

Time Series as Stochastic Processes

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Forecasting & Time Series Analysis – Fall 2023



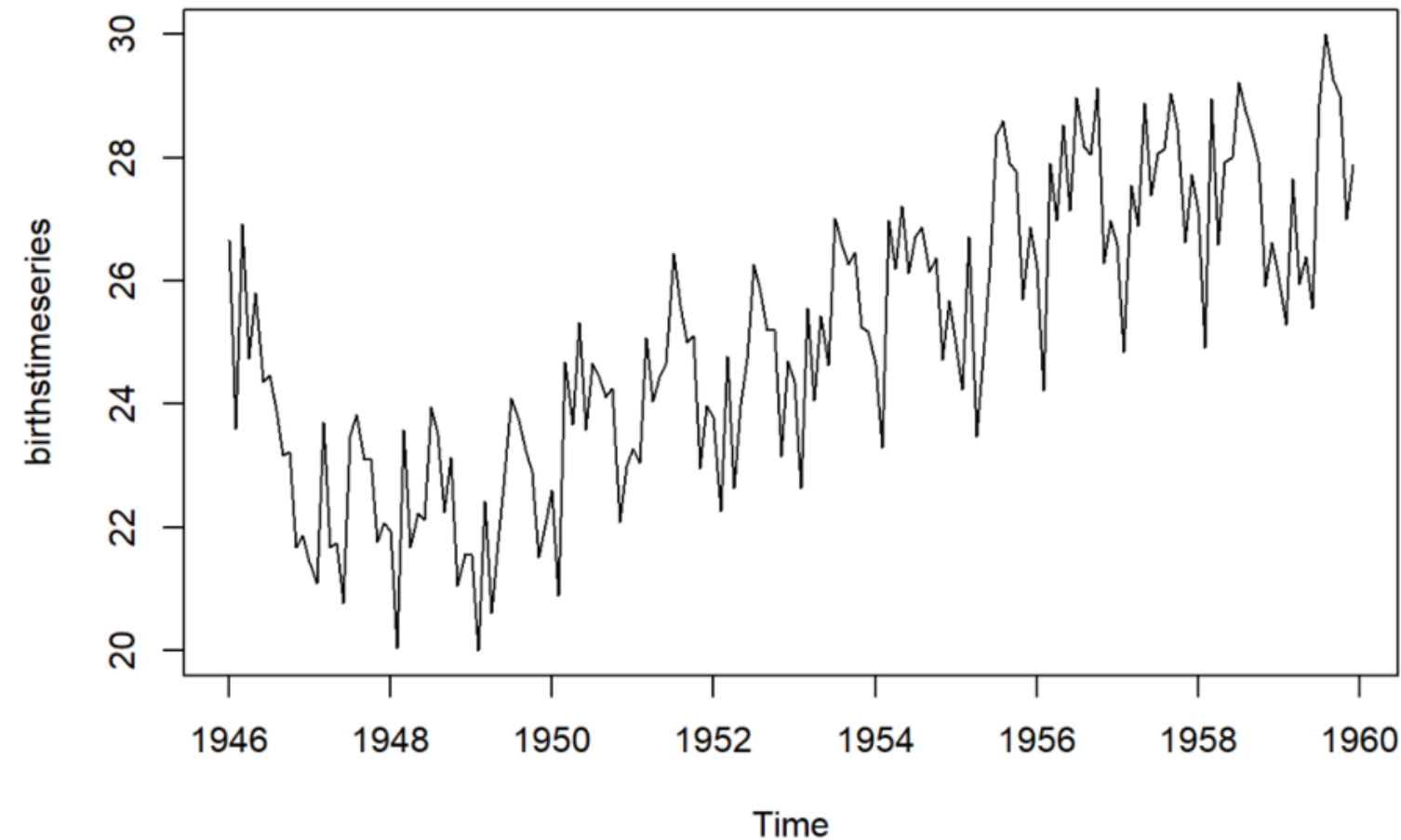
01 TS as stochastic processes

02 TS and time correlation

03 Key takeaways

Modelling Time Series as a Stochastic Process

Time Series Data: sequential observations of one or several variables over time

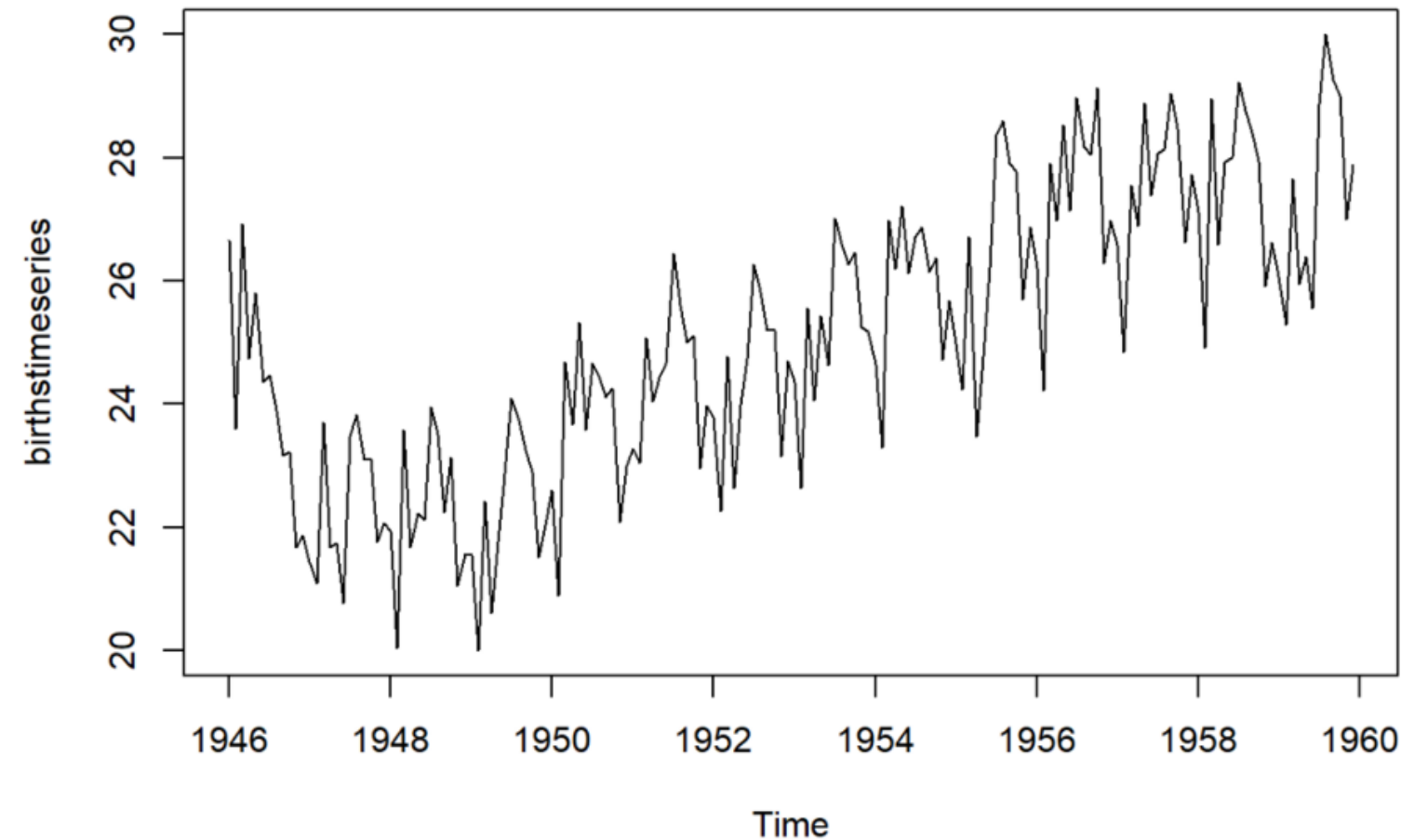


For modelling purposes lets consider:

- **Future values of the time series:** a random variable that has not yet been realized.
- **Past values of the time series:** realizations of a random variable

Modelling Time Series as a Stochastic Process

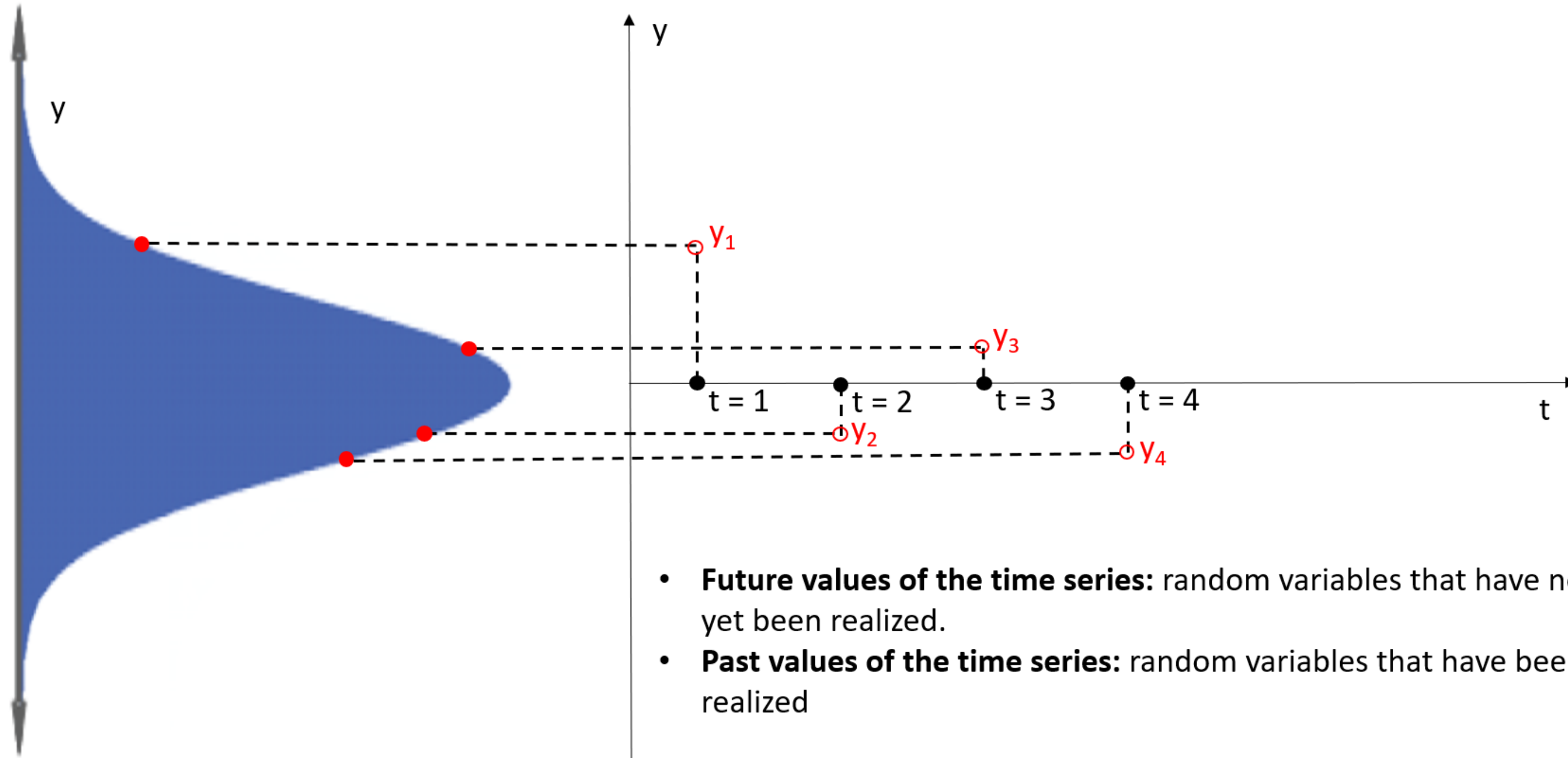
Time Series Data: sequential observations of one or several variables over time



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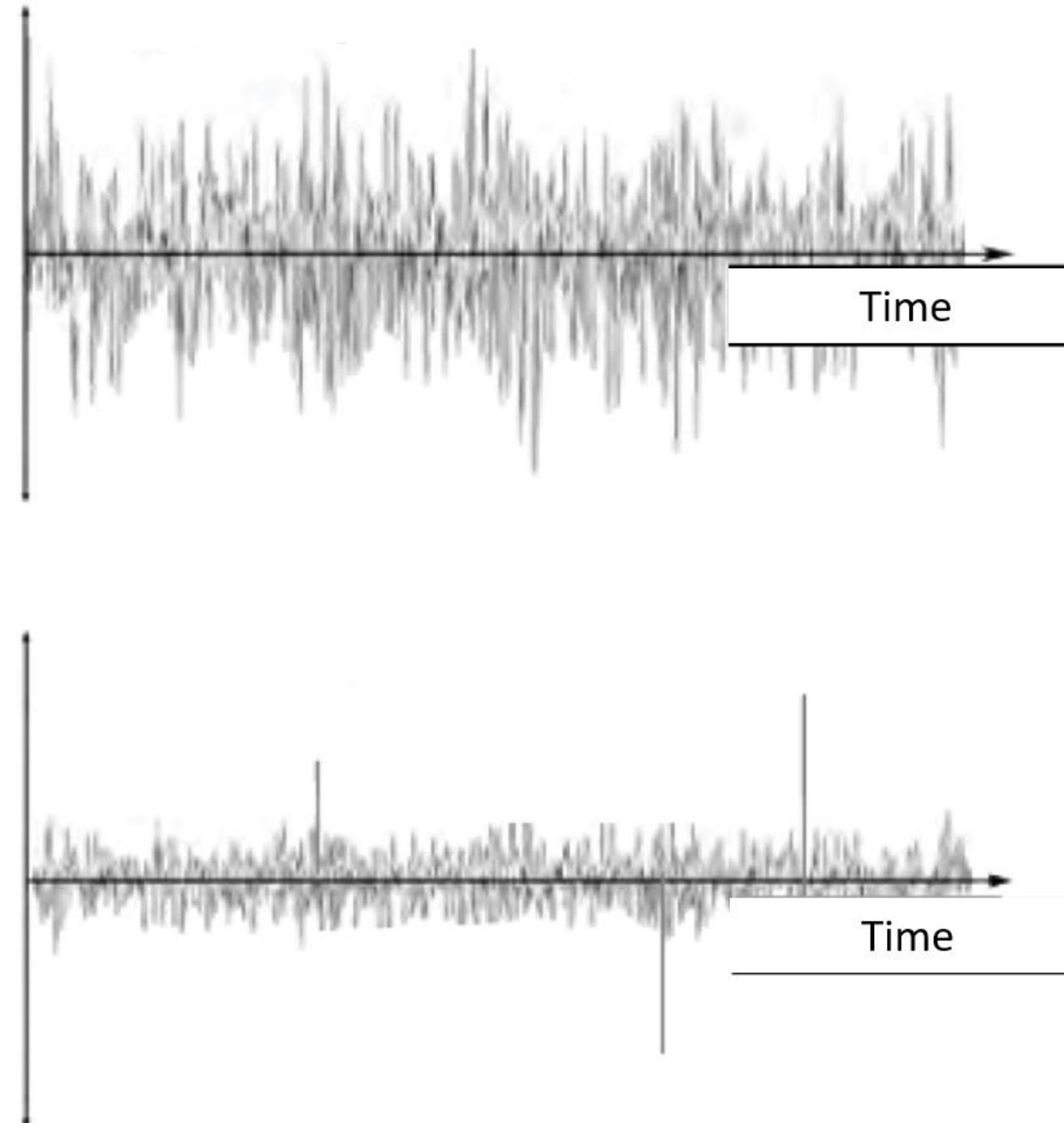
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Example



- **Future values of the time series:** random variables that have not yet been realized.
- **Past values of the time series:** random variables that have been realized

In this particular time series all the random variables are identically distributed (follow the same distribution).



Modelling Time Series as a Stochastic Process

Stochastic vs Random: these two words are many times used interchangeably. However, it is normally the case that:

- **Random** is reserved to refer to random variables.
- **Stochastic** is reserved to refer to processes involving a family of random variables indexed by a set. In the case of time series, indexed over time.

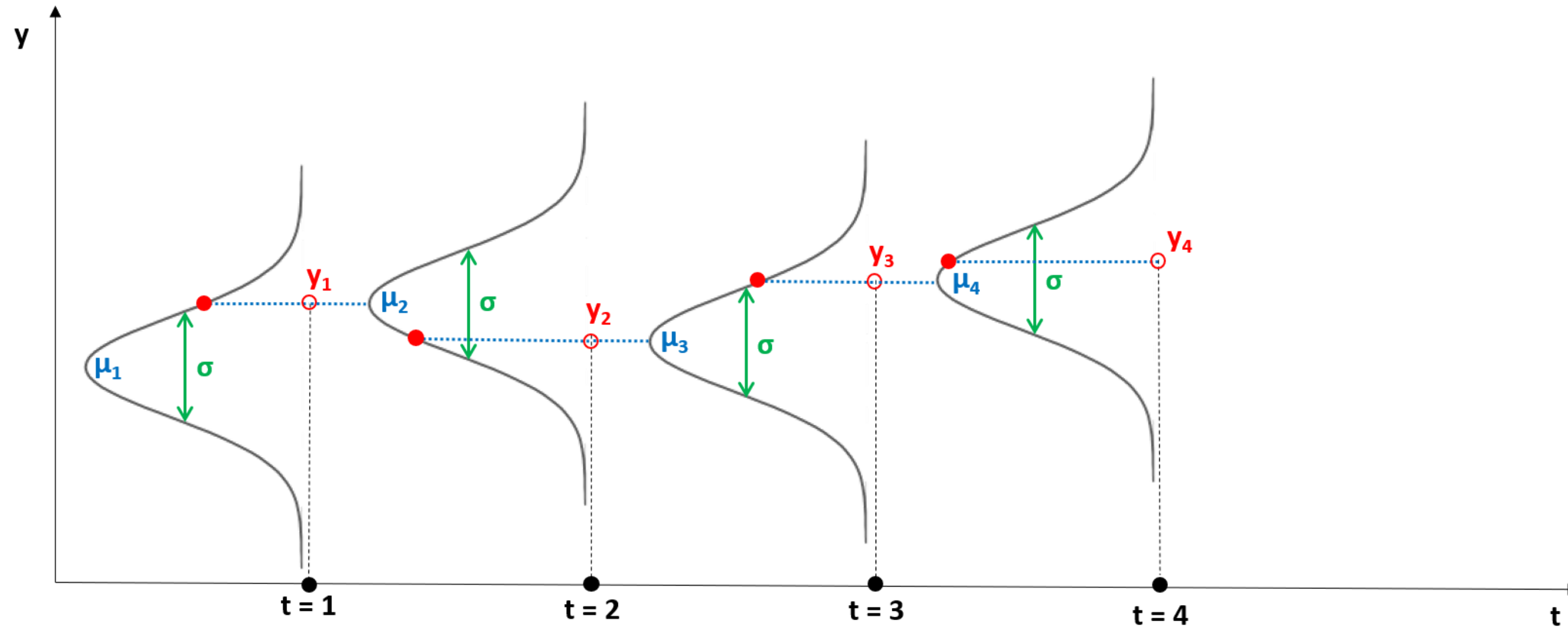
In our previous example:

- **For each value of time** $t_1, t_2, t_3 \dots$ we had a **random variable** $y_1, y_2, y_3 \dots$
- This **collection of random variables $\{y_t\}$ indexed by time** is a stochastic process

NOTE

- There are **different kinds of stochastic processes**. Time series are just one of these.

Time series, correlation and independence



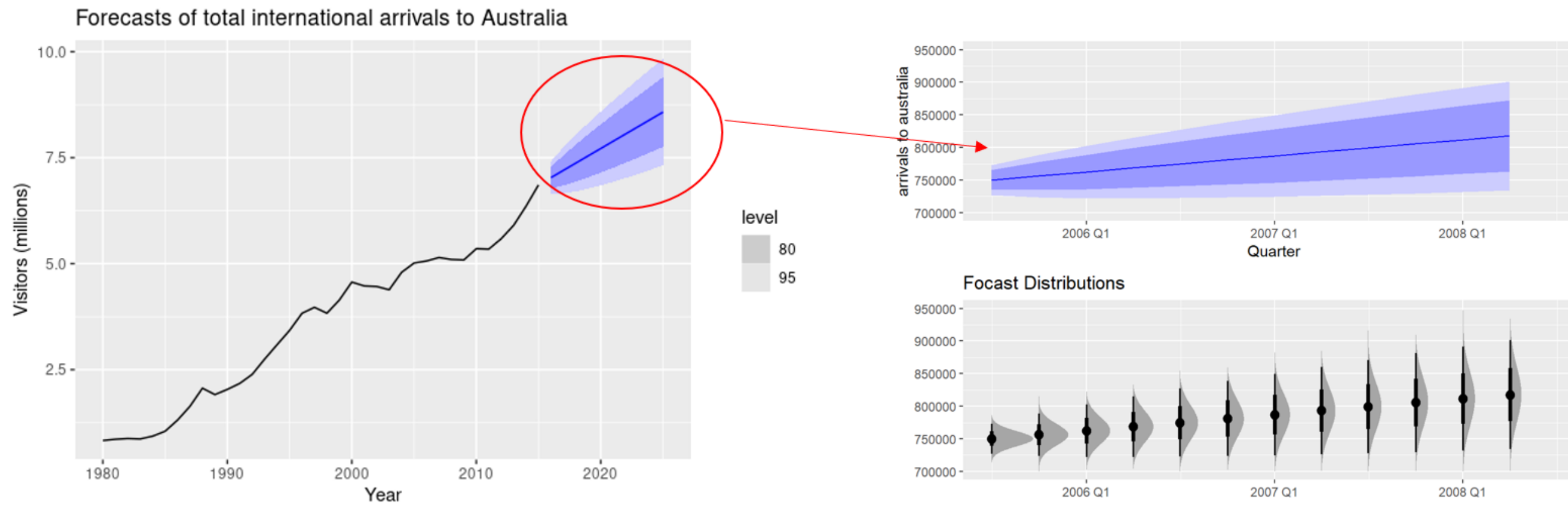
Since **past events** tend to **influence future events**, there is a correlation between the values of the time series at different points in time.

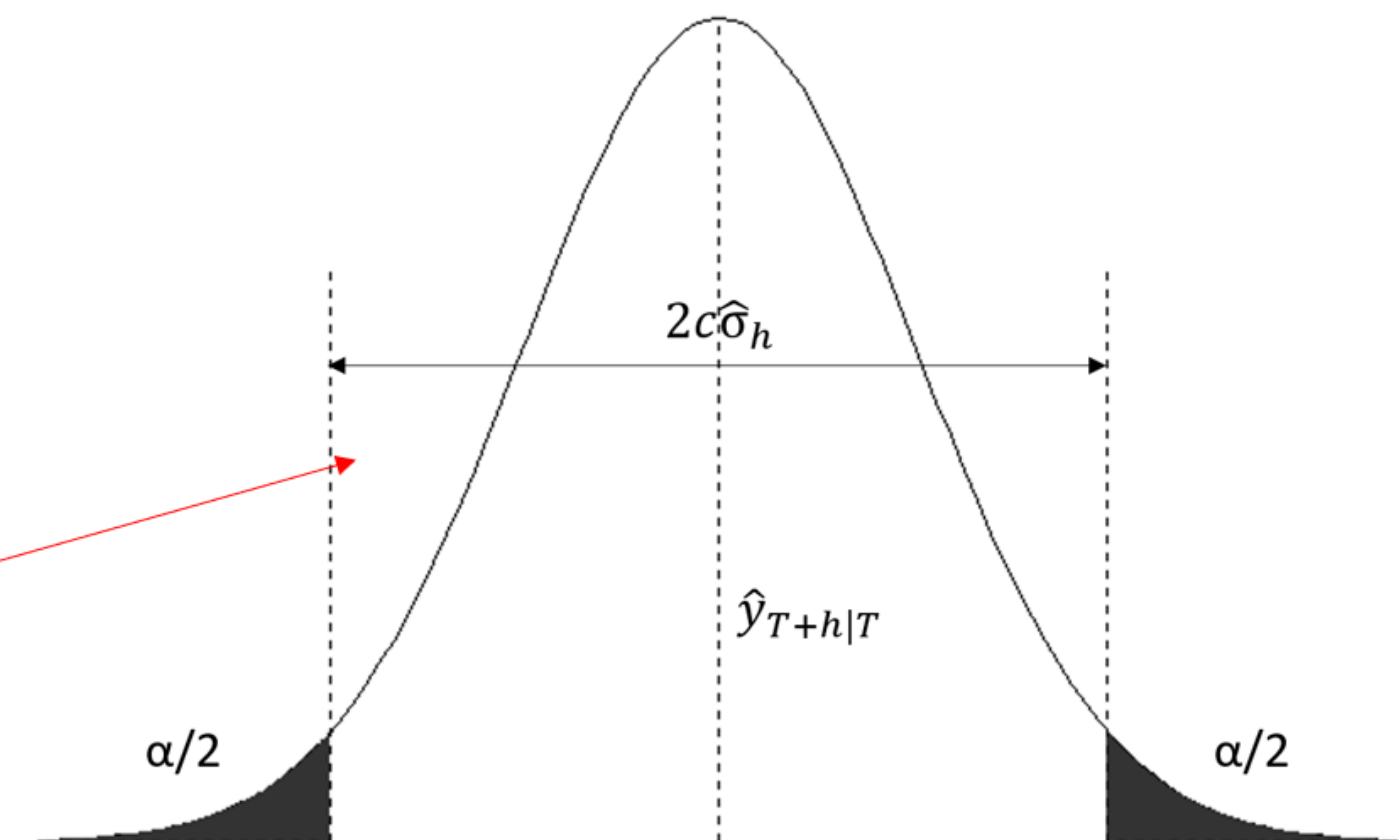
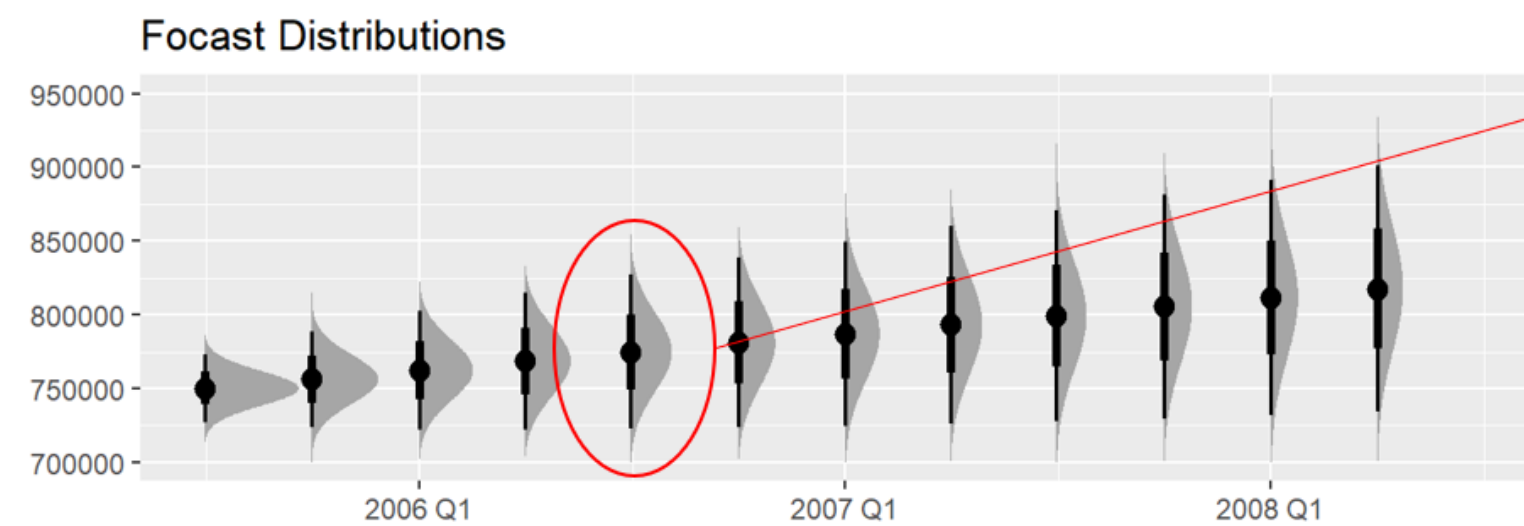
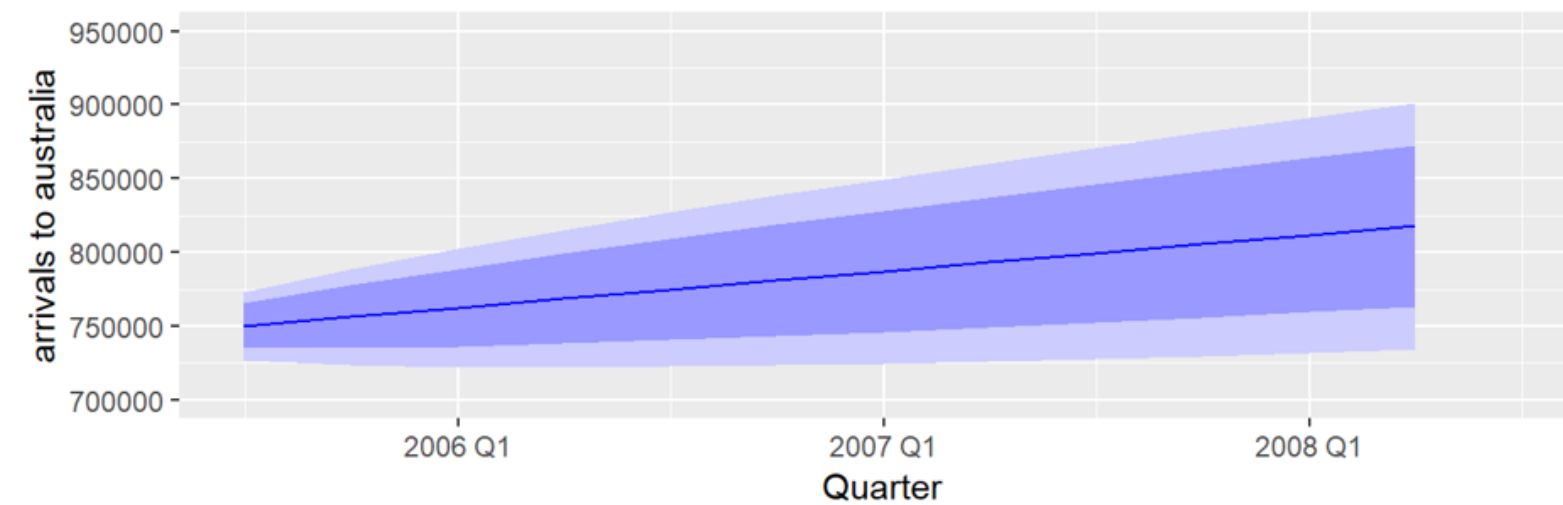
The **random variables** of the **time series process** are **correlated in general** (and therefore NOT independent).

Key takeaway 1

Time series as stochastic processes (a **collection of random variables indexed in time**):

- **Past values:** realizations of random variables.
 - They are no longer random, they are a specific number, an outcome of the random variable.
- **Forecast or future values:** forecasting a random variable that has not yet realized.
 - The concept of **confidence intervals** therefore applies to time series forecasting.
 - You should think of the values to forecast as a random variable.





Key takeaway 2

The random variables of a **time series process** are in general:

- **Correlated** (and therefore **not independent**).
 - In most phenomena studied in time series, past values tend to affect future values.
- **Not identically distributed.**

The ***independent and identically distributed hypothesis*** used in most conventional statistical methods (e.g. Central Limit Theorem) does **not apply**.

Time Series can be referred to as the *systematic approach to answer the mathematical and statistical questions posed by these time correlations.*

