

## Variable Elimination (CE).

From the graph we have.

$$P(S_m, S_t, P_r, F, P_a) = P(S_m) P(S_t) P(P_r | S_m, S_t) P(F) P(P_a | S_m, P_r, F).$$

$$P(P_a) = \sum_{S_m, S_t, P_r, F} P(S_m, S_t, P_r, F, P_a).$$

$$= \sum_{S_m, S_t, P_r, F} P(S_m) P(S_t) P(P_r | S_m, S_t) \underbrace{P(F) P(P_a | S_m, P_r, F)}_{f_F}$$

$$f_F = \sum_F P(F) P(P_a | S_m, P_r, F).$$

where

$f_F =$	$S_m$	$P_r$	$P_a$	$f_F(S_m, P_r, P_a)$
	T	T	T	$0.9 \times 0.9 + 0.1 \times 0.1 = 0.82$
	T	T	F	$0.9 \times 0.1 + 0.1 \times 0.9 = 0.18$
	T	F	T	$0.9 \times 0.7 + 0.1 \times 0.1 = 0.64$
	T	F	F	$0.9 \times 0.3 + 0.1 \times 0.9 = 0.36$
	F	T	T	$0.9 \times 0.7 + 0.1 \times 0.1 = 0.64$
	F	T	F	$0.9 \times 0.3 + 0.1 \times 0.9 = 0.36$
	F	F	T	$0.9 \times 0.2 + 0.1 \times 0.1 = 0.19$
	F	F	F	$0.9 \times 0.8 + 0.1 \times 0.9 = 0.81$

$$P(P_a) = \sum_{S_m, P_r, S_t} P(S_m) f_F(S_m, P_a, P_r) \underbrace{P(S_t) P(P_r | S_m, S_t)}_{f_{St}}$$

$f_{St}$	$S_m$	$P_r$	$f_{St}(P_r, S_m)$	$f_{St} = \sum_{S_t} P(S_t) P(P_r   S_m, S_t)$
	T	T	$0.6 \times 0.9 + 0.4 \times 0.5 = 0.74$	
	T	F	$0.6 \times 0.1 + 0.4 \times 0.5 = 0.26$	
	F	T	$0.6 \times 0.7 + 0.4 \times 0.1 = 0.46$	
	F	F	$0.6 \times 0.3 + 0.4 \times 0.9 = 0.54$	

$$P(Pa) = \sum_{Pr, Sm} P(Sm) f_F(Sm, Pa, Pr) f_{St}(Pr, Sm)$$

$$f_{Sm} = \sum_{Sm} P(Sm) f_F(Sm, Pa, Pr) f_{St}(Pr, Sm)$$

where

$f_{Sm} =$

$Pr$	$Pa$	$f_{Sm}(Pr, Pa)$
T	T	$0.8 \times 0.82 \times 0.74 + 0.2 \times 0.64 \times 0.46 = 0.5443$
T	F	$0.8 \times 0.18 \times 0.74 + 0.2 \times 0.36 \times 0.46 = 0.1396$
F	T	$0.8 \times 0.64 \times 0.26 + 0.2 \times 0.19 \times 0.54 = 0.1536$
F	F	$0.8 \times 0.36 \times 0.26 + 0.2 \times 0.81 \times 0.54 = 0.1623$

$$P(Pa) = \sum_{Pr} f_{Sm}(Pr, Pa)$$

$Pa$	$P(Pa)$
T	$0.5443 + 0.1536 = 0.6979$
F	$0.1396 + 0.1623 = 0.3016$

Probability that student will pass

$$= \underline{\underline{0.6979}}$$