Juan Pablo Ludueña Zakka

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Ejercicio 3)

a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}\, 3^n} (3x-1)^n$

criterio de la razón: $L = \lim_{n \to \infty}$

$$L \left| \frac{\frac{(3x-1)^{n+1}}{\sqrt[4]{(n+1)}\; 3^{n+1}}}{\frac{(3x-1)^n}{\sqrt[4]{n}\; 3^n}} \right| = L \left| \frac{\cancel{(3x-1)^n}(3x-1)\sqrt[4]{n}\; \cancel{3^n}}{\sqrt[4]{n+1}\; \cancel{3^n}\; 3\; \cancel{(3x-1)^n}} \right| =$$

$$= L \left| \frac{\sqrt[4]{n}\,(3x-1)}{\sqrt[4]{n+1}\; 3} \right| = L \underbrace{\sqrt[4]{\frac{n}{n+1}}}_{\text{siempre positivo}} \; \frac{|3x-1|}{\underbrace{3}_{|3|=3}} = L \sqrt[4]{\frac{\cancel{n}/\cancel{n}}{\cancel{n}/\cancel{n} + \frac{1}{n}}} \; \frac{|3x-1|}{3} =$$

$$= \sqrt{\frac{1}{1+0}} \; \frac{|3x-1|}{3} = \frac{|3x-1|}{3} \Rightarrow \boxed{K < 1 \text{ Converge}} \Rightarrow$$

$$\Rightarrow \frac{|3x-1|}{3} < 1 \Rightarrow |3x-1| < 3 \Rightarrow -3 < 3x - 1 < 3$$

$$\Rightarrow -2 < 3x < 4$$

$$\Rightarrow -\frac{2}{3} < x < \frac{4}{3}$$

$\therefore$ Radio de conv. = 1

Intervalo de conv. = $x \in \left[-\frac{2}{3}, \frac{4}{3} \right)$

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b) $\frac{1}{x^2}$ centrada en $a = -3$ 7

$f(x) = \frac{1}{x^2}$

$F(x) = -\frac{1}{x} = -\frac{1}{x+3-3} = -\frac{1}{-3+(x+3)} = \frac{1}{3-(x+3)} =$

$= \frac{1}{3} \cdot \frac{1}{1-\left(\frac{x+3}{3}\right)} = \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{x+3}{3}\right)^n = \sum_{n=0}^{\infty} \frac{(x+3)^n}{3^{n+1}} + c$

$f(x) = F(x) + c \iff \left|\frac{x+3}{3}\right| < 1$

$\Rightarrow f(x) = \frac{d}{dx}(F(x)+c)$

$f(x) = \frac{1}{x^2} = \sum_{n=0}^{\infty} \frac{d}{dx}\left(\frac{1}{3^{n+1}}(x+3)^n\right) = \boxed{\sum_{n=1}^{\infty} \frac{n}{3^{n+1}}(x+3)^{n-1}}$

Por criterio de la razón: $\lim_{n\to\infty} = L$

$L\left|\frac{\frac{(n+1)(x+3)^n}{3^{n+2}}}{\frac{n(x+3)^{n-1}}{3^{n+1}}}\right| = L\left|\frac{(n+1)\cancel{(x+3)^n}\,\cancel{3^n}\,\cancel{3}}{\cancel{3^n}\,3^{\cancel{2}}\,n\,\cancel{(x+3)^n}\,(x+3)^{-1}}\right| = L\left|\frac{(n+1)}{\frac{3n}{(x+3)}}\right| =$

$= L\left|\frac{(n+1)(x+3)}{3n}\right| = L\ \underbrace{\frac{n+1}{n}}_{\text{siempre positivo}}\left|\frac{x+3}{3}\right| = L\ \frac{\cancel{\frac{n}{n}}^{1} + \cancel{\frac{1}{n}}^{0}}{\cancel{\frac{n}{n}}_{1}}\left|\frac{x+3}{3}\right| =$

$= \left|\frac{x+3}{3}\right| \Rightarrow k<1$ converge $\Rightarrow \left|\frac{x+3}{3}\right| < 1 \Rightarrow |x+3| < 3$

$\Rightarrow -3 < x+3 < 3$

$\Rightarrow -6 < x < 0$

∴ Radio de conv. $= 3$

Intervalo de conv. $= x \in (-6, 0)$