Juan Pablo Ludueña Zatka

Ejercicio 2) ④

a)

i) $\lim_{n \to \infty} n \arctan\left(\frac{1}{n}\right)$ $L = \lim_{n \to \infty}$

$$L\ n \arctan\left(\frac{1}{n}\right) = L \frac{\arctan\left(\frac{1}{n}\right)}{\frac{1}{n}} \underset{L'H}{=} (*)$$

$f(x) = \arctan\left(\frac{1}{x}\right)$ $f'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \left(-\frac{1}{x^2}\right) =$

$g(x) = \frac{1}{x}$

$g'(x) = \frac{0 \cdot 1 - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$ $= -\frac{1}{\left(1+\frac{1}{x^2}\right)x^2} =$

$$= -\frac{1}{x^2+1}$$

$$(*) = L \frac{-\frac{1}{x^2+1}}{-\frac{1}{x^2}} = L \frac{x^2}{x^2+1} = L \frac{\cancel{x^2}}{\cancel{x^2}\left(1+\frac{1}{x^2}\right)} =$$

$$= L \frac{1}{1+\frac{1}{x^2}} = \frac{1}{1+\cancel{\frac{1}{\infty}}^{\,0}} = \boxed{1}$$

ii) $\lim_{n \to \infty} \sqrt{n^2+2n} - n$ $L = \lim_{n \to \infty}$

$$L \sqrt{n^2+2n} - n = L \left(\sqrt{n^2+2n} - n\right) \cdot \left(\frac{\sqrt{n^2+2n}+n}{\sqrt{n^2+2n}+n}\right) =$$

$$= L \frac{\left(\sqrt{n^2+2n}\right)^2 - n^2}{\sqrt{n^2+2n}+n} = L \frac{\cancel{n^2}+2n-\cancel{n^2}}{\sqrt{n^2+2n}+n} = L \frac{2n}{\sqrt{n^2+2n}+n} =$$

$$= L \frac{2n}{\sqrt{n^2\left(1+\frac{2}{n}\right)}+n} = L \frac{2n}{\sqrt{n^2}\sqrt{1+\frac{2}{n}}+n} = L \frac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\cancel{\frac{2}{n}}^{\,0}}+1\right)} = \frac{2}{\sqrt{1}+1} = \boxed{1}$$

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b) $\sum_{n=1}^{\infty} \frac{n^{1/3}}{n^3+3n}$ $\quad a_n = \frac{n^{1/3}}{n^3+3n} \leq \frac{n^{1/3}}{n^3} = b_n$ (5)

Por criterio de la integral:

$$\int_1^{\infty} \frac{n^{1/3}}{n^3}\, dn = \int_1^{\infty} n^{1/3} \cdot n^{-3}\, dn = \int_1^{\infty} n^{1/3-3}\, dn = \int_1^{\infty} n^{-8/3}\, dn =$$

$$= \lim_{t \to \infty} \left. \frac{n^{-5/3}}{-\frac{5}{3}} \right|_1^t = \left(\lim_{t \to \infty} \frac{t^{-5/3}}{-\frac{5}{3}} \right) - \left(\frac{1^{-5/3}}{-\frac{5}{3}} \right) =$$

$$= \left(\lim_{t \to \infty} \cancel{\frac{1}{-\frac{5}{3}\, t^{5/3}}}^{\to 0} \right) - \left(\frac{1}{-\frac{5}{3}} \right) = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

Como b_n converge y por criterio de comparación dice que si $\sum_{n=1}^{\infty} b_n$ converge, entonces $\sum_{n=1}^{\infty} a_n$ converge; se llega a la conclusión de que $\sum_{n=1}^{\infty} \frac{n^{1/3}}{n^3+3n}$, converge.