1.) Every odd integer n is differene of two squares

odd: $n = 2k+1$

Direct Proof

$n = 2k+1 = 2k+1+k^2-k^2$

$= k^2+2k+1-k^2$, sine expanding a square

$(\text{i.e: } (x+y)^2 = x^2+2xy+y^2)$

$= (k+1)^2 - y^2$

odd int n is the differene of 2 squares

2.) Since $\forall x > 0$

$x = y(y+1) = y^2+y$

$0 = y^2+y-x$ by quadratic formula

$$y = \frac{-1 \pm \sqrt{1-4(-x)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1+4x}}{2}$$

for $\forall x > 0$, $y =$ a real number.

3.) $2y + 3x = 3y - 4x$

$\Rightarrow$ $x \neq 0$, $y \neq 0$ same time

$2y + 3x = 3y - 4x$

$\boxed{y = 7x}$

and $y = 0$ only if $x = 0$

as $y = 7x$.

as both x and y camot be zero at tee same time, $y \neq 0$

as only case of $y = 0$ would also mean $x = 0$ as well.

4.) Theorem:

$x \wedge y$, $x \wedge y = \text{real}$, $x+y=14 \rightarrow x \neq 5 \wedge y \neq 12$

when the proof is assuming that the conclusion doesn't hold, proof should do $\neg(x \neq 5 \wedge y \neq 12)$ which would be equivalent of $\neg x \neq 5 \quad \vee \neg y \neq 12$, but the proof uses $x = 5 \wedge y = 12$ instead which goes against the idea of proof by contradiction.

5.) Theorem: any real $x \rightarrow x^2 \geq 0$

The proof is flawed as the proof only proves the theorem in the case where $x = 5$, rather than proving all real number x.

6.) if x and y are real, then

$$|x| \leq y \longleftrightarrow -y \leq x \leq y$$

if $x > 0$, $|x| = x$, $|x| \leq y \rightarrow x \leq y$

if $x < 0$, $|x| = -x$, $|x| \leq y \rightarrow -x \leq y$
$\rightarrow -y \leq x$

thus

$$|x| \leq y \longrightarrow -y \leq x \leq y$$

Converse:

$-y \leq x \leq y \longrightarrow |x| \leq y$

assuming $y > 0$, and $-y \leq x \leq y$

$x \leq y$ and $-x \leq y$

and since $y > 0$, $0 \leq y$

which means for all x, y is $\leq$

meaning: $|x| \leq y$

thus $-y \leq x \leq y \longrightarrow |x| \leq y$

and since $|x| \leq y \longrightarrow -y \leq x \leq y$

$x \leq y \longleftrightarrow -y \leq x \leq y$

7.) • if $x < 0$, then $x = -|x|$,

and $x < |x|$, thus

$x < |x|$

• if $x = 0$, then

$-|x| = x = |x|$

• if $x > 0$, then $-|x| < 0$, and

$x = |x|$, thus you get

$-|x| < x$

Since:

$x < |x|$

$-|x| < x$

$-|x| = x = |x|$

$-|x| \leq x \leq |x|$

8.) if $x \wedge y$ are real,

$$|x+y| \leq |x| + |y|$$

$$|x+y| = \begin{cases} x+y & a+b \geq 0 \\ -(x+y) & a+b < 0 \end{cases}$$

$$x \leq |x| \qquad y \leq |y|$$

$$-x \leq |x| \qquad -y \leq |y|$$

$$\underline{x+y \leq |x| + |y|}_{1}$$

$$-(x+y) = \underline{-x + -y \leq |x| + |y|}_{2}$$

1+2

$|x+y| \leq |x| + |y|$ since

$\underline{x+y \leq |x| + |y|}_{1}$ and

$\underline{-x + -y \leq |x| + |y|}_{2}$

9.)

Statement:

$\forall$ coloring, $x < y < z \wedge y - x = z - y$

if we have:

1	2	3	4	5	6	7	8
B	B	R	R	B	B	R	R

for Blue, all available combinations/cases are:

case 1: $1 < 2 < 5 \wedge 2 - 1 \neq 5 - 2$

case 2: $1 < 2 < 6 \wedge 2 - 1 \neq 6 - 2$

case 3: $1 < 5 < 6 \wedge 5 - 1 \neq 6 - 1$

case 4: $2 < 5 < 6 \wedge 5 - 2 \neq 6 - 5$

→ There does not exist a case where 3 ints $x < y < z$ of same color $\wedge$ $y - x = z - y$

meaning that the given statement is false

10.) if x = int, $x^2 + x$ = even

•) if x = even, $x = 2k$

$= (2k)^2 + 2k$

$= 4k^2 + 2k$

$= 2(2k^2 + k)$

since its in $2k$ format

it is even

• if x = odd, $x = 2k+1$

$= (2k+1)^2 + 2k+1$

$= 4k^2 + 6k + 2$

$= 2(2k^2 + 3k + 1)$

since its in $2k$ format

it is even