

Modelamiento Avanzado con Programación Entera Mixta

Parte 3/3

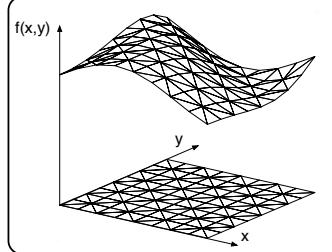
Juan Pablo Vielma

University of Pittsburgh

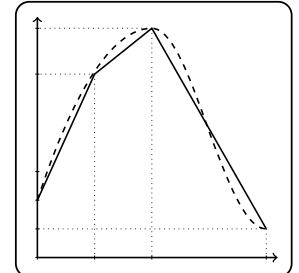
Universidad de Antofagasta, 2011 – Antofagasta, Chile

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Funciones Lineales por Trazos (FLT)



Aproximación

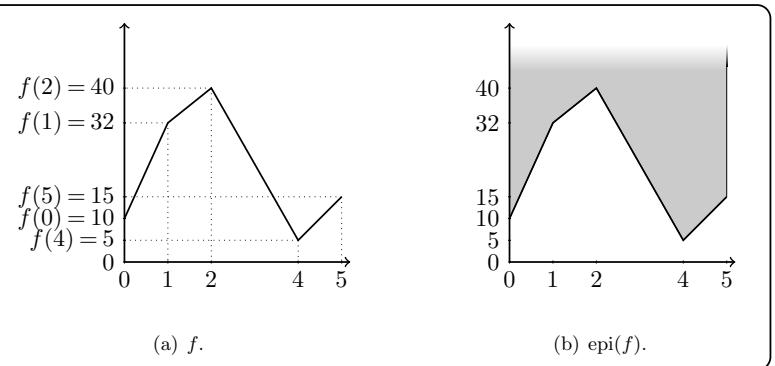


Economías de escala

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Modelar Funciones = Epigrafo

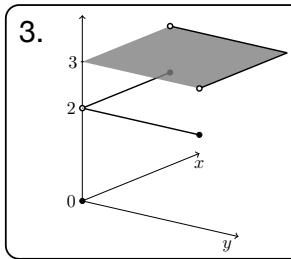
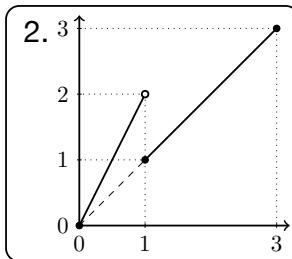
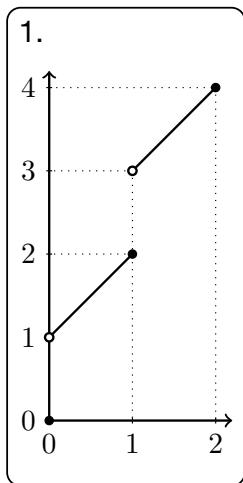


● Ejemplo:

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Costos Fijos y Descuentos



1. Costos Fijos.
2. Descuentos (e.g. Remates).
3. Descuentos en costos Fijos

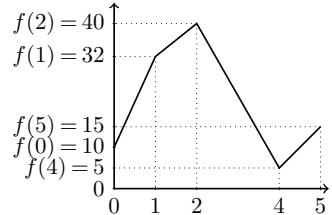
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Funciones Lineales por Trazos



$$f(x) := \begin{cases} 22x + 10 & x \in [0, 1] \\ 8x + 24 & x \in [1, 2] \\ -17.5x + 75 & x \in [2, 4] \\ 10x - 35 & x \in [4, 5] \end{cases}$$

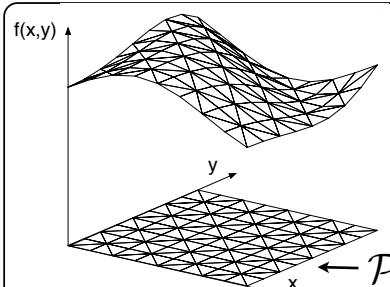
DEFINITION 1. Piecewise Linear $f : D \subset \mathbb{R}^n \rightarrow \mathbb{R}$:

$$f(x) := \begin{cases} m_P x + c_P & x \in P \quad \forall P \in \mathcal{P}. \end{cases}$$

for finite family of polytopes \mathcal{P} such that $D = \bigcup_{P \in \mathcal{P}} P$

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Funciones Lineales por Trazos



$$f(x, y) := \begin{cases} 0.48x + 0.03y + 6 & (x, y) \in P_1 \\ \vdots & \vdots \\ -0.4x - 0.04y + 8.45 & (x, y) \in P_{128} \end{cases}$$

$$P_1 := \{(x, y) \in \mathbb{R} : y \geq 0, x \leq 1, y - x \leq 0\}$$

$$\vdots$$

$$P_{128} := \{(x, y) \in \mathbb{R} : y \geq 0, x \geq 7, x + y \leq 8\}$$

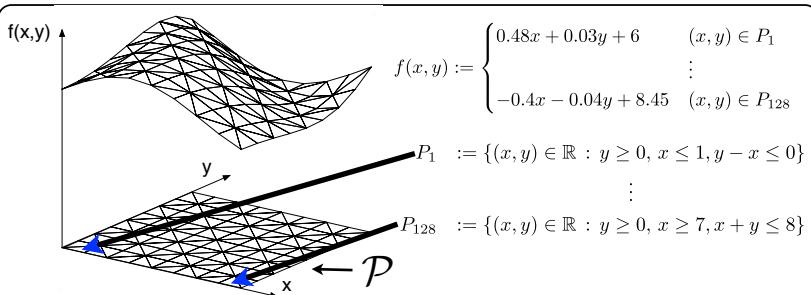
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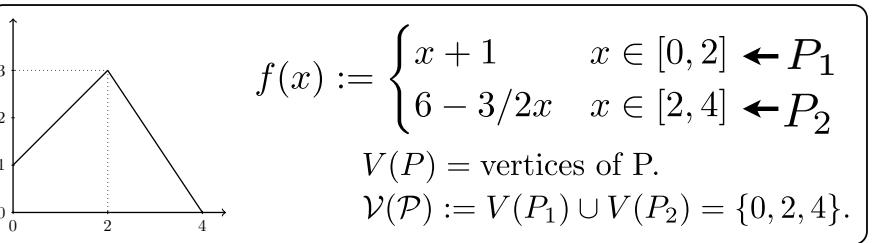
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5

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Formulacion Tradicional 1 variable



$$f(x) := \begin{cases} x + 1 & x \in [0, 2] \leftarrow P_1 \\ 6 - 3/2x & x \in [2, 4] \leftarrow P_2 \end{cases}$$

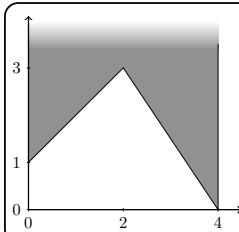
$V(P)$ = vertices of P .

$$\mathcal{V}(\mathcal{P}) := V(P_1) \cup V(P_2) = \{0, 2, 4\}.$$

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6

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Formulacion Tradicional 1 variable



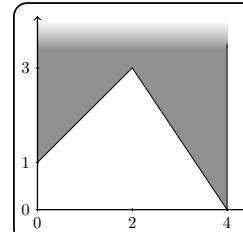
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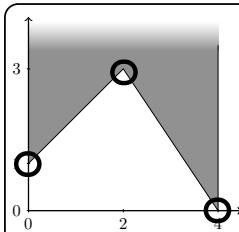
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idea: write $(x, y) \in \text{epi}(f)$
as convex combination of
 $(v, f(v))$ for $v \in \mathcal{V}(\mathcal{P})$.

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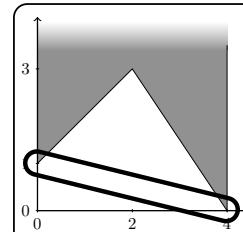
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idea: write $(x, y) \in \text{epi}(f)$
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 $(v, f(v))$ for $v \in \mathcal{V}(\mathcal{P})$.

$$\begin{aligned} x &= 0\lambda_0 + 2\lambda_2 + 4\lambda_4 \\ z &\geq 1\lambda_0 + 3\lambda_2 + 0\lambda_4 \\ 1 &= \lambda_0 + \lambda_2 + \lambda_4, \quad \lambda_0, \lambda_2, \lambda_4 \geq 0 \end{aligned}$$

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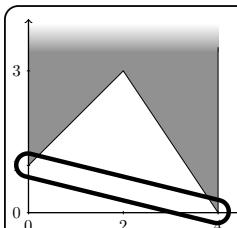
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λ_0 and λ_4 cannot be nonzero at the same time.

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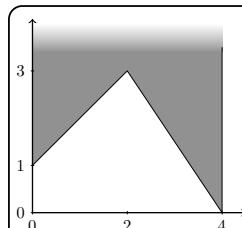
$$1 = \lambda_0 + \lambda_2 + \lambda_4, \quad \lambda_0, \lambda_2, \lambda_4 \geq 0$$

$$\begin{aligned} \lambda_0 &\leq y_{P_1}, \quad \lambda_2 \leq y_{P_1} + y_{P_2}, \quad \lambda_4 \leq y_{P_2} \\ 1 &= y_{P_1} + y_{P_2}, \quad y_{P_1}, y_{P_2} \in \{0, 1\} \end{aligned}$$

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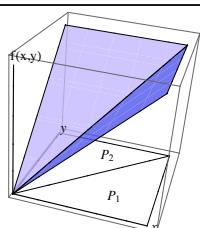
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Formulacion Tradicional 2 variables

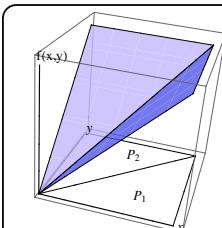


$$f(x, y) := \begin{cases} x & (x, y) \in P_1 \\ y & (x, y) \in P_1 \end{cases}$$

$$\begin{aligned} P_1 &:= \{(x, y) : x \leq 1, 0 \leq y \leq x\}, \quad P_2 := \dots \\ \mathcal{V}(\mathcal{P}) &:= \{(0,0), (1,0), (0,1), (1,1)\}. \end{aligned}$$

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$$x = 0\lambda_{(0,0)} + 1\lambda_{(1,0)} + 0\lambda_{(0,1)} + 1\lambda_{(1,1)}$$

$$y = 0\lambda_{(0,0)} + 0\lambda_{(1,0)} + 1\lambda_{(0,1)} + 1\lambda_{(1,1)}$$

$$z \geq 0\lambda_{(0,0)} + 1\lambda_{(1,0)} + 1\lambda_{(0,1)} + 1\lambda_{(1,1)}$$

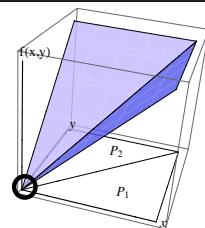
$$1 = \lambda_{(0,0)} + \lambda_{(1,0)} + \lambda_{(0,1)} + \lambda_{(1,1)}, \quad \lambda_{(0,0)}, \dots, \lambda_{(1,1)} \geq 0$$

$$\dots \quad y_{P_1} + y_{P_1} = 1, \quad y_{P_1}, y_{P_1} \in \{0, 1\}$$

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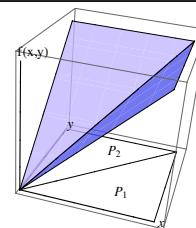
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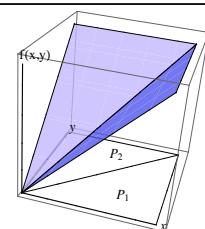
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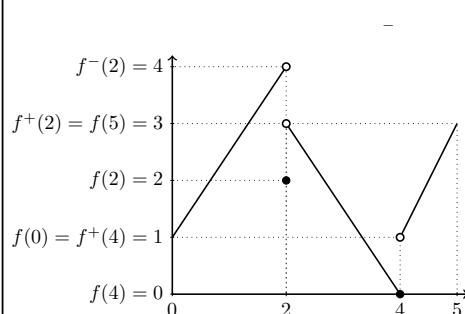
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Semi-continuidad Inferior

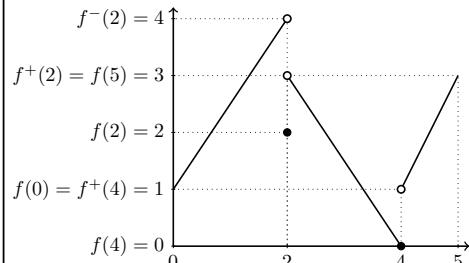


$$f^-(d) = \lim_{\substack{x \rightarrow d \\ x \leq d}} f(x)$$

$$f^+(d) = \lim_{\substack{x \rightarrow d \\ x \geq d}} f(x).$$

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Semi-continuidad Inferior

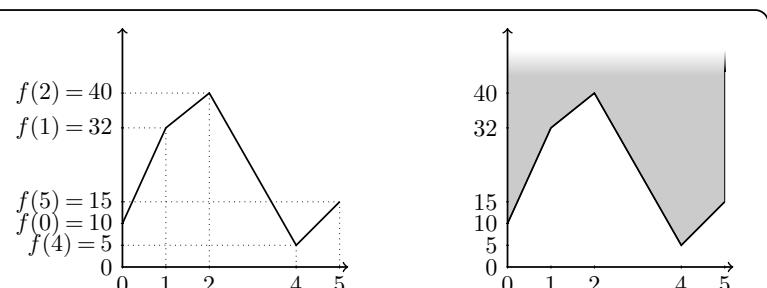


● Semi-continuidad Inferior:

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Semi-continua = epígrafo cerrado



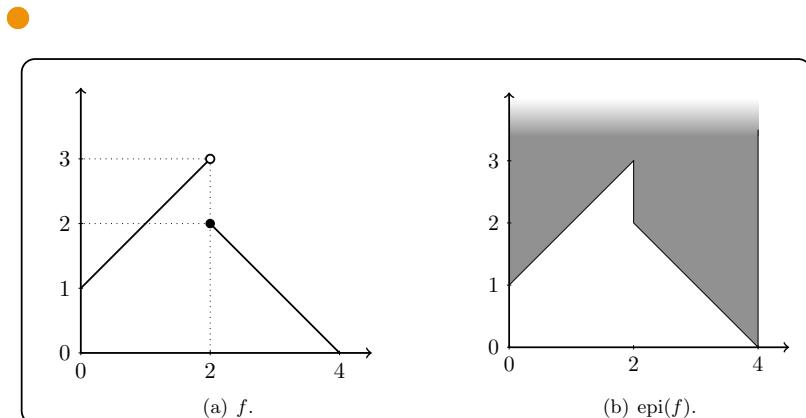
(a) f .

(b) $\text{epi}(f)$.

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Semi-continua = epígrafo cerrado



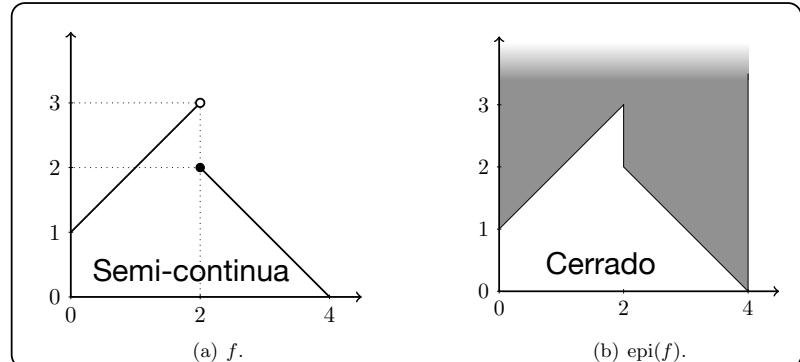
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9

Semi-continua = epígrafo cerrado



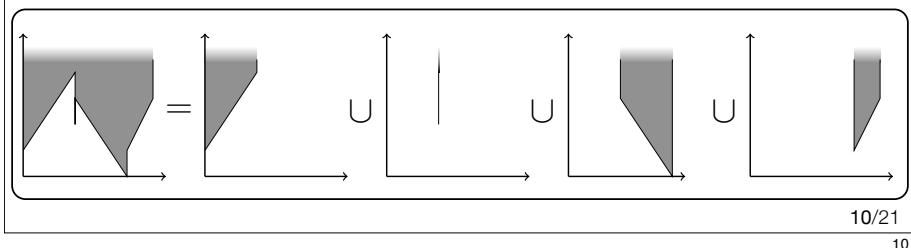
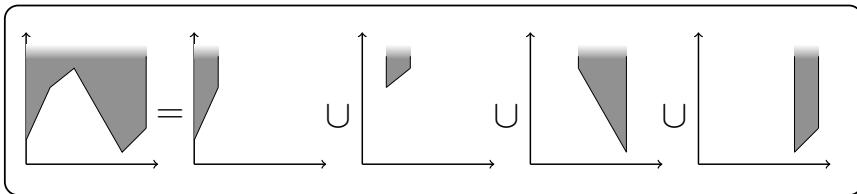
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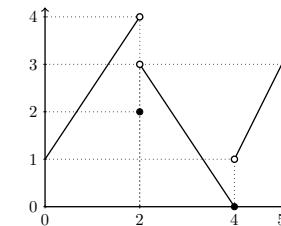
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Epígrafe de FLT es unión the PL's



Funciones LT semi-continuas



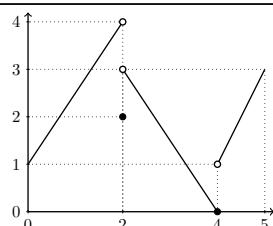
$$f(x) := \begin{cases} 1.5x + 1 & x \in [0, 2) \\ 2 & x \in [2, 2] \\ -1.5x + 6 & x \in (2, 4] \\ 2x - 7 & x \in (4, 5] \end{cases}$$

$$f(x) := \begin{cases} m_P x + c_P & x \in P \quad \forall P \in \mathcal{P} \end{cases}$$

$P = \{x \in \mathbb{R}^n : a_i x \leq b_i \forall i \in \{1, \dots, p\}, a_i x < b_i \forall i \in \{p, \dots, m\}\}$

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Funciones LT semi-continuas



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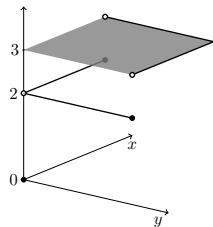
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Funciones LT semi-continuas



$$f(x, y) := \begin{cases} 3 & (x, y) \in (0, 1]^2 \\ 2 & (x, y) \in \{(x, y) \in \mathbb{R}^2 : x = 0, y > 0\} \\ 2 & (x, y) \in \{(x, y) \in \mathbb{R}^2 : y = 0, x > 0\} \\ 0 & (x, y) \in \{(0, 0)\}. \end{cases}$$

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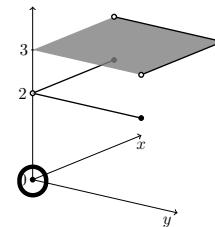
Finite family of copolytopes

$$P = \{x \in \mathbb{R}^n : a_i x \leq b_i \forall i \in \{1, \dots, p\}, a_i x < b_i \forall i \in \{p, \dots, m\}\}$$

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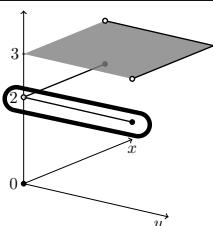
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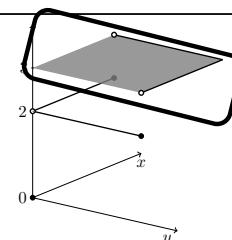
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Funciones LT semi-continuas



$$f(x, y) := \begin{cases} 3 & (x, y) \in (0, 1]^2 \\ 2 & (x, y) \in \{(x, y) \in \mathbb{R}^2 : x = 0, y > 0\} \\ 2 & (x, y) \in \{(x, y) \in \mathbb{R}^2 : y = 0, x > 0\} \\ 0 & (x, y) \in \{(0, 0)\}. \end{cases}$$

$$f(x) := \begin{cases} m_P x + c_P & x \in P \quad \forall P \in \mathcal{P} \end{cases}$$

Finite family of copolytopes

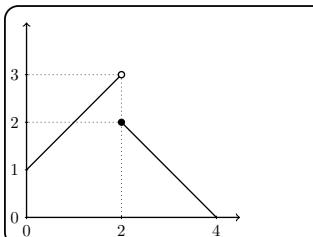
$$P = \{x \in \mathbb{R}^n : a_i x \leq b_i \forall i \in \{1, \dots, p\}, a_i x < b_i \forall i \in \{p, \dots, m\}\}$$

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Discontinuous Case

OOOO

Modelo para FLT semi-continuas

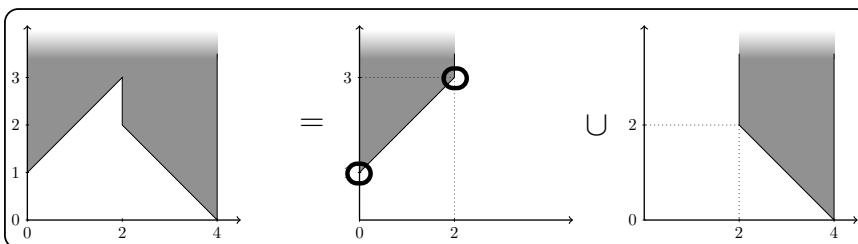
(DCC)

$$f(x) := \begin{cases} x + 1 & x \in [0, 2) \\ 4 - x & x \in [2, 4] \end{cases}$$

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Discontinuous Case

OOOO

Modelo para FLT semi-continuas

(DCC)

$$P_1 \downarrow$$

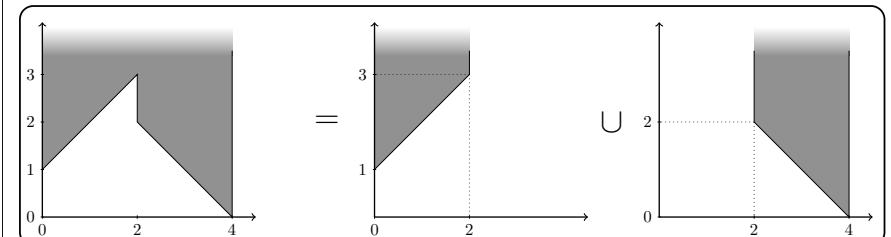
$$f(x) := \begin{cases} x + 1 & x \in [0, 2) \\ 4 - x & x \in [2, 4] \end{cases}$$

$$\begin{aligned} x &= 0\lambda_{P_1,0} + 2\lambda_{P_1,2} \\ z &\geq 1\lambda_{P_1,0} + 3\lambda_{P_1,2} \\ 1 &= \lambda_{P_1,0} + \lambda_{P_1,2}, \quad \lambda_{P_1,0}, \lambda_{P_1,2} \geq 0 \end{aligned}$$

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12

Discontinuous Case

OOOO

Modelo para FLT semi-continuas

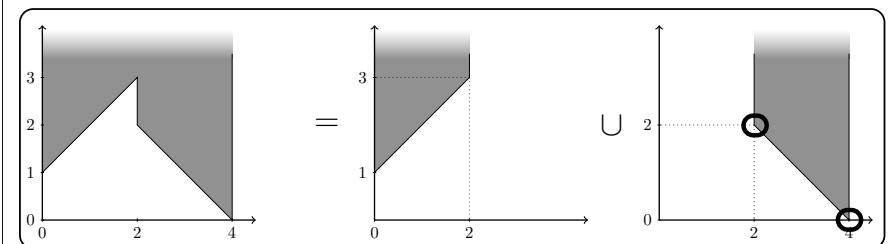
(DCC)

$$f(x) := \begin{cases} x + 1 & x \in [0, 2) \\ 4 - x & x \in [2, 4] \end{cases}$$

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Discontinuous Case

OOOO

Modelo para FLT semi-continuas

(DCC)

$$x =$$

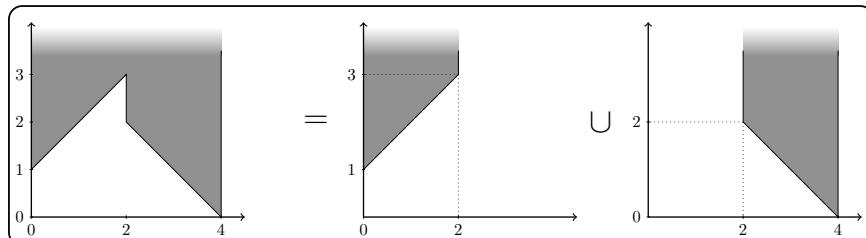
$$z \geq$$

$$P_2 \uparrow$$

$$f(x) := \begin{cases} x + 1 & x \in [0, 2) \\ 4 - x & x \in [2, 4] \end{cases}$$

$$\begin{aligned} 2\lambda_{P_2,2} + 4\lambda_{P_2,4} \\ 2\lambda_{P_2,2} + 0\lambda_{P_2,4} \\ 1 = \lambda_{P_2,2} + \lambda_{P_2,4}, \quad \lambda_{P_2,2}, \lambda_{P_2,4} \geq 0 \end{aligned}$$

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12

Modelo para FLT semi-continuas

(DCC)

$$f(x) := \begin{cases} x+1 & x \in [0, 2) \\ 4-x & x \in [2, 4] \end{cases}$$

$$x = 0\lambda_{P_1,0} + 2\lambda_{P_1,2} + 2\lambda_{P_2,2} + 4\lambda_{P_2,4}$$

$$z \geq 1\lambda_{P_1,0} + 3\lambda_{P_1,2} + 2\lambda_{P_2,2} + 0\lambda_{P_2,4}$$

$$1 = \lambda_{P_1,0} + \lambda_{P_1,2}, \quad \lambda_{P_1,0}, \lambda_{P_1,2} \geq 0$$

$$1 = \lambda_{P_2,2} + \lambda_{P_2,4}, \quad \lambda_{P_2,2}, \lambda_{P_2,4} \geq 0$$

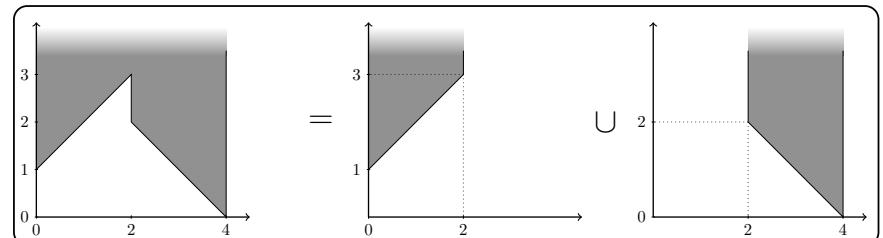
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Intermedio

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Modelo para FLT semi-continuas

(DCC)

$$f(x) := \begin{cases} x+1 & x \in [0, 2) \\ 4-x & x \in [2, 4] \end{cases}$$

$$x = 0\lambda_{P_1,0} + 2\lambda_{P_1,2} + 2\lambda_{P_2,2} + 4\lambda_{P_2,4}$$

$$z \geq 1\lambda_{P_1,0} + 3\lambda_{P_1,2} + 2\lambda_{P_2,2} + 0\lambda_{P_2,4}$$

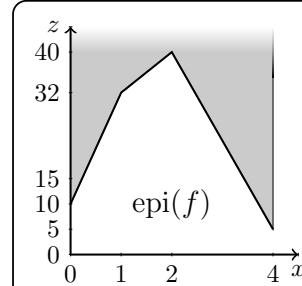
$$y_{P_1} = \lambda_{P_1,0} + \lambda_{P_1,2}, \quad \lambda_{P_1,0}, \lambda_{P_1,2} \geq 0$$

$$y_{P_2} = \lambda_{P_2,2} + \lambda_{P_2,4}, \quad \lambda_{P_2,2}, \lambda_{P_2,4} \geq 0$$

$$1 = y_{P_1} + y_{P_2}, \quad y_{P_1}, y_{P_2} \in \{0, 1\}$$

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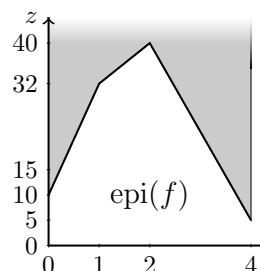
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Formulación Tradicional de FLT

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Formulación Tradicional de FLT



$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

$$\sum_{i=1}^4 \lambda_i = 1, \quad \lambda_i \geq 0 \quad \forall i \in \{1, \dots, 4\}$$

$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

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Disjunciones Especiales

$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

$$\lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1$$

$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

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Disjunciones Especiales

$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

$$\left\{ \lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1 \right\} =: \Delta^4$$

$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

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Disjunciones Especiales

$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

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$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

$$y_1 = 1 \Rightarrow \lambda \in P_1 := \{\lambda \in \Delta^4 : \lambda_3, \lambda_4 \leq 0\}$$

$$y_2 = 1 \Rightarrow \lambda \in P_2 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_4 \leq 0\}$$

$$y_3 = 1 \Rightarrow \lambda \in P_3 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_2 \leq 0\}$$

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15

Disjunciones Especiales

$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

$$\left\{ \lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1 \right\} =: \Delta^4$$

$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

$$\left. \begin{array}{l} \left\{ \lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1 \right\} =: \Delta^4 \\ \lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3, \\ \lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3 \end{array} \right\} \lambda \in \bigcup_{i=1}^4 P_i$$

$$y_1 = 1 \Rightarrow \lambda \in P_1 := \{\lambda \in \Delta^4 : \lambda_3, \lambda_4 \leq 0\}$$

$$y_2 = 1 \Rightarrow \lambda \in P_2 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_4 \leq 0\}$$

$$y_3 = 1 \Rightarrow \lambda \in P_3 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_2 \leq 0\}$$

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15

Disjunciones Especiales

$$x = 0\lambda_1 + 1\lambda_2 + 2\lambda_3 + 4\lambda_4$$

$$z \geq 10\lambda_1 + 32\lambda_2 + 40\lambda_3 + 5\lambda_4$$

$$\left\{ \lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1 \right\} =: \Delta^4$$

$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3,$$

$$\lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3$$

$$\left. \begin{array}{l} \left\{ \lambda \in [0, 1]^4 : \sum_{i=1}^4 \lambda_i = 1 \right\} =: \Delta^4 \\ \lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 + y_3, \\ \lambda_4 \leq y_3, \quad \sum_{i=1}^3 y_i = 1, \quad y \in \{0, 1\}^3 \end{array} \right\} \lambda \in \bigcup_{i=1}^4 P_i$$

$$y_1 = 1 \Rightarrow \lambda \in P_1 := \{\lambda \in \Delta^4 : \lambda_3, \lambda_4 \leq 0\}$$

$$y_2 = 1 \Rightarrow \lambda \in P_2 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_4 \leq 0\}$$

$$y_3 = 1 \Rightarrow \lambda \in P_3 := \{\lambda \in \Delta^4 : \lambda_1, \lambda_2 \leq 0\}$$

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Disjunciones Especiales (DE)

$$\lambda \in \bigcup_{i=1}^m P(F_i)$$

$$P(F_i) := \{\lambda \in \Delta^n : \lambda_j \leq 0 \forall j \in F_i\}$$

$$\Delta^n := \left\{ \lambda \in [0, 1]^n : \sum_{i=1}^n \lambda_i = 1 \right\}$$

● SOS1:

● SOS2:

● Funciones Lineales por trazos.

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Formulación Estándar

$$x \in \bigcup_{i=1}^m P_i \subset \mathbb{R}^n$$

$$P_i := \{x \in \mathbb{R} : A^i x \leq b^i\}$$

$$A^i \in \mathbb{R}^{r \times n}, b^i \in \mathbb{R}^{r_i}$$

$$A^i x^i \leq b^i y_i \quad \forall i$$

$$\sum_{i=1}^m x^i = x$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^M$$



- Sharp y localmente Ideal.
- variables y restricciones.

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Formulación Estándar para DE

$$\lambda \in \bigcup_{i=1}^m P(F_i) \quad P(F_i) := \{\lambda \in \Delta^n : \lambda_j \leq 0 \forall j \notin F_i\}$$

$$\Delta^n := \left\{ \lambda \in [0, 1]^n : \sum_{i=1}^n \lambda_i = 1 \right\}$$

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

- Sharp y localmente Ideal.

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SOS1, SOS2 and Piecewise Linear

Eliminar copias de en formulacion

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

SOS1, SOS2 and Piecewise Linear

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$$\sum_{j=1}^n \lambda_j^i = y_i$$

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$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

$$\sum_{j=1}^n \lambda_j = 1,$$

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Eliminar copias de λ en formulacion

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda \geq 0,$$

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Eliminar copias de λ en formulacion

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda \geq 0, \quad \lambda_j \leq \sum_{i:j \notin F_i} y_i, \quad \sum_{i=1}^m y_i = 1, \quad y \in \{0, 1\}^m$$

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Eliminar copias de λ en formulacion

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda \geq 0, \quad \lambda_j \leq \sum_{i:j \notin F_i} y_i,$$

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Eliminar copias de λ en formulacion

$$\sum_{j=1}^n \lambda_j^i = y_i$$

$$0 \leq \lambda_j^i \leq y_i \quad \forall j \notin F_i$$

$$0 \leq \lambda_j^i \leq 0 \quad \forall j \in F_i$$

$$\sum_{i=1}^m \lambda^i = \lambda$$

$$\sum_{i=1}^m y_i = 1$$

$$y \in \{0, 1\}^m$$

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda \geq 0, \quad \lambda_j \leq \sum_{i:j \notin F_i} y_i, \quad \sum_{i=1}^m y_i = 1, \quad y \in \{0, 1\}^m$$

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Eliminar copias de en formulacion

$$\begin{aligned}\sum_{j=1}^n \lambda_j^i &= y_i \\ 0 \leq \lambda_j^i &\leq y_i \quad \forall j \notin F_i \\ 0 \leq \lambda_j^i &\leq 0 \quad \forall j \in F_i\end{aligned}$$

$$\begin{aligned}\sum_{i=1}^m \lambda^i &= \lambda \\ \sum_{i=1}^m y_i &= 1 \\ y &\in \{0, 1\}^m\end{aligned}$$

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda \geq 0, \quad \lambda_j \leq \sum_{i:j \notin F_i} y_i, \quad \sum_{i=1}^m y_i = 1, \quad y \in \{0, 1\}^m$$

- Formulación Tradicional para FLT
- Sharp pero no localmente ideal.

19/21
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Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}, S_1 = \{1\}, S_2 = \{2\}, S_3 = \{3\}$.20/21
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Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}, S_1 = \{1\}, S_2 = \{2\}, S_3 = \{3\}$.

| i | S_i | $B(i)$ |
|-----|-------|--------|
| 0 | {0} | 0 0 |
| 1 | {1} | 1 0 |
| 2 | {2} | 0 1 |
| 3 | {3} | 1 1 |

$w_1 w_2 \in \{0, 1\}$

- Injective function:
- Variables:
- Idea:

20/21
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Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}, S_1 = \{1\}, S_2 = \{2\}, S_3 = \{3\}$.

| i | S_i | $B(i)$ |
|-----|-------|--------|
| 0 | {0} | 0 0 |
| 1 | {1} | 1 0 |
| 2 | {2} | 0 1 |
| 3 | {3} | 1 1 |

$w_1 w_2 \in \{0, 1\}$

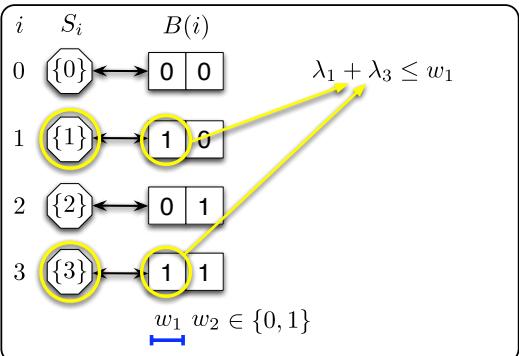
- Injective function:
- Variables:
- Idea:

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20

Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}$, $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$.



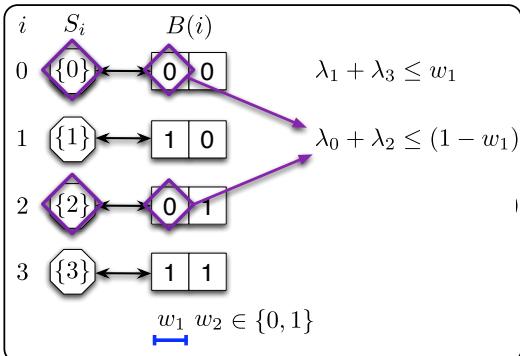
- Injective function:
- Variables:
- Idea:

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20

Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}$, $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$.



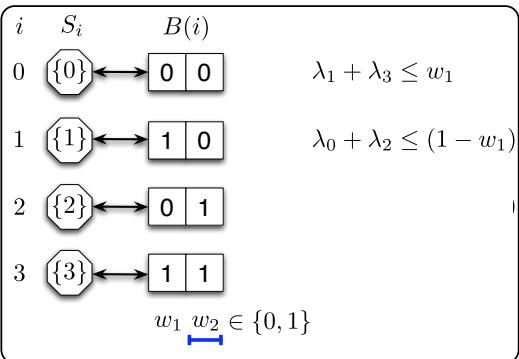
- Injective function:
- Variables:
- Idea:

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20

Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

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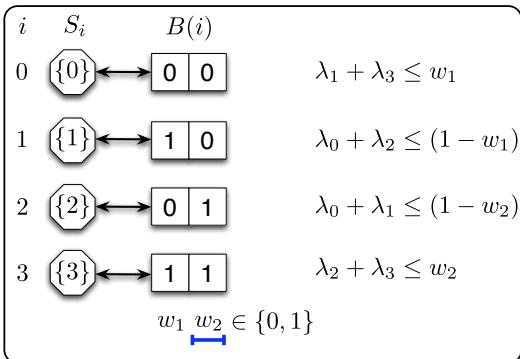
- Injective function:
- Variables:
- Idea:

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20

Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}$, $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$.



- Injective function:
- Variables:
- Idea:

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20

Formulacion Logaritmica para SOS1

$$\sum_{j=0}^3 \lambda_j = 1, \quad \lambda_0, \lambda_1, \lambda_2, \lambda_3 \geq 0, \text{ at most 1 } \lambda_j \text{ is nonzero.}$$

Allowed sets: $S_0 = \{0\}$, $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$.

| i | S_i | $B(i)$ |
|-----|-------------|---------|
| 0 | $\{\cdot\}$ | $0 \ 0$ |
| 1 | $\{1\}$ | $1 \ 0$ |
| 2 | $\{2\}$ | $0 \ 1$ |
| 3 | $\{3\}$ | $1 \ 1$ |

$w_1, w_2 \in \{0, 1\}$

● In general:

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Formulacion Logaritmica para SOS2

$$\sum_{j=0}^4 \lambda_j = 1, \quad \lambda_0, \dots, \lambda_4 \geq 0, \text{ only 2 adjacent } \lambda_j \text{'s are nonzero.}$$

Allowed sets: $S_i = \{i, i+1\}$ for $i \in \{0, \dots, 3\}$.

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Formulacion Logaritmica para SOS2

$$\sum_{j=0}^4 \lambda_j = 1, \quad \lambda_0, \dots, \lambda_4 \geq 0, \text{ only 2 adjacent } \lambda_j \text{'s are nonzero.}$$

Allowed sets: $S_i = \{i, i+1\}$ for $i \in \{0, \dots, 3\}$.

| i | S_i | $B(i)$ |
|-----|------------|---------|
| 0 | $\{0, 1\}$ | $0 \ 0$ |
| 1 | $\{1, 2\}$ | $1 \ 0$ |
| 2 | $\{2, 3\}$ | $0 \ 1$ |
| 3 | $\{3, 4\}$ | $1 \ 1$ |

$w_1, w_2 \in \{0, 1\}$

● Injective function:

● Variables:

● Idea:

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Formulacion Logaritmica para SOS2

$$\sum_{j=0}^4 \lambda_j = 1, \quad \lambda_0, \dots, \lambda_4 \geq 0, \text{ only 2 adjacent } \lambda_j \text{'s are nonzero.}$$

Allowed sets: $S_i = \{i, i+1\}$ for $i \in \{0, \dots, 3\}$.

| i | S_i | $B(i)$ |
|-----|------------|---------|
| 0 | $\{0, 1\}$ | $0 \ 0$ |
| 1 | $\{1, 2\}$ | $1 \ 0$ |
| 2 | $\{2, 3\}$ | $0 \ 1$ |
| 3 | $\{3, 4\}$ | $1 \ 1$ |

$w_1, w_2 \in \{0, 1\}$

● Injective function:

● Variables:

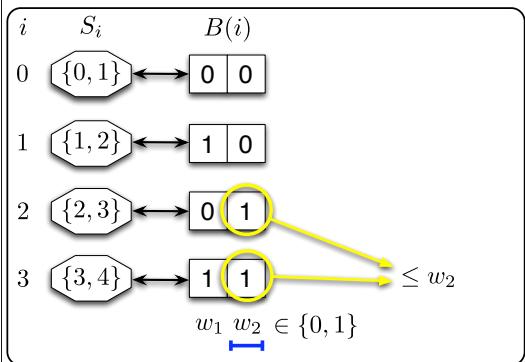
● Idea:

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Formulacion Logaritmica para SOS2

$$\sum_{j=0}^4 \lambda_j = 1, \quad \lambda_0, \dots, \lambda_4 \geq 0, \text{ only 2 adjacent } \lambda_j \text{'s ar nonzero.}$$

Allowed sets: $S_i = \{i, i + 1\}$ for $i \in \{0, \dots, 3\}$.



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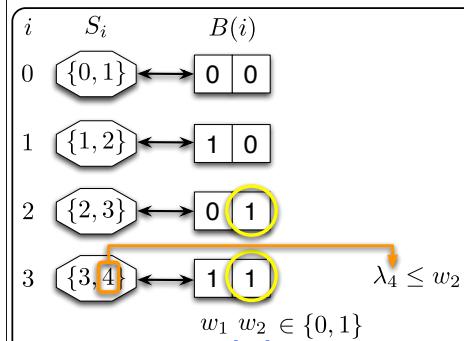
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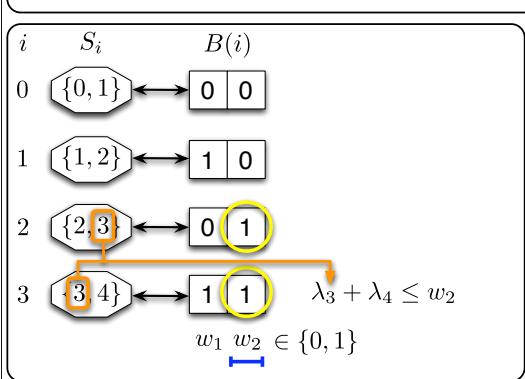


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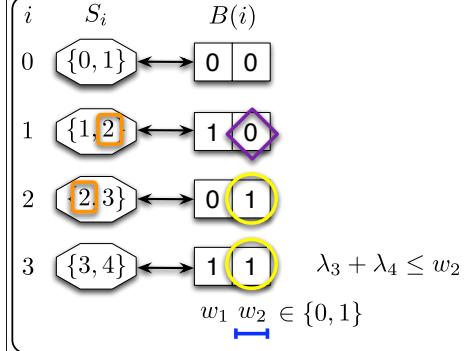


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| i | S_i | $B(i)$ | |
|-----|-------|--------|--|
| 0 | | | $\lambda_0 \leq w_1$ |
| 1 | | | $\lambda_4 \leq (1 - w_1)$ |
| 2 | | | $\lambda_0 + \lambda_1 \leq (1 - w_2)$ |
| 3 | | | $\lambda_3 + \lambda_4 \leq w_2$ |
| | | | $w_1, w_2 \in \{0, 1\}$ |

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● Where is ?!

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| i | S_i | $B(i)$ | |
|-----|-------|--------|--|
| 0 | | | $\lambda_0 \leq w_1$ |
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Formulacion Logaritmica para SOS2

$$\sum_{j=0}^4 \lambda_j = 1, \quad \lambda_0, \dots, \lambda_4 \geq 0, \text{ only 2 adjacent } \lambda_j \text{'s are nonzero.}$$

Allowed sets: $S_i = \{i, i + 1\}$ for $i \in \{0, \dots, 3\}$.

| i | S_i | $B(i)$ |
|-----|----------------|---------|
| 0 | $\{\{0, 1\}\}$ | $0 \ 0$ |
| 1 | $\{\{1, 2\}\}$ | $1 \ 0$ |
| 2 | $\{\{2, 3\}\}$ | $1 \ 1$ |
| 3 | $\{\{3, 4\}\}$ | $0 \ 1$ |

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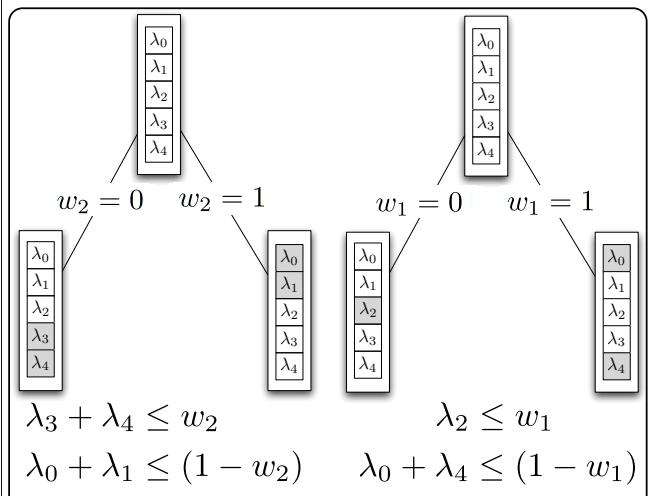
Where is ?!

In general:

Gray Code.

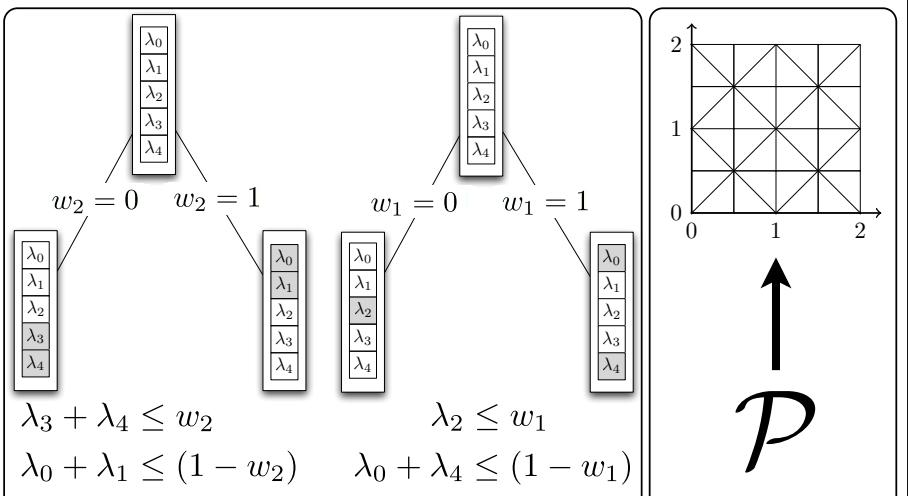
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Ramificacion Independiente



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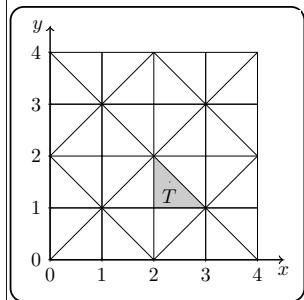
Ramificacion Independiente



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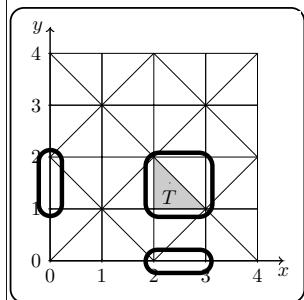
Ramificacion Independiente para FLT

- Seleccionar triángulo prohibiendo vértices.
- 2 etapas:
 - Seleccionar cuadrado con SOS2 por variable.
 - Seleccionar 1 triángulo de cada cuadrado.

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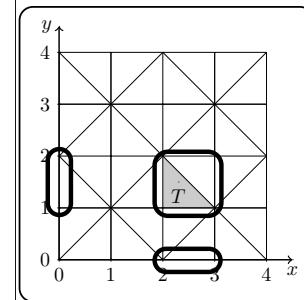
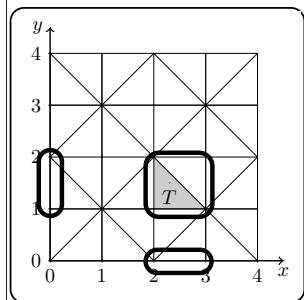
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$$\begin{aligned} \bar{L} &= \{(r,s) \in J : \\ &\quad r \text{ even and } s \text{ odd}\} \\ &= \{\text{square vertices}\} \\ \bar{R} &= \{(r,s) \in J : \\ &\quad r \text{ odd and } s \text{ even}\} \\ &= \{\text{diamond vertices}\} \end{aligned}$$