

Modelamiento Avanzado con Programación Entera Mixta

Parte 1/3

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Vendedor Viajero y Programación Entera

Podemos Enumerar las Rutas?

Nro. de Ciudades	Nro. de Soluciones Factibles
10	$10^{5.5}$
100	10^{156}
1,000	$10^{2,565}$
33,810	$10^{138,441}$
85,900	$10^{456,000}$

Edad del universo (en segundos)	10^{18}
Número de átomos en el universo	$< 10^{100}$

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Vendedor Viajero y Programación Entera

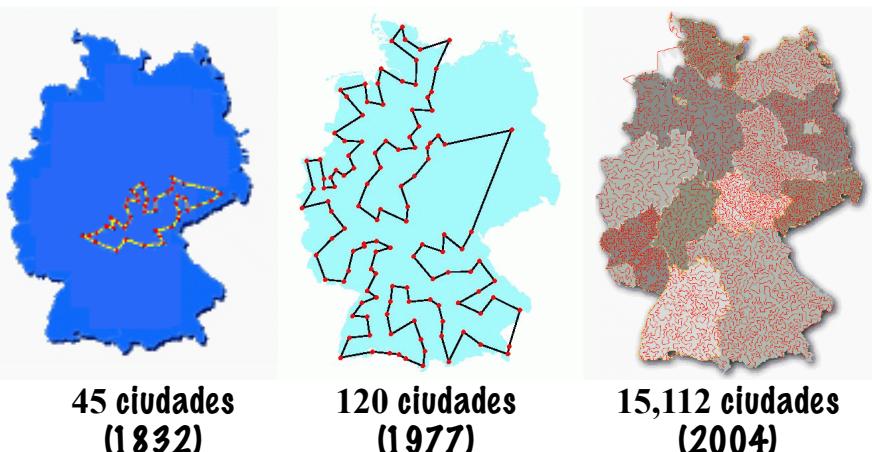
Modelo Programación Entera (PE)

$$\begin{aligned} \min \quad & \sum_{e \in E} d_e x_e \\ st \quad & \sum_{e \in \delta(i)} x_e = 2 \quad \forall i \in V \\ & \sum_{e \in \delta(S)} x_e \geq 2 \quad \forall S \subsetneq V \\ & x_e \in \{0, 1\} \quad \forall e \in E \end{aligned}$$

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Recorrer Alemania con PE



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Problema de Programación Entera

$$\begin{aligned} \max \quad & \sum_{i=1}^N a_i x_i \\ \text{s.t.} \quad & Ax \leq b \\ & x_i \in \mathbb{Z} \quad \forall i \in I \subset \{1, \dots, N\} \end{aligned}$$

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Mina de Tajo/Rajo Abierto



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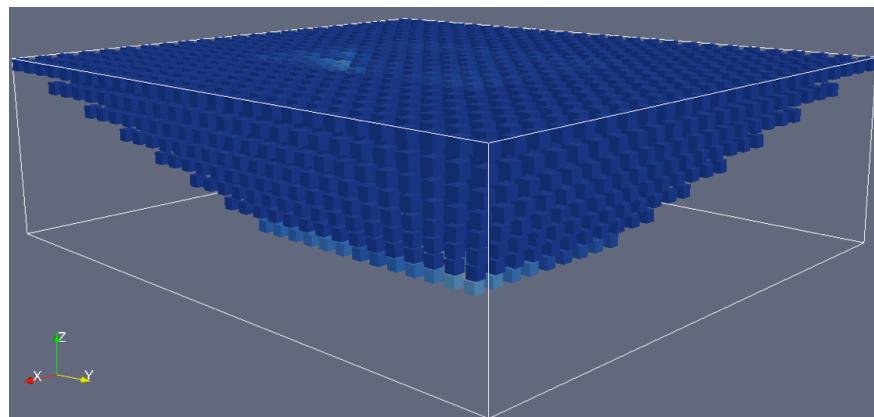
Ejemplos y Aplicaciones

- Combinatorial o 0-1
 - Minería
 - Forestal
 - Computación Biológica
- Entera Mixta
 - Alternativas, Funciones No-lineales, Restricciones Probabilísticas

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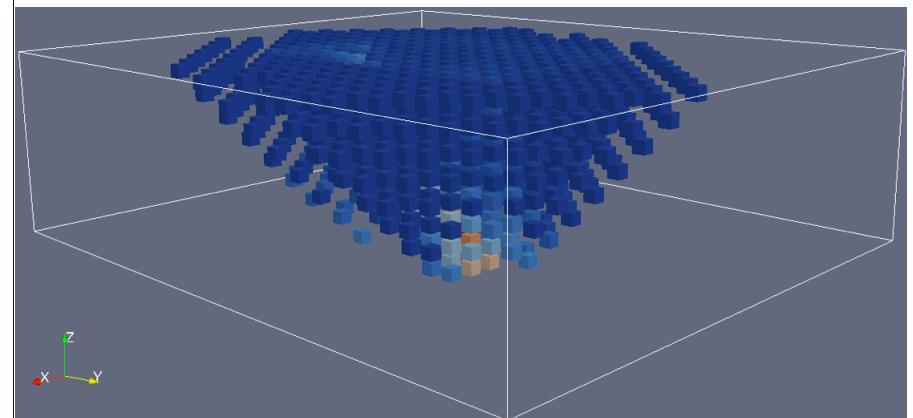
Modelo de Bloque



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Que bloques extraer?



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Variables: Extraer o No

1	2	3
4	5	6

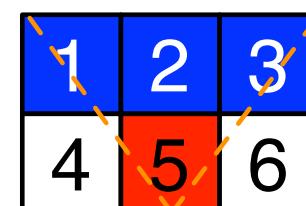
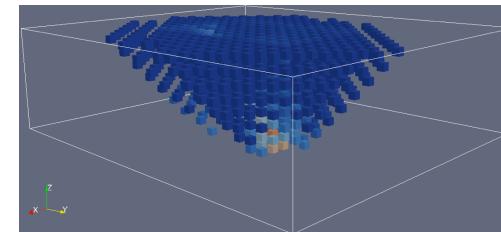
$$x_i = \begin{cases} 1 & \text{si el bloque } i \\ & \text{es extraido} \\ 0 & \text{si no} \end{cases}$$

- Restricciones y Objetivos Lineales:
 - VPN, capacidad de extracción/procesamiento, etc.
- Restricciones Combinatoriales:
 - Reglas de precedencia

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Extraer = Reglas de Precedencia

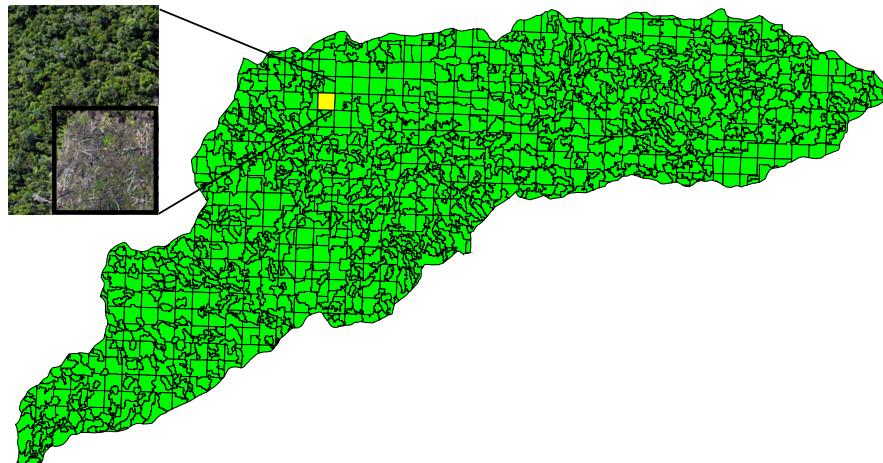


$$\begin{aligned} x_5 &\leq x_1 \\ x_5 &\leq x_2 \\ x_5 &\leq x_3 \end{aligned}$$

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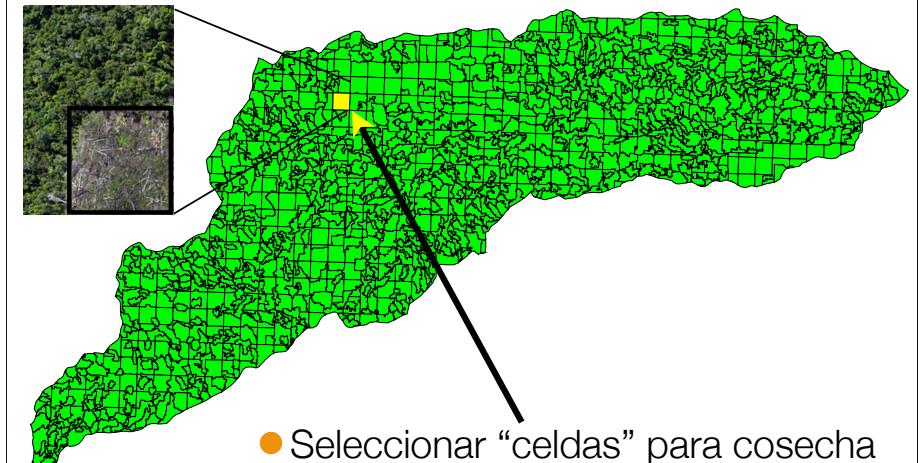
Problema de Planificación Forestal



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Problema de Planificación Forestal

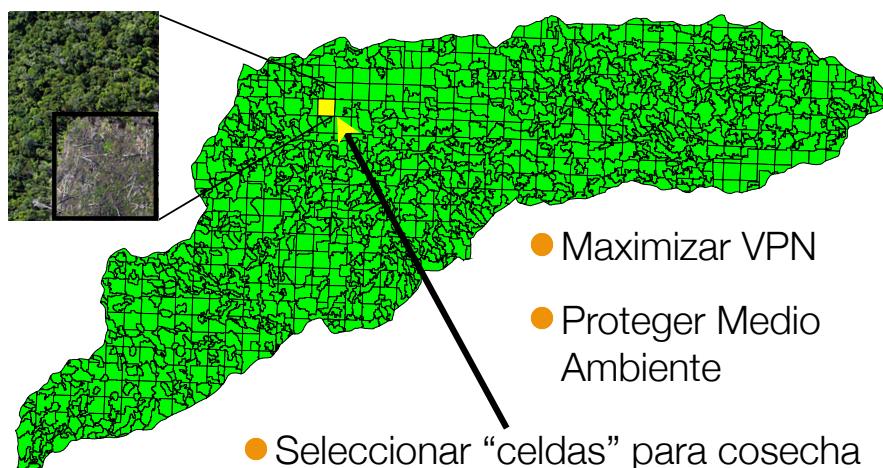


- Seleccionar “celdas” para cosecha

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Problema de Planificación Forestal

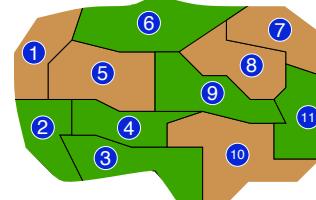


- Maximizar VPN
- Proteger Medio Ambiente
- Seleccionar “celdas” para cosecha

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Variables: Cuando Cosechar



$$x_{v,t} = \begin{cases} 1 & \text{si celda } v \text{ es cosechada en el periodo } t. \\ 0 & \text{si no} \end{cases}$$

- Restricciones y Objetivos Lineales:
 - VPN, flujo de madera, edad final del bosque, etc.
- Restricciones Combinatoriales:
 - Proteger el Medio Ambiente

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Formas de Proteger el Ambiente

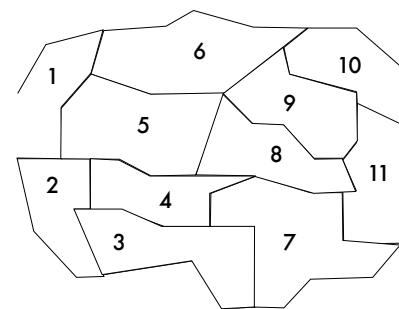
- No cosechar grandes áreas contiguas
 - erosión
 - animales no cruzan
 - belleza escénica
- Reservas contiguas



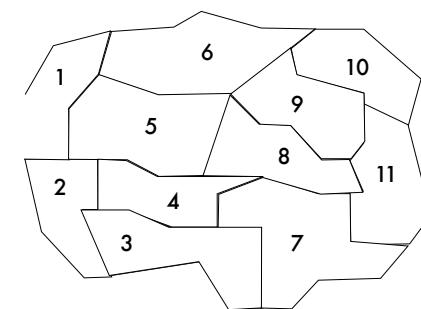
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No cosechar grandes áreas contiguas



● Aceptable

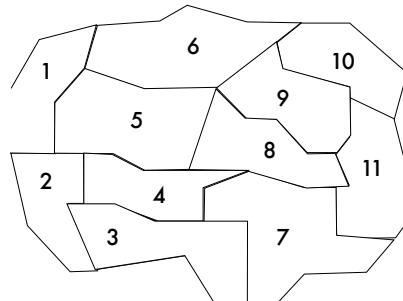


● No Aceptable

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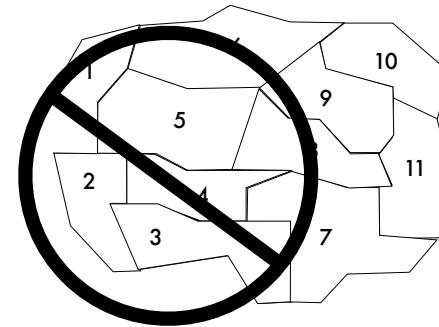
Prohibir Cosechas No Aceptables



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Prohibir Cosechas No Aceptables

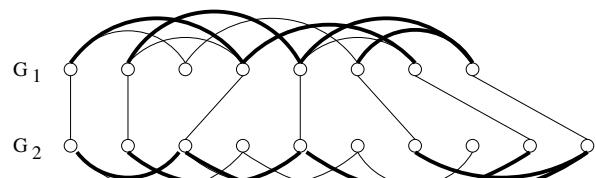
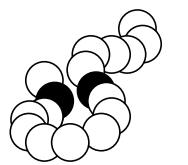
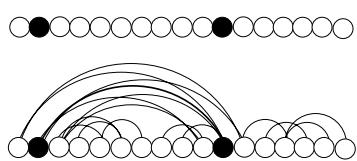


$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 4$$

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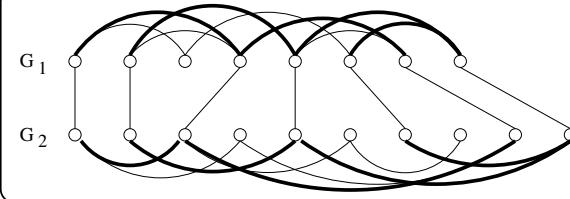
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Alineamiento de Proteinas



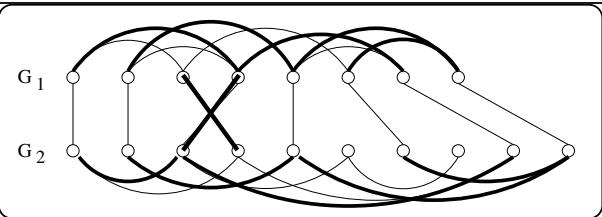
● Carr y Lancia 2004

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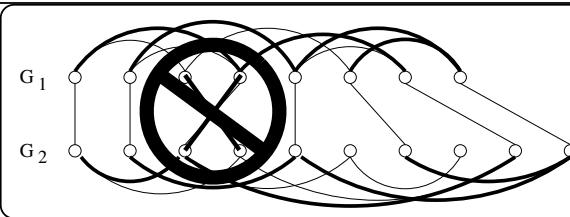
$$\begin{aligned} \max & \sum_{(i,j) \in E_1, (u,v) \in E_2} y_{(i,j),(u,v)} \\ & y_{(i,j),(u,v)} \leq x_{i,u} \\ & y_{(i,j),(u,v)} \leq x_{j,v} \\ & \sum_{(u,v) \in F} x_{u,v} \leq 1 \quad \forall F \in \mathcal{C} \\ & x_{u,v}, y_{(i,j),(u,v)} \in \{0, 1\} \quad \forall u, v, i, j \end{aligned}$$

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$$\begin{aligned} \max & \sum_{(i,j) \in E_1, (u,v) \in E_2} y_{(i,j),(u,v)} \\ & y_{(i,j),(u,v)} \leq x_{i,u} \\ & y_{(i,j),(u,v)} \leq x_{j,v} \\ & \sum_{(u,v) \in F} x_{u,v} \leq 1 \quad \forall F \in \mathcal{C} \\ & x_{u,v}, y_{(i,j),(u,v)} \in \{0, 1\} \quad \forall u, v, i, j \end{aligned}$$

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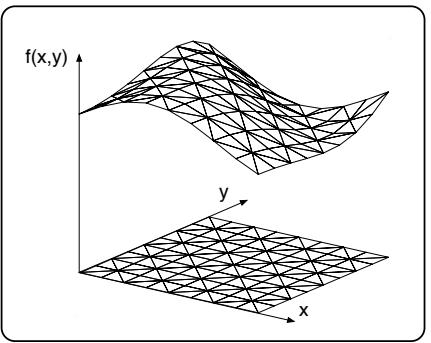
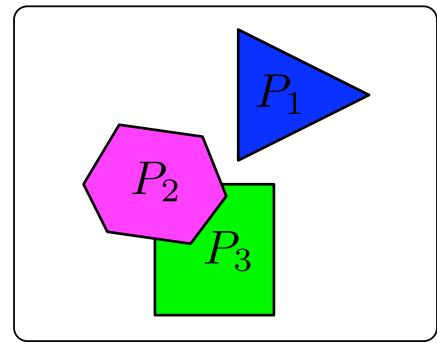


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Disjunciones o Alternativas

$$x = 0 \quad \vee \quad 1 \leq x \leq 2$$



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Restricciones Probabilísticas

$$Q := \{x \in \mathbb{R}^d : \mathbb{P}(x \geq \xi) \geq 1 - \delta\} \quad \xi \sim U(\{\xi^s\}_{s=1}^S)$$

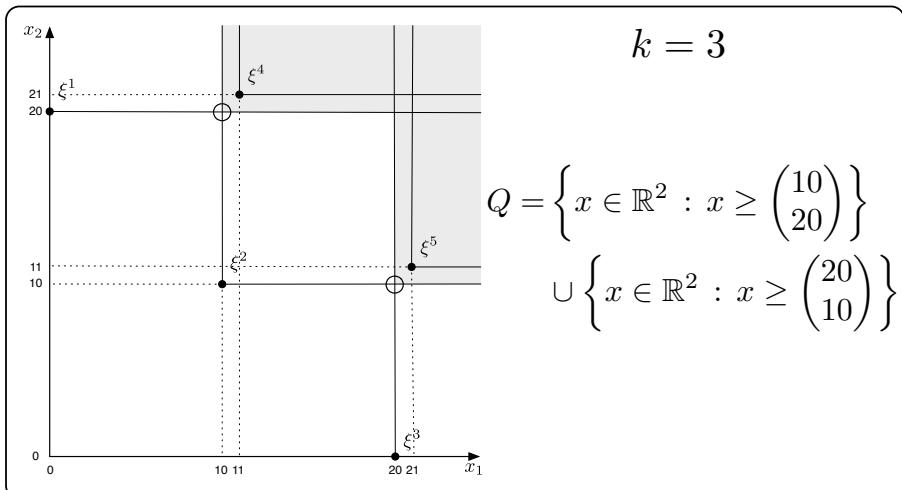
$$Q = \{x \in \mathbb{R}^d : |v(x)| \leq \lfloor \delta D \rfloor =: k\}$$

$$v(x) := \{s \in \{1, \dots, S\} : x \not\geq \xi^s\}$$

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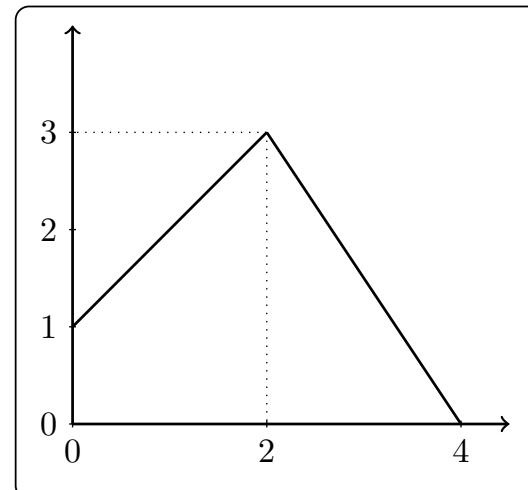
Ejemplo: Violar 3 Restricciones



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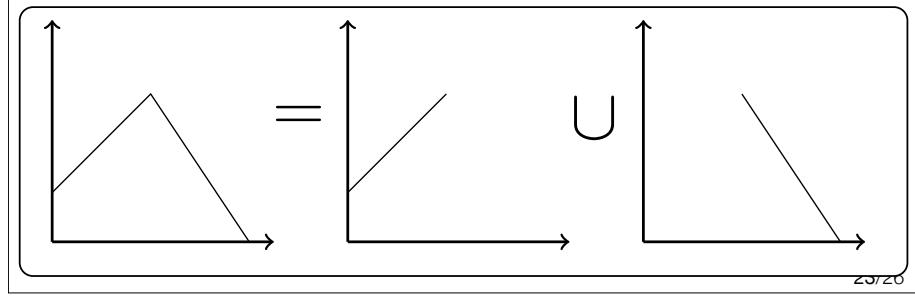
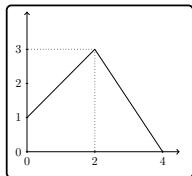
Lineales por trazo = Unión de líneas



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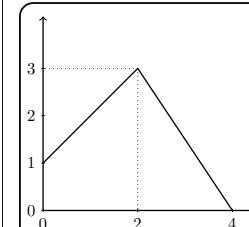
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Lineales por trazo = Union de lineas



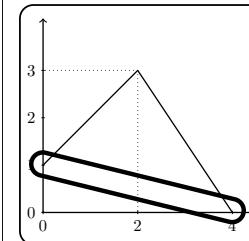
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Modelo Combinacion Convexa



$$f(x) := \begin{cases} x + 1 & x \in [0, 2] \\ 6 - 3/2x & x \in [2, 4] \end{cases}$$

Modelo Combinacion Convexa



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$$\begin{aligned} x &= 0\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ f(x) &= 1\lambda_1 + 3\lambda_2 + 0\lambda_3 \end{aligned}$$

idea: escribir $(x, f(x))$
como combinacion convexa
 $(0, f(0)), (2, f(2)), (4, f(4))$

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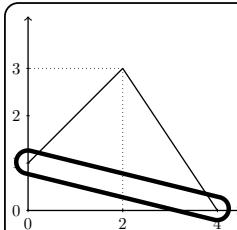
λ_1 y λ_3 no pueden ser > 0 al mismo tiempo.

$$\begin{aligned} x &= 0\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ f(x) &= 1\lambda_1 + 3\lambda_2 + 0\lambda_3 \\ 1 &= \lambda_1 + \lambda_2 + \lambda_3, \quad \lambda_1, \lambda_2, \lambda_3 \geq 0 \end{aligned}$$

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Modelo Combinación Convexa



$$f(x) := \begin{cases} x + 1 & x \in [0, 2] \\ 6 - 3/2x & x \in [2, 4] \end{cases}$$

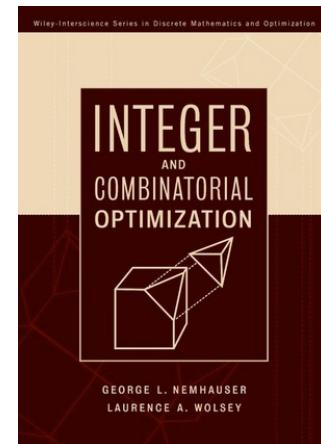
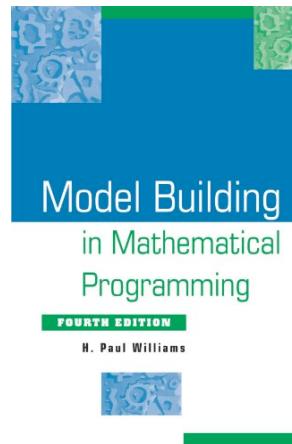
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$$\begin{aligned} x &= 0\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ f(x) &= 1\lambda_1 + 3\lambda_2 + 0\lambda_3 \\ 1 &= \lambda_1 + \lambda_2 + \lambda_3, \quad \lambda_1, \lambda_2, \lambda_3 \geq 0 \\ \lambda_1 &\leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2 \\ 1 &= y_1 + y_2, \quad y_1, y_2 \in \{0, 1\} \end{aligned}$$

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Libros con Modelamiento Básico



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Que significa modelar con PE?

$$x \in S \subset \mathbb{R}^n$$



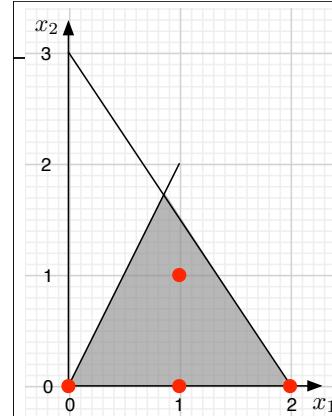
$$\exists y \in \mathbb{R}^m \text{ t.q.}$$

$$Ax + Dy \leq b,$$

$$x_i \in \mathbb{Z} \quad \forall i \in I \subset \{1, \dots, n\}$$

$$y_j \in \mathbb{Z} \quad \forall j \in J \subset \{1, \dots, m\}$$

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$$\min z := x_2$$

$$3x_1 + 2x_2 \leq 6$$

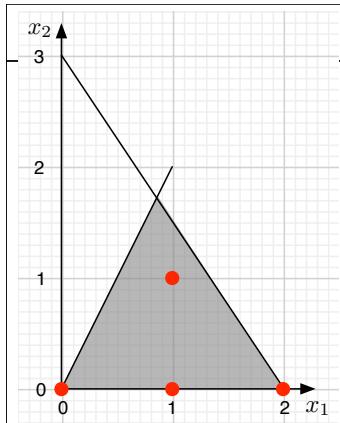
$$-2x_1 + x_2 \leq 0$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2 \in \mathbb{Z}$$

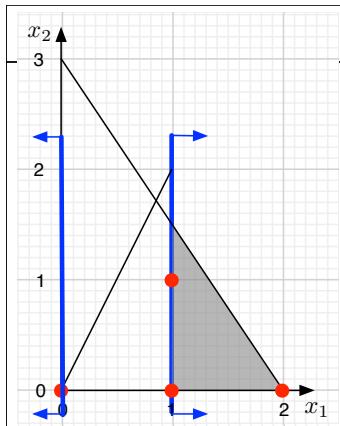
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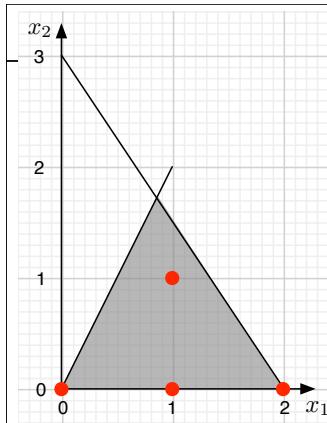
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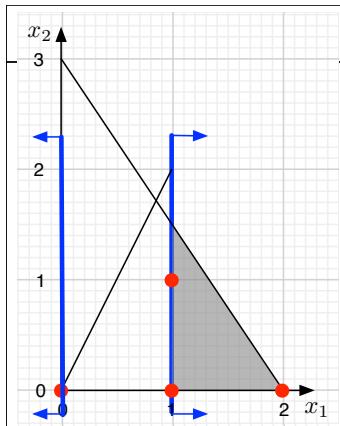
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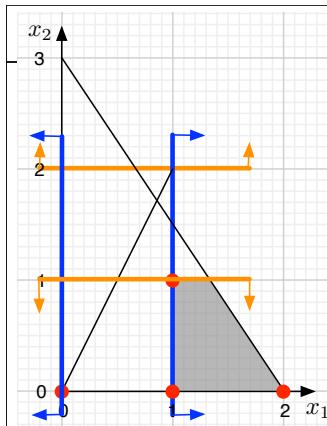
(1) FRAC
z=1.71

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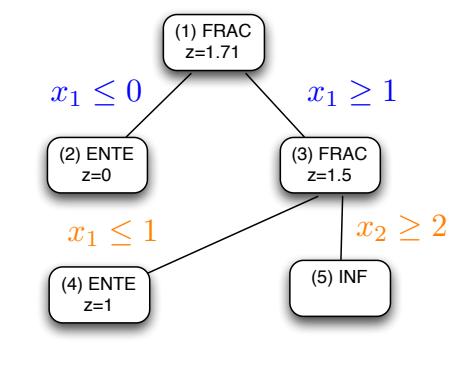


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