Solución Parcial 1 – Señales y Sistemas 2024-I – Juan Jerónimo Castaño Rivera

Punto a:

a) $d(x_1,x_2) = \bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\, \partial t$

$x_1(t) = Ae^{jW_0t}$ $\quad x_2(t) = Be^{j5W_0t}$

$|x_1(t)-x_2(t)|^2 = |x_1(t)|^2 - 2\,x_1(t)x_2(t) + |x_2(t)|^2$

Por lo tanto

$$d(x_1,x_2) = \lim_{T\to\infty}\left[\frac{1}{T}\int_T |x_1(t)|^2\,\partial t - \frac{2}{T}\int_T x_1(t)x_2(t)\,\partial t + \frac{1}{T}\int_T |x_2(t)|^2\,\partial t\right]$$

Para $|x_1(t)|^2 = |Ae^{jW_0t}|^2 = (Ae^{jW_0t})(Ae^{jW_0t})^*$

$|x_2(t)|^2 = |Be^{j5W_0t}|^2 = (Be^{j5W_0t})(Be^{j5W_0t})^*$

$$d(x_1,x_2) = \lim_{T\to\infty}\left[\frac{1}{T}\int_T (Ae^{jW_0t})(Ae^{-jW_0t})\,\partial t - \frac{2}{T}\int_T (Ae^{jW_0t})(Be^{j5W_0t})\,\partial t + \dots\right.$$

$$\left.\dots + \frac{1}{T}\int_T (Be^{j5W_0t})(Be^{-j5W_0t})\,\partial t\right]$$

Podemos resolver el problema por partes

$$\int_T |x_1(t)|^2\,\partial t = \int_T A^2 e^{jW_0t-jW_0t}\,\partial t = A^2\int_T e^0\,\partial t = A^2 t\Big|_0^T = A^2T$$

$$\int_T |x_2(t)|^2\,\partial t = \int_T B^2 e^{j5W_0t-j5W_0t}\,\partial t = B^2\int_T e^0\,\partial t = B^2 t\Big|_0^T = B^2T$$

$$\int_T x_1(t)x_2(t)\,\partial t = \int_T (Ae^{jW_0t})(Be^{j5W_0t})\,\partial t = AB\int_T e^{jW_0t+j5W_0t}\,\partial t$$

$$\int_T x_1(t)x_2(t)\,\partial t = AB\int_T e^{6jW_0t}\,\partial t \qquad u = 6jW_0t \qquad du = 6jW_0\,\partial t$$

$$\partial t = \frac{du}{6jW_0}$$

$$\int_T x_1(t)x_2(t)\,\partial t = \frac{AB}{6jW_0}\, e^{6jW_0t}\Big|_0^T$$

$$\int_T x_1(t)x_2(t)\,\partial t = \frac{AB}{6jW_0}\left[e^{6j\frac{2\pi}{T}T} - e^0\right]$$

$$e^{6j2\pi} = \overset{1}{\cos(6\cdot 2\pi)} + j\,\overset{0}{\text{Sen}(6\cdot 2\pi)}$$

Por lo tanto $e^{6j2\pi} = 0$

$$\int_T x_1(t)\,x_2(t)\,\partial t = \frac{AB}{6jw_0}\left[1-1\right] = 0$$

$$d(x_1,x_2) = \lim_{T\to\infty}\left[\frac{A^2T}{T} - \frac{2}{T}\cdot 0 + \frac{B^2T}{T}\right]$$

$$d(x_1,x_2) = \lim_{T\to\infty}\left[A^2 + B^2\right] = A^2 + B^2$$

Punto b:

b) $F_s = 5\,kHz$

$$x(t) = 3\cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$$

$w_1 = 1000\pi$ $\quad T_1 = \frac{2\pi}{w_1} = \frac{1}{500}$ $\quad F_1 = 500\ Hz$

$w_2 = 2000\pi$ $\quad T_2 = \frac{2\pi}{w_2} = \frac{1}{1000}$ $\quad F_2 = 1000\ Hz$

$w_3 = 11000\pi$ $\quad T_3 = \frac{2\pi}{w_3} = \frac{1}{5500}$ $\quad F_3 = 5500\ Hz$

Dado que

$\frac{w_1}{w_2} = \frac{1}{2} \in \mathbb{Q}$ $\quad \frac{w_1}{w_3} = \frac{1}{11} \in \mathbb{Q}$ $\quad \frac{w_2}{w_3} = \frac{2}{11} \in \mathbb{Q}$

La señal es cuasiperiódica. Hallamos el periodo

$T = kT_1 = lT_2 = rT_3 \quad k, l, r \in \mathbb{Z}$

$T = \frac{k}{500} = \frac{l}{1000} = \frac{r}{5500}$

$11.000T = 22k = 11l = 2r$

El MCM entre 2, 11 y 22 es 22

$11000T = 22$

$T = \frac{1}{500}$

Por Nyquist

$$F_s \geq 2 F_{max}$$

$$F_{max} = 5500 \text{ Hz}$$

Por lo tanto

$$F_s \geq 11000 \text{ Hz} \Rightarrow \text{No cumple}$$

Para la discretización $t = nT_s$; $T_s = \frac{1}{F_s}$

$$x[t = nT_s] = 3\cos\left[1000\pi \frac{n}{F_s}\right] + 5\,\text{sen}\left[2000\pi \frac{n}{F_s}\right] + 10\cos\left[11000\pi \frac{n}{F_s}\right]$$

$$x[n] = 3\cos\left[\frac{1000}{5000}\pi n\right] + 5\,\text{sen}\left[\frac{2000}{5000}\pi n\right] + 10\cos\left[\frac{11000}{5000}\pi n\right]$$

$$= 3\cos\left[\frac{\pi n}{5}\right] + 5\,\text{sen}\left[\frac{2\pi}{5} n\right] + 10\cos\left[\frac{11}{5}\pi n\right]$$

$$\Omega_1 = \frac{\pi}{5} < 2\pi \qquad \Omega_2 = \frac{2\pi}{5} < 2\pi \qquad \Omega_3 = \frac{11\pi}{5} > 2\pi \Rightarrow \text{Copia}$$

$$\hat{\Omega}_3 = \frac{11\pi}{5} - 2\pi = \frac{\pi}{5}$$

Por lo tanto

$$x[n] = 3\cos\left[\frac{\pi n}{5}\right] + 5\,\text{sen}\left[\frac{2\pi}{5} n\right] + 10\cos\left[\frac{\pi}{5} n\right]$$

$$= 13\cos\left[\frac{\pi n}{5}\right] + 5\,\text{sen}\left[\frac{2\pi}{5} n\right]$$

Punto c:

c) $x(t) = 20(\cos(t/3) + \cos(t/4))$

$W_1 = \frac{1}{3}$ $\quad$ $W_2 = \frac{1}{4}$

$\frac{W_1}{W_2} = \frac{4}{3} \in \mathbb{Q}$ Es cuasiperiódica. Hallamos el periodo

$T_1 = \frac{2\pi}{W_1} = 6\pi$ $\quad$ $T_2 = \frac{2\pi}{W_2} = 8\pi$ $\quad$ $F_1 = \frac{1}{6\pi}$ $\quad$ $F_2 = \frac{1}{8\pi}$

$T = k\,6\pi = l\,8\pi$

$MCM(6,8) = 24$

Por lo tanto $T = 24\pi$

Para discretizar $F_s \geq 2F_{max}$

$F_s \geq \frac{1}{3\pi}$

Podemos hacer $F_s = 10 \cdot \frac{1}{3\pi} = \frac{10}{3\pi}$