Inspire Create Transform

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On the geometry of deep generative models

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Content

Problem definition

I like images of cats. Are 30k images enough for me?

AutoEncoders

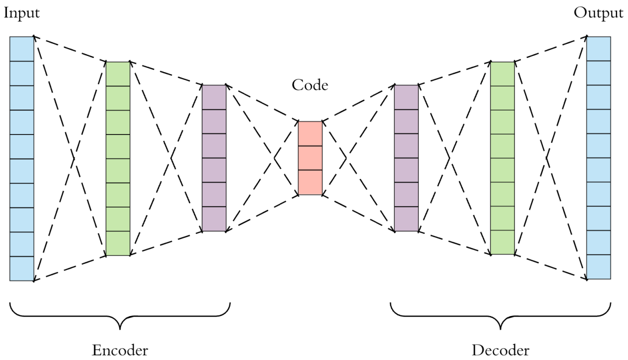

Figure 1: Basic AutoEncoder architecture

Latent Space Interpolations

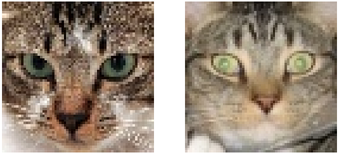

Figure 2: two cute cats.

- Data generation is not always straightforward, sampling can be arbitrary.
- Generates intermediate states between data points.
- Produces supervised data [4].

Differential Geometry

Let $Z \subseteq \mathbb{R}^n$ and $X \subseteq \mathbb{R}^N$ be the latent and ambient spaces.

$\gamma : Z \to X$ is a generator.

$J = \dfrac{\partial \gamma}{\partial z}$ is the jacobian of the generator.

$J^T J$ is a metric tensor.

- Changes on the latent space where the metric tensor is large will represent big changes in the ambient space.
- This discourages interpolations between classes and through regions with few training data.

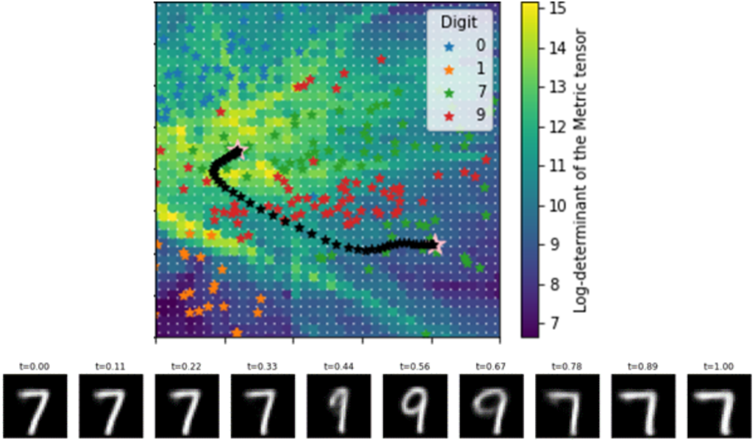

Figure 3: A geodesic connecting two MNIST sevens in the latent space of a VAE.

Latent Space Interpolations

Difficulties:

- Convergence issues.
- The jacobian is ill-conditioned [2].
- The jacobian is not well defined away from data [1].

Jacobians

An estimation of g will greatly depend on the jacobian.

- Magnitude of the tensor: $\det(J^T J)$
- Condition number of the jacobian:

$$\kappa(A) = \frac{|\lambda_{max}|}{|\lambda_{min}|}$$

  Where λ_{max}, λ_{min} are the largest and smaller singular values of A.

A smooth jacobian is necessary for optimisers to converge.

Generative Adversarial Networks

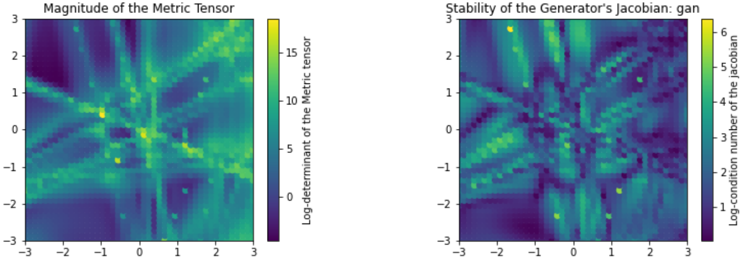

Figure 4: Magnitude of the metric tensor and condition number of the jacobian for a GAN with 128 and 32 hidden dimensions and a Tanh activation.

Variational AutoEncoders

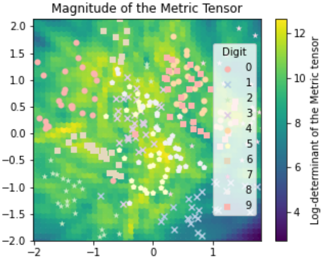

Figure 5: Magnitude of the metric tensor and condition number of the jacobian for a VAE with 128 and 32 hidden dimensions and a Sigmoid activation.

Observations

	Tensor's smoothness	Jacobian's conditioning
GANs	Unsmooth	Poor
DAEs	Adequate	Adequate
VAEs	Adequate	Great

Table 1: Behaviour of the jacobians of different deep generative models.

- VAEs also have good theoretical properties [3].
- Results do not vary with the size or activation function.
- Throughout, jacobians are poorly defined away from data.

Conclusions

- VAEs present a well conditioned jacobian.
- The tensor must improve away from training data. Current solutions are poor [3].
- Ideally, the jacobian should be intervened during the VAE's training.
- Alternatively, it could be modified after training.

Thank you!

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[2] Georgios Arvanitidis, Søren Hauberg, Philipp Hennig, and Michael Schober. Fast and robust shortest paths on manifolds learned from data. *arXiv preprint arXiv:1901.07229*, 2019.

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[4] Vikas Verma, Alex Lamb, Christopher Beckham, Amir Najafi, Ioannis Mitliagkas, David Lopez-Paz, and Yoshua Bengio. Manifold mixup: Better representations by interpolating hidden states. In *International Conference on Machine Learning*, pages 6438–6447, 2019.