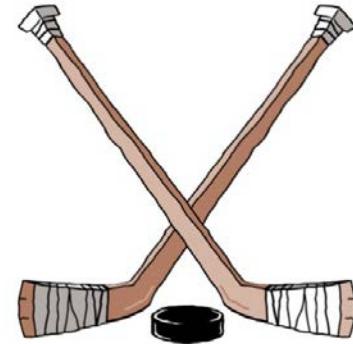


# Winning at Daily Fantasy Hockey Using Analytics

David Hunter,  
Juan Pablo Vielma (@J\_P\_Vielma), and  
Tauhid Zaman (@zlisto)



# Example Entry

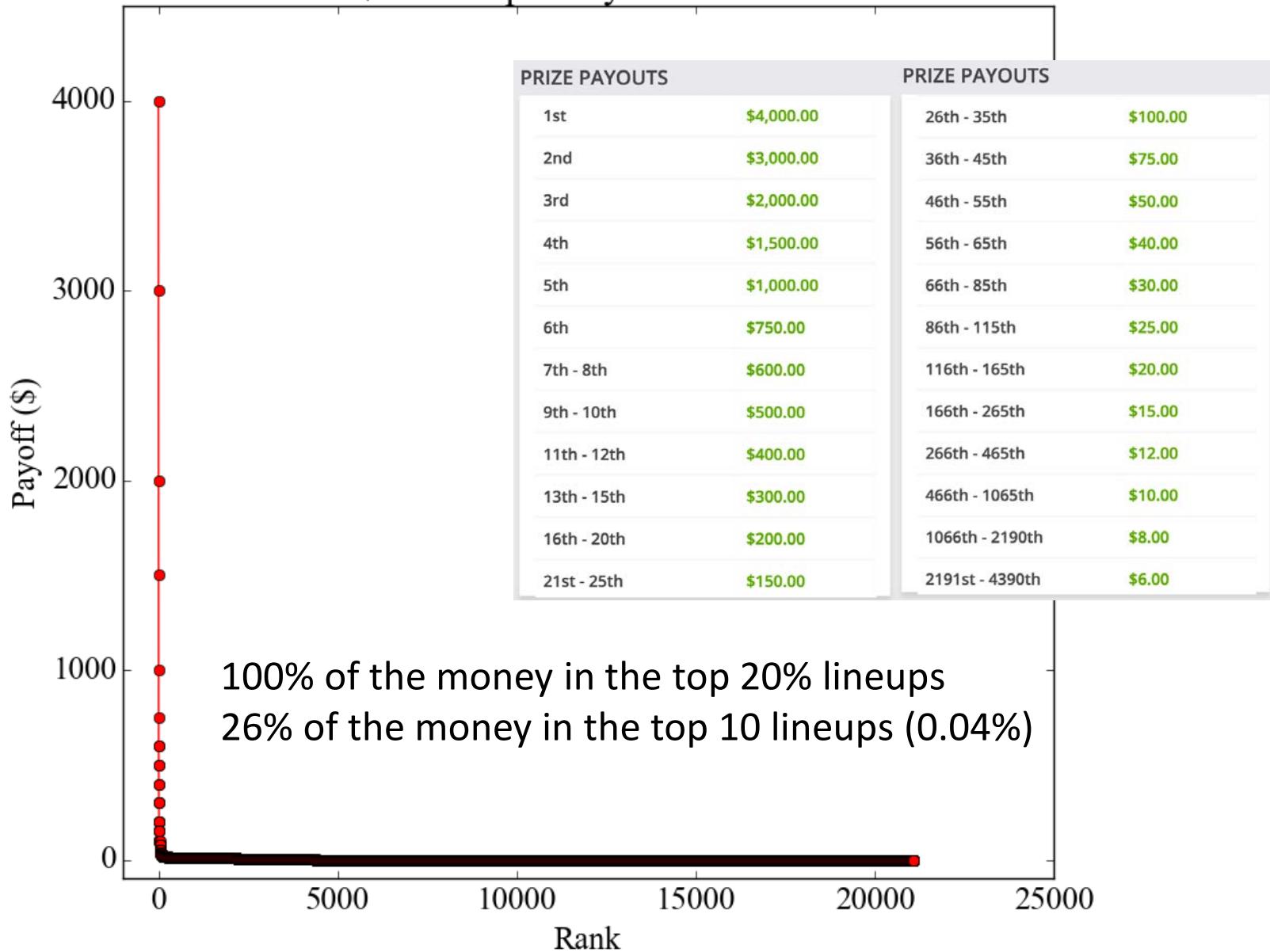


Avg. Rem. / Player: \$0  
Rem. Salary: \$0

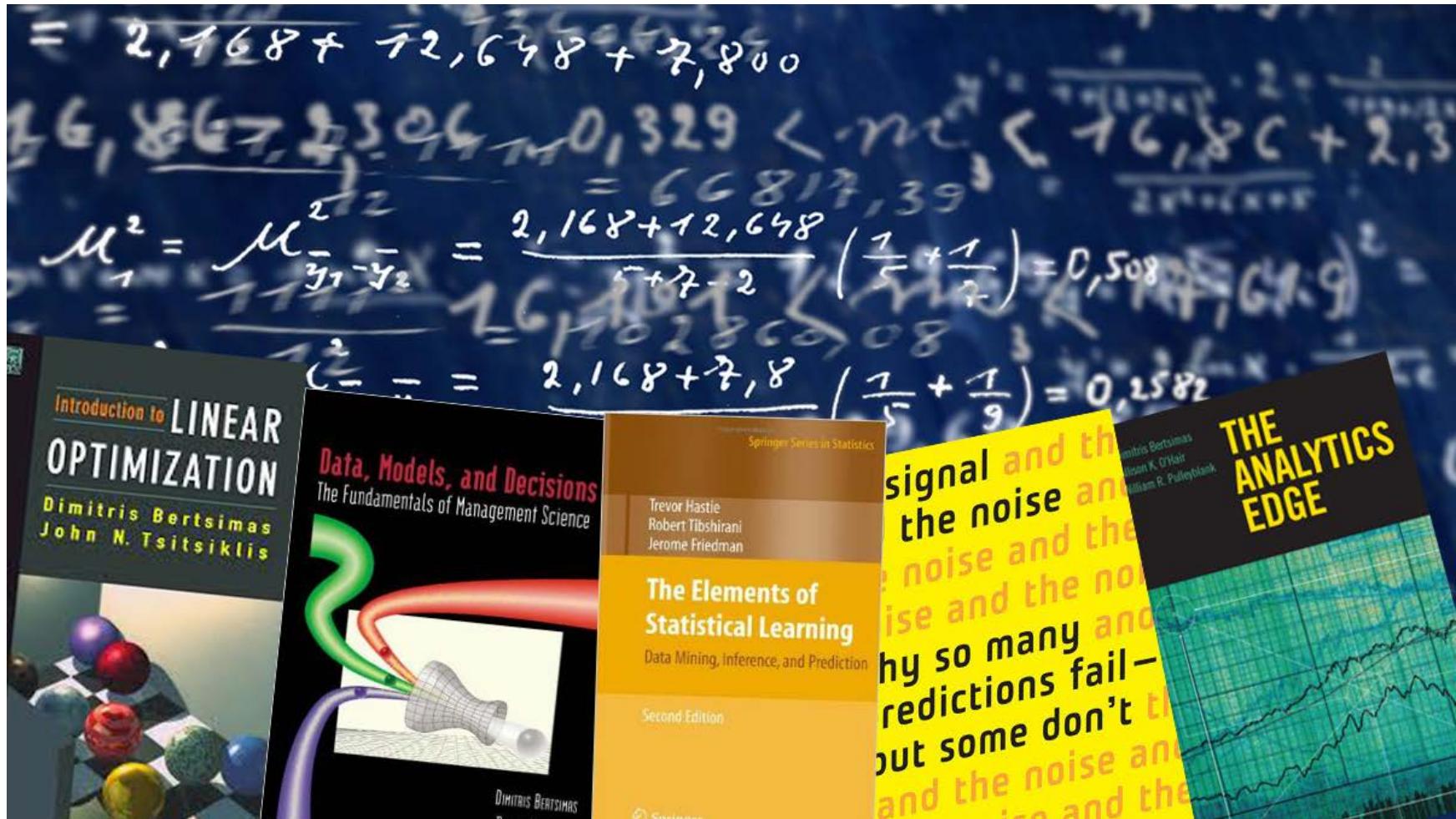
## LINEUP

POS	PLAYER	OPP	FPPG	SALARY	
C	Jussi Jokinen	Fla@Anh	3.1	\$5,300	X
C	Brandon Sutter	Pit@Van	3.0	\$4,400	X
W	Nikolaj Ehlers	Wpg@Tor	3.9	\$4,800	X
W	Daniel Sedin	Pit@Van	3.8	\$6,400	X
W	Radim Vrbata	Pit@Van	3.4	\$5,800	X
D	Brian Campbell	Fla@Anh	2.6	\$4,100	X
D	Morgan Rielly	Wpg@Tor	3.5	\$4,200	X
G	Corey Crawford	StL@Chi	6.3	\$7,800	X
UTIL	Blake Wheeler	Wpg@Tor	4.8	\$7,200	X

## \$55K Sniper Payoff Structure

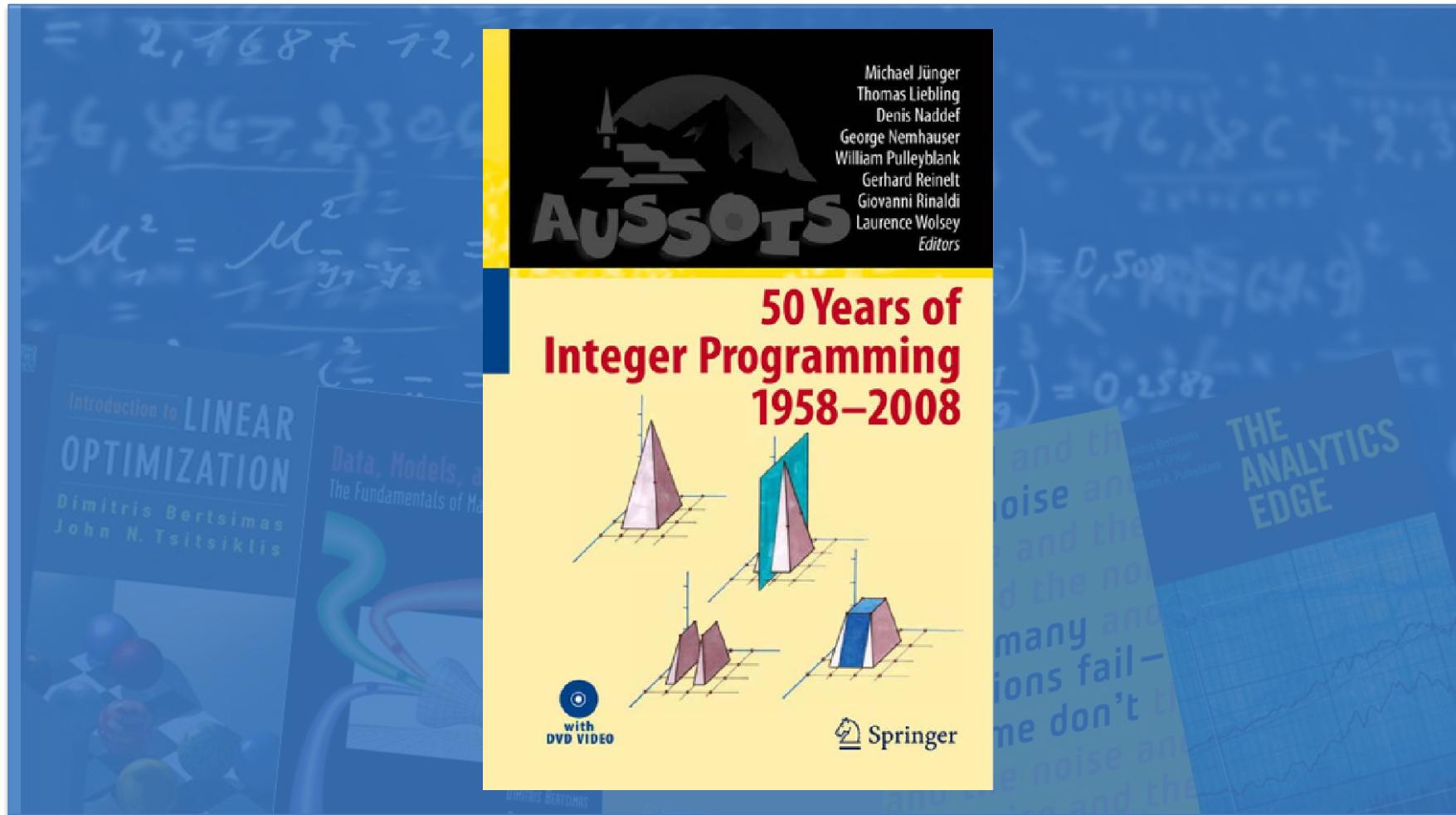


# Previous Knowledge: Analytics



# Previous Knowledge: Analytics

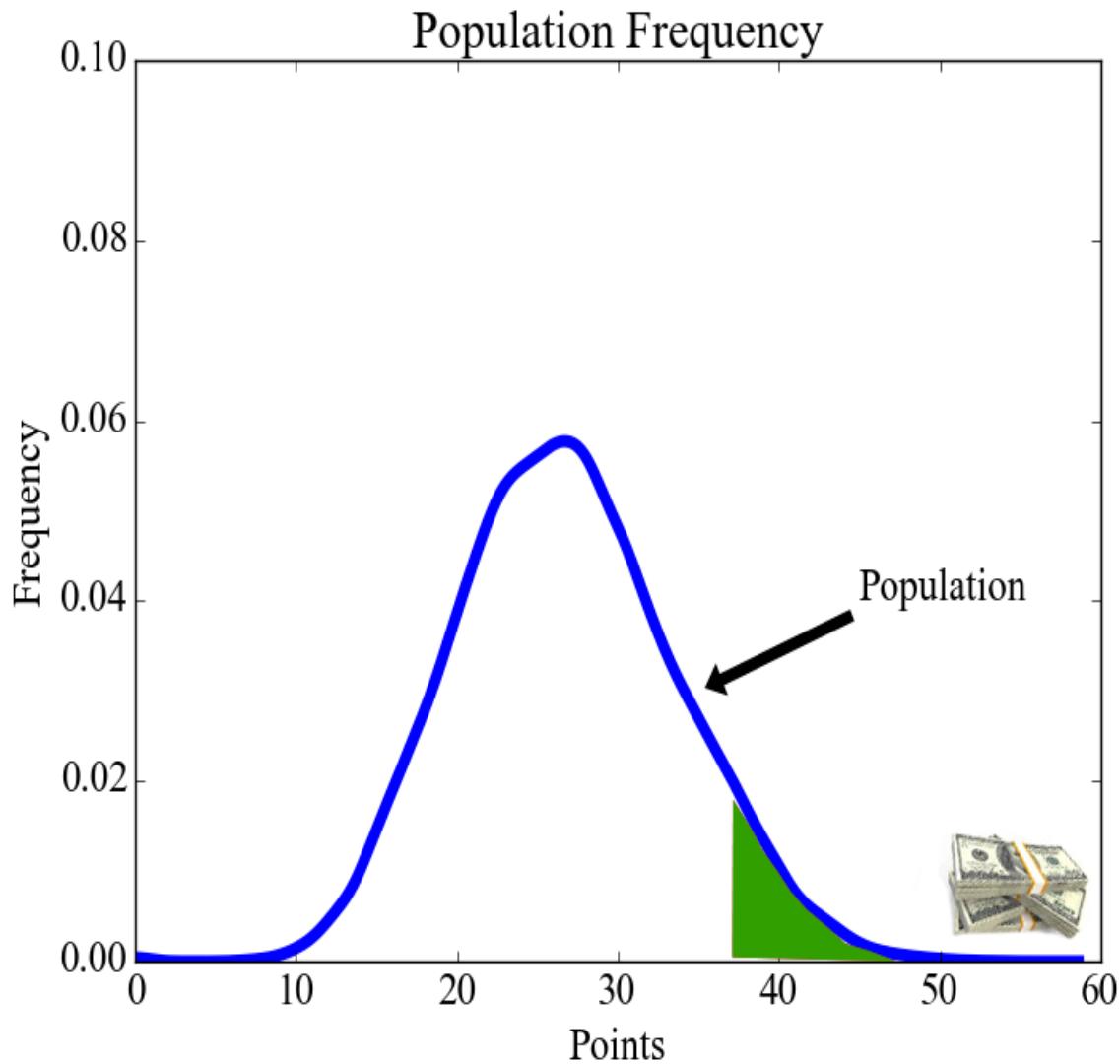
#KnightsOfMIP



# Building a Lineup



# Using this knowledge...



# Were we able to do it?

NHL \$2K Sniper [\$2,000 Guaranteed]			
STANDINGS	ENTRIES	DETAILS	GAMES
1st	 \$150.00	54.50	PMR 0
3rd	 \$90.00	51.50	PMR 0
9th	 \$30.00	49.50	PMR 0
23rd	 \$18.75	46.00	PMR 0
28th	 \$15.00	45.50	PMR 0
28th		45.50	

November 15, 2015

NHL \$40K Sniper [\$40,000 Guaranteed]			
STANDINGS	ENTRIES	DETAILS	GAMES
2nd	 \$2,000.00	61.30	PMR 0
21st	 \$50.00	57.30	PMR 0
21st	 \$50.00	57.30	PMR 0
40th	 \$40.00	56.10	PMR 0
42nd	 \$40.00	55.70	PMR 0
81st		54.10	

November 16, 2015

NHL \$80K Tuesday Special [\$80,000 Guaranteed]			
STANDINGS	ENTRIES	DETAILS	GAMES
3rd	 \$3,000.00	54.60	PMR 0
6th	 \$1,000.00	52.80	PMR 0
7th	 \$800.00	52.30	PMR 0
10th	 \$600.00	50.60	PMR 0
11th	 \$500.00	50.30	PMR 0
15th		50.10	

November 17, 2015

NHL \$45K Sniper [\$45,000 Guaranteed]			
STANDINGS	ENTRIES	DETAILS	GAMES
1st	 \$3,000.00	52.60	PMR 0
8th	 \$275.00	49.60	PMR 0
57th	 \$50.00	45.60	PMR 0
57th	 \$50.00	45.60	PMR 0
83rd	 \$40.00	44.60	PMR 0
83rd		44.60	

November 23, 2015

200 lineups

# Policy Change

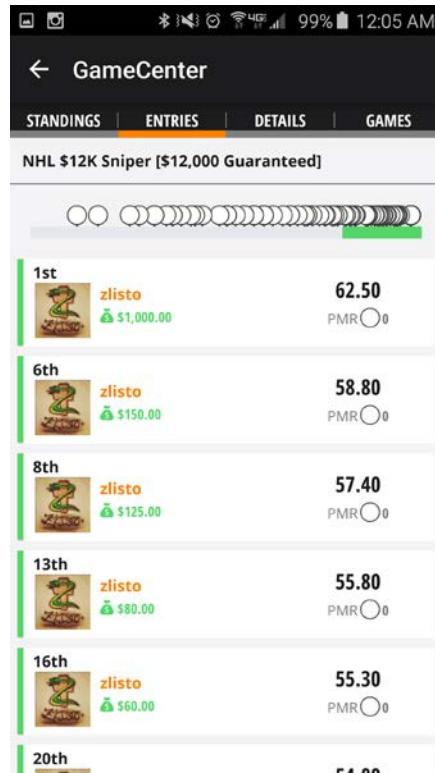


# Policy Change



200 lineups -> 100 lineups

# Were we able to continue it?



December 12, 2015

100 lineups

# Legal Disclaimer:

All profits are in the  
process of being  
donated to charity.

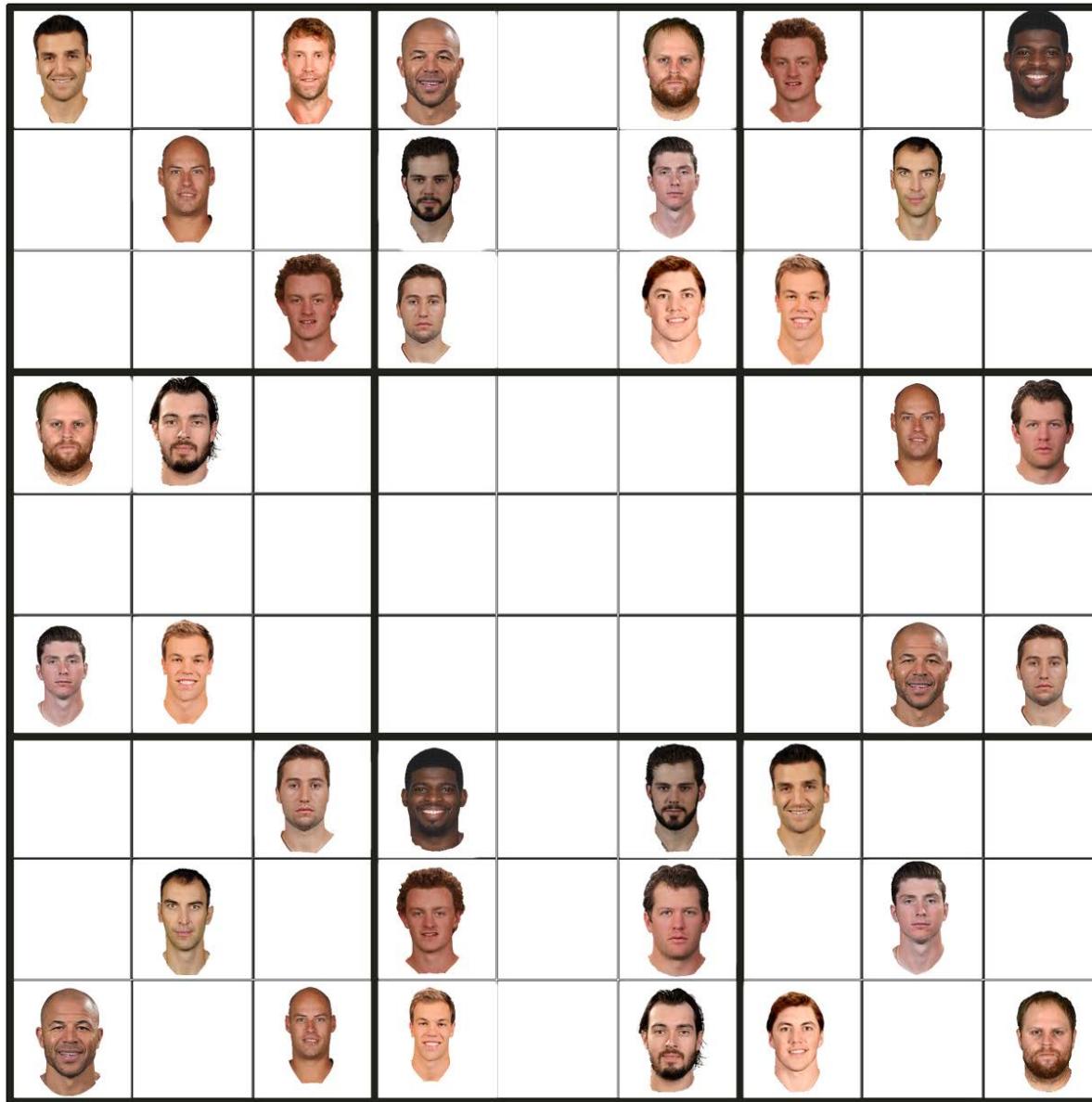
# Integer Programming Formulation

- We will make a bunch of lineups consisting of 9 players each
- Use an integer programming approach to find these lineups

Decision variables

$$x_{pl} = \begin{cases} 1, & \text{if player } p \text{ in lineup } l \\ 0, & \text{otherwise} \end{cases}$$

# First Attempt...



# Basic Feasibility

- 9 different players
- Salary less than \$50,000

## Basic constraints

$$\sum_{p=1}^N c_p x_{pl} \leq \$50,000, \quad (\text{budget constraint})$$

$$\sum_{p=1}^N x_{pl} = 9, \quad (\text{lineup size constraint})$$

$$x_{pl} \in \{0, 1\}, \quad 1 \leq p \leq N.$$

# Position Feasibility

- Between 2 and 3 centers
- Between 3 and 4 wingers
- Between 2 and 3 defensemen
- 1 goalie

## Position constraints

$$2 \leq \sum_{p \in C} x_{pl} \leq 3, , \quad (\text{center constraint})$$

$$3 \leq \sum_{p \in W} x_{pl} \leq 4, \quad (\text{winger constraint})$$

$$2 \leq \sum_{p \in D} x_{pl} \leq 3, \quad (\text{defensemen constraint})$$

$$\sum_{u \in G} x_{pl} = 1 \quad (\text{goalie constraint})$$

# Team Feasibility

- At least 3 different NHL teams

## Team constraints

$$t_i \leq \sum_{p \in T_i} x_{pl}, \quad \forall i \in \{1, \dots, N_T\}$$

$$\sum_{i=1}^{N_T} t_i \geq 3,$$

$$t_i \in \{0, 1\}, \quad \forall i \in \{1, \dots, N_T\}.$$

# First Attempt...

\$6400	\$7200	\$4200	\$4100	\$5300	\$4400	\$4800	\$5800	\$7800
W	UTIL	D	D	C	C	W	W	G

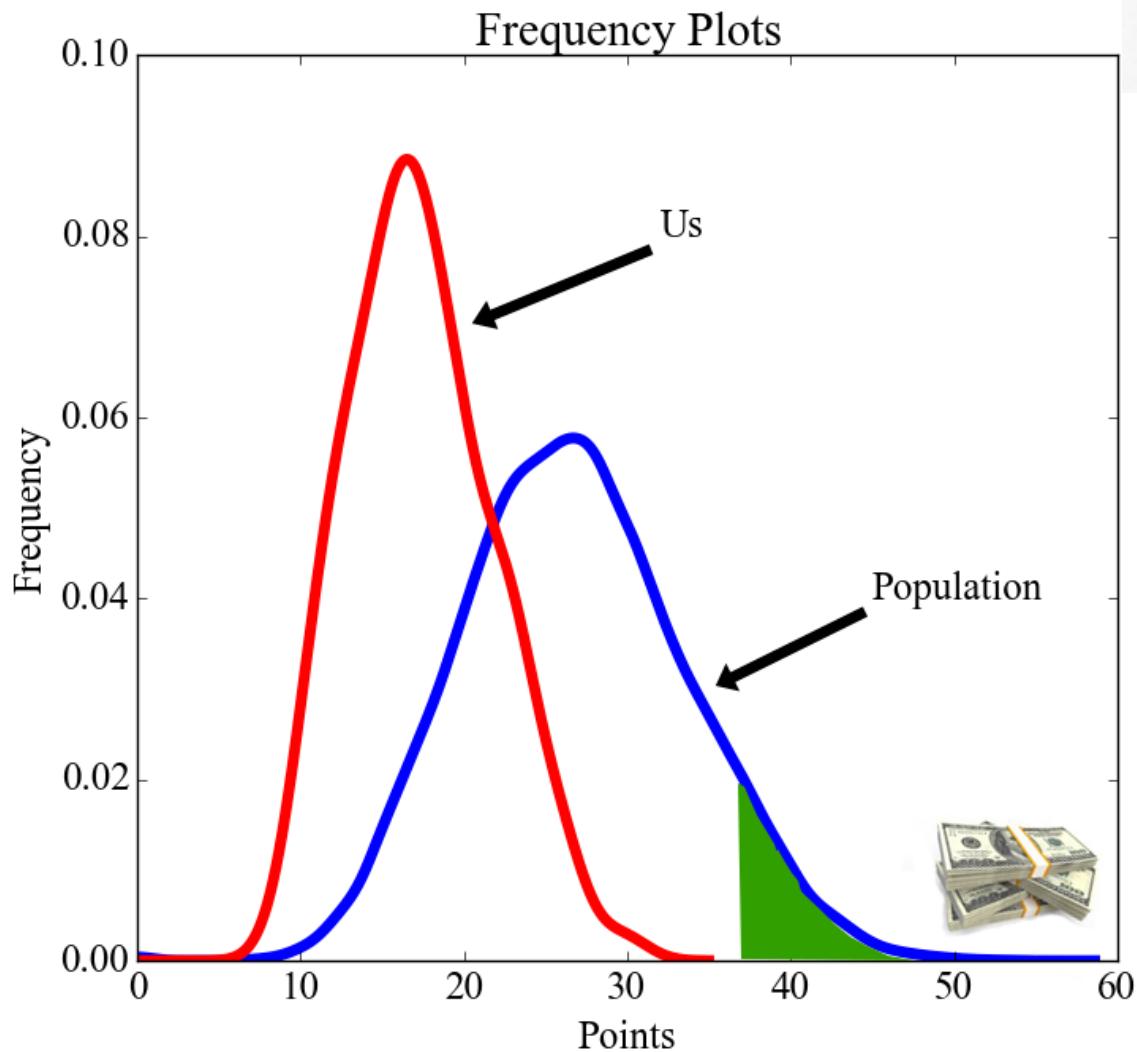


Feasible...

But Not Good

> 3 Different Teams

# First Attempt...



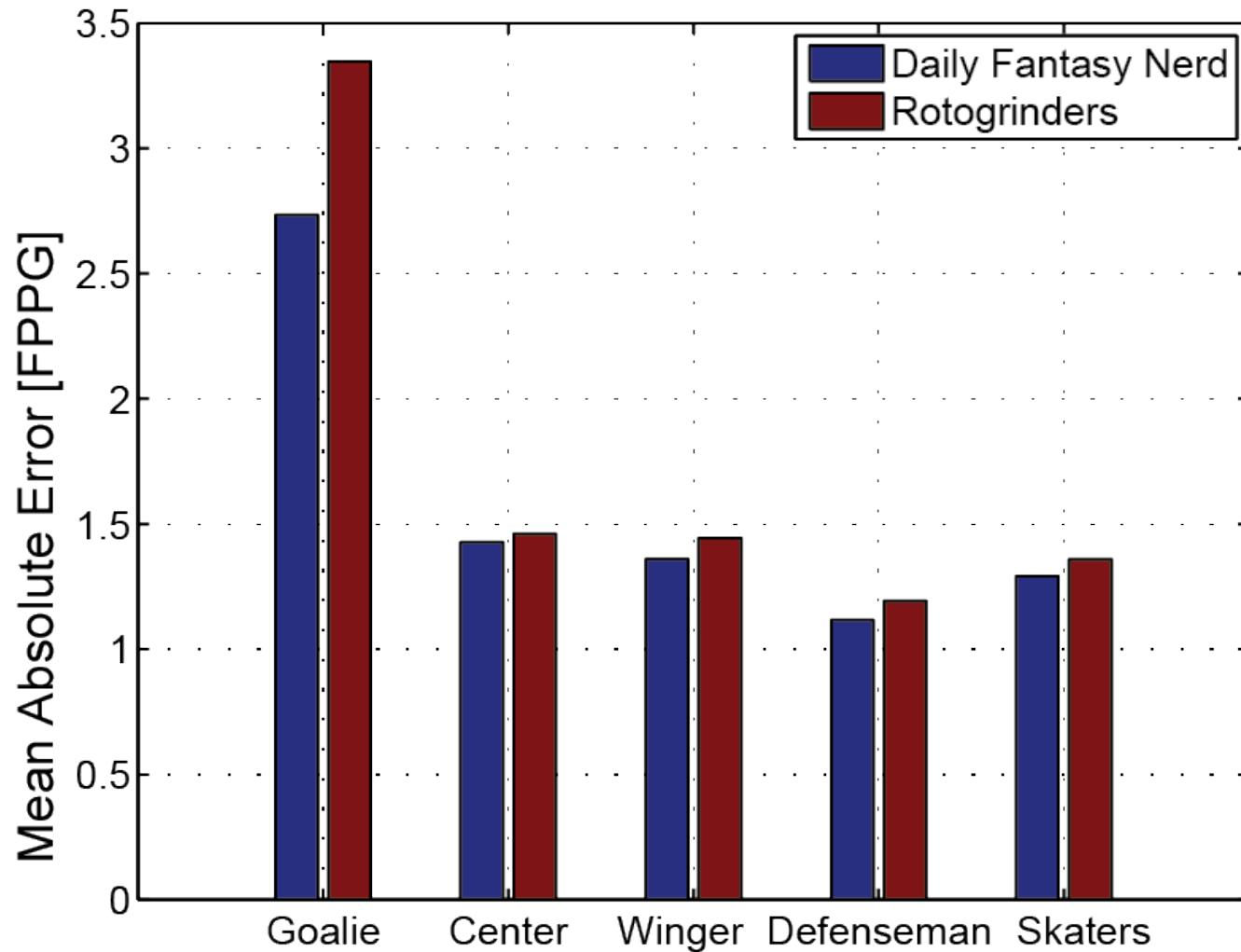
# Second Attempt...

- Must increase our mean points...
- Solution:

**USE EXPERT PREDICTIONS**



# Prediction Errors



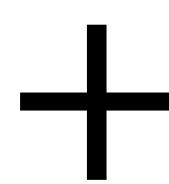
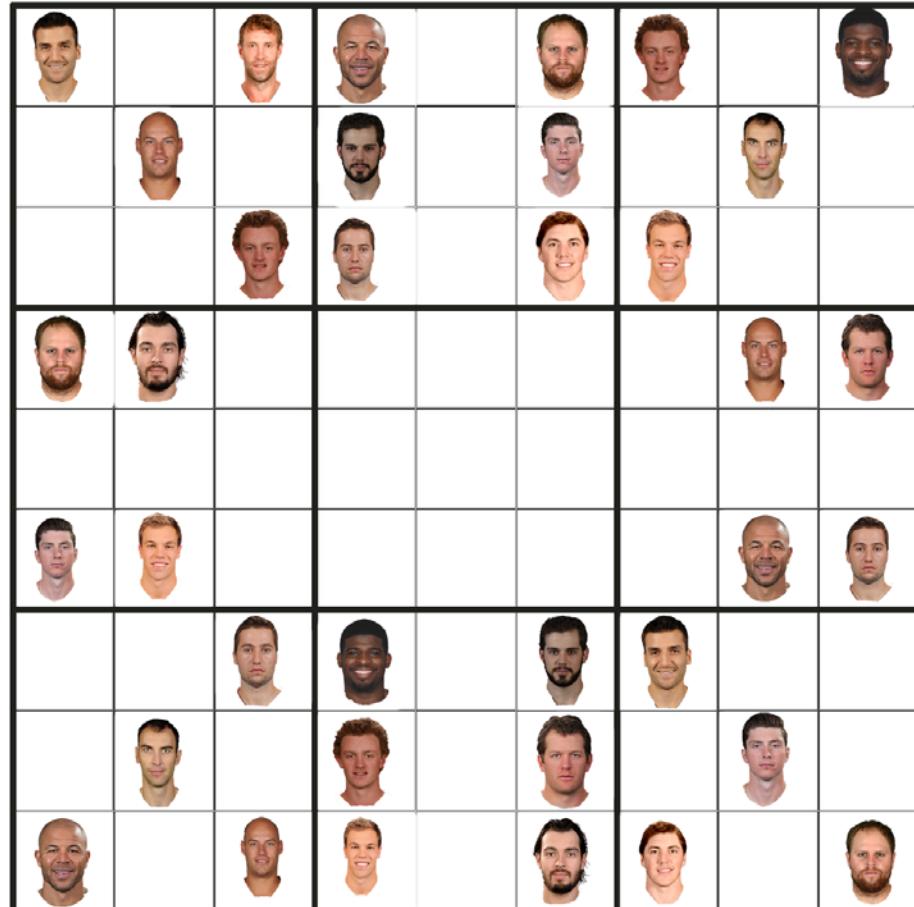
# Maximize Points

- Forecasted points for player p:  $f_p$
- You get to choose what the forecasts are

## Points Objective Function

$$\sum_{p=1}^N f_p x_{pl}$$

# Second Attempt...



Maximize  
points

$$\max_x \sum_{p=1}^N f_p x_{pl}$$

# Old Lineup

\$6400 \$7200 \$4200 \$4100 \$5300 \$4400 \$4800 \$5800 \$7800  
W UTIL D D C C W W G



Feasible...

But Not Good

12 points on average

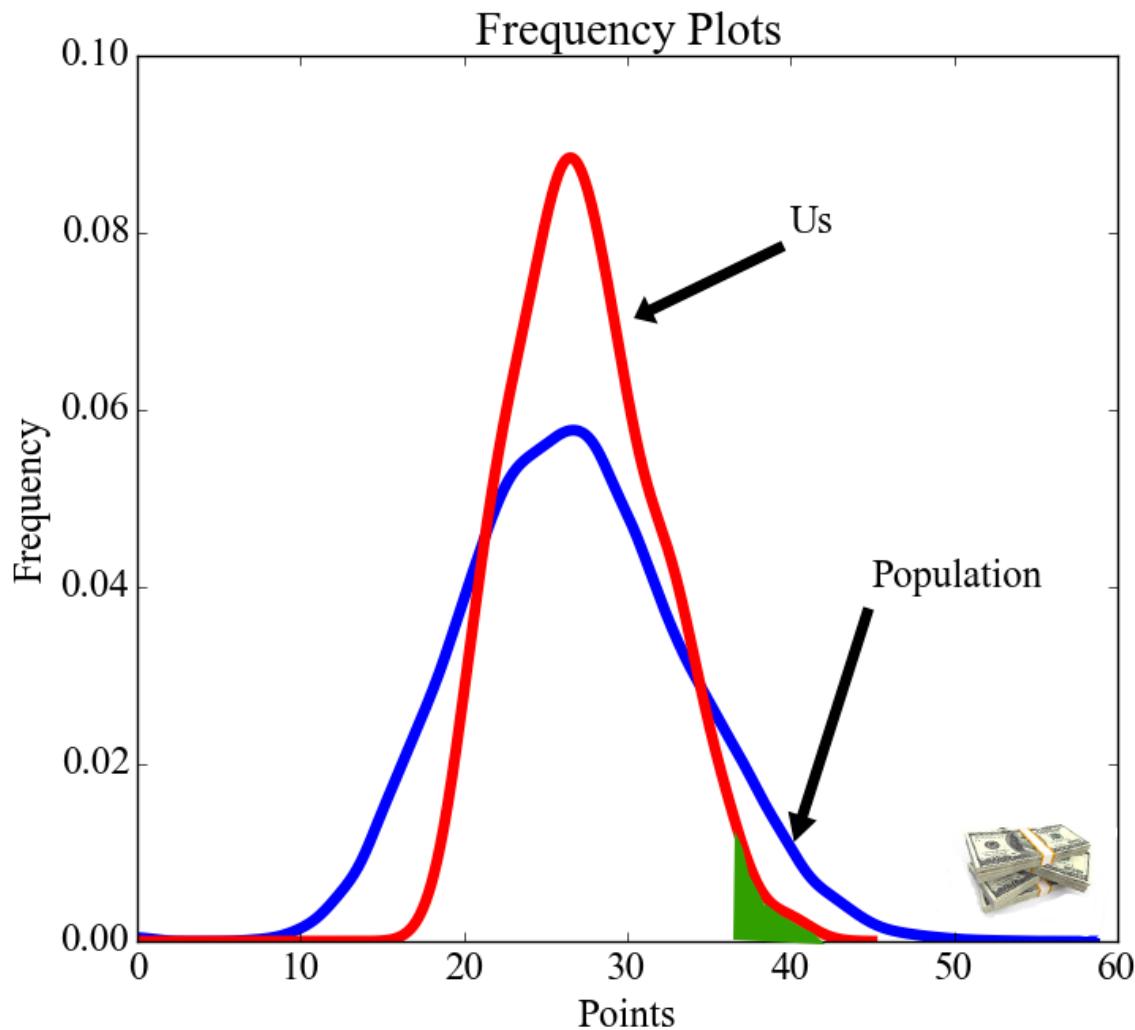
# New Lineup

Projections: 5.4 2.5 3.4 3.0 3.2 4.2 3.5 3.4 5.7  
\$9500 \$2700 \$4600 \$3800 \$4600 \$6400 \$5200 \$5100 \$8000  
W UTIL D D C C W W G



23 points on average

# Second Attempt...



# How can we do better?

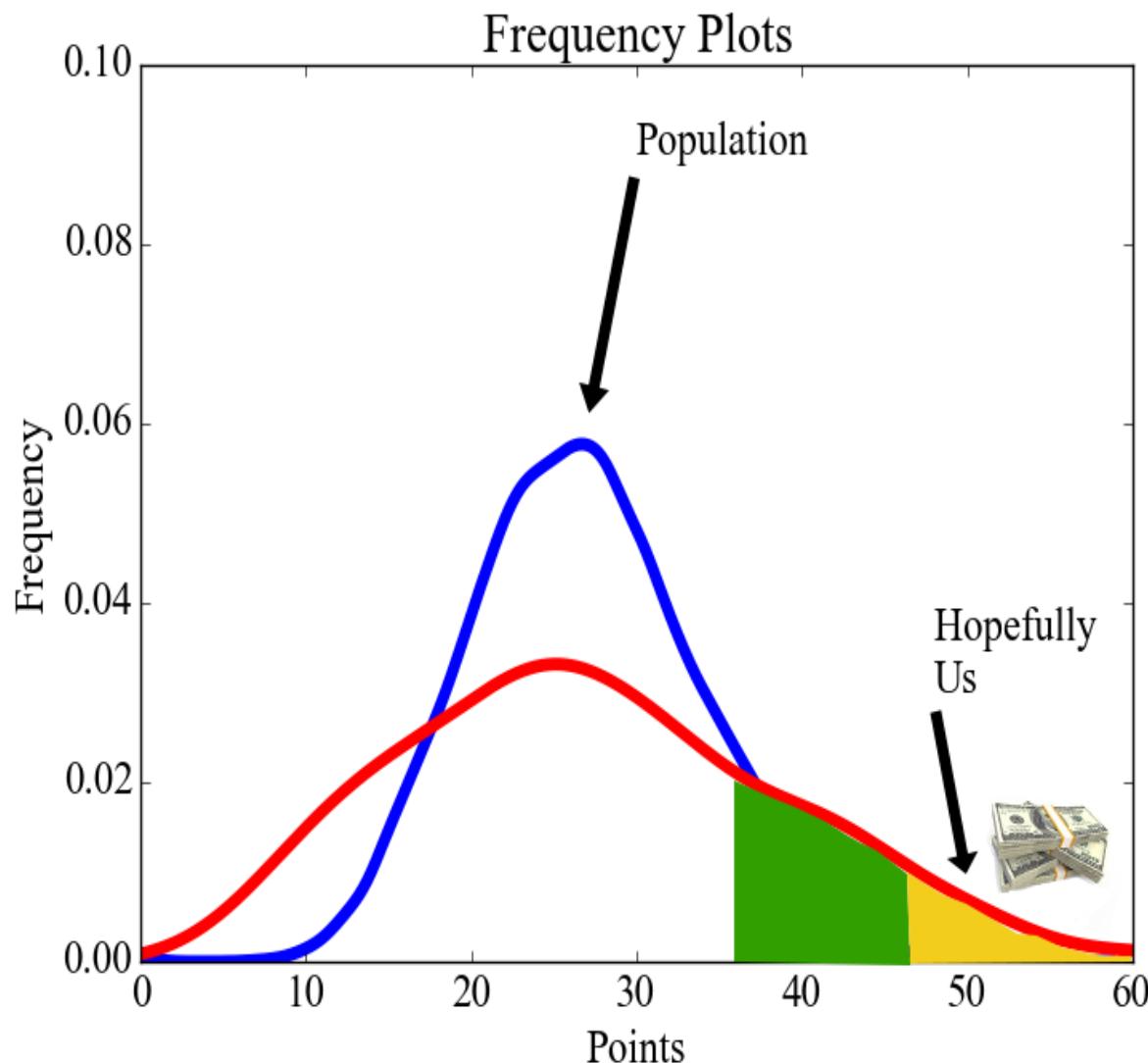
- Three D's of Finance.



# So what do we do?



# By doing this...



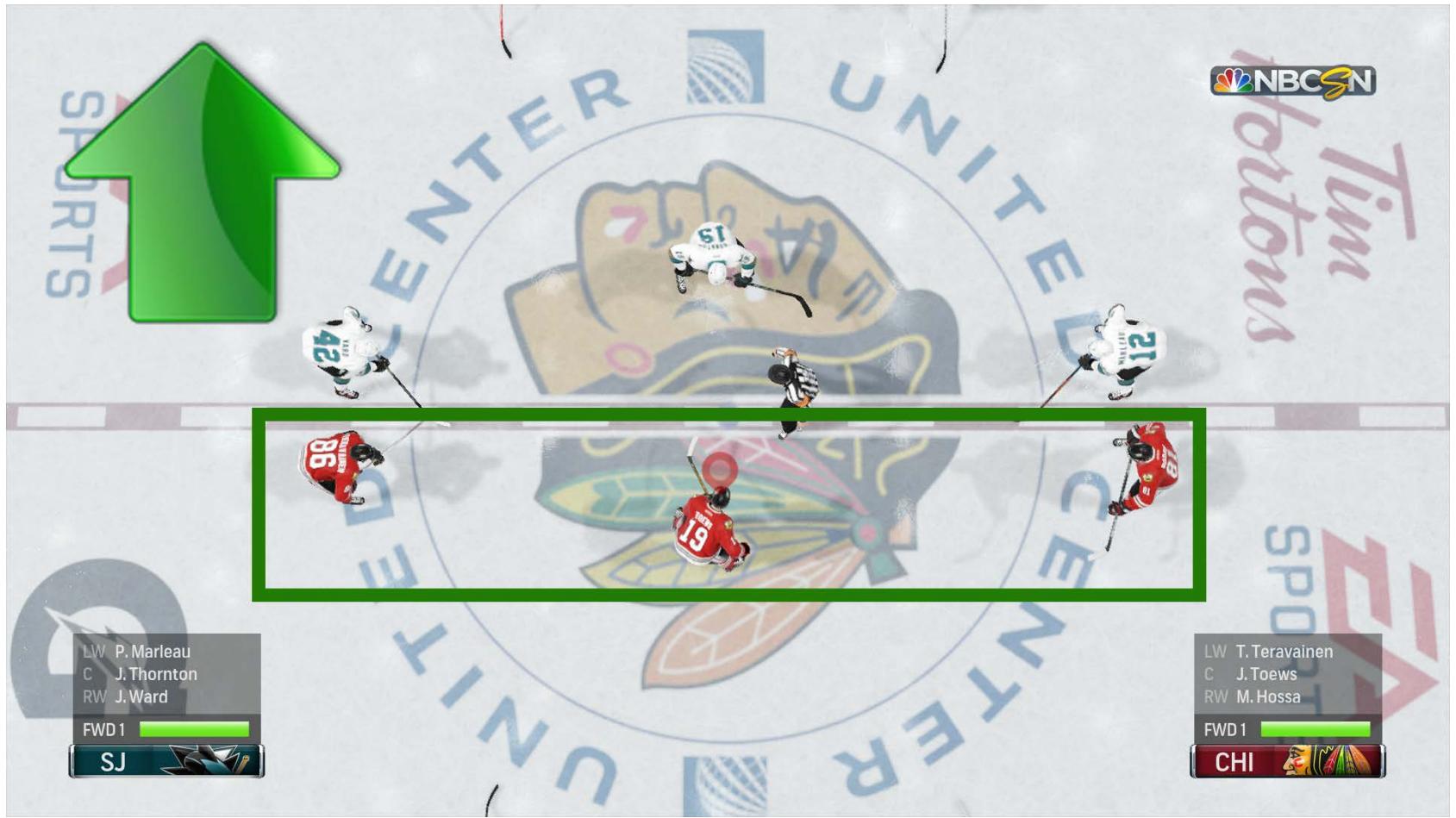
# Stacking Lineups

- Stacking means putting players on a single lineup that have a positive correlation
- Either the players pop off together -> tons of points
- Or the players crap out -> few points
- We stack using “structural correlations”

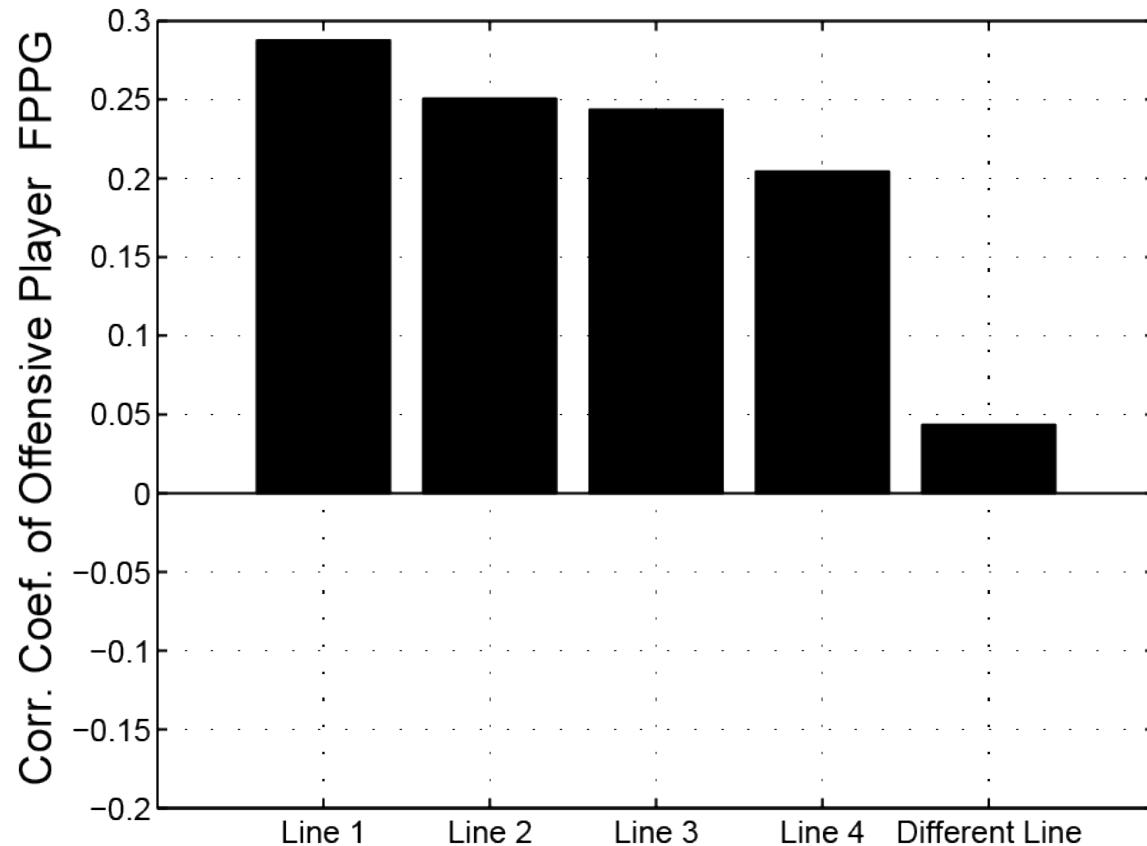
# Structural Correlations - Teams



# Structural Correlations - Lines



# Structural Correlations - Lines



# Structural Correlations - Lines

- At least 1 complete line (3 players per line)
- At least 2 partial lines (at least 2 players per line)

1 complete line constraint

$$3v_i \leq \sum_{p \in L_i} x_{pl}, \quad \forall i \in \{1, \dots, N_L\}$$

$$\sum_{i=1}^{N_L} v_i \geq 1$$

$$v_i \in \{0, 1\}, \quad \forall i \in \{1, \dots, N_L\}.$$

2 partial lines constraint

$$2w_i \leq \sum_{p \in L_i} x_{pl}, \quad \forall i \in \{1, \dots, N_L\}$$

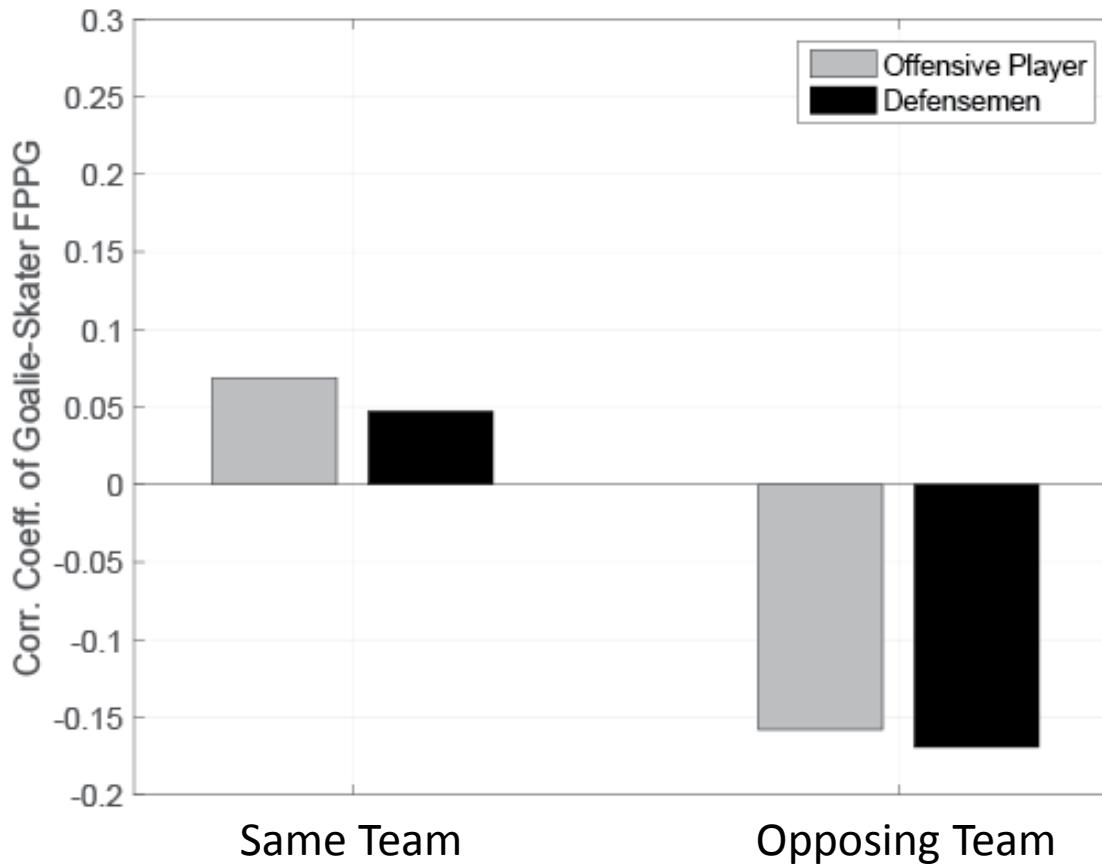
$$\sum_{i=1}^{N_L} w_i \geq 2$$

$$w_i \in \{0, 1\}, \quad \forall i \in \{1, \dots, N_L\}.$$

# Structural Correlations – Goalie Against Opposing Players



# Structural Correlations – Goalie Against Opposing Players



# Structural Correlations – Goalie Against Skaters

- No skater against goalie

No skater against goalie constraint

$$6x_{pl} + \sum_{q \in Opponents_p} x_{ql} \leq 6, \quad \forall p \in G$$

Feasible

Line

Team

Line

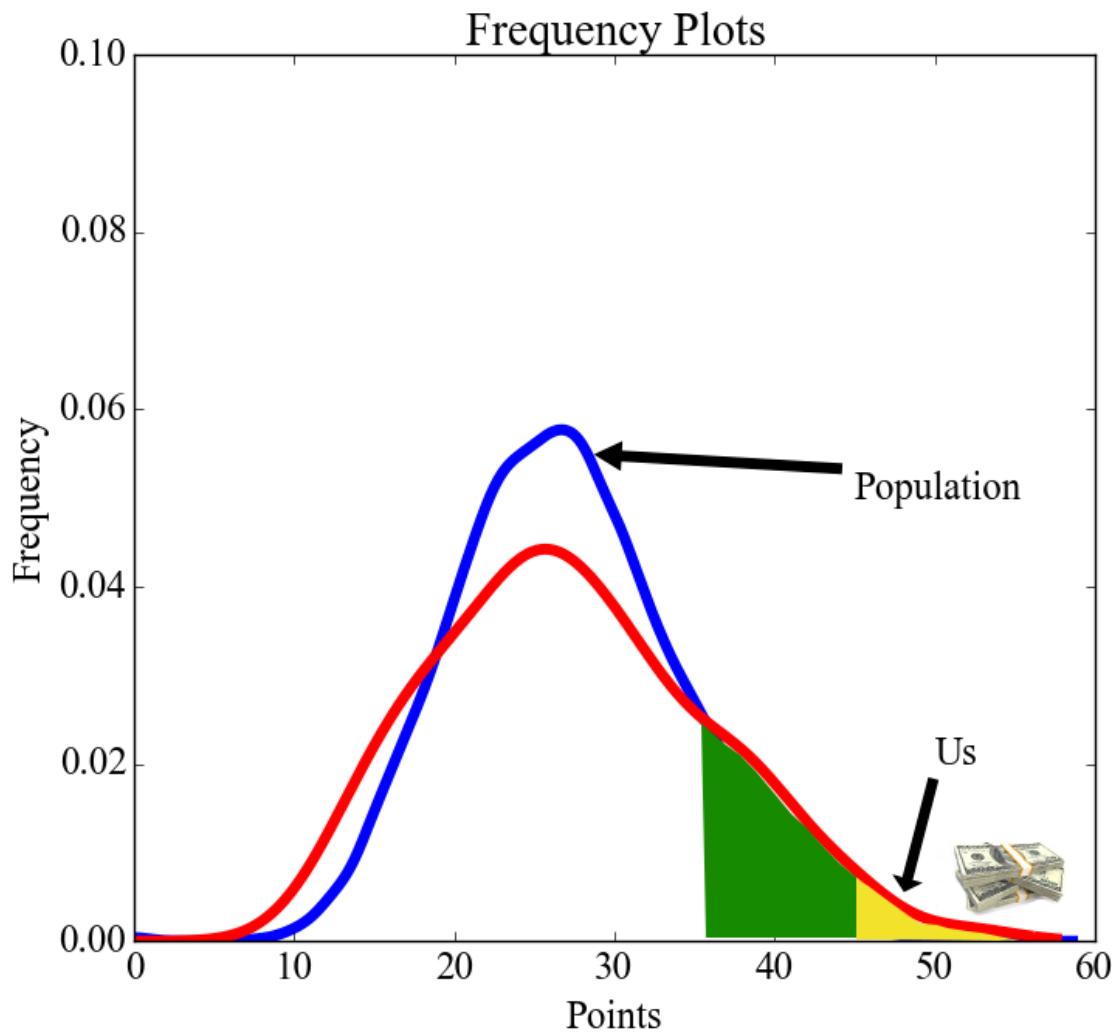
Goalie

Not

Against



# Second Attempt...





# Lineup Diversity

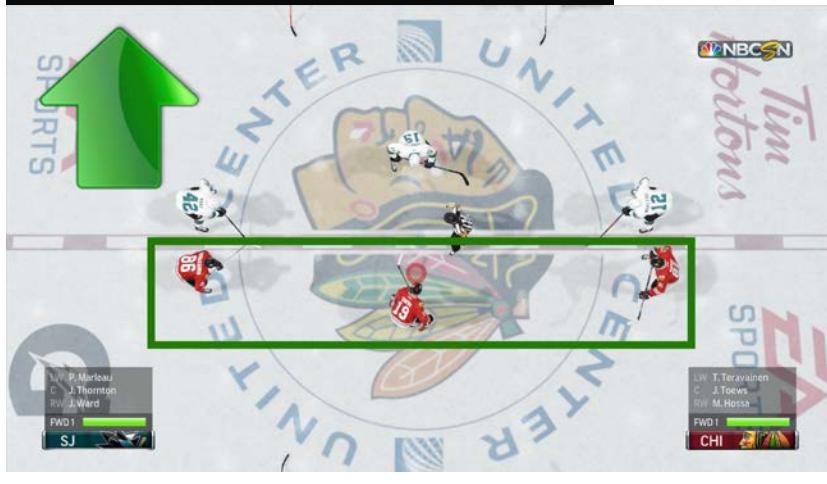
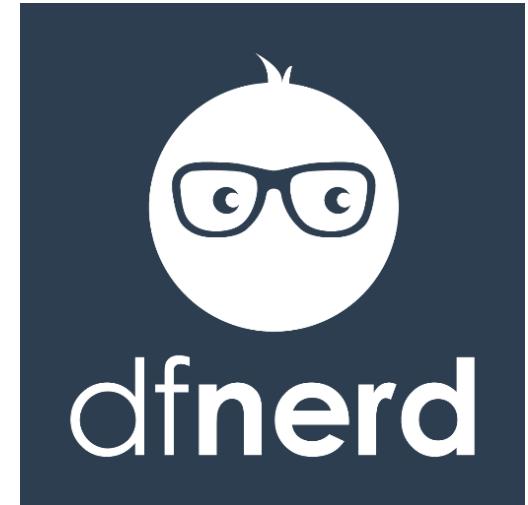
- Make sure lineup  $l$  has no more than  $\gamma$  players in common with lineups 1 to  $l-1$

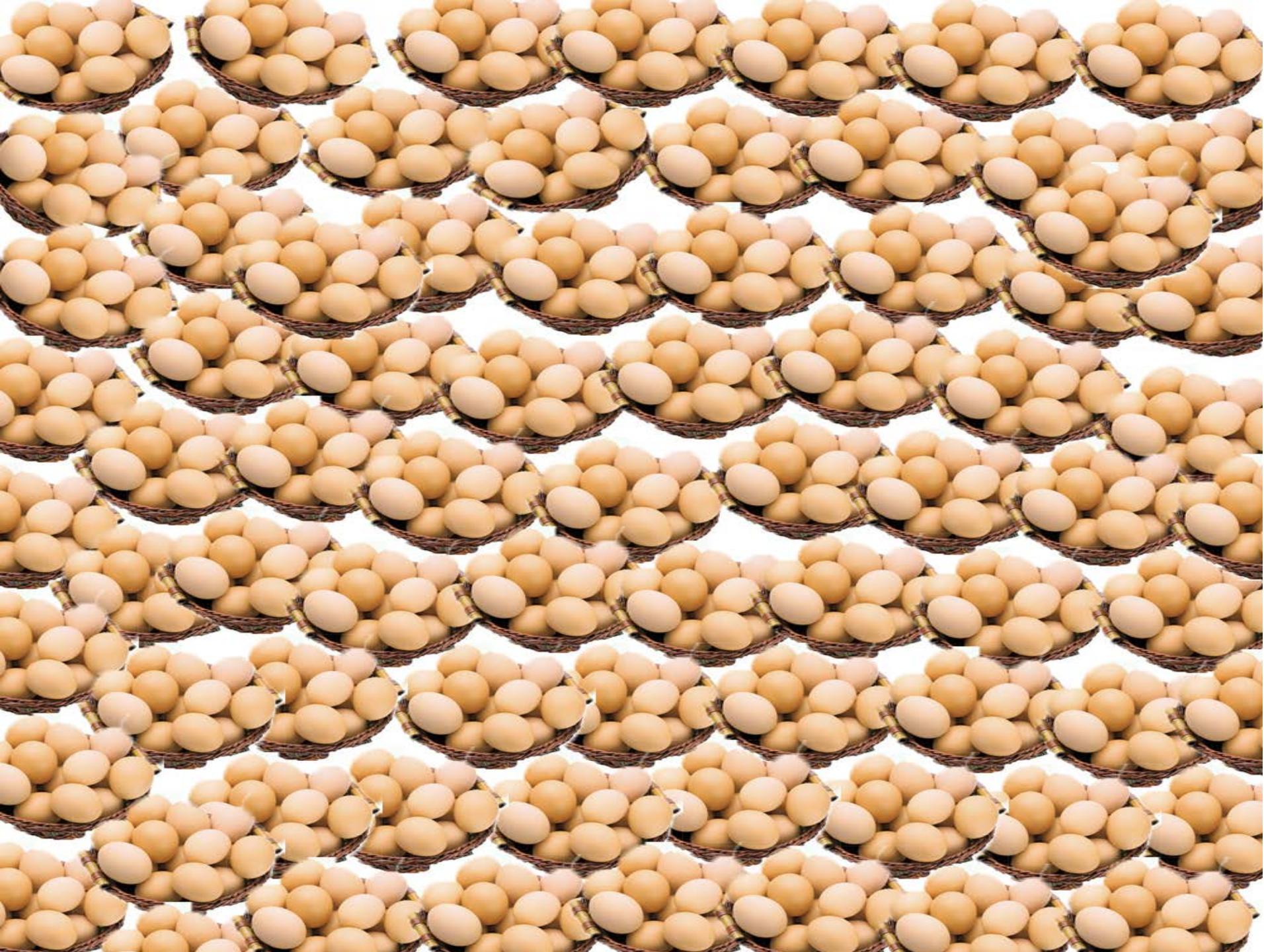
Diversity constraint

$$\sum_{p=1}^N x_{pk}^* x_{pl} \leq \gamma, k = 1, \dots, l-1$$

# To Review...

LINEUP					Avg. Rem. / Player: \$0
POS	PLAYER	OPP	FPPG	SALARY	
C	Jussi Jokinen	Fla@Anh	3.1	\$5,300	X
C	Brandon Sutter	Pit@Van	3.0	\$4,400	X
W	Nikolaj Ehlers	Wpg@Tor	3.9	\$4,800	X
W	Daniel Sedin	Pit@Van	3.8	\$6,400	X
W	Radim Vrbata	Pit@Van	3.4	\$5,800	X
D	Brian Campbell	Fla@Anh	2.6	\$4,100	X
D	Morgan Rielly	Wpg@Tor	3.5	\$4,200	X
G	Corey Crawford	StL@Chi	6.3	\$7,800	X
UTIL	Blake Wheeler	Wpg@Tor	4.8	\$7,200	X

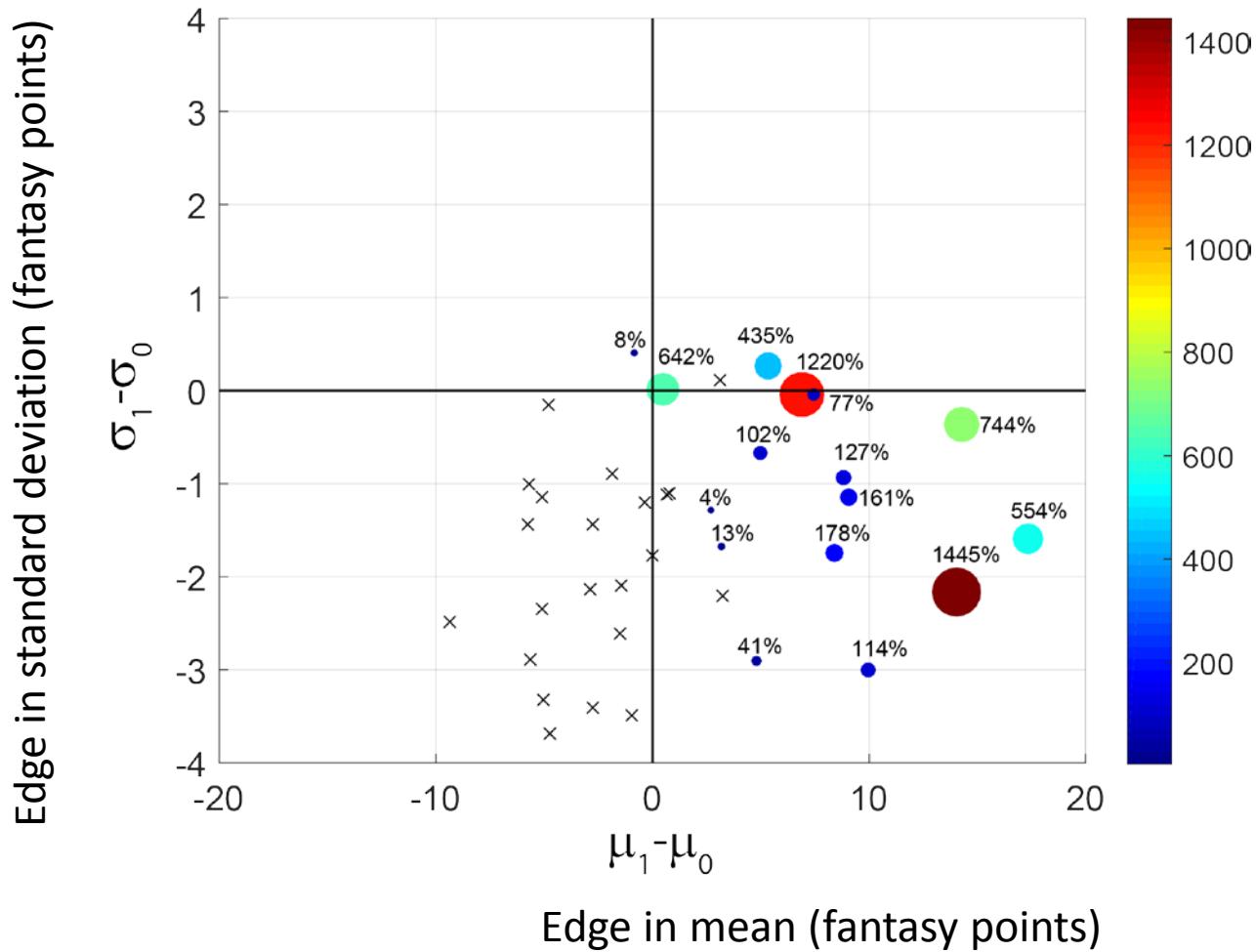




# **PERFORMANCE ON REAL CONTESTS**

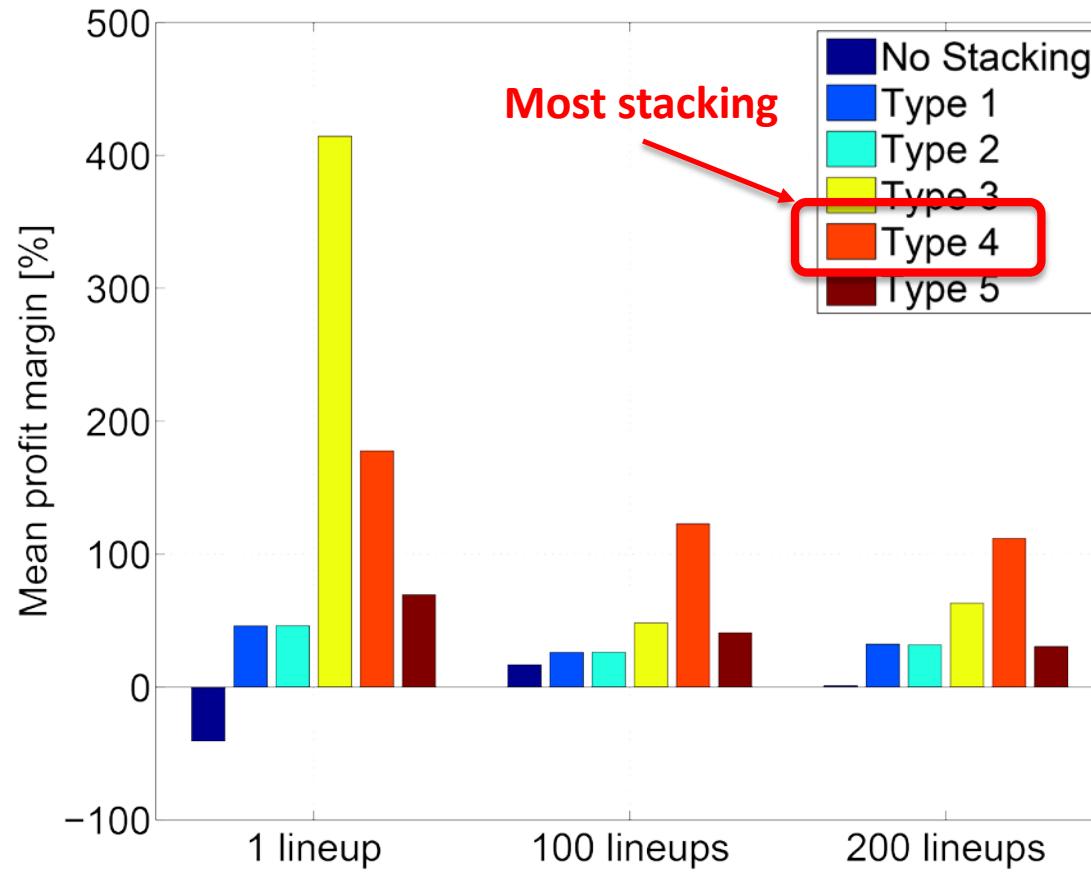
# Performance on Real Contests

- Each point is a contest, with profit margin shown
- Used all stacking, a maximum overlap of 7, and 200 lineups



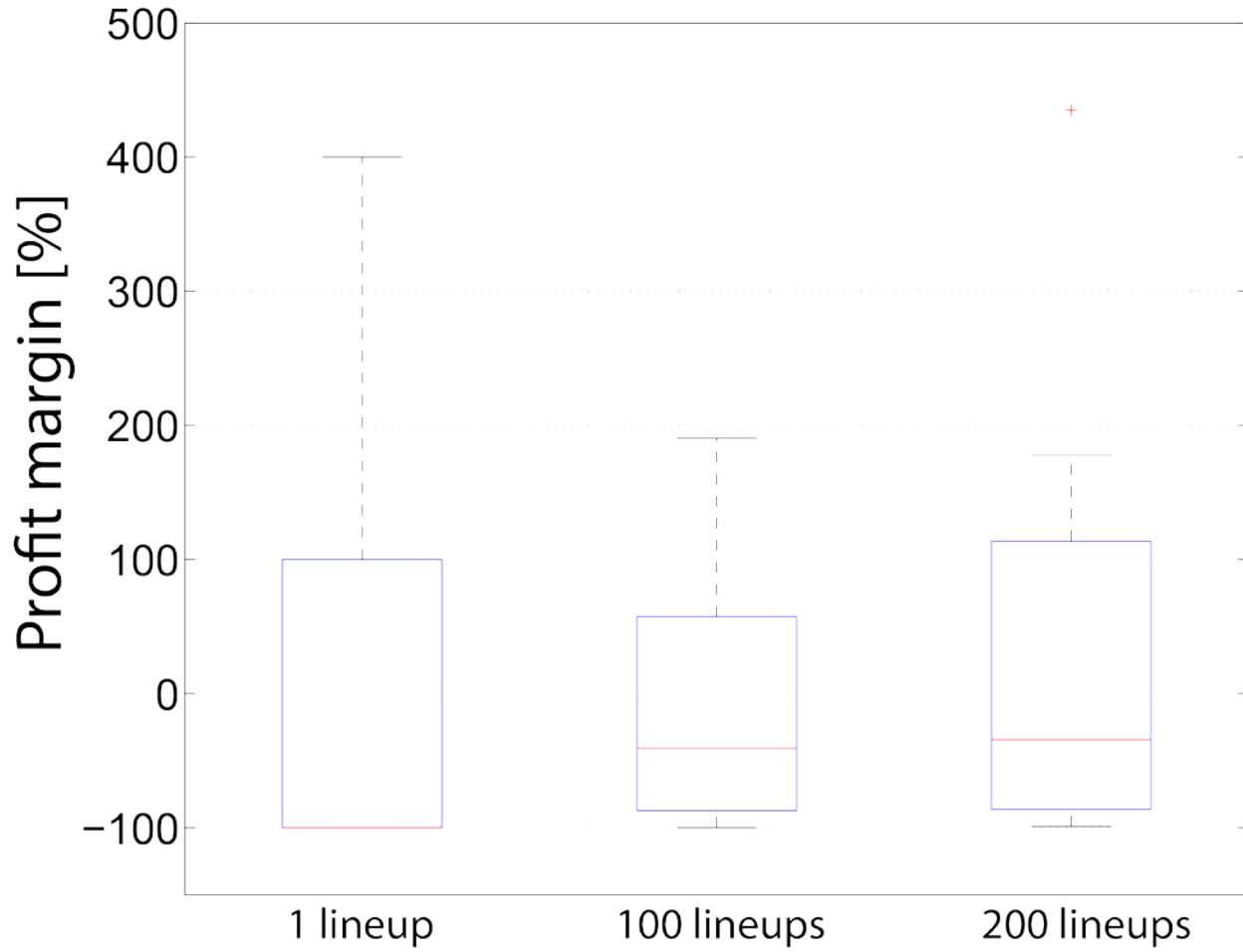
# Impact of Stacking

- Used a maximum overlap of 7, and 200 lineups



More stacking is better

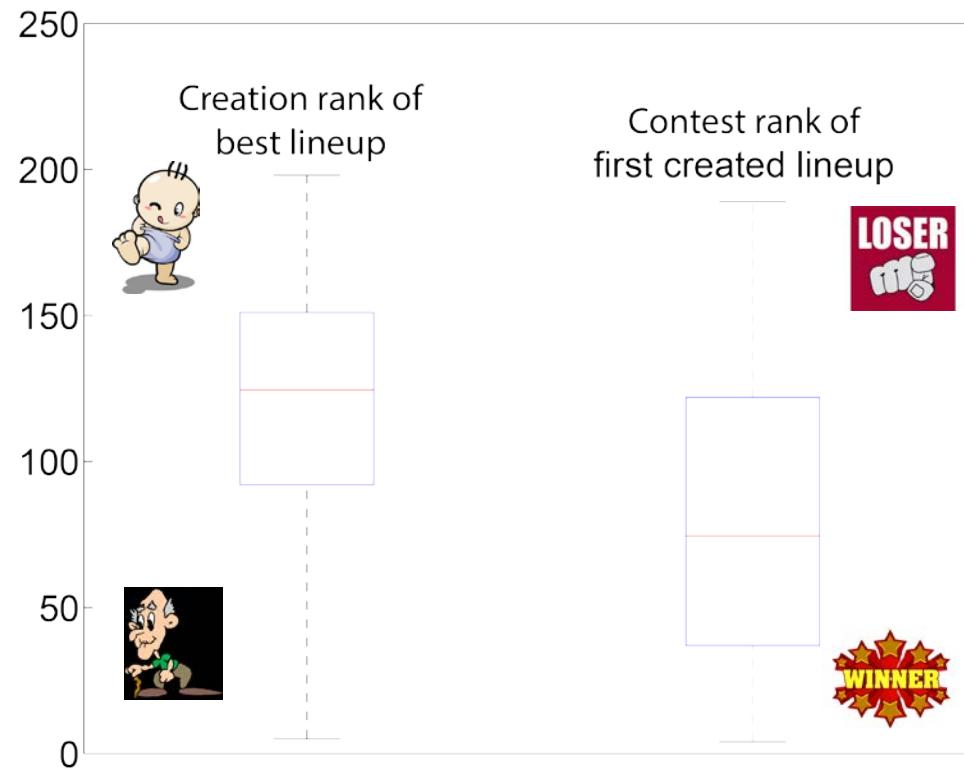
# Impact of Number of Lineups



**More lineups is better**

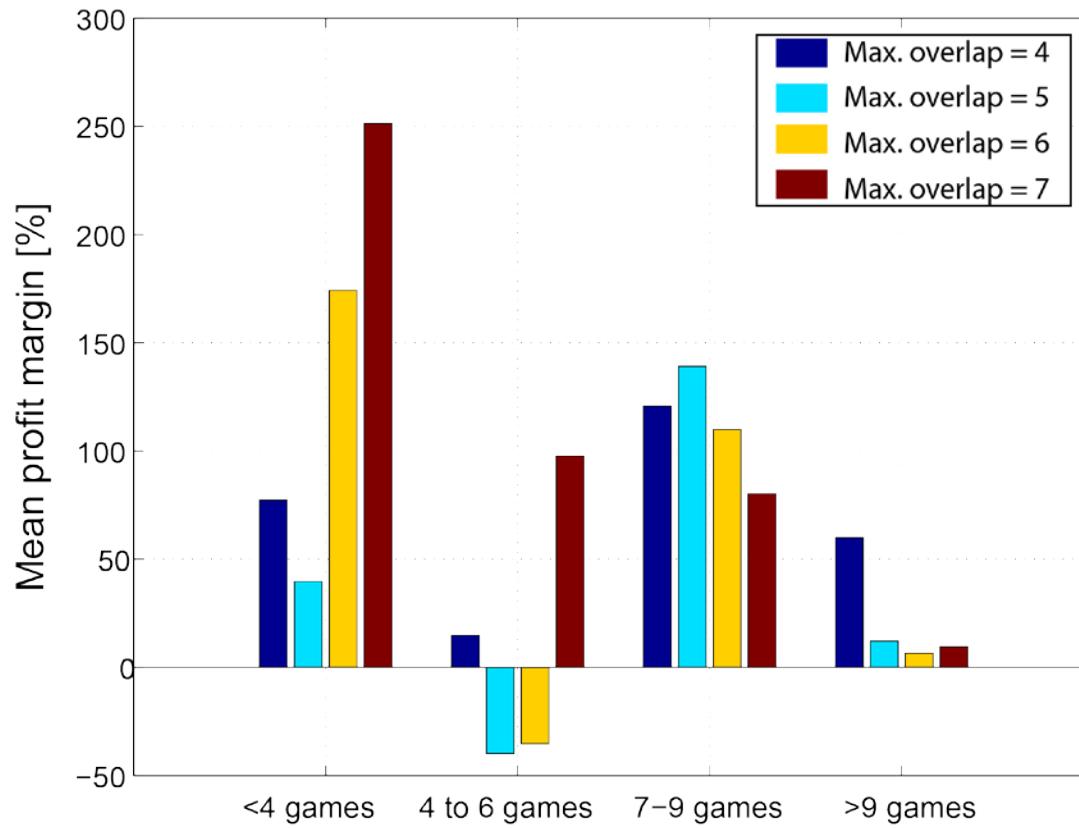
# Impact of Lineup Birth Order

- We create lineups sequentially
- Are the best lineups the “oldest” lineups?



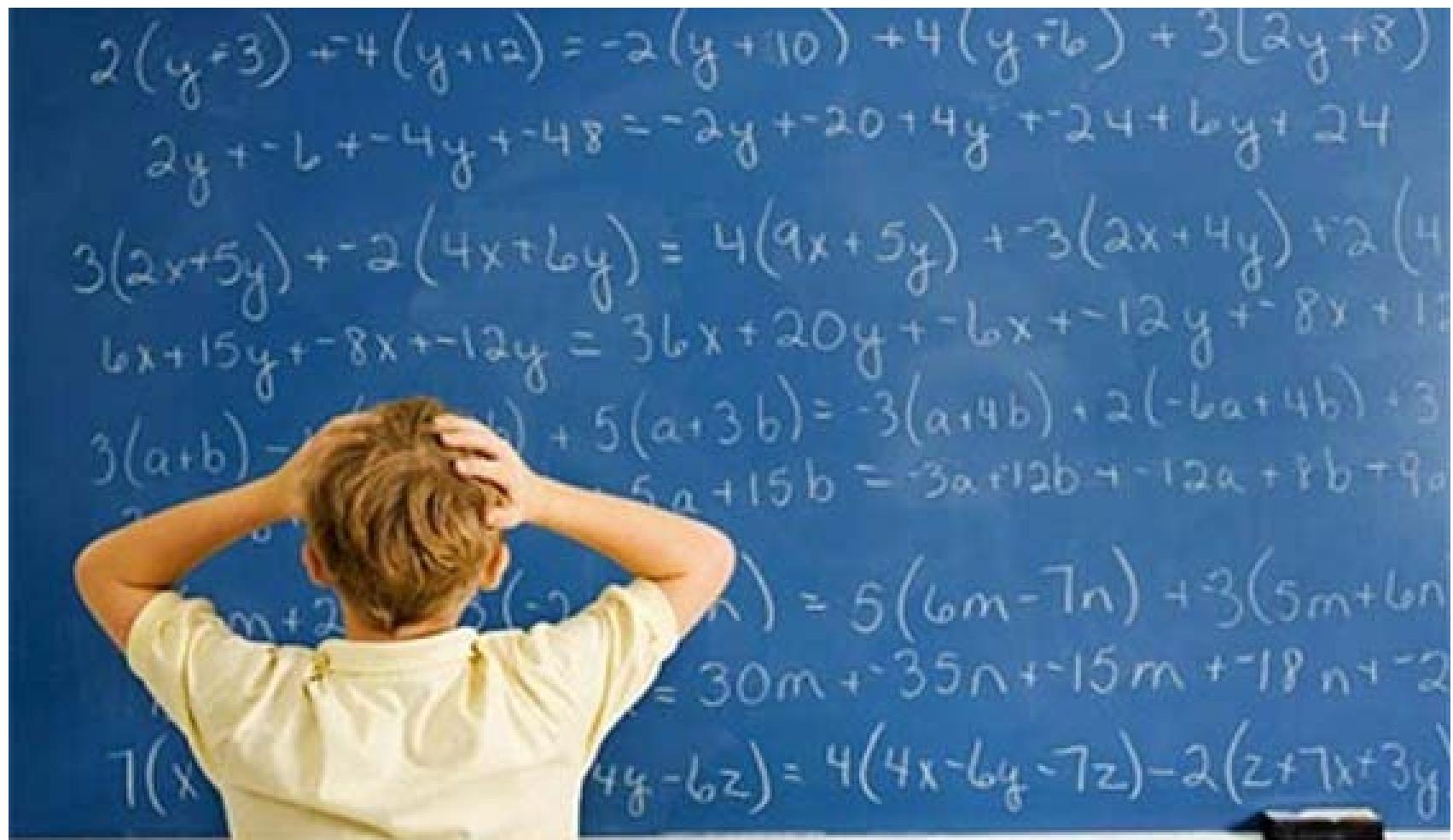
**First lineup isn't usually the best lineup**

# Impact of Diversity



**More games -> Use more diversity**

# How can you do it?



# Lineup Construction Procedure

- Get projection data
  - Make sure you wait until the starting goalies are announced
- Solve integer program for each lineup one at a time
  - But add in the new diversity constraints for each new lineup



< 30 Minutes



# How can you do it?

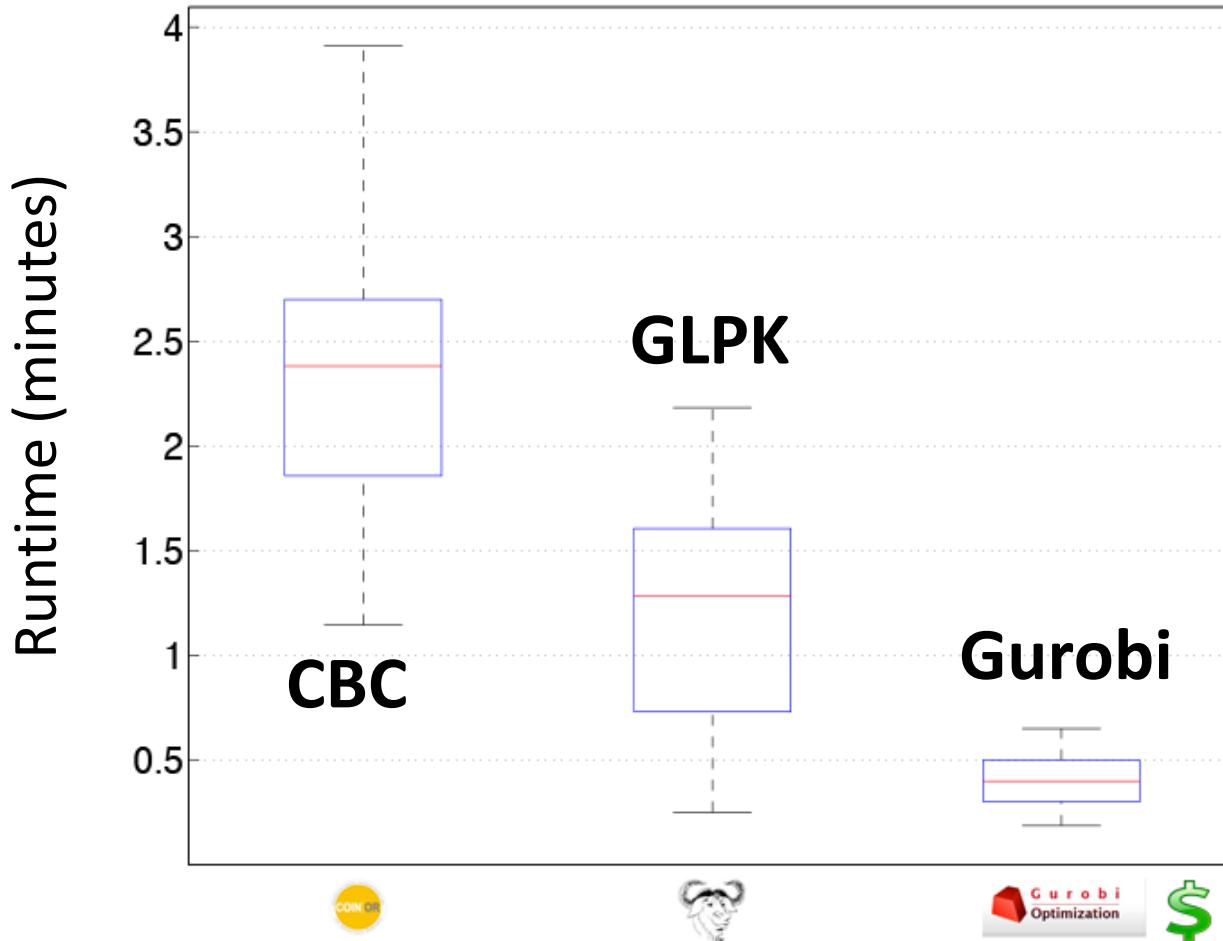


Download Code from Github:

<https://github.com/dscotthunter/Fantasy-Hockey-IP-Code>

```
376 function one_lineup_Type_4(skaters, goalies, lineups, num_overlap, num_skaters, num_goalies, centers, wingers, defenders, num_teams, skaters_teams, goalie_opponents, team_lines, num_lines, PI_info)
377     m = Model(solver=GurobiSolver())
378
379     # Variable for skaters in lineup
380     skaters_lineup[i::num_skaters], Bin
381     gdeVar{m, goalies_lineup[i::num_goalies]}, Bin
382     # one goalie
383     gdeConstraint{m, sum(goalies_lineup[i], i::num_goalies) == 1}
384
385     gdeConstraint{m, sum(skaters_lineup[i], i::num_skaters) == 8}
386
387     # between 2 and 3 centers
388     gdeConstraint{m, sum(centers[i], skaters_lineup[i], i::num_skaters) == 3}
389     gdeConstraint{m, 2 == sum(centers[i], skaters_lineup[i], i::num_skaters)}
390
391     # between 3 and 4 wingers
392     gdeConstraint{m, sum(wingers[i], skaters_lineup[i], i::num_skaters) == 4}
393     gdeConstraint{m, 3 == sum(wingers[i], skaters_lineup[i], i::num_skaters)}
394
395     # between 2 and 3 defenders
396     gdeConstraint{m, sum(defenders[i], skaters_lineup[i], i::num_skaters) == 3}
397     gdeConstraint{m, sum(defenders[i], skaters.lineup[i], i::num_skaters) == 3}
398
399     # Financial Constraint
400     gdeConstraint{m, sum(skaters[i].$salary * skaters.lineup[i], i::num_skaters) + sum(goalies[i].$salary * goalies.lineup[i], i::num_goalies) == 50000}
401
402     # exactly 3 different teams for the 8 skaters constraint
403     gdeVar{m, num_on_team_pos[i::num_teams] == 0}
404     gdeVar{m, num_on_team_neg[i::num_teams] == 0}
405     gdeConstraint{m, xconstr1{m, num_on_team_pos[i] - num_on_team_neg[i] == sum(skaters_teams[i], skaters.lineup[i], t::num_skaters) - 0.5}}
406     gdeVar{m, used_team[i::num_teams], Bin}
407     gdeConstraint{m, sum(used_team[i], i::num_teams) == 3}
408     gdeConstraint{m, sum(used_team[i], num_on_team_pos[i] == used_team[i])}
409     gdeConstraint{m, constr1{m, num_on_team_neg[i] == 1 - used_team[i]}}
410     gdeConstraint{m, sum(used_team[i], i::num_teams) == 3}
411
412     # No goalies going against skaters
413     gdeConstraint{m, constr1{m, true}, #goalies_lineup[i] + sum(goalie_opponents[k], k::skaters_lineup[k], k::num_skaters) == 0}
414
415     # Must have at least one complete line in each lineup
416     gdeVar{m, pos_num_in_line[i::num_lines] == 0}
417     gdeVar{m, neg_num_in_line[i::num_lines] == 0}
418     gdeConstraint{m, constr1{m, pos_num_in_line[i] + neg_num_in_line[i] == sum(team_lines[k], skaters_lineup[k], k::num_skaters) - 2.5}}
419     gdeVar{m, line_stack[i::num_lines], Bin}
420     gdeConstraint{m, constr1{m, num_lines == line_stack[i]}}
421     gdeConstraint{m, constr1{m, pos_num_in_line[i] == line_stack[i]}}
422     gdeConstraint{m, constr1{m, neg_num_in_line[i] == 3 - line_stack[i]}}
423
424     # Must have at least 3 lines per team at least 8 people
425     gdeVar{m, pos_num_in_line2[i::num_lines] == 0}
426     gdeVar{m, neg_num_in_line2[i::num_lines] == 0}
427     gdeConstraint{m, constr1{m, pos_num_in_line2[i] + neg_num_in_line2[i] == sum(team_lines[k], skaters_lineup[k], k::num_skaters) - 1.5}}
428     gdeVar{m, line_stack2[i::num_lines], Bin}
429     gdeConstraint{m, constr1{m, num_lines == 2 * line_stack2[i]}}
430     gdeConstraint{m, constr1{m, pos_num_in_line2[i] == 2 * line_stack2[i]}}
431     gdeConstraint{m, constr1{m, neg_num_in_line2[i] == 2 * line_stack2[i]}}
432
433
434     # The defenders must be on P1
435     gdeConstraint{m, sum(defenders[i], PI_info[i], skaters.lineup[i], i::num_skaters), f::num_teams} == sum(defenders[i], skaters.lineup[i], i::num_skaters)
436
437     # Overlap Constraint
438     gdeConstraint{m, constr1{m, size(lineups)[i]}, sum(lineups[i], skaters.lineup[i], j::num_skaters) + sum(lineups[num_skaters-j], goalies.lineup[i], j::num_goalies) == num_overlap}
439
440
441     # Objective
442     setObjective(m, Max, sum(skaters[i], iHOT_PROJECTED_SKATERS[i]) * skaters.lineup[i], i::num_skaters) + sum(goalies[i], iHOT_PROJECTED_GOALIES[i]) * goalies.lineup[i], i::num_goalies)
443     println("Solving Problem...")
444     print("Status: ")
445     status = solve(m);
446
447     if status == Optimist
448         skaters_lineup_copy = Array(Int64, 8)
449         for i in 1:num_skaters
450             skaters_lineup_copy[i] = gdeValue(skaters.lineup[i])
451             if gdeValue(skaters.lineup[i]) == 0.0 then gdeValue(skaters.lineup[i]) == 1.0
452             else skaters_lineup_copy[i] = vcat(skaters.lineup_copy, fill(0, 1))
453             end
454         end
455         for i in 1:num_goalies
456             skaters_lineup_copy[i] = gdeValue(goalies.lineup[i])
457             if gdeValue(goalies.lineup[i]) == 0.0 then gdeValue(goalies.lineup[i]) == 1.0
458             else skaters_lineup_copy[i] = vcat(skaters.lineup_copy, fill(0, 1))
459             end
460         end
461     end
462
463     Lineup = skaters_lineup_copy
```

# Performance Time



# In the paper...

- Consider several strategies
- Different Integer Programming formulations
- Varying prediction models
- Number of lineups
- <http://arxiv.org/pdf/1604.01455v1.pdf>



**YOU CAN DO IT**