

Cuts for Nonlinear MIP: Closed Form Expressions and Extended Formulations

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Massachusetts Institute of Technology

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Outline

- Introduction
- Two Techniques
- Summary

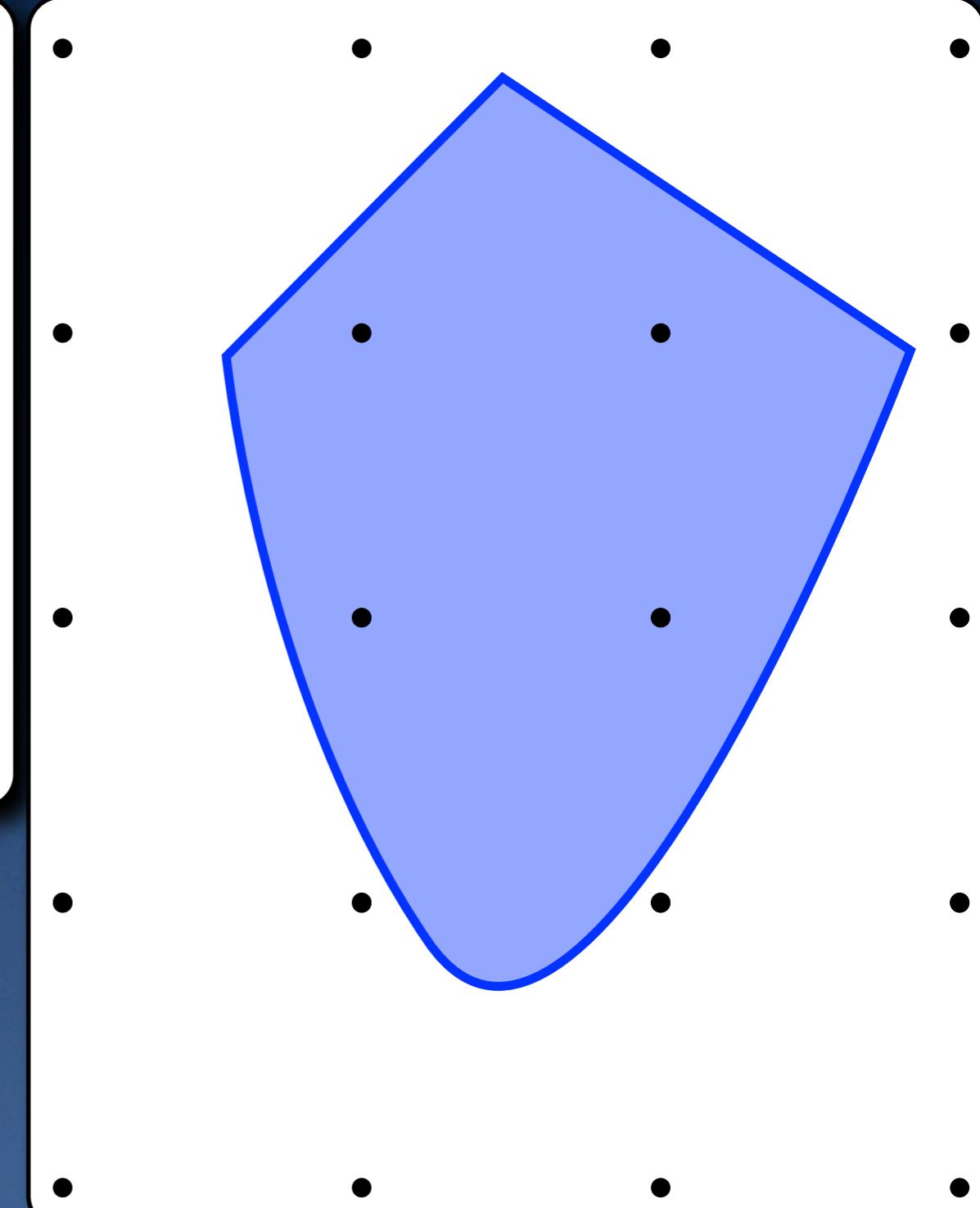
“Intersection” Cuts:

Continuous Relaxation

$$C := \{x \in \mathbb{R}^n : f(x) \leq 0\}$$

Open Set S without “interesting”
points e.g. $S \cap \mathbb{Z}^n = \emptyset$

$$\text{conv}(C \setminus S) = \{x : f(x) \leq 0, \\ g_j(x) \leq 0, j \in J\}$$



“Intersection” Cuts: Ex. 1 Split Cuts

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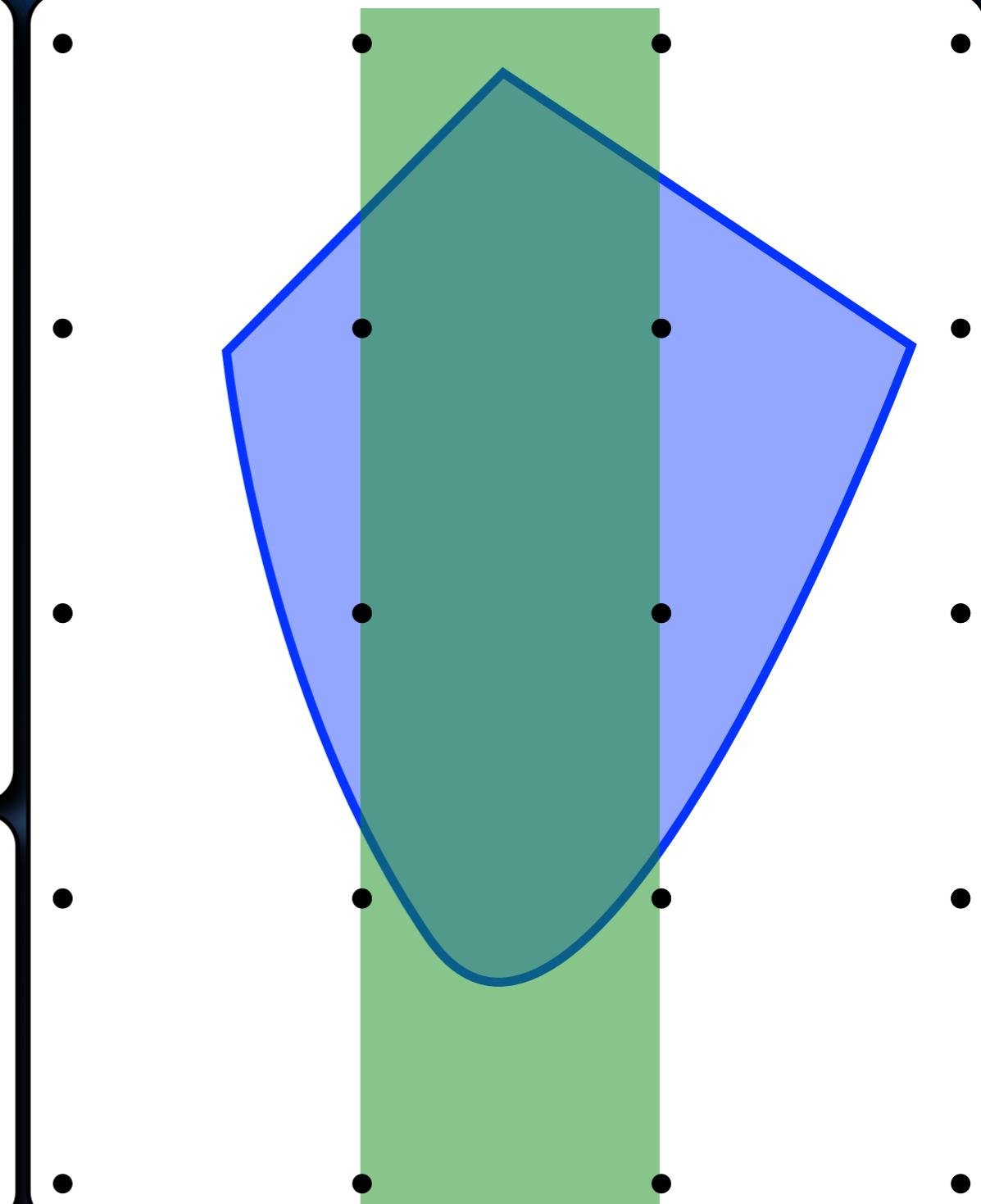
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Split Disjunction

$$S = \{x \in \mathbb{R}^n : \pi_0 < \pi^T x < \pi_1\}$$

$$\pi \in \mathbb{Z}^n, \quad \pi_0 \in \mathbb{Z}, \quad \pi_1 = \pi_0 + 1$$



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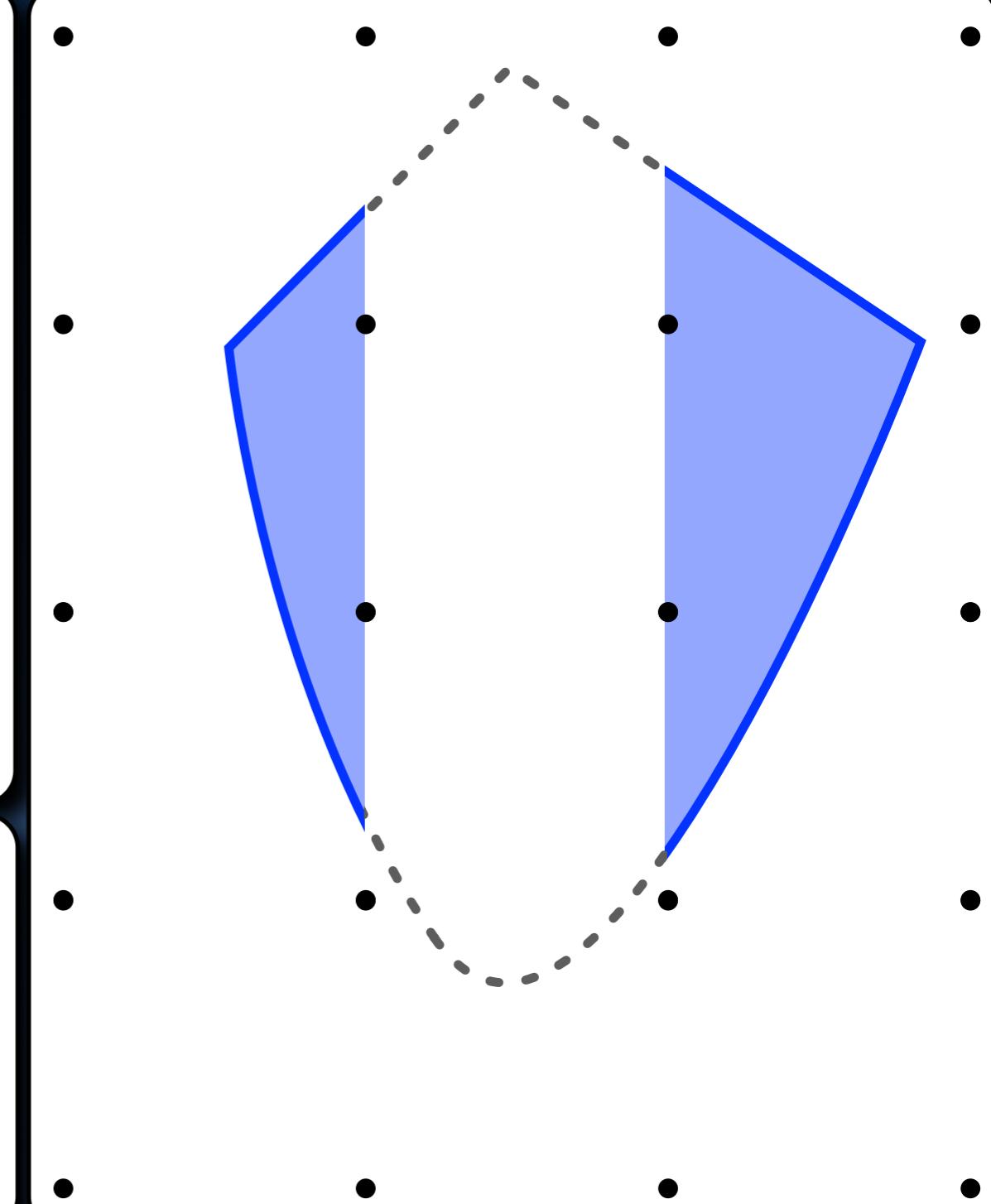
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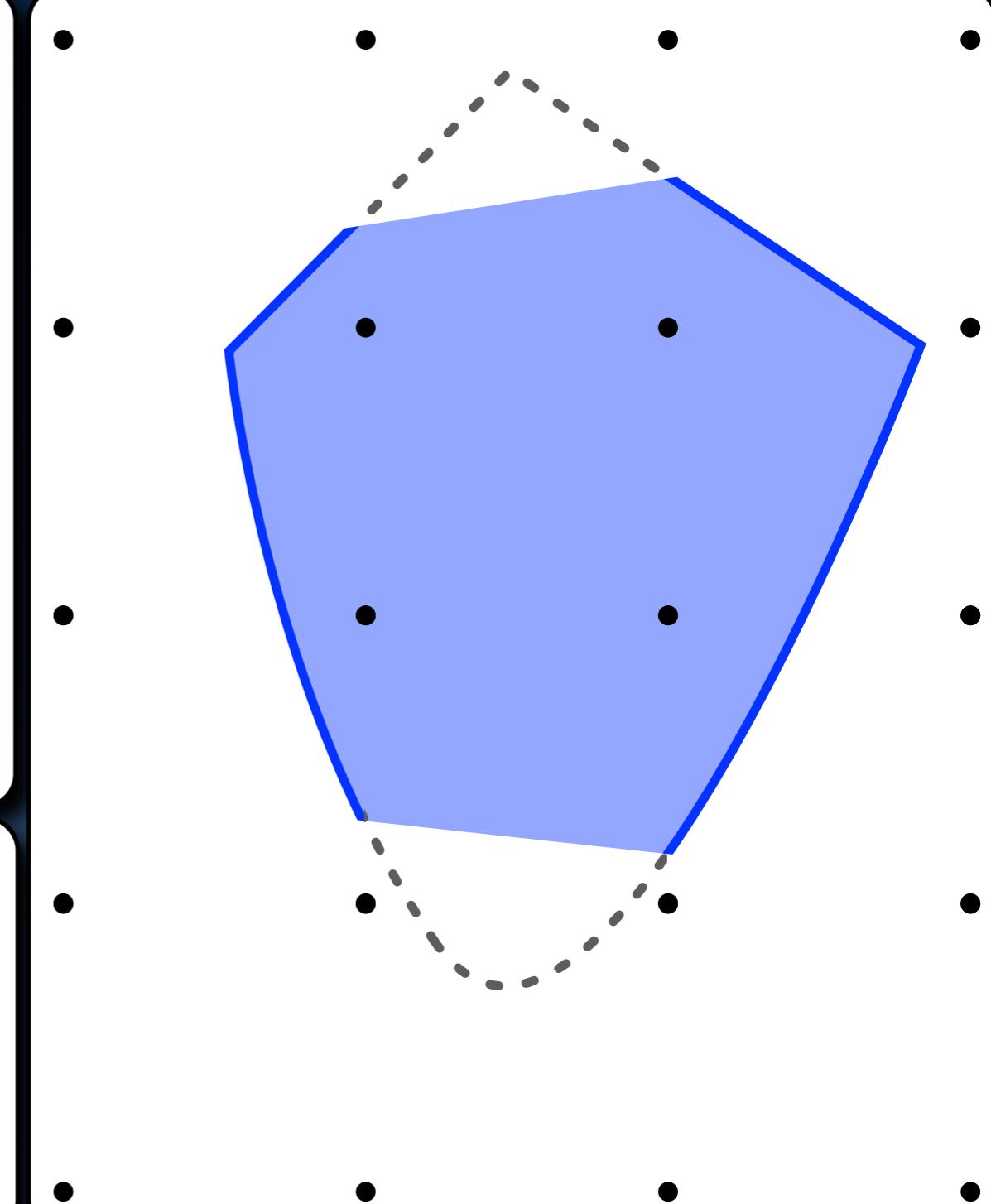
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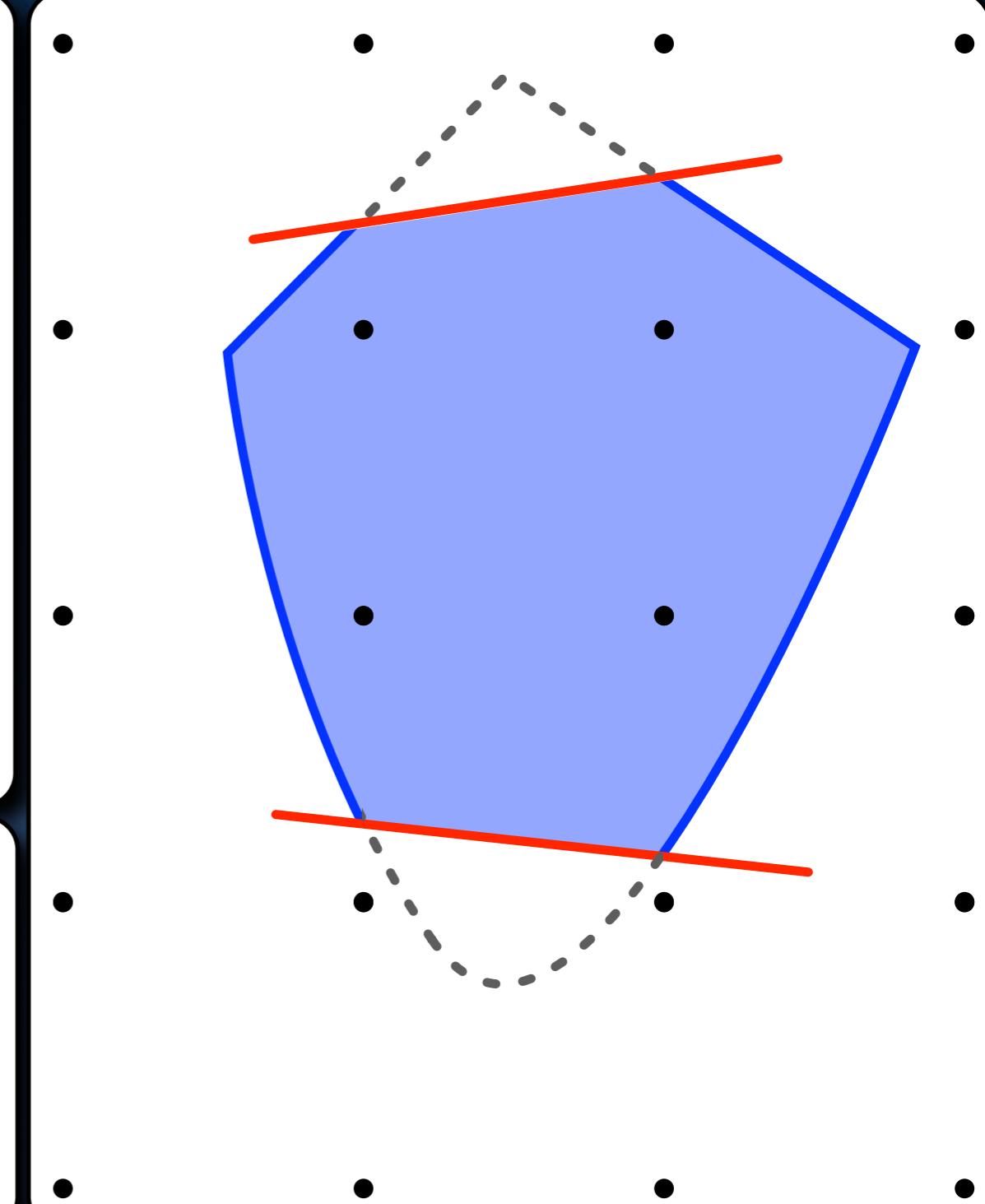
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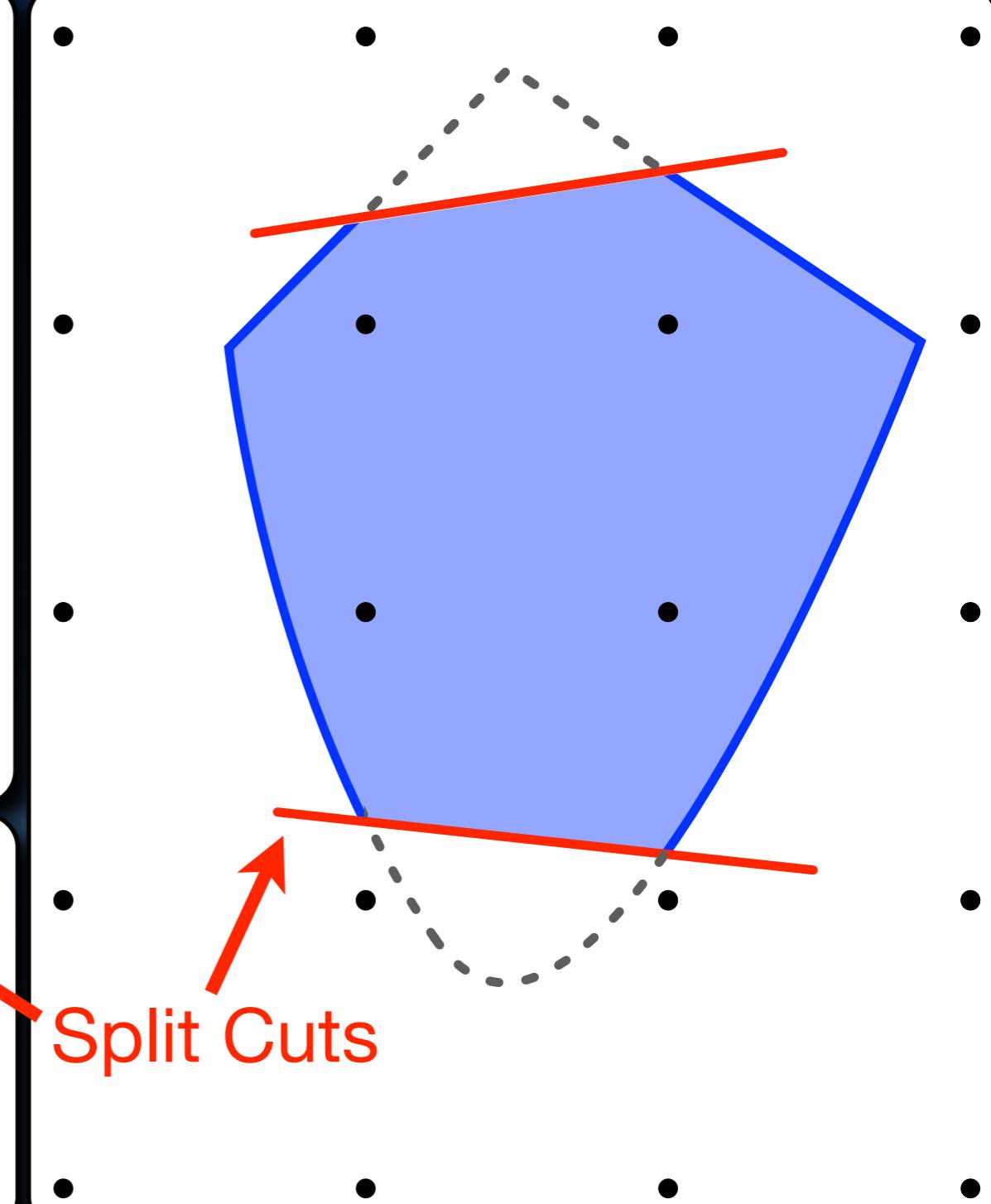
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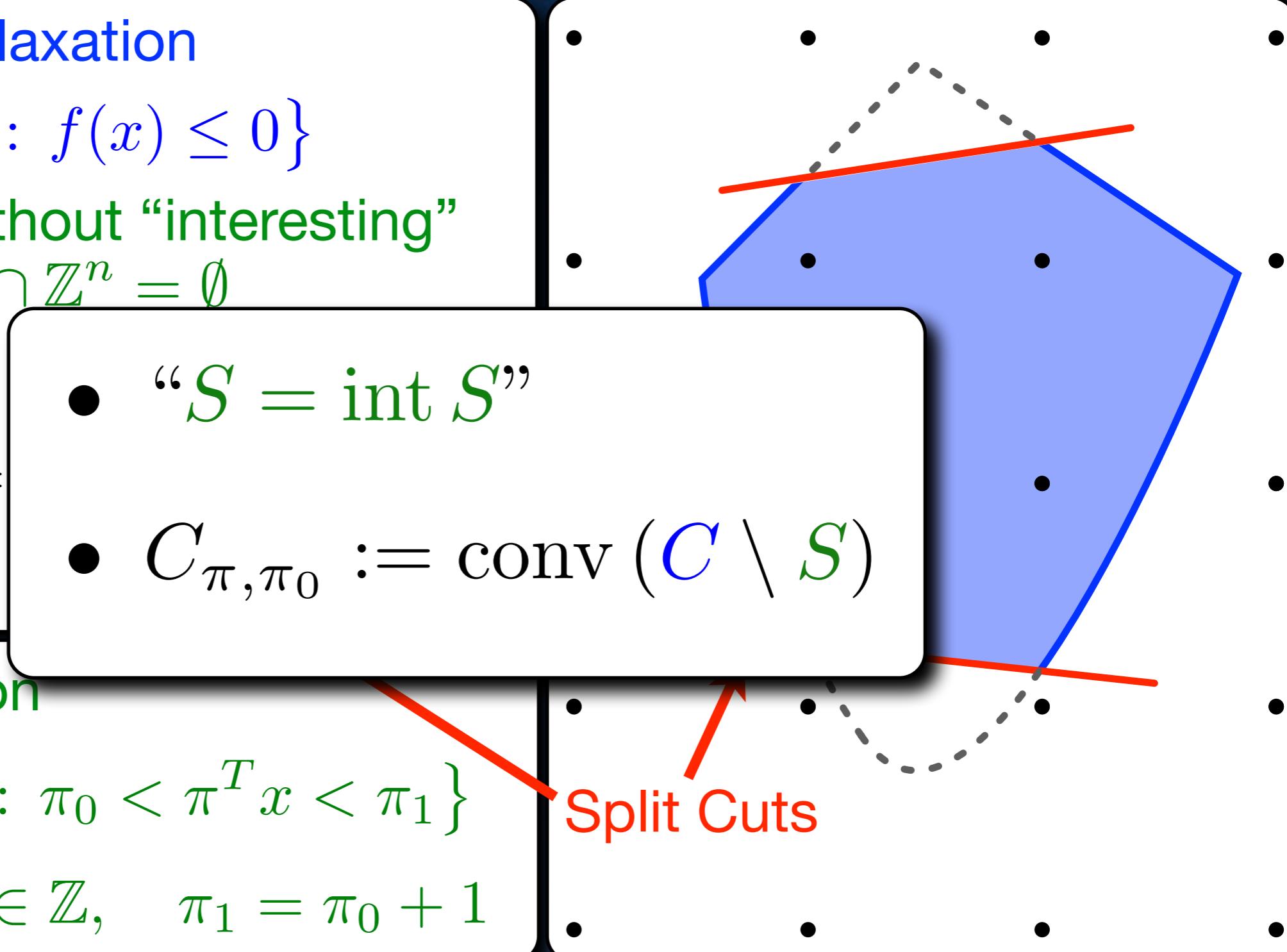
- “ $S = \text{int } S$ ”
- $C_{\pi, \pi_0} := \text{conv}(C \setminus S)$

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“Intersection” Cuts: Ex. 2 n-Split Cuts

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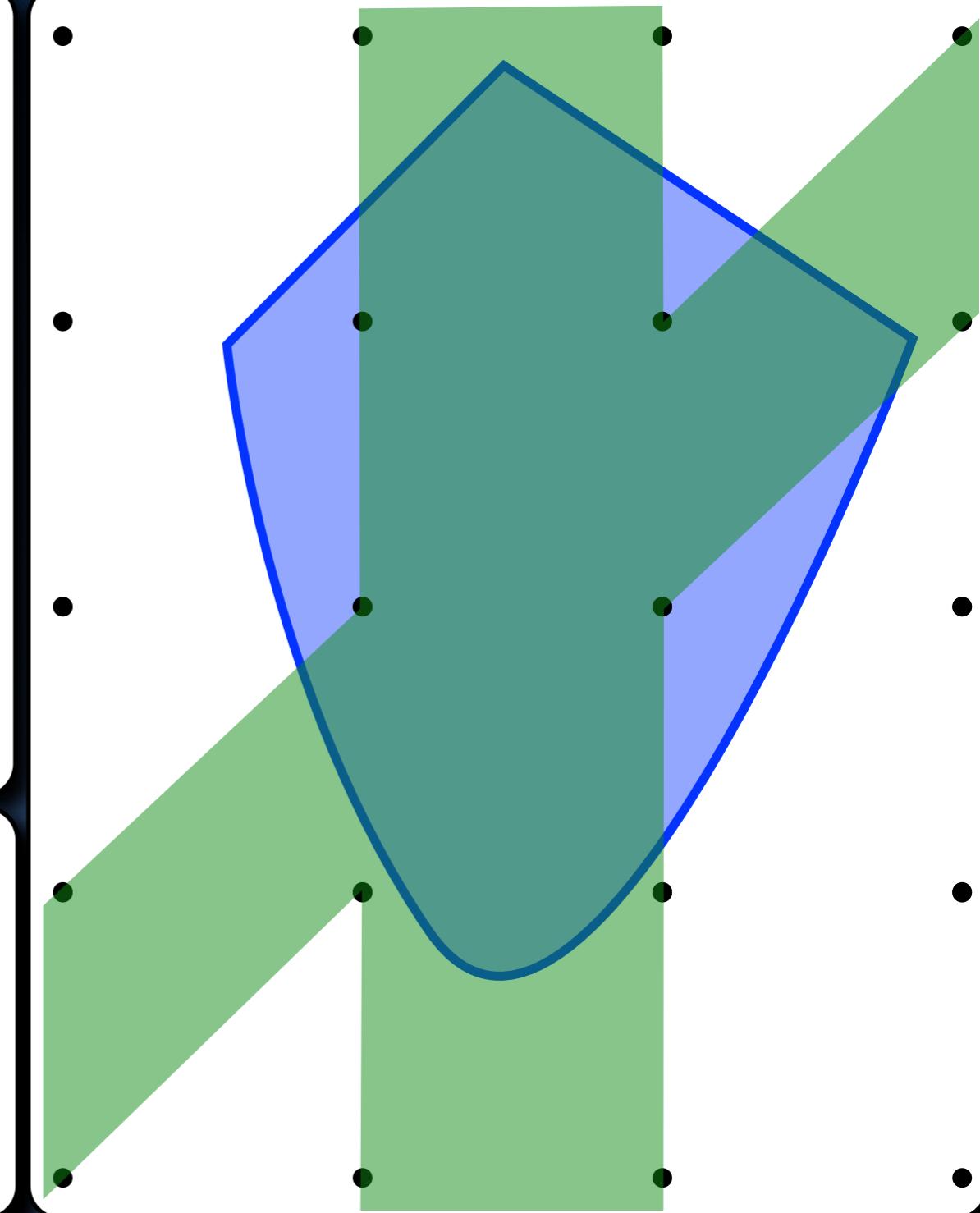
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Multi-branch Split Disjunctions

$$S = \bigcup_{i \in I} \{x : \pi_0^i < \pi_i^T x < \pi_1^i\}$$

$$I \subseteq [n] := \{1, \dots, n\}$$



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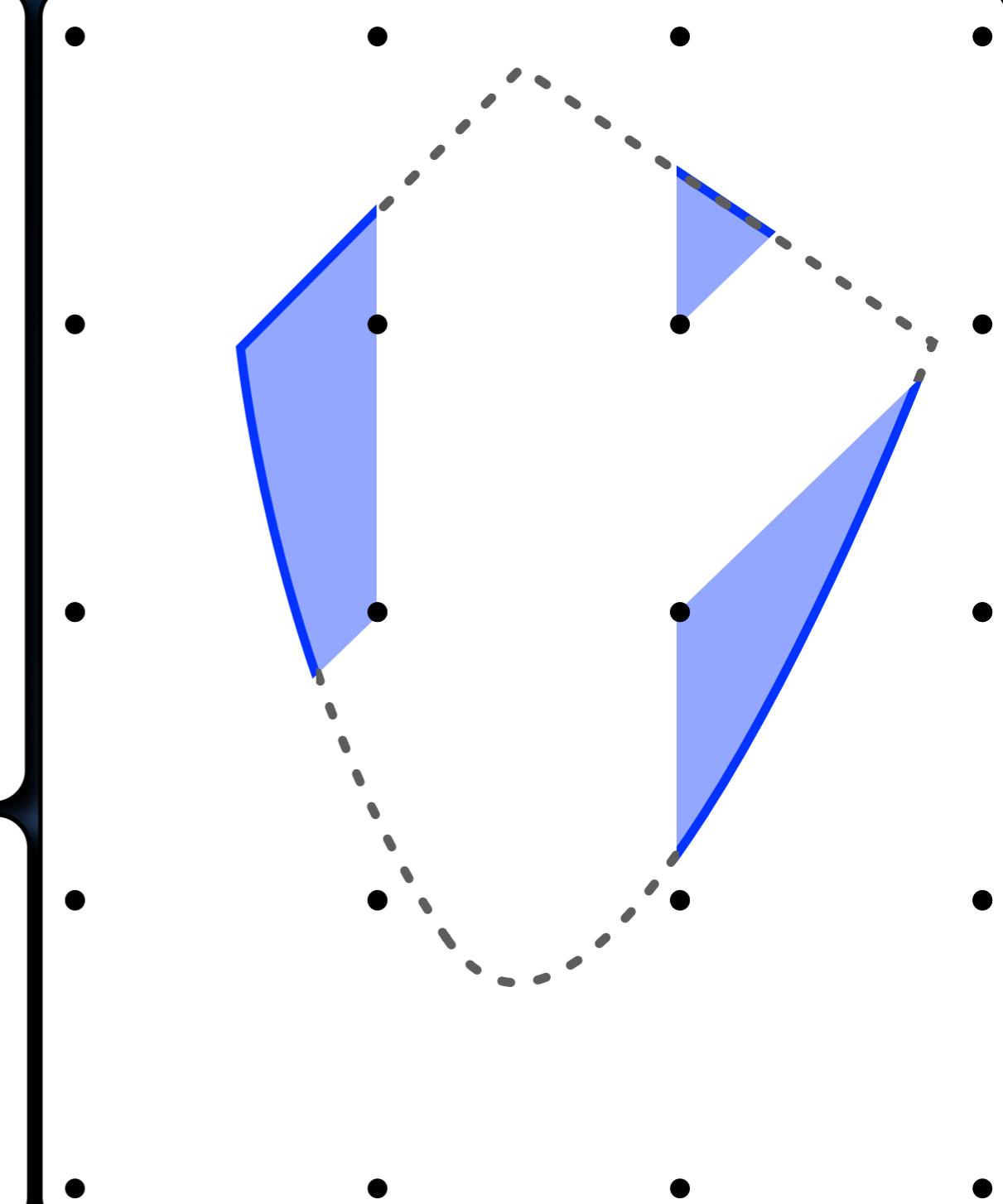
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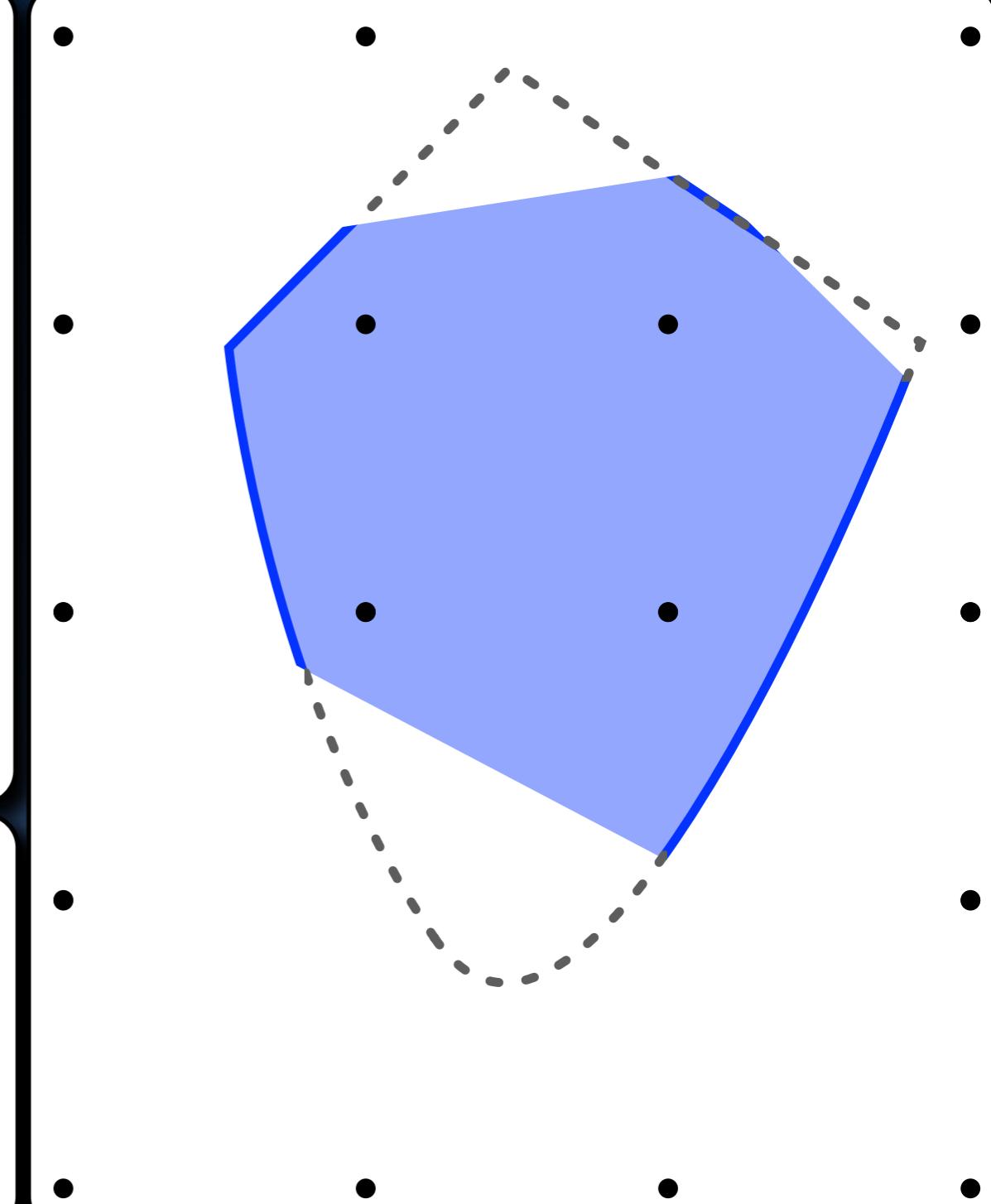
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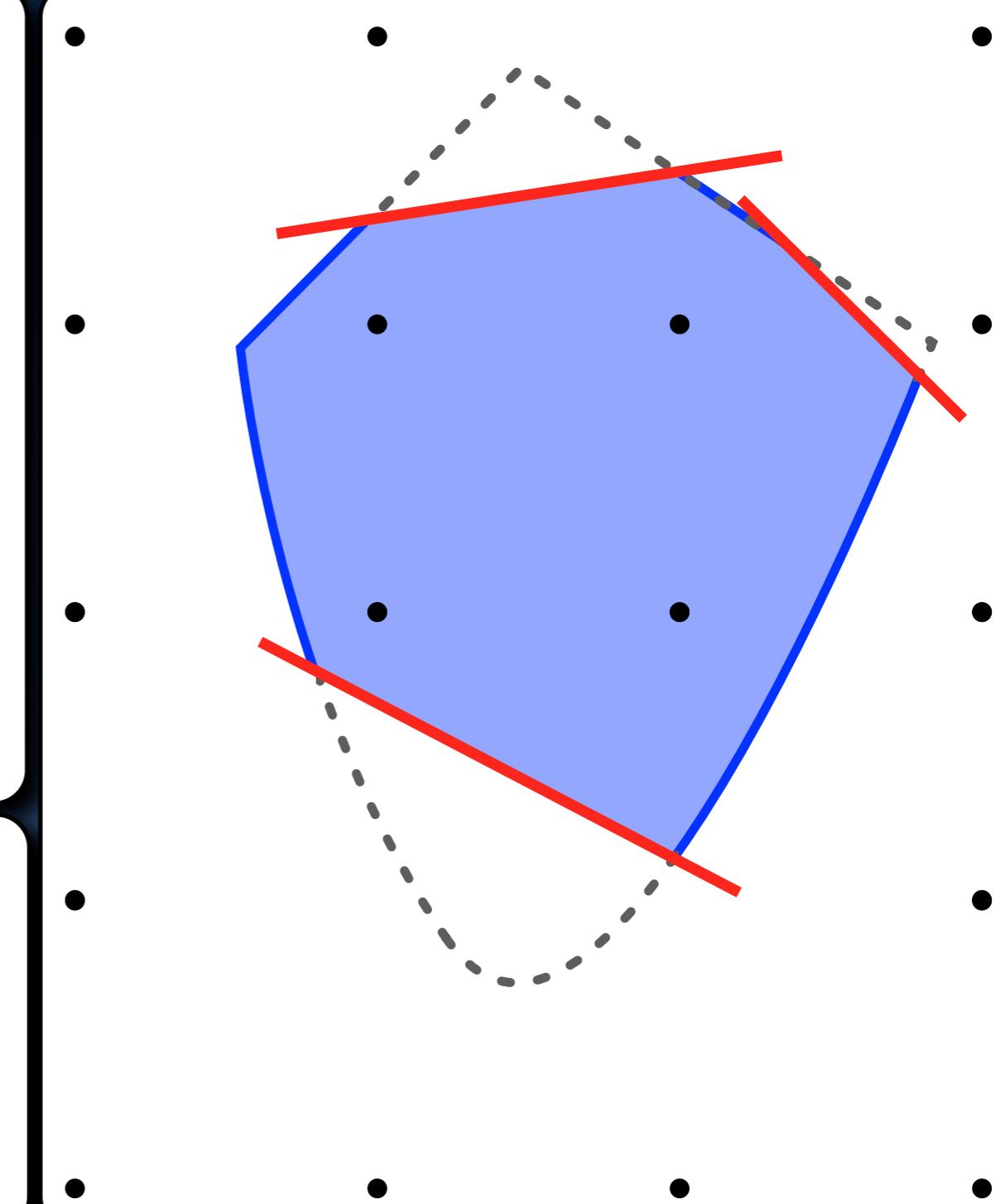
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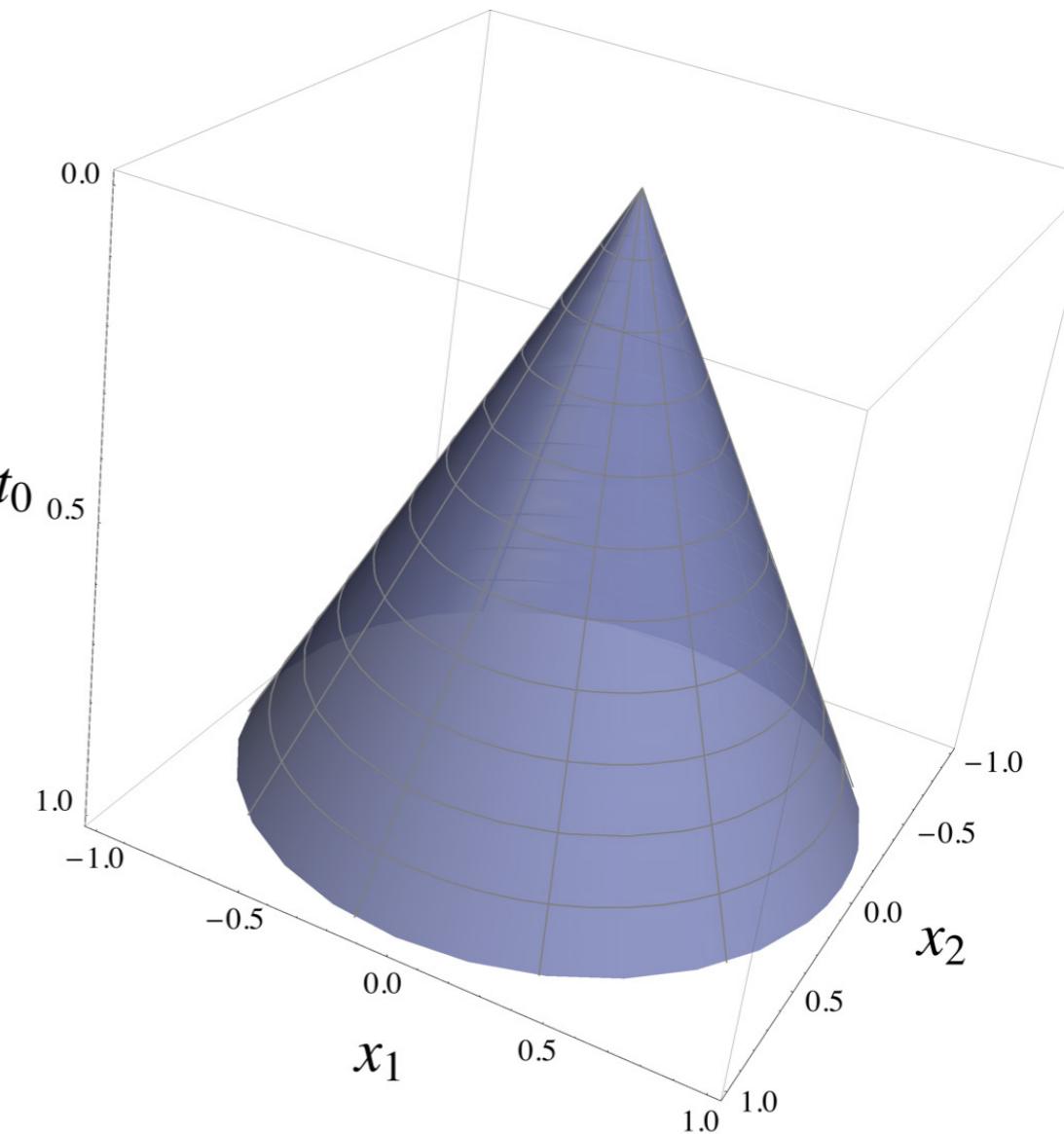
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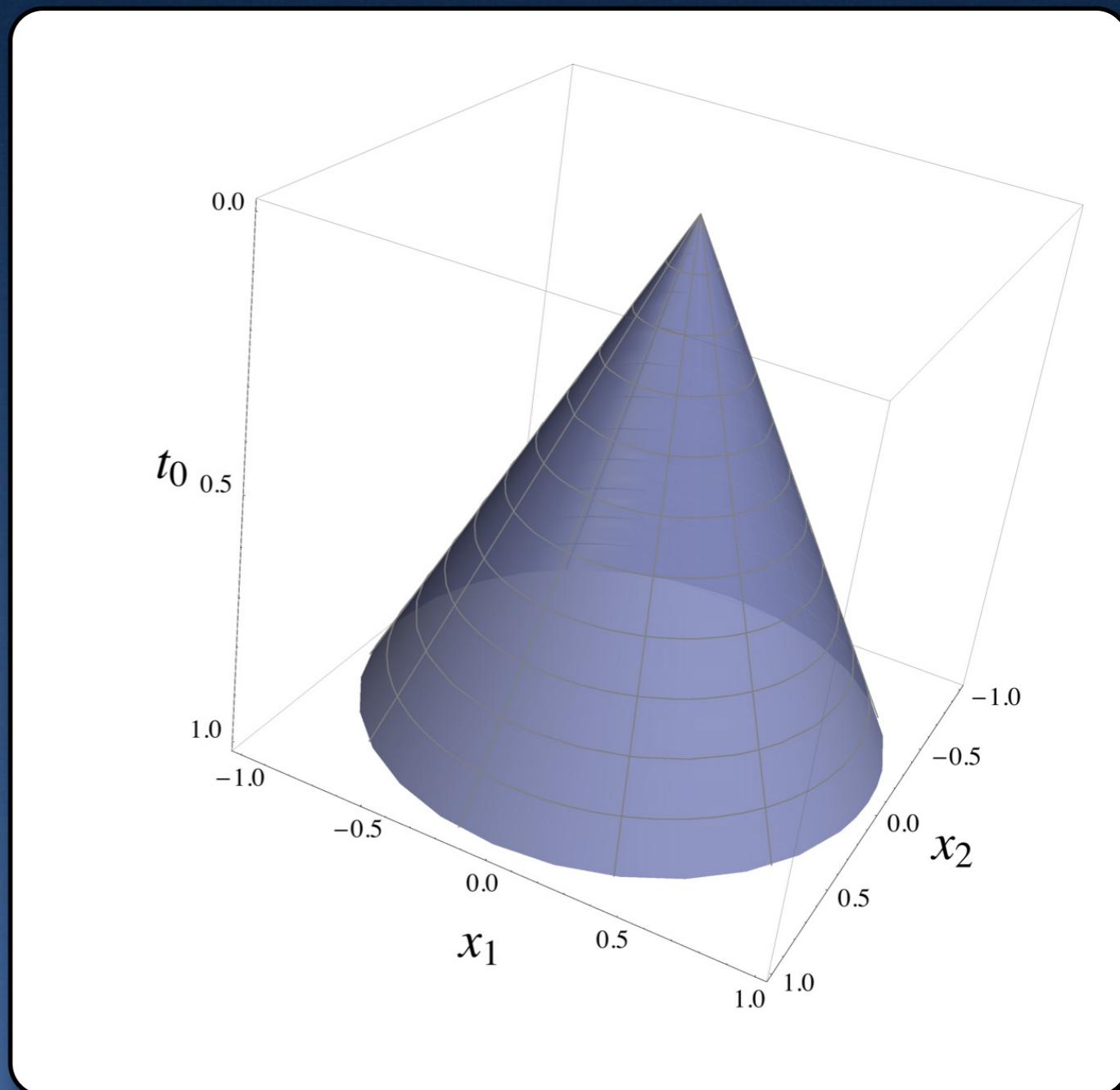


Cuts Can be Non-linear



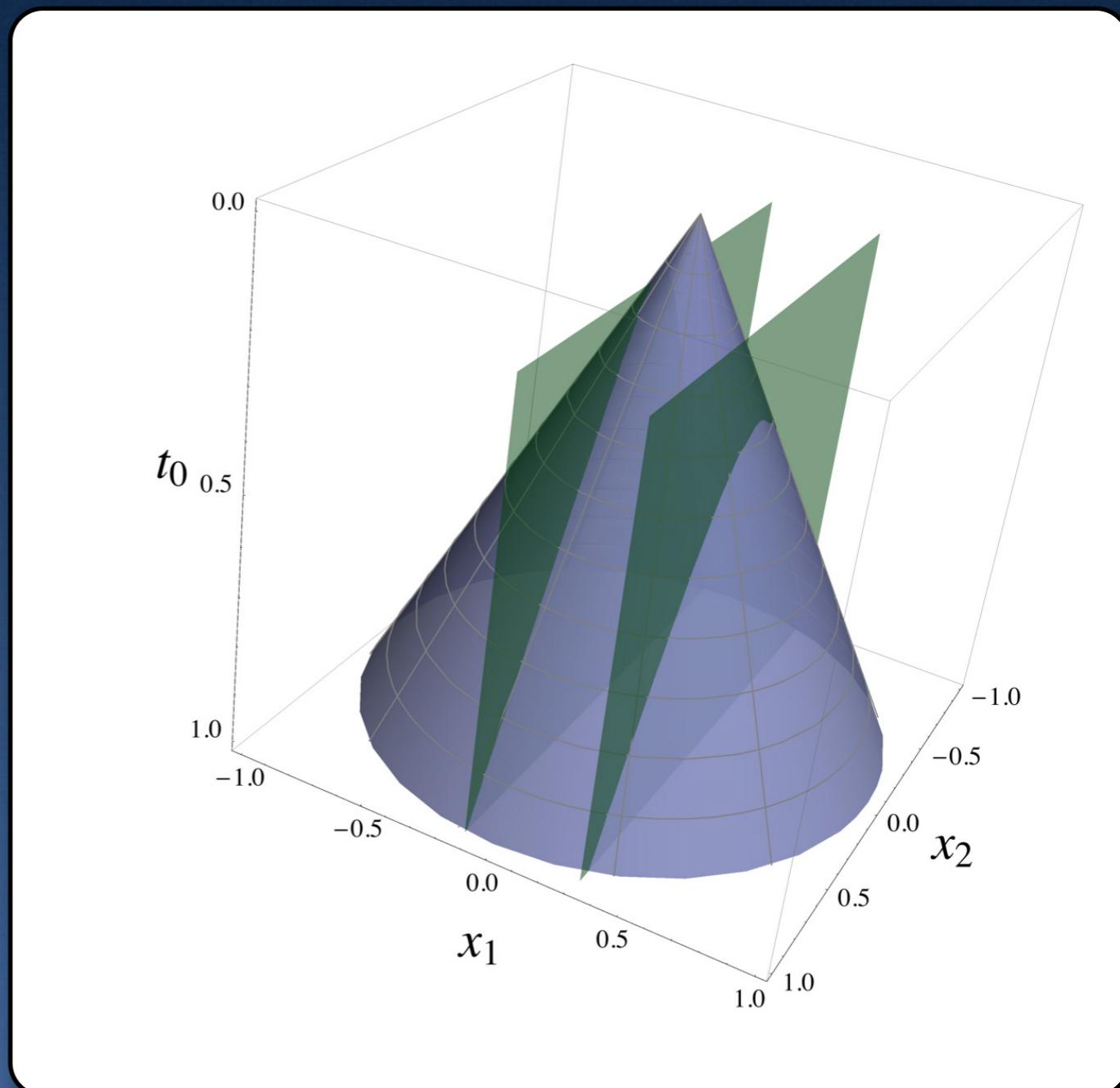
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- Split Cuts for Cones



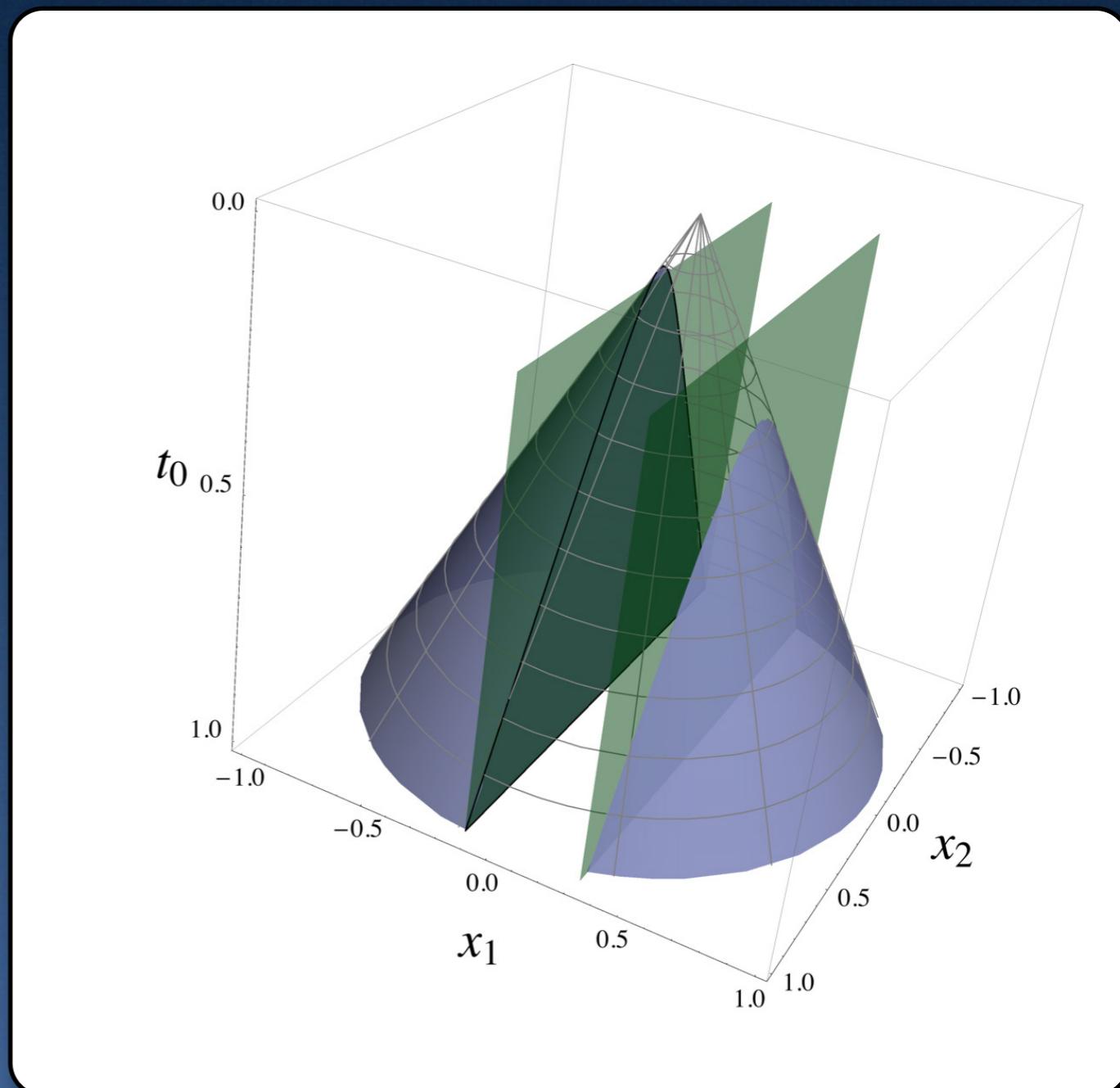
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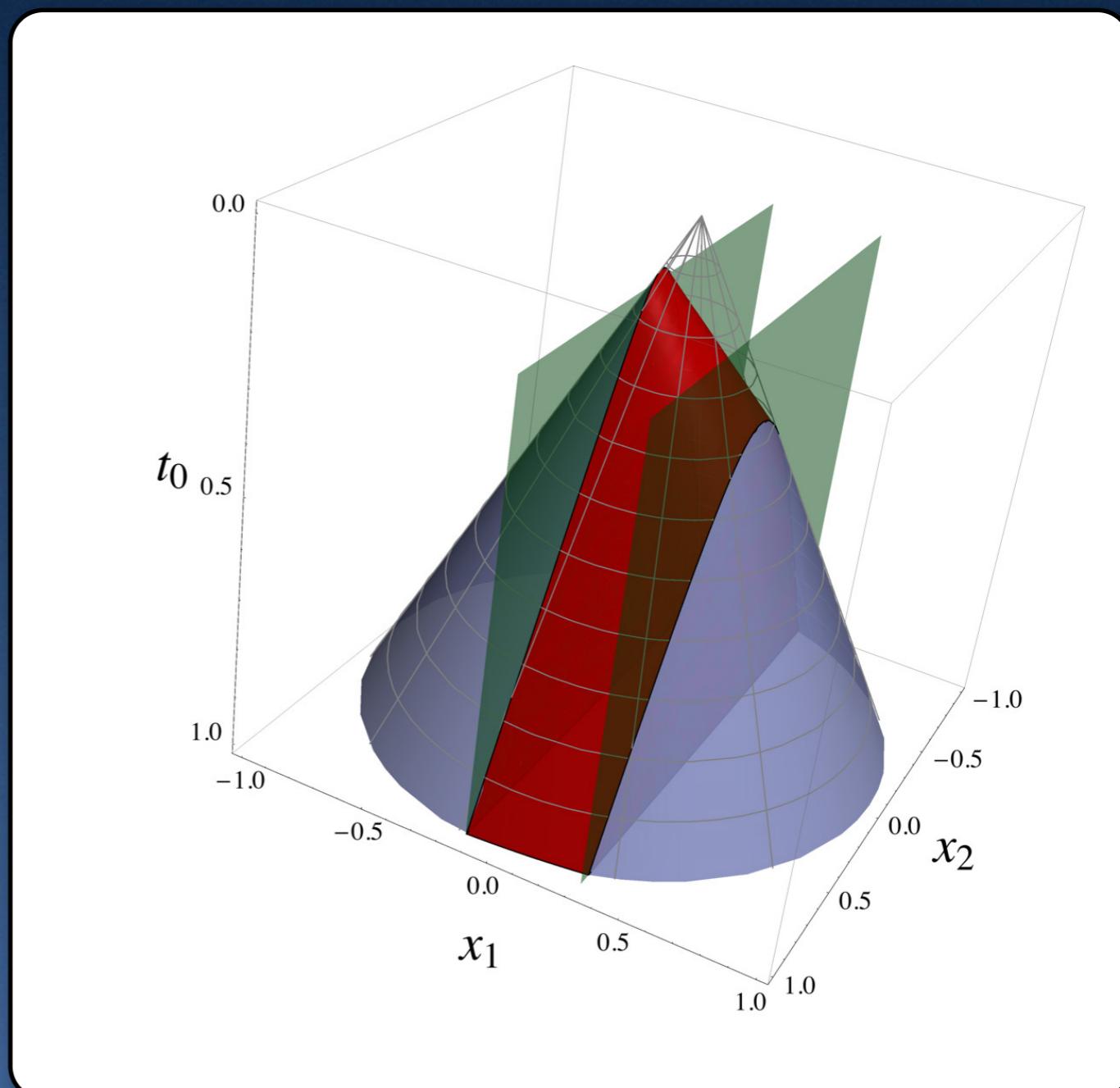
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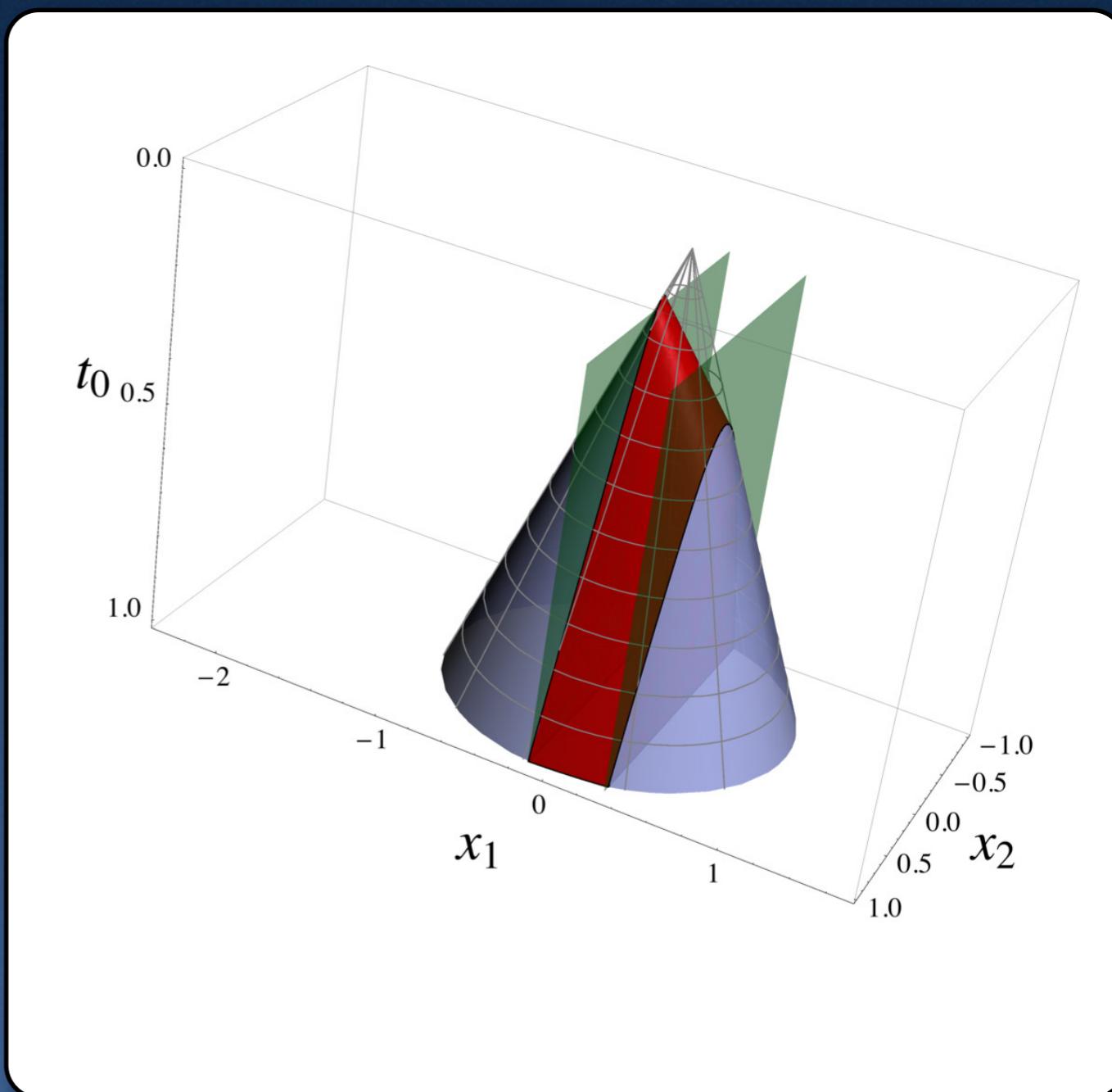
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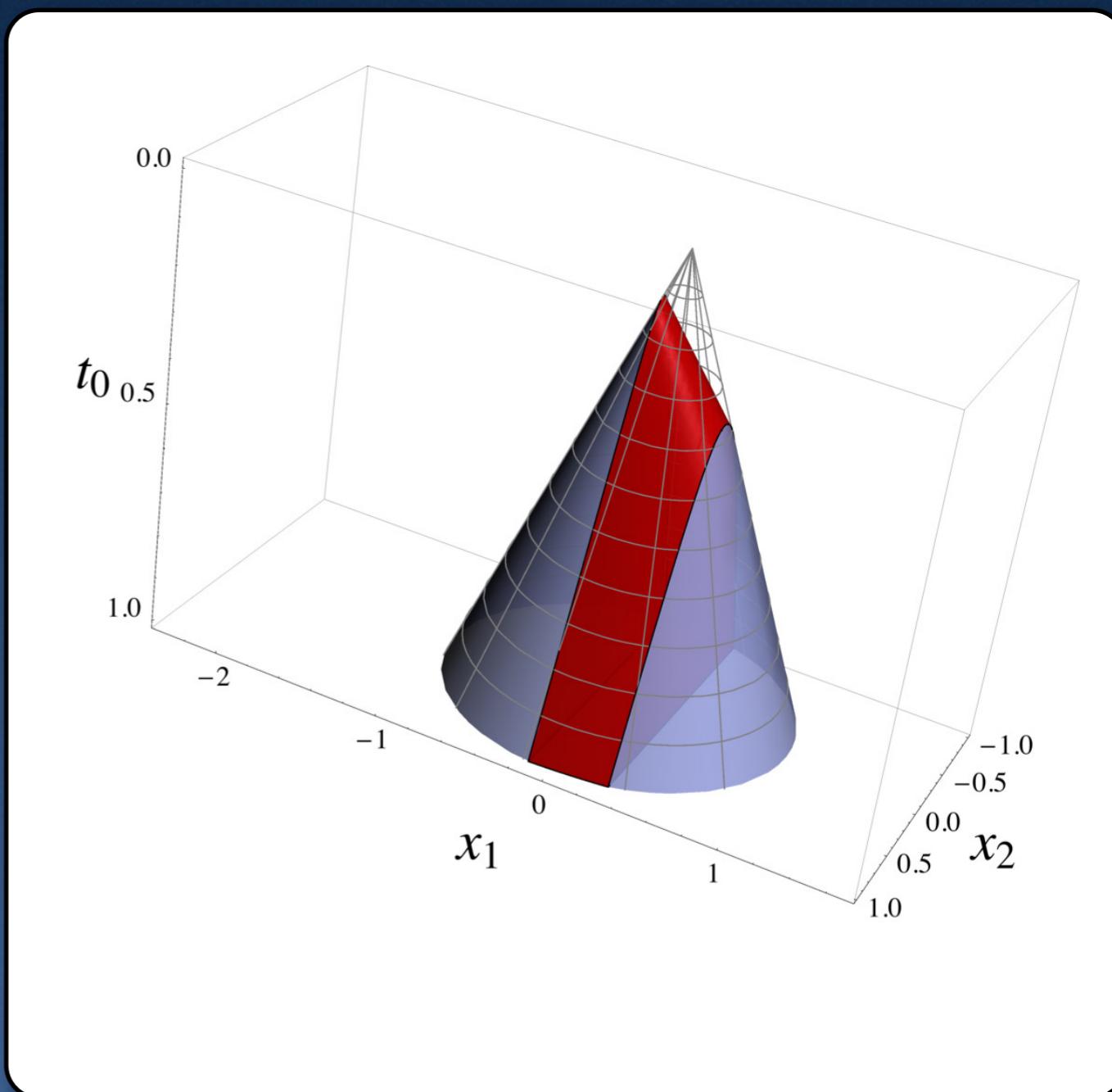
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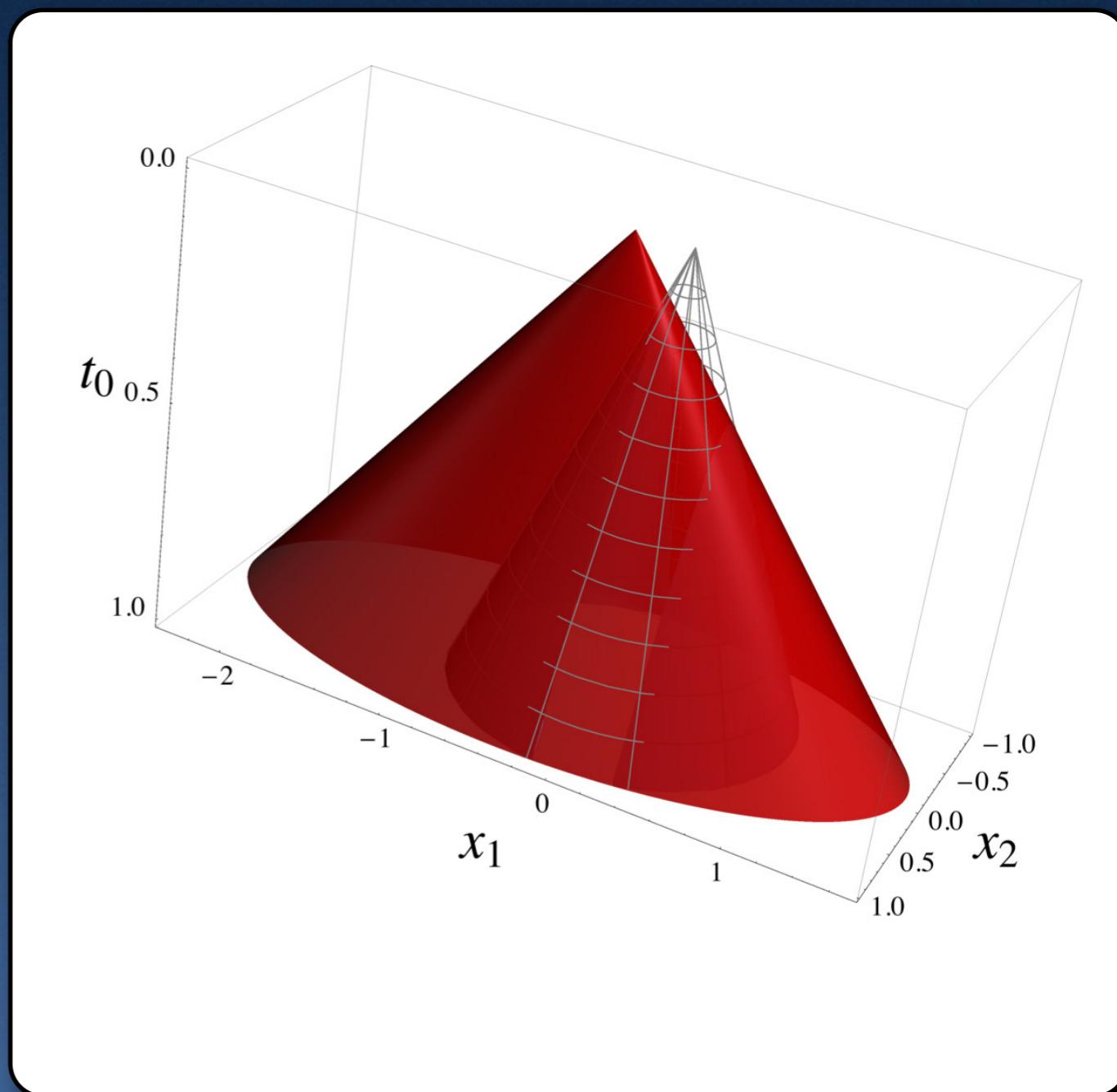
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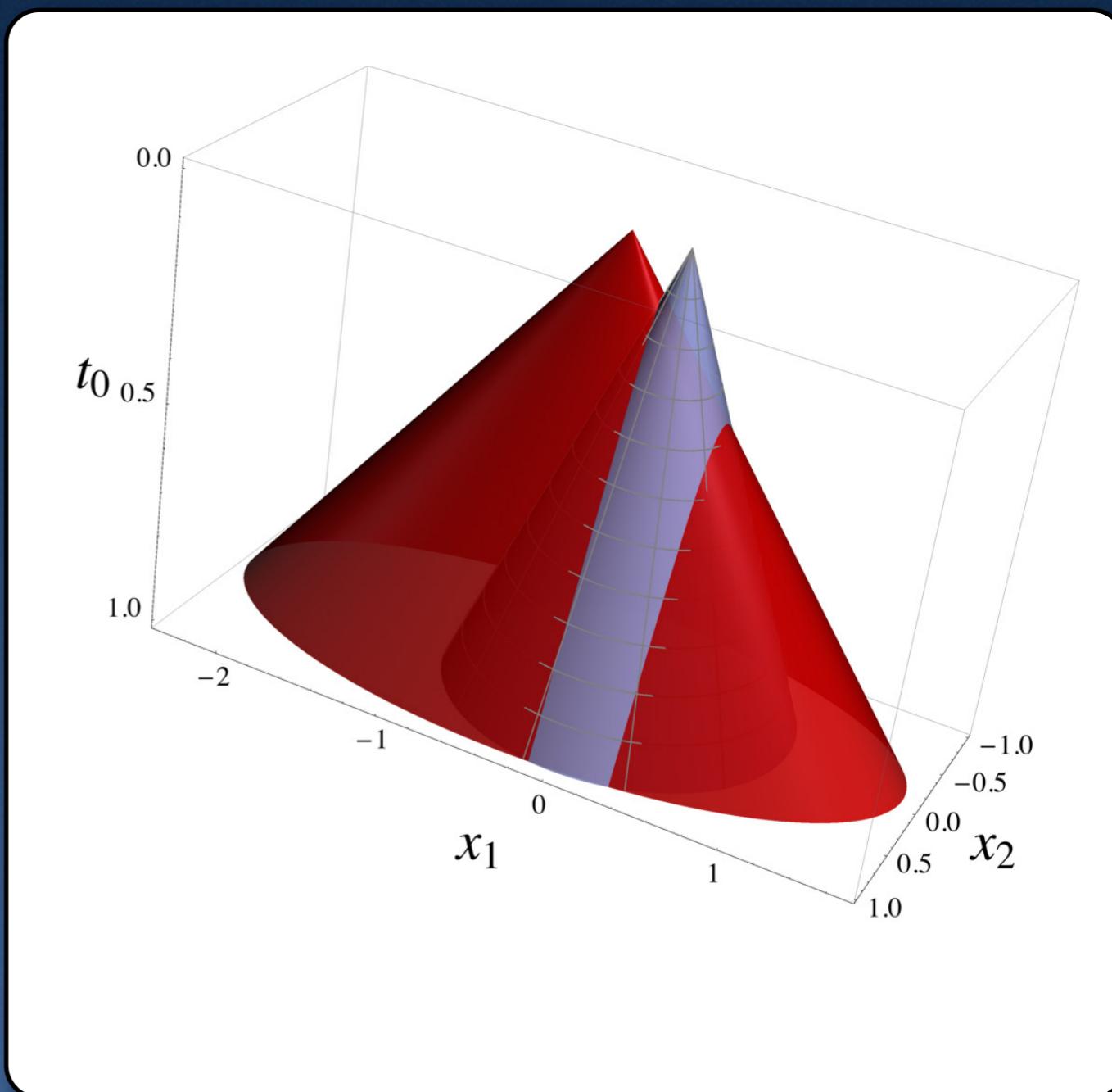
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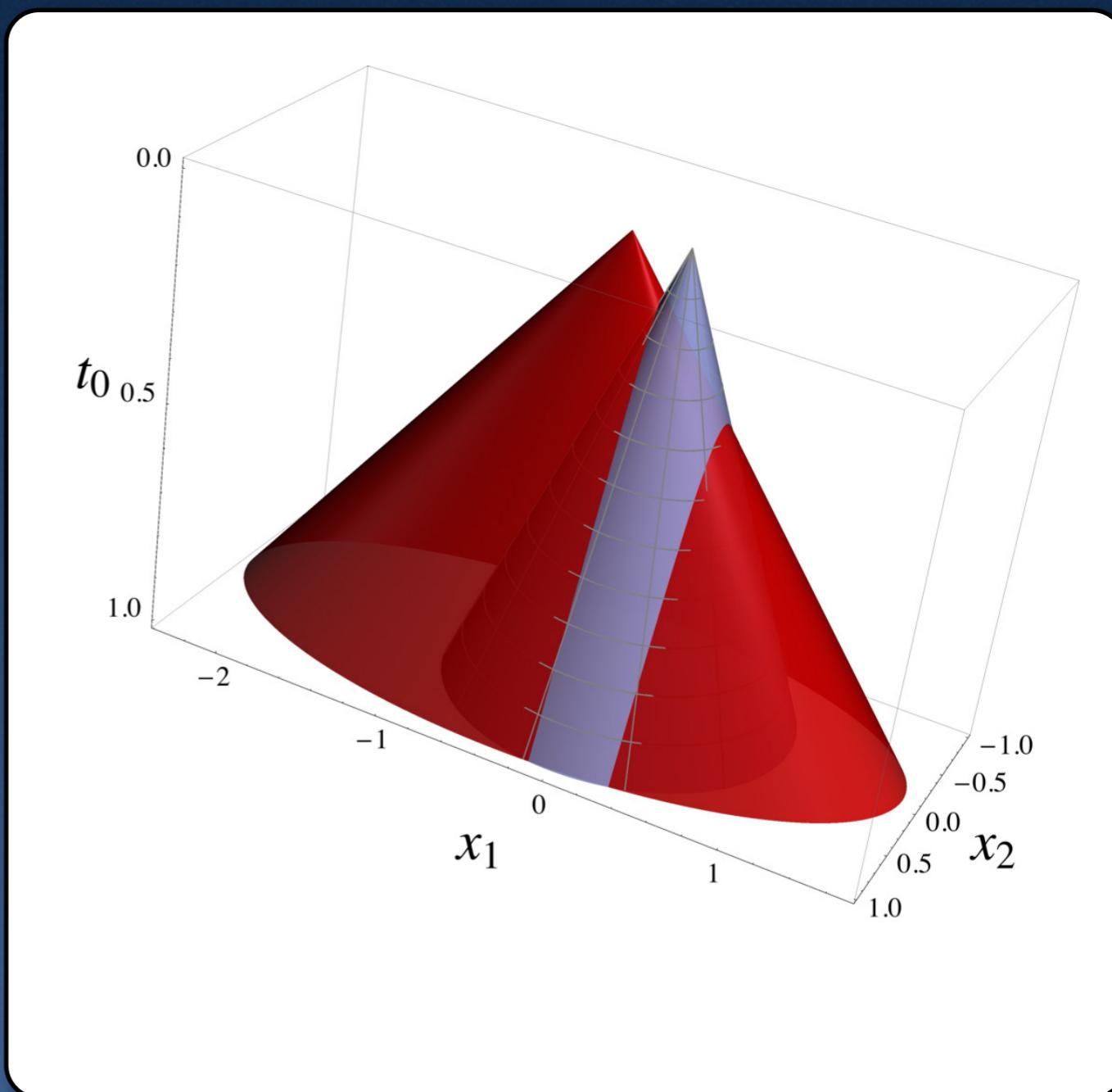
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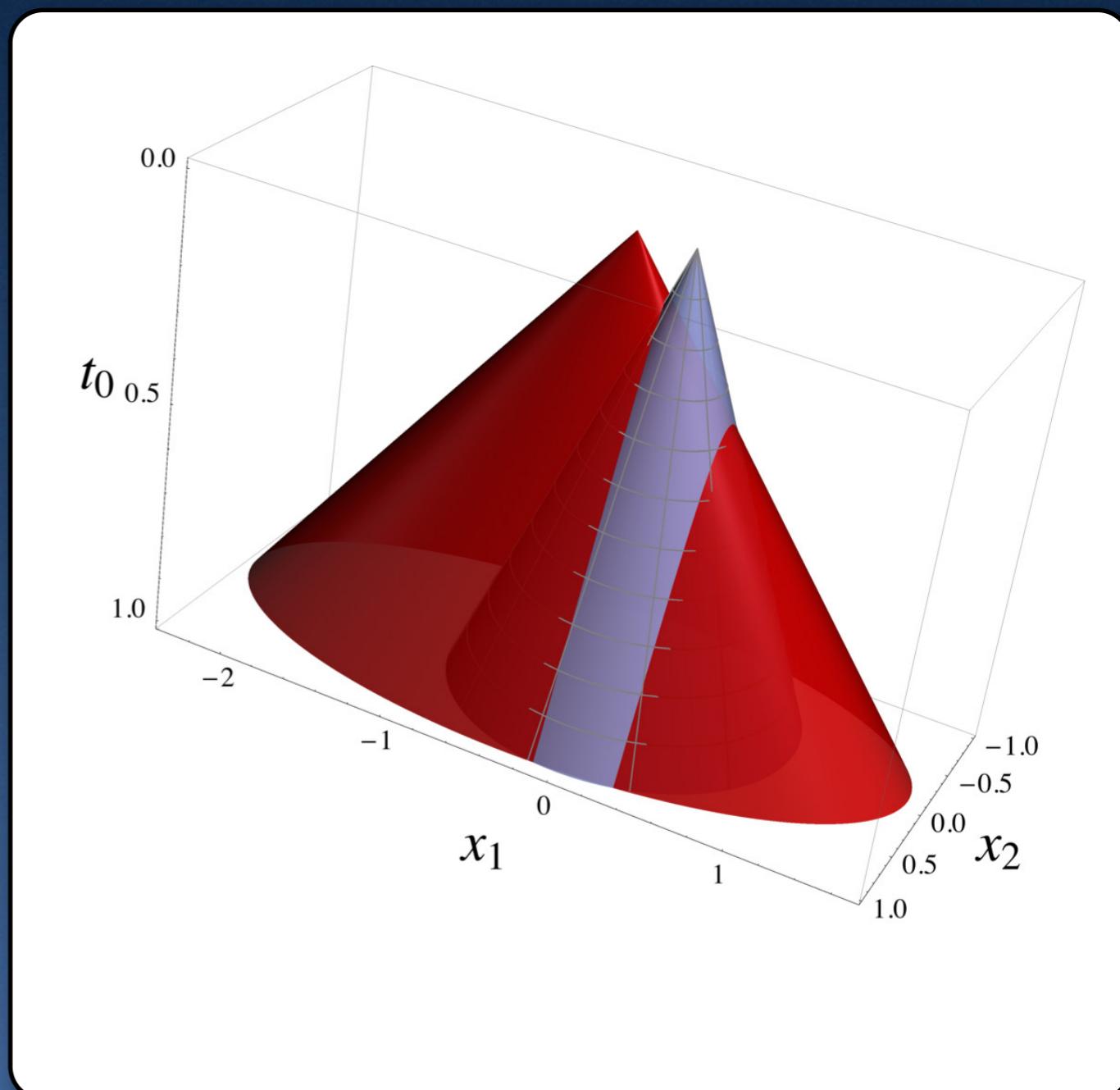
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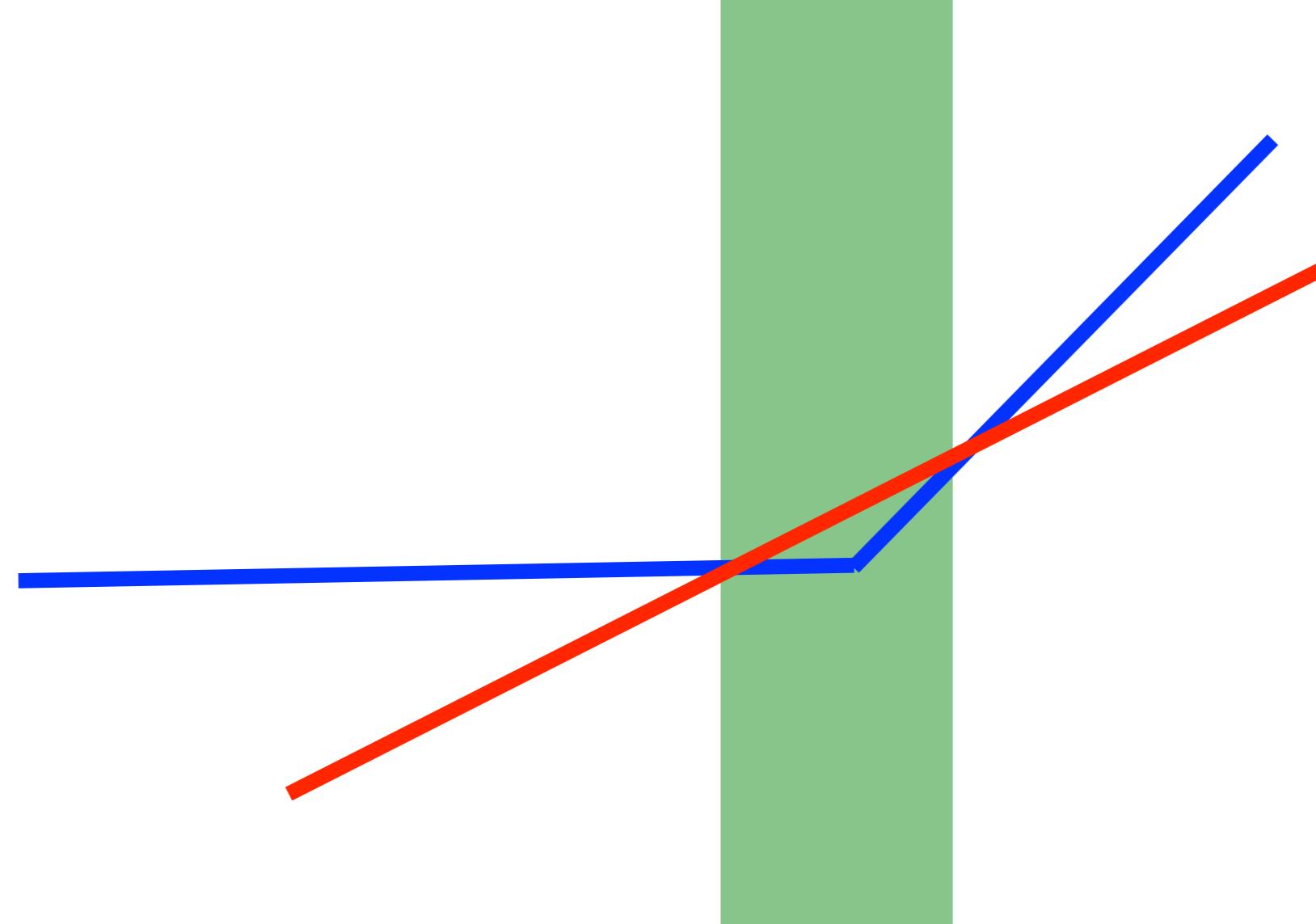


Cuts Can be Non-linear

- Split Cuts for Cones
 - Non-linear, but ...
 - “Class Preserving” &
 - Simple Formulas?
- General Convex Hulls
 - Blekherman, et al. 2013
 - Horst and Tuy. 2003



Follow the MIR way!



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- Imply general cuts for quadratic sets.
- Techniques:
 - (1) Interpolation, (2) Aggregation.

Interpolation

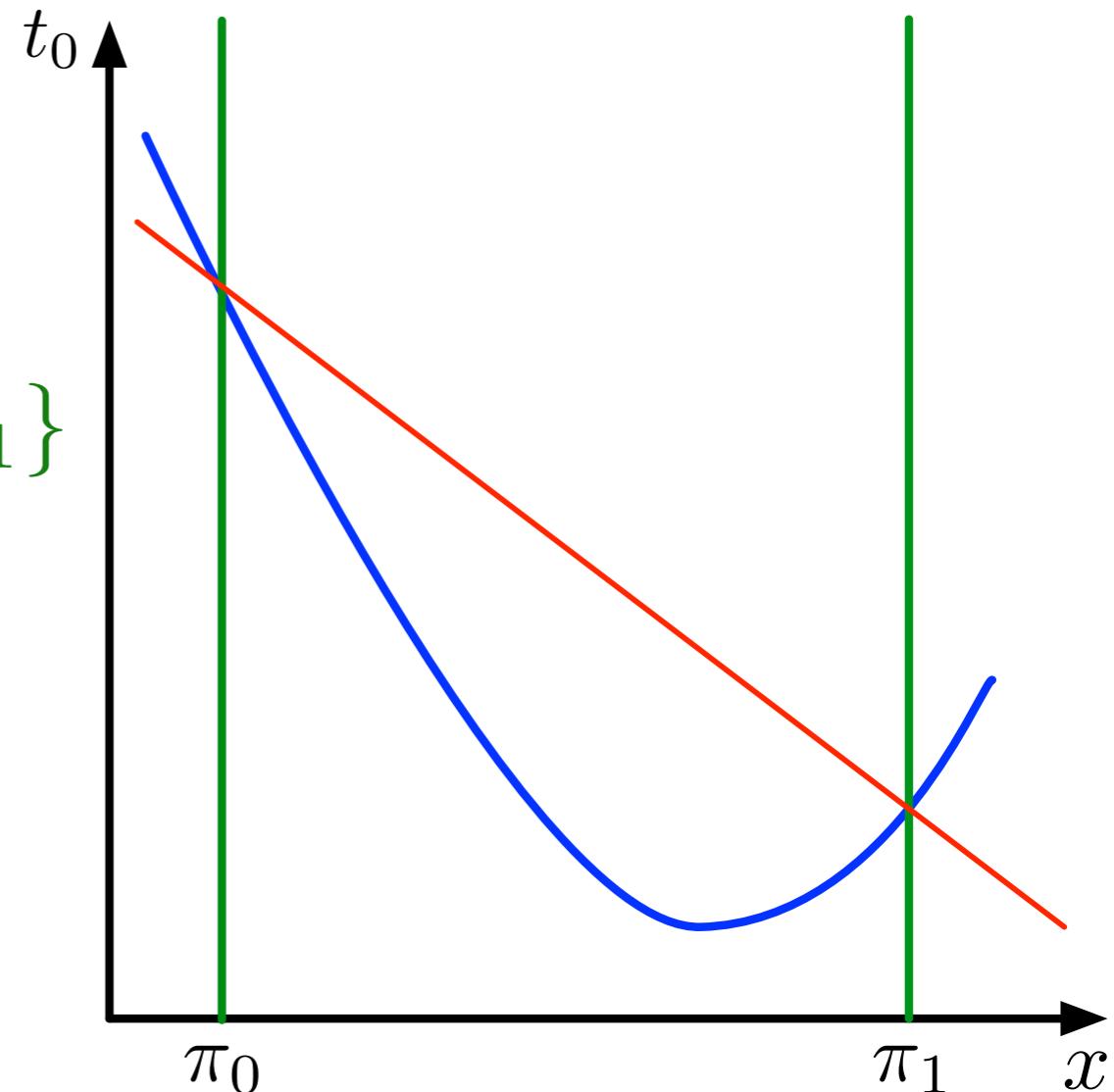
Interpolation for Separable Functions

- Modaresi, Kılınç, V. 2013:

$$C := \{(x, t_0) \in \mathbb{R} \times \mathbb{R} : f(x_1) \leq t_0\}$$

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$$C_{e^1, \pi_0} = \{(x, t_0) \in \mathbb{R} \times \mathbb{R} : f(x_1) \leq t_0 \text{ and } ax_1 + b \leq t_0\}$$



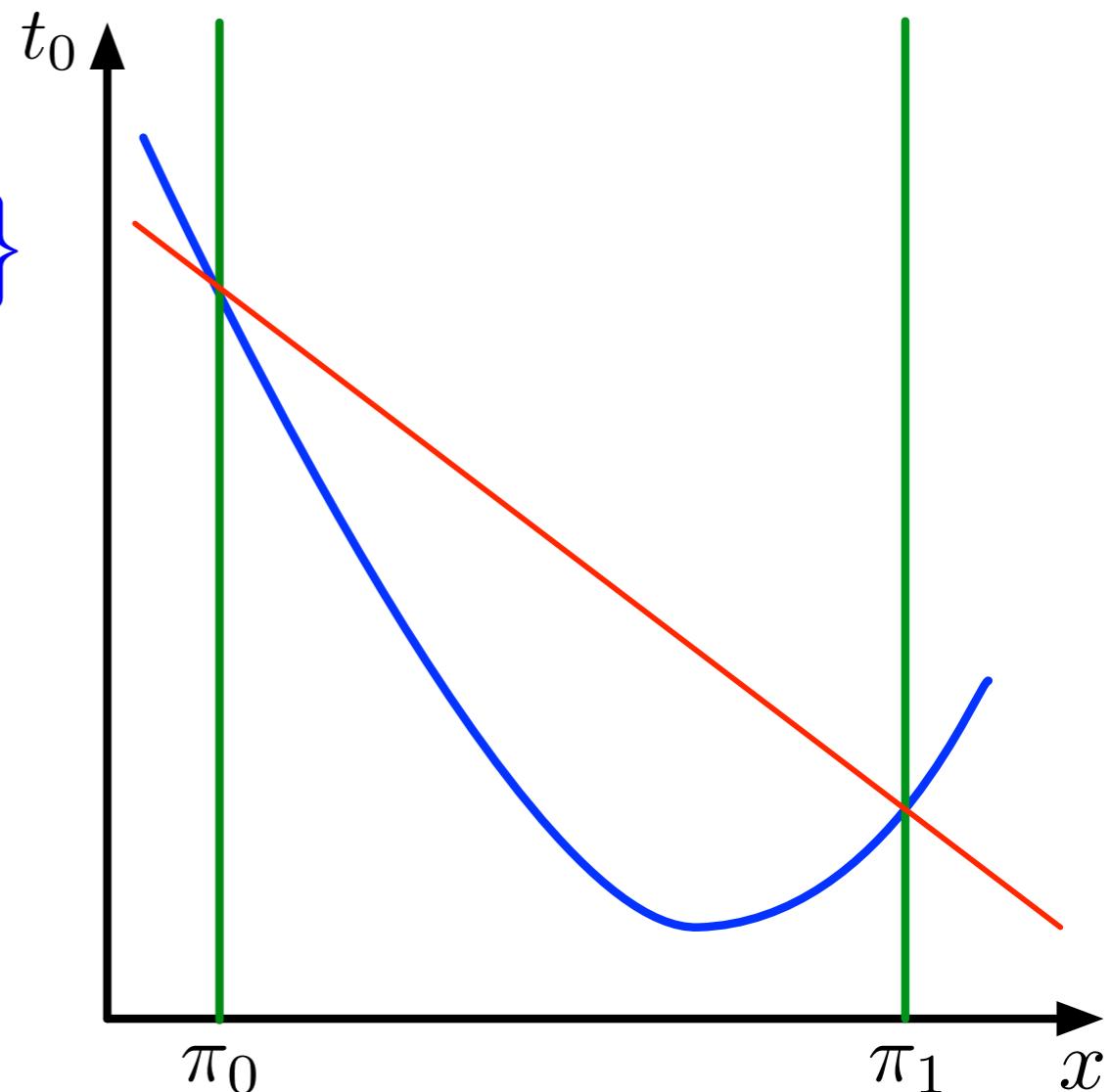
“Simple” Split = Ignores t_0 : $\pi^T x \leq \pi_0 \vee \pi^T x \geq \pi_1$

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$$P_\pi^\perp := I - \left(1/\|\pi\|_2^2\right) \pi \pi^T$$



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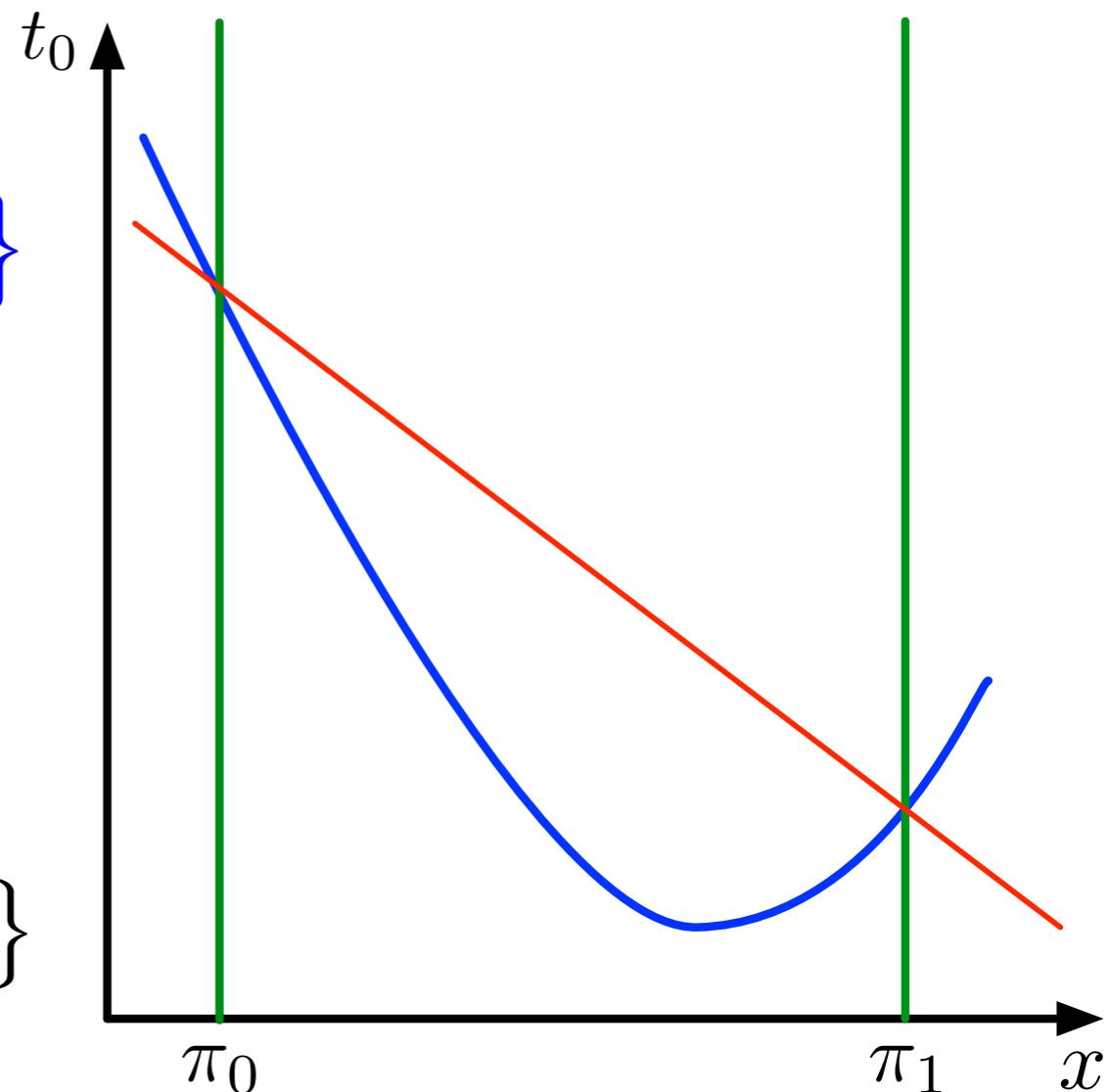
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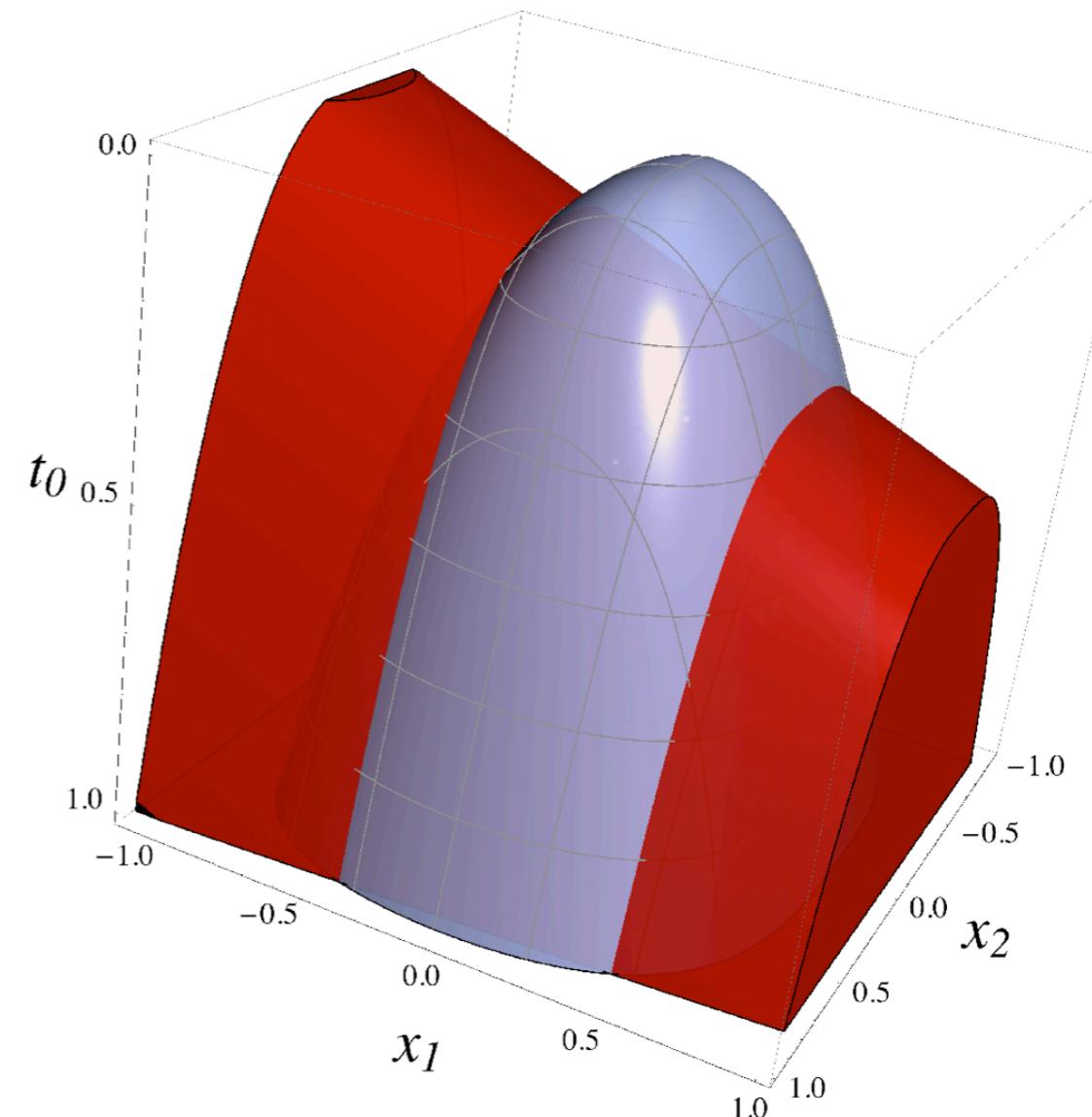


“Simple” Split = Ignores $t_0 : \pi^T x \leq \pi_0 \vee \pi^T x \geq \pi_1$

Cuts for Some General Functions

$$C := \{(x, t_0) \in \mathbb{R}^2 \times \mathbb{R} : \\ \exp(x_1^2) - \exp(0) \\ + |x_2|^3 \leq t_0\}$$

$$C_{e^1, \pi_0} = \{(x, t_0) \in \mathbb{R}^2 \times \mathbb{R} : \\ \exp(x_1^2) - \exp(0) \\ + |x_2|^3 \leq t_0 \\ ax_1 + b + |x_2|^3 \leq t_0\}$$



Cuts for All Paraboloids (Simple Split)

- Modaresi, Kılınç, V. 2012:

$$C := \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \\ \|B(x - c)\|_2^2 \leq t_0\}$$

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- $P_\pi := \left(1/\|\pi\|_2^2\right) \pi \pi^T, P_\pi^\perp := I - P_\pi$
- $\|x\|_2^2 = \|P_\pi x\|_2^2 + \|P_\pi^\perp x\|_2^2$
- $\|P_\pi x\|_2^2 = (1/\|\pi\|)^2 (\pi^T x)^2$

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- $\|x\|_2^2 = \|P_\pi x\|_2^2 + \|P_\pi^\perp x\|_2^2$
- $\|P_\pi x\|_2^2 = (1/\|\pi\|)^2 (\pi^T x)^2$

$$\|x\|_2^2 \leq t_0 \Leftrightarrow f(\pi^T x) + g(P_\pi^\perp x) \leq t_0$$

Cuts for All Paraboloids (Simple Split)

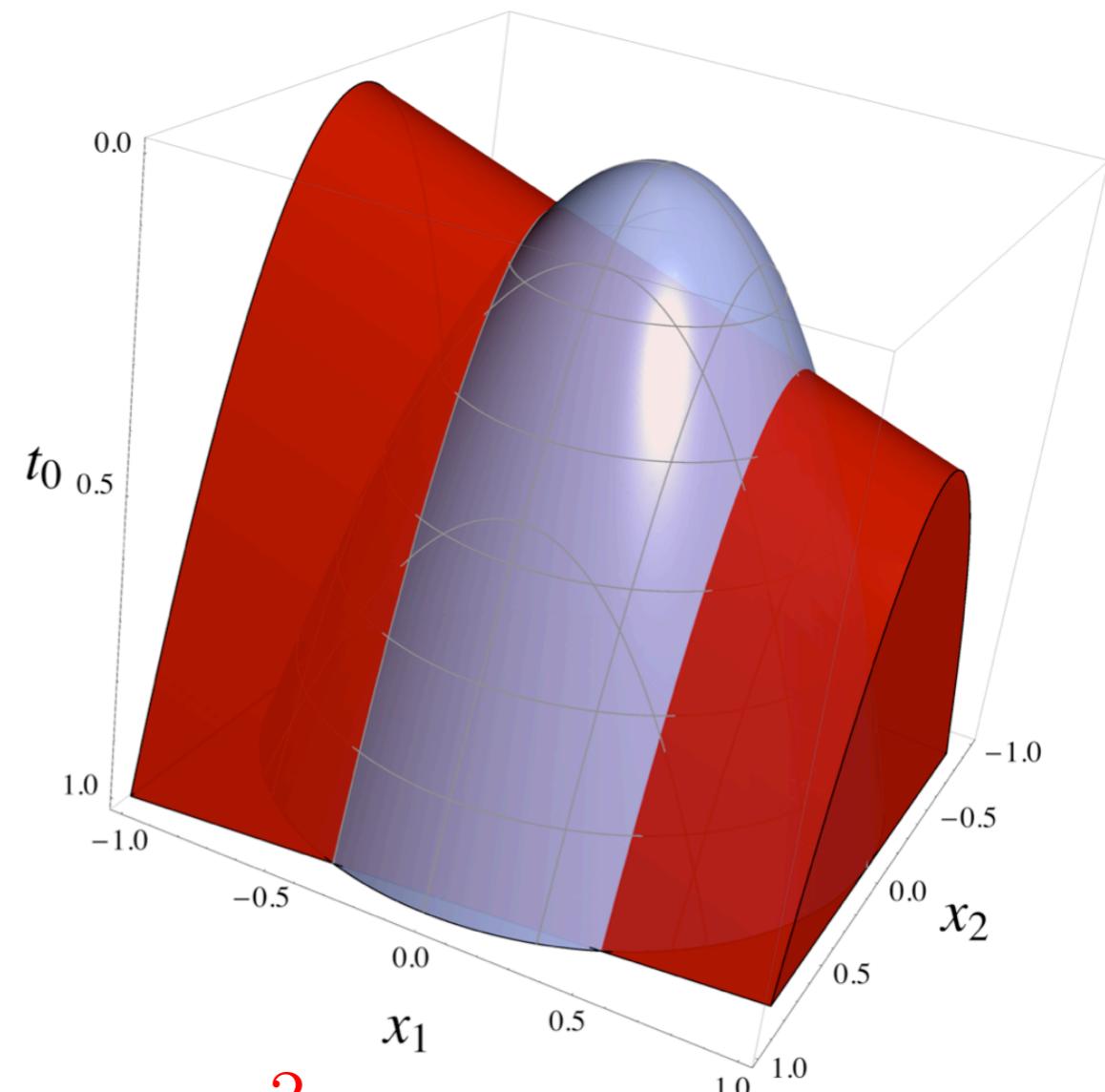
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$$C_{\pi, \pi_0} = \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} :$$

$$\|B(x - c)\|_2^2 \leq t_0$$

$$a\pi^T x + b + \|P_{B^{-T}\pi}^\perp B(x - c)\|_2^2 \leq t_0\}$$



Interpolation for Conic Functions

- Modaresi, Kılınç, V. 2013:

- g is positively homogeneous in addition to convex.

$$C := \left\{ (x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \left(|\pi^T x|^p + g(P_\pi^\perp x)^p \right)^{1/p} \leq t_0 \right\}$$

$$C_{\pi, \pi_0} := \left\{ (x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \left(|\pi^T x|^p + g(P_\pi^\perp x)^p \right)^{1/p} \leq t_0 \right. \\ \left. \left(|a\pi^T x + b|^p + g(P_\pi^\perp x)^p \right)^{1/p} \leq t_0 \right\}$$

- See also Atamturk and Narayanan 2010.

Cuts for Some General Functions

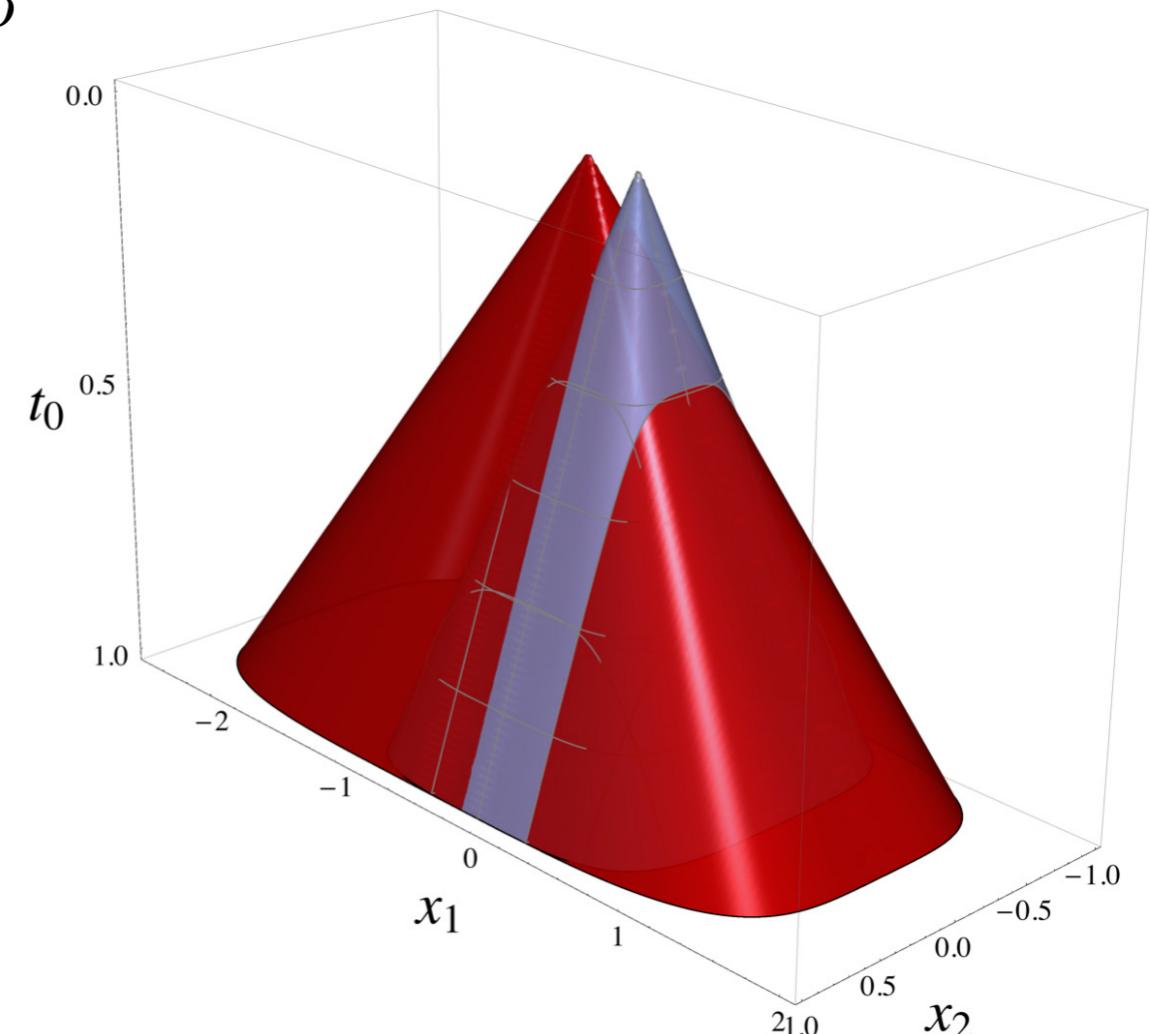
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$$\|x - c\|_p := \left(\sum_{i=0}^n (x_i - c_i)^p \right)^{1/p}$$

$$C := \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \|x - c\|_p \leq t_0\}$$

$$C_{e^1, \pi_0} := \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \|x - c\|_p \leq t_0,$$

$$\left((\alpha(x_1 - d_1) + \beta)^p + \sum_{i=2}^n (x_i - d_i)^p \right)^{1/p} \leq t_0 \}$$



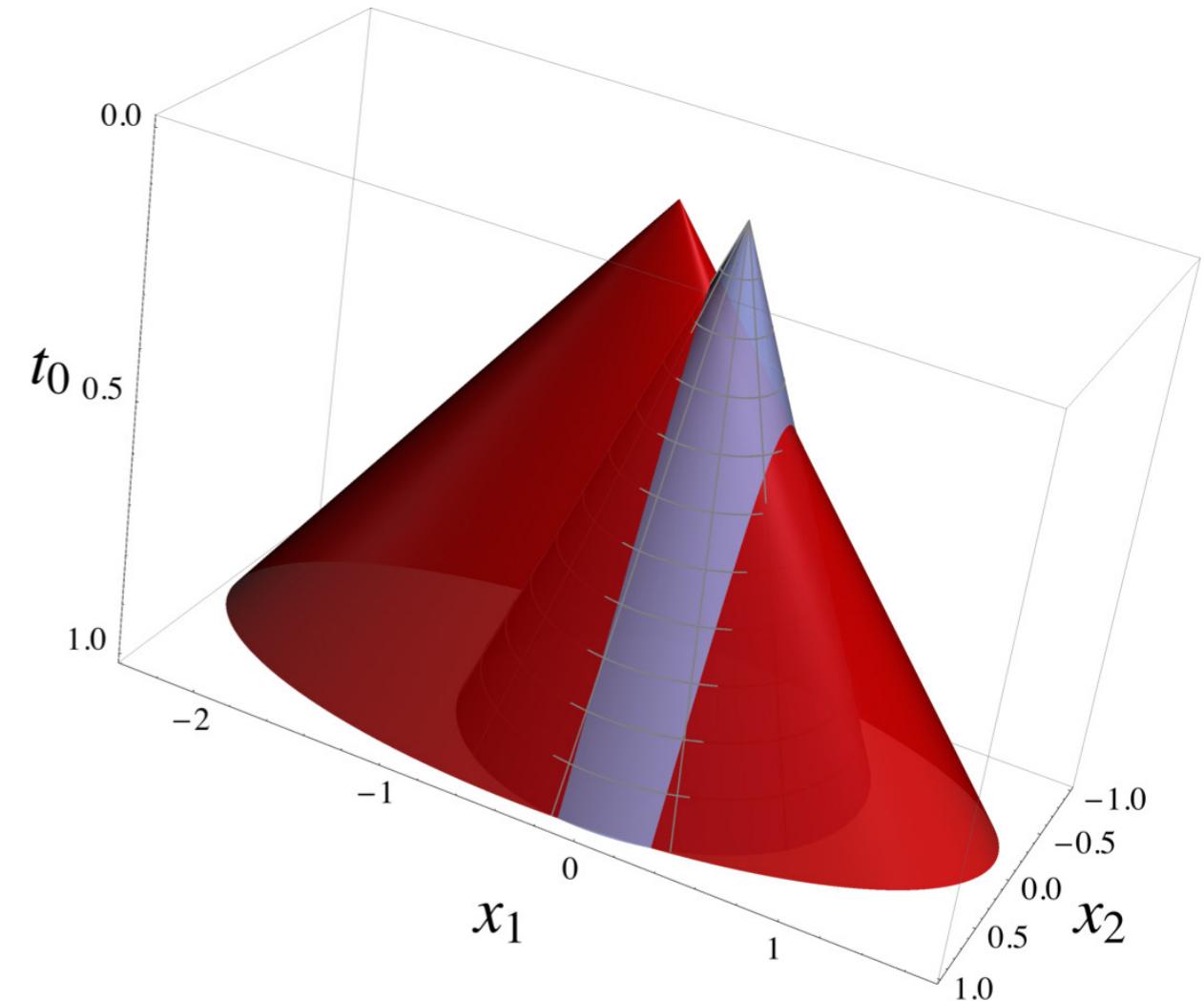
Cuts for All Quadratic Cones (Simple)

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$$C_{\pi, \pi_0} = \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \|B(x - c)\|_2 \leq t_0$$

$$\left((a\pi^T x + b)^2 + \|P_{B-T}^\perp \pi B(x - c)\|_2^2 \right)^{1/2} \leq t_0 \}$$

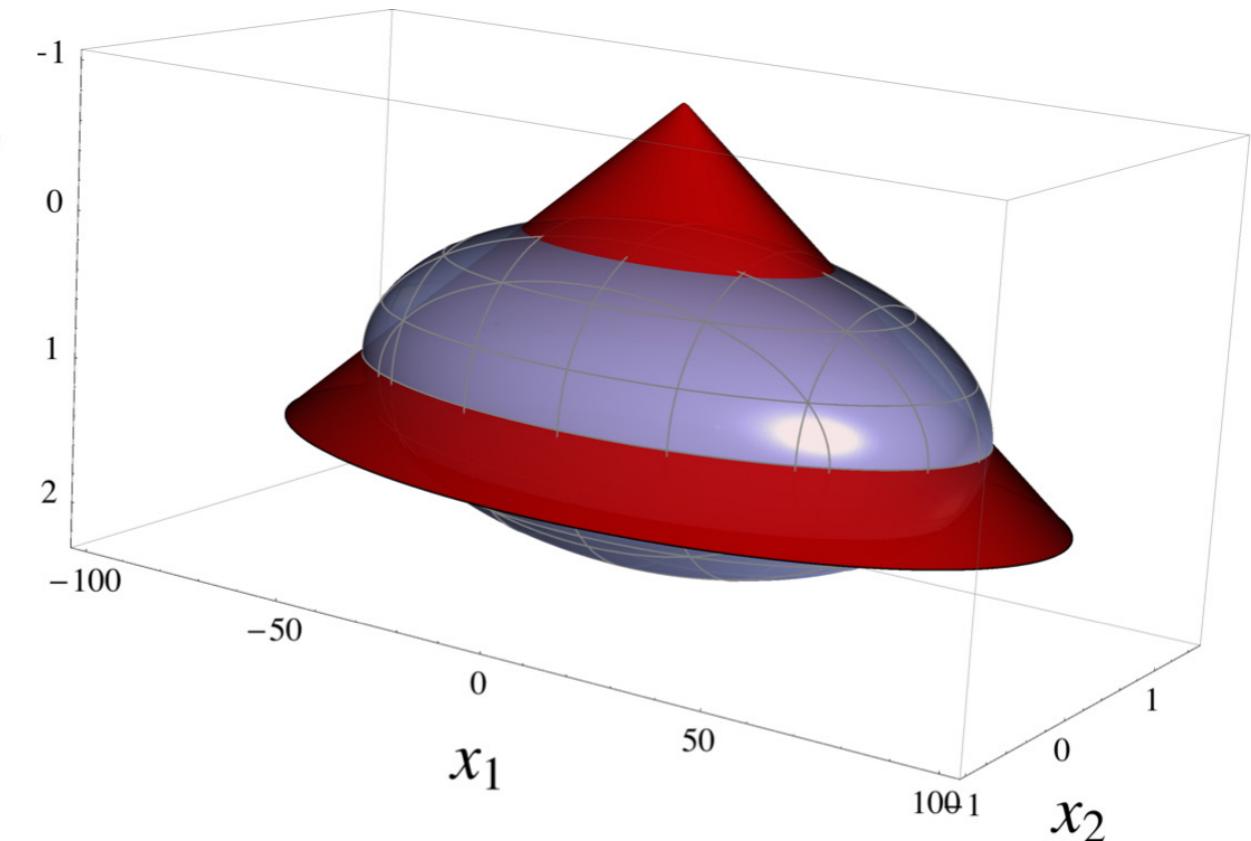
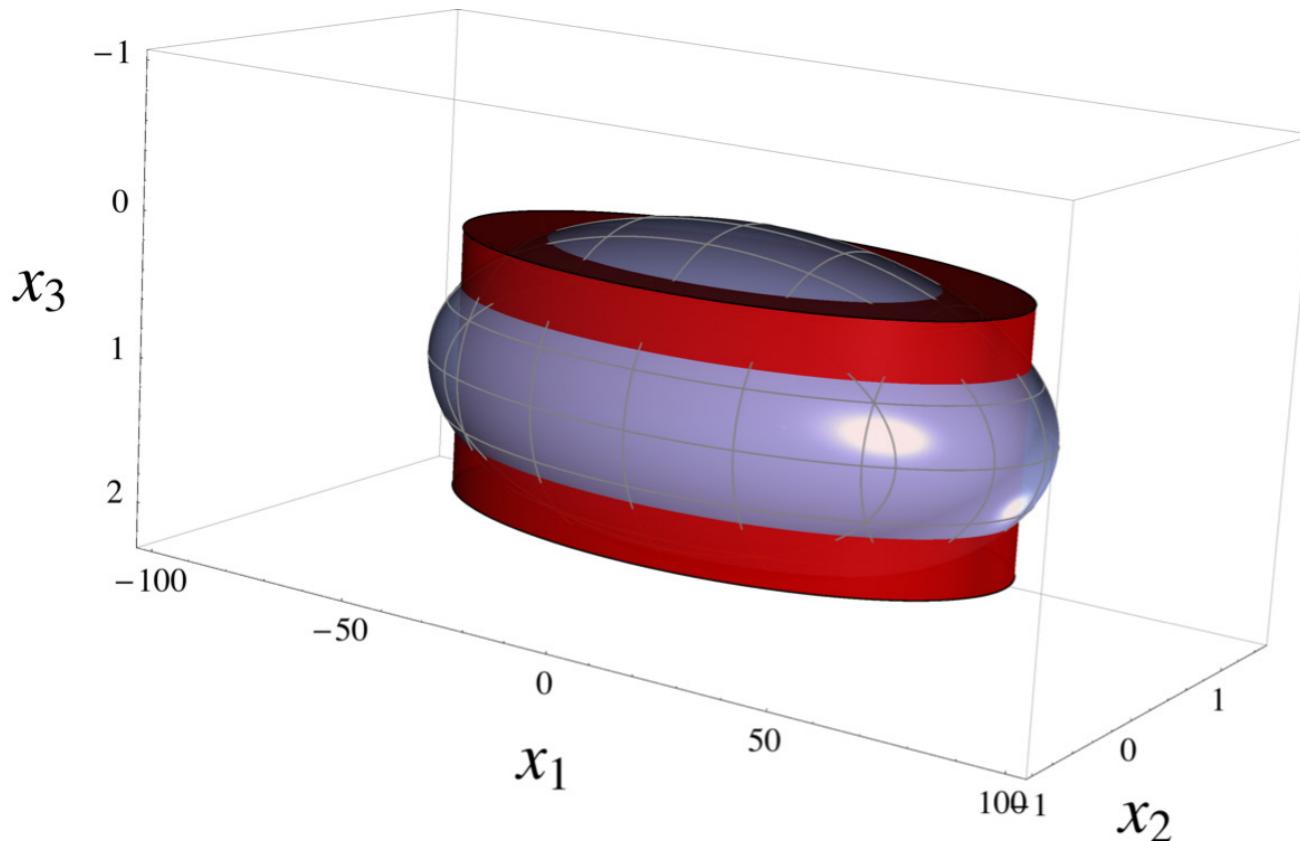


Similarly: Cuts for All Ellipsoids

- Dadush, Dey and V. 2011

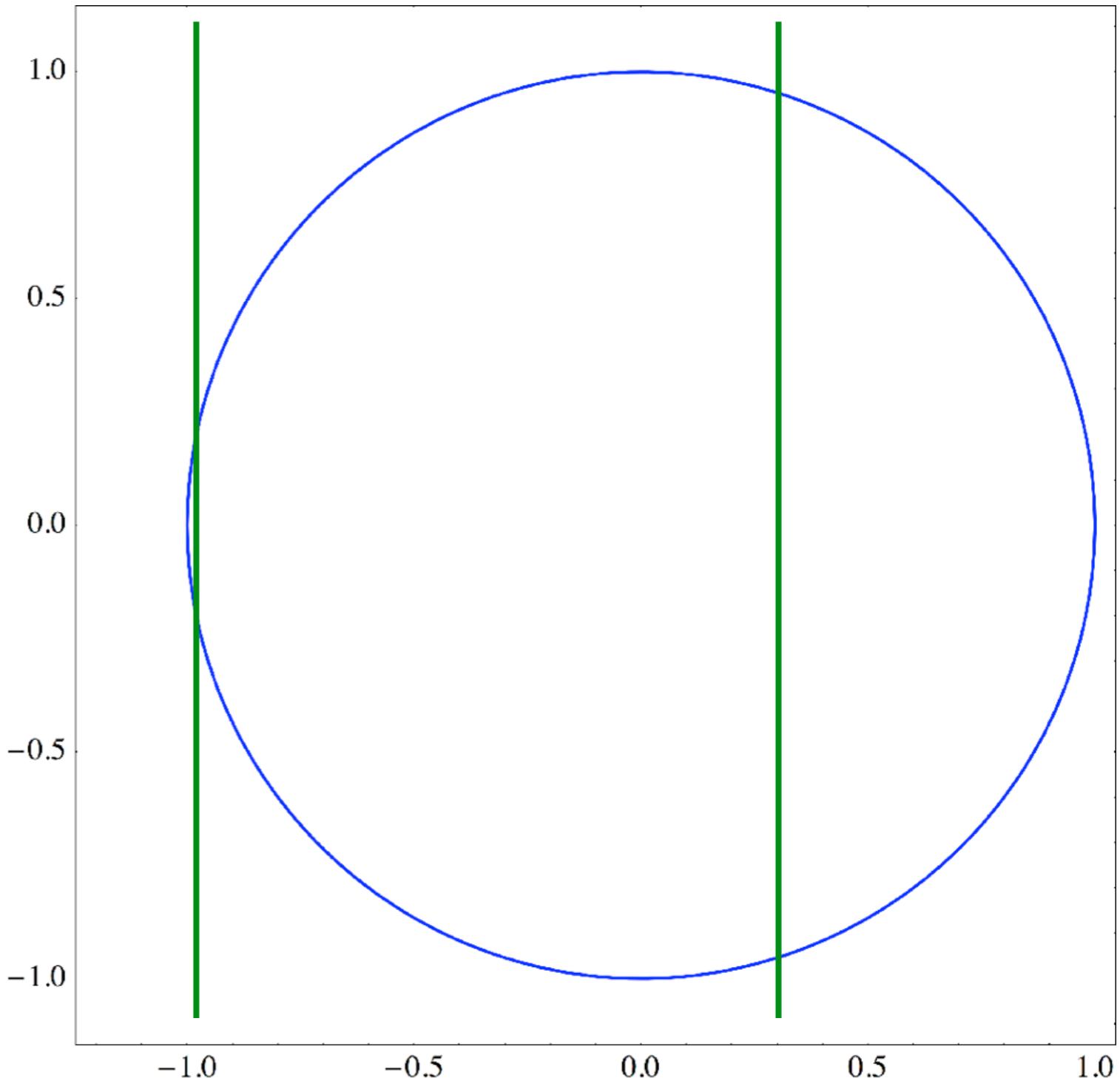
- g is positively homogeneous in addition to convex.

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Interpolation “Form” is Crucial

$$\sqrt{x_1^2 + x_2^2} \leq t_0, \quad t_0 = 1$$

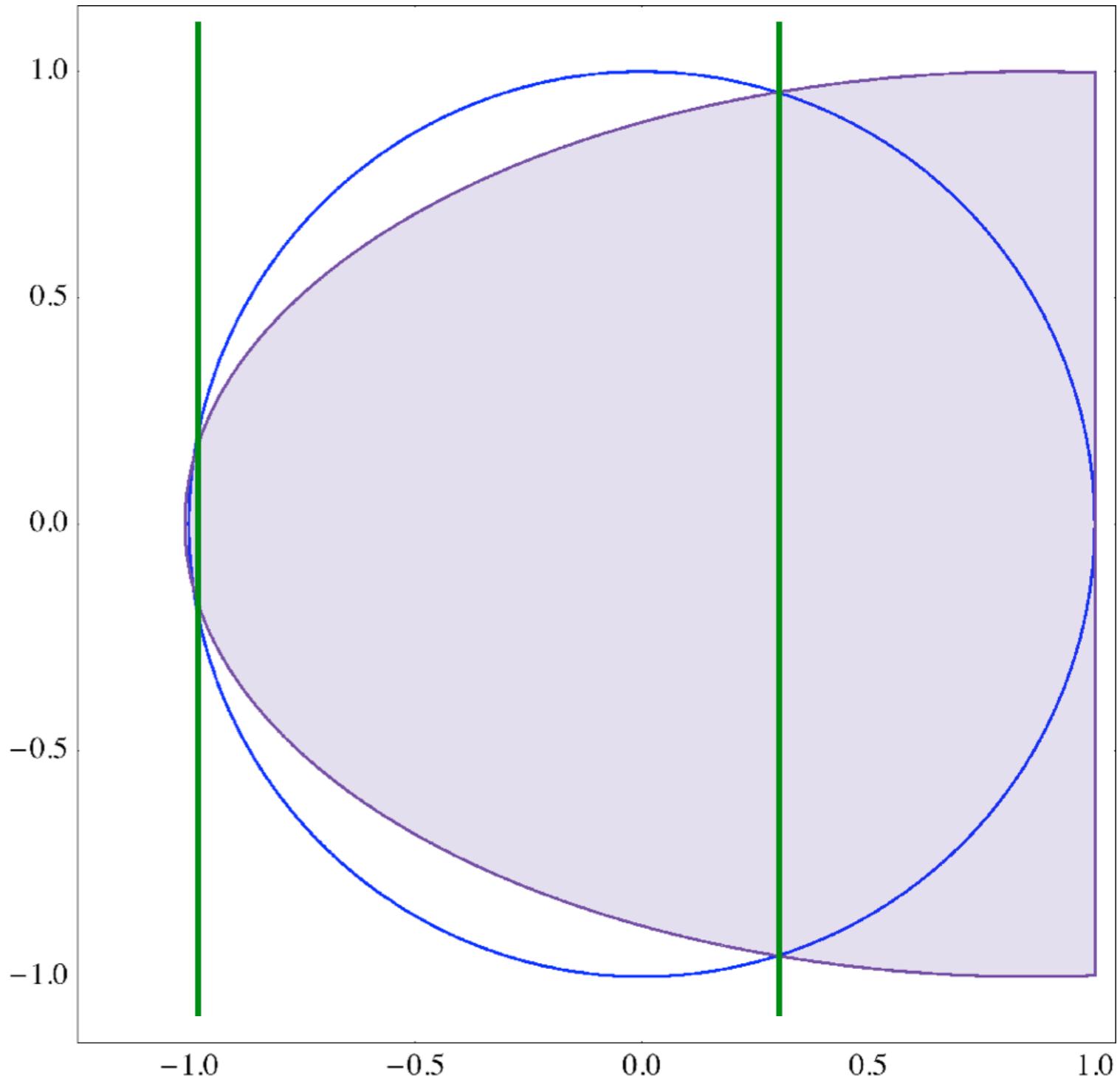


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Cone

$$\sqrt{(ax_1 + b)^2 + x_2^2} \leq 1$$

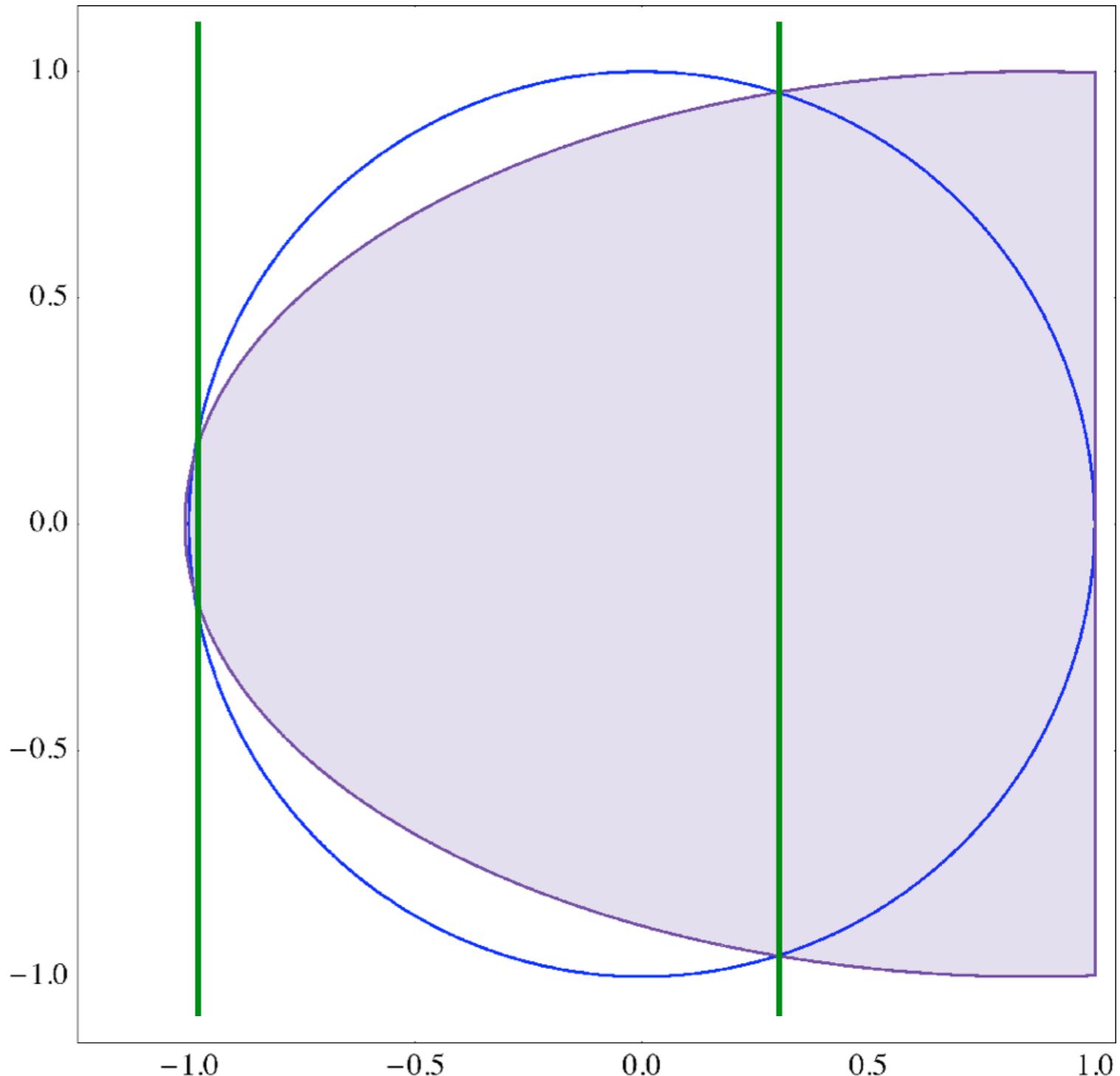


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Interpolation “Form” is Crucial

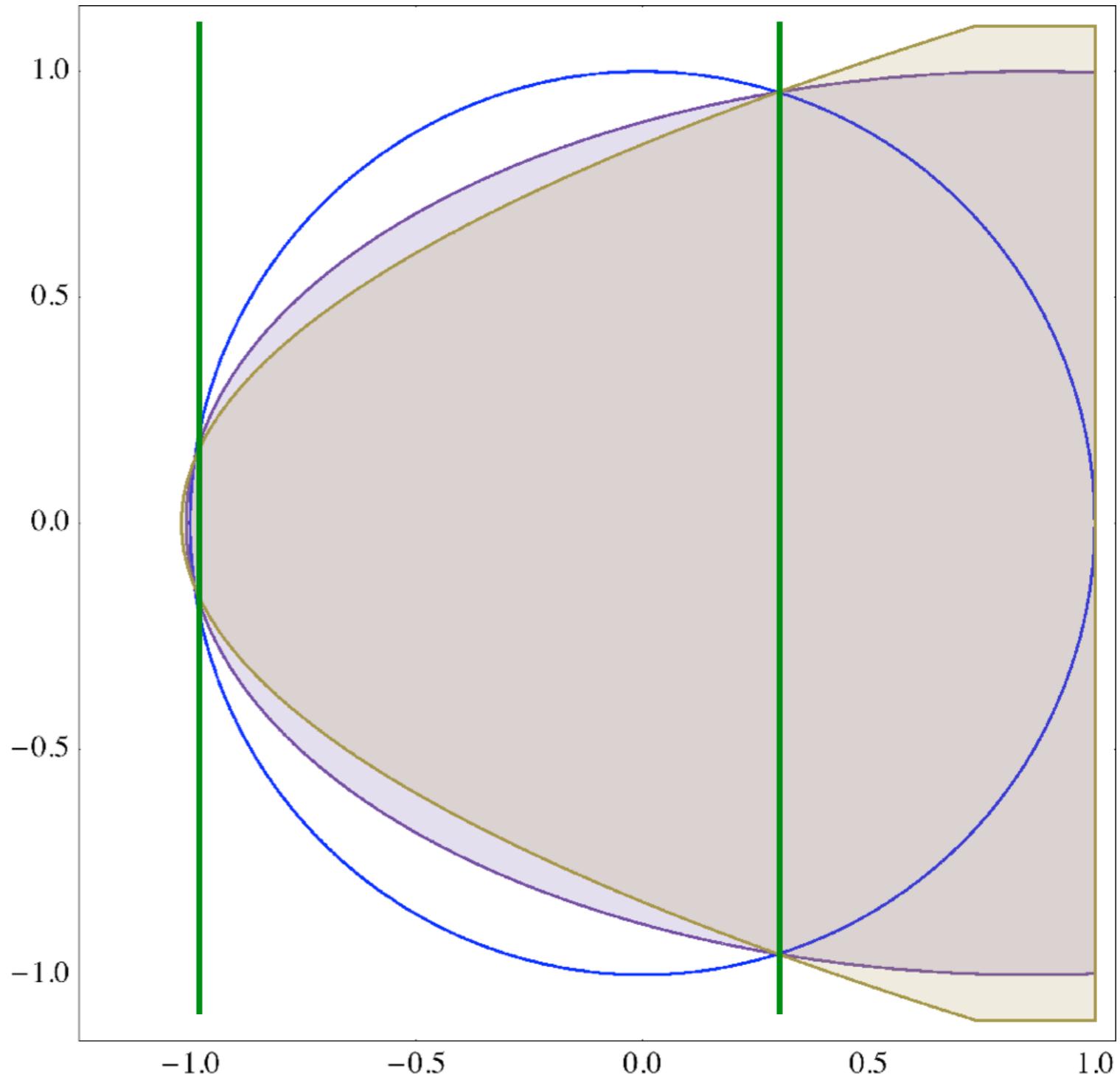
$$x_1^2 + x_2^2 \leq t_0, \quad t_0 = 1$$

Cone

$$\sqrt{(ax_1 + b)^2 + x_2^2} \leq 1$$

Paraboloid

$$ax_1 + b + x_2^2 \leq 1$$



Interpolation “Form” is Crucial

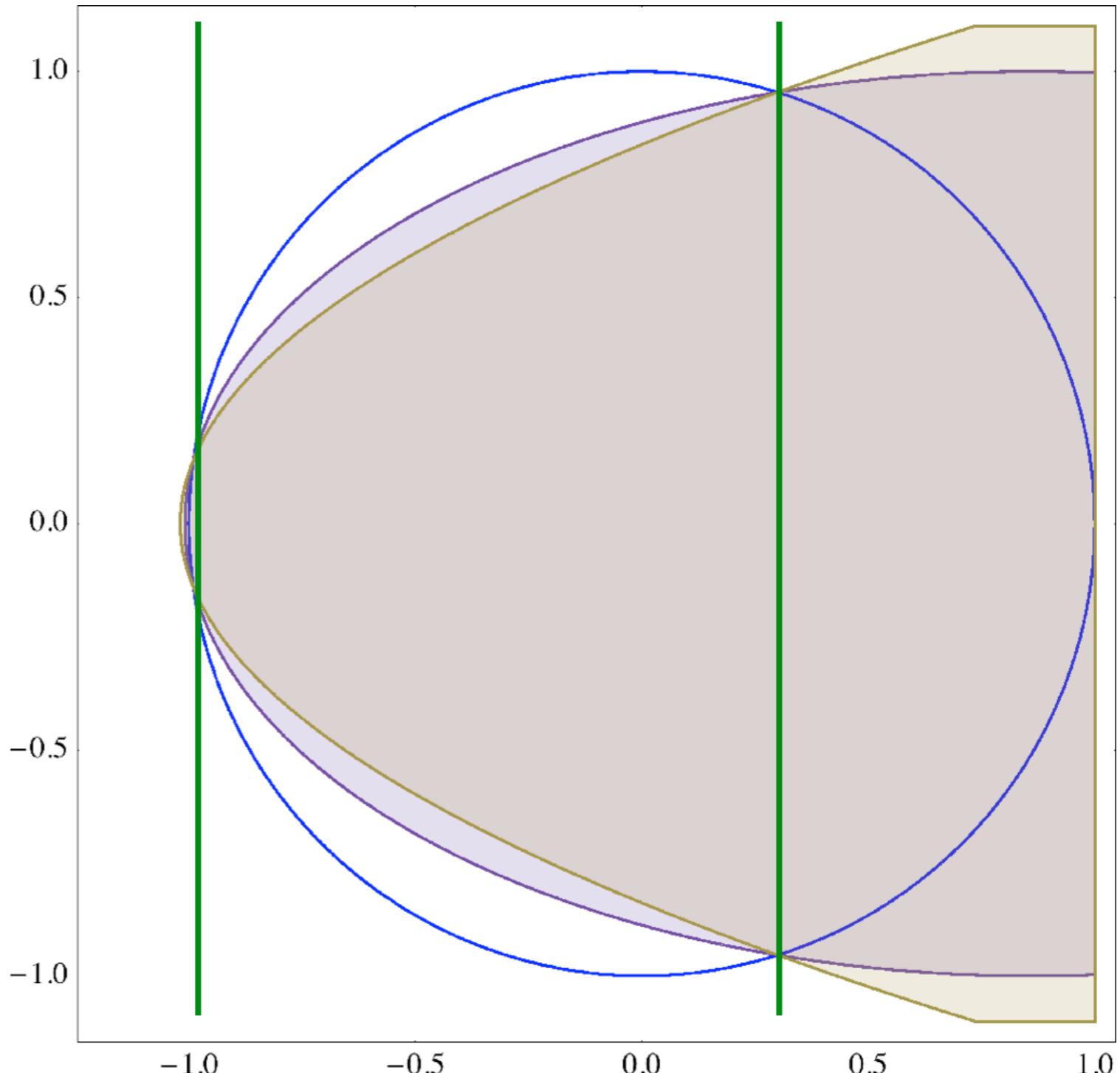
$$-\sqrt{1 - x_1^2} + |x_2| \leq t_0$$

Cone $t_0 = 0$

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Paraboloid

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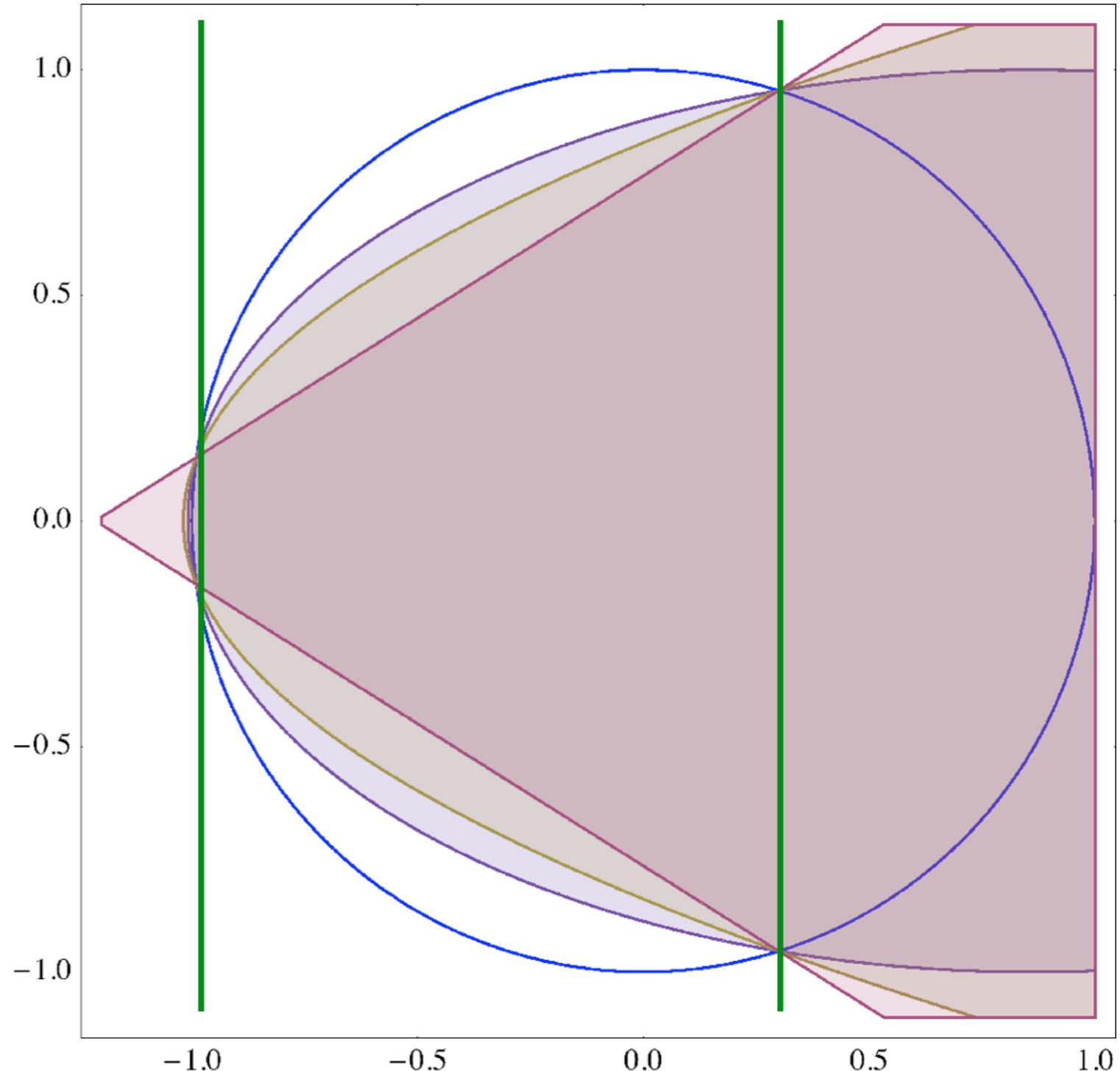
$$\sqrt{(ax_1 + b)^2 + x_2^2} \leq 1$$

Paraboloid

$$ax_1 + b + x_2^2 \leq 1$$

Ellipsoid

$$ax_1 + b + |x_2| \leq 0$$



Extensions 1: Other QP Split Cuts

- General Split Cuts: $\pi^T x + \hat{\pi}t_0 \leq \pi_0 \vee \pi^T x + \hat{\pi}t_0 \geq \pi_1$

- Paraboloid $\|x\|_2^2 \leq t_0$ and Cone $\|x\|_2 \leq t_0$

$$\left\| P_\pi^\perp x + (a\pi^T x + b) \pi \right\|_2 \leq c\pi^T x + dt_0 + e$$

- Other “Simple” Split Cuts: $\pi^T x \leq \pi_0 \vee \pi^T x \geq \pi_1$

- Hyperboloids: $\sqrt{\|x\|_2^2 + l^2} \leq t_0$

$$\left\| P_\pi^\perp x + (a\pi^T x + b) \pi \right\|_2 \leq t_0$$

Extensions 2: Parabolic n-split cuts

$$C := \{(x, t_0) : \|B(x - c)\|_2^2 \leq t_0\}$$

$$B^{-T} \pi_i \perp B^{-T} \pi_j$$

$$S := \bigcup_{i=1}^n \{(x, t_0) : \pi_0^i \leq \pi_i^T x \leq \pi_1^i\}$$

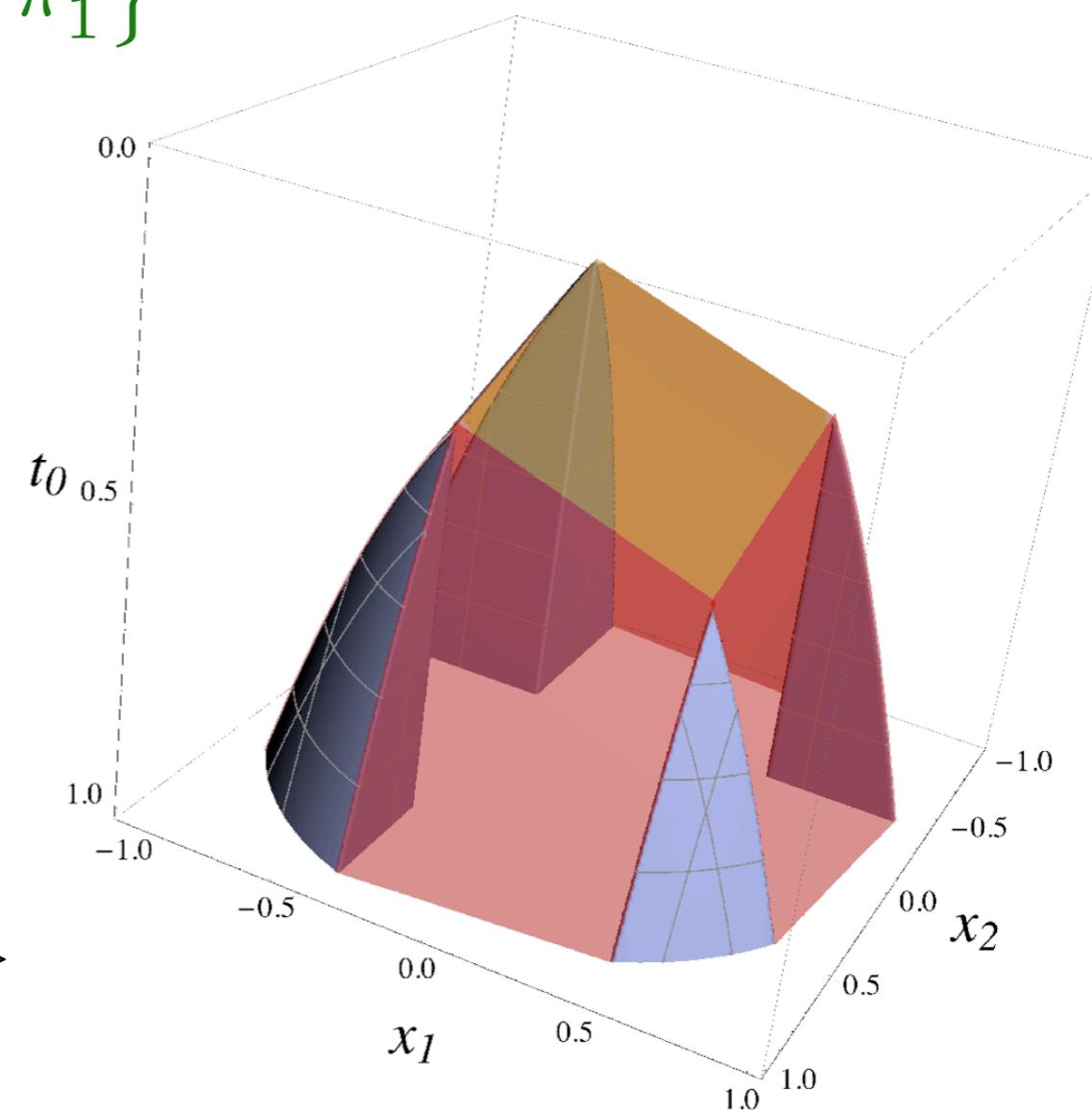
$$\text{conv}(C \setminus S) =$$

$$\{(x, t_0) : \|B(x - c)\|_2^2 \leq t_0,$$

$$\sum_{j \in J} (a_j \pi_j^T x + b_j)$$

$$+ \sum_{j \notin J} d_j (\pi_j^T (x - c))^2 \leq t_0$$

$$J \subseteq \{1, \dots, n\}\}$$



Extended QCP: Parabolic n-split cuts

$$C := \{(x, t_0) : \|B(x - c)\|_2^2 \leq t_0\} \quad B = I, \quad c_i = 1/2, \quad \pi_i = e^i$$

$$S := \bigcup_{i=1}^n \{(x, t_0) : \pi_0^i \leq \pi_i^T x \leq \pi_1^i\}$$

$$\text{conv}(C \setminus S) =$$

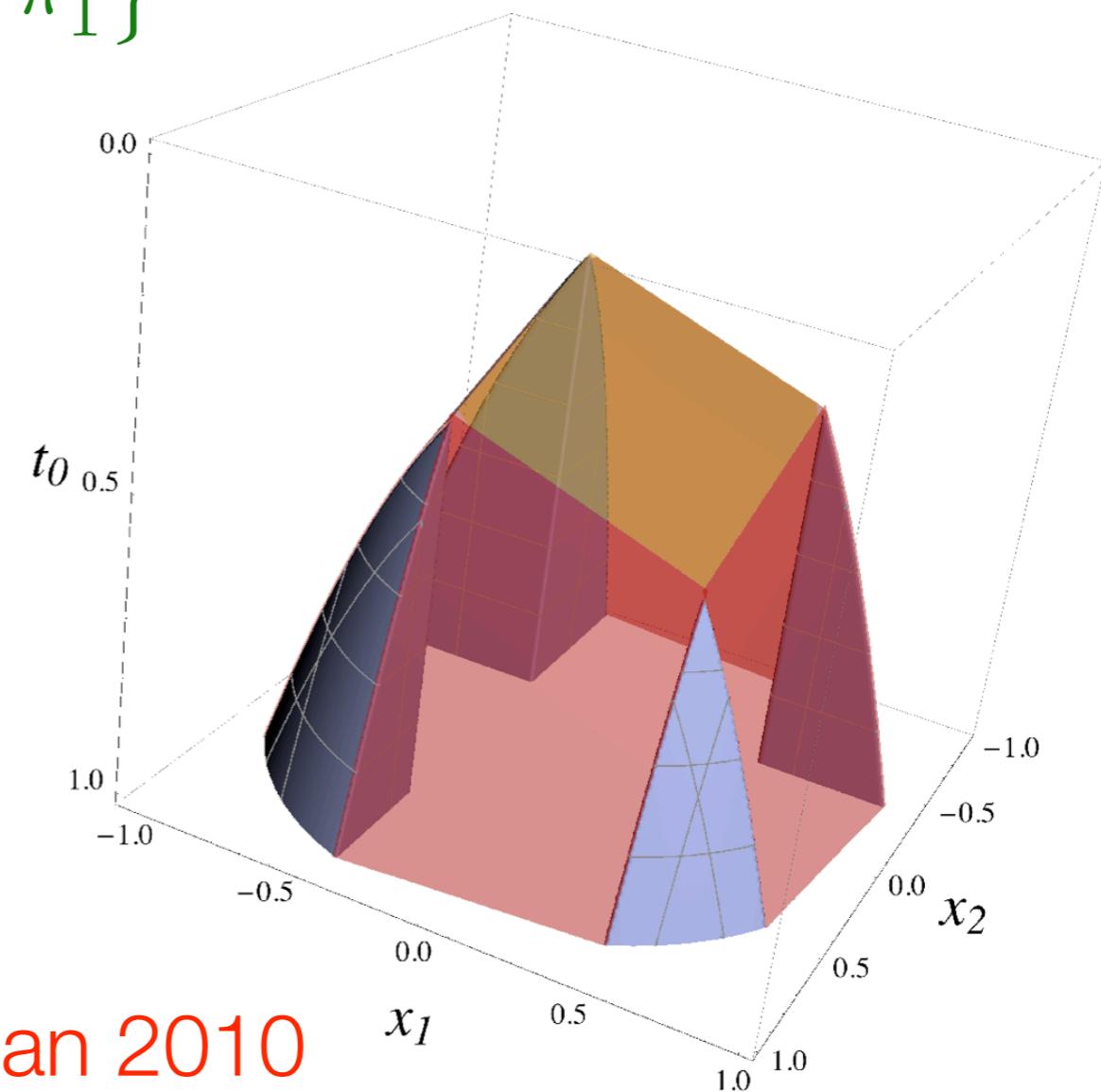
$$\{(x, t, t_0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} :$$

$$\|t\|_2^2 \leq t_0,$$

$$|x_i - c_i| \leq t_i, \forall i$$

$$(1 - 2F(c_i))(x_i - \lfloor c_i \rfloor)$$

$$+ F(c_i) \leq t_i, \forall i\}$$



Conic MIR of Atamturk and Narayanan 2010

Aggregation

Aggregation \approx SDP Approach

$$C := \{(x, t_0) \in \mathbb{R}^{n+1} : f(x) \leq t_0\}$$

$$S := \{(x, t_0) \in \mathbb{R}^{n+1} : g(x) > t_0\}$$

f convex, g concave.

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Aggregation \approx SDP Approach

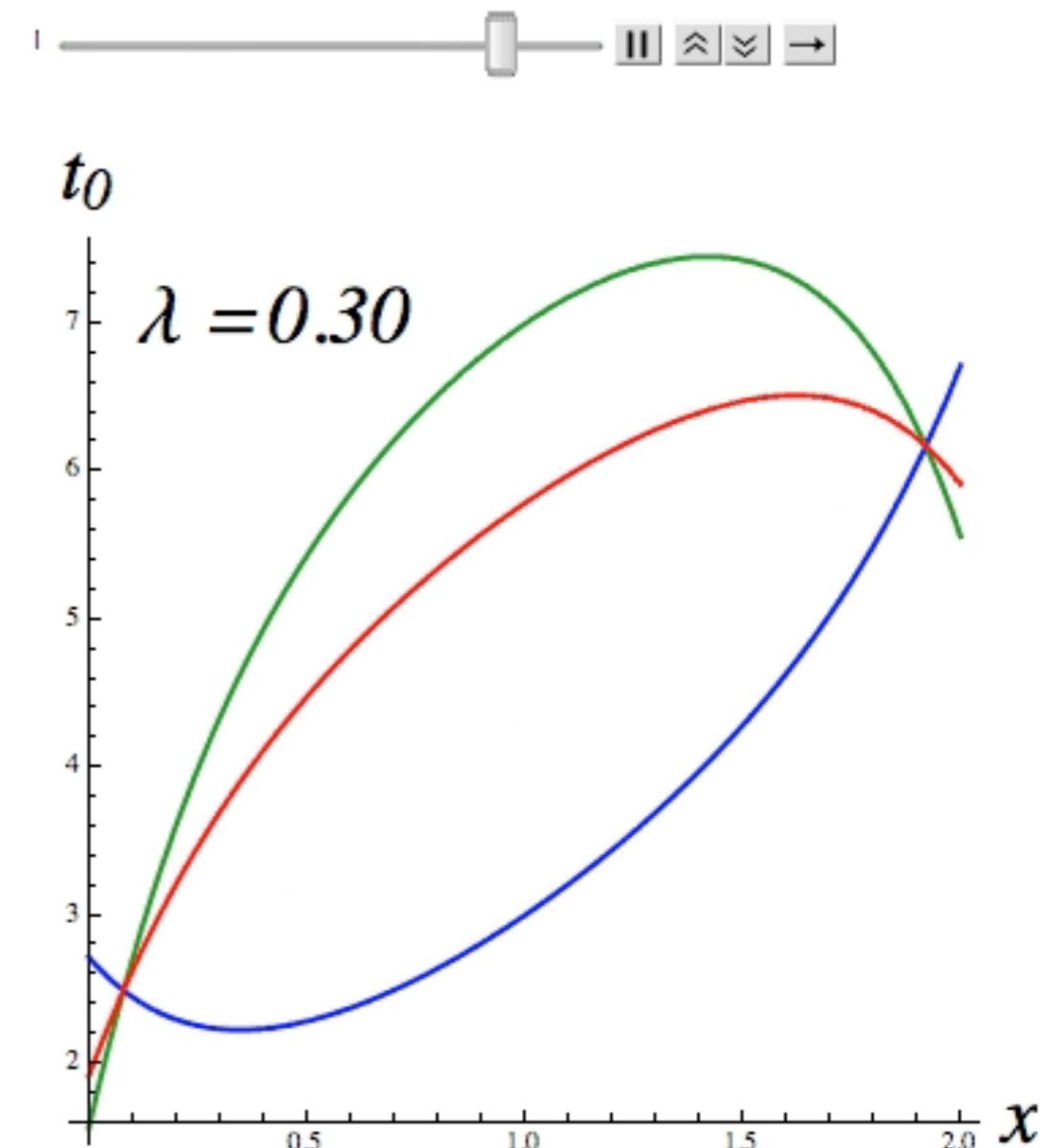
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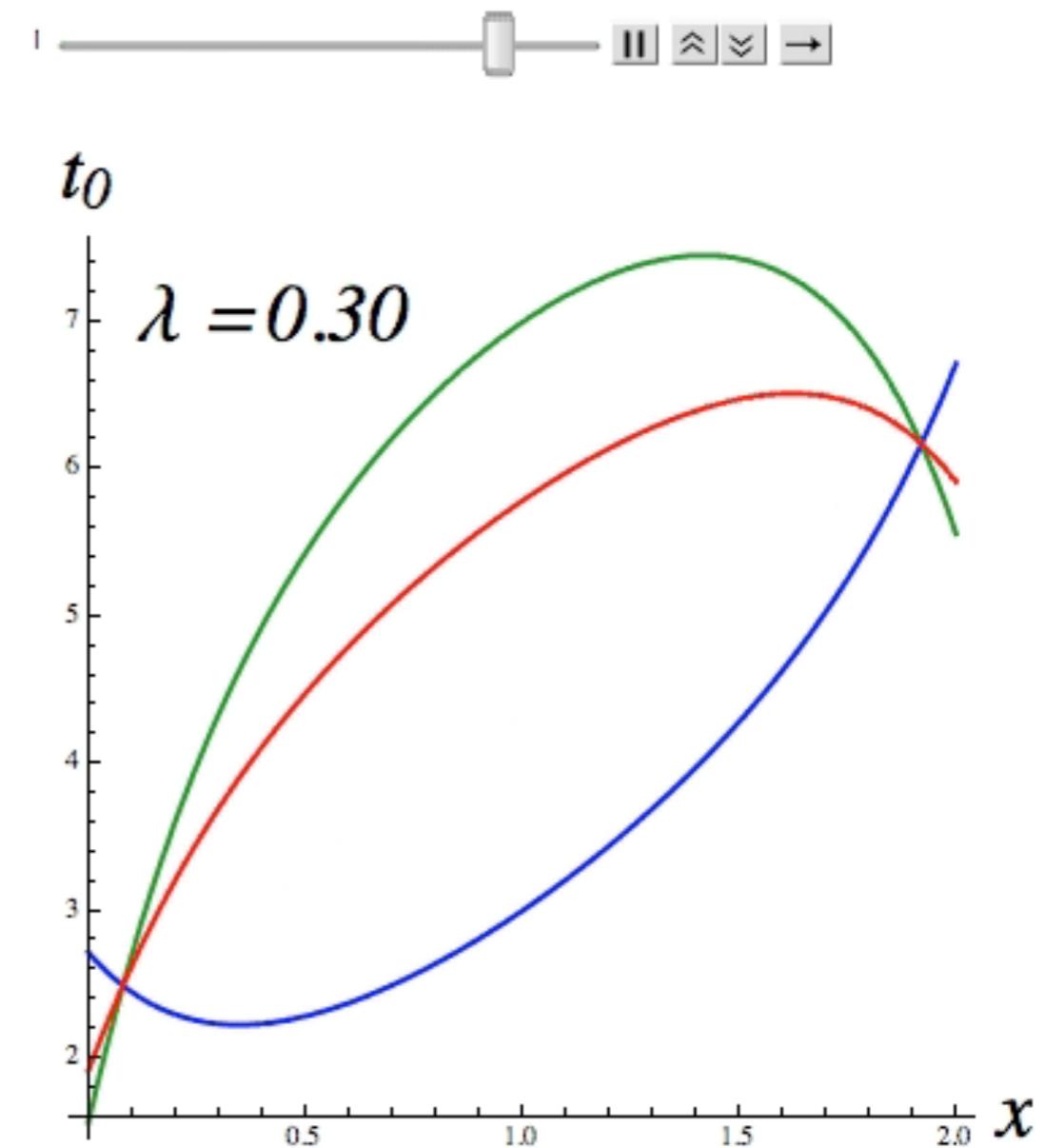
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$$\lambda^* := \min\{\lambda \in [0, 1] : h_\lambda \text{ convex}\}$$



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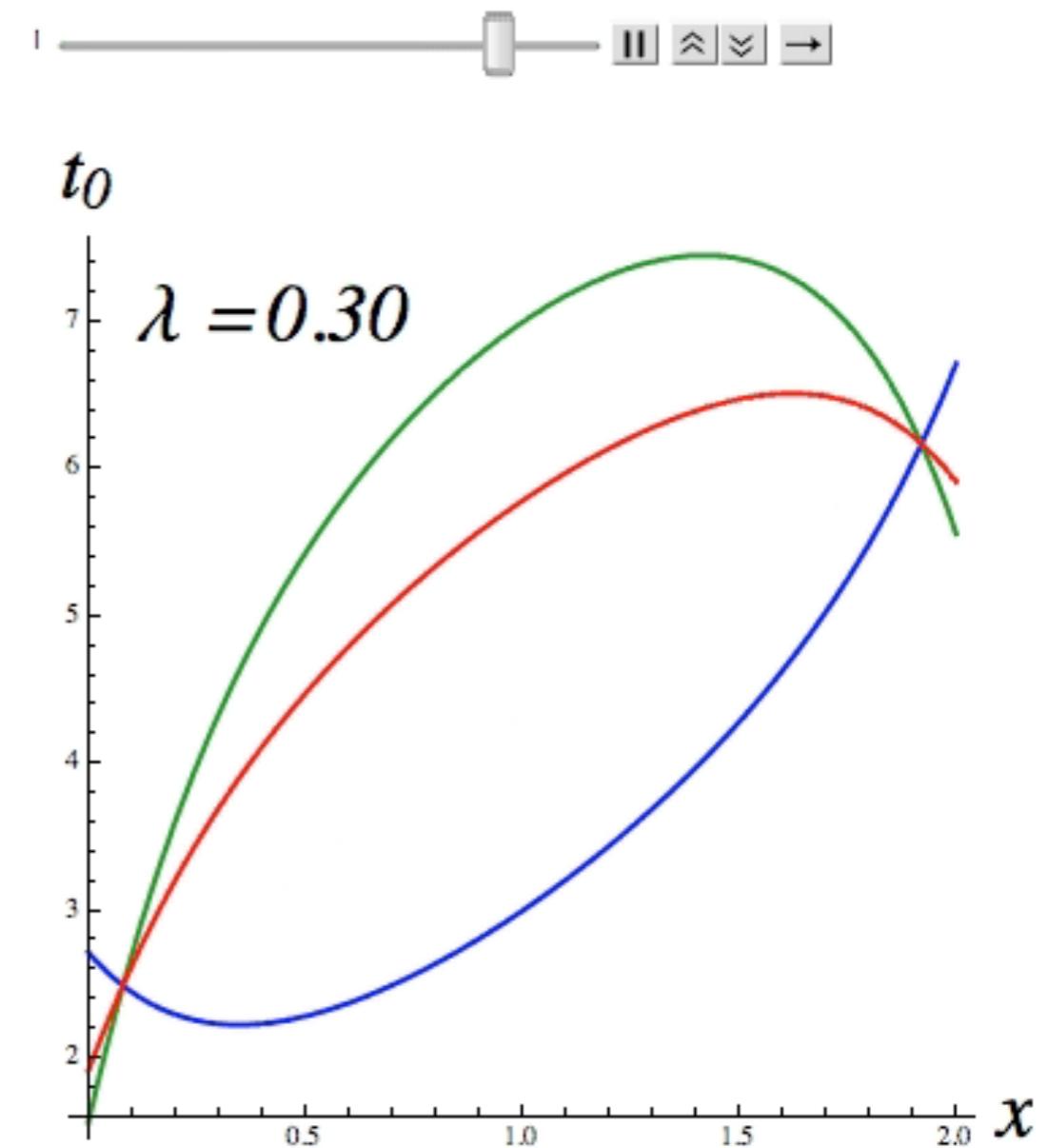
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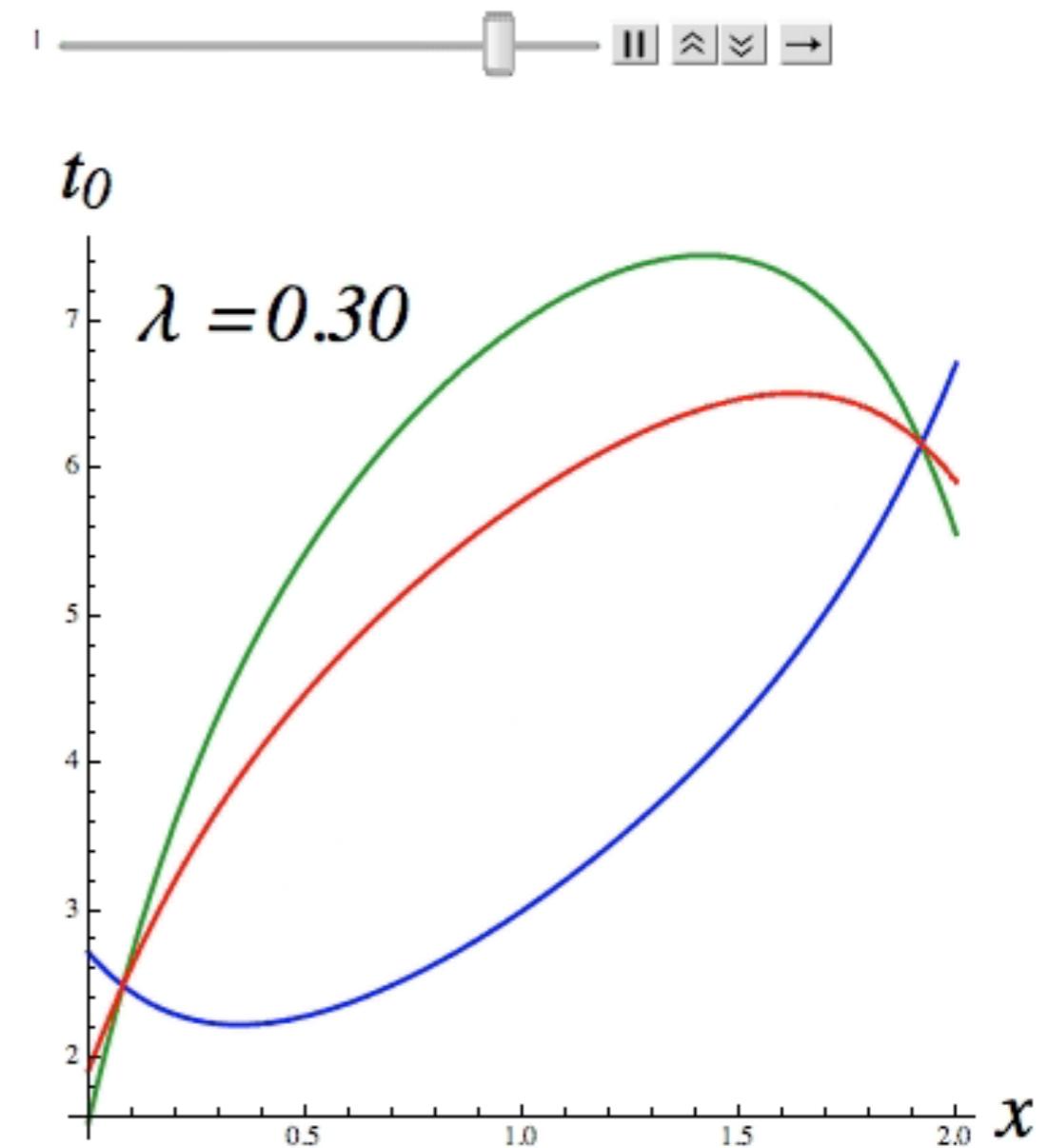
$$S := \{(x, t_0) \in \mathbb{R}^{n+1} : g(x) > t_0\}$$

f convex, g concave.

$$\text{conv}(C \setminus S) = \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \\ f(x) \leq t_0 \\ h_{\lambda^*}(x) \leq t_0\}$$

$$h_\lambda(x) := \lambda f(x) + (1 - \lambda)g(x)$$

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A Case Where Aggregation Works

- Modaresi, Kılınç, V. 2013:

$$C := \{(x, t_0) \in \mathbb{R}^{n+1} : f(x) \leq t_0\}$$

$$S := \{(x, t_0) \in \mathbb{R}^{n+1} : g(x) > \gamma t_0\}$$

$$f(x) = \sum_{i=1}^n w_i (a_i^T x) + m^T x + r$$

$$g(x) = - \sum_{i=1}^n \alpha_i w_i (a_i^T x) - l^T x - q$$

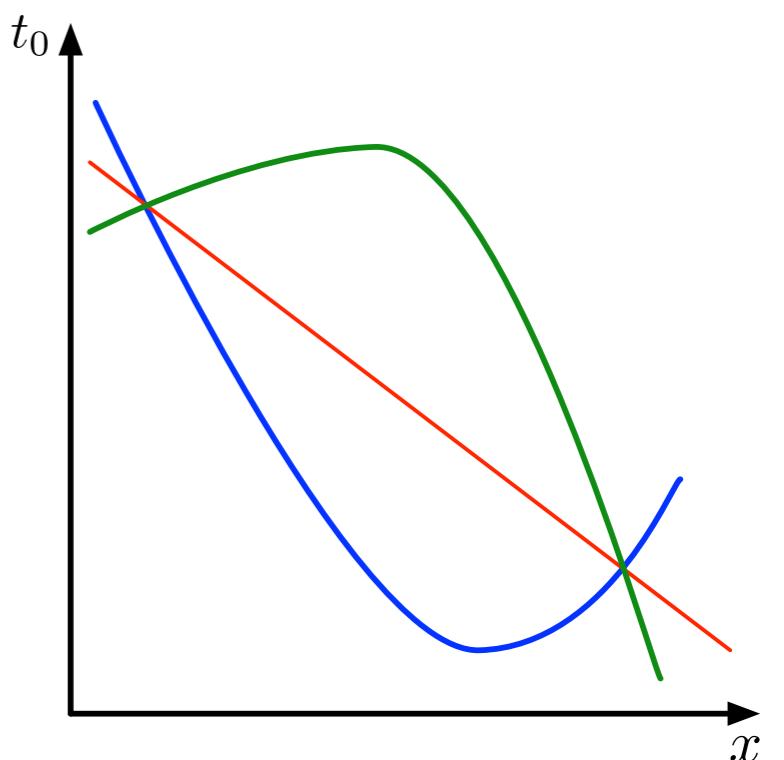
$$h_{\lambda^*}(x) := \frac{f(x) + (1/\alpha_n)g(x)}{1 + \gamma/\alpha_n}$$

$$a_i \perp a_j, \quad i \neq j$$

$$0 \neq \alpha_n \geq \alpha_i$$

w_i are convex

w_n is “coercive”



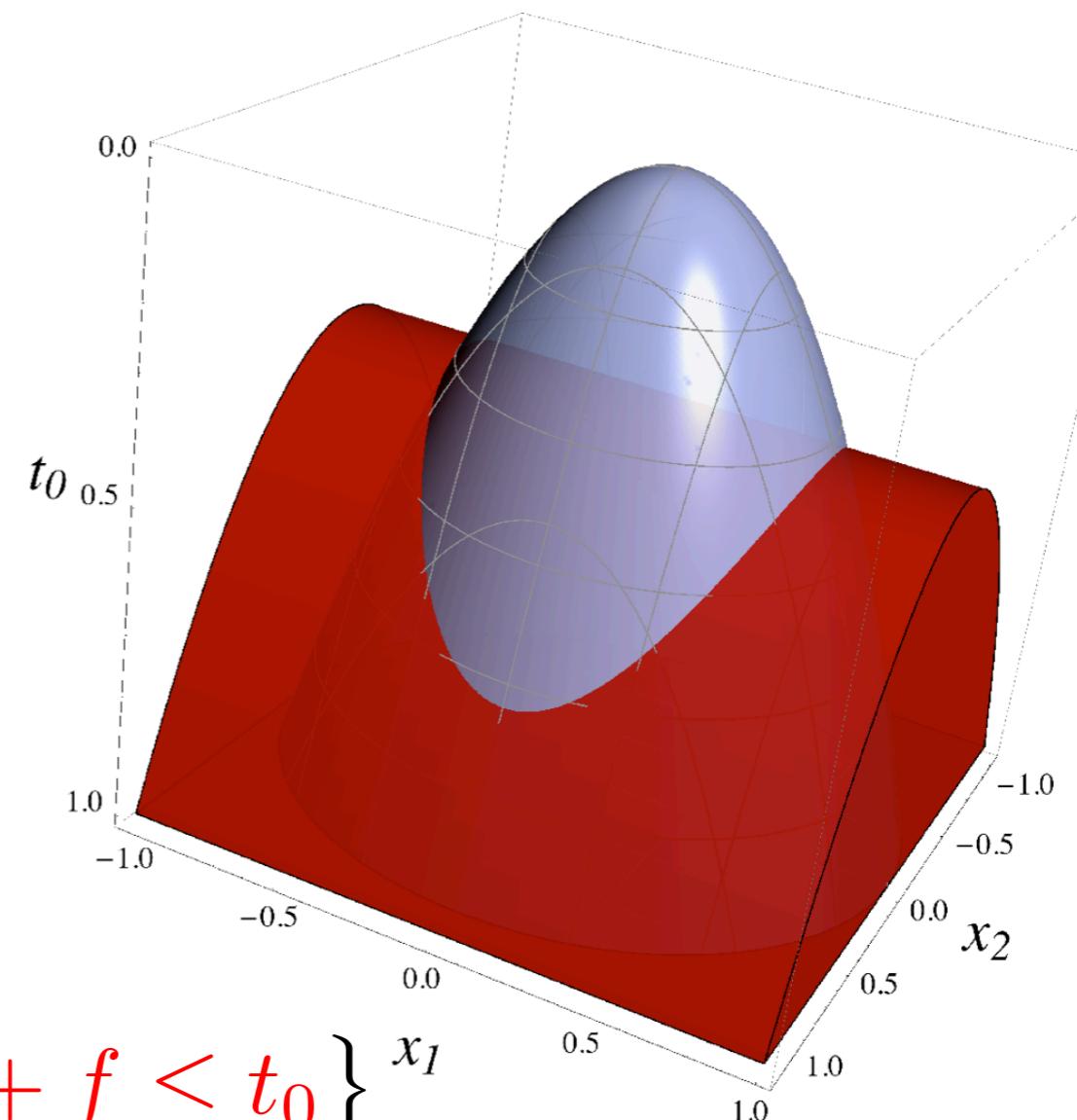
Case Gives Cuts for All Paraboloids

- Modaresi, Kılınç, V. 2013:

$$C := \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \|B(x - c)\|_2^2 \leq t_0\}$$

$$S := \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : \|A(x - d)\|_2^2 \leq 1\}$$

$$\text{conv}(C \setminus S) = \{(x, t_0) \in \mathbb{R}^n \times \mathbb{R} : f(x) \leq t_0, x^T E x + a^T x + f \leq t_0\}$$



- Also Bienstock and Michalka 2011 and 2013.

Summary

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- Two techniques for non-linear cuts:
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 - General geometry for quadratic sets.
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 - Aggregation technique cannot yield conic cuts.
- Cut Name? Intersection/Concavity Cuts?