1. Dar expresiones regulares para los lenguajes de los ejercicios 1 a 3 de la práctica 1.

Te amo ♡

a) Cadenas sobre $\Sigma = \{0\}$ de longitud par. $(00)^*$

b) Cadenas sobre $\Sigma = \{0, 1\}$ con cantidad par de ceros. $1^*(01^*0)^*1^*$

c) Cadenas sobre $\Sigma = \{0, 1\}$ con cantidad par de ceros y cantidad impar de unos.

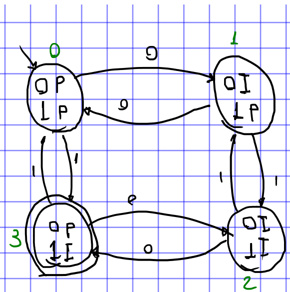
d) Cadenas sobre $\Sigma = \{0, 1\}$ que, interpretadas como un número binario, sean congruentes a cero módulo 5.

Proposición

Dados $\mathrm{R}, \alpha, \beta$ lenguajes sobre Σ,

si $\mathrm{R} = \alpha.\mathrm{R}|\beta \wedge \lambda \notin \alpha$, entonces $\mathrm{R} = \alpha^*.\beta$

c)



CONVERTIMOS $\Longrightarrow$

$L_0 = 1L_3 | 0L_1$
$L_1 = 0L_0 | 1L_2$
$L_2 = 0L_3 | 1L_1$
$L_3 = 0L_2 | 1L_0 \Rightarrow L_3 = 0(0L_3 | 1L_1) | 1L_0$
$= 00L_3 | 01L_1 | 1L_0$
$L_3 = (00)^*(01L_1 | 1L_0)$

$(0x + x) = (0+1)x \qquad y(x+z)+w = (yx+yz).w$

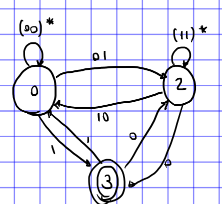
$L_0 = 1(00)^*(01L_1 | 1L_0) | 0L_1$
$= 1(00)^*1L_0 | 1(00)^*01L_1 | 0L_1$
$= 1(00)^*1L_0 | (1(00)^*01 | 0)L_1$
$L_0 = (1(00)^*1)^*(1(00)^*01 | 0)L_1$

$L_1 = 0L_0 | 1L_2$
$= 0(1(00)^*1)^*(1(00)^*01 | 0)L_1 | 1L_2$
$L_1 = (0(1(00)^*1)^*(1(00)^*01 | 0))^*1L_2$

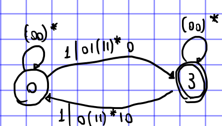
$L_2 = 1L_1 | 0L_3$
$= 1(0(1(00)^*1)^*(1(00)^*01 | 0))^*1L_2 | 0L_3$
$L_2 = (1(0(1(00)^*1)^*(1(00)^*01 | 0))^*1)^*0L_3$

HORRIBLE

PLAN B $\Rightarrow$ sacamos 1



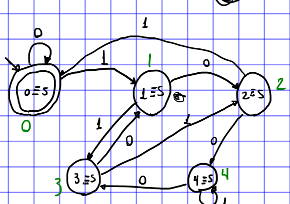
sacamos 2



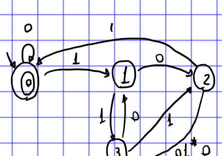
$\Downarrow$ REGEXP

$((00)^* + (1+01(11)^*0)(00)^*(1+0(11)^*10))^*(1+01(11)^*0)(00)^*$

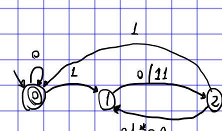
d)



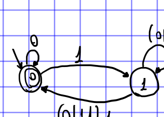
$\Rightarrow$ sacamos 4



sacamos 3



sacamos 2



$\Rightarrow$ sacamos 1 (Regex final)

$[0 | 1((0|11)01^*00)^*(0|11)1]$

a) Cadenas que comiencen con 010. $010(0|1)^*$

b) Cadenas que terminen con 010. $(0|1)^*010$

c) Cadenas que contengan la subcadena 000. $(0|1)^*000(0|1)^*$

d) Cadenas que no contengan la subcadena 000.

e) Cadenas que contengan la subcadena 000 exactamente una vez. (la cadena 0000 no pertenece a este lenguaje).

f) Cadenas que no contengan la subcadena 000 ni la 010.

2. Calcular las siguientes derivadas:

ej 2)

a) $\partial_1(10^*1)$

b) $\partial_\lambda(10^*1)$

c) $\partial_0(10^*1)$ $= 0^*1$

d) $\partial_a(ab^*|ac|c^+)$ $= 10^*1$

$= \partial_a(ab^*) | \partial_a(ac) | \partial_a(c^+) = b^* | c | \emptyset = b^*c$

e) $\partial_a(a^+ba)$ $= \emptyset$

f) $\partial_a(a^*ba)$ $= a^*ba$

g) $\partial_{01}(0(1|\lambda)|1^+)$ $= a^*ba$

$= \partial_1(\partial_0(0(1|\lambda)) | \partial_0(1^+)) = \partial_1(\cancel{\partial_0}(0)(1|\lambda) | \epsilon(0)\partial_0(1|\lambda) | \cancel{\emptyset}) = \partial_1((1|\lambda) | \cancel{\emptyset\emptyset}) = \partial_1(1) | \partial_1(\lambda) = \lambda | \cancel{\emptyset} = \lambda$

3. Pasar las siguientes expresiones regulares a autómatas finitos (mediante el método de las derivadas) y 3)

   a) $(0|1)^*01$

   b) $(a(b|\lambda)|b^+)$

a) $\partial_0(L_0) = \partial_0((0|1)^*01) = \partial_0((0|1)^*)01 \mid \epsilon(\overset{\lambda}{\cancel{(0|1)^*}})\,\partial_0(01) = \partial_0(0|1)(0|1)^*01 \mid 1 = (\overset{\lambda}{\cancel{\partial_0(0)}} \mid \overset{\emptyset}{\cancel{\partial_0(1)}})(0|1)^*01 \mid 1 = (0|1)^*01|1]$ L_1

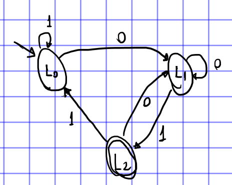
$\partial_1(L_0) = \partial_1((0|1)^*01) = \partial_1((0|1)^*)01 \mid \epsilon(\overset{\lambda}{\cancel{(0|1)^*}})\,\overset{\emptyset}{\cancel{\partial_1(01)}} = \partial_1\overset{\lambda}{\cancel{(0|1)}}(0|1)^*01 \mid \emptyset = (0|1)^*01]$ L_0

$\partial_0(L_1) = \partial_0((0|1)^*01|1) = \partial_0((0|1)^*01) \mid \partial_0(1) = (0|1)^*01 \mid 1 \mid \overset{\emptyset}{\cancel{\partial_0(1)}} = (0|1)^*01 \mid 1]$ L_1

$\partial_1(L_1) = \partial_1((0|1)^*01|1) = \partial_1((0|1)^*01) \mid \partial_1(1) = (0|1)^*01 \mid \lambda]$ L_2

$\partial_0(L_2) = \partial_0((0|1)^*01) \mid \overset{\emptyset}{\cancel{\partial_0(\lambda)}} = (0|1)^*01 \mid 1]$ L_1

$\partial_1(L_2) = \partial_1((0|1)^*01 \mid \partial_1(\lambda) = (0|1)^*01]$ L_0



b) $\partial_a(L_0) = \partial_a(a(b|\lambda)|b^+) = \partial_a(a(b|\lambda)) \mid \partial_a(b^+) = \partial_a(a)(b|\lambda) \mid \epsilon(a)\,\partial_a(b|\lambda) \mid \partial_a(b^+) = (b|\lambda) \mid \emptyset\,\emptyset \mid \emptyset = b|\lambda]$ L_1

$\partial_b(L_0) = \partial_b(a(b|\lambda)|b^+) = \partial_b(a(b|\lambda)) \mid \partial_b(b^+) = \partial_b(a)(b|\lambda) \mid \epsilon(a)\,\partial_b(b|\lambda) \mid \partial_b(b^+) = \emptyset \ldots \mid \emptyset \ldots \mid b^* = b^*]$ L_2

$\partial_a(L_1) = \partial_a(b|\lambda) = \partial_a(b) \mid \partial_a(\lambda) = \emptyset \mid \emptyset = \emptyset]$ T

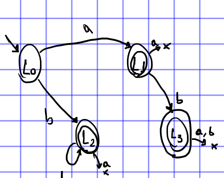
$\partial_b(L_1) = \partial_b(b|\lambda) = \partial_b(b) \mid \partial_b(\lambda) = \lambda \mid \emptyset = \lambda]$ L_3

$\partial_a(L_2) = \partial_a(b^*) = \partial_a(b)\,b^* = \emptyset]$ T

$\partial_b(L_2) = \partial_b(b^*) = \partial_b(b)\,b^* = b^*]$ L_2

$\partial_a(L_3) = \partial_a(\lambda) = \emptyset]$ T

$\partial_b(L_3) = \partial_b(\lambda) = \emptyset]$ T

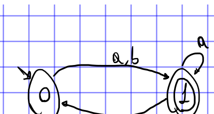


4. Pasar del autómata finito a la expresión regular los siguientes autómatas (mediante el método de las ecuaciones):

a) $A = \langle Q_1, \Sigma_1, \delta_1, q_1, F_1 \rangle$, donde:
$Q_1 = \{0,1\}, \Sigma_1 = \{a,b\}, q_1 = 0, F_1 = \{1\}$,

$\delta_1 =$

	a	b
0	1	1
1	1	0



$L_1 = aL_1 \mid bL_0 \mid \lambda = a^*(bL_0|\lambda) = a^*bL_0 \mid a^*]$

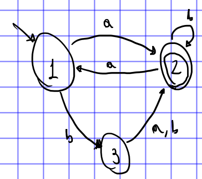
$L_0 = aL_1 \mid bL_1 = a^+bL_0 \mid a^+ \mid ba^*bL_0 \mid ba^*$

$= (a^+b \mid ba^*b)L_0 \mid a^+ \mid ba^* = (a^+b|ba^*b)^*(a^+|ba^*)]$

b) $A = \langle Q_2, \Sigma_2, \delta_2, q_2, F_2 \rangle$, donde:
$Q_2 = \{1,2,3\}, \Sigma_2 = \{a,b\}, q_2 = 1, F_2 = \{2\}$,

$\delta_2 =$

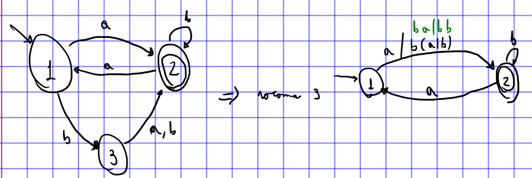
	a	b
1	2	3
2	1	2
3	2	2



$L_2 = bL_2 \mid aL_1 \mid \lambda = b^*(aL_1 \mid \lambda)$

$L_1 = aL_2 \mid bL_3 = ab^*aL_1 \mid ab^* \mid b(a|b)b^*aL_1 \mid b(a|b)b^* = (a \mid b(a|b))b^*aL_1 \mid (a \mid b(a|b))b^* = ((a \mid b(a|b))b^*a)^*(a \mid b(a|b))b^*$

$L_3 = aL_2 \mid bL_2 = (a|b)L_2 = (a|b)b^*(aL_1 \mid \lambda)$



$\Rightarrow ER = ((a \mid ba \mid bb)b^*a)^*(a \mid ba \mid bb)b^*$

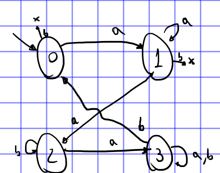
c) El autómata no determinístico $A = \langle Q_3, \Sigma_3, \delta_3, q_3, F_3 \rangle$, donde:
$Q_3 = \{0,1,2,3\}, \Sigma_3 = \{a,b\}, q_3 = 0, F_3 = \{2\}$,

$\delta_3 =$

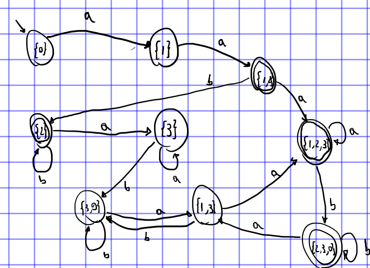
	a	b
0	1	–
1	1, 2	–
2	3	2
3	3	3, 0

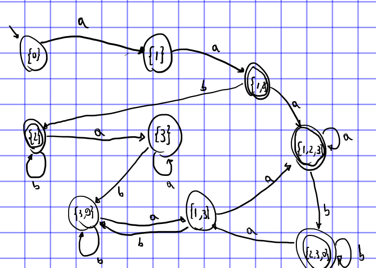
$\Rightarrow$ AFD

	a	b
$\rightarrow \{0\}$	$\{1\}$	$\{T\}$
$\{1\}$	$\{1,2\}$	$\{T\}$
$\{2\}^*$	$\{3\}$	$\{2\}$
$\{3\}$	$\{3\}$	$\{3,0\}$
$\{1,2\}^*$	$\{1,2,3\}$	$\{2\}$
$\{3,0\}$	$\{1,3\}$	$\{3,0\}$
$\{1,3\}$	$\{1,2,3\}$	$\{3,0\}$
$\{1,2,3\}^*$	$\{1,2,3\}$	$\{2,3,0\}$
$\{2,3,0\}^*$	$\{1,3\}$	$\{2,3,0\}$



$\Longrightarrow$ AFD





$L_0 = aL_1$

$L_1 = aL_{12}$

$L_{12} = aL_{123} \mid bL_2 \mid \lambda$

✓ $L_{123} = aL_{123} \mid bL_{230} \mid \lambda = a^*(bL_{230} \mid \lambda)^{*}$

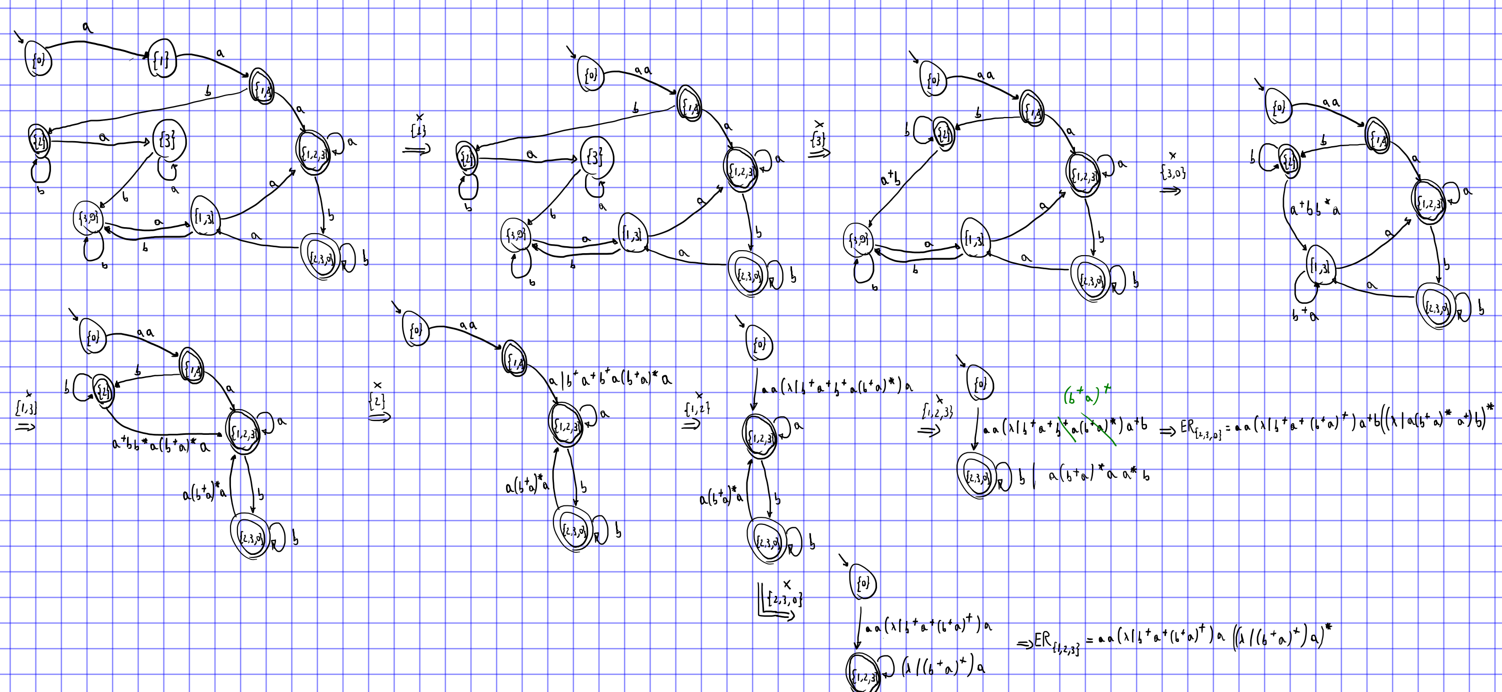
✓ $L_2 = bL_2 \mid aL_3 \mid \lambda = b^*(aL_3 \mid \lambda) \overset{\#}{=} b^*(a^+b^+a((b^+a)^*a^+b^+a)^*(b^+a)^*a^+b^+ \mid \lambda)$

✓ $L_{230} = bL_{230} \mid aL_{13} \mid \lambda = b^*(aL_{13} \mid \lambda)^{*}$

✓ $L_3 = aL_3 \mid bL_{30} = a^*bL_{30} \overset{\#}{=} a^*b^+a((b^+a)^*a^+b^+a)^*(b^+a)^*a^+b^+]$ #

✓ $L_{13} = aL_{123} \mid bL_{30} \overset{*}{=} b^+aL_{13} \mid aL_{123} = (b^+a)^*aL_{123} \overset{*}{=} (b^+a)^*a^+(bL_{230} \mid \lambda) \overset{*}{=} (b^+a)^*a^+(b^+(aL_{13} \mid \lambda)) = (b^+a)^*a^+b^+aL_{13} \mid (b^+a)^*a^+b^+ = ((b^+a)^*a^+b^+a)^*(b^+a)^*a^+b^+]$ #

✓ $L_{30} = bL_{30} \mid aL_{13} = b^*aL_{13} \overset{*\#}{=} b^*a((b^+a)^*a^+b^+a)^*(b^+a)^*a^+b^+]$ #



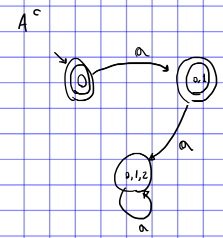
9. Para el AF $A = \langle \{0, 1, 2\}, \{a, b, c\}, \delta, 0, \{2\} \rangle$, donde

$$\delta = \begin{array}{c|c|c|c} & a & b & c \\ \hline 0 & 0,1 & - & - \\ 1 & 2 & - & - \\ 2 & - & 2 & - \end{array}$$

	a	b	c
0	0,1	T	T
× 1	2	T	T
× 2*	T	2	T
0,1	0,1,2	T	T
0,1,2*	0,1,2	T	T

A

Sea $L = L(A)$. Encontrar una expresión regular que denote el lenguaje $(L^c)^3$ (donde L^c es el complemento de L).



$L_0 = a L_1 | \lambda = a | \lambda]$

$L_1 = a L_2 | \lambda = a \emptyset | \lambda = \lambda]$

$L_2 = a L_2 | \emptyset = a^* \emptyset = \emptyset]$

$ER_{\{L^c\}} = (a|\lambda)^3$

$= (a|\lambda)(a|\lambda)(a|\lambda)$