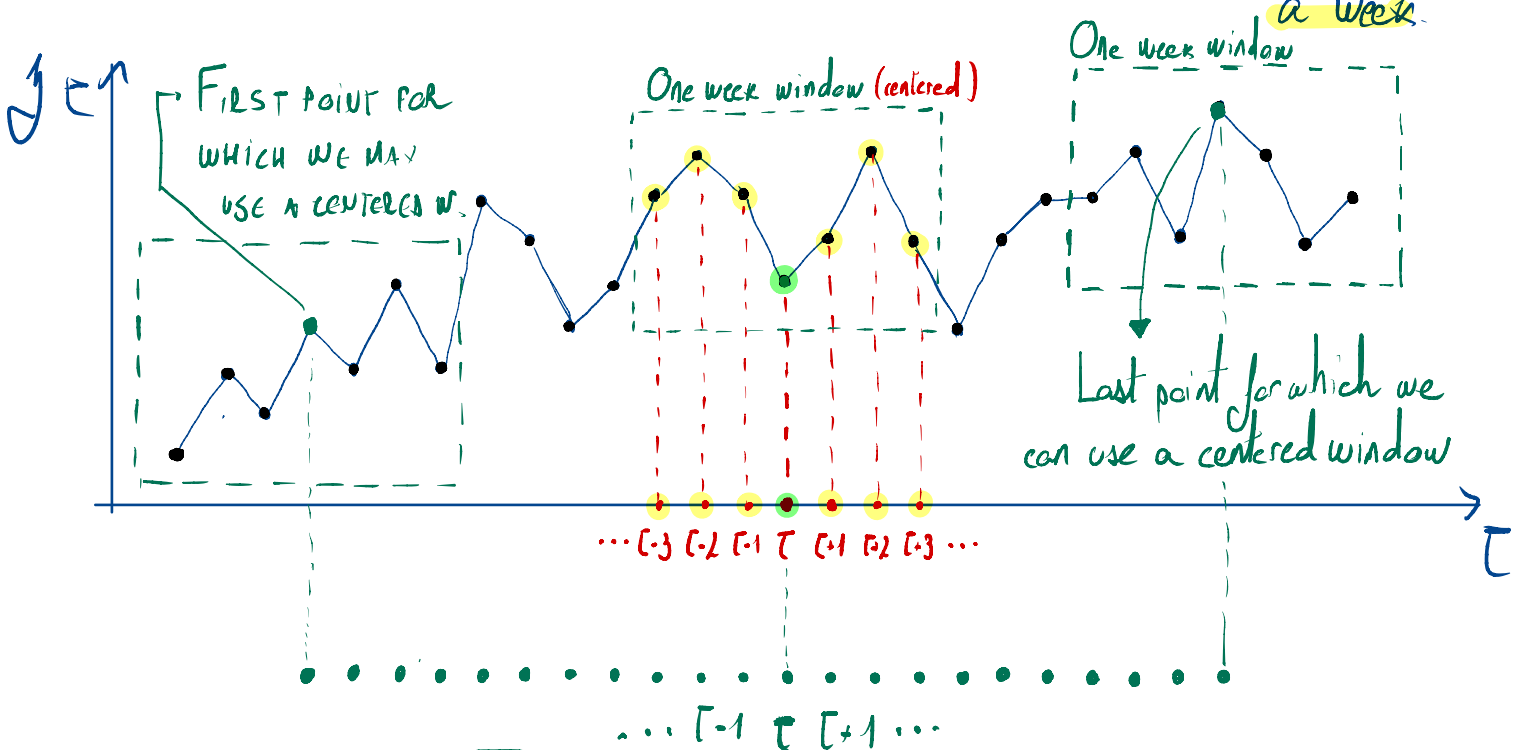
Moving Averages - Centered

Uneven number of points - example: daily data → average over a week



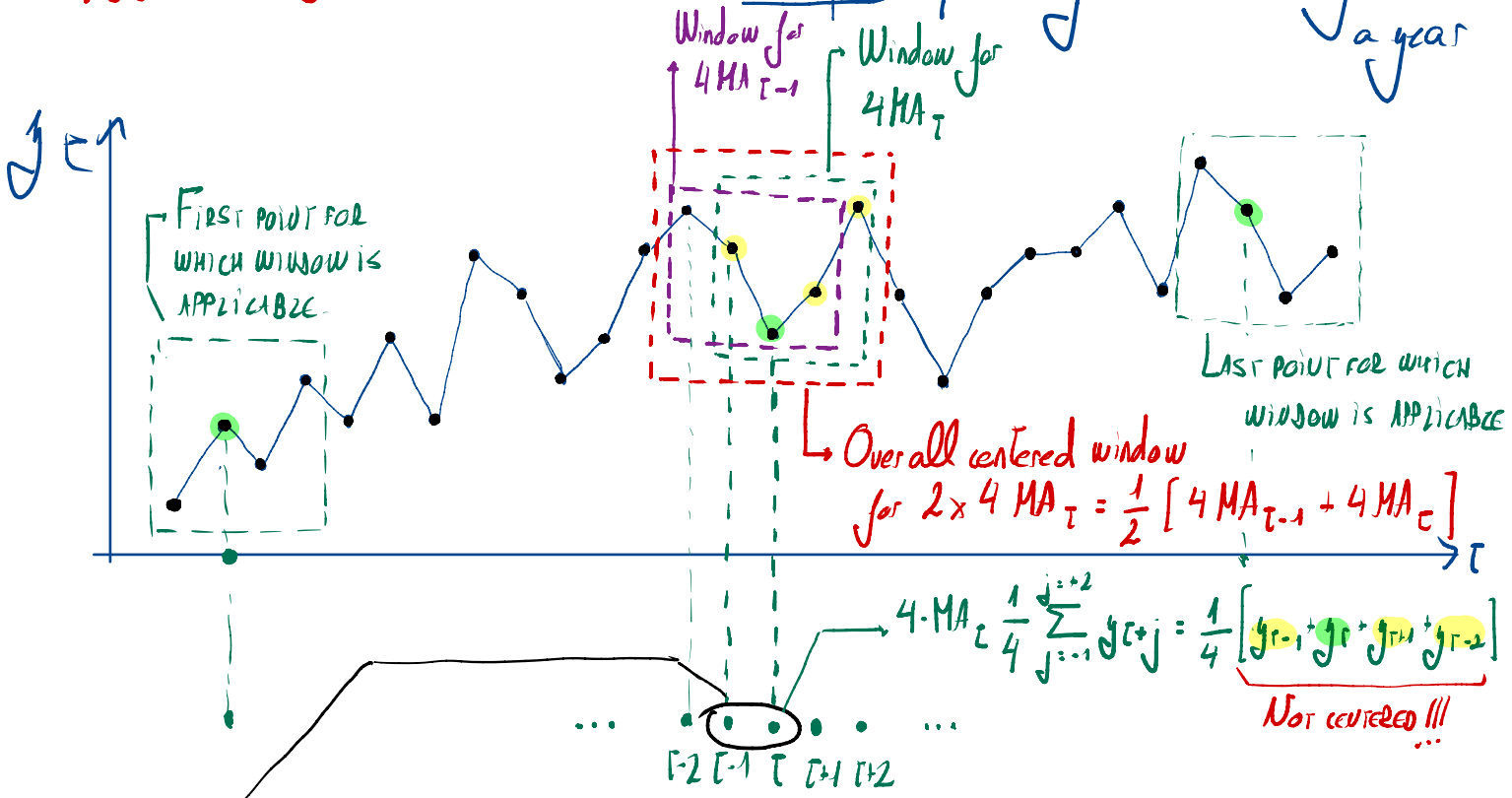
$$7\text{-MA}_\tau = \frac{1}{7}\sum_{j=-3}^{j=+3} y_{\tau+j} = \frac{1}{7}\left[y_{\tau-3} + y_{\tau-2} + y_{\tau-1} + y_\tau + y_{\tau+1} + y_{\tau+2} + y_{\tau+3}\right]$$

- Daily data – 1 point = 1 day
- Week has 7 days (uneven number)

} Centered window of 7 points is directly applicable.

Moving Averages - Centered

Even number of points - example: quarterly data → average over a year



$$\frac{1}{2}\left[4MA_{t-1} + 4MA_t\right] =$$

$$= \frac{1}{2}\left[\underbrace{\frac{1}{4}\left[y_{t-2}+y_{t-1}+y_t+y_{t-1}\right]}_{4MA_{t-1}} + \underbrace{\frac{1}{4}\left[y_{t-1}+y_t+y_{t+1}+y_{t+2}\right]}_{4MA_t}\right]$$

$$= \frac{1}{8}\left[\underbrace{y_{t-2} + 2y_{t-1} + 2y_t + 2y_{t+1} + y_{t+2}}_{\text{Centered window}}\right]$$

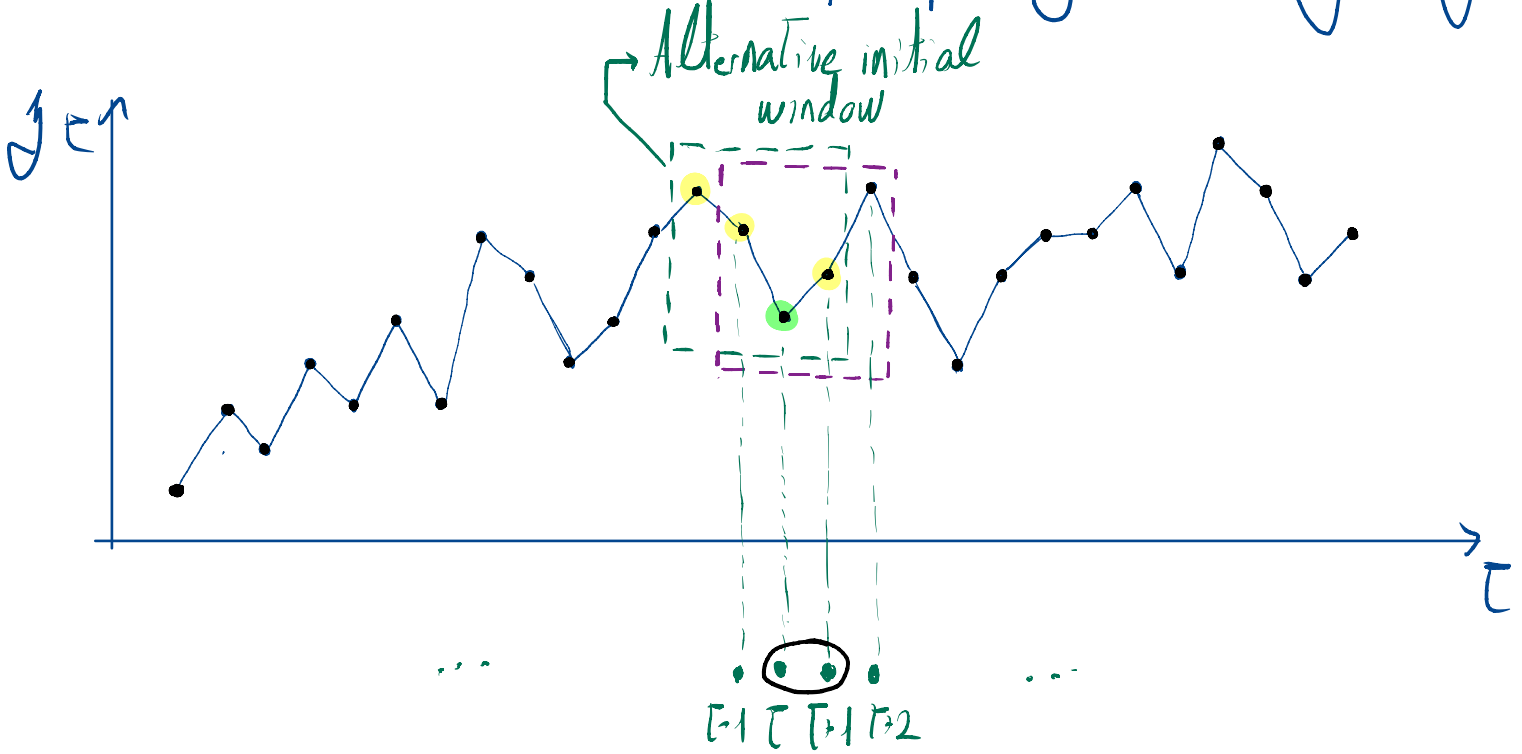
Weighted MA

Coefficients: $a_j = \left[\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}\right]$ ($a_{-2}, a_{-1}, a_0, a_1, a_2$) $\rightarrow \sum_{j=-2}^{j=2} a_j = 1$

$$2 \times 4MA_t = \sum_{j=-2}^{j=2} a_j y_{t+j}$$

Moving Averages - Centered

Even number of points - example: quarterly data // average over year



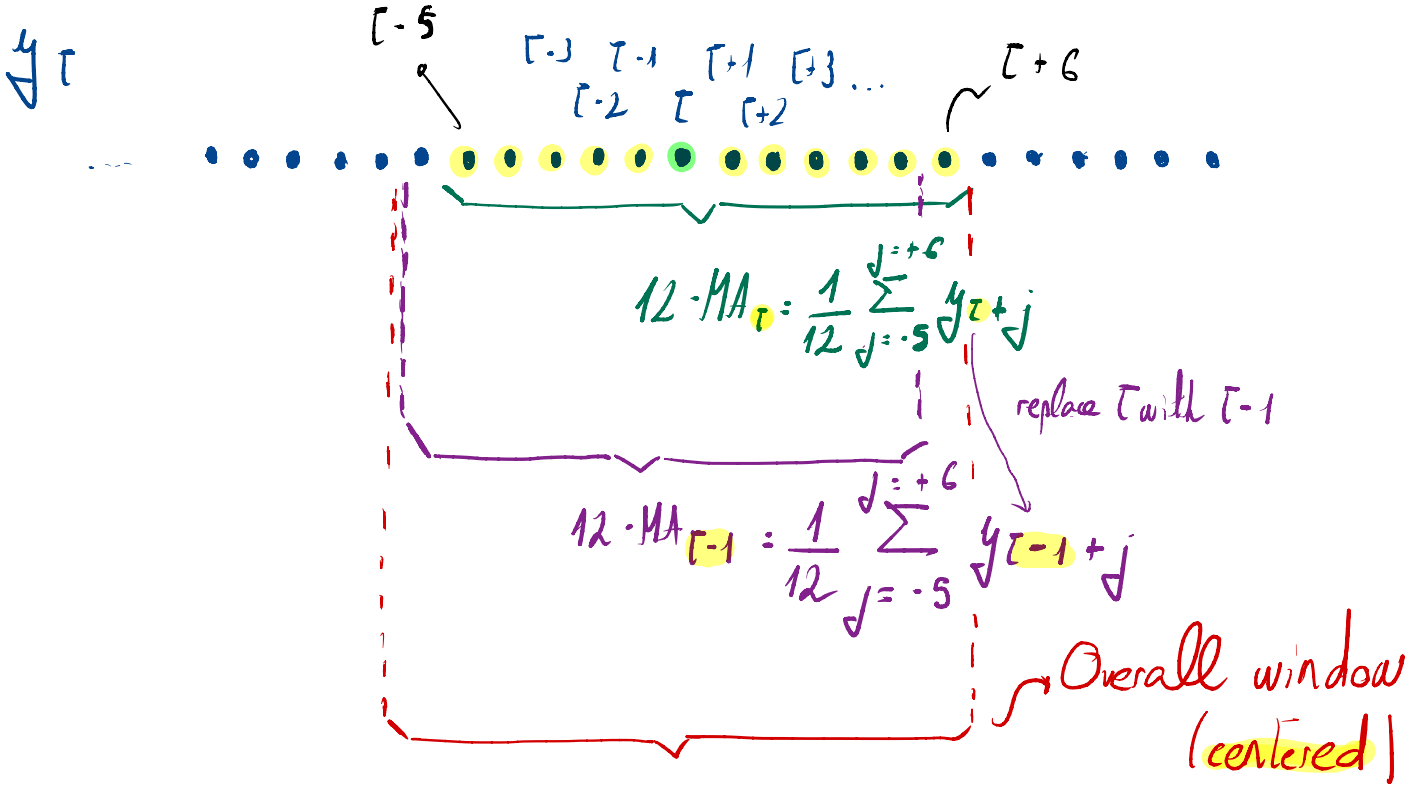
$$4MA_t = \frac{1}{4}\left[y_{t-2} + y_{t-1} + y_t + y_{t+1}\right]$$

$$2 \times 4MA_t = \frac{1}{2}\left[4MA_t + 4MA_{t+1}\right]$$

↳ If you develop this formula, you will reach the exact same result

Moving Averages - Centered

Even number of points → Example: monthly data → average over year



$$2\times 12\,MA_t = \frac{1}{2}\left[12\,MA_{t-1} + 12\,MA_t\right] =$$

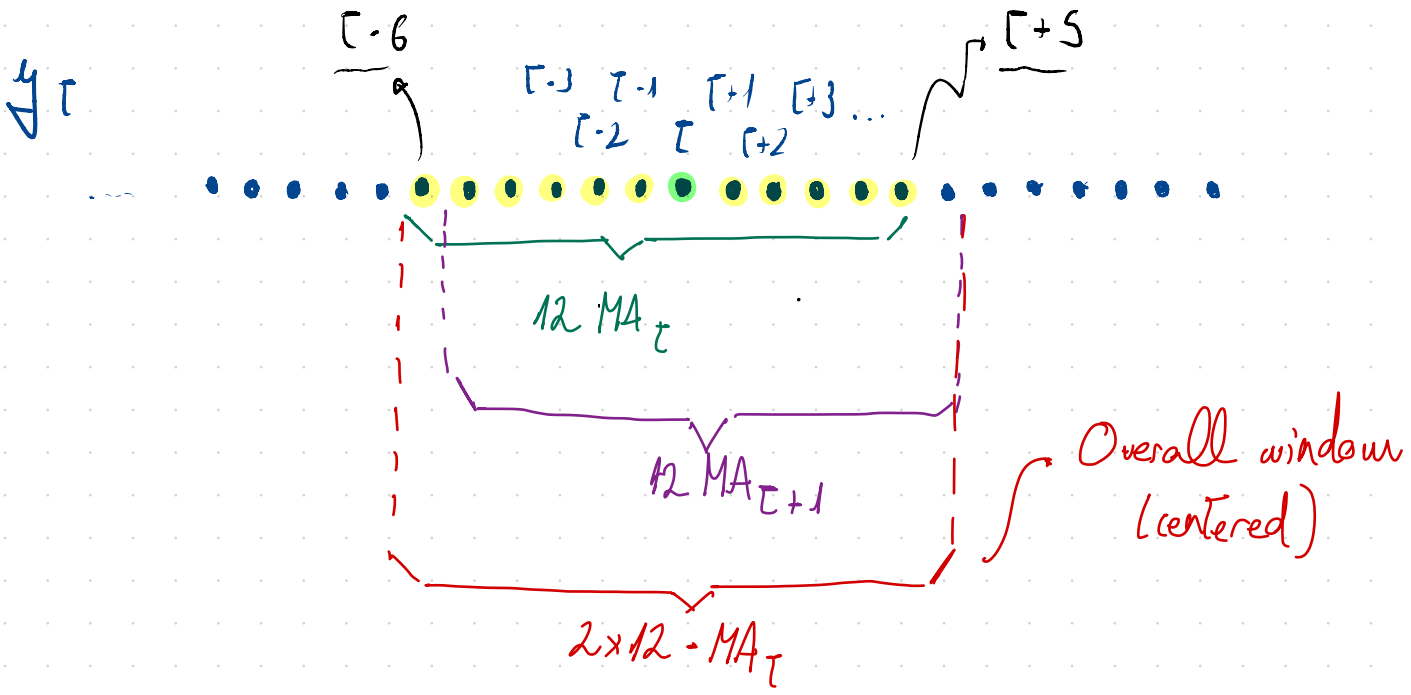
$$= \frac{1}{2}\cdot\frac{1}{12}\Big[y_{t-6} + 2\big[y_{t-5}+y_{t-4}+y_{t-3}+y_{t-2}+y_{t-1}+y_t+y_{t+1}+y_{t-2}+y_{t+3}+y_{t+4}+y_{t+5}\big] + y_{t+6}\Big] = \sum_{j=-6}^{j=6} a_j\, y_{t+j}$$

All these points appear in both windows (=)

$$\left[\frac{1}{24}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{24}\right]$$

j:	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6

$$\sum_{j=-6}^{j=6} a_j = 1$$



$$2 \times 12\,MA_t = \frac{1}{2}\left[MA_t + MA_{t+1} \right] = \dots$$

Had we picked the alternative initial window for MA_t, the formula needs to be adapted as shown above. But, again, the end result would be the same