Foundations of Computer Science (COMP109)

Tutorial III, Week 02.11.2020 – 06.11.2020

A reasonable attempt at answering Question (III.4.) should be submitted on Canvas by ***23:59 on Tuesday 03.11.2020*** *either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten answer. This assignment makes up 1% of your final mark. We would like to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board, but do not copy your answer from anybody else.*

III.1. Use proof by contradiction to show that if p is rational and r is irrational then $p+r$ is irrational.
Hint: Use the fact that the difference of two rational numbers is rational.

III.2. Prove or disprove that for any irrational numbers r and s their product rs is also irrational.

III.3. Use induction to prove that, for every integer $n \geq 2$, the sum of any n even positive integers is even. Hint: Use the fact that if positive integers x and y are even, then so is $x+y$.

III.4. In the lectures we have seen that the sum of the first n positive odd integers is n^2. Establish the formula for the sum of the first n positive **even** integers and use proof by mathematical induction to prove its correctness.

III.5. Find an error in the "proof" by induction of the following statement: All horses are the same colour.
Base case: One horse is obviously of the same colour.
Induction step: Assume that it holds for $n = m$ that n horses are the same colour. Consider the case of $n = m+1$. Let us consider two ways of grouping horses. First, consider the first m horses (that is, exclude the $(m+1)$-th horse from the consideration). By induction hypothesis, they all are the same colour.

Now consider all but the first horse. There are m horses in that group, so by induction hypothesis they all are the same colour.

A horse in the middle belongs to both groups, so it has the same colour as all the horses in the first group and all the horses in the second group. Thus, all $m+1$ horses are the same colour.