

# Hard Shadows Aliasing and Remedies

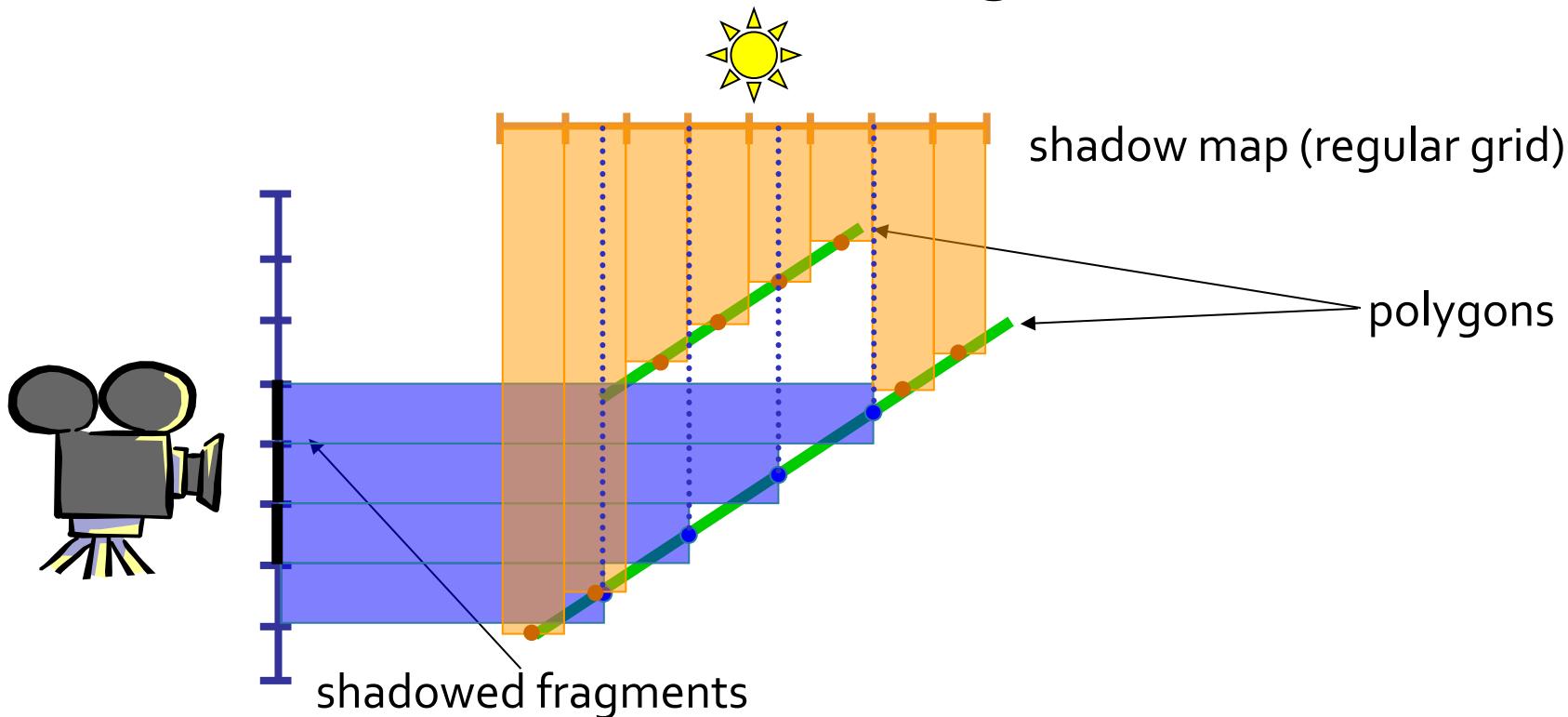
Michael Wimmer

[www.realtimeshadows.com](http://www.realtimeshadows.com)



# Shadow Map as Signal Reconstruction

- Initial sampling: shadow-map rendering
- Resampling: determined by view
- Reconstruction: nearest neighbor, PCF, ...



# Main Types of Error

Initial sampling:

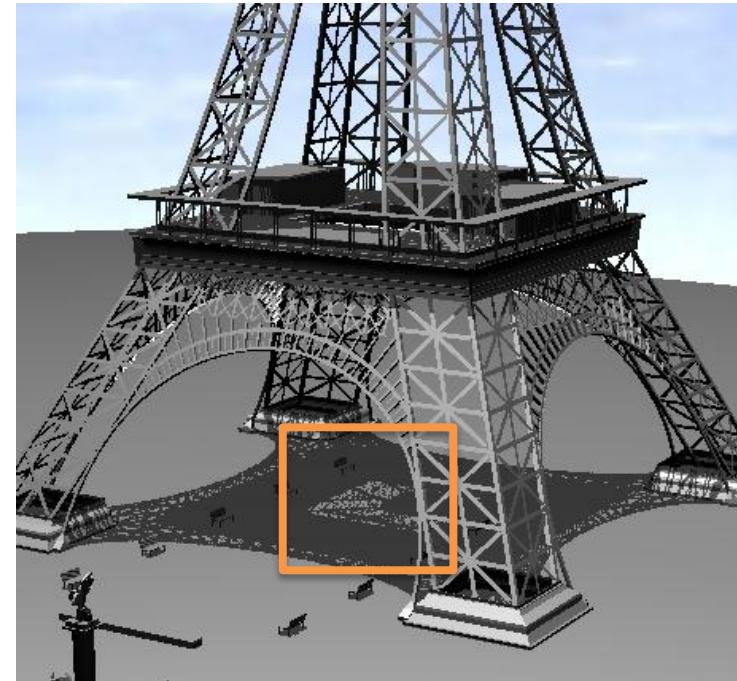
**Undersampling**



25 sample Poisson PCF

Resampling:

**Oversampling**



Reconstruction:

**Reconstruction error  
(with undersampling)**



nearest neighbor

# Main Types of Error

## ■ Undersampling

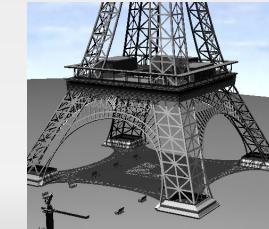
- No bandlimiting (e.g., „super-sampled shadow map“, possible!)
- Improve initial sampling!

Main topic for next  
30 minutes!



## ■ Oversampling

- Use bandlimiting filters in reconstruction (VSM, CSM, ... → later)

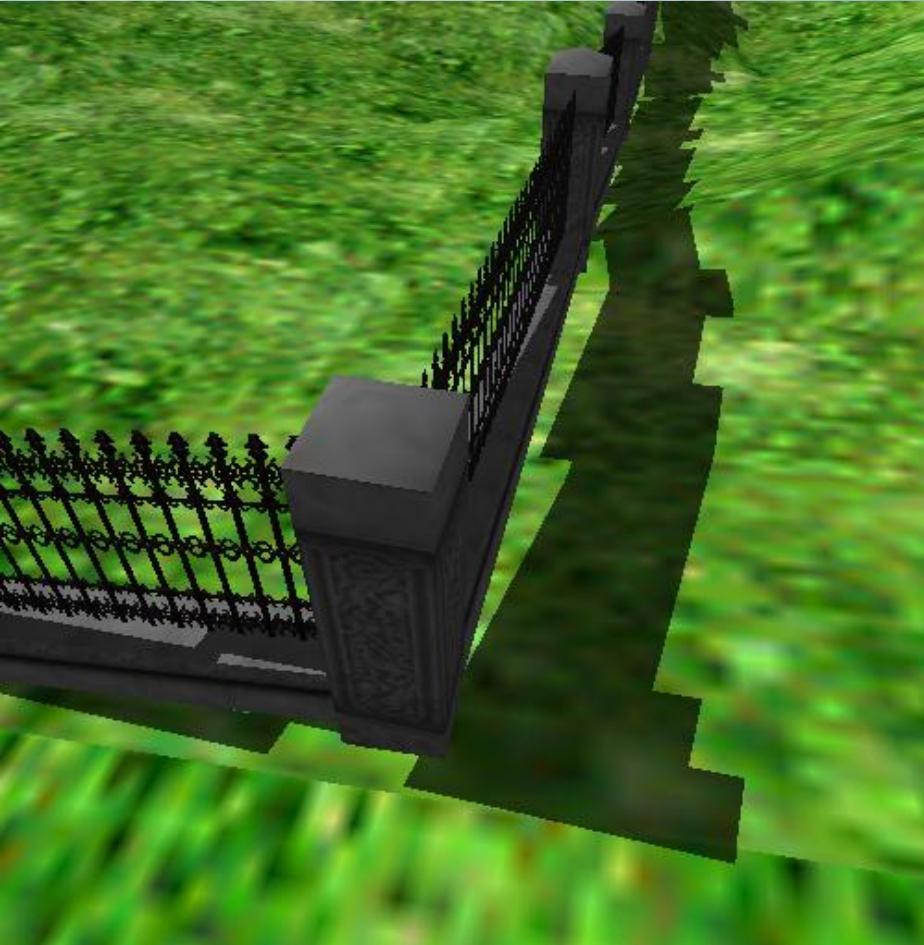


## ■ Reconstruction error

- Use better reconstruction filters (PCF, ...)
- Use different reconstruction algorithm (Silhouette Shadow Maps)



- Improving Initial Sampling/Undersampling
  - Fitting (Focusing)
  - Error Analysis
  - Warping
  - Partitioning
  - Irregular Sampling
  - Temporal Coherence
- Better reconstruction



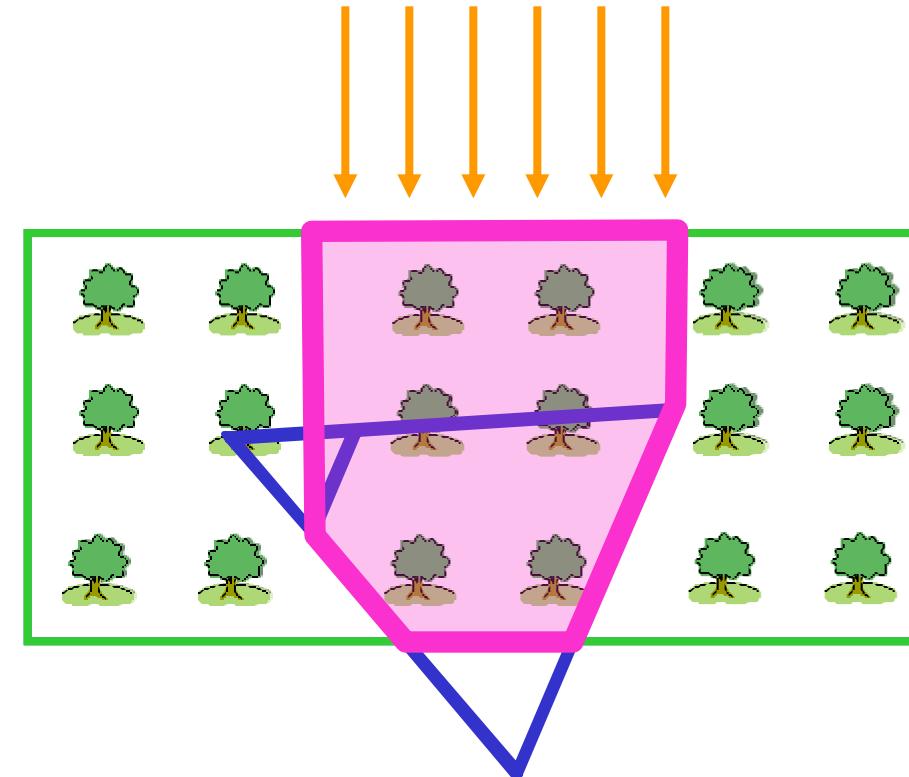
# Hard Shadows

Fighting Undersampling - Fitting

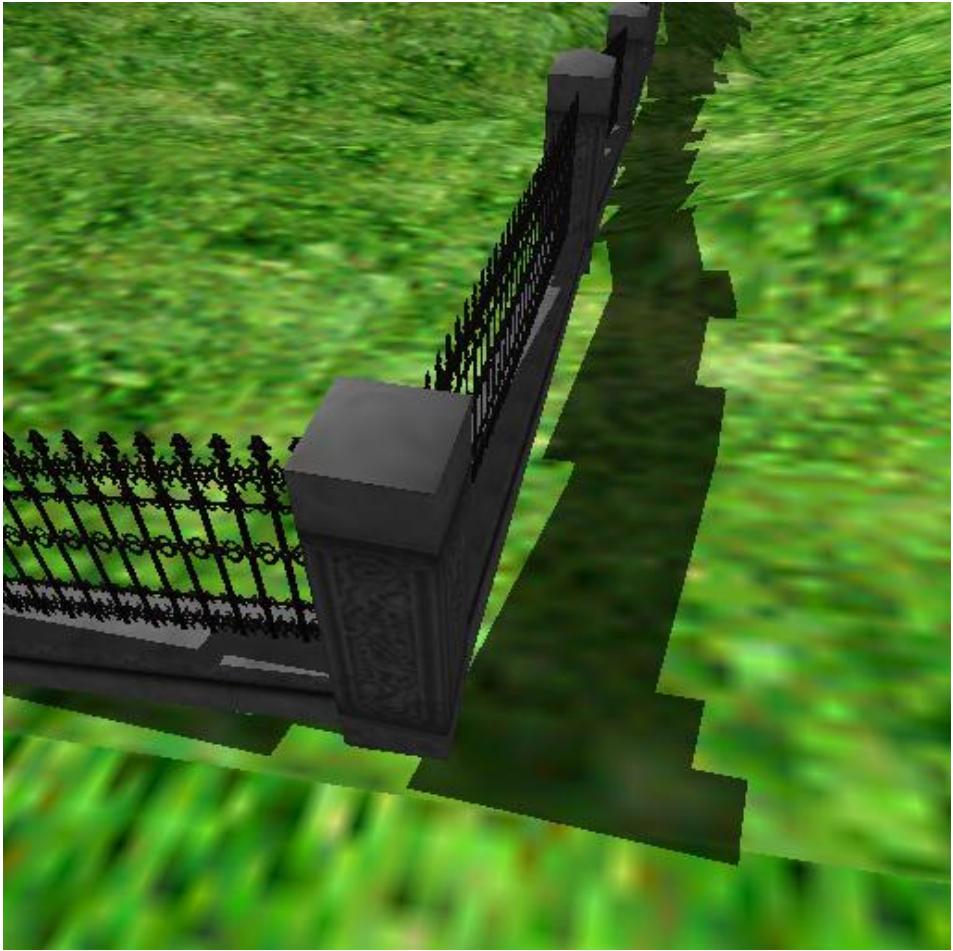
# Fitting: Focus the Shadow Map

[Brabec et al. 2002]

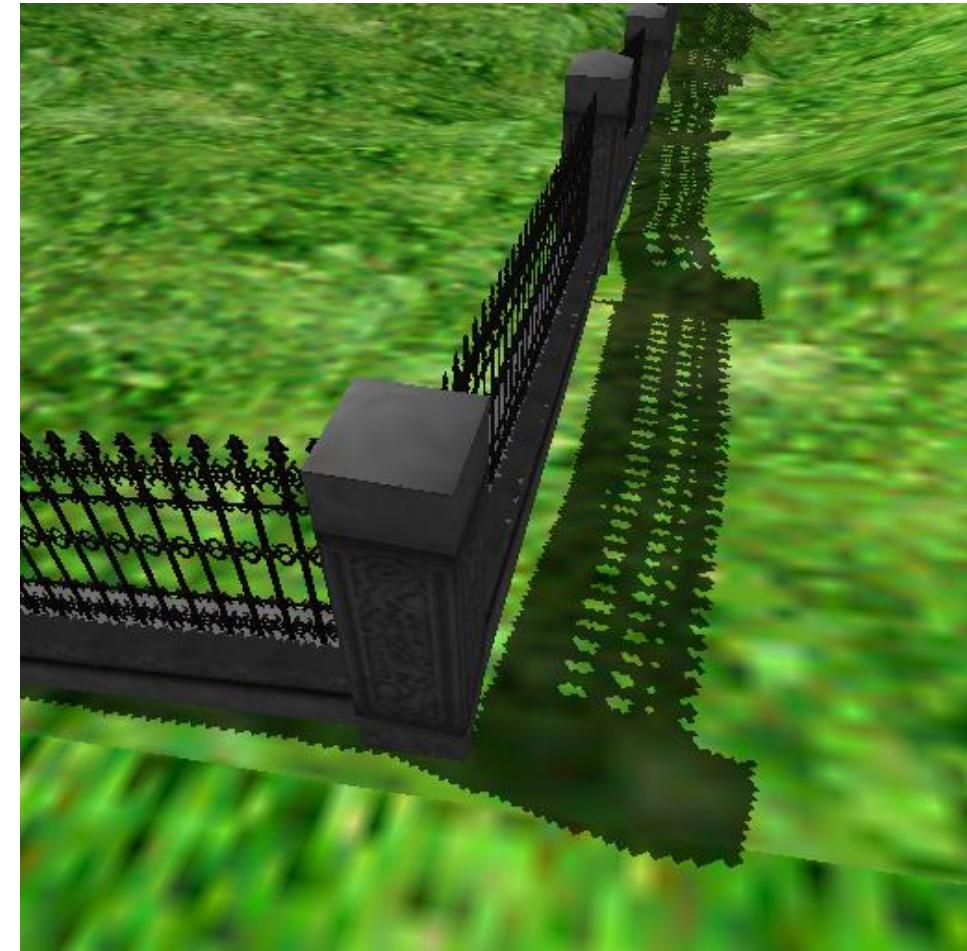
- Only include relevant objects
- Intersection body determined by:
  - Shadow casters
  - Light source frustum
  - View frustum



# Fitting: Focus the Shadow Map



Unfocused



Focused

# Fitting Calculation

- Model matrix  $\mathbf{M}$ , light view matrix  $\mathbf{Lv}$ , light proj. matrix  $\mathbf{Lp}$
- Transform intersection body by  $\mathbf{Lp} \mathbf{Lv}$
- Calculate bounds (xmin,ymin,xmax,ymax)
- Calculate fitting matrix  $\mathbf{F}$  („viewport transform“):

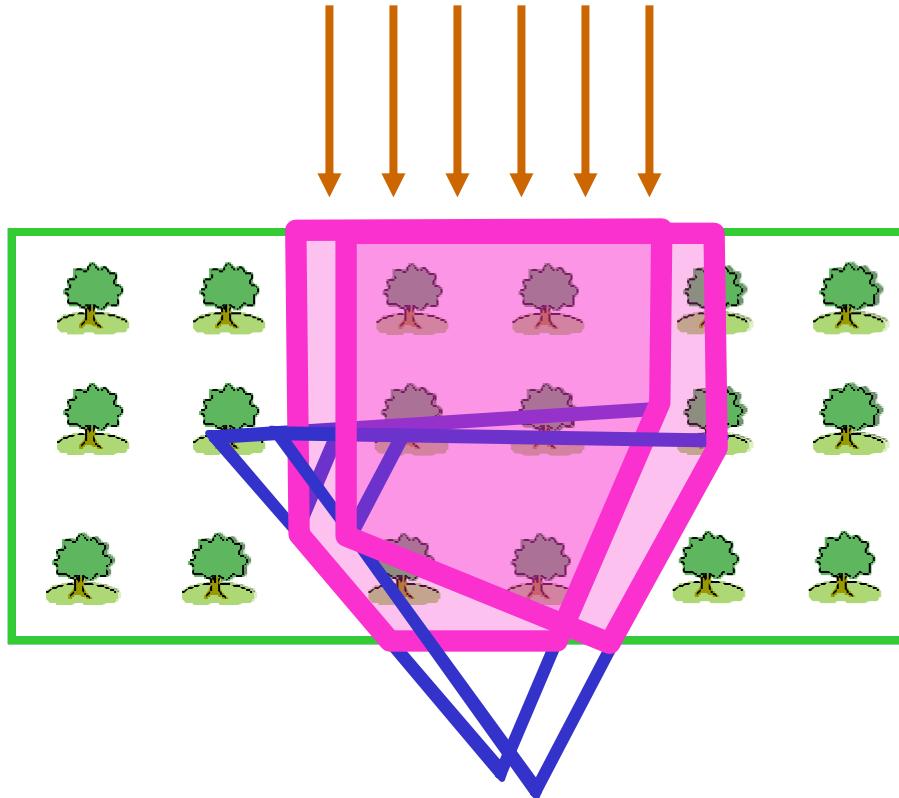
- Shadow matrix:  

$$\mathbf{S} = \mathbf{F} \mathbf{Lp} \mathbf{Lv} \mathbf{M}$$

$$\mathbf{F} = \begin{pmatrix} s_x & 0 & 0 & o_x \\ 0 & s_y & 0 & o_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$s_x = \frac{2}{x_{\max} - x_{\min}}, \quad o_x = -\frac{s_x(x_{\max} + x_{\min})}{2}$$

# Fitting: Temporal Aliasing



# Fitting: Temporal Aliasing

## ■ Solutions:

- Increase initial sampling frequency
- Make fitting adhere to texel boundaries:

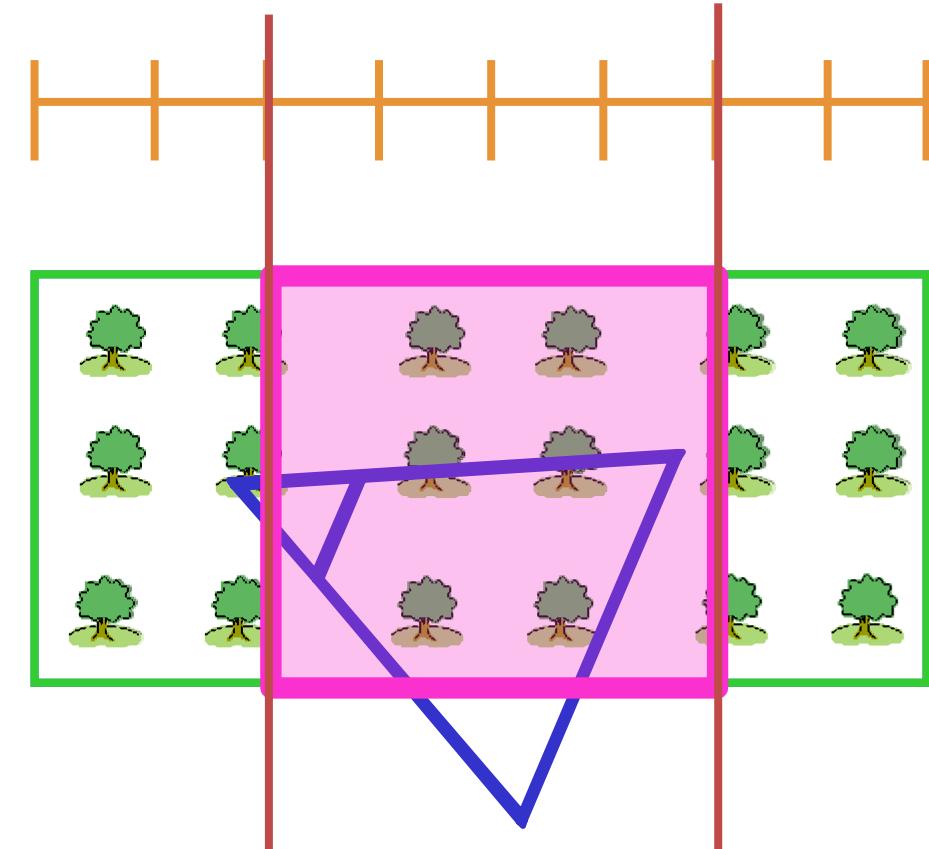
- Adjust fitting matrix  $\mathbf{F}$ :

- $r$ : half shadow-map resolution

- $o$ :  $ox/oy$

$$o' = \frac{\text{ceil}(or)}{r}$$

- Will not work for warping



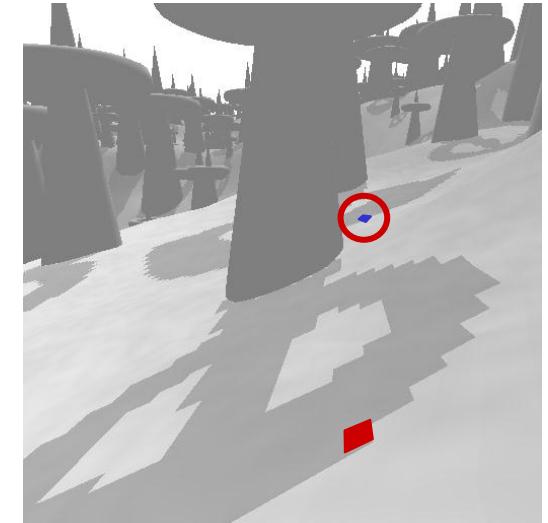
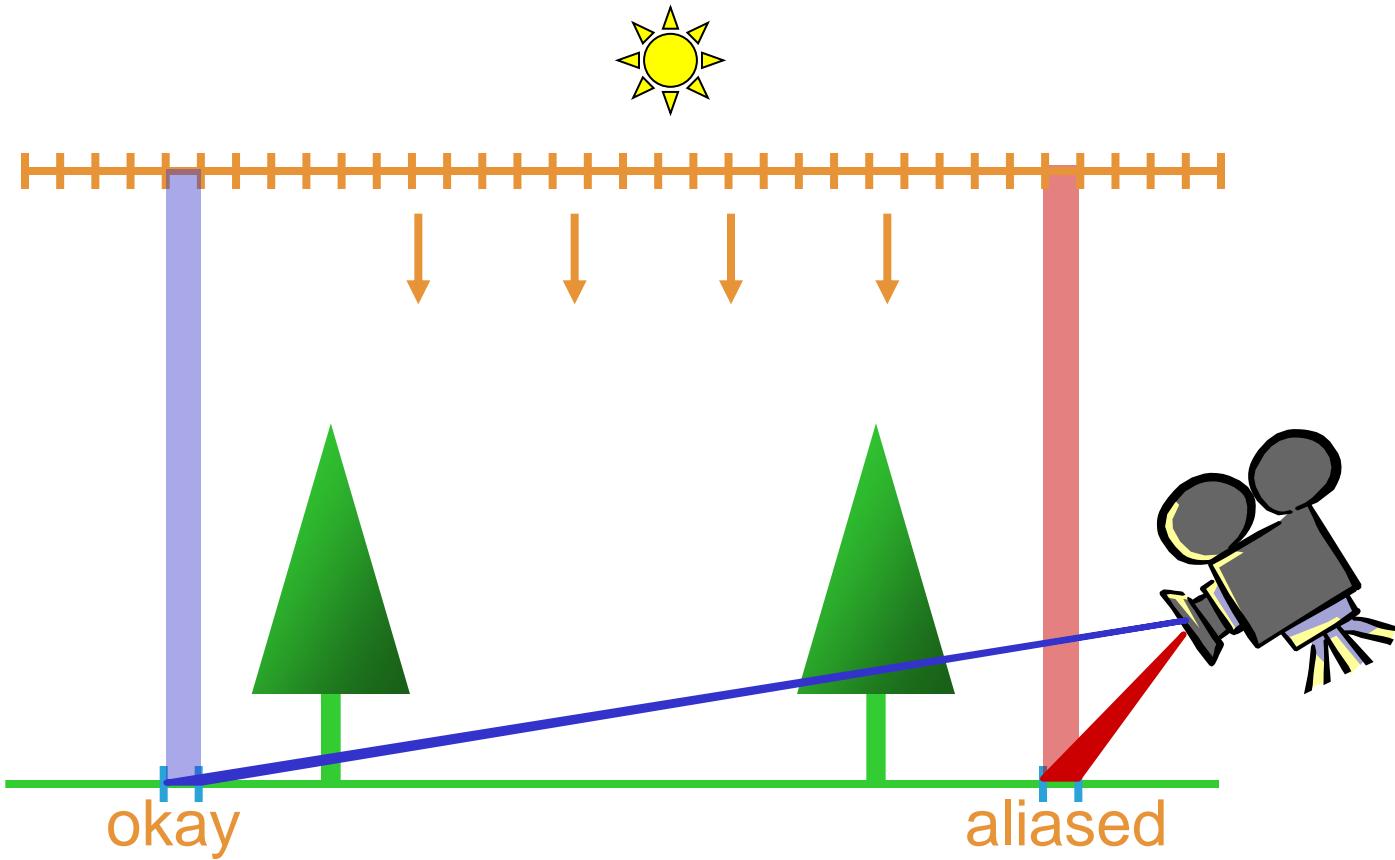
$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos \alpha}{\cos \beta}$$

# Hard Shadows

Error Analysis

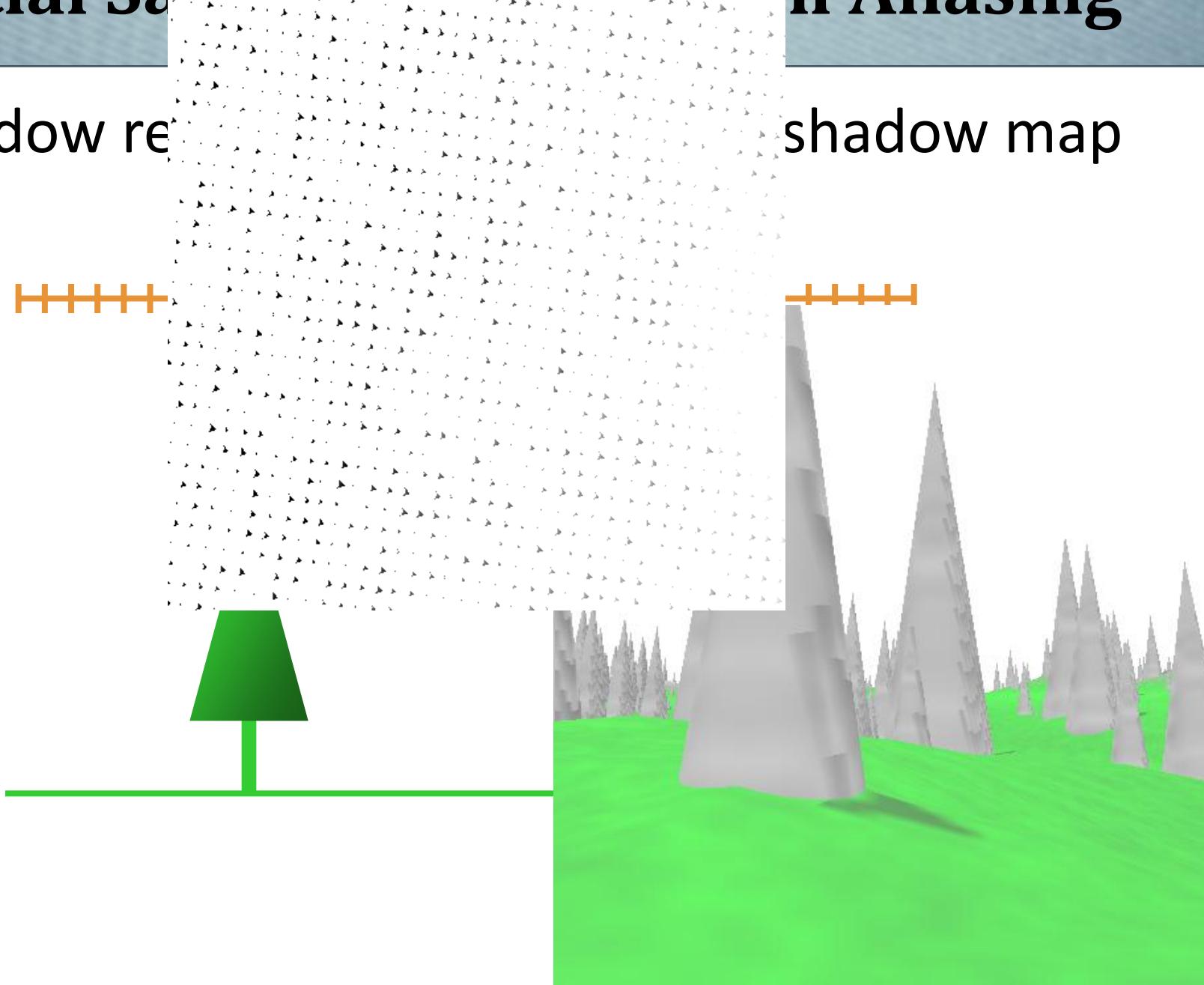
# Initial Sampling - Perspective Aliasing

- **Sufficient** resolution far from the observer
- **Insufficient** resolution near the observer



# Initial Sampling - Projection Aliasing

- Shadow rendering shadow map



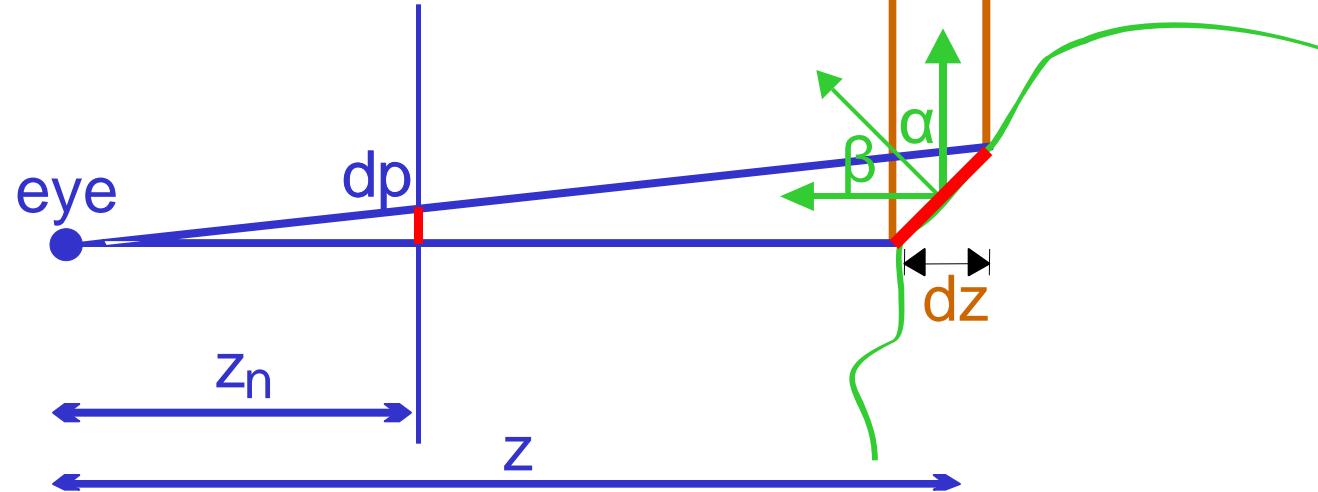
# (Simple) Initial Sampling Error Analysis

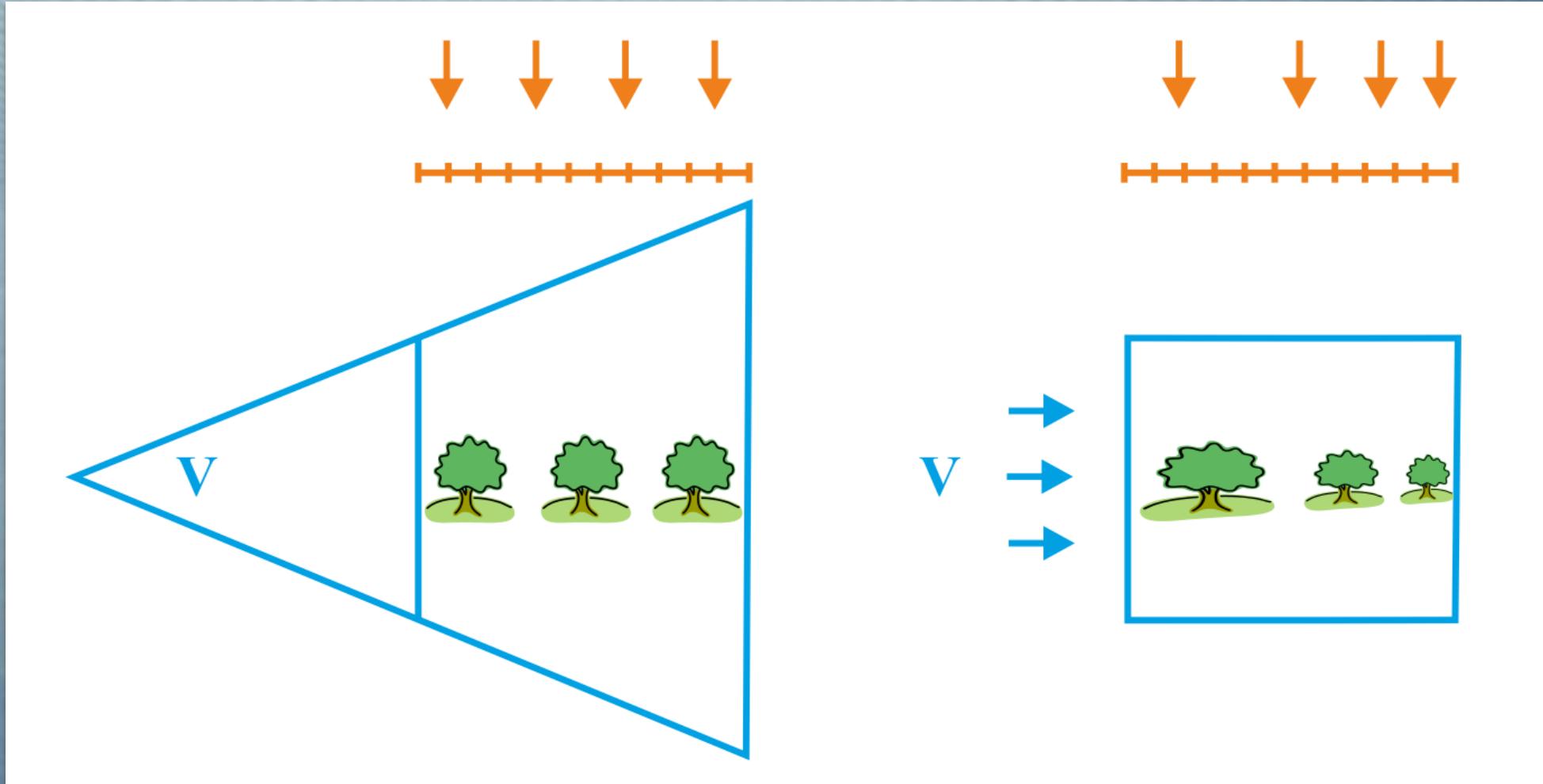
## ■ Aliasing error:

- Parameterization
- Perspective
- Projection

$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos \alpha}{\cos \beta}$$

ds



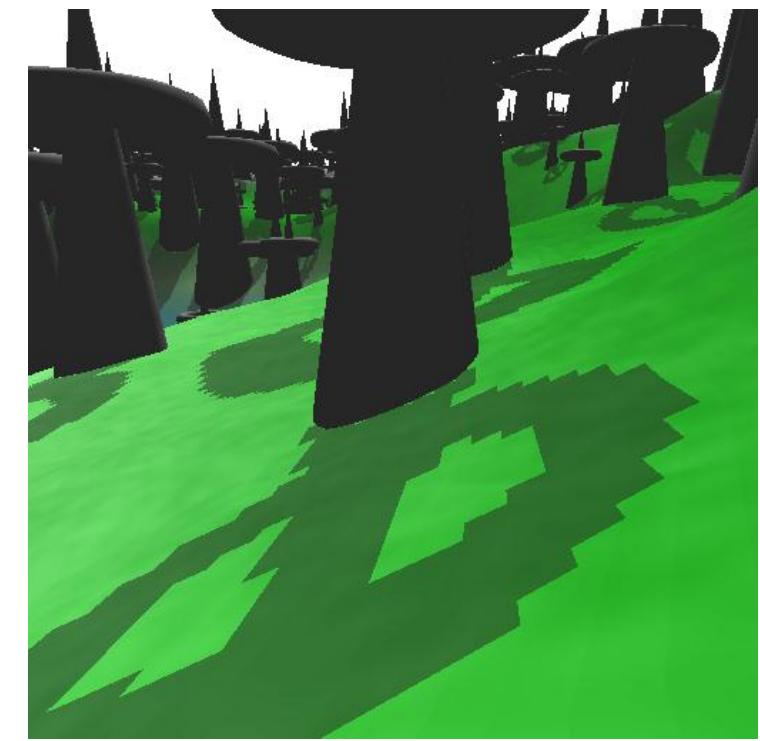
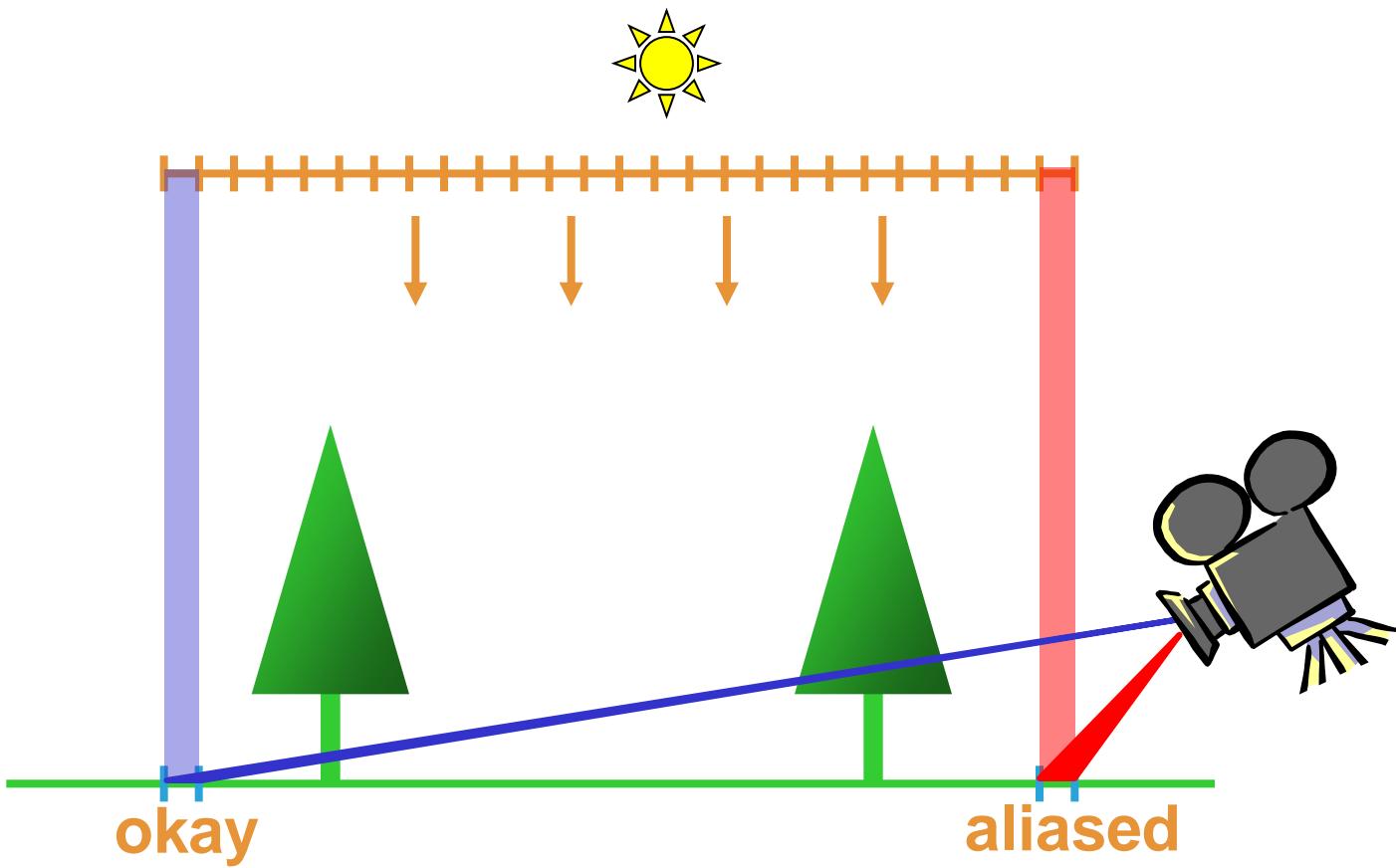


# Hard Shadows

Fighting Undersampling – Warping

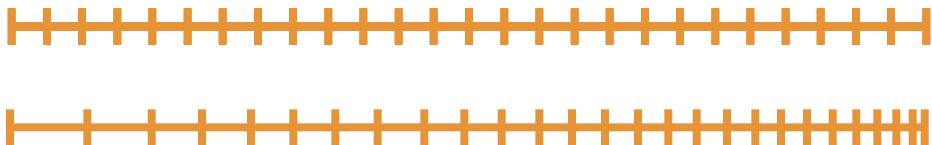
# Solution for Perspective Aliasing

- **Insufficient** resolution near eye



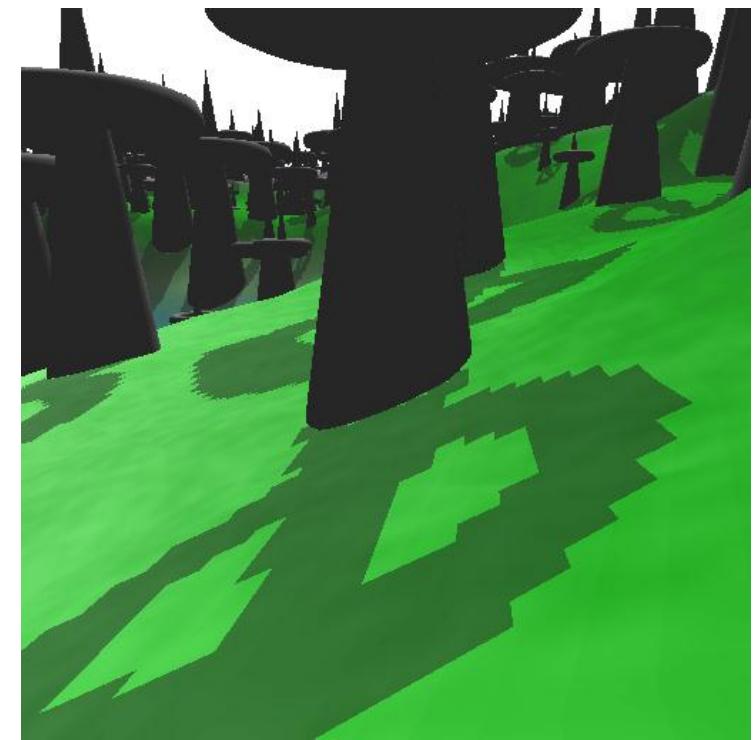
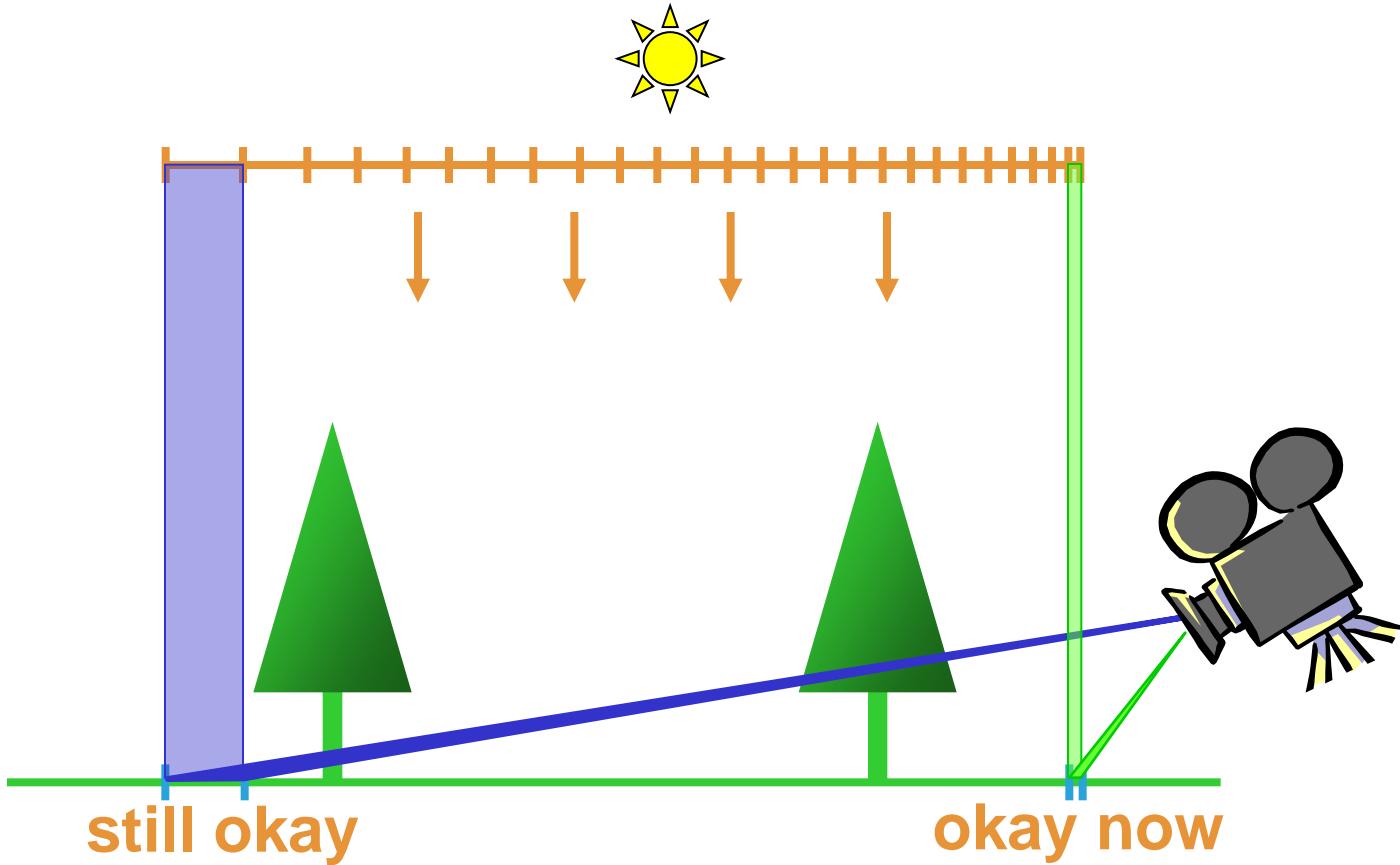
# Solution for Perspective Aliasing

- **Insufficient** resolution near eye
- **Redistribute** values in shadow map



# Solution for Perspective Aliasing

- **Sufficient** resolution near eye
- **Redistribute** values in shadow map

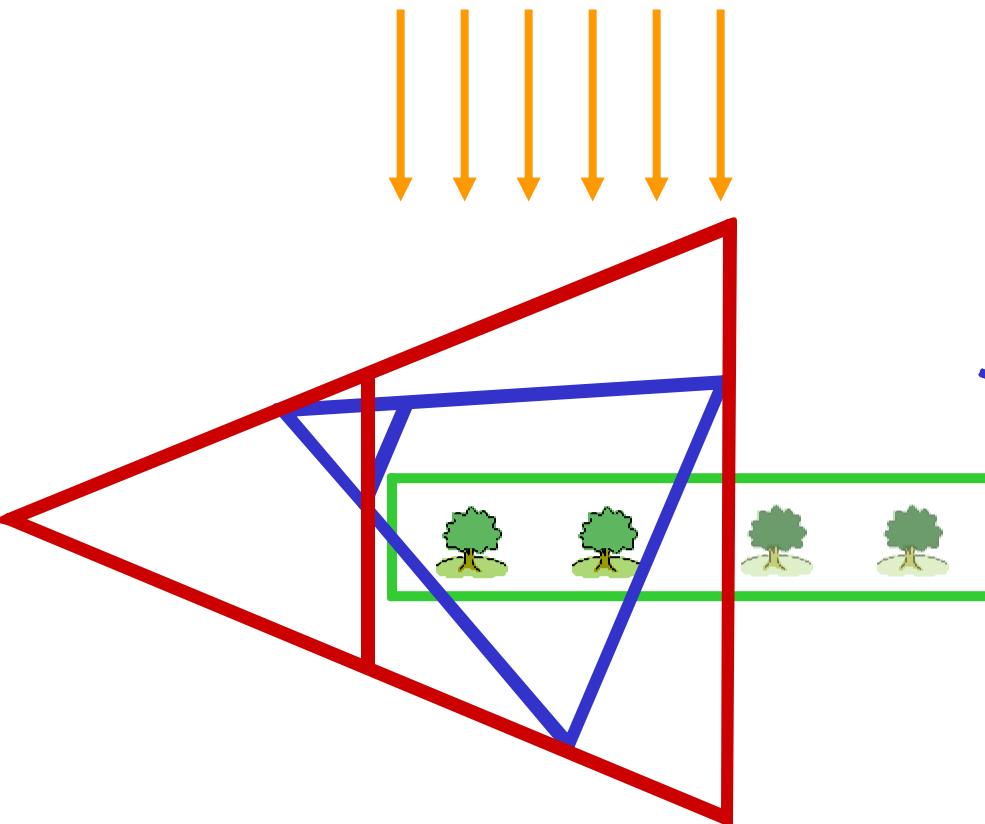


# Shadow Map Warping

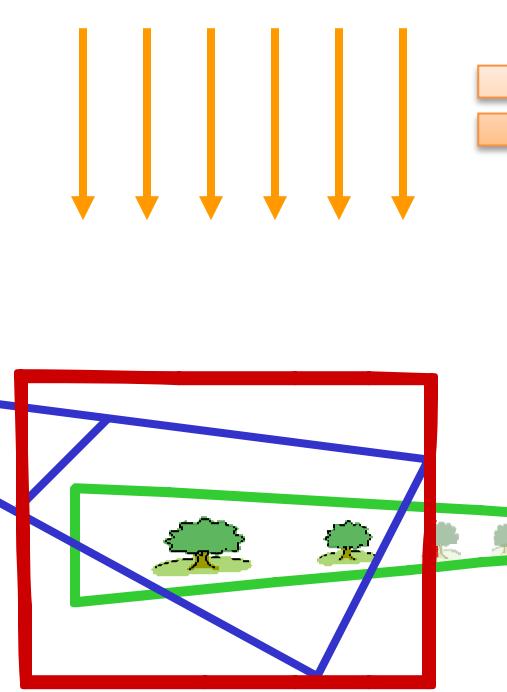
- Use an **additional perspective frustum**

- Perspective Shadow Maps (PSM) [Stamminger & Drettakis 2002]

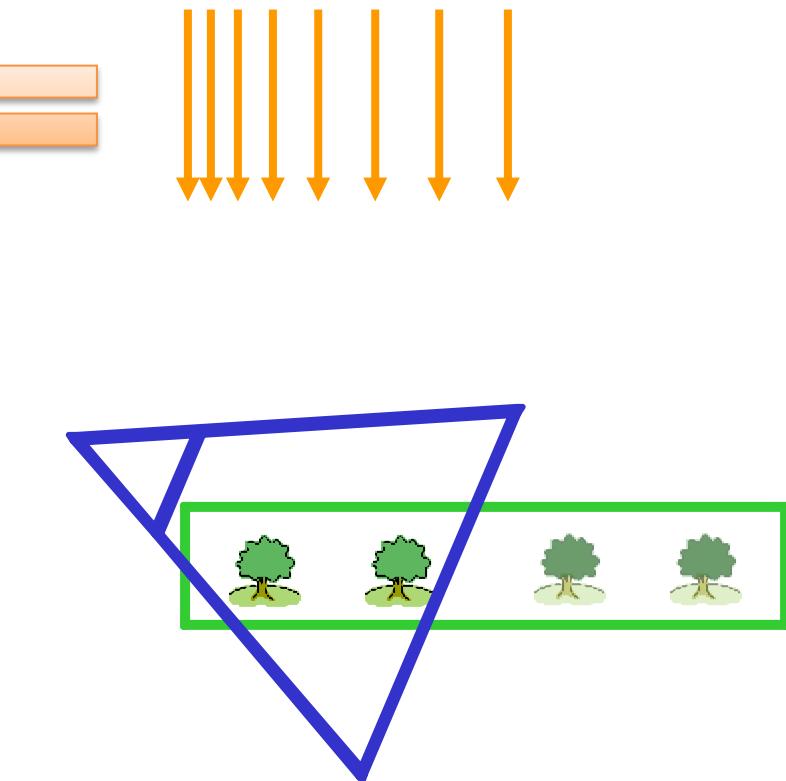
uniform shadow map



warped scene



warped shadow map

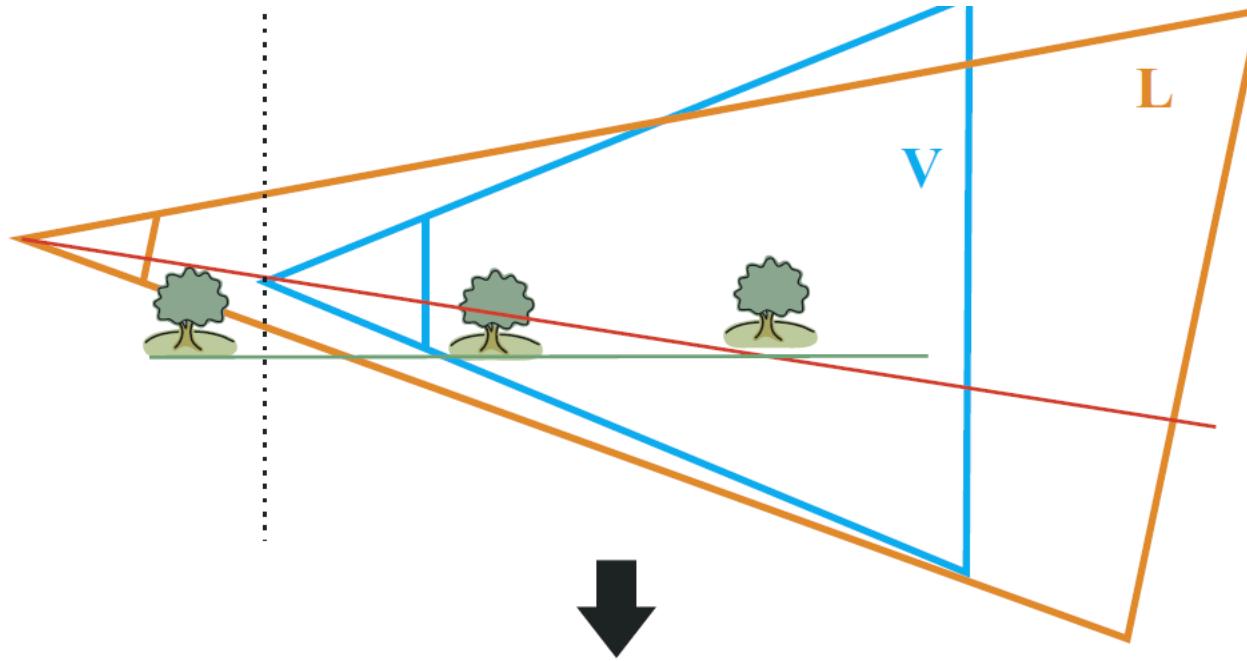


# PSM: Warping Frustum = View Frustum!



→ Shadows from behind

world space

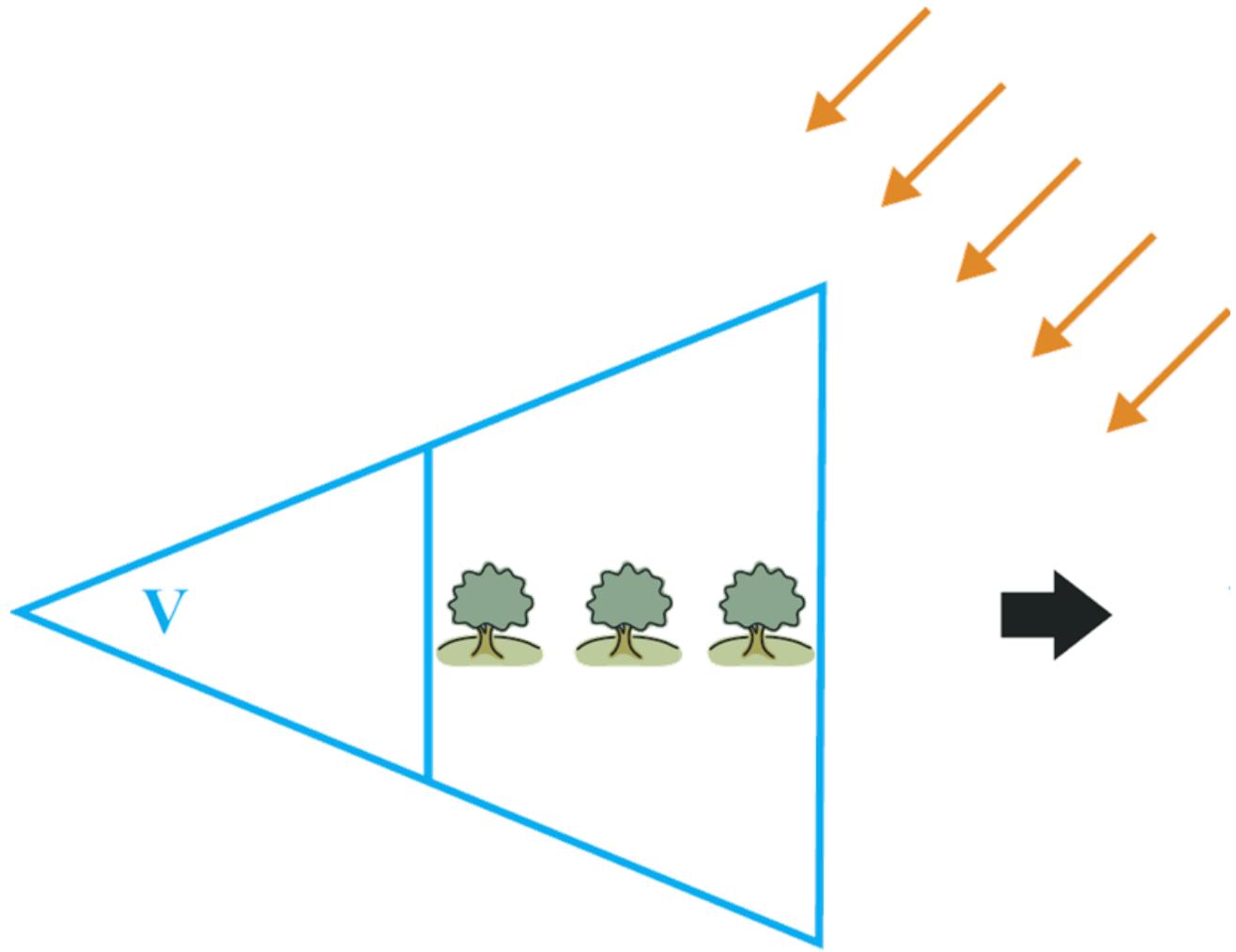


post-perspective space

# PSM: Warping Frustum = View Frustum!



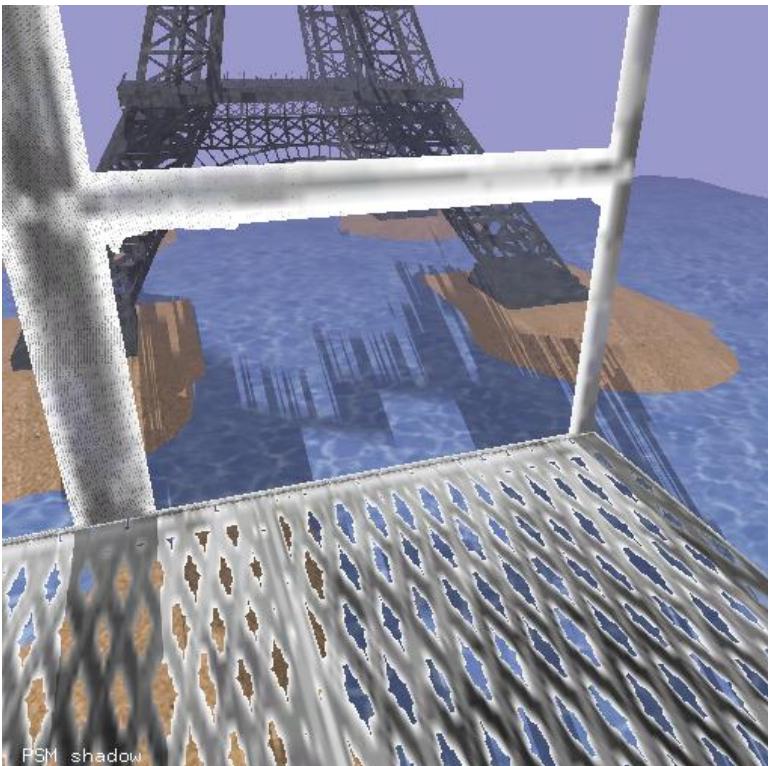
→ Light changes type



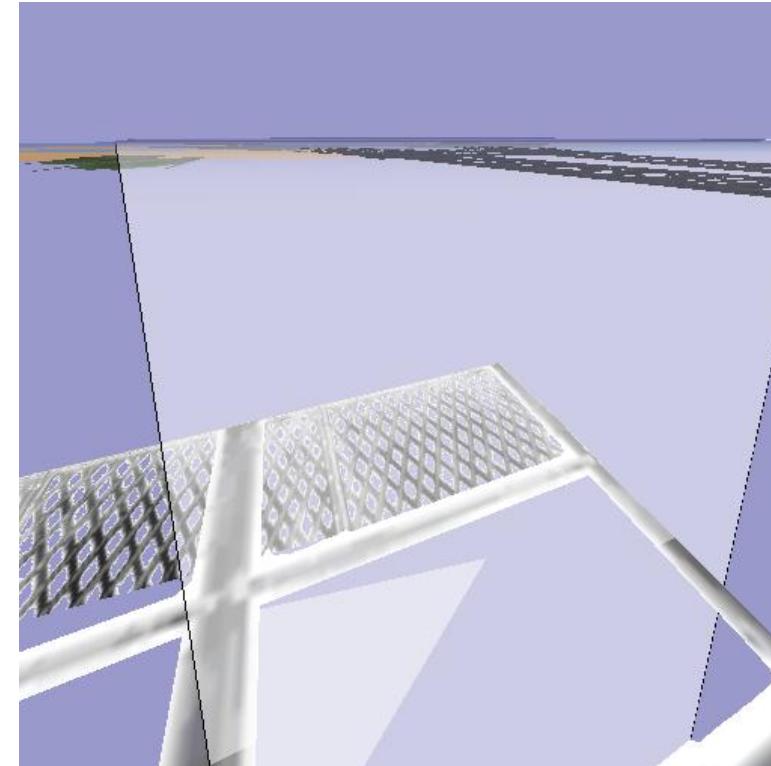
# PSM: Warping Frustum = View Frustum!

→ Most severe: uneven z-distribution

- Good near viewer, very bad far away!
- Can be reduced by pushing near plane away



world space



post-perspective space

## ■ Enter Light-Space Perspective Shadow Maps (LiSPSM)

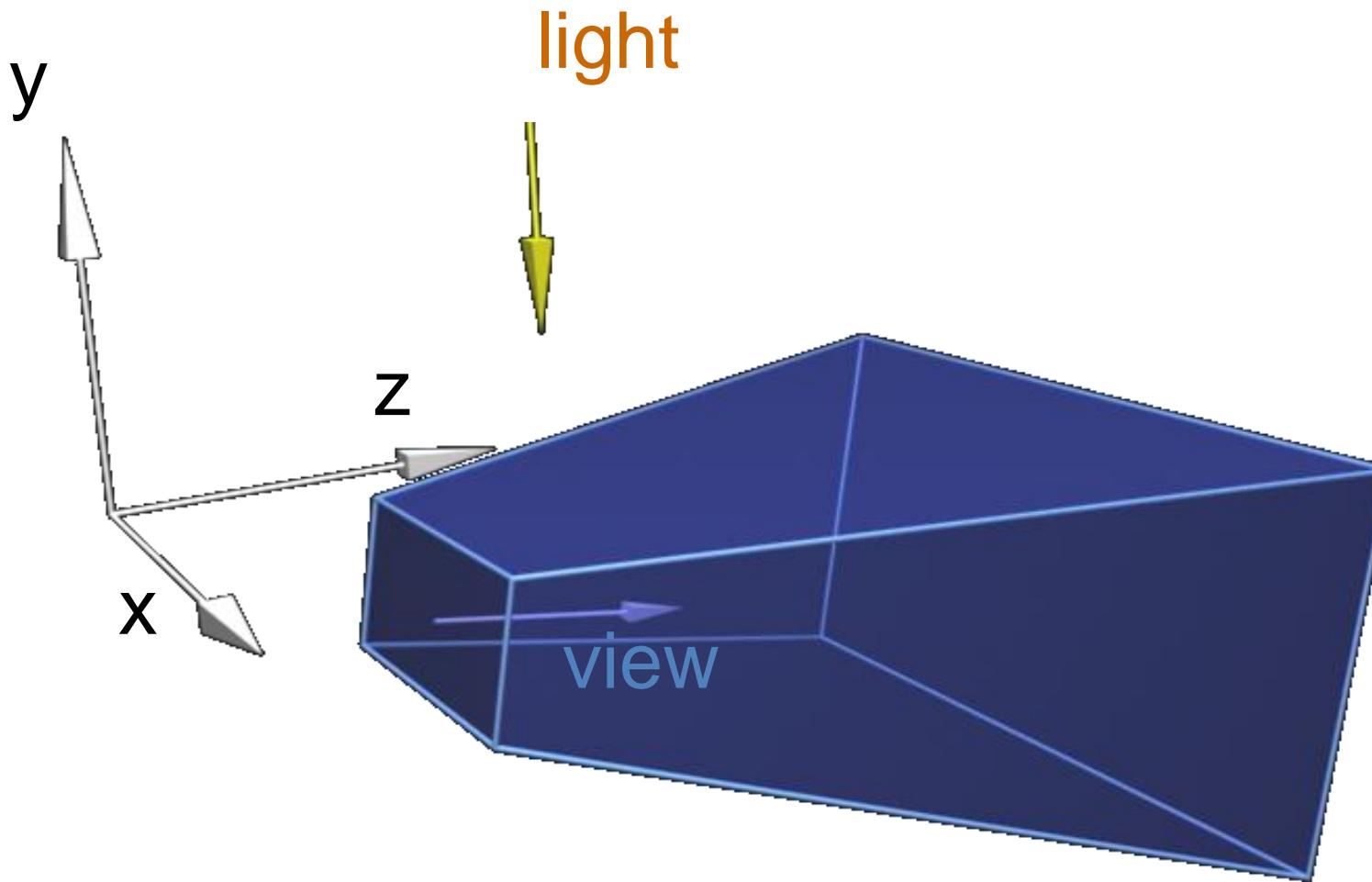
[Wimmer et al. 2004]

- Perspective frustum defined relative to light space
- Optimize warping strength to improve z-distribution
- Similar to Trapezoidal Shadow Maps (TSM)

[Martin & Tan 2004]

# Light-Space Perspective Shadow Mapping (LiSPSM)

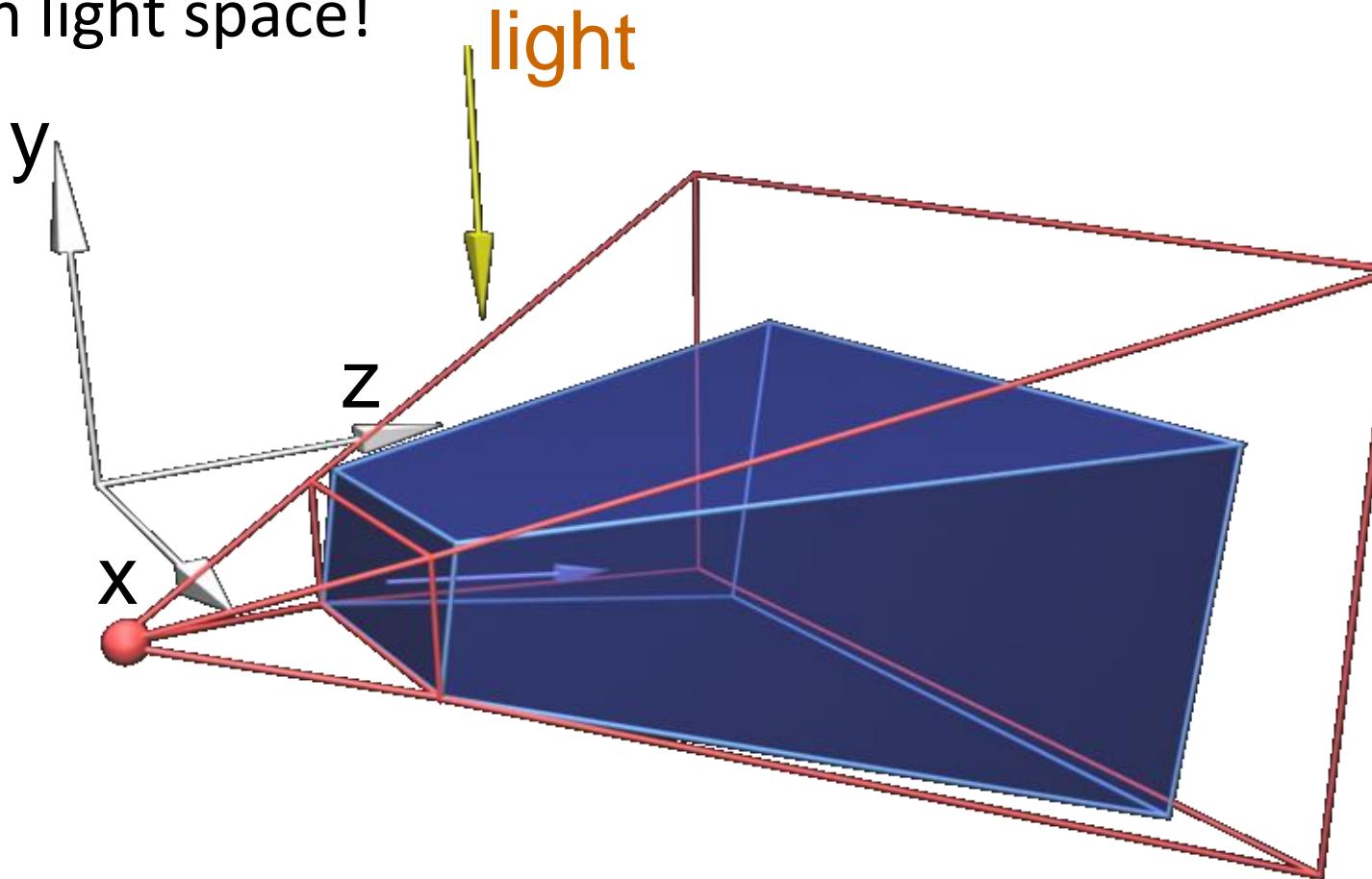
- Light and view vector define yz-plane



# Light-Space Perspective Shadow Mapping (LiSPSM)

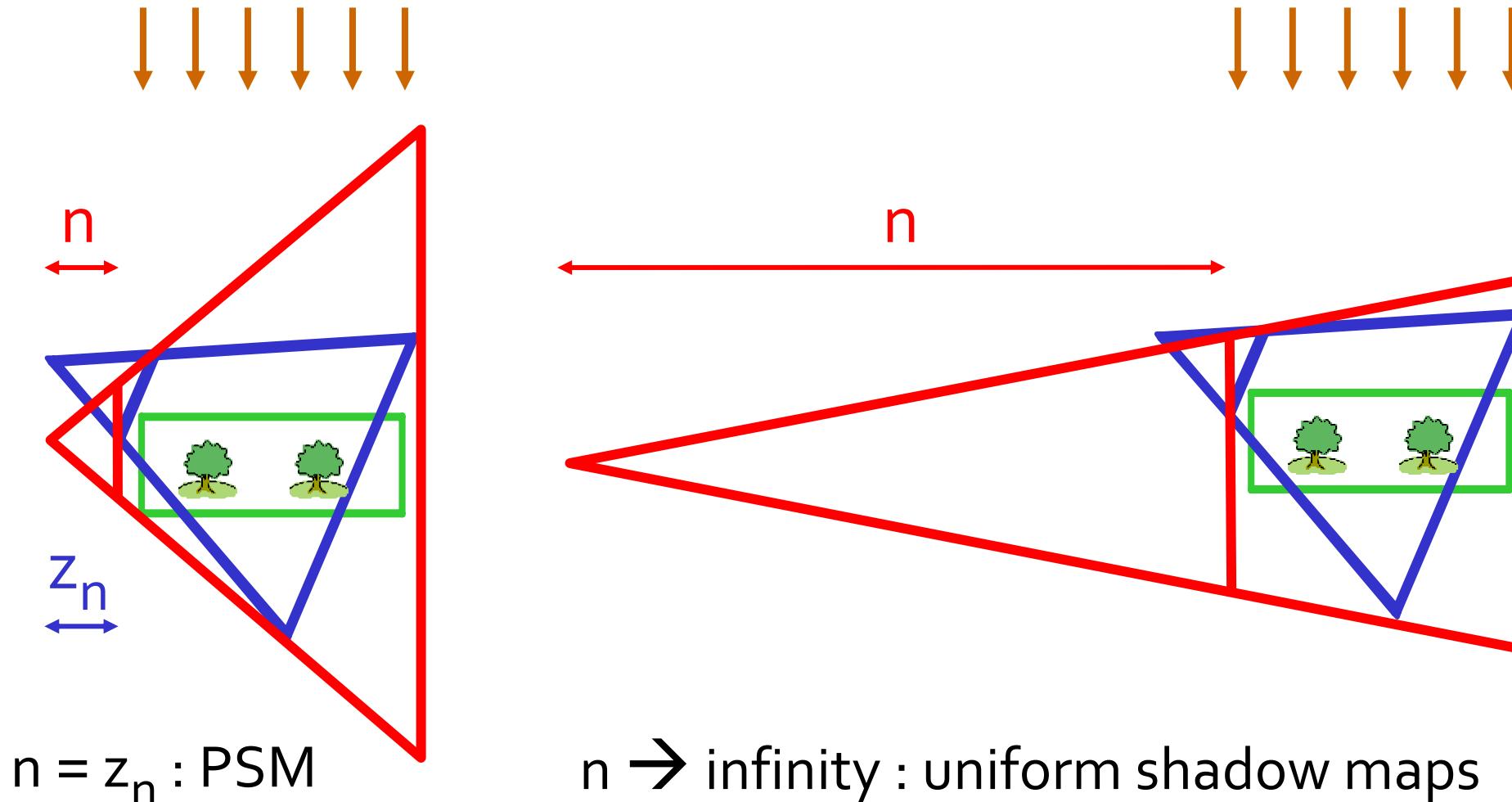
- Find a **tight perspective frustum** on focused region

- In light space!

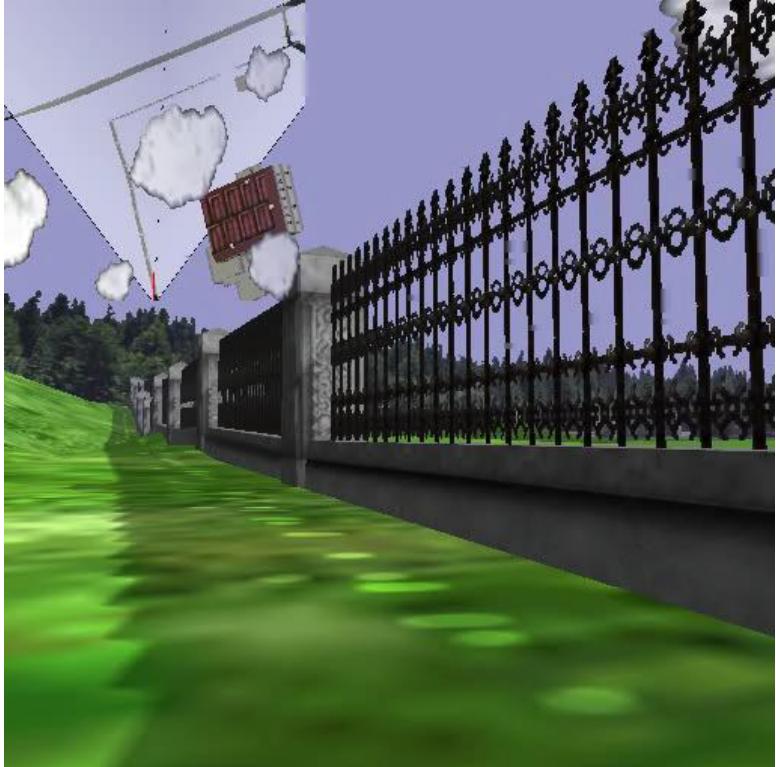


# Free Parameter $n$

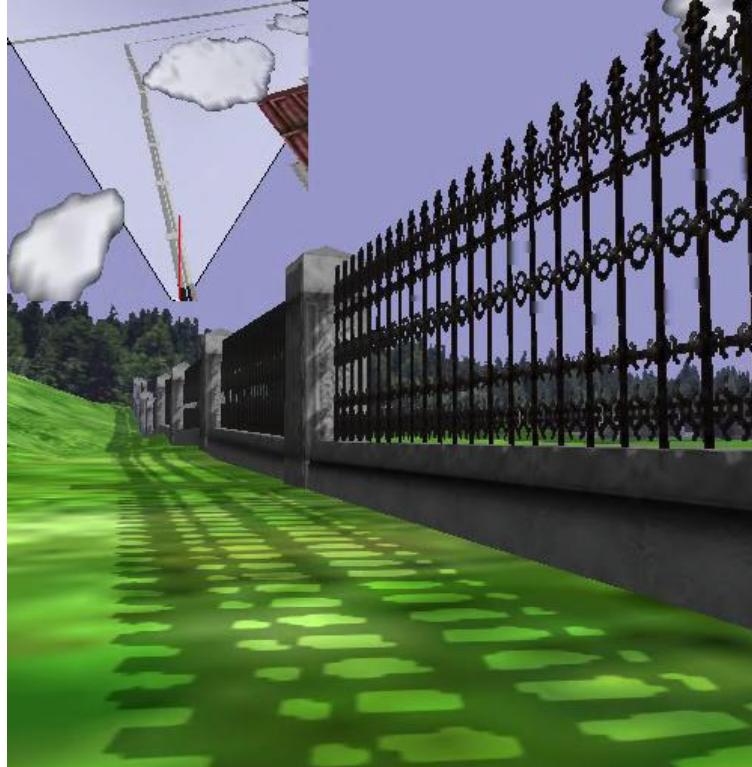
- Controls warping effect



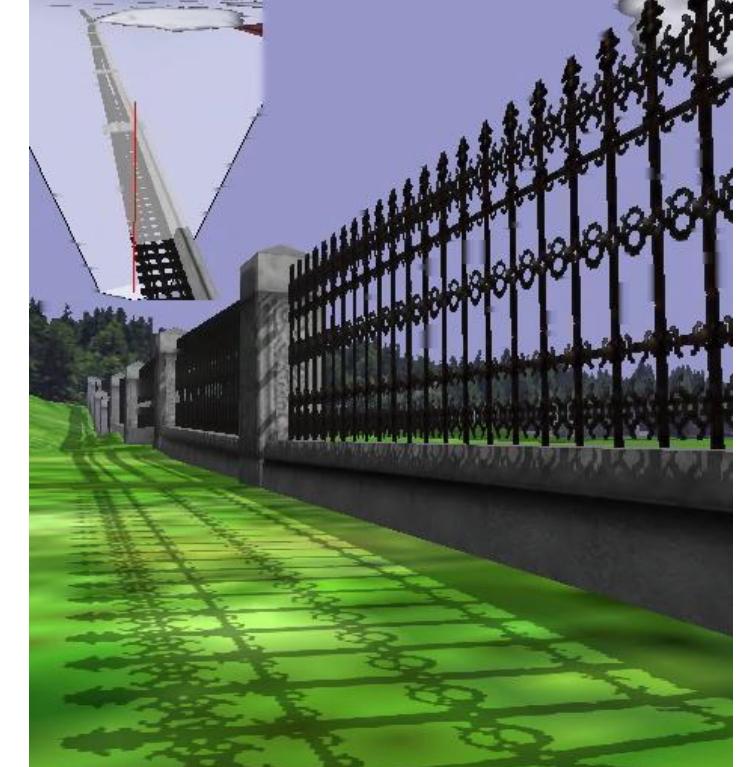
# Free Parameter $n$



very big  $n$



optimal  $n$ ?



very small  $n$

# LiSPSM Matrix

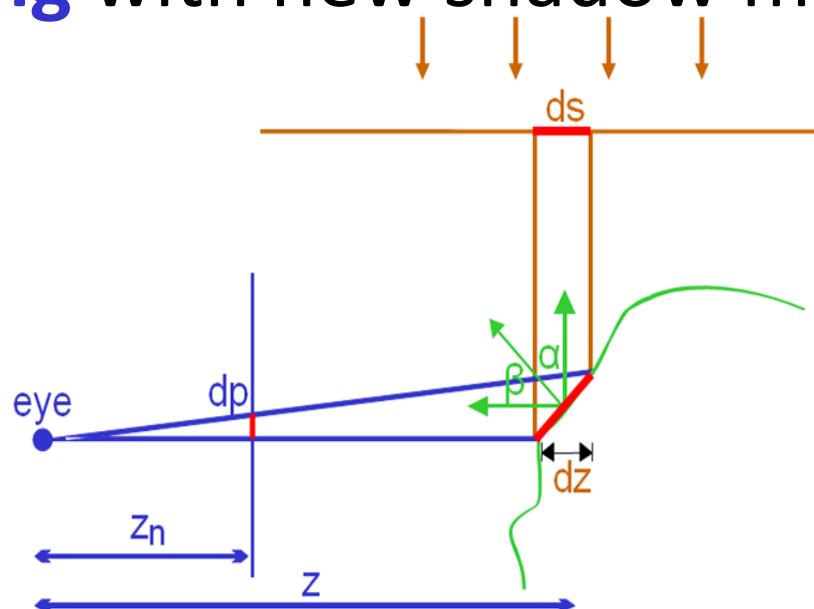


- Standard light matrix:  $S = F L_p L_v M$
- Orient shadow map along view vector:  $R$
- Transform intersection body by  $R L_p L_v$
- Find near/far, and choose warping strength  $n$
- Calculate warping matrix:  $W_p W_v$
- Calculate  $F$  using  $W_p W_v R L_p L_v$
- Warped shadow matrix  $S_w = F W_p W_v R L_p L_v M$
- Use  $S_w$  both for shadow-map generation and rendering

# How to Choose the Free Parameter $n$ ?

- Recall error analysis
- $\frac{dp}{ds} > 1 \rightarrow$  shadow map undersampling
- **Projection aliasing** cannot be changed
- Counter **perspective aliasing** with new shadow map **parameterization**  $s(z, \dots)$
- Goal:  $\frac{dp}{ds} \sim 1$

$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos \alpha}{\cos \beta}$$



# Error Analysis

$$\frac{dp}{ds} = \frac{z_n}{z} \frac{dz}{ds} \frac{\cos\alpha}{\cos\beta}$$



- Perfect: logarithmic re-parameterization

$$s \sim \log z \quad \Rightarrow \quad \frac{ds}{dz} \sim \frac{1}{z} \quad \Rightarrow \quad \frac{dp}{ds} \sim 1$$

- Hardware support? [Lloyd 2007, 2008]

- Uniform shadow maps

$$s \sim z \quad \Rightarrow \quad \frac{ds}{dz} \sim 1 \quad \Rightarrow \quad \frac{dp}{ds} \sim \frac{1}{z}$$

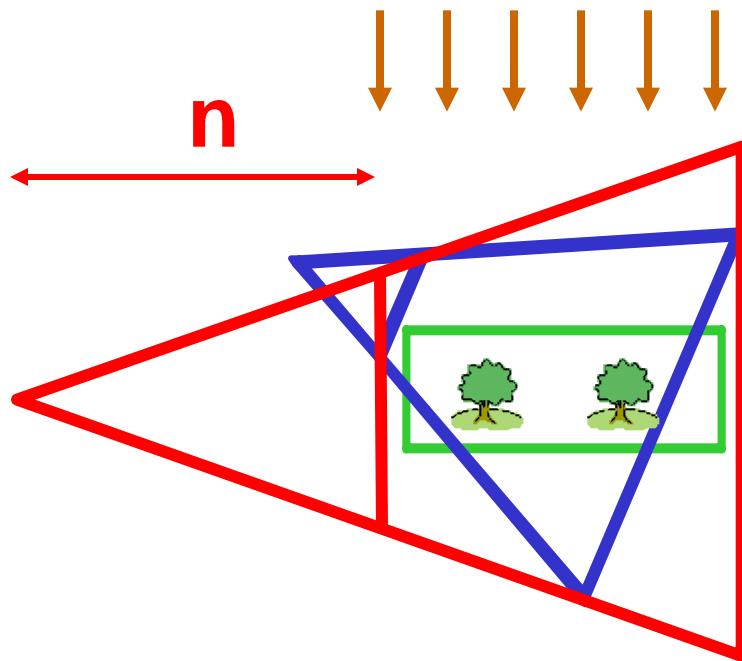
- Perspective shadow maps

$$s \sim \frac{1}{z} \quad \Rightarrow \quad \frac{ds}{dz} \sim \frac{1}{z^2} \quad \Rightarrow \quad \frac{dp}{ds} \sim z$$

- Linear increase in error!

# Error Analysis: LiSPSM Optimal Choice

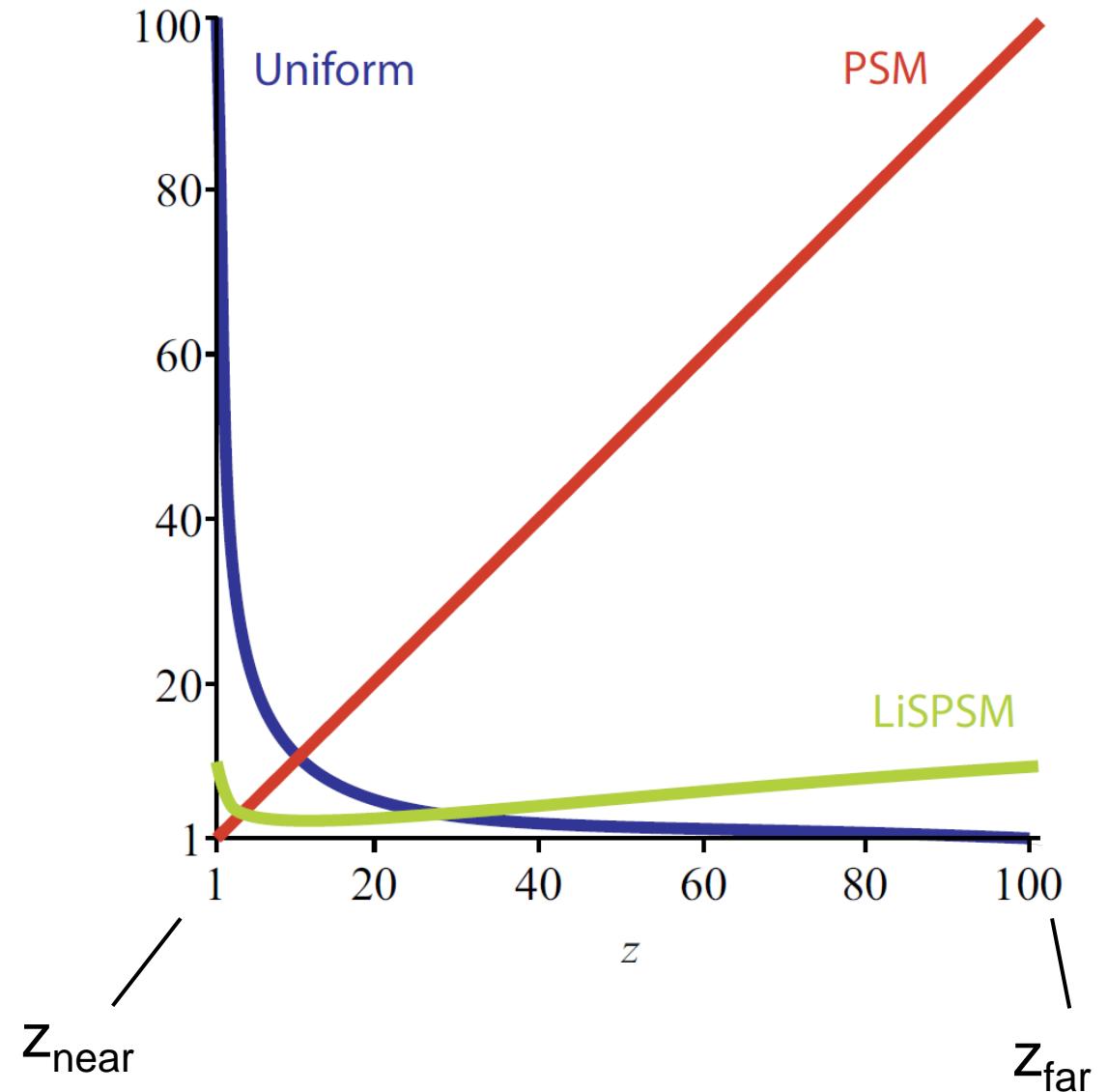
- For LiSPSM,  $\frac{dp}{ds}$  depends on n
  - Gives  $\frac{dp}{ds}$  between uniform and perspective
- Optimal choice:



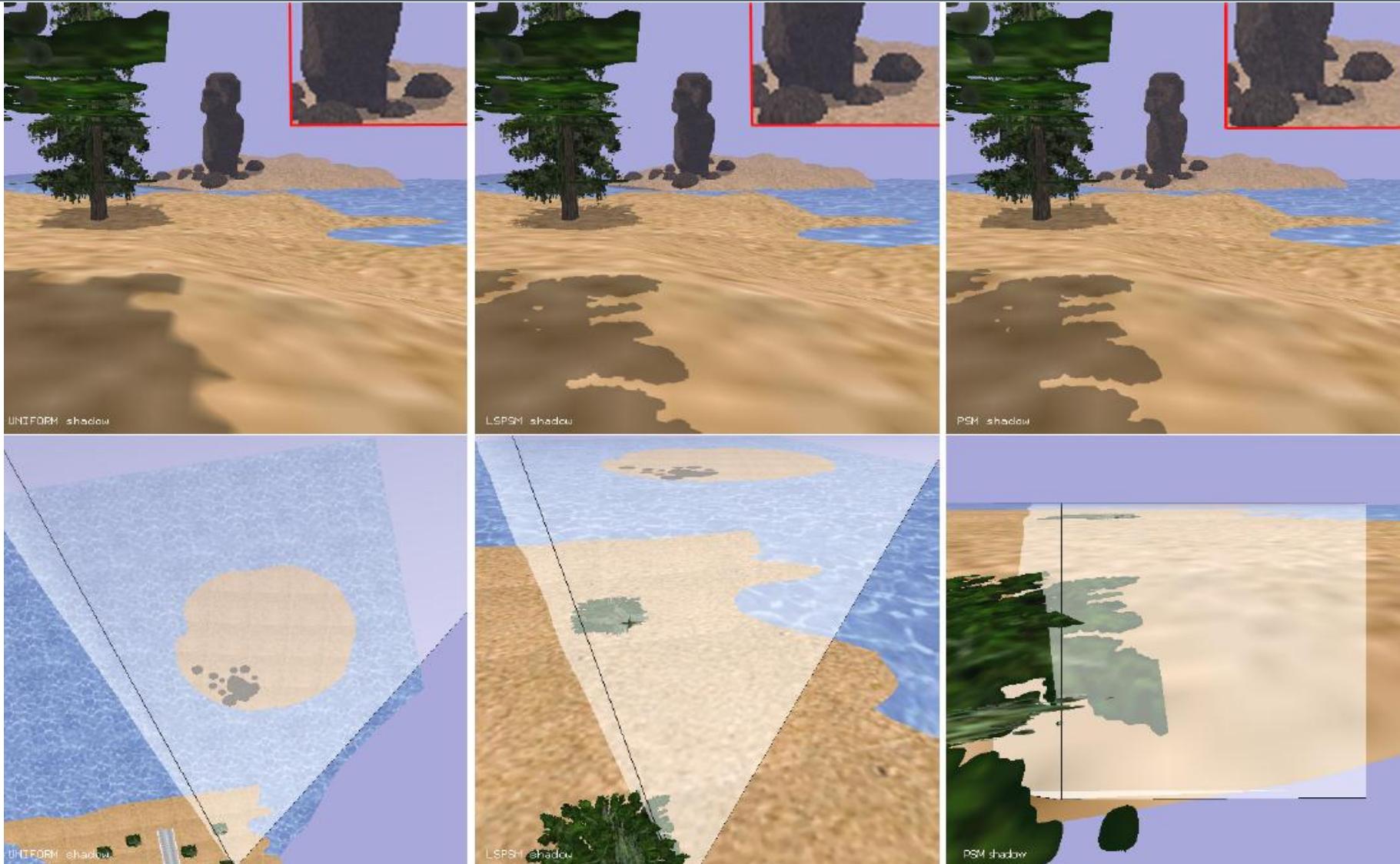
$$n_{opt} = z_n + \sqrt{z_f z_n}$$

# Error Comparison

- LiSPSM optimal choice
- Measured along view dir
- LiSPSM vs PSM
  - for same depth range, LiSPSM error much lower



# Comparison



Uniform

LiSPSM

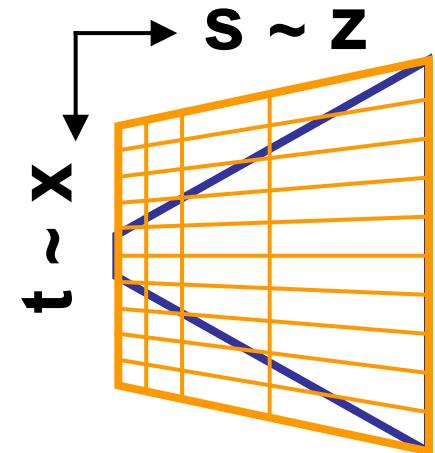
PSM

# Error Comparison

- Caveat: only measured along view direction
- What about  $\frac{dp}{dt}$ ?

- $\frac{dp}{dt} \sim 1$  for PSM, slightly worse for LiSPSM

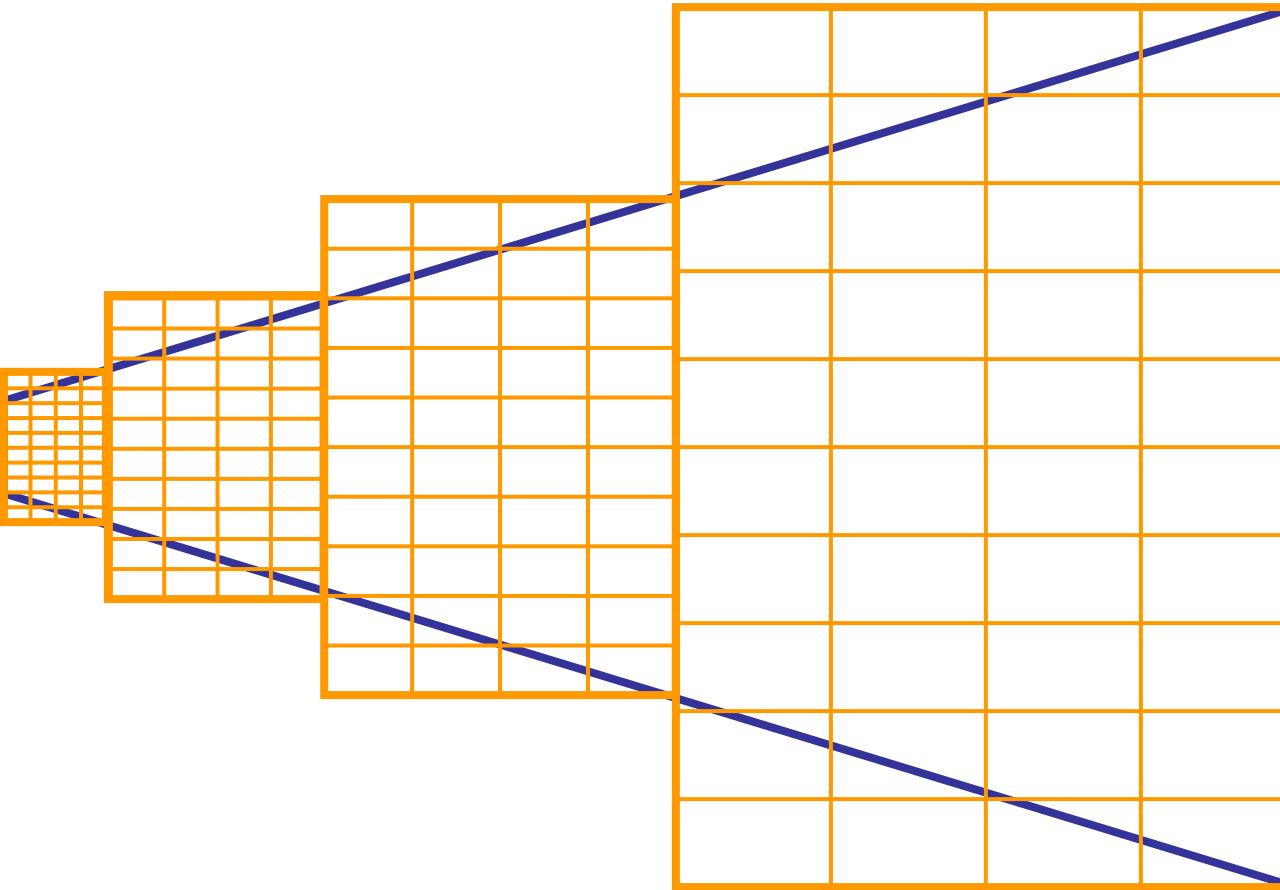
- More advanced analysis was done in [Lloyd 2006]
  - Result: “storage factor” constant for  $n$  in  $[z_n, n_{opt}]$
  - But: best error distribution for  $n_{opt}$



# Warping: Problems

- Only works if large z-range visible from light
- Dueling frusta case



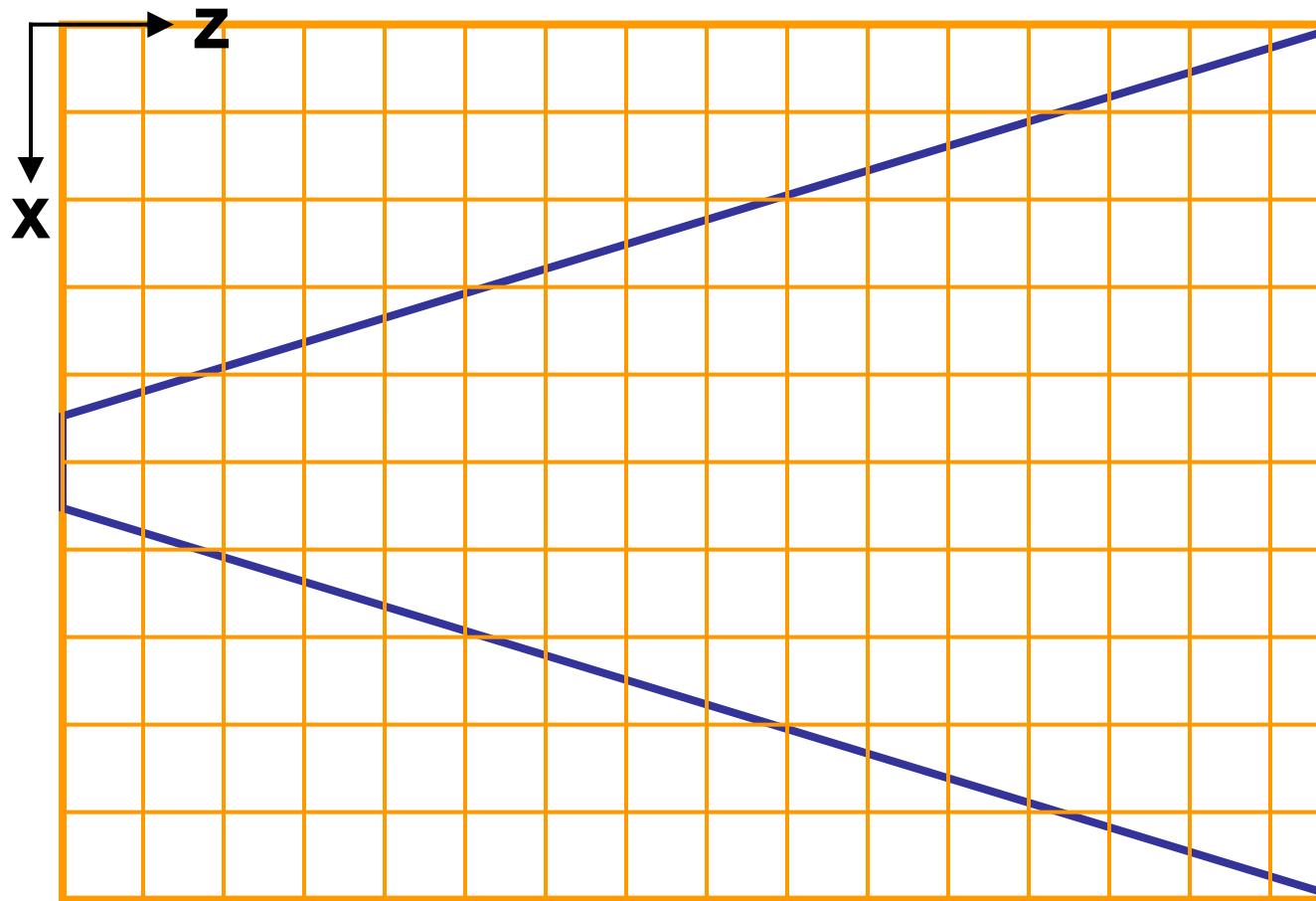


# Hard Shadows

Fighting Undersampling – Partitioning

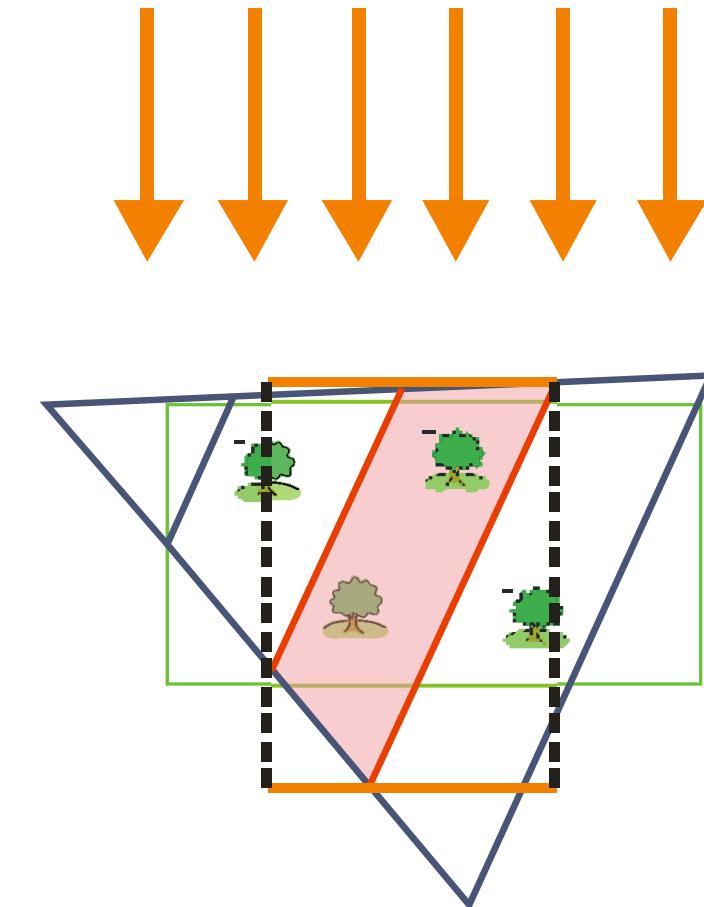
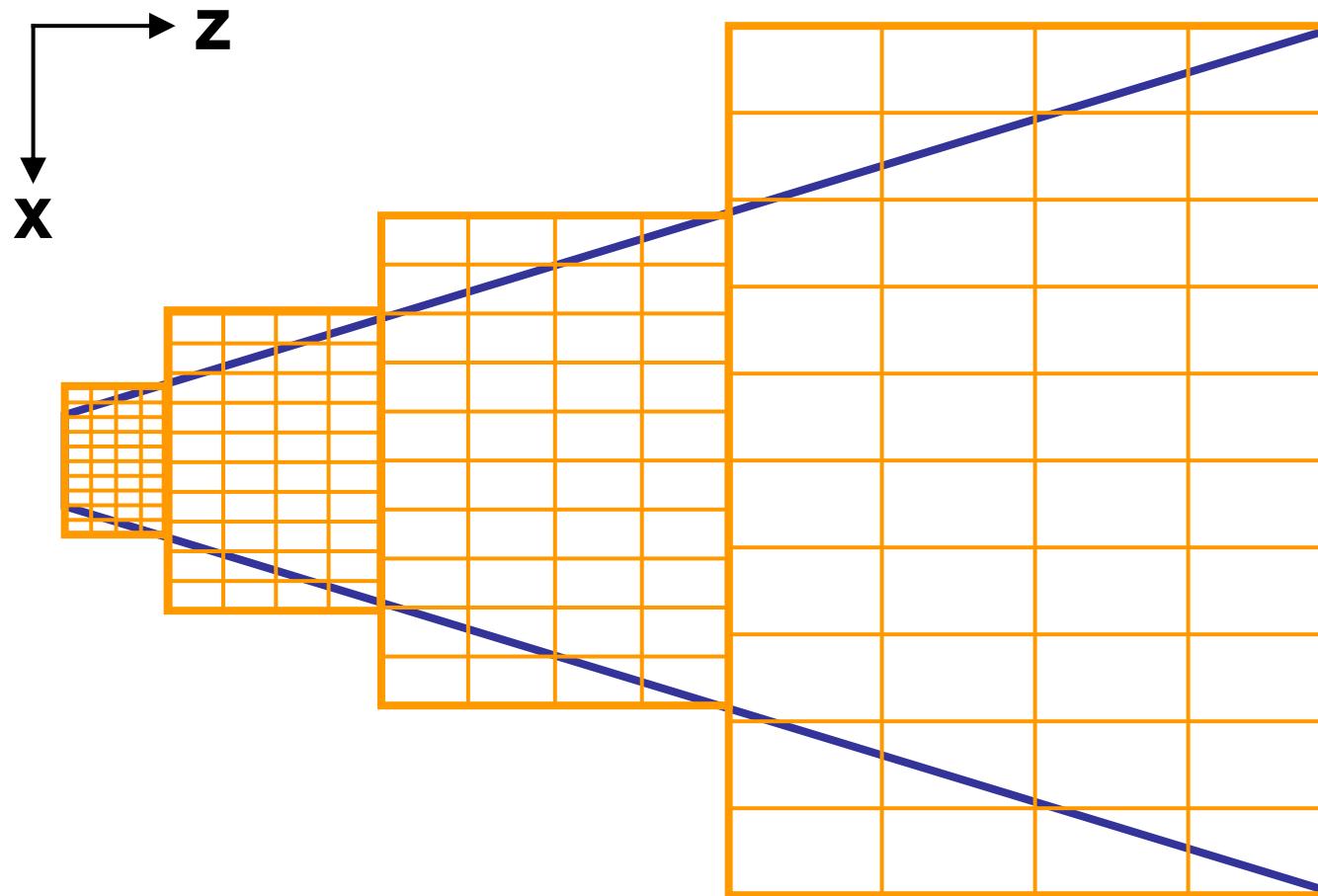
# Z-Partitioning: Idea

- Parallel Split Shadow Maps [Zhang 2007]
- Cascaded Shadow Maps [Engel 2007][Zhang 2009]



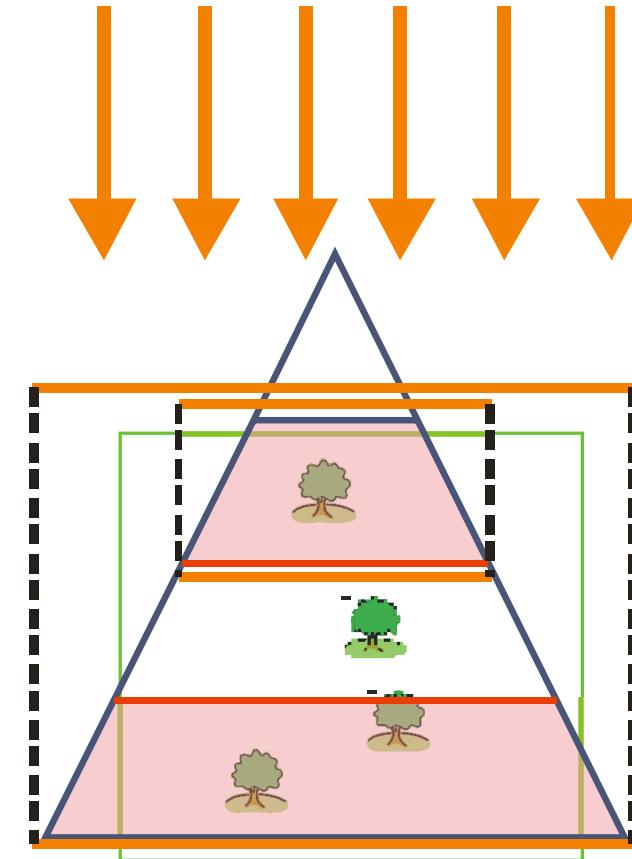
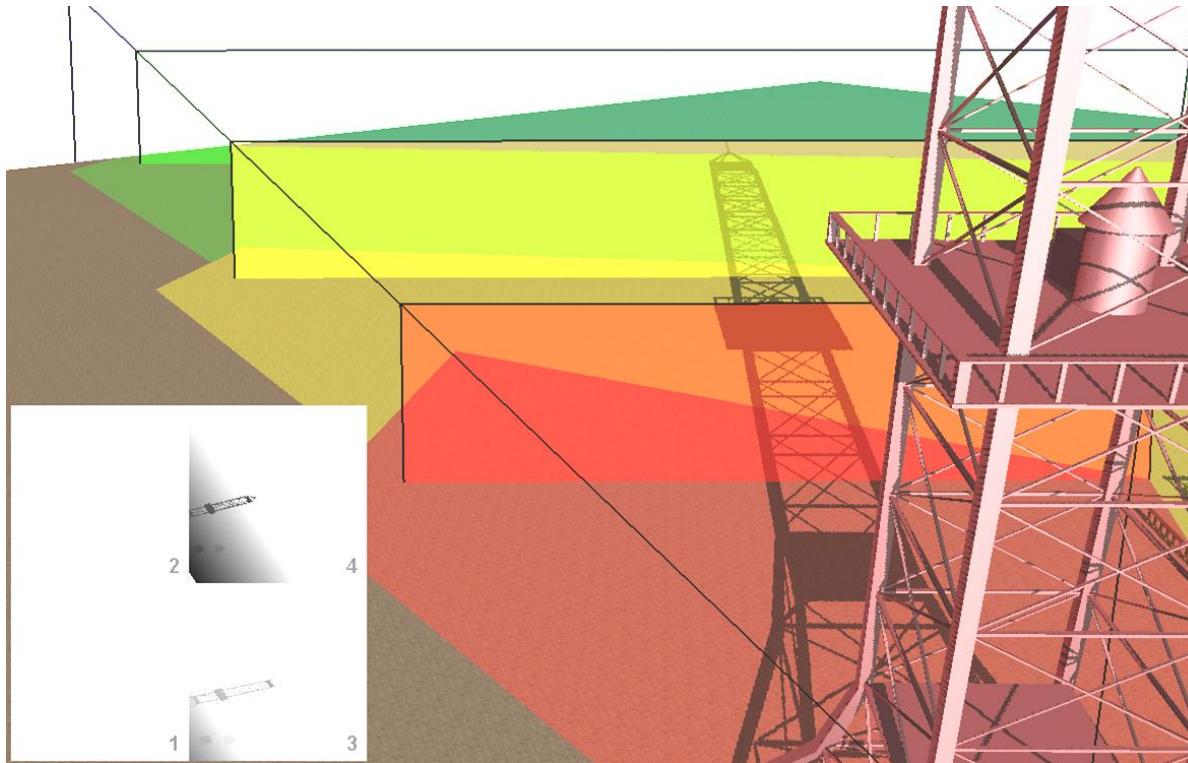
# Z-Partitioning

- Partition view frustum into n sub-frusta
- Calculate separate shadow map for each



# Z-Partitioning

- Works even in cases where warping fails
  - Light from behind, dueling frusta



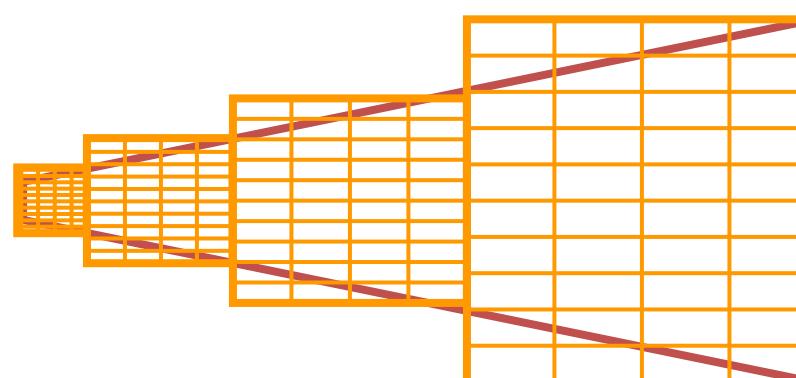
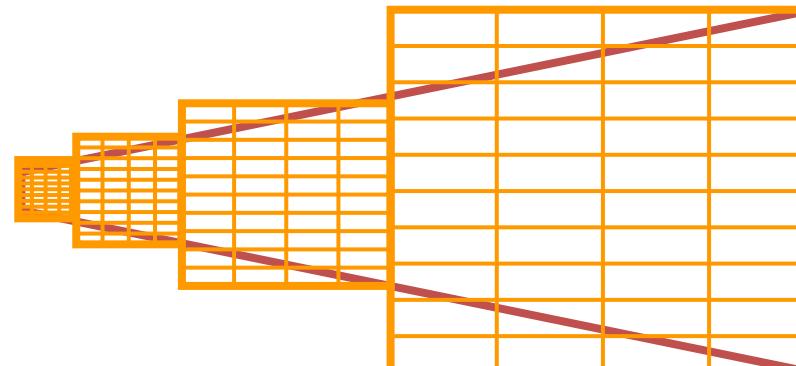
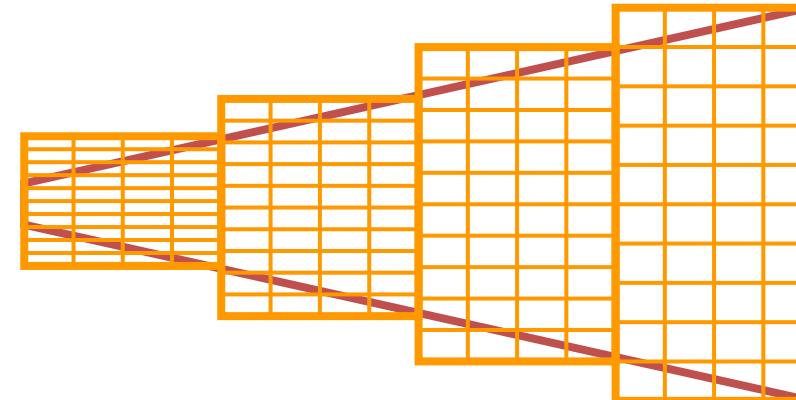
# Z-Partitioning

- How to choose partition sizes?

- Uniform
- Logarithmic/self-similar

$$C_i = z_n \left( \frac{z_f}{z_n} \right)^{\frac{i}{m}}$$

- Linear blend between the two

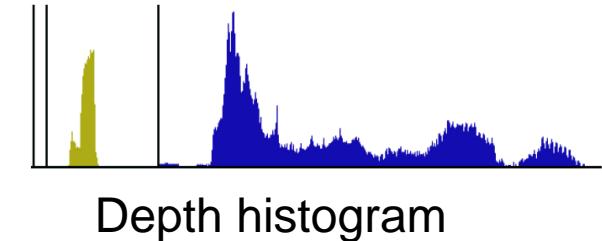
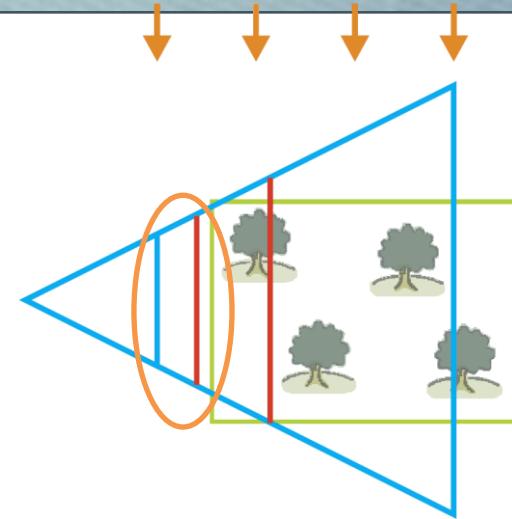


# Z-Partitioning – Sample Distribution Maps

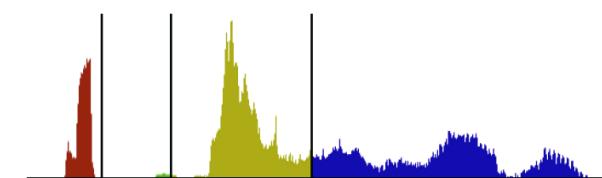
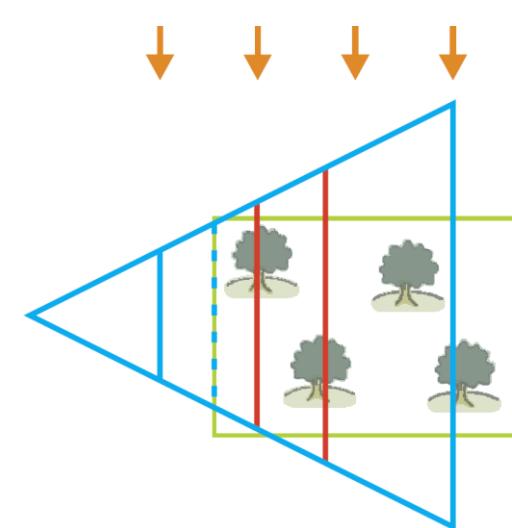
[Lauritzen et al. 2011]



- Optimal partition:  
logarithmic
  - Problem: empty areas



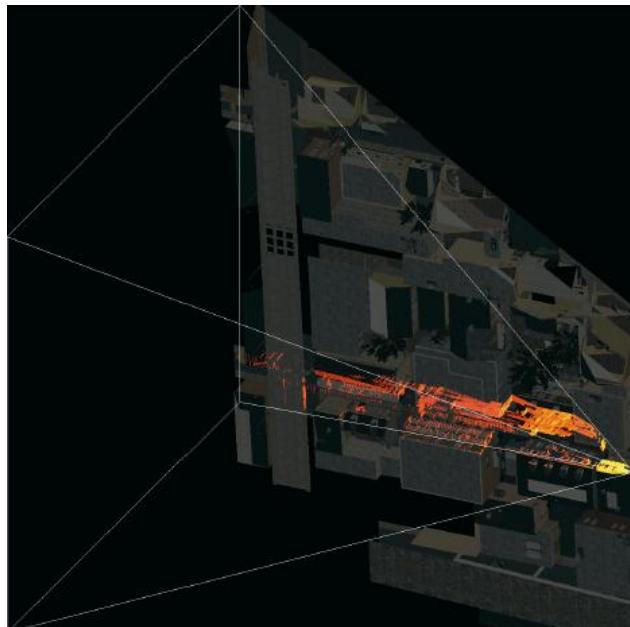
- Solution: scene analysis
  - Get min/max-z
  - Adjust depth bounds



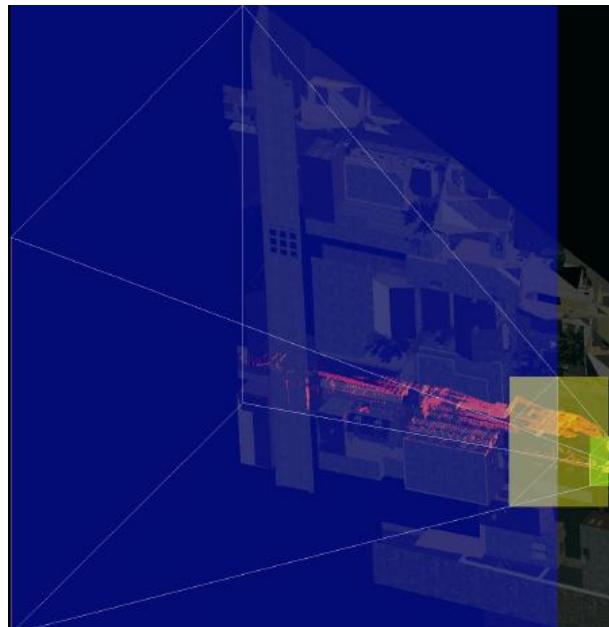
# Z-Partitioning – Sample Distribution Maps

[Lauritzen et al. 2011]

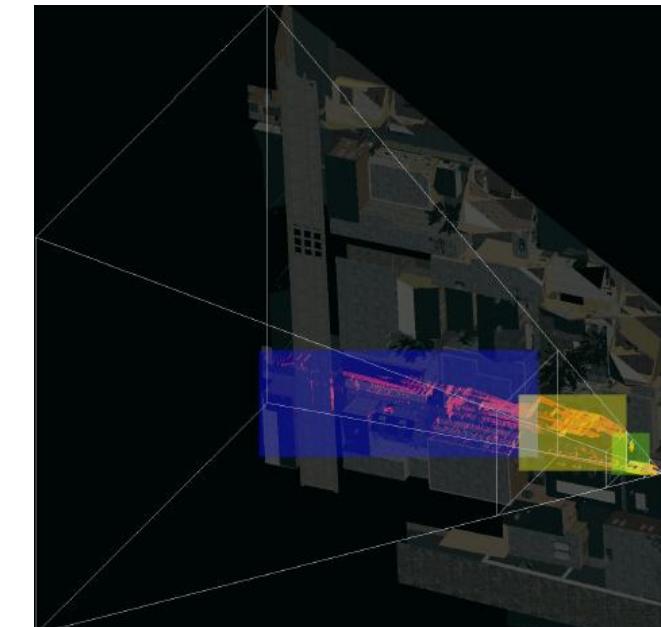
- Extension: analyze fragment distribution in light space
  - Tightly fit partitions in s and t



Shadow map



Standard Z-partitions

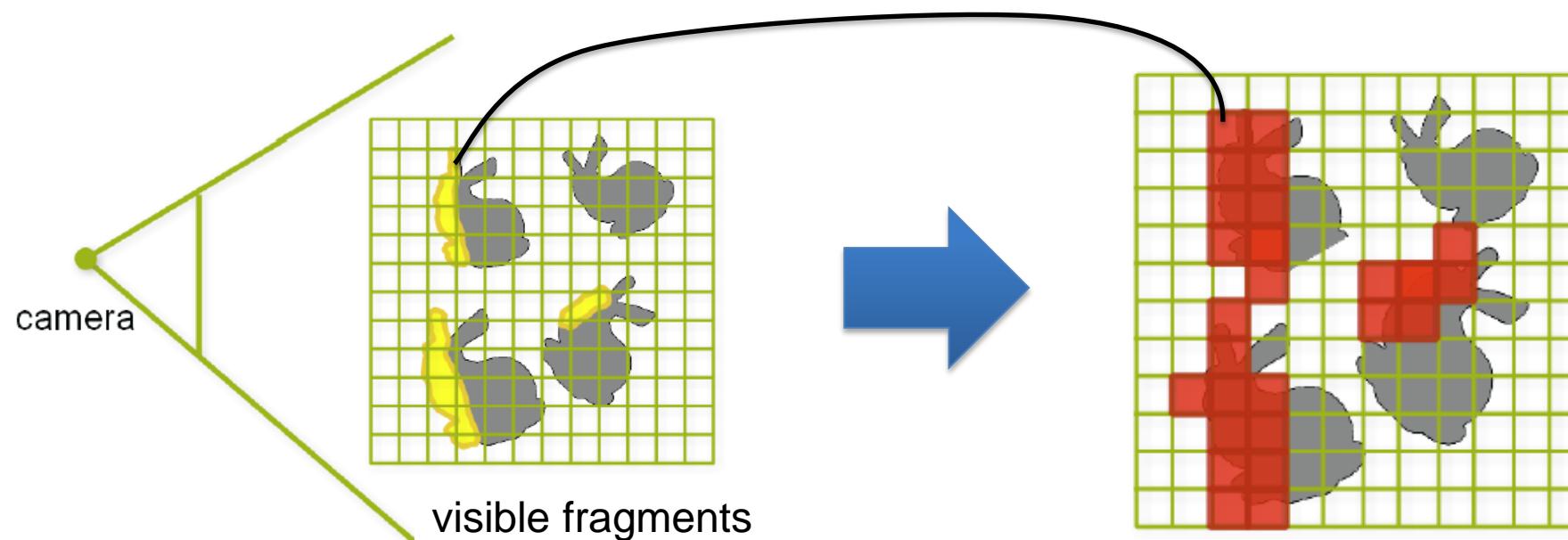


Bounded partitions

# Shadow Caster Culling

## [Mattausch et al. 2011]

- How to determine relevant fragments in light space?
  - Determine shadow receivers (camera pass)
  - Render shadow receivers into light-space mask (light pass)
    - Fragment-level check using reverse shadow lookup

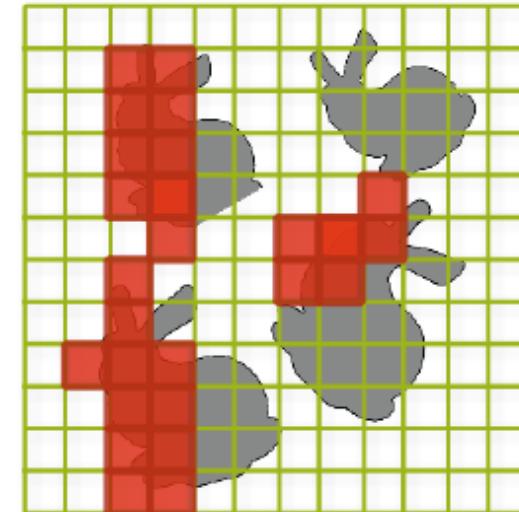


# Shadow Caster Culling

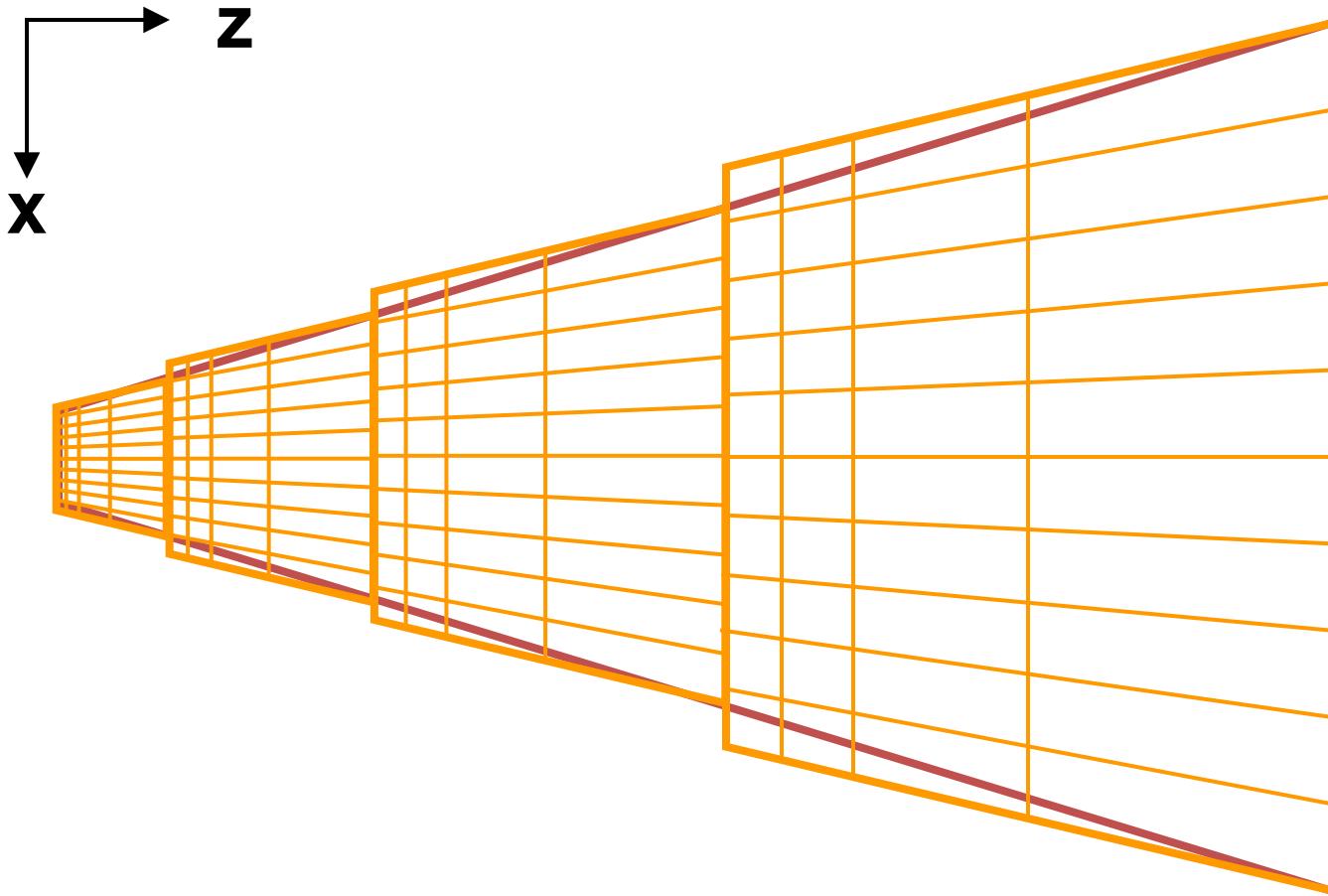
## [Mattausch et al. 2011]

### ■ Culling:

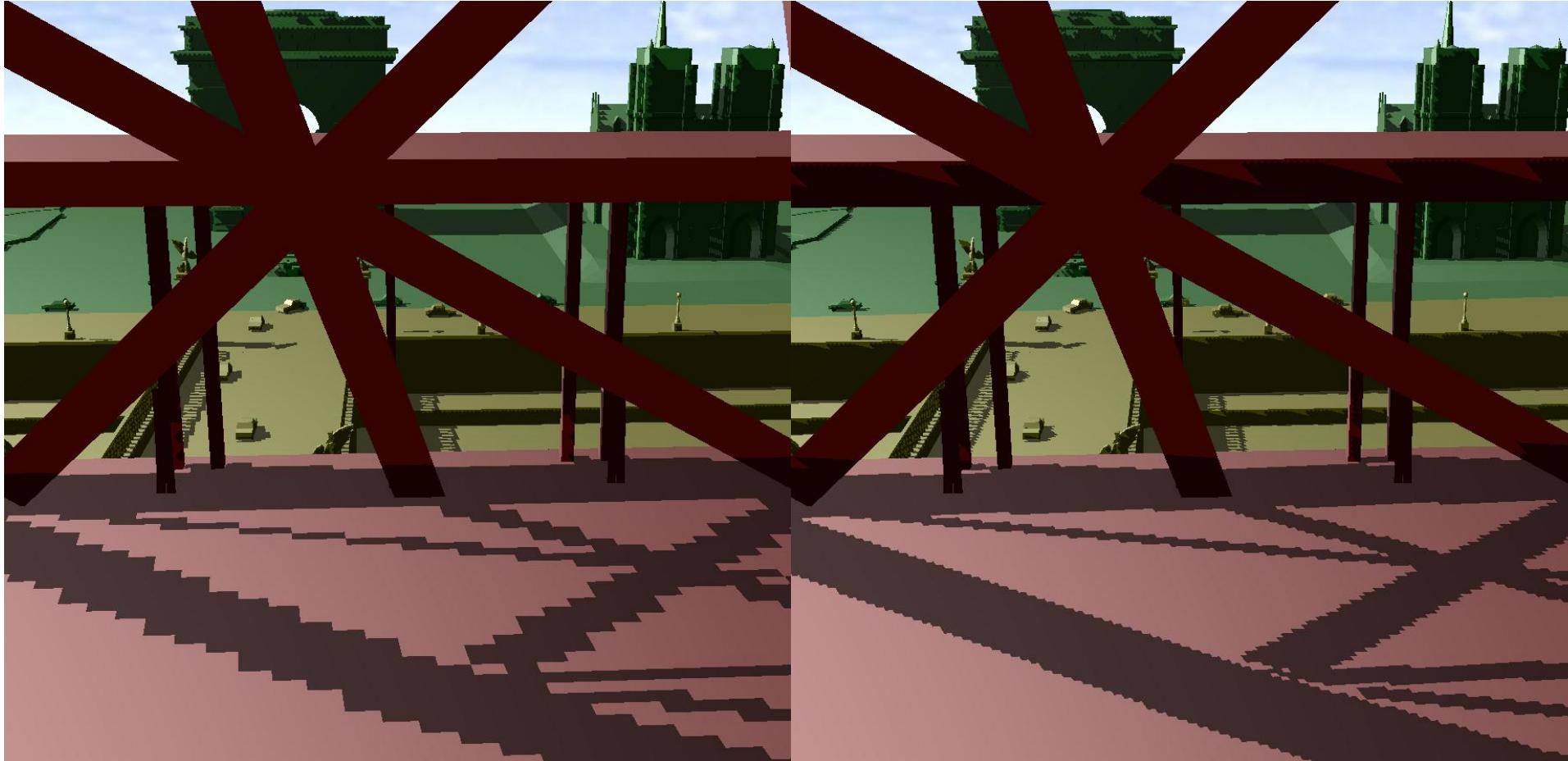
- Use mask for occlusion culling for depth map
- Large performance gain for outdoor scenes!



# Z-Partitioning and Warping



# Z-Partitioning and Warping

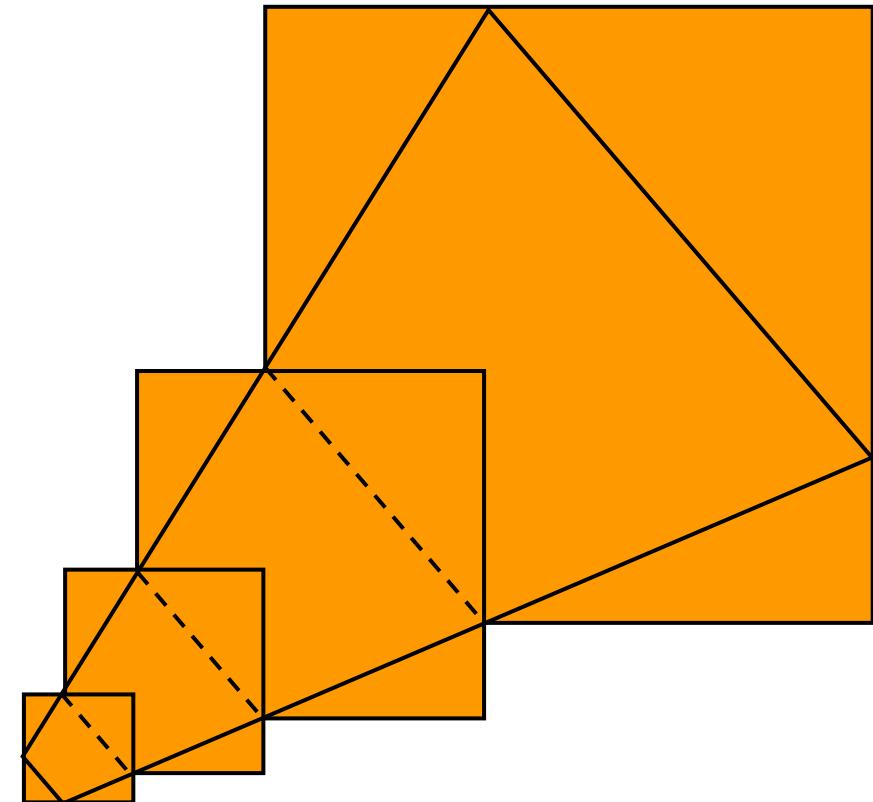


Partitioning

Partitioning + warping

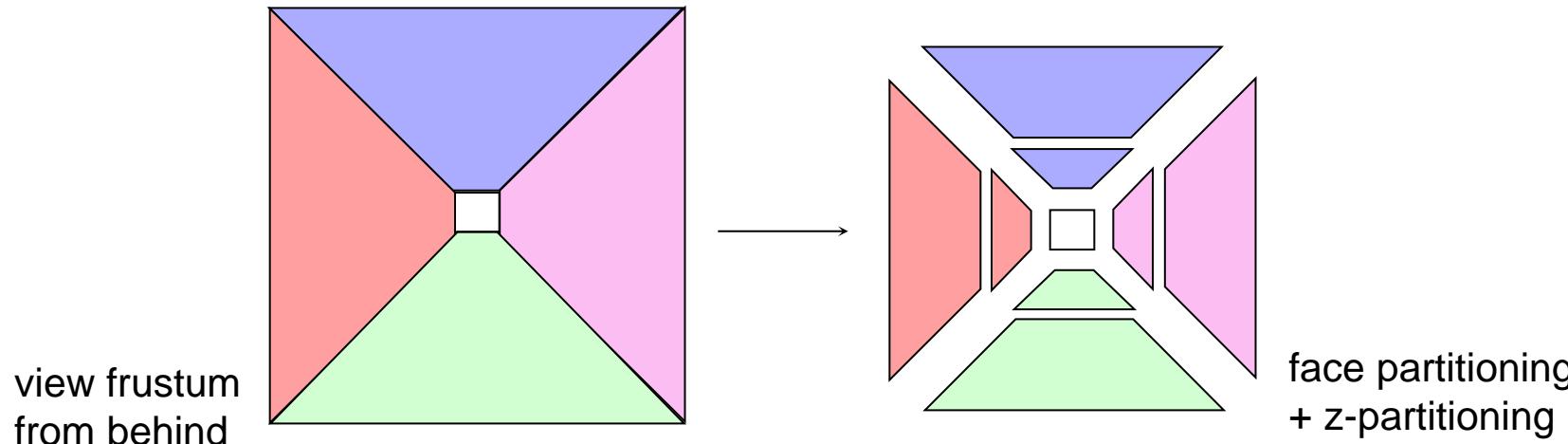
# Temporal coherence

- Fix coordinate system orientation in world space
- Shadow maps move at integral multiples of a texel width (see fitting)
- Gives up warping

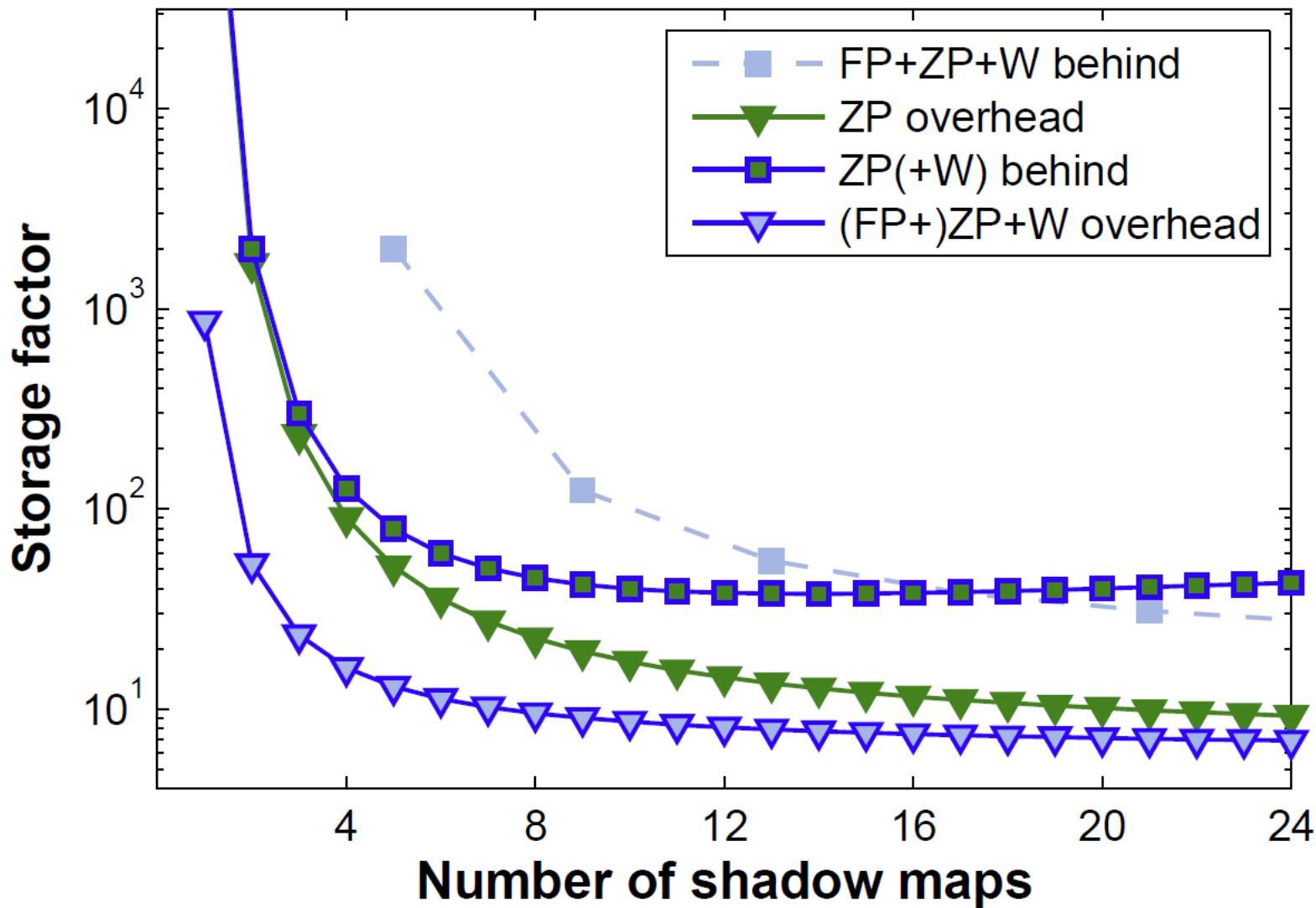


# Face Partitioning [Lloyd et al. 2006]

- Alternative to z-partitioning
  - Partition frustum according to faces
- Can be combined with z-partitioning (see later)
  - Reduces redundancy
  - Also works without warping (“oblique projection”)

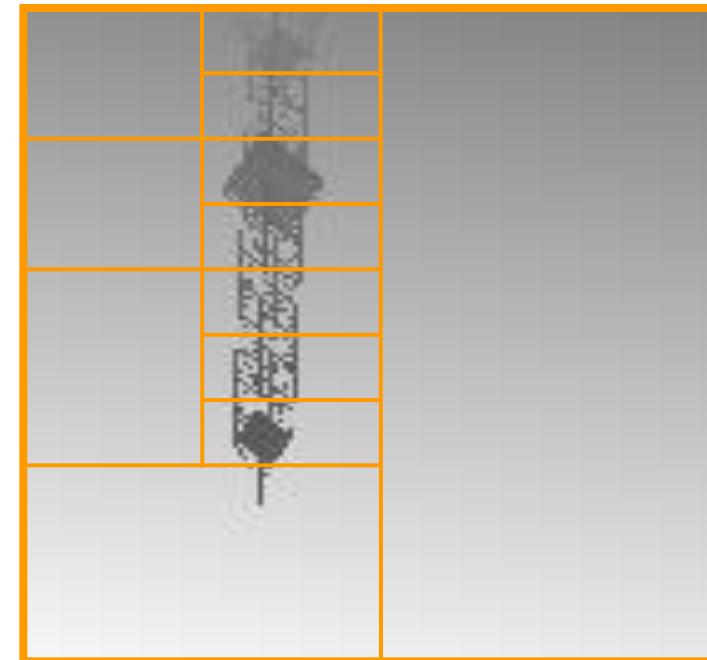


# Full Error Analysis [Lloyd et al. 2006]



# Adaptive Partitioning

- Warping and z-partitioning are **global** resampling schemes
  - Deal with perspective aliasing
  - Projection aliasing needs local scene adaptive resampling!
- Adaptive partitioning adaptively splits shadow map
  - Usually quad-tree subdivision
  - Algorithms mainly differ in termination criteria

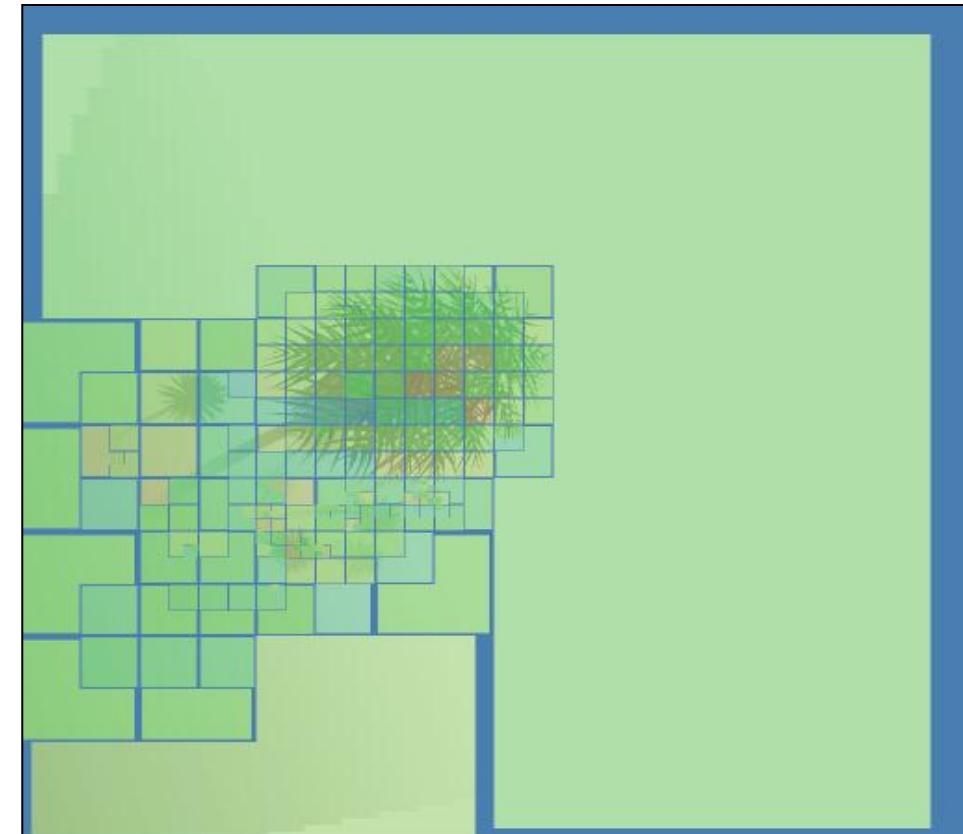
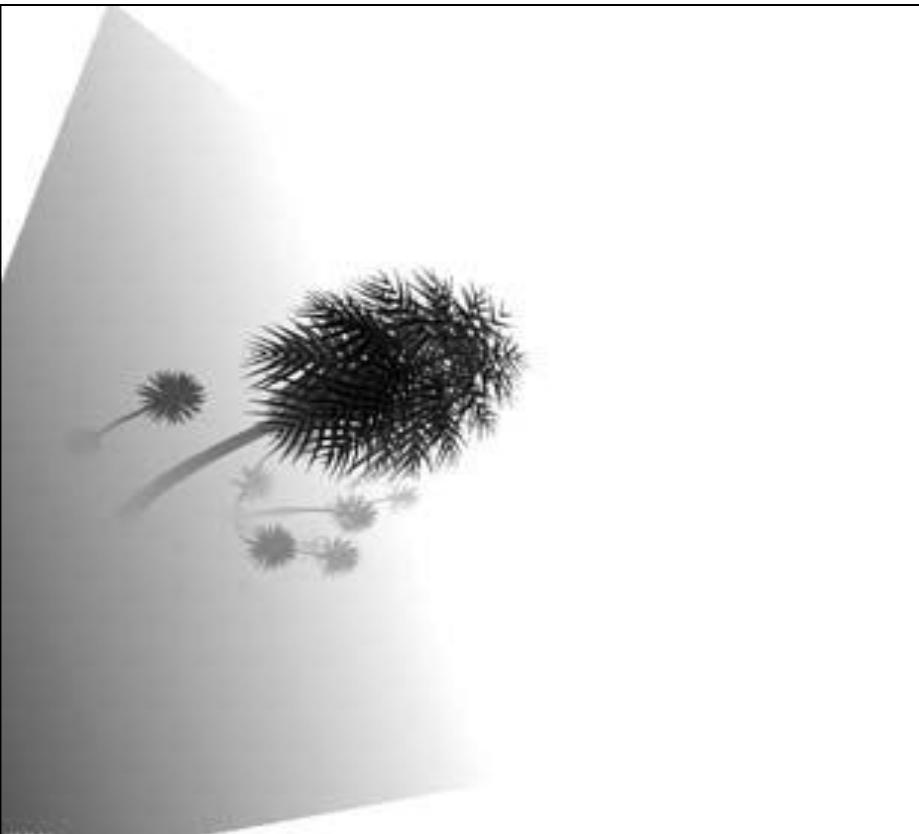


# Adaptive Shadow Maps

[Fernando et al. 2001; Lefohn et al. 2006]



- High resolution only needed at edges
- Search for edge (slow)
  - If edge → split



# Fitted Virtual Shadow Maps/Resolution Matched SM

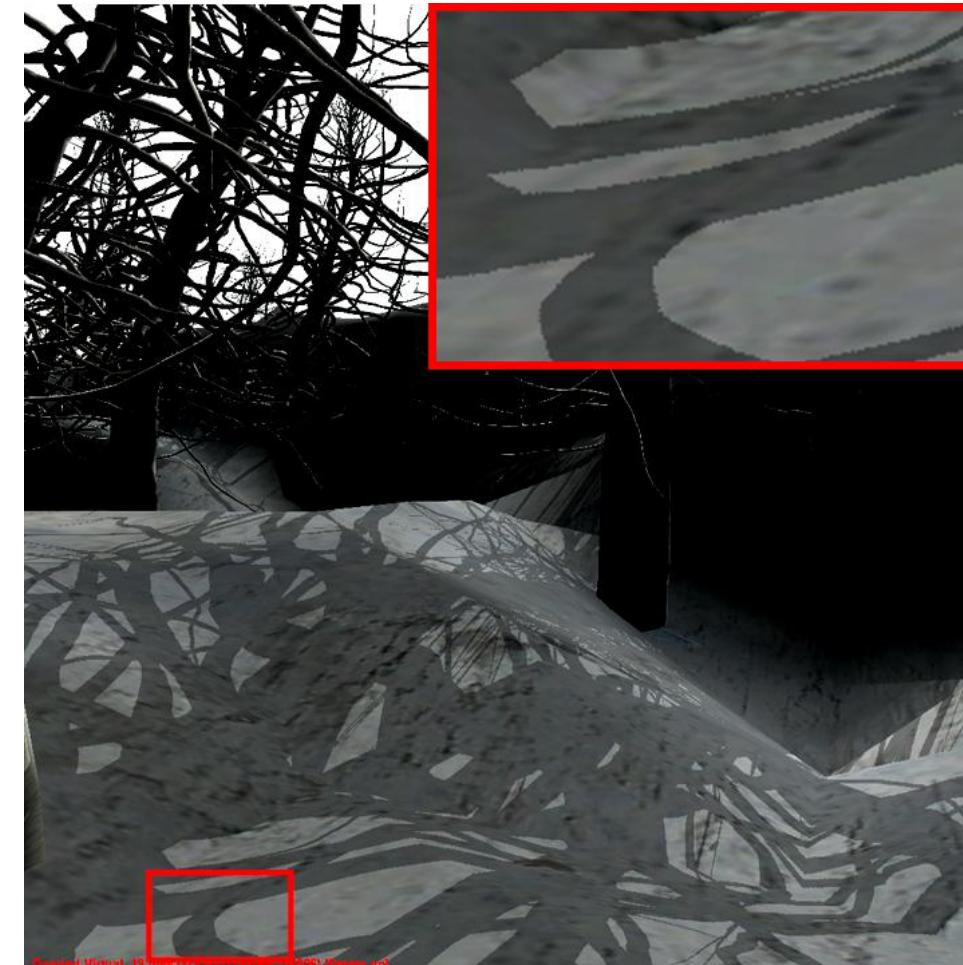
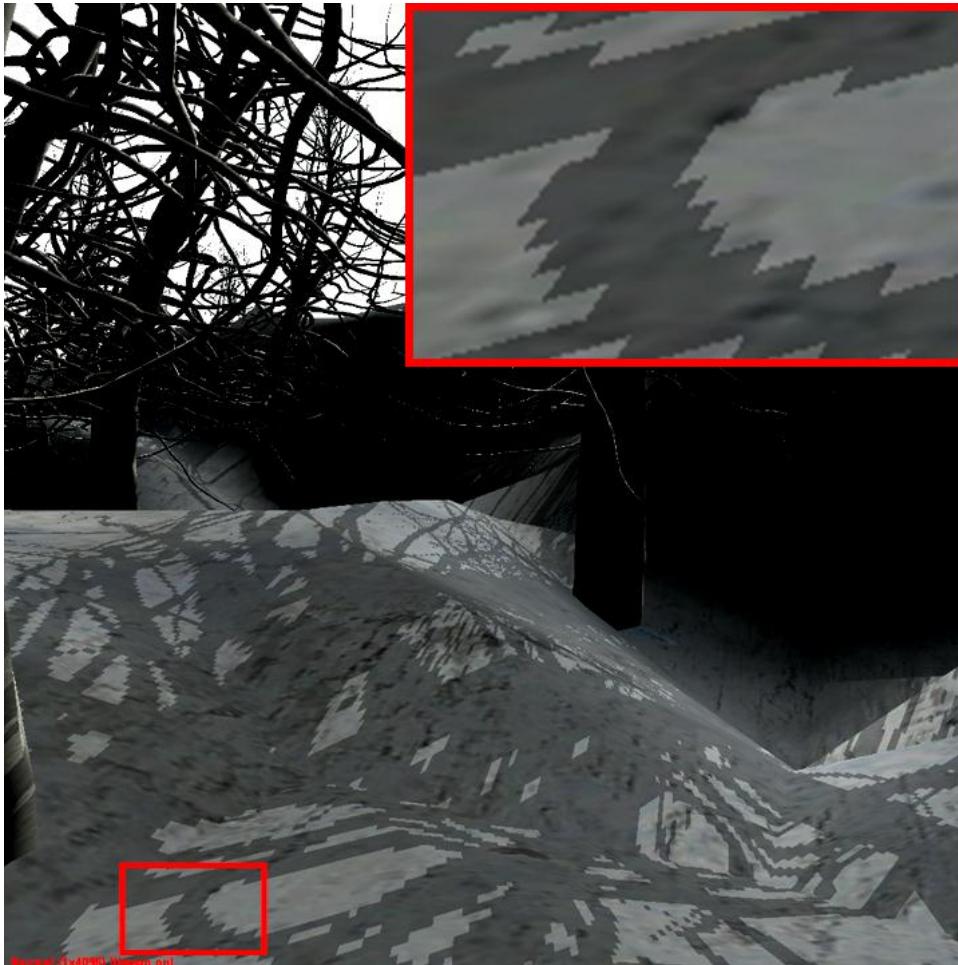
[Gieg'l & Wimmer 2007], [Lefohn et al. 2007]



- Do not calculate all quad-tree levels, but determine finest levels necessary
  - Camera prepass
  - Analyzed on CPU (FVSM) or GPU (RMSM)

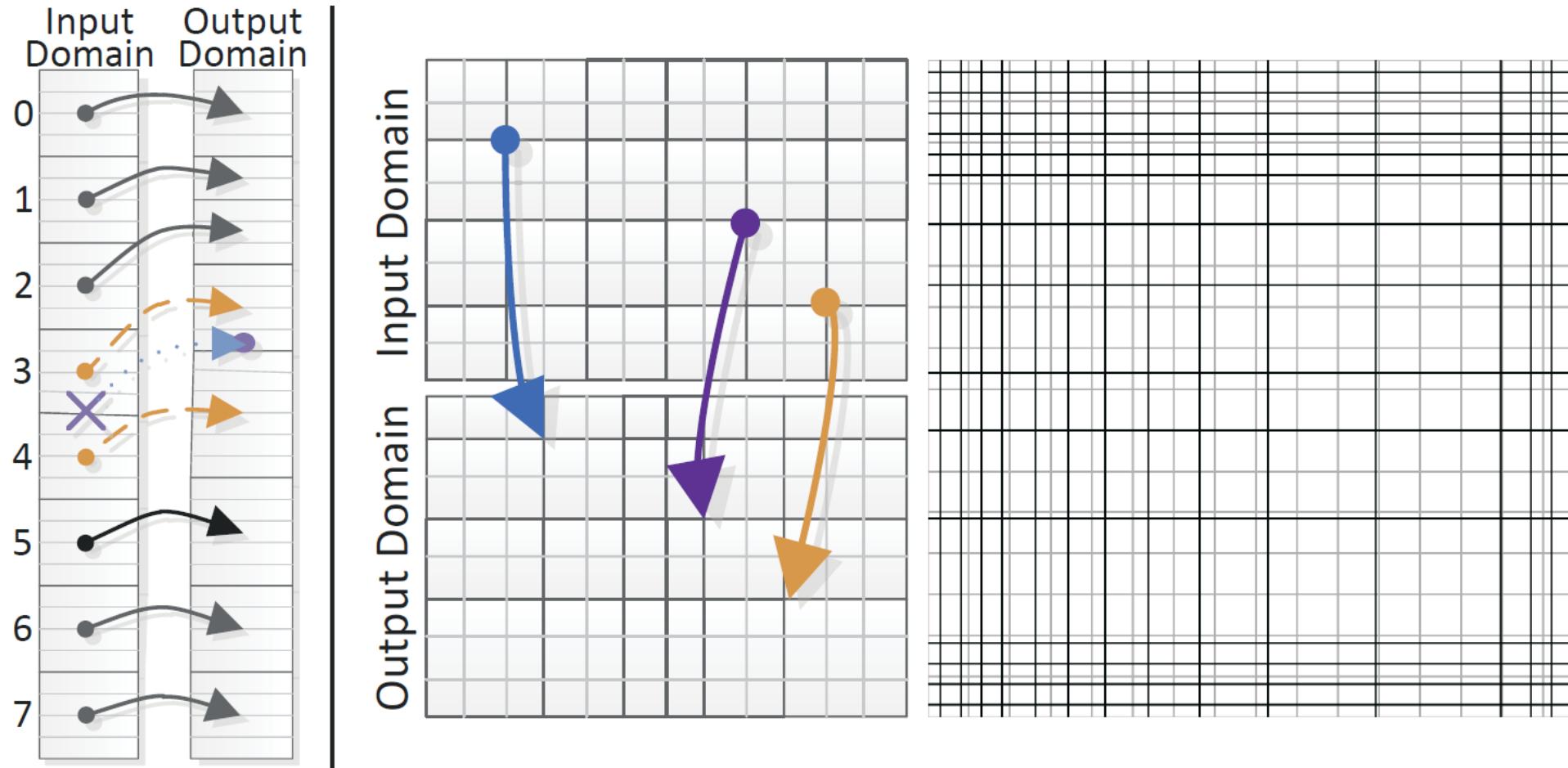
# Fitted Virtual Shadow Maps

[Giegel & Wimmer 2007]



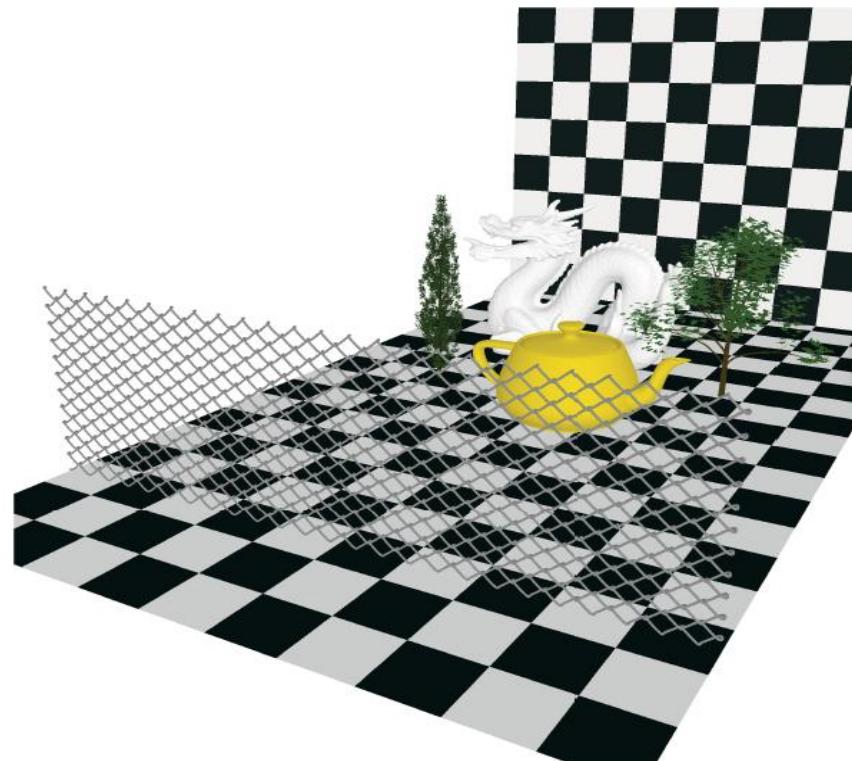
# Rectilinear Warping [Rosen et al. 2012]

- Apply 2 separate 1D-warps based on scene content

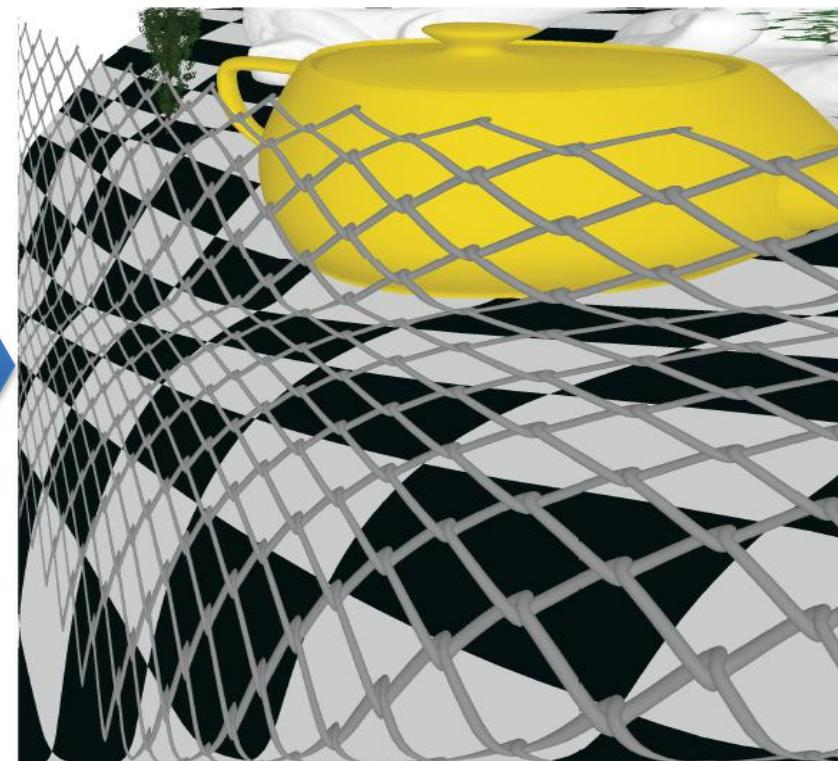


# Rectilinear Warping [Rosen et al. 2012]

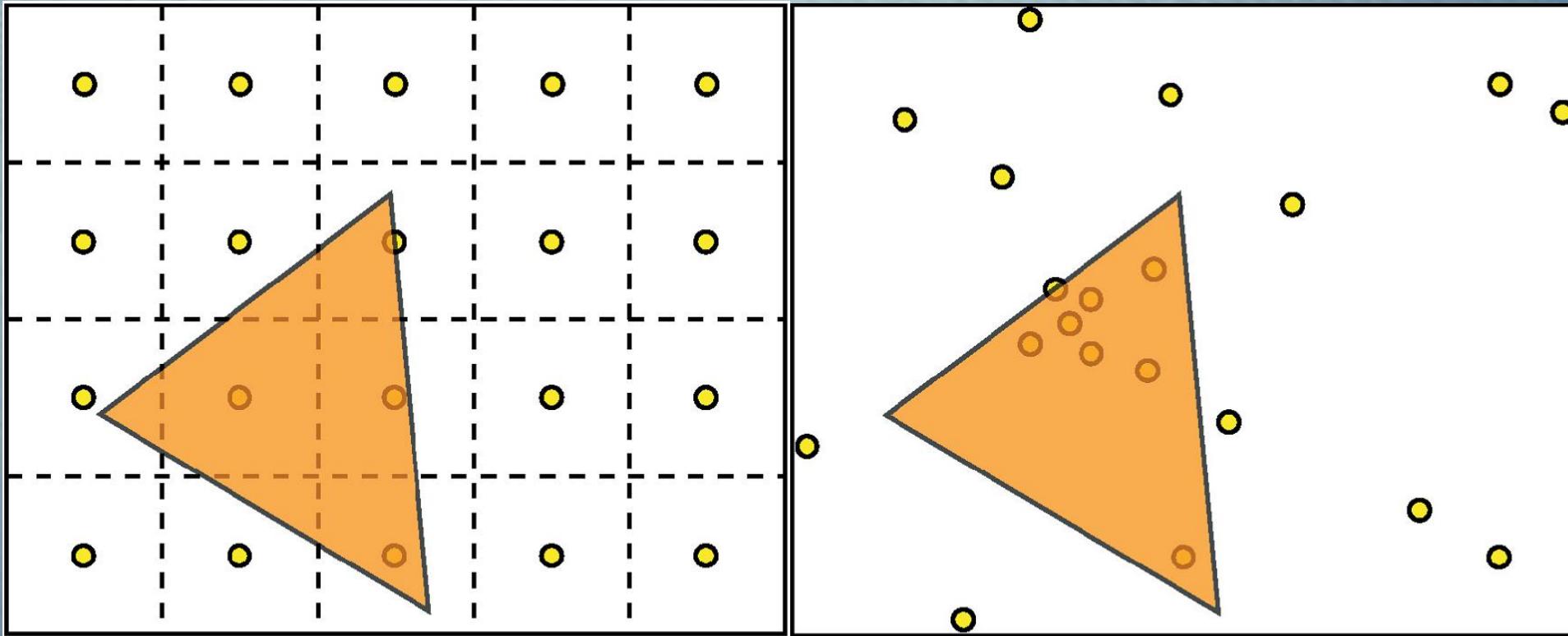
- Requires scene analysis pass
- Requires tessellation to create shadow map



Standard shadow map



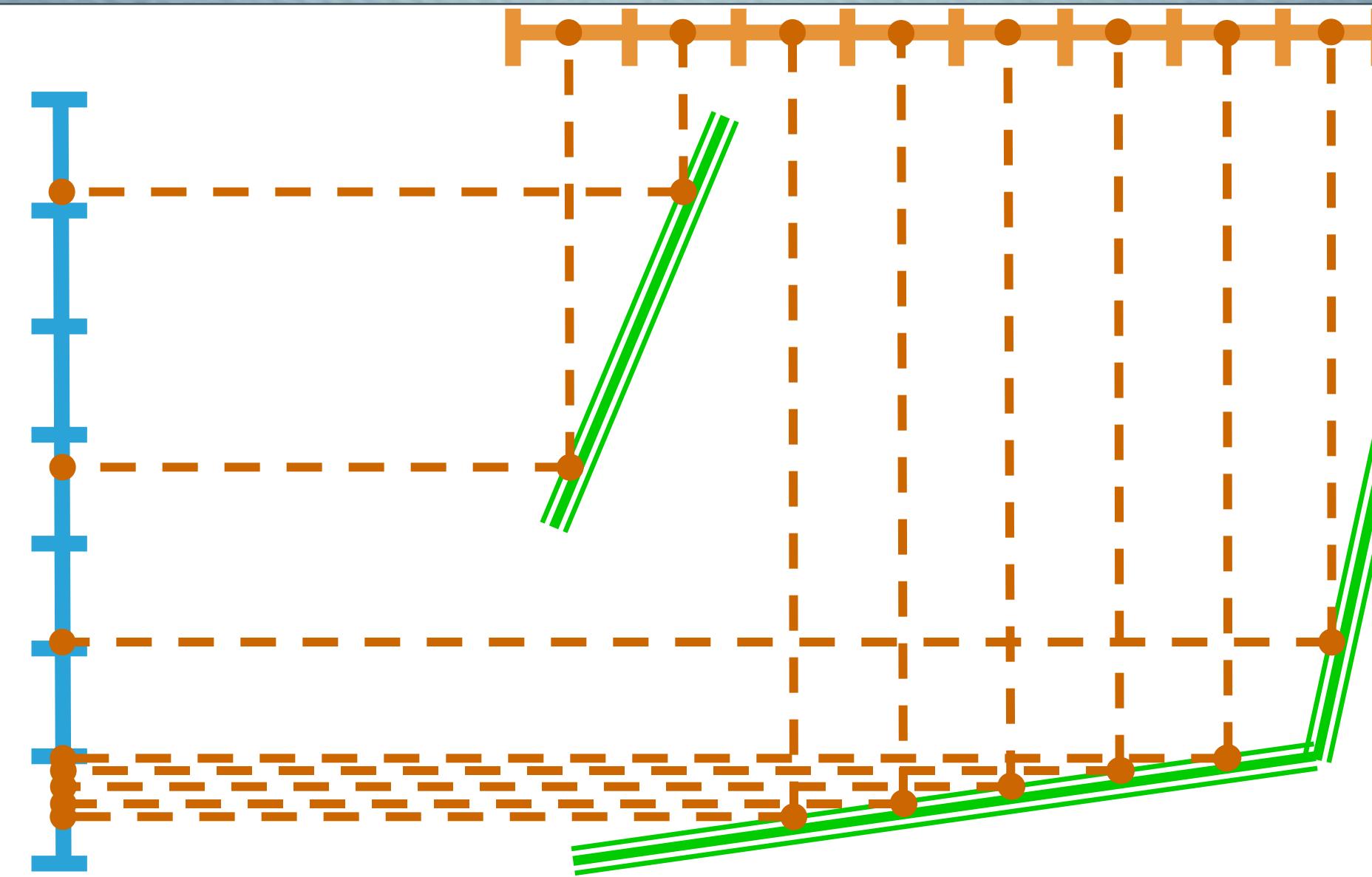
Rectilinearly warped shadow map



# Hard Shadows

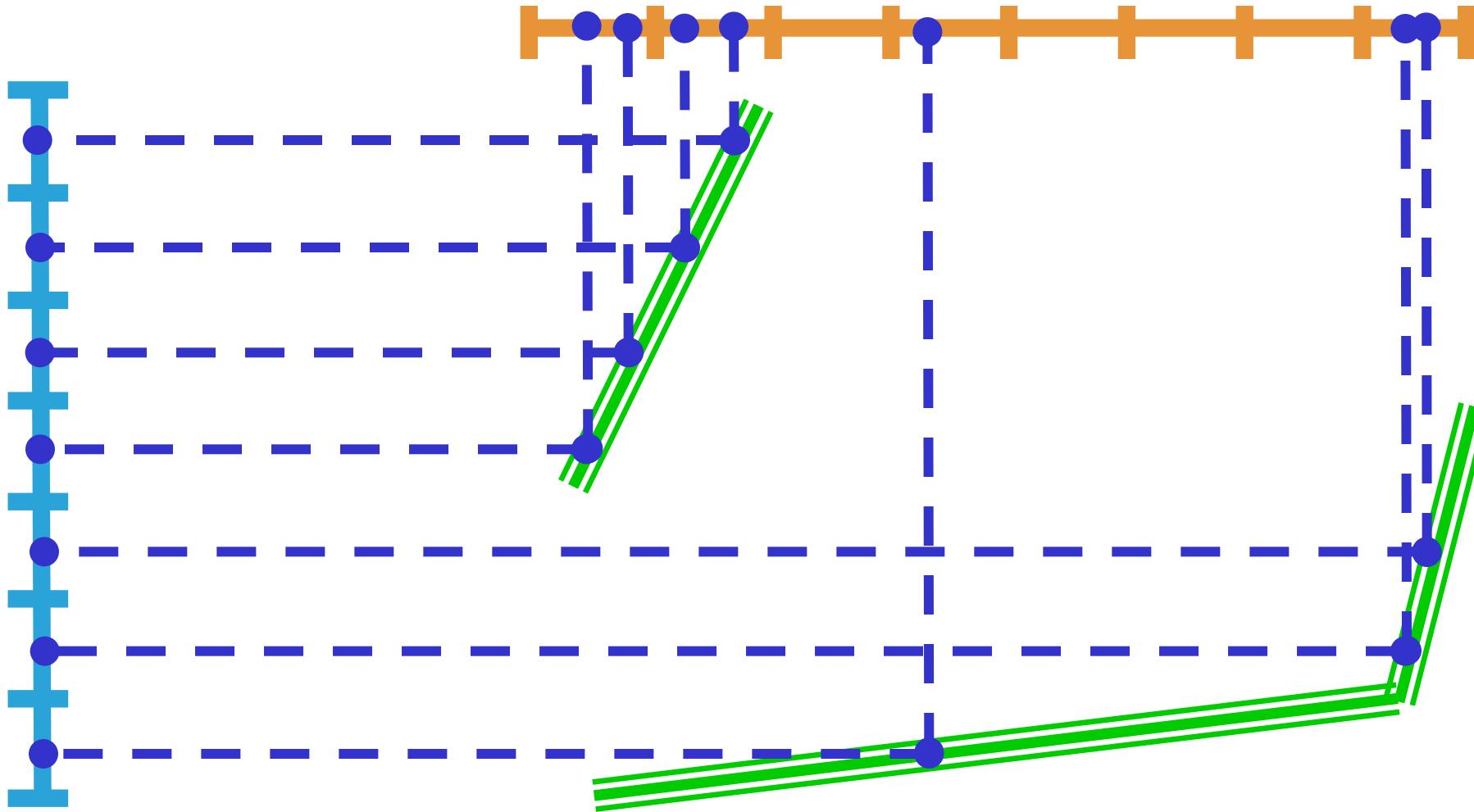
Fighting Undersampling – Irregular Sampling

# Shadow Mapping Sampling



# What Samples Do We Want?

- Idea: use eye space samples to generate shadow map samples

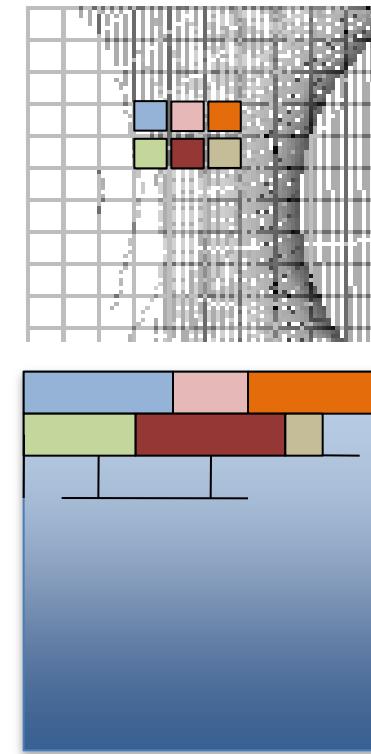
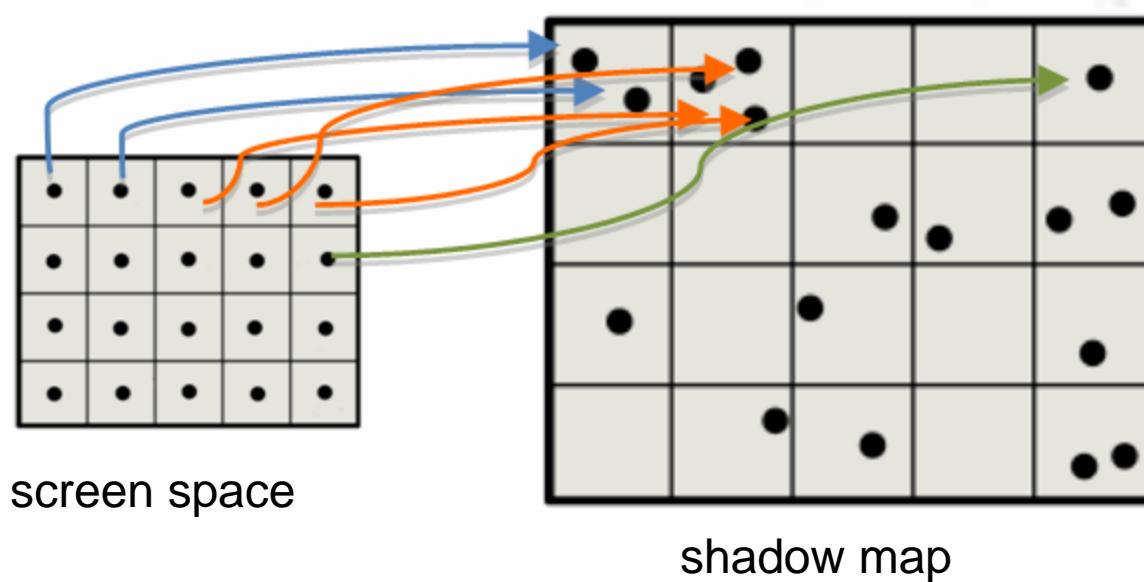


# Alias Free Hard Shadows

[Aila and Laine 2004][Sintorn et al. 2008]



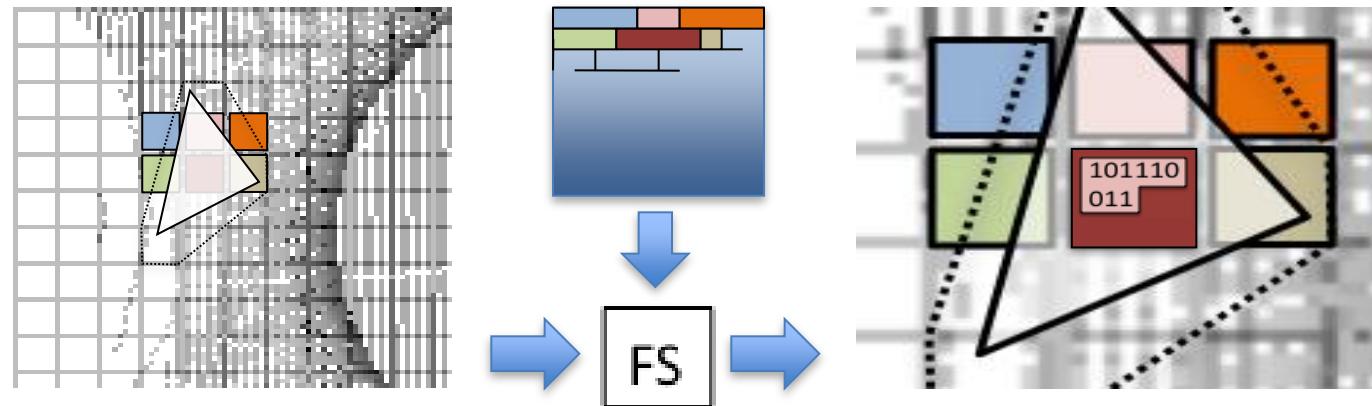
- Camera pass: transform and project view-samples into light-space
  - Store in a compact data structure with a list per light-space texel



# Alias Free Hard Shadows

[Sintorn et al. 2008]

- Light pass: render all geometry (conservatively)
  - For each generated fragment, test all view-samples in list against triangle
  - Set corresponding output bit depending on occlusion

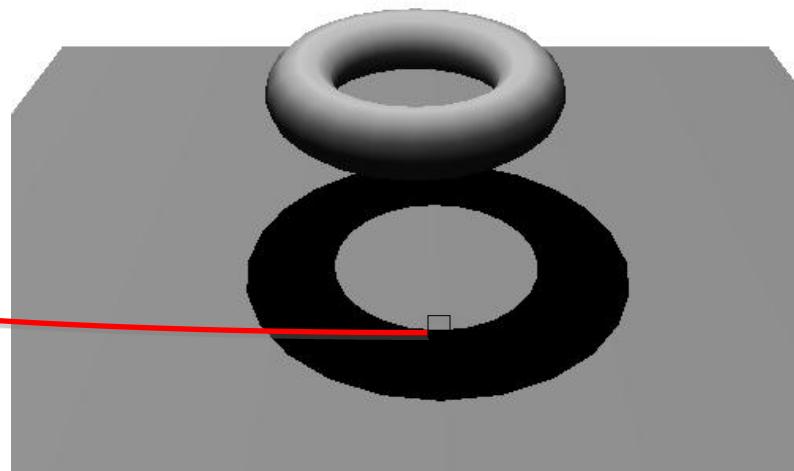
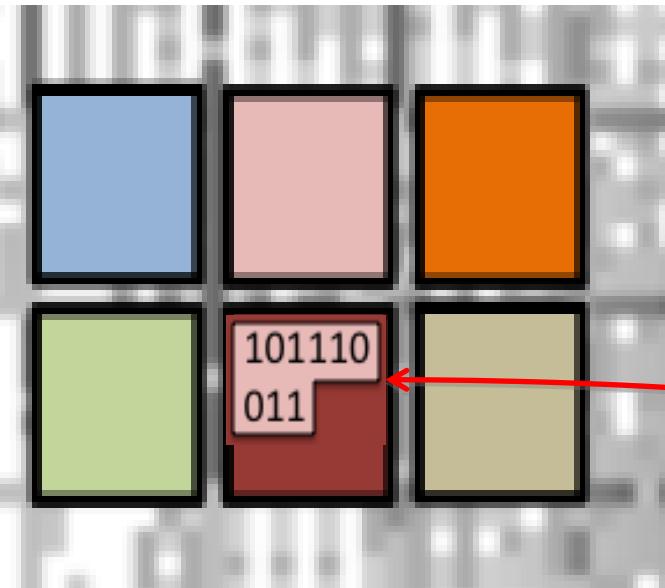


# Alias Free Hard Shadows

[Sintorn et al. 2008]



- Final screen-space pass
- Use bitmask from previous pass for shadowing

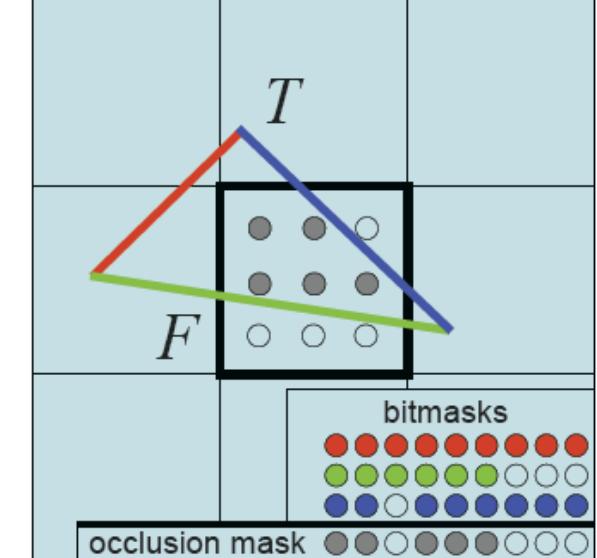
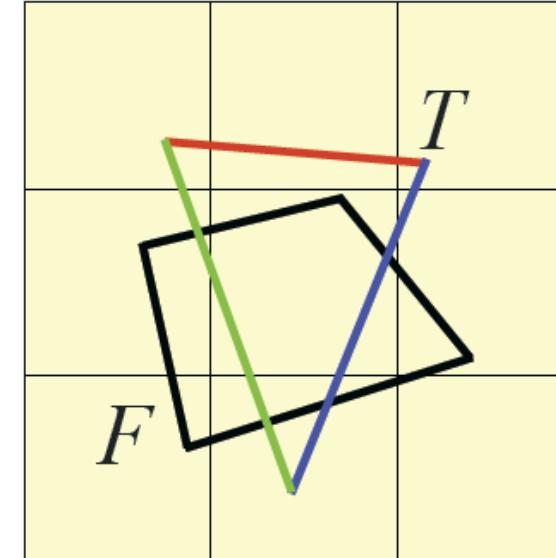
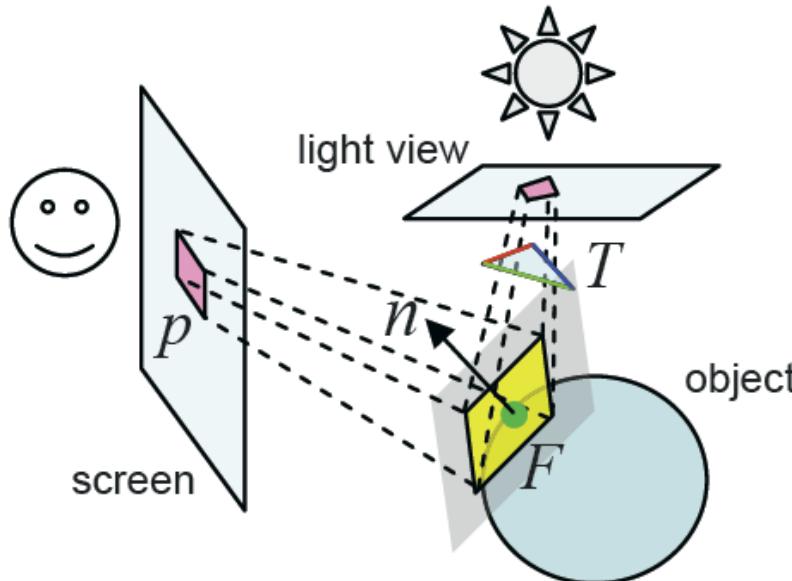


# Subpixel Alias Free Shadow Maps

[Zhou et al. 2009]



- Problem: no antialiasing (supersampling)
- Solution:
  - Project facets instead of samples
  - Precompute subsample tests (constant time [Eisemann 2007])



$$cache(n) := \text{conf} * s(n) + (1 - \text{conf}) * cache(n-1)$$

# Hard Shadows

Temporal Reprojection

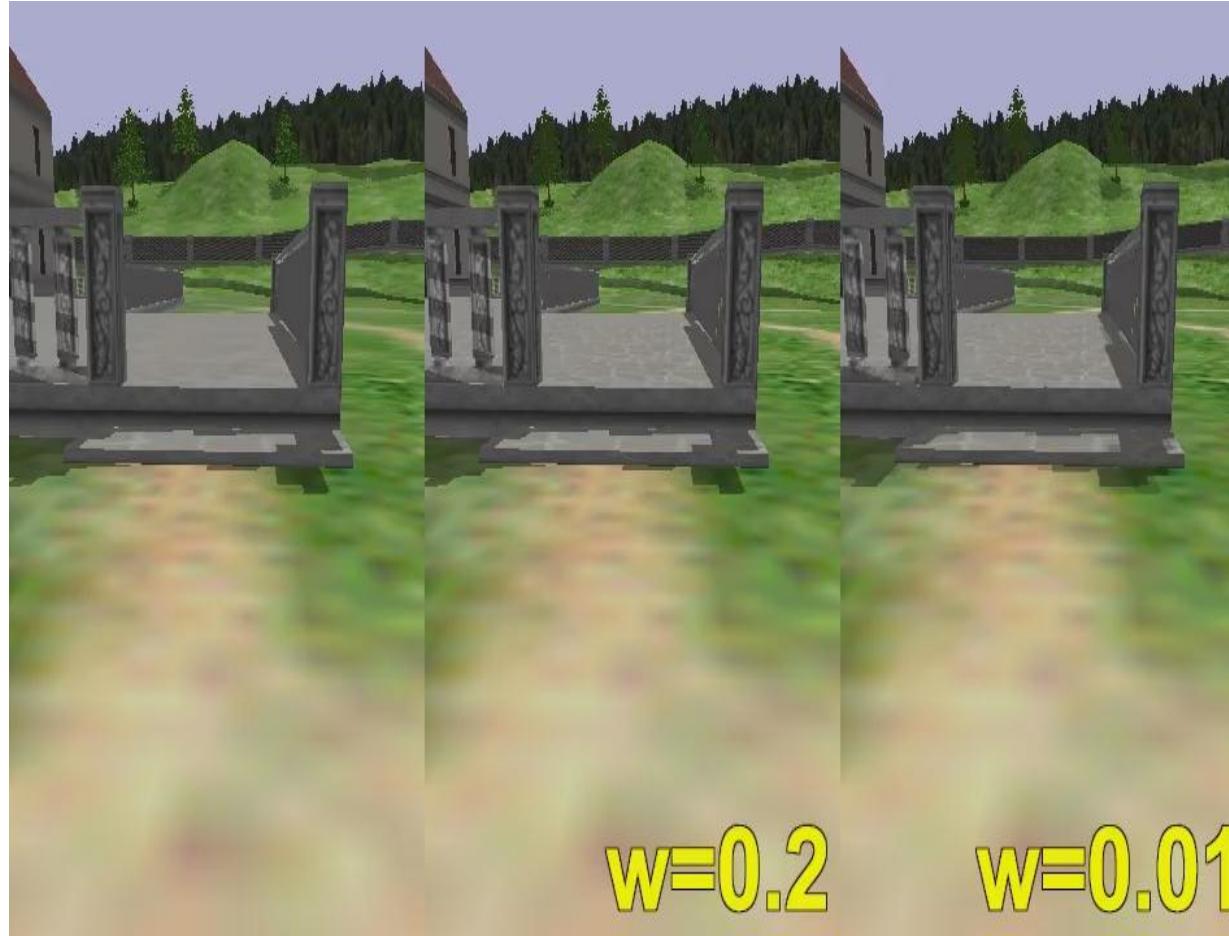
# Temporal Smoothing

- Shadowing result of previous frame ( $n-1$ ) is stored in *cache*

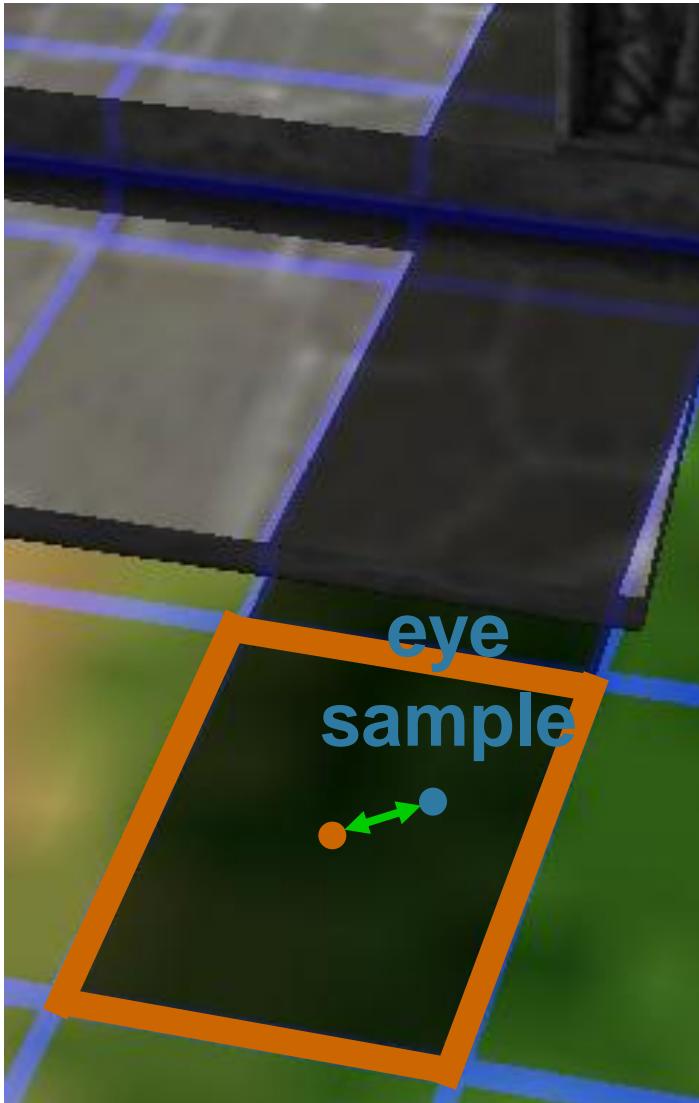


# Temporal Smoothing

- Shadowing result of previous frame ( $n-1$ ) is stored in *cache*
- Current frame ( $n$ ):  $cache(n) := w * s(n) + (1-w) * cache(n-1)$



# Weight? Confidence Estimation!

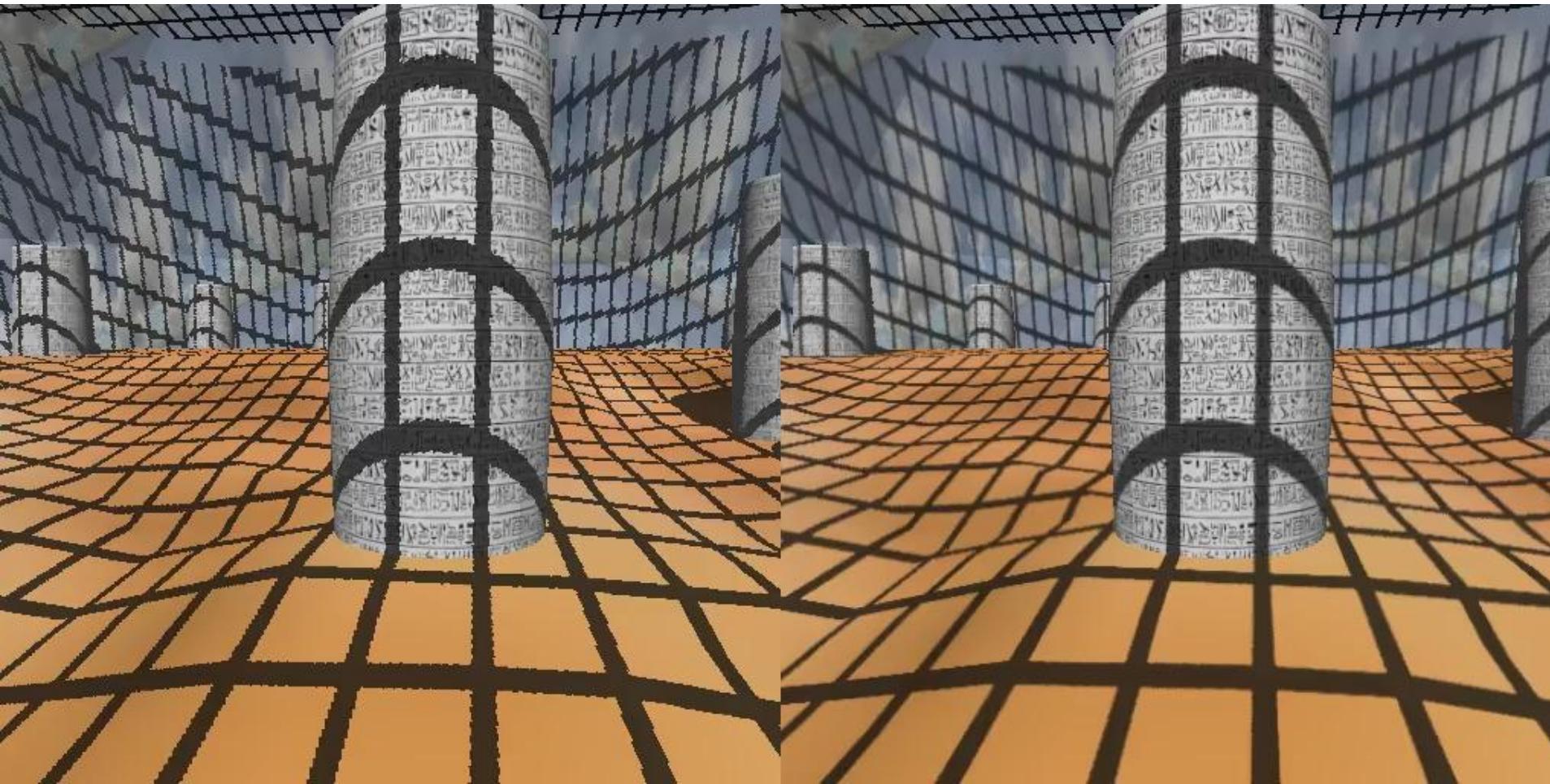


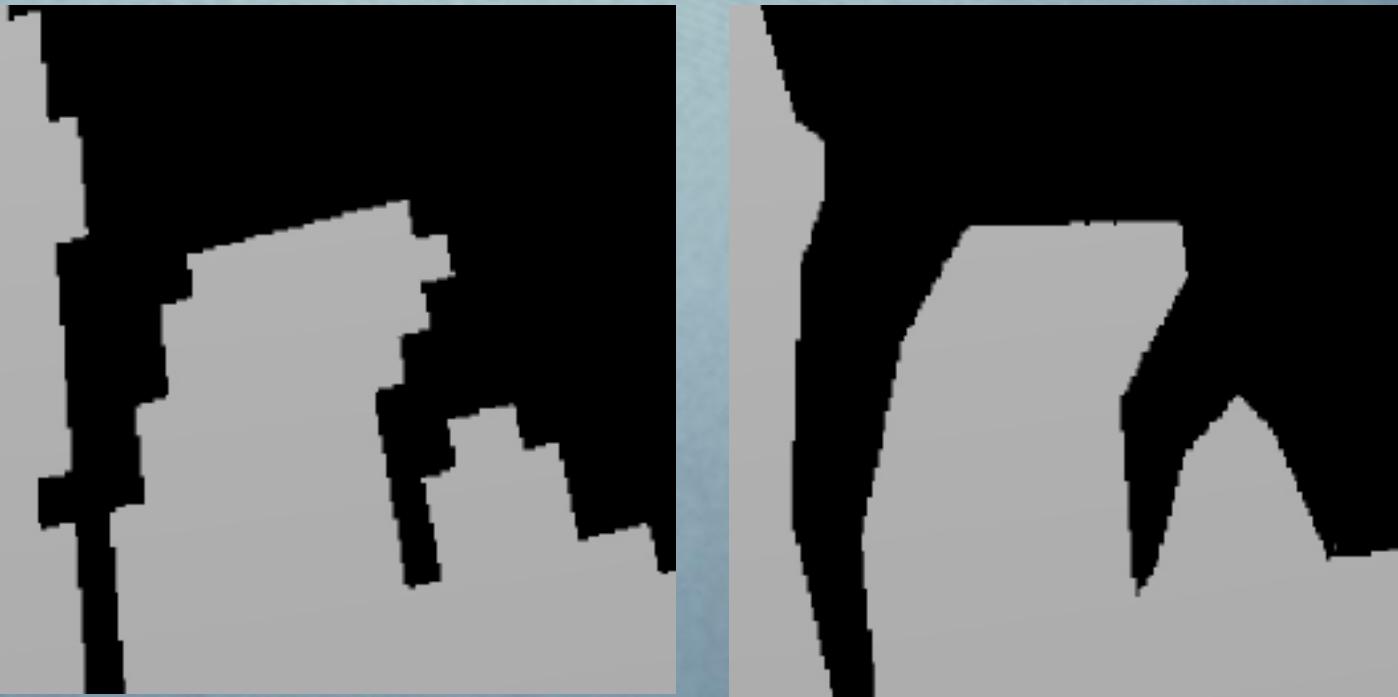
- Shadow map test probably more correct closer to texel center
- Confidence:  
 $\text{conf} := 1 - \text{dist}(\text{eye}, \text{texel})$
- Greater confidence  
→ greater impact

# Confidence and Temporal Smoothing

- Confidence is weight for temporal smoothing

$$cache(n) := \text{conf} * s(n) + (1 - \text{conf}) * cache(n-1)$$





# Hard Shadows

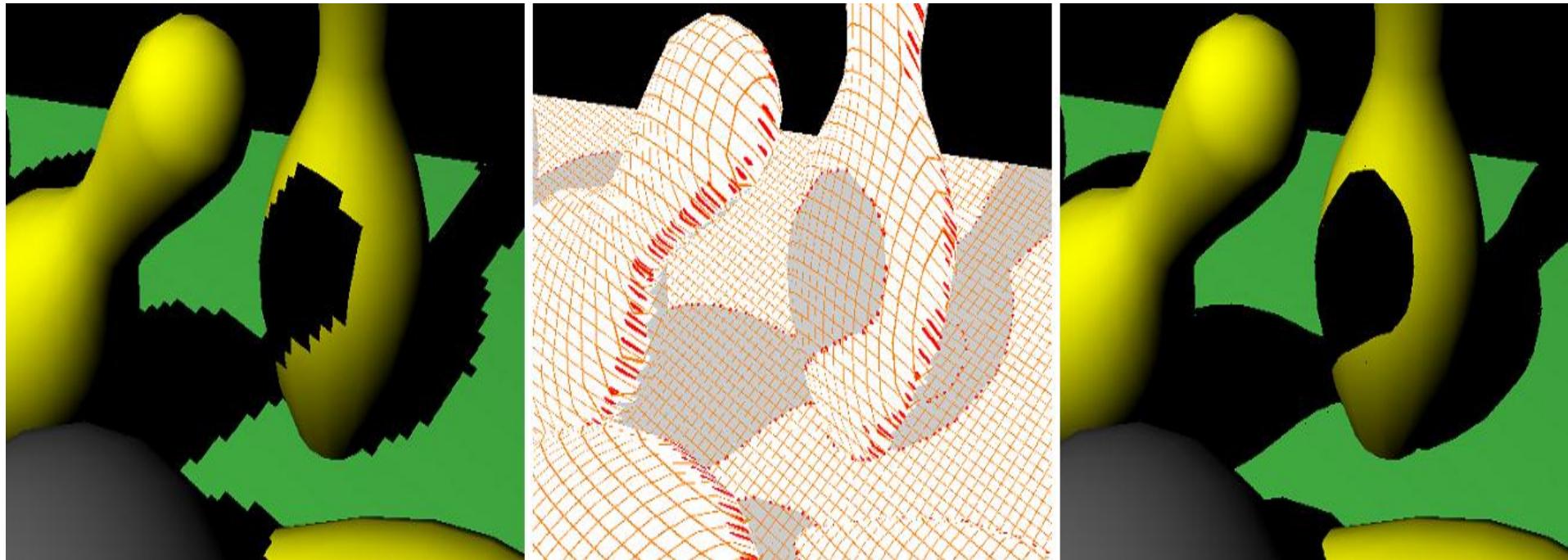
Better Reconstruction

# Shadow Silhouette Maps

[Sen et al. 2003]



- Use a better silhouette approximation
- Store additional information of shadow edge

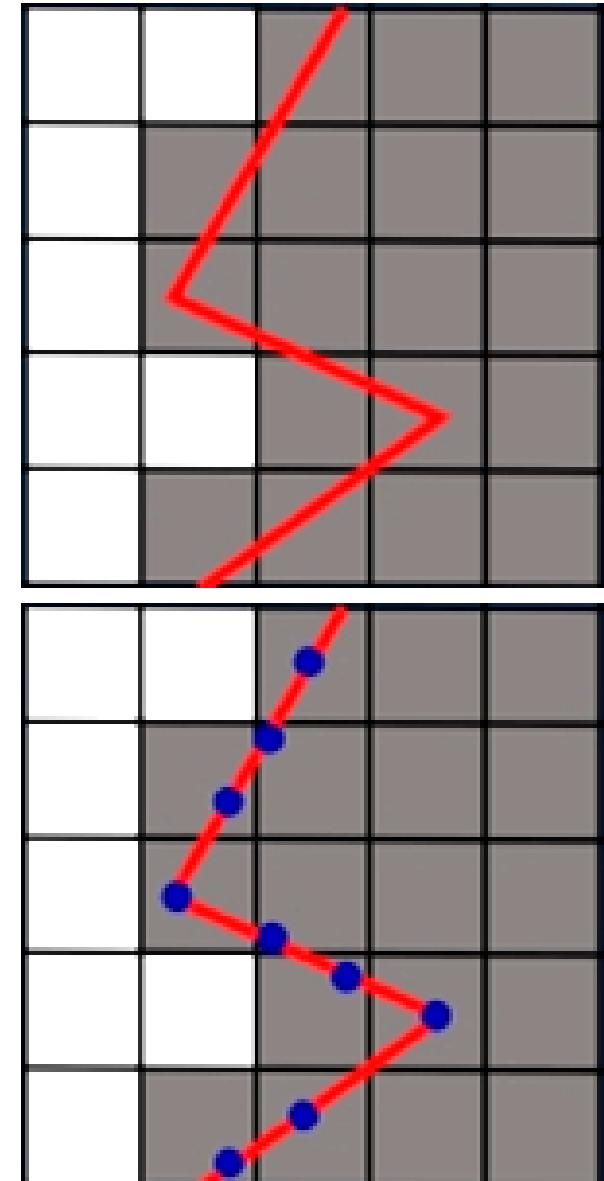


# Shadow Silhouette Maps

[Sen et al. 2003]



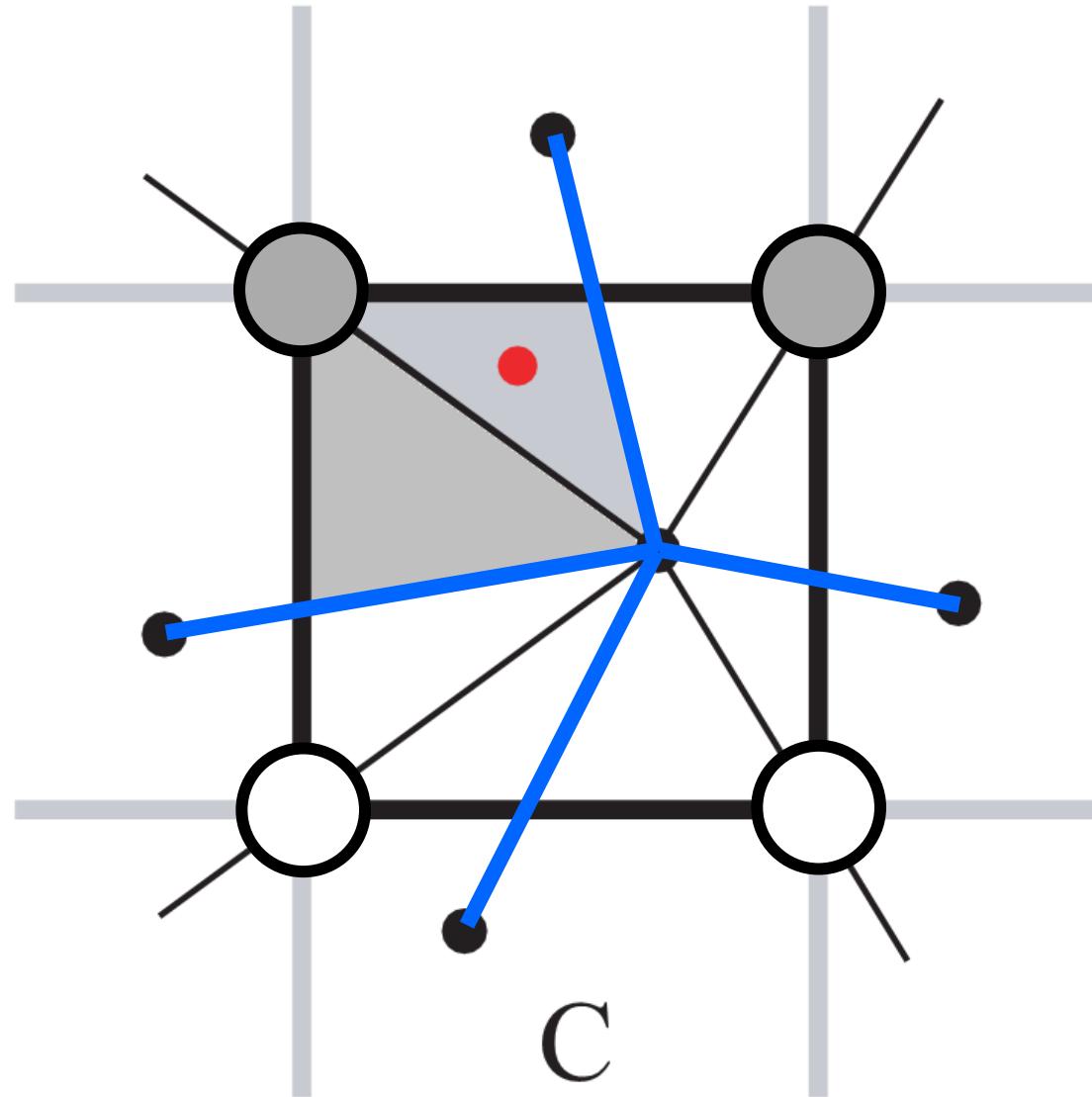
1. Create shadow map
2. Find silhouette edges
3. Rasterize silhouettes
  - a. Find points that lie on silhouette edges
  - b. Store such points into silhouette map
4. Compute shadows
  - a. Non-silhouette pixels use standard shadow map
  - b. Silhouette pixels use silhouette map



# Shadow Silhouette Maps

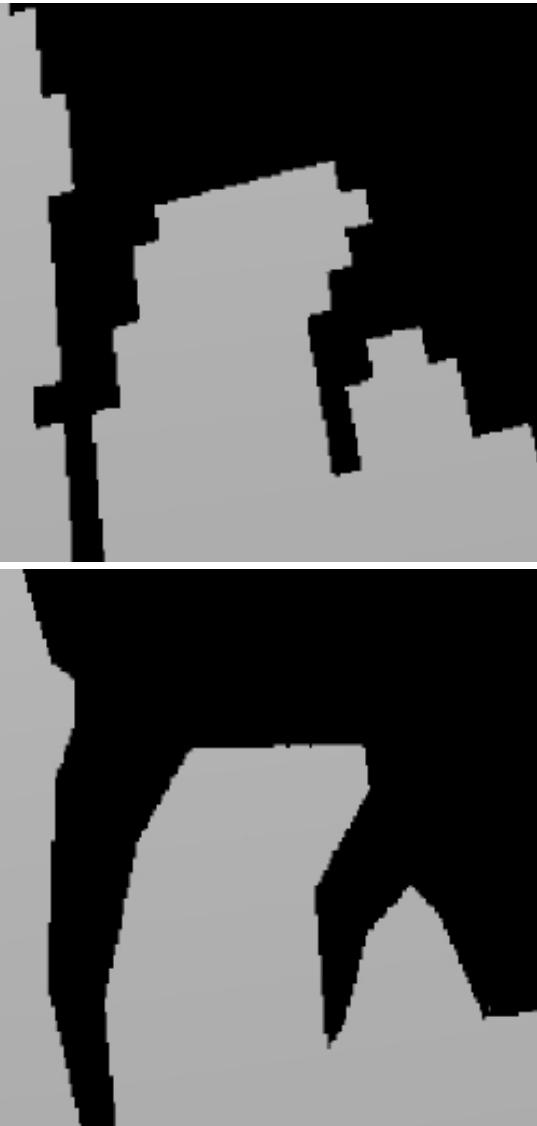
[Sen et al. 2003]

- Fragment
- Silhouette points
  - 1 interior + 4 external
- Create quadrants
- Shade **fragment** according to shadow test result at corner point



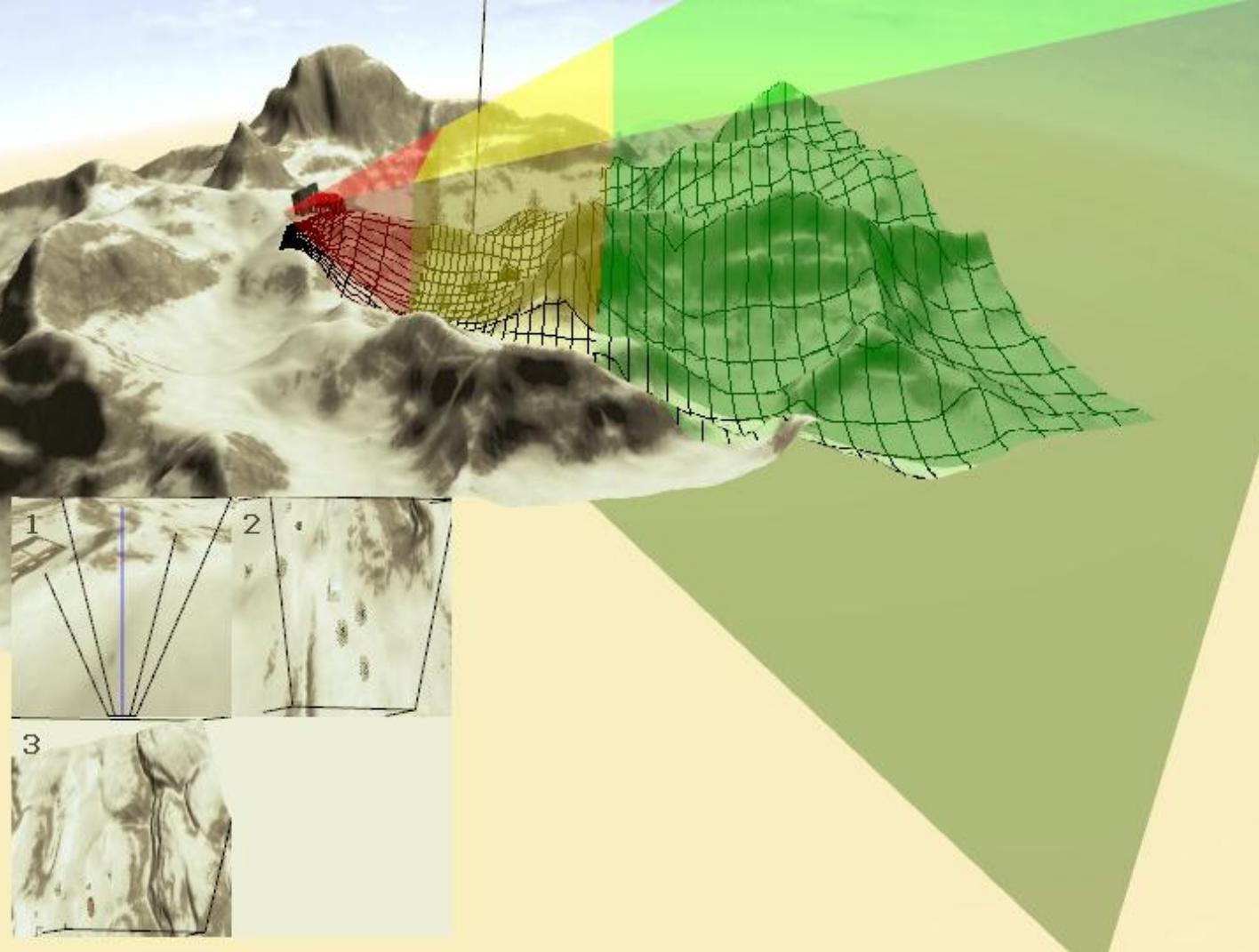
# Shadow Silhouette Maps

[Sen et al. 2003]



shadow map

silhouette map



# Hard Shadows

Conclusion

- Fastest speed, single shadow map: **warping**
  - Good for outdoor
- Fast speed, better quality, >1 shadow maps:  
**z-partitioning (+warping)**
  - Add “lightweight” scene analysis to tightly bound partitions and cull shadow casters
- High quality, lower speed: **adaptive partitioning**
- Reference quality, even slower: **irregular sampling**
  - Combine with subpixel sampling