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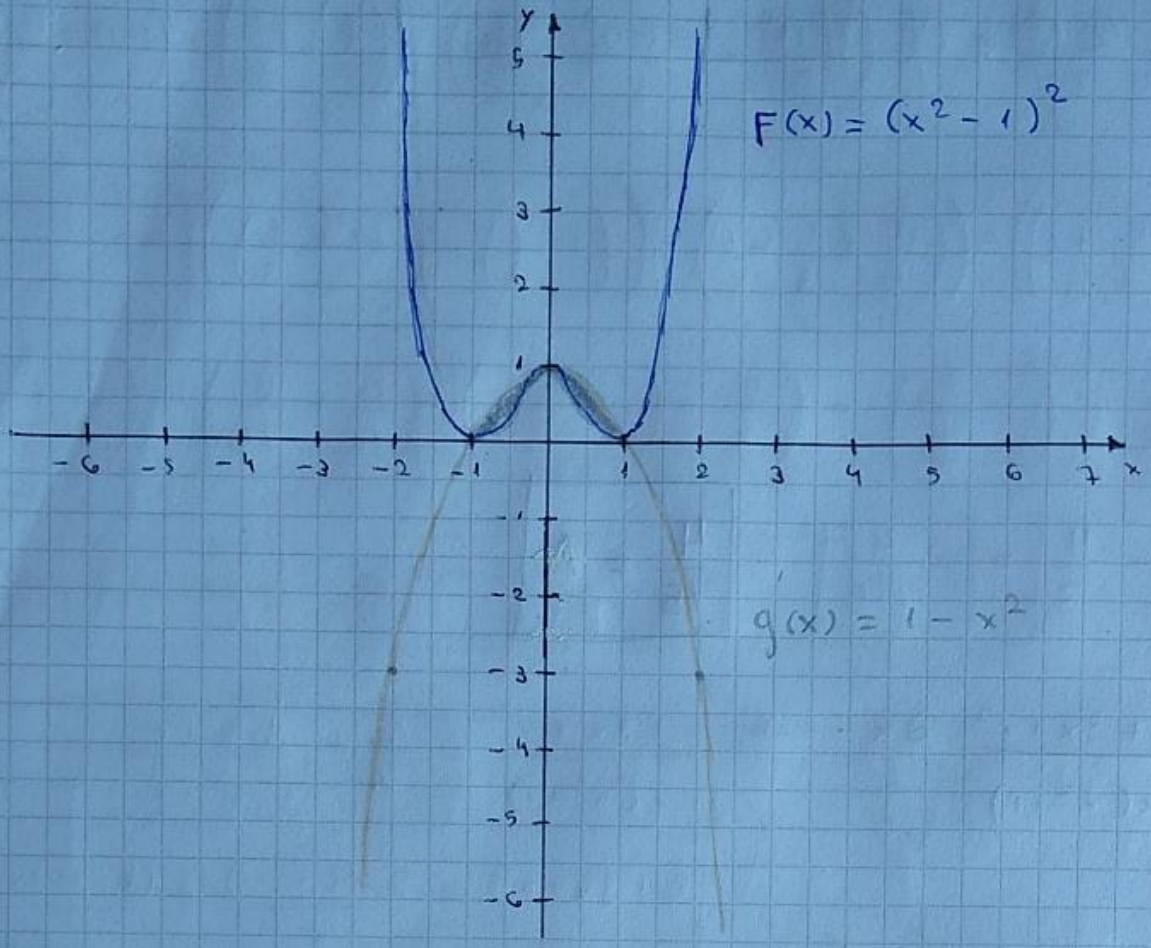
Ejercicio 1) (1)

a)



$F(x) = (x^2 - 1)^2$

$g(x) = 1 - x^2$

$$A = \int_{-1}^{1} F(x) - \int_{-1}^{1} g(x)$$

$$\int (x^2 - 1)^2 \, dx = \int x^4 - 2x^2 + 1^2 \, dx = \int x^4 \, dx - \int 2x^2 \, dx + \int 1 \, dx$$

$$= \left(\frac{x^5}{5}\right) - \left(2\int x^2 \, dx\right) + x = \frac{x^5}{5} - \frac{2x^3}{3} + x + C = F(x)$$

$$\int 1 - x^2 \, dx = \int 1 \, dx - \int x^2 \, dx = x - \frac{x^3}{3} + C = F_2(x)$$

$$\left| \int_{-1}^{1} (x^2-1)^2 - \int_{-1}^{1} 1-x^2 \right| = \left| (F(1)-F(-1)) - (F_2(1)-F_2(-1)) \right| =$$

$$= \left| \left(\frac{1^5}{5} - \frac{2\cdot 1^3}{3} + 1\right) - \left(\frac{(-1)^5}{5} - \frac{2(-1)^3}{3} + (-1)\right) - \left(\left(1-\frac{1^3}{3}\right) - \left((-1) - \frac{(-1)^3}{3}\right)\right) \right| =$$

$$= \left| \left(\frac{1}{5} - \frac{2}{3} + 1\right) - \left(-\frac{1}{5} + \frac{2}{3} - 1\right) - \left(\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right) \right| =$$

$$= \left| \frac{8}{15} - \left(-\frac{8}{15}\right) - \left(\frac{2}{3} - \left(-\frac{2}{3}\right)\right) \right| = \left| \left(\frac{8}{15} + \frac{8}{15}\right) - \left(\frac{2}{3} + \frac{2}{3}\right) \right| =$$

$$= \left| \frac{16}{15} - \frac{4}{3} \right| = \left| -\frac{4}{15} \right| = \boxed{\frac{4}{15} = A}$$

b) $\int \frac{2+3x+x^2}{x(x^2+1)}\,dx \rightarrow \frac{2+3x+x^2}{x(x^2+1)} = \frac{a}{x} + \frac{bx+c}{x^2+1}$

$$2+3x+x^2 = \frac{a\cancel{x}(x^2+1)}{\cancel{x}} + \frac{(bx+c)\cancel{x}(x^2+1)}{\cancel{x^2+1}}$$

$$2+3x+x^2 = ax^2 + a + bx^2 + cx$$

$$2+3x+x^2 = (a+b)x^2 + cx + a$$

$$\left.\begin{array}{l} \boxed{a=2} \\ \boxed{c=3} \\ 1 = a+b \\ 1 = 2+b \\ \boxed{b=-1} \end{array}\right\} \frac{2}{x} + \frac{-x+3}{x^2+1}$$

$$\int \frac{2}{x} + \frac{-x+3}{x^2+1}\,dx =$$

$$= \int \frac{2}{x}\,dx + \int \frac{-x+3}{x^2+1}\,dx =$$

$$= 2\int \frac{1}{x}\,dx + \int -\frac{x}{x^2+1} + \frac{3}{x^2+1}\,dx =$$

$$= 2\ln|x| + \left(-\int \frac{x}{x^2+1}\,dx\right) + \int \frac{3}{x^2+1}\,dx =$$

$$= 2\ln|x| - \underbrace{\int \frac{x}{x^2+1}\,dx}_{①} + 3\underbrace{\int \frac{1}{x^2+1}\,dx}_{②} = (*)$$

→ otro lado

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③

① $\int \frac{x}{x^2+1}\,dx \rightarrow v = x^2+1$

$dv = 2x\,dx$

$\Rightarrow$

$\frac{1}{2}\int \frac{1}{v}\,2x\,dx$

$\Rightarrow$

$\frac{1}{2}\int \frac{1}{v}\,dv \Rightarrow \frac{1}{2}\ln|v| \Rightarrow \frac{1}{2}\ln|x^2+1| + c$

② $\int \frac{1}{x^2+1}\,dx = \arctan(x) + c$

$(*) = \boxed{2\ln|x| - \frac{1}{2}\ln|x^2+1| + 3\arctan(x) + c}$