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On the geometry of deep generative models

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Content

Problem definition

I like images of cats. Are 30k images enough for me?

AutoEncoders

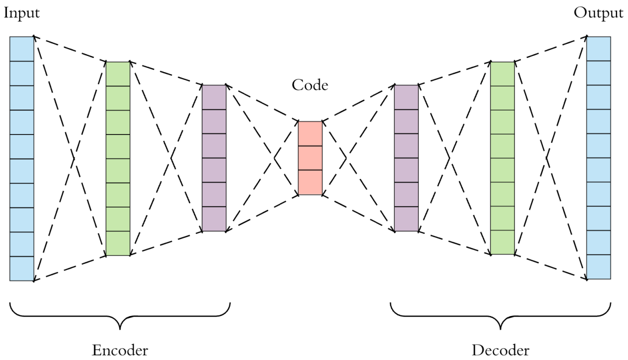
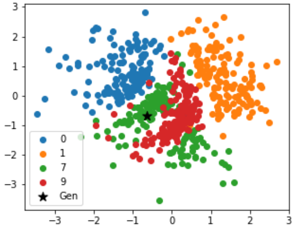


Figure 1: Basic AutoEncoder architecture

Generating data

They allow dimensionality reduction and data generation.



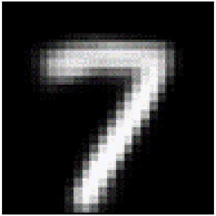


Figure 2: two dimensional latent space (left) and a generated seven (right) via a Variational AutoEncoder [6] with two hidden layers.

Latent Space Interpolations

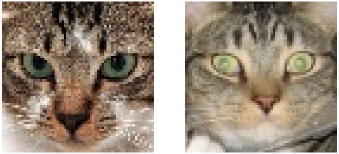


Figure 3: two cute cats.

- Data generation is not always straightforward, sampling can be arbitrary [5].
- Generates intermediate states between data points.
- Produces supervised data [7].

Linear Interpolation

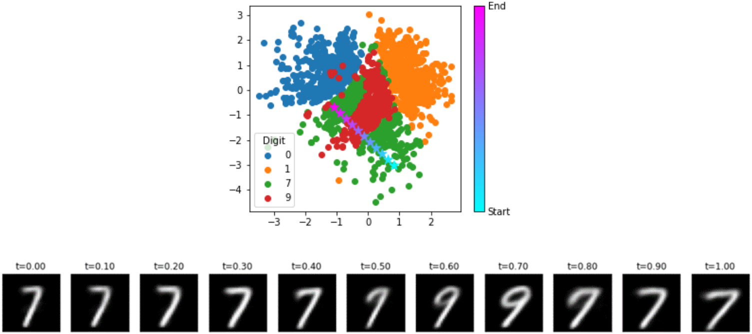


Figure 4: Linear interpolation between two images.

Linear Interpolation Pitfalls

- The latent space and the original space are not isometric.
- Interpolations can traverse areas of high uncertainty, where few real data fell.
- They disregard class divisions.

Distances on the latent space do not appropriately represent distances between data points!

Differential Geometry

Let $Z \subseteq \mathbb{R}^n$ and $X \subseteq \mathbb{R}^N$ be the latent and ambient spaces.

$\gamma : Z \to X$ is a generator.

$J = \dfrac{\partial \gamma}{\partial z}$ is the jacobian of the generator.

$J^T J$ is a metric tensor.

- Changes on the latent space where the metric tensor is large will represent big changes in the ambient space.
- This discourages interpolations between classes and through regions with few training data.

Geodesics

Let $g : [0, 1] \to Z$ be a gesodesic and z_0 and z_1 be interpolation boundaries. An optimal geodesic satisfies the boundary value problem:

$$g^* = \underset{g}{\operatorname{argmin}} \int_0^1 \sqrt{g'(t) J^T J g'(t)} dt$$
$$\text{subject to } g(0) = z_0, \; g(1) = z_1$$

State of the Art

How to model the geodesic:

- As a differential equation [1].
- As a neural network [4].
- As a nearest-neighbour graph [3].

Throughout, the jacobian of the generator is essential, yet numerically unstable and non-smooth [2].

Our Approach

$$g(t) = (1 - t)z_0 + tz_1 + t(1 - t)\psi(t, \theta)$$

$\psi(t, \theta)$ is a feed-forward neural network with parameters θ.

- Trivially satisfies the boundary conditions.
- Is initialised as an approximate linear interpolation.
- Optimisation of θ is focused away from z_0 and z_1.

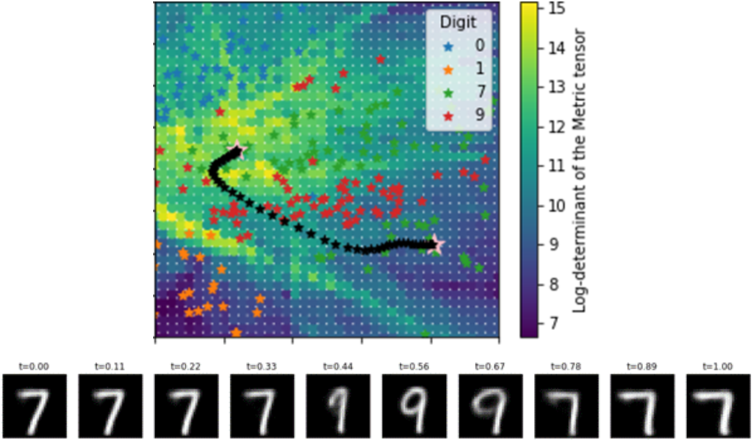


Figure 5: A geodesic connecting two MNIST sevens in the latent space of a VAE.

Difficulties

- The geodesic may not follow the data manifold.
- Convergence issues.
- There is no straightforward way to evaluate the quality of a geodesic.

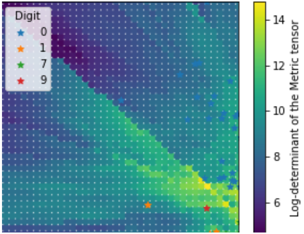


Figure 6: Magnitude of the tensor far away from data.

Future work

- Smooth the metric tensor on areas of high uncertainty [4].
- Adjust the loss function to improve convergence.
- Implement benchmark method from the literature [1].
- Test on a harder dataset [8].

Thank you!

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