Measuring profitability of the Acme Corp Portfolio

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Abstract
We show how Internal Rate of Return is calculated for a given portfolio on lending securities Portfolio.

Notations and Definitions

We place us on the point of view of a lender which is our position. At Acme Corp, we finance digital enterprises by allocating them funds on the form of a credit that client can use at will. Most of them employ the funds for financing advertisement, stock replenishment or a mix of both. At Acme Corp we aggregate the cash flows by week. The *terms and conditions* of a lending are as follows:

- The sum of M is made available to the client.
- Client expends M on (unspecified number of) weekly spaced flows M_t,
$$\sum_{t=0} M_t = M$$
- At contract inception, clients pays upfront a Commission S.
- At time t, client reimburses the credit by chunks $N_t = \rho R_{t-\tau}$ (Again on an unspecified number) on cash flows based on a percentage share $0 < \rho < 1$ of her revenue $R_{t-\tau}$ of the former τ weeks.
- Credit is fully reimbursed when
$$\sum_{t=0} N_t = \sum_{t=0} \rho R_{t-\tau} = M + S(1 + vat) \quad (1)$$
vat being the *Value Added Tax*

So for each entity where Acme Corp has allowed a credit there are two suites of cash flows. An *outbound* cash flow: $\{M_t\}$ and an *inbound* cash flows, $\{N_t\}$. We call $t_0 = 0$ the time where the first cash flow arrives, whether negative of positive. We align the cash flows by date and wrote:

$$F_t = N_t - M_t$$

The suite $F = \{F_i : 1 = 0 \dots N\}$ of cash flows is a *credit security*. Each individual cash flow can be either negative, either positive, let's say there are q of them. The value r such that

$$\sum_{t=0}^{q-1} \frac{F_t}{(1+r)^t} = 0 \quad (2)$$

Is called the *Internal Rate of Return* of the investment (*Actuarial* or *periodic* names are often used as well). Note that the periods on former IRR can any regularly spaced amount of time, weeks, months or years, in our case they are weeks. For converting to years, we remark that placing \$1 at a periodic rate r_p on $365.25/7$ weeks produces by compounding $\$(1+r_p)^{365.25/7}$. On the other hand \$1 at an annual rate r_a produces by the year $\$(1+r_a)$. As amount are equal we get:

$$r_a = (1+r_p)^{365.25/7} - 1$$

Fundings

Very often a client needs another funding, at Acme Corp we have three versions of fundings, V1, V2 and V3. The difference between them is on the terms and conditions. Namely, fundings of type V1 and V2 are ranked according to the date they were obtained and repayments are allocated primarily to the most ancient funding until full repayment. Then we continue with the next funding until full repayment. The V3 is of another kind, that funding does not wait for others to be fully paid: it starts immediately.

Forecasting

If the funding is finished (allocated funds expended and reimbursement done), we can easily calculate the irr. For fundings which are alive, cash flows are not known, as fundings at Acme Corp are *Revenue Based* credit securities. One challenge here is to forecast client's revenue in order to calculate reimbursement cash flows. We use models like the simple moving average on revenue and a more elaborate one, Auto-Regressive Integrated Moving Average (ARIMA) for forecasting revenue.

Appendix

We use the financial library **numpy financial** and sometimes the irr calculation falls upon negative rates. Take the following example, (Which by the way is a real one)

$$\begin{aligned} F = \{&-137.7, -1229.04, -1253.67, -1207.88, \\ &-1098.08, -998.25, -907.5, -825, -750, -1500, \\ &1139.83, 715.8, 1181.34, 1696.57, 2628.88, 1365.41, \\ &1255.11, 1033.67, 1089.47, 823.72, 624.25, 711.98, \\ &933.97, -750, -1500, -1204.78, -638.1\} \end{aligned}$$

One solution we got for this series of cash flows is $r = -0.015$. On the figure, we plot the net present value for a set of rates.

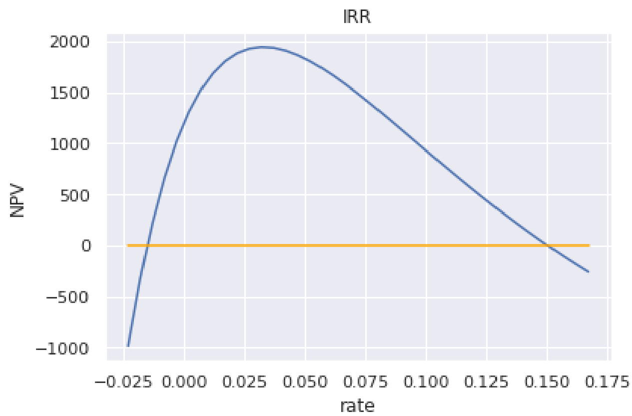

Figure 1. Negative and positive IRR

We clearly see two roots. Put $x = 1/(1+r)$ on the equation 2 and we get a polynomial of degree $q-1$,

$$p(x) = \sum_{i=0}^{q-1} F_i x^i$$

From algebra, we know that such polynomial has exactly $q-1$ roots. We can search for another rate of return if the obtained one is negative. For that we use the companion matrix C:

$$C = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -F_0 & -F_1 & \cdots & -F_{q-1} \end{bmatrix}$$

As it turns out, the roots of the characteristic polynomial of C (i.e., the roots of $\det(C – xI) = 0$) are precisely the roots of the polynomial p. So we use linear algebra for finding all roots. If all new ones are purely complex, we stay with the negative IRR. Whereas if we find purely real and positive ones, we define the IRR as the *minimum* of all of them.

As said, on the former example, we get $r = -0.015$ and further applying the former algorithm, we get $r = 0.15$.

References

[1] Franck Fabozzi, Steven Mann The Handbook of Fixed Income Securities *Mc Graw Hill*

[2] B. Cooperstein Advanced Linear Algebra *CRC Press*