$$\min_x \frac{1}{2}\|\underbrace{\boxed{A}}_{\phi''}x - y\|^2 + \lambda \overset{\|\cdot\|_1}{R(x)} \iff \min_{\|Ax-y\| \le \varepsilon} \|x\|_1$$

$\varepsilon \to \lambda$

$$R(x) = \begin{cases} \frac{1}{2}\|x\|^2 & \text{Ridge} \\ \|x\|_1 = \sum_i |x_i| & \begin{cases} \text{LASSO} \\ \text{Basis Pursuits} \end{cases} \end{cases}$$

Lasso $\rightsquigarrow$ ISTA

"Compressed sensing" $\approx$ A random

go beyond ISTA $\longrightarrow$ Proximal $\begin{cases} \text{Algo} \\ \text{Splitting} \end{cases}$

In ML: $A = \phi$ data (feature)

In Signal Processing (CS): $A = \underline{\underline{\phi}} = K \times \Psi$

K: Imaging op (Random)

Ψ: "Tool"
- Wavelet (JPL2K)
- DCT (JPEG)
- [DeepGenNet]

no theory

Set Up

$f_0 \in \mathbb{R}^N$ → large.

$f_0 \simeq \Psi x_0$ (1% err.)

f_0: image

x_0: coefficient.

x_0 is sparse.

↳ # non-zero coef

$S \ll N$.

k → JPEG2K))??

↳ CS : $f_0 \mapsto Rf_0 = y \in \mathbb{R}^P$

$P \ll N$ (CS)

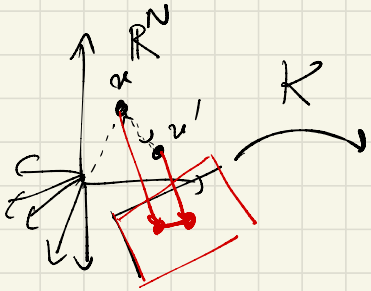
$P \approx S$?!

Emanuel Candes + Terrence Tao. (Fields)

(Watterman Award) Justin Romberg.

PHD

David Donoho → Lasso



δ

$\|Kx - Kx'\| \simeq \|x - x'\|$

$\|K(x - x')\|$

True → Sparse vect.