On the radius of ionised bubbles

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ABSTRACT
The use of the Cen&Haiman expression for computing the radius of an ionised bubble has been extensively used by many authors in recent years. We propose to use a new interpretation, or rather a new formulation, to get the radius of an ionised bubble. This is done by using the comoving volume as the value we set constant to derive the radius equation.

Key words: Galaxies:High-z – Galaxies: Starburst–Cosmology: Reionisation – Cosmology: early Universe

1 INTRODUCTION

Many authors are using either the (Cen & Haiman 2000) equation or some variation thereof, to derive the radius of an ionised bubble, (Yan et al. 2010; Harikane et al. 2018; Jiang et al. 2018; Jung et al. 2020; Tilvi et al. 2020; Yajima et al. 2018). Lately most of these authors have used the Yajima et al. (2018) model to derived the radius of, typically individual bubbles for each of their sources. Indeed Yajima et al. (2018) derived a direct relation between the Lα luminosity and the radius of an ionised bubble. It is important to note that the original expression for the radius of an ionised bubble was first given by Spitzer (1978) or Osterbrock (1989) in their books. In the case of the high redshift Universe, adapted to work with typically larger luminosities, Hydrogen clumping factors, as well as escape factions and over-densities in the case of proto-clusters (Rodríguez Espinosa et al. 2020).

This study originated as we were trying to solve the differential equation given by Cen & Haiman (2000). We realised that the value to set constant was the volume rather than the radius. By doing so, the equation from Cen & Haiman (2000) became the well known equation given by Spitzer (1978) or Osterbrock (1989), particularised for its use at higher redshifts. Section 3 compare the radii derived with the Yajima et al. (2018) model with those obtained with our equation. Section 4 explains why the Yajima et al. (2018) model works and section 5 gives the conclusions.

2 RADIUS VERSUS VOLUME

When trying to know whether an ionised bubble is growing, the correct value to examine is its comoving volume. Certainly the relation between the amount of hydrogen atoms and the number of ionising photons emitted is of tantamount importance to ascertain the growth of a bubble. However, we need to think of comoving volumes of bubbles, and not the radii of these bubbles. Indeed, all we have to do is to require that the ionised bubble matches the photons emitted with recombinations, in comoving coordinates. As even if there is equilibrium between photons and recombinations, the physical volume can still increase due to the expansion of the Universe. Thus we need to require that the volume does not vary with the expansion, i.e., if the source of photons is constant the comoving volume of the bubble is constant.

The equation, used in recent years, to derive the radius of an ionised bubble is Cen & Haiman (2000), which is rendered below

$$\frac{dR^3}{dt} = 3H(z)R^3 + \frac{3\dot{N}_{ion}}{4\pi < n_H >} - C_H < n_H > \alpha_B R^3 \tag{1}$$

where the derivative of R^3 with respect to the time is equated to a first term, that account for the expansion of the Universe, plus a second term that is the ionising part, minus a third term that account for recombinations. Now, we need to require that the comoving volume is constant, i.e. its derivative is zero. Then the equation to use is not that of Cen & Haiman (2000) but that of the volume, namely,

$$V_c = \frac{4\pi}{3}\frac{R^3}{a^3} \tag{2}$$

where R is the size of the ionised bubble and a is the Universe scale factor.

Then, to derive the size of an ionised bubble, one has to check for the maximum comoving volume achievable given the available number of ionising photons, and discounting the recombinations. Thus we derive the equation for the comoving volume as in 3.

$$\frac{dV_c}{dt} = \frac{4\pi}{3}\frac{d\frac{R^3}{a^3}}{dt} = \frac{4\pi}{3}\frac{dR^3}{dt} + 3H(z)\frac{R^3}{a^3} \tag{3}$$

where H(z) is the Hubble function. If we now multiply the Cen &

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Haiman (2000) equation by $\frac{a^3}{\frac{4\pi}{3}}$, and we make $dV_c/dt = 0$, to find the point where the ionised bubble reaches its equilibrium, we get

$$0 = \frac{\dot{N}}{a^3 < n_H >} - \frac{4\pi}{3} C_H < n_H > \alpha_B \frac{R^3}{a^3} \tag{4}$$

Where we can see that the expansion term disappears. Thus the size of an ionised bubble can be readily derived as

$$R = \left\{ \frac{3\dot{N}}{4\pi C_H < n_H >^2 \alpha_B} \right\}^{\frac{1}{3}} \tag{5}$$

This is the equation that gives the radius of an ionised bubble in equilibrium between the number of ionising photons emitted and the number of recombinations. It is in fact an equation that can be seen in Spitzer (1978), and Osterbrock (1989), albeit with different symbols.Summarising, all we have done is to require that the ionised bubble matches the photon emitted with recombinations, in comoving coordinates.

3 SOME COMPARISONS

3.1 The BDF

We can test our calculation for some cases to check whether the results agree. The first result we will check are those of the BDF (Rodríguez Espinosa et al. 2021). In this case, to achieve an ionised volume similar to those in Rodríguez Espinosa et al. (2021), and taking into account the over-density (3.5) of the BDF, and the escape fraction we derived $\approx$ 10% the radius comes out as for group1 0.65 pMpc and 0.63 pMpc fr group2. These result in volumes for group 1 and 2 of 611 and 561.5 cMpc^3 respectively. We require escape fractions of the order of 30% to recover the volumes given in Rodríguez Espinosa et al. (2021). Incidentally, this may be a result of the fashion we computed the volumes of the BDF. Indeed, we assumed rather flat volumes instead of spherical volumes, as the sources seem to be rather in a plane.

3.2 The LAE cluster in Hu et al. 2021

Hu et al. (2021) discovered a Lyα protocluster at redshift 6.9. They used the Yajima et al. (2018) method to estimate the size of the ionised bubble of each individual source.

For the radius calculated by us, we have used densities and clumping factor from the AMIGA model (Salvador-Solé et al. 2017), and the recombination parameter is $\alpha_B = 2.78 \times 10^{-70}$ Mpc^3 Gyr−1.

Note that the radius as calculated by us is quite similar to the radius given by Hu et al. (2021), with some minor out-layers such as LAE-12, and LAE-15, which are discrepant. LAE-12 according to Hu et al. (2021) has a NB964 magnitude of 24.76, which is relatively bright for such a small radius. As for LAE-15 was probably discovered in the NB973 filter. Our calculation was done with the magnitudes given in the NB964 which were complete. Thus there may be some discrepancy with our calculation. Note that our radii are in most cases larger than those of Hu et al. (2021). While we are estimating the total number of ionising continuum photons, Hu et al. (2021) uses the Lyα luminosity only, which would imply a number of ionising continuum photons, which should be smaller than our number. Alternatively, the escape fraction used by Yajima et al. (2018) was 20%. If all the photons were used to ionise the bubble, perhaps our and their values would be closer.

Name	z	Q* 10^{54} s^{1}	R_{Hu} cMpc	Density 10^{69} Mpc^{-3}	R_{us} cMpc
LAE-1	6.938	5.153	14.50	2.940	15.23
LAE-2	6.932	2.076	11.50	2.933	11.26
LAE-3	6.923	4.000	13.50	2.923	14.03
LAE-4	6.900	1.508	8.50	2.898	10.16
LAE-5	6.899	1.297	8.00	2.897	9.67
LAE-6	6.915	0.824	7.30	2.915	8.29
LAE-7	6.945	0.407	6.10	2.948	6.529
LAE-8	6.931	0.621	7.20	2.932	7.53
LAE-9	6.920	0.509	6.40	2.920	7.06
LAE-10	6.922	0.365	5.80	2.922	6.32
LAE-11	6.962	0.569	6.40	2.967	7.28
LAE-12	6.931	6.992	7.00	2.932	16.88
LAE-13	6.936	0.555	6.40	2.938	7.25
LAE-14	6.931	0.688	6.90	2.932	7.79
LAE-15	6.971	0.253	12.10	2.977	5.55
LAE-16	6.915	1.195	7.30	2.915	9.39
LAE-17	6.917	1.873	7.90	2.917	10.90
LAE-18	6.953	1.229	7.30	2.957	9.43
LAE-19	6.943	0.484	6.40	2.946	6.92
LAE-20	6.931	0.631	7.00	2.932	7.57
LAE-21	6.931	0.575	6.90	2.932	7.34

Table 1. The columns are 1) Name, 2) redshift, 3) Number of continuum ionising photons, 4) radious as in Hu et al. (2021), 5) density and 6) Radius calculated by us.

Name	z	Q* 10^{54} s^{-1}	R(Jung) cMpc	C_H	n_H 10^{69} cMpc^{-3}	R(us) cMpc
GND44088	7.13	1.153	7.16	2.26	3.163	9.09
GND22233	7.34	1.042	7.68	2.24	3.415	8.56
GND18626	7.42	0.178	4.38	2.38	3.515	4.71
GND42912	7.51	1.025	8.17	2.23	3.617	8.35
GND6330	7.55	0.263	5.30	2.23	3.669	5.28
GND16863	7.60	1.636	9.20	2.23	3.737	9.66
GND34204	7.61	2.763	12.40	2.23	3.749	11.49
GND7376	7.77	0.203	4.73	2.22	3.962	4.73
GND39781	7.88	2.610	9.24	2.20	4.117	10.93
GND10402	7.94	0.229	5.27	2.20	4.199	4.82
GND6451	7.25	0.475	5.94	2.25	3.296	6.67
GND45190	7.27	0.186	4.55	2.26	3.319	4.87
GND41470	7.31	0.593	6.73	2.25	3.375	7.12
GND41247	8.04	5.559	9.67	2.19	4.336	13.82
GND7157	8.13	0.203	5.11	2.19	4.470	4.54

Table 2. Comparison between the radii computed by Jung and ours. Column 1) name, 2) redshift, 3) number of ionising continuum photons, 4) radius from Jung, 5)Clumping factor for Hydrogen, 6) density at z=7.7, and 7) our own radius.

3.3 The sources from Jung et al. 2020

Jung et al. (2020) published a set of sources from the Texas Spectroscopic Search for Lyα Emission at the end of Reionisation Survey. They published also a set of radii using the Yajima et al. (2018) model. The agreement between our exact calculation and the derivation using the Yajima model is very good, as will be explained in the following section.

It can be seen that the results computed by Jung et al. (2020) are very similar to our own radii, in spite of some discrepancies. In fact for the mean we derive 4.54, while the radii computed by Jung et al. (2020) gives a mean of 4.38. Note that the model from Yajima et al.

(2018) could give larger values were it not have used a 20% escape fraction.

4 DISCUSSION

We have developed an exact expression, that gives the radius of an ionised bubble. Note that this expression was seen in Spitzer (1978) and Osterbrock (1989), in the context of the interstellar medium and referring to ionised bubbles produced by stars. The expression we have derived from the equation in Cen & Haiman (2000) includes the clumping factor, as hydrogen at the early times could be distributed in clumps. It could also include an escape fraction and an overdensity factor, in the form of $(1+\delta)$ multiplying the density, were δ is the density contrast value.

Regarding bubbles at the epoch of reionisation, note that the radius of the bubble is proportional to the cubic root of the number of continuum ionising photons per second emitted by the starburst galaxies that produced the photons. Note that whatever photons are produced are used to either maintain the bubble, if an equilibrium has been reached, or to enlarge the bubble if the equilibrium has not been reached. So discussing about the escape fraction of ionising continuum photons is not warranted. If there are more photons, they are used to enlarge the bubble first, and only if the bubbles have merged one can talk about photons escaping from the bubbles, when these have reached a size such that the Lyα photons, if redshifted enough, are able to escape from the merged bubbles.

4.1 Why does the Yajima model work?

There is a direct relation between the Lyα luminosity and the number of ionising continuum photons per second available to ionise the medium. This relation is $L_{Ly\alpha} = 1.18 \times 10^{-11}$ Q*, where the Lyα luminosity is in erg per second. Therefore using either the Lyα luminosity or the number of ionising continuum photons is equivalent, up to a constant.

5 SUMMARY

We have derived an exact equation to compute the radius of an ionised bubble. This equation is readily usable for determining the proper radius of an ionised bubble. The expression was derived by Spitzer (1978) and Osterbrock (1989) in a context of stellar HII regions. The equation can be adapted to work for bubbles ionised by starbursts and primordial galaxies. Besides the expression can be used for individual galaxies or groups of galaxies, protoclusters, etc. And it can be adapted to account for its use with over-densities and clumping factors.

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DATA AVAILABILITY

The data used in this paper are freely available from the papers referred to in the text.

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APPENDIX A: SOME EXTRA MATERIAL

If you want to present additional material which would interrupt the flow of the main paper, it can be placed in an Appendix which appears after the list of references.