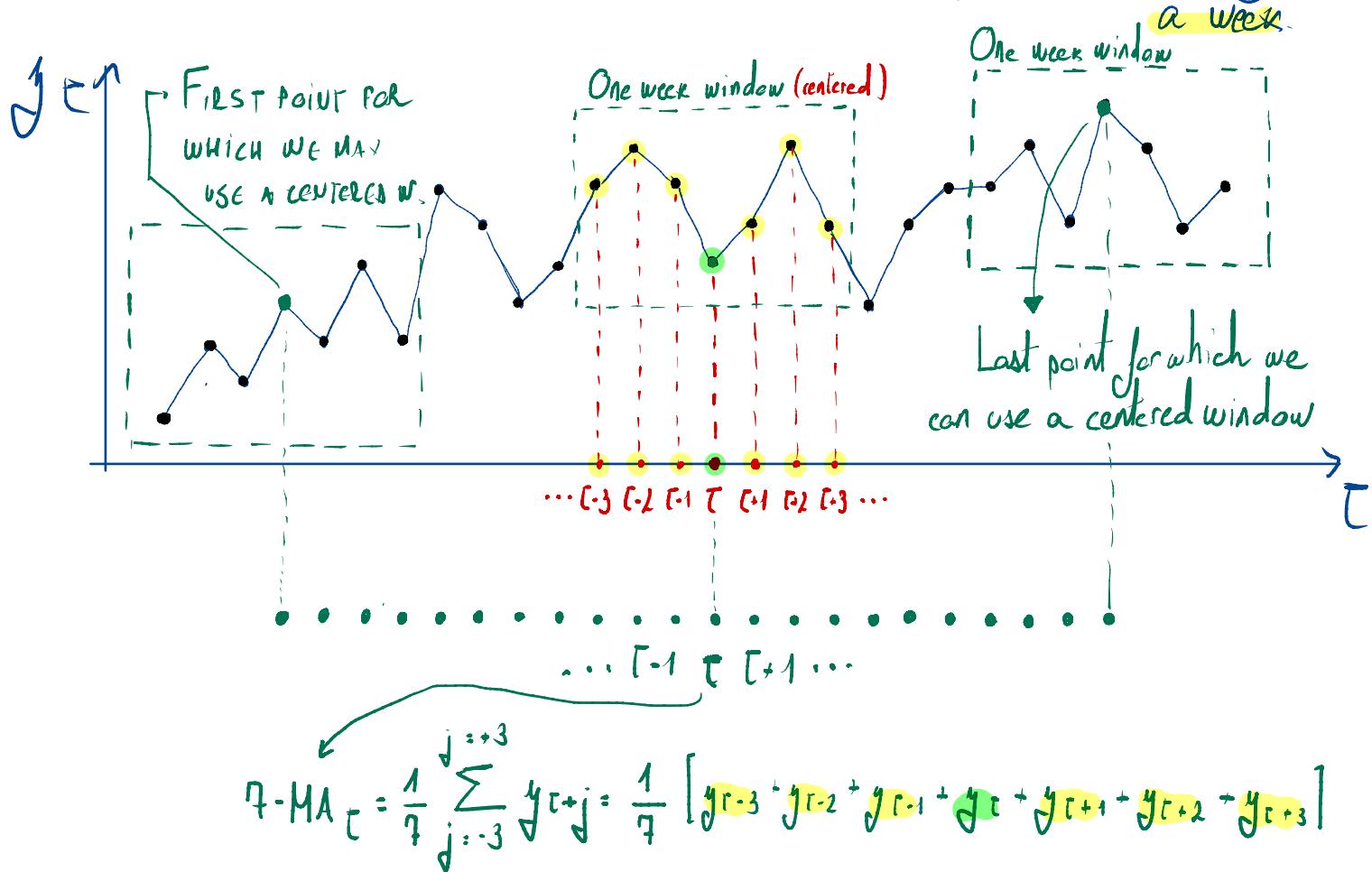


# Moving Averages - CENTERED

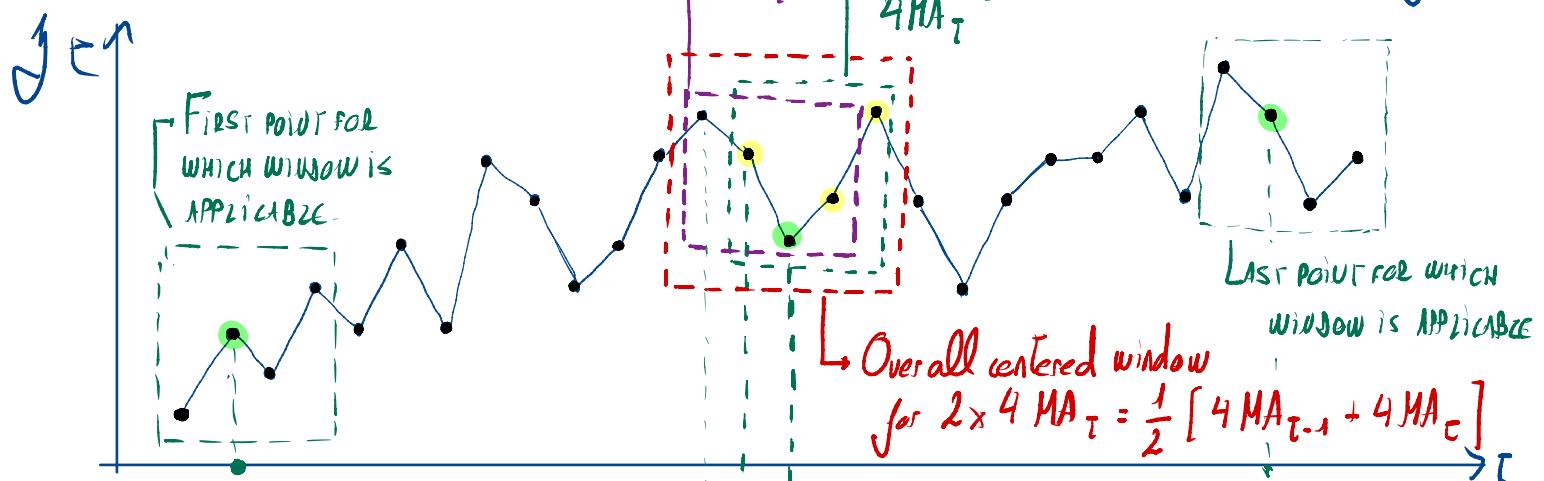
UNEVEN NUMBER OF POINTS - example: daily data → average over



- Daily data - 1 point = 1 day
  - Week has 7 days (uneven number)
- } Centered window of 7 points is directly applicable.

# Moving Averages - CENTERED

Even NUMBER OF POINTS - example: quarterly data → average over a year



$$\text{Overall centered window for } 2 \times 4 \text{ MA}_t = \frac{1}{2} [4 \text{ MA}_{t-1} + 4 \text{ MA}_t]$$

$$4 \cdot \text{MA}_t \cdot \frac{1}{4} \sum_{j=-2}^{j=+2} y_{t+j} = \frac{1}{4} [y_{t-2} + y_{t-1} + y_{t+1} + y_{t+2}]$$

Not centered!!!

$$\rightarrow \frac{1}{2} [4 \text{ MA}_{t-1} + 4 \text{ MA}_t] =$$

$$= \frac{1}{2} \left[ \underbrace{\frac{1}{4} [y_{t-2} + y_{t-1} + y_t + y_{t+1}]}_{4 \text{ MA}_{t-1}} + \underbrace{\frac{1}{4} [y_{t-1} + y_t + y_{t+1} + y_{t+2}]}_{4 \text{ MA}_t} \right]$$

$$= \frac{1}{8} [y_{t-2} + 2y_{t-1} + 2y_t + 2y_{t+1} + y_{t+2}]$$

CENTRED WINDOW

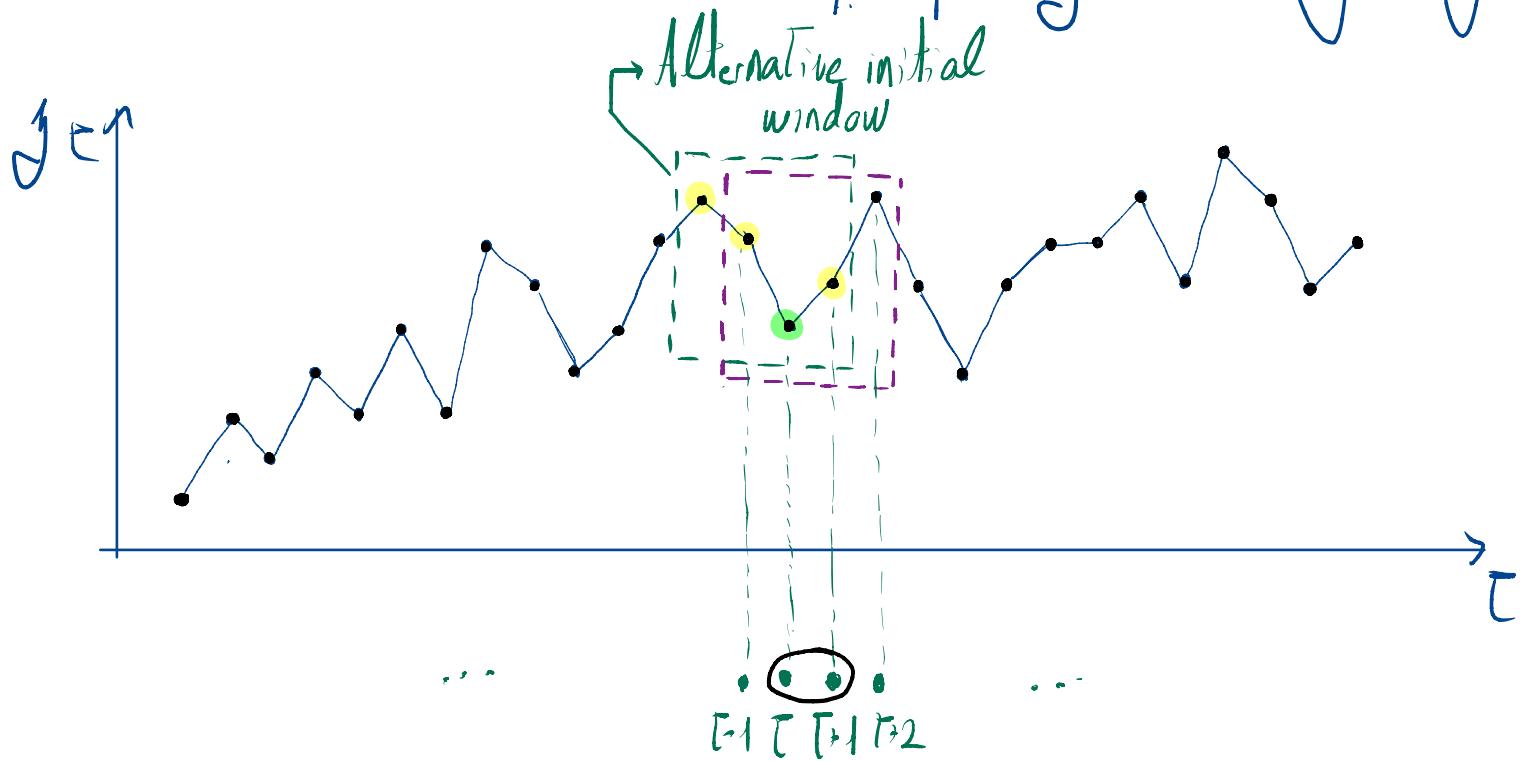
Weighted MA

**COEFFICIENTS:**  $a_j = \left[ \begin{array}{c} \frac{1}{8} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{8} \end{array} \right] \rightarrow \sum_{j=-2}^{j=2} a_j = 1$

$$2 \times 4 \text{ MA}_t = \sum_{j=-2}^{j=2} a_j y_{t+j}$$

# Moving Averages - CENTERED

Even NUMBER OF POINTS - example: quarterly data // average over year



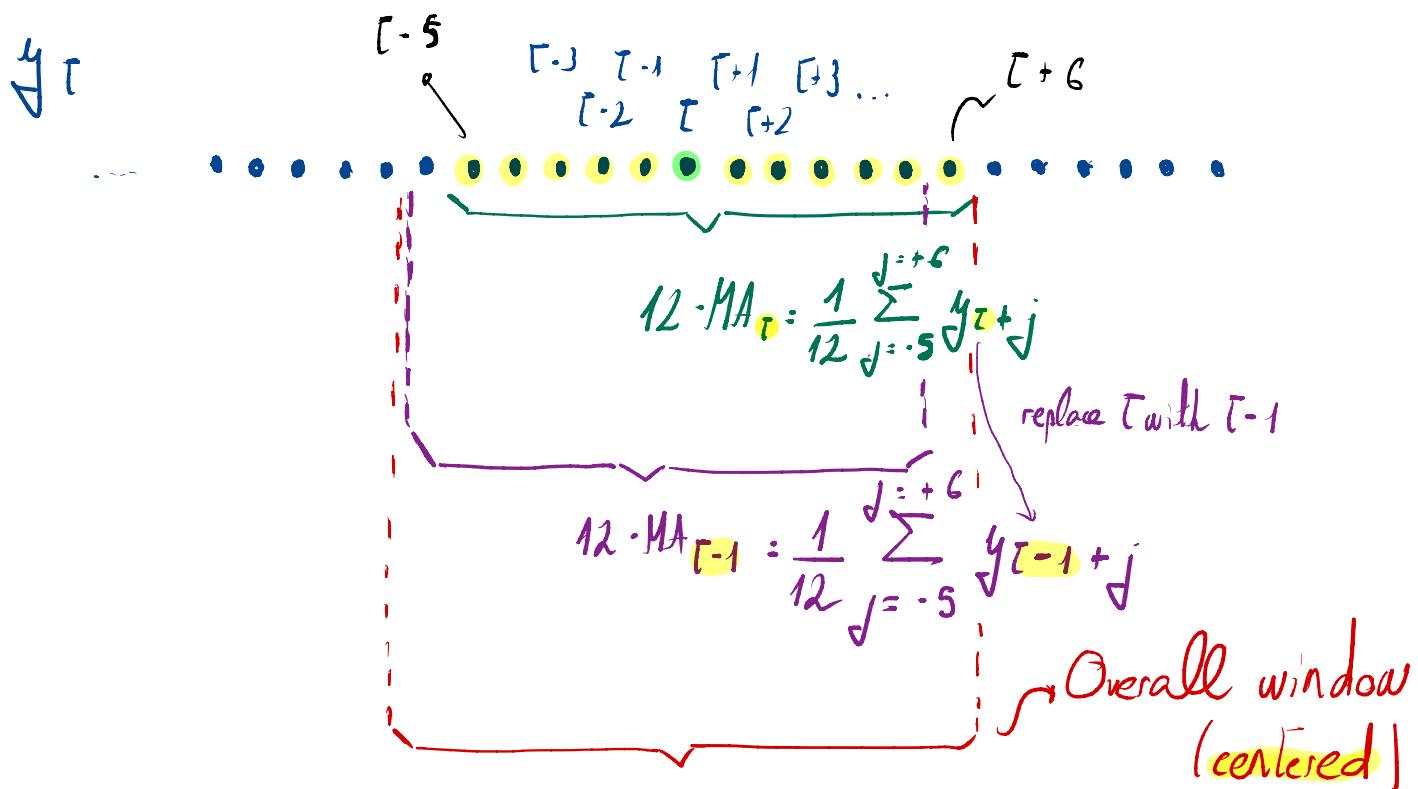
$$4MA_t = \frac{1}{4} [y_{t-2} + y_{t-1} + y_t + y_{t+1}]$$

$$2 \times 4MA_t = \frac{1}{2} [4MA_t + 4MA_{t+1}]$$

↳ Homework: develop this and show you reach the exact same formula as with the other initial window.

# Moving Averages - Centered

EVEN NUMBER OF POINTS → EXAMPLE: monthly data → average over year



$$2 \times 12 \cdot MA_t = \frac{1}{2} [12 \cdot MA_{t-1} + 12 \cdot MA_t] =$$

$$\begin{aligned} &= \frac{1}{2} \cdot \frac{1}{12} \left[ y_{t-6} + \right. \\ &\quad \left. + 2[y_{t-5} + y_{t-4} + y_{t-3} + y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2} + y_{t+3} + y_{t+4} + y_{t+5}] + \right. \\ &\quad \left. + y_{t+6} \right] = \sum_{j=-6}^{j=6} a_j y_{t+j} \end{aligned}$$

All these points appear in both windows (=)

$$\left[ \frac{1}{24}, \frac{1}{12}, \frac{1}{24} \right]$$

$$j = -6 \ -5 \ -4 \ -3 \ -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$$

$$\sum_{j=-6}^{j=6} a_j = 1$$



QUESTION : how does the formula for  $2 \times MA_t$  change if we choose the alternative initial window?

$$2 \times MA_t = \frac{1}{2} [$$