odd: 
$$n = 2k + 1$$

$$n = aktl = aktl tk^2 - k^2$$

= 
$$K^2 + 2K + 1 - K^2$$
, sine expanding a square

(i.e: 
$$(x+y)^2 = x^2 + \lambda xy + y^2$$
)

$$= (\kappa + 1)^2 - \gamma^2$$

$$= (K+1)^{2} - \gamma^{2}$$
 odd int n is the sifference of d squares

Since 
$$\forall x > 0$$

$$x = y (y + 1) = y^{2} + y$$

$$0 = \chi^2 + \gamma - \chi$$

 $0 = x^2 + y - x$  y quadratic formula

$$y = -1 \pm \sqrt{1-4(-x)} = -1 \pm \sqrt{1+4x}$$

$$2 \cdot 1$$

for 
$$\forall x > 0$$
,

for 
$$\forall x > 0$$
,  $y = a real number.$ 

3.) 
$$dy + 3x = 3y - 4x$$
  
 $\Rightarrow x \neq 0, y \neq 0$  Same time

and 
$$y = 0$$
 only if  $x = 0$ 

as y = 7x.

as both x and y camet be zero at the same time,  $x \neq 0$  as only case of y = 0 would also mean x = 0 as well.

4.) Theorem:

 $X\LambdaY$ ,  $X\LambdaY = real$ ,  $X+Y=14 \rightarrow X \neq S\Lambda Y \neq Q$ 

when the proof is assuming that the conclusion doesn't hold, proof should do  $7(x \neq 5 \land y \neq 12)$  which would be equivalent of  $7x \neq 5 \lor 7y \neq 12$ , but the proof uses  $x = 5 \land y = 12$  instead which goes against the idea of proof by contradiction.

5.) Theorem: any real  $x \rightarrow x^2 \ge 0$ 

The proof is flawed as the proof only proves the theorem in the case where X = 5, rother than proving all real number X.

6.) if x and y are real, teen  $1x1 \leq y \iff -y \leq x \leq y$ 

if x > 0, (x = x),  $|x| \le y \rightarrow x \le y$ if x < 0, |x| = -x,  $|x| \le y \rightarrow -x \le y$  $\Rightarrow -y \le x$ 

thus

IXI = Y - Y = X = Y

Converse:

$$-y \leq x \leq y \longrightarrow |x| \leq y$$
assuming  $y > 0$ , and  $-y \leq x \leq y$ 

$$x \leq y \text{ and } -x \leq y$$
and since  $y > 0$ ,  $0 \leq y$ 
which means for all  $x$ ,  $y \in y \leq y$ 

$$\text{meaning:} |x| \leq y$$

$$\text{thus } -y \leq x \leq y \longrightarrow |x| \leq y$$

$$\text{and since } |x| \leq y \longrightarrow -y \leq x \leq y$$

$$x \leq y \longleftrightarrow -y \leq x \leq y$$

```
7.) If x < 0, then x = -lxl,
       and X < 1x1, thus
           \times < l \times l
 • if x = 0, then
   -|X| = X = |X|
 * if x > 0, then -1x1 < 0, and
  x = 1x1, thus you get
       -[\chi] < \chi
  Since:
               x < 1x1
         -1X1 < X
         -|x| = x = |x|
```

$$-1x1 \leq x \leq 1x1$$

8.) if 
$$x \wedge y$$
 are real,  

$$|x + y| \leq |x| + |y|$$

$$|x + y| = \begin{cases} x + y & a + b \geq 0 \\ -(x + y) & a + b \leq 0 \end{cases}$$

$$- \times \leq 1 \times 1 \qquad - \lambda \leq 1 \lambda 1$$

$$- \times \leq 1 \times 1 \qquad \lambda \leq 1 \lambda 1$$

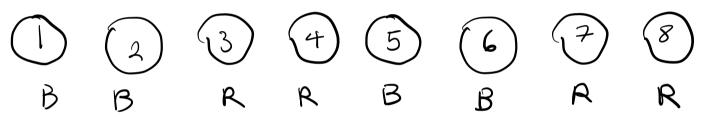
$$-(x+y) = -x+-y \leq |x|+|y|$$

$$1x+y$$
  $1 \le |x|+|y|$  Since  $x+y \le |x|+|y|$ , and  $-x+-y \le |x|+|y|$ 

9,) Stakment:

 $\forall x$  coloring,  $x < y < z \land y - x = z - y$ 

if we have:



for 81/10, all available combinations/coses are:

(case 1: 1 < 2 < 5  $\Lambda$   $2-1 \neq 5-2$ (case 2: 1 < 2 < 6  $\Lambda$   $2-1 \neq 6-2$ (case 3: 1 < 5 < 6  $\Lambda$   $5-1 \neq 6-1$ (case 4: 2 < 5 < 6  $\Lambda$   $5-2 \neq 6-5$ There does not exist a case where 3 ints

x < y < z of same color  $\Lambda y - x = z - y$ meaning that the given statement is false

10.) if 
$$x = int$$
,  $x^2 + x = even$   
if  $x = even$ ,  $x = 2K$   

$$= (2K)^2 + 2K$$

$$= 4k^2 + 2K$$

$$= 2(2K^2 + K)$$
Sine its in  $2K$  format
$$if \quad x = odd, \quad x = 2K + 1$$

$$= (2K+1)^2 + 2K+1$$

$$= 4k^2 + 6k + 2$$

$$= 2(2K^2 + 3K + 1)$$
Sine its in  $2K$  format

it is even