Lista Aulas 13 e 14 Luís Felipe Cabran Domingues 202069025A

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① $|\beta| > |\alpha| > 0$

a) $x_1[n] = \alpha^n u[n+1] + \beta^n u[n+2]$

$$X_1[z] = \underbrace{\sum_{n=-1}^{\infty} \alpha^n z^{-n}}_{n+1=M} + \underbrace{\sum_{n=-2}^{\infty} \beta^n z^{-n}}_{n+2=M}$$

$$X_1[z] = \sum_{M=0}^{\infty} (\alpha z^{-1})^{m+1} + \sum_{M=0}^{\infty} (\beta z^{-1})^{m+2}$$

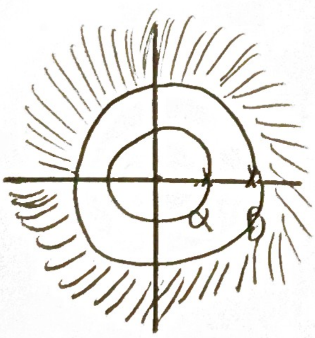
$$X_1[z] = \sum_{M=0}^{\infty} (\alpha z^{-1})^{m} \cdot (\alpha z^{-1})^{1} + \sum_{M=0}^{\infty} (\beta z^{-1})^{m} \cdot (\beta z^{-1})^{2}$$

$$X_1[z] = \underbrace{(\alpha z^{-1}) \cdot \frac{1}{1-\alpha z^{-1}}}_{|\alpha z^{-1}| < 1} + \underbrace{(\beta z^{-1})^2 \cdot \frac{1}{1-\beta z^{-1}}}_{|\beta z^{-1}| < 1}$$

$|z| > |\alpha|$

$|z| > |\beta|$ → ROC: $|z| > |\beta|$

↳ POLO MAIS DISTANTE



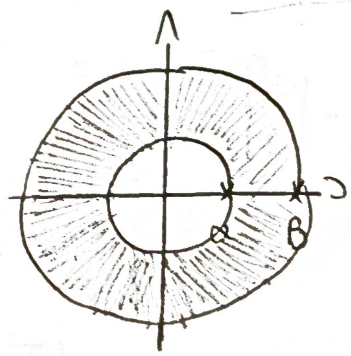
b) $x_2[n] = \alpha^n u[n-2] + \beta^n u[-n-1]$

$$X_2[z] = \underbrace{\sum_{n=2}^{\infty} (\alpha z^{-1})^n}_{n-2=M_1} + \underbrace{\sum_{n=-1}^{-\infty} (\beta z^{-1})^n}_{M=-n} \to \underbrace{\sum_{M=1}^{\infty} (\beta z^{-1})^{-m}}_{m-1=M_2}$$

$$X_2[z] = \sum_{M_1=0}^{\infty} (\alpha z^{-1})^{m-2} + \sum_{M_2=0}^{\infty} (\beta^{-1} z)^{m-1}$$

$$X_2[z] = \underbrace{(\alpha z^{-1})^{-2} \cdot \frac{1}{1-\alpha z^{-1}}}_{|\alpha z^{-1}|<1 \to \left|\frac{\alpha}{z}\right|<1 \to |z|>|\alpha|} + \underbrace{(\beta^{-1} z)^{-1} \cdot \frac{1}{1-\beta^{-1} z}}_{|\beta^{-1} z|<1 \to \left|\frac{z}{\beta}\right|<1 \,;\, |\beta|>|z|}$$

$\to$ ROC: $|\alpha| < |z| < |\beta|$

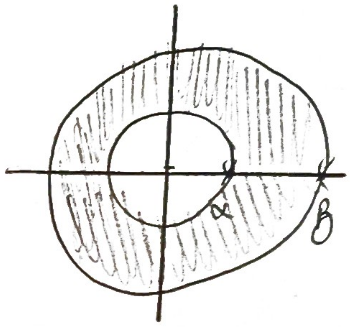


c) $X_3[n] = \alpha^n u[n+2] + \beta^n u[-n-1]$

$$X_3[z] = \underbrace{(\alpha z^{-1})^{2} \frac{1}{1-\alpha z^{-1}}}_{|\alpha z^{-1}|<1 \rightarrow \left|\frac{\alpha}{z}\right|<1} + \underbrace{(\beta^{-1}z)^{-1} \cdot \frac{1}{1-\beta^{-1}z}}_{|\beta^{-1}z|<1 \rightarrow \left|\frac{z}{\beta}\right|<1}$$

$|z| > |\alpha|$ $\qquad$ $|\beta| > |z|$

$\rightarrow$ ROC: $|\alpha| < |z| < |\beta|$



② $\left.\begin{array}{l} x[n] = \{1,2,2,1\} \\ h[n] = \{1,2,1\} \end{array}\right\} \; x[n] * h[n] = X(z) \cdot Y(z)$

$$\rightarrow X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = \left(1 \cdot z^{0} + 2 \cdot z^{-1} + 2 \cdot z^{-2} + 1 \cdot z^{-3}\right)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n} = \left(1 \cdot z^{-0} + 2 \cdot z^{-1} + 1 \cdot z^{-2}\right)$$

$$\rightarrow X(z) \cdot H(z) = \left(1 + 2z^{-1} + z^{-2} + 2z^{-1} + 4z^{-2} + 2z^{-3} + 2z^{-2} + 4z^{-3} + 2z^{-4} + z^{-3} + 2z^{-4} + z^{-5}\right)$$

$$= \left(1 + 4z^{-1} + 7z^{-2} + 7z^{-3} + 4z^{-4} + z^{-5}\right)$$

$\rightarrow x[n] = \{1, 4, 7, 7, 4, 1\}$

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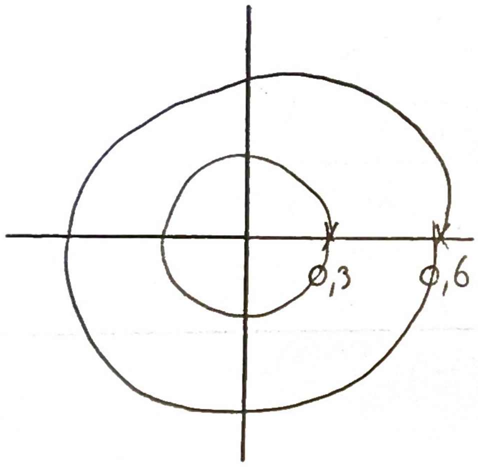
$$H(z) = \underbrace{\frac{7 + 3{,}6z^{-1}}{1 + 0{,}9z^{-1} + 0{,}18z^{-2}}}_{}= \frac{Y(z)}{X(z)}$$

POLOS DA FUNÇÃO TRANSFERÊNCIA = LIMITES DA ROC

$$\rightarrow \frac{7 + 3{,}6z^{-1}}{1 + 0{,}9z^{-1} + 0{,}18z^{-2}} \cdot \frac{z^2}{z^2} = \frac{7z^2 + 3{,}6z}{\underbrace{z^2 + 0{,}9z + 0{,}18}_{\text{RAÍZES} = \{0{,}3 \text{ e } 0{,}6\}}}$$



ROC:
$$\begin{cases} |z| < 0{,}3 \\ 0{,}3 < |z| < 0{,}6 \\ |z| > 0{,}6 \end{cases}$$

→ INVERSAS:

$$Y(z) + 0{,}9z^{-1}Y(z) + 0{,}18z^{-2}Y(z) = 7X(z) + 3{,}6z^{-1}X(z)$$

$$y[n] + 0{,}9\,y[n-1] + 0{,}18\,y[n-2] = 7X[n] + 3{,}6\,X[n-1]$$

$$\rightarrow y[n] = 7X[n] + 3{,}6X[n-1] - 0{,}9\,y[n-1] - 0{,}18\,y[n-2]$$

$$\rightarrow X[n] = \frac{y[n] + 0{,}9y[n-1] + 0{,}18\,y[n-2] - 3{,}6\,X[n-1]}{7}$$

4) $y[n] = h[n] * x[n]$ $\begin{cases} h[n] = (-0{,}2)^n u[n] \\ x[n] = (0{,}3)^n u[n] \end{cases}$

$$\rightarrow H(z) = \sum_{n=0}^{\infty} (-0{,}2z^{-1})^n = \frac{1}{1-(-0{,}2z^{-1})} = \frac{1}{1+0{,}2z^{-1}}$$

$$|-0{,}2z^{-1}| < 1 \;, \quad \left|\frac{-0{,}2}{z}\right| < 1 \;, \quad \underbrace{|z| > |-0{,}2|}$$

$$\rightarrow X(z) = \sum_{n=0}^{\infty} (0{,}3z^{-1})^n = \frac{1}{1-0{,}3z^{-1}}$$

$$|0{,}3z^{-1}| < 1 \;, \quad \left|\frac{0{,}3}{z}\right| < 1 \;, \quad \underbrace{|z| > |0{,}3|}$$

$$\rightarrow \begin{matrix} h[n] * x[n] = \\ H(z) \cdot X(z) \end{matrix} \frac{1}{1+0{,}2z^{-1}} \cdot \frac{1}{1-0{,}3z^{-1}} = \frac{1}{(1-0{,}3z^{-1}+0{,}2z^{-1}-0{,}06z^{-2})}$$

$$\rightarrow Y(z) = \frac{1}{(1-0{,}1z^{-1}-0{,}06z^{-2})}$$