

# TIME SERIES RESIDUALS DIAGNOSTICS

## ① INNOVATION RESIDUALS ARE UNCORRELATED

$$\text{cov}(e_t, e_{t-s}) : \text{cov}(e_{t-j}, e_{t-j-s}) = 0$$

→ IS THERE INFORMATION IN THE RESIDUALS?

→ CHECK

• ACF Plot

• TEST → { Box-Pierce TEST  
Ljung-Box TEST } - Section 5.4 Book

→ SOLUTIONS: harder → { Introduce Regressors  
...  
CHANGE Model }

→ If a model does not satisfy ① + ②

↓  
it can be improved

## ② INNOVATION RESIDUALS HAVE 0 MEAN

$$E[e_t] = E[y_t] - E[\hat{y}_t] = 0 \leftrightarrow E[y_t] = E[\hat{y}_t]$$

→ ARE THE PREDICTIONS BIASED?

→ CHECK: compute the average of the residuals.

→ Formal statistical TEST ON THE MEAN (C-test)

→ SOLUTION: subtract the mean of the residuals from the forecasts

→ If a model satisfies ① + ②

↓  
it may be possible to improve it

(Multiple models may satisfy ① + ② and we need to select among them)

## ③ CONSTANT VARIANCE OF THE RESIDUALS (HOMOSKEDASTICITY)

$$\text{var}(e_t) = \text{var}(e_{t-1}) = \dots = \sigma^2$$

→ EASIER CALCULATION OF PREDICTION INTERVALS

→ CHECK

• ③ { → VISUAL INSPECTION (Time-plot) → for now  
→ TESTS } → Breusch-Pagan  
→ McLeod-Li } later

• ④ { → VISUAL INSPECTION { HISTOGRAM / KERNEL  
QQ-Plots  
Box-Plots (SYMMETRY) }

→ TESTS: Shapiro, Diagnóstico, Jarque-Bera...

→ SOLUTION

→ Box-Cox TRANSFORMATION

↳ DEAL WITH IT (alternatives to obtain prediction intervals)

→ CHANGE MODEL

→ If a model does not satisfy ③ + ④

↓  
Calculation of the prediction intervals more elaborate

⊕

Prediction INTERVALS MAY BECOME  
TOO WIDE (HETEROSENCOASTICITY)