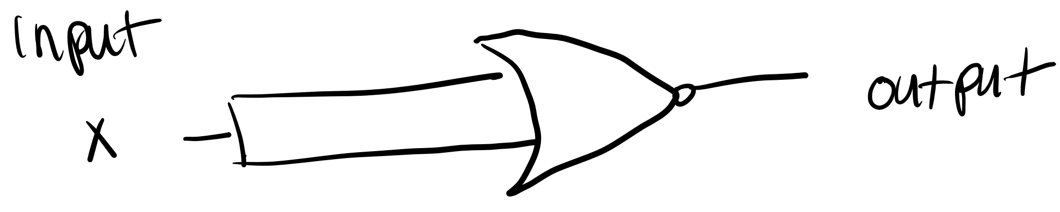
are all H-primes,

441 is an H-prime.

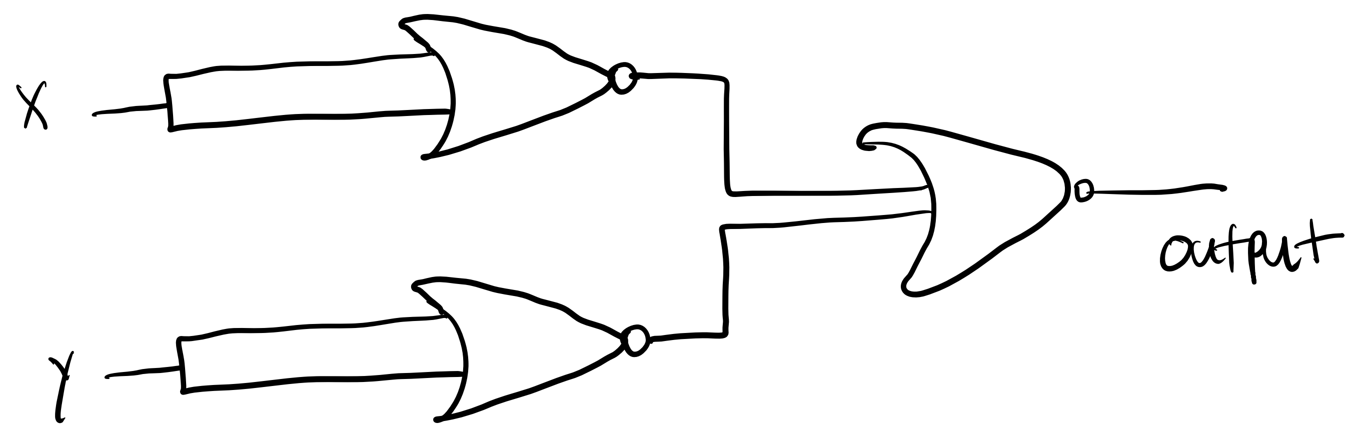
(proven in 2 different ways)

5.) Use NOR gate and x, y only

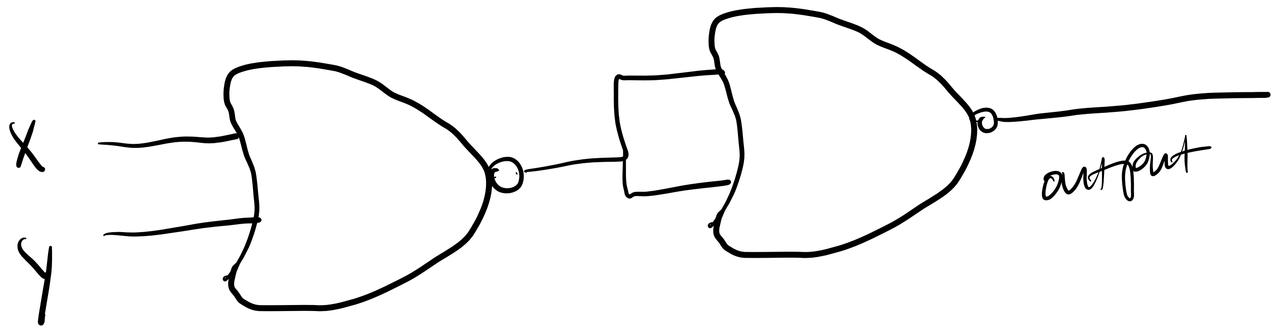
a. Not (x)



b. And (x) (y)



c. OR (X)(Y)



6.)

$$\overset{1}{\bar{w}\bar{x}\bar{y}z} + \overset{3}{\bar{w}\bar{x}yz} + \overset{4}{\bar{w}x\bar{y}\bar{z}} + \overset{6}{\bar{w}xy\bar{z}} +$$

$$\overset{12}{wx\bar{y}\bar{z}} + \overset{14}{wxy\bar{z}} + \overset{9}{w\bar{x}\bar{y}z} + \overset{11}{w\bar{x}yz}$$

Equivalents:

$f(w,x,y,z) : (1, 3, 4, 6, 9, 11, 12, 14)$

wx \ yz	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

$x\bar{z}$

$\bar{x}z$

Thus,

minimal expression is

$f(w,x,y,z) : x\bar{z} + \bar{x}z$

7.) Quine - McClusky

00100   00110   01101   10100

$\bar{v}\bar{w}x\bar{y}\bar{z} + \bar{v}\bar{w}xy\bar{z} + \bar{v}wx\bar{y}z + v\bar{w}x\bar{y}\bar{z} +$

10110   11101

$v\bar{w}xy\bar{z} + vwx\bar{y}z$

00100 = 4

00110 = 6

01101 = 13

10100 = 20

10110 = 22

11101 = 29

minimal expression:

4, 6, 13, 20, 22, 29

Grp. 1: 00100 = 4

Grp. 2: 00110 = 6
10100 = 20

Grp. 3: 01101 = 13
10110 = 22

Grp. 4: 11101 = 29

Merge groups based on min.

Grp. 1 + 4 = (1)	4, 6	001_0
	4, 20	_0100
	6, 22	_0110
Grp. 2 + 3 = (2)	20, 22	101_0
Grp. 3 + 4 =	13, 29	_1101

(1) + (2) = 4, 6, 20, 22 _01_0

Meaning, if we set up prime implication chart,

	4	6	13	20	22	29	vwxyz
1 + 2	✓	✓		✓	✓		_01_0
Grp. 3+4			✓			✓	_1101

Now, if we take the implications and combine...

_01_0 , _1101

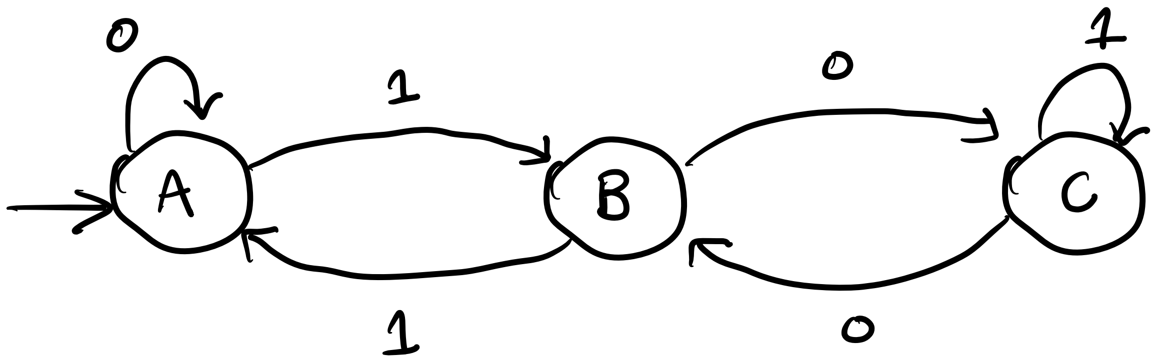
then express it in minimal Quine-McClusky expression...

$$= \bar{w}x\bar{z} + wx\bar{y}z$$

8.) Regular Expression for odd number of 1s

$$0^*10^*(0^*10^*10^*)^*$$

9.) DFA wlec divisible by 3



10.) given

$\langle S \rangle ::= \varepsilon$ ① empty str

$\quad | \ \langle S \rangle \langle S \rangle$ ② rule 1

$\quad | \ (\langle S \rangle)$ ③ rule 2

Since an input may generates more than one parse tree (given rule 1 and 2) making the grammar ambiguous.

if an input chooses rule 1 to expand, there would be more than one parse tree.

To modify this to an unambiguous grammar, rule 1 must be removed.

$\langle S \rangle ::= \varepsilon$ ① empty str

$\quad | \ (\langle S \rangle)$ ② rule 2