Fundamentals of Machine Learning - Worksheet 1

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1 Derivation of Exercise 1.1

Theorem 1 *If $F(x)$ is a CDF with inverse $F^{-1}(u) = \inf\{x : F(x) \geq u\}$ and $U \sim \mathbf{U}(0,1)$, then $X = F^{-1}(U)$ is a random variable with CDF F*

This leads us to the inverse transform algorithm.

Inverse Transform Algorithm:

1. Generate $U \sim \mathbf{U}(0,1)$
2. Return $X = F^{-1}(U)$

Following the algorithm:

1. We calculate the CDF for $Y = 0$ and $Y = 1$. $\forall x \in [0,1]$:

$$F(x) = \text{CDF} = \begin{cases} \int_0^x 2 - 2y \text{ d}y & \text{if } p(X = x|Y = 0) \\ \int_0^x 2y \text{ d}y & \text{if } p(X = x|Y = 1) \end{cases} = \begin{cases} 2x - x^2 & \text{if } p(X = x|Y = 0) \\ x^2 & \text{if } p(X = x|Y = 1) \end{cases}$$

2. We can compute the inverse of the CDF's:

$$F^{-1}(x) = \begin{cases} 1 - \sqrt{1-x} & \text{if } p(X = x|Y = 0)\ \forall x \in [0,1] \\ \sqrt{x} & \text{if } p(X = x|Y = 1)\ \forall x \in [0,1] \end{cases}$$