

# Errors

$x \rightarrow$  TRUE VALUE

$\hat{x} \rightarrow$  APPROXIMATION

• Differences

Units

e.g.: absolute error:  $|x - \hat{x}|$   $\rightarrow$  Units of  $[x]$

squared error:  $(x - \hat{x})^2$   $\rightarrow$  Units of  $[x^2]$

• Ratios:

No units

e.g.: relative error:  $\frac{|x - \hat{x}|}{|x|}$   $\rightarrow$  No units (units cancel)

(percentage error)

# TS ERRORS

Error at time  $\tau$

$$e_{\tau} = y_{\tau} - \hat{y}_{\tau}$$

↓  
AT A SPECIFIC  
POINT IN TIME

- TRAINING  $\tau \leq T$  OR LAST POINT IN TRAINING

$$e_{\tau} = y_{\tau} - \hat{y}_{\tau | \tau-1}$$

RESIDUALS  
(RESIDUAL ERRORS)

- TEST OR FORECAST  $\tau > T \Rightarrow \tau = T + h$  (HORIZON)

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h | T}$$

FORECAST  
ERRORS

# Metrics to summarize errors (ONLY THOSE USED FOR THIS COURSE)

SUMMARIES OF THE VECTORS OF ERRORS AT DIFFERENT POINTS

$$e_t = y_t - \hat{y}_t$$

SCALED DEPENDENT (SAME UNITS AS  $y$ )

MEAN ABS ERROR :  $MAE = \text{mean}(|e_t|)$

ROOT MEAN SQUARED ERROR :  $RMSE = \sqrt{\text{mean}(e_t^2)}$

SCALE INDEPENDENT (NO UNITS)

PERCENTAGE ERROR AT  $t$

$$p_t = 100 \frac{e_t}{y_t} = 100 \frac{y_t - \hat{y}_t}{y_t}$$

MEAN ABSOLUTE PERCENTAGE ERROR

$$MAPE = \text{mean}(|p_t|)$$