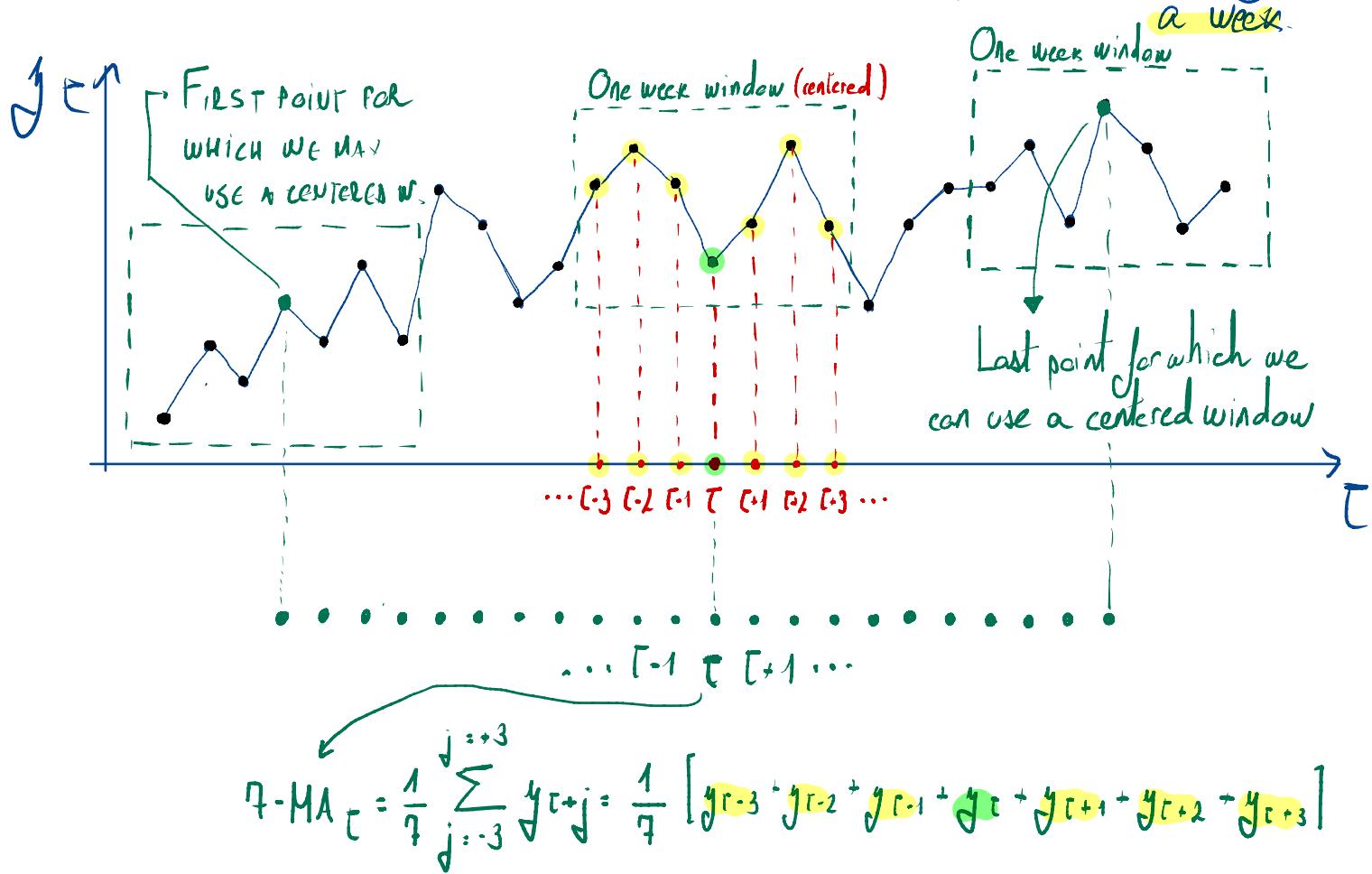


Moving Averages - CENTERED

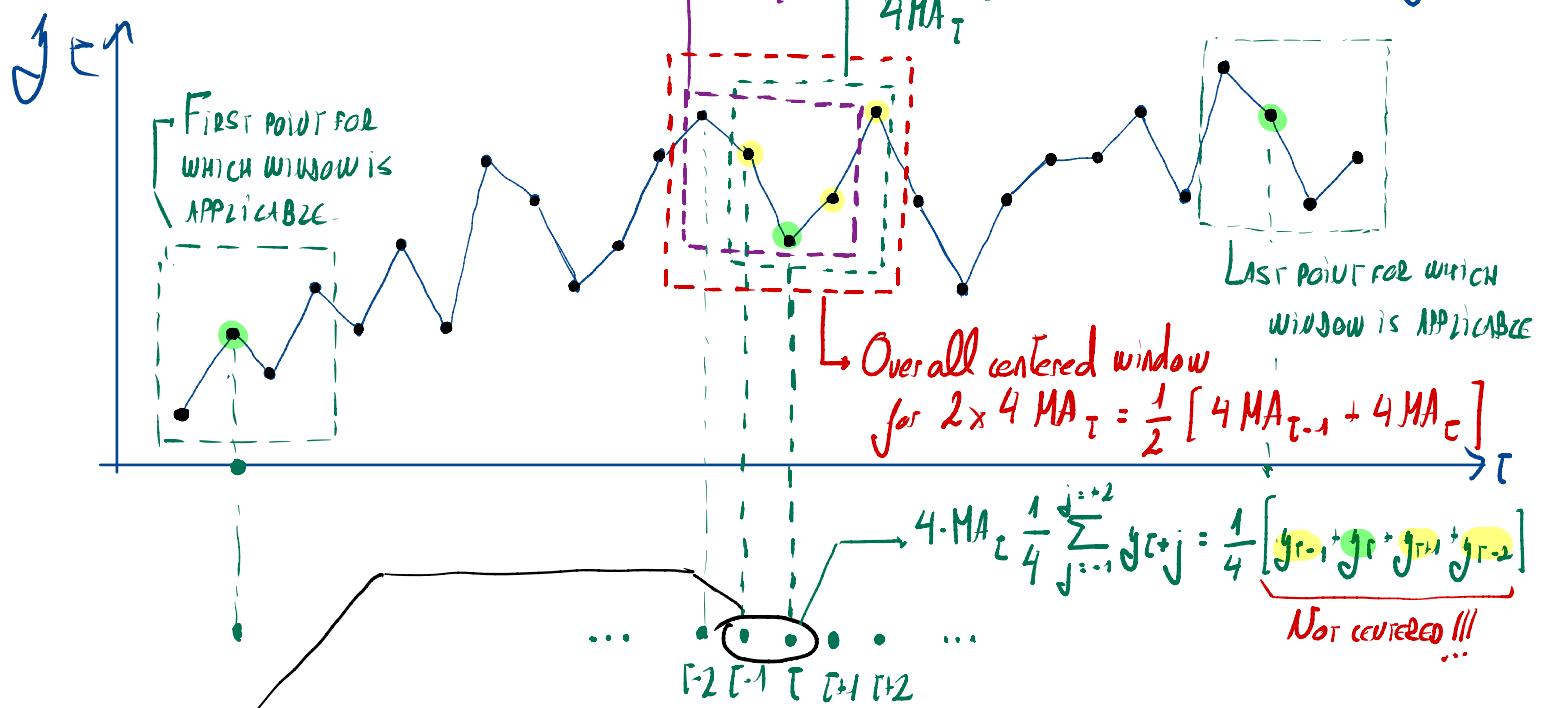
UNEVEN NUMBER OF POINTS - example: daily data → average over



- Daily data - 1 point = 1 day
 - Week has 7 days (uneven number)
- } Centered window of 7 points is directly applicable.

Moving Averages - CENTERED

Even NUMBER OF POINTS - example: quarterly data → average over a year



$$\begin{aligned} & \rightarrow \frac{1}{2} [4 MA_{t-1} + 4 MA_t] = \\ & = \frac{1}{2} \left[\underbrace{\frac{1}{4} [y_{t-2} + y_{t-1} + y_t + y_{t+1}]}_{4 MA_{t-1}} + \underbrace{\frac{1}{4} [y_{t-1} + y_t + y_{t+1} + y_{t+2}]}_{4 MA_t} \right] \end{aligned}$$

$$= \frac{1}{8} [y_{t-2} + 2y_{t-1} + 2y_t + 2y_{t+1} + y_{t+2}]$$

CENTERED WINDOW

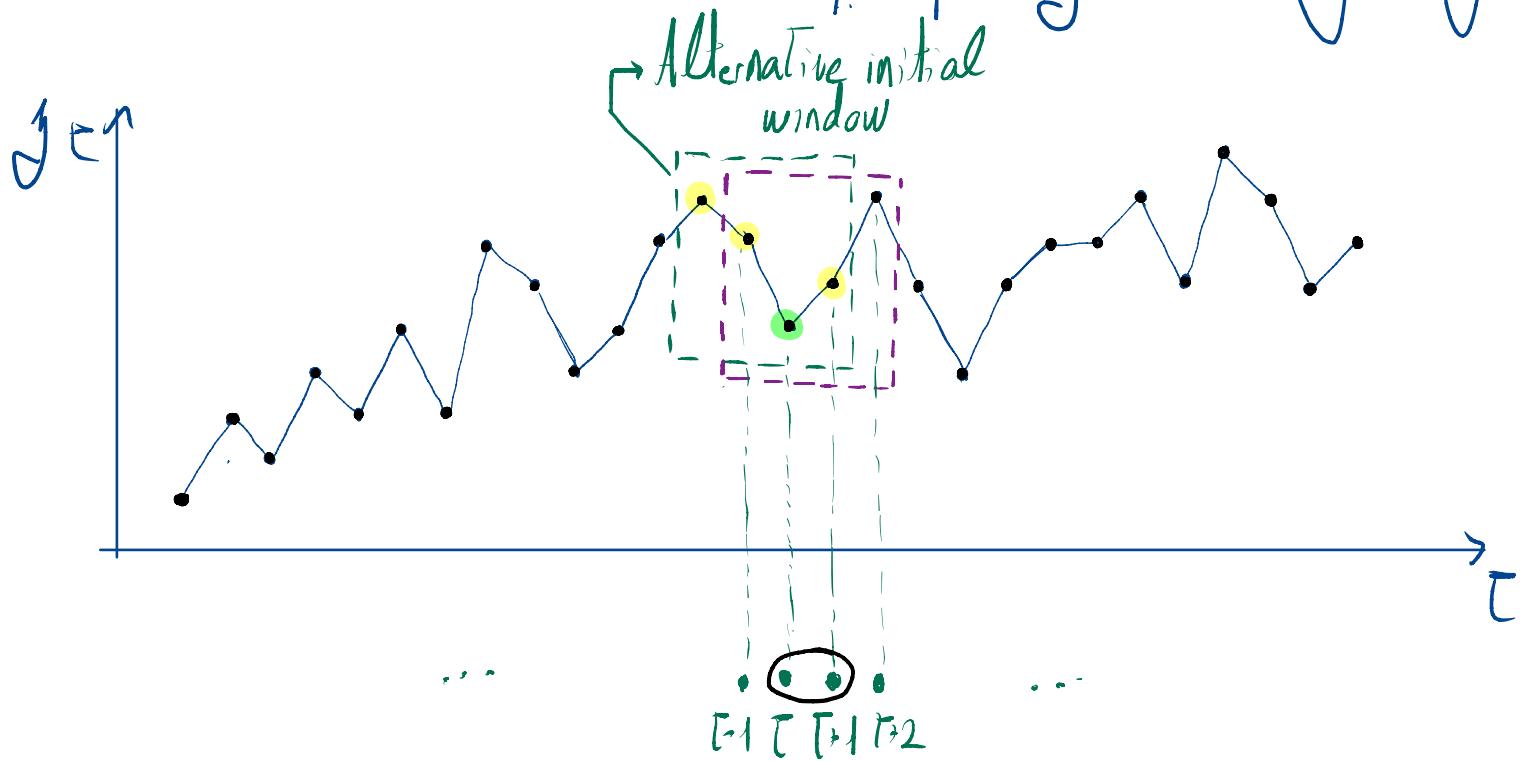
Weighted MA

$$\text{COEFFICIENTS: } a_j = \left[\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8} \right] \xrightarrow{j=-2} \sum_{j=-2}^{j=2} a_j = 1$$

$$2 \times 4 MA_t = \sum_{j=-2}^{j=2} a_j y_{t+j}$$

Moving Averages - CENTERED

Even NUMBER OF POINTS - example: quarterly data // average over year



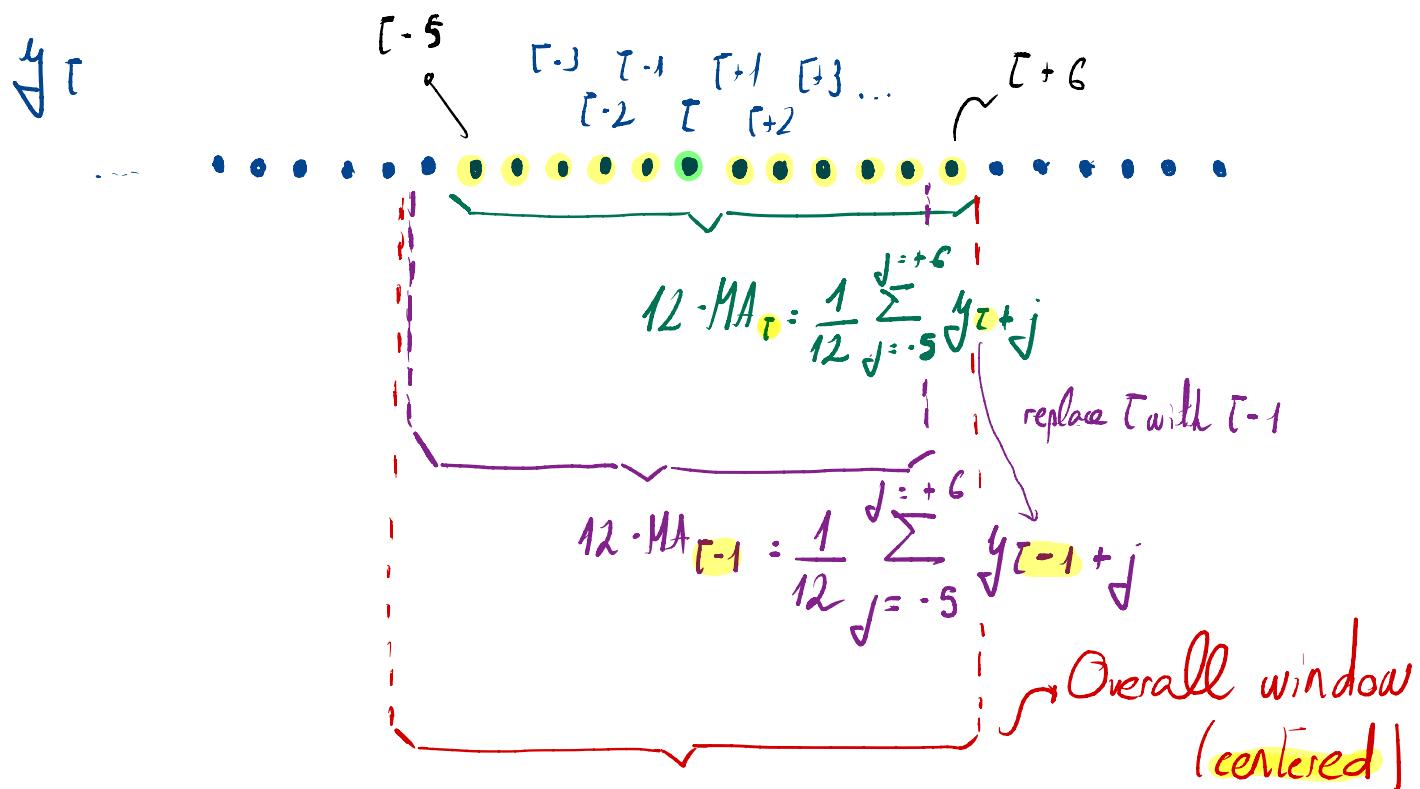
$$4MA_t = \frac{1}{4} [y_{t-2} + y_{t-1} + y_t + y_{t+1}]$$

$$2 \times 4MA_t = \frac{1}{2} [4MA_t + 4MA_{t+1}]$$

If you develop this formula, you will reach the exact same result

Moving Averages - Centered

EVEN NUMBER OF POINTS → EXAMPLE: monthly data → average over year



$$2 \times 12 \cdot MA_t = \frac{1}{2} [12 \cdot MA_{t-1} + 12 \cdot MA_t] =$$

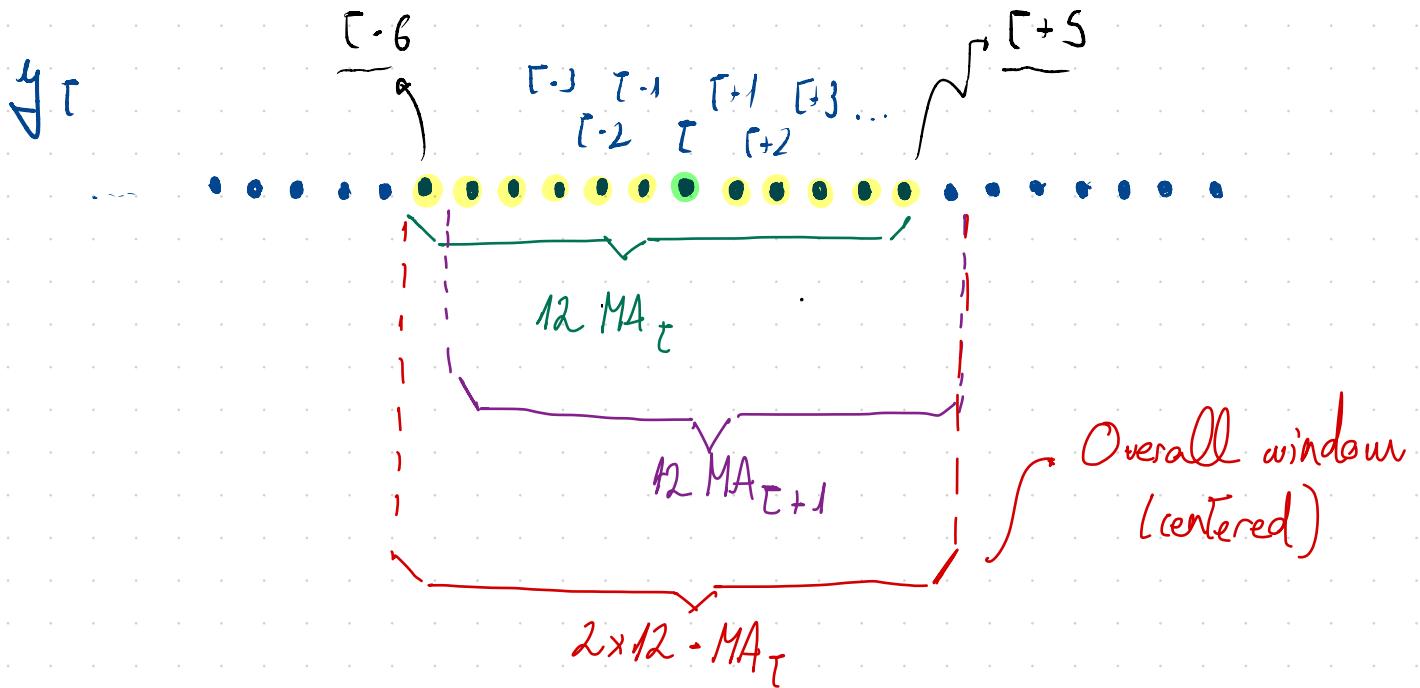
$$\begin{aligned} &= \frac{1}{2} \cdot \frac{1}{12} \left[y_{t-6} + \right. \\ &\quad \left. + 2[y_{t-5} + y_{t-4} + y_{t-3} + y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2} + y_{t+3} + y_{t+4} + y_{t+5}] + \right. \\ &\quad \left. + y_{t+6} \right] = \sum_{j=-6}^{j=6} a_j y_{t+j} \end{aligned}$$

All these points appear in both windows (=)

$$\left[\frac{1}{24}, \frac{1}{12}, \frac{1}{24} \right]$$

$$j = -6 \ -5 \ -4 \ -3 \ -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$$

$$\sum_{j=-6}^{j=6} a_j = 1$$



$$2 \times 12\text{ MA}_t = \frac{1}{2} [MA_t + MA_{t+1}] = \dots$$

If we picked the alternative initial window for MA_t , the formula needs to be adapted as shown above. But, again, the end result would be the same.