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# Planning of electric vehicle charging station based on queuing theory

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**Abstract:** EV charging station is a very important link in urban planning, and the design of the position and capacity of charging stations are not only to meet the public demand for traveling, but also to maintain the stability of operation and management in power grid. Considering the non-simultaneity and randomness of the EVs arriving at the charging station, the capacity of EV charging stations is calculated by using queuing theory in this study. This study proposes a model which has considered the EVs' charging requirement, economy, and power grid safety to solve the problem of planning EV charging station. Firstly, determine the location of charging station by traffic satisfaction. Secondly, the EV charging stations' charging capacity is optimized with the minimum cost as the optimization target, with the EV user' waiting time patience limit and the distribution network running safety as the constraint condition. Finally, the service area of charging station is divided by the Voronoi diagram. In this study, a 25_nodes traffic network with 24_nodes distribution network underground is modelled in the MATLAB environment to demonstrate the effectiveness of the proposed method. In addition the authors solve the model by the genetic algorithm in MATLAB.

## 1 Introduction

With the development of society, environment pollution problem has become more serious. While electric vehicles (EVs) have the characteristics of low pollution and sustainability, so many countries have considered EVs as one of national strategic plans [1–4]. EV charging station is a very important connection between EVs and the corresponding power system. Thus, this paper presents a model based on queuing theory to solve the problem of how to plan the charging station economically.

A large volume of studies have been researched on the issue of EVs charging station planning. For example, in [5], the Voronoi diagram was used to locate charging stations from the view of the load density distribution, and proposed an optimisation models to resolve the planning problem. Lam *et al.* [6] summarised a basic charging station placement model, and four potential solutions were reviewed. In [7], a mixed-integer non-linear optimisation approach for optimal placing and sizing charging stations was proposed. Above all, different types of charging infrastructures were considered in a social cost-based planning model [8]. Wang *et al.* [9] and Liu *et al.* [10] proposed a model to solve the problem of charging station planning based on traffic flow.

These literatures proposed different planning models for charging stations. However, some literatures take the randomness of EVs into account, and the law that describe how EVs reach the charging station is attributed to the M/M/S model, but there is no sufficient consideration for the maximum patience of EVs. In addition, some literatures only considered the problem of charging station location by maximum coverage location model or P-median model. Meanwhile how to get the initial sites selection is not described. Thus, this paper proposes a model to plan EV charging stations, which provide remedy for the above deficiencies. First, based on the parking demand model, this paper will get the temporal and spatial distribution of EVs. Secondly, the fuzzy clustering is used to cluster the space-time distribution of EVs, and the load centre is obtained. The initial station set of charging station is determined near the centre of the load, and the optimal site can be obtained by the traffic satisfaction model according to traffic rate. Thirdly, according to the EV users' waiting time patience limit and the safety of distribution network, the EV charging stations' charging capacity is optimised with the minimum cost as the optimisation target. As this is a constrained non-linear integer optimisation problem, this paper chooses the genetic algorithm to solve this problem. Select the appropriate population size, algebra, reorganisation probability, mutation probability, so as to improve the efficiency of the solution. Finally, use the Voronoi diagram to divide the service area of charging station. The specific model is shown in Fig. 1, which will be described in detail later.

## 2 Location planning of EV charging station

Planning EV charging stations need to consider many factors, including the requirement of satisfying the charging demand of EVs, high traffic satisfaction, as well as the safety and stability of distribution network. The space-time distribution of EVs has certain regional characteristics. The fuzzy clustering method is used to cluster the fuzzy centres and classification areas of the space-time distribution of EVs. The initial site set is constructed according to the load centre. The optimal location of the EV charging station is obtained accurately by the traffic satisfaction model.

The charging station needs to cover the traffic flow as much as possible in traffic network, and it is a typical location allocation (LA) problem. At the same time, the most common method to solve the LA problem is the P-median model and the maximum coverage location model [11]. Among them, the P-median model is designed to keep the travel distance of all vehicles as short as possible; meanwhile the maximum coverage location model ensures that the charging station services EVs as much as possible, which are also charging station location purpose. Therefore, this paper will combine the *P*-median model and the maximum coverage

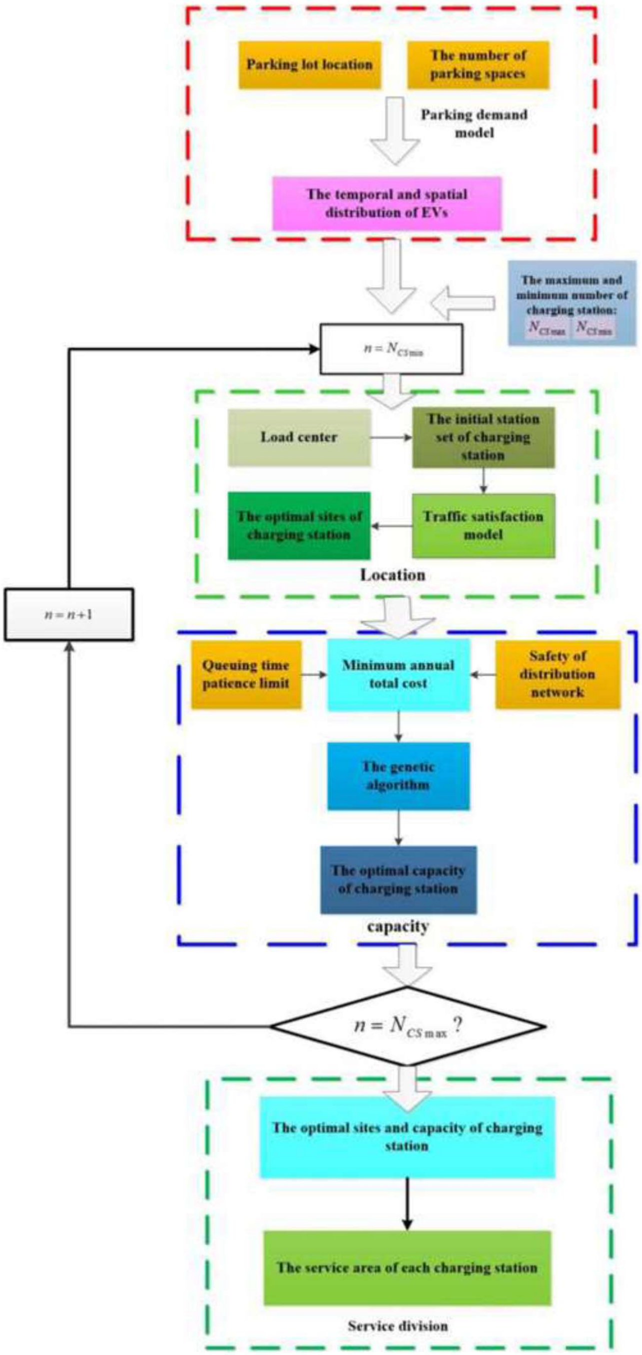


**Fig. 1** *Model of planning EV charging station*

location model to establish a traffic satisfaction model, defined as traffic satisfaction $z$

$$z = \omega \sum_{i=1}^{n} \sum_{j=1}^{n} P_i d_{ij} X_{ij} - (1-\omega) \sum_{q=1}^{n_q} f_q Y_q \tag{1}$$

$$f_q = \frac{P_{\rm O} P_{\rm D}}{1.5 d_q} \tag{2}$$

where $\omega$ is the compromise weight, take the decimal from the (0,1) interval, which combines the *P*-median model and the maximum coverage location model effectively. The first half part of formula (1) represents network demand weight distance, $P_i$ is the traffic demand of the start point and the demand is abstracted into a weight representation, which means the number of vehicles on the node. $d_{ij}$is the shortest distance between nodes $i$ and $j$, which is calculated by Dijkstra algorithm in this paper. $X_{ij}$ indicates whether there is a facility between node $ij$, which exists at 1 otherwise at 0. The second part represents the network demand satisfaction, where $f_q$is the traffic flow for the O–D path $q$ in the network. If the network satisfies $f_q$, $Y_q$ is 1, otherwise $Y_q$is 0. This paper assumes if there is a charging station on the O–D path, the network satisfies the $f_q$. $P_{\rm O}$ and $P_{\rm D}$ are the starting and terminal points' requirements. From the P-median model and the maximum coverage location model, we can see that it is necessary to keep the network demand weight as much as possible and keep the network demand satisfaction as small as possible in the traffic network. Therefore, with the minimum satisfaction as the optimisation target, this paper gets the optimal site of the charging station.

## 3 Capacity planning of EV charging station

The capacity planning of EV charging stations should consider the construction cost, the operating cost of the charging station, power loss, and the needs of EVs. This paper will take full account of the interests of the Power Grid Company and users. The EV charging station's charging capacity is optimised with the minimum cost as the optimisation target, according to the EV user' waiting time patience limit and the distribution network running safety as the constraint condition.

### *3.1 Constant capacity optimisation objective*

In this paper, the optimal capacity of EV charging station is obtained by optimising the annual total cost. The total annual cost includes annual fixed investment, annual operating costs and the annual cost for power loss. The annual fixed investment is mainly the investment cost of charging motor, transformer, land cost and other auxiliary equipment. The annual operating cost mainly refers to EV charging station maintenance personnel salaries and equipment maintenance cost [11]. Through analysis, we can know that charging motor is the main factor to determine the fixed investment, and the number of charging motors reflects the size and capacity of the charging station. The more the charging motor, the more the EVs charging station can serve; the stronger the service capacity, the larger the area, the greater the investment cost of the distribution transformer and other auxiliary equipment. If the scale is larger, the annual operating costs of the charging station will be greater. To sum up, we can conclude that fixed annual investment costs and annual operating costs are a function of the number of charged motors ($N_{\rm cm}$). The annual total cost can be expressed as follows:

$$\min C_{\rm cost} \sum_{i \in J_{\rm cs}}^{i=1} \left[ f_{\rm cs}(N_{{\rm cm}i}) \frac{r_0 (1+r_0)^{ms}}{(1+r_0)^{ms}-1} + u_{\rm cs}(N_{{\rm cm}i}) \right] + g_{\rm loss} \tag{3}$$

$$f_{\rm cs}(N_{{\rm cm}i}) = w + qN_{{\rm cm}i} + eN_{{\rm cm}i}^2 \tag{4}$$

$$u_{\rm cs}(N_{{\rm cm}i}) = \alpha f_{\rm cs}(N_{{\rm cm}i}) \tag{5}$$

$$g_{\rm loss} = 8760 c P_{\rm Loss}(f_i P_{si}, f_i Q_{si}) \tag{6}$$

where $f_{\rm cs}(N_{{\rm cm}i})$ indicates the annual fixed investment cost of the charging station $i$, $u_{\rm cs}(N_{{\rm cm}i})$ indicates the annual operating cost of the charging station $i$, $g_{\rm loss}$ is the annual cost for power loss. $N_{{\rm cm}i}$ is the number of charging motors for charging station $i$, $r_0$ is the discount rate, $J_{\rm cs}$ is the collection of charging station, $w$ is the fixed investment which mainly includes auxiliary construction costs such as business buildings and roads, $q$ is the equivalent investment coefficient for the unit price of the charging motor, $e$ is the equivalent investment coefficient related to the number of charging motors which includes the area, distribution transformer capacity, cable and other investment cost. Where $f_i$represent whether node $i$ have charging station, $P_{si}$ and $Q_{si}$ are the active and reactive power of node $i$, $c$ representing the power price, respectively.

### 3.2 Constraints

In this paper, the capacity optimisation process of EV charging station is restricted from two aspects, such as EV charging user psychology and grid safety.

*3.2.1 EV queuing time:* If the number of vehicles in the area is constant, the waiting time in the charging station decreases as the number of charging motors in the charging station increases. Through the psychological research of EV users, most users have tolerant waiting time for queuing, so the number of charging motors should be matched under the condition of the waiting time of queuing [12, 13].

EVs arrived at charging stations to charge were independent of each other, meet the character of stationarity and no aftereffect, so EVs can be charging in charging station according to M/M/s model of queue theory. This paper uses parameters $\lambda$ as the passion flow to describe the rules of EVs to the charging station, the service time of EVs in the charging station is subject to the negative exponential distribution of the parameter $\mu$

$$\lambda_i = \frac{n_{\text{ev}i}p}{t_{\text{c}}} \tag{7}$$

$$\rho_i = \frac{\lambda_i}{\mu} \tag{8}$$

$$\mu = \frac{1}{t_{\text{s}}} \tag{9}$$

$$P_{0i} = \left[ \sum_{k=0}^{N_{\text{cm}i}-1} \frac{\rho_i^k}{k!} + \frac{N_{\text{cm}i}\rho_i^{N_{\text{cm}i}}}{N_{\text{cm}i}!\left(N_{\text{cm}i} - \rho_i\right)} \right]^{-1} \tag{10}$$

$$W_{\text{q}i} = \frac{N_{\text{cm}i}\rho_i^{N_{\text{cm}i}+1}P_{0i}}{\lambda_i N_{\text{cm}i}!\left(N_{\text{cm}i} - \rho_i\right)^2} \tag{11}$$

where $n_{\text{ev}i}$ is the number of EVs in the initial service area of the charging station $i$, $p$ is the charging probability of EV to the charging station, $t_s$is the charging time. Where $\rho_i$ is charging motor service strength, $P_{0i}$ is the total idle probability of the charging station $i$, $W_{\text{q}i}$ is queuing waiting time expectation.

Get the minimum number of charge motors by queuing patient time, and gets the maximum number of charging motors by charging station planning requirements, the charging motors number constraint is shown as follows:

$$N_{i,\min} \le N_i \le N_{i,\max} \tag{12}$$

*3.2.2 Distribution network:* As the power of the fast charging station is large, it will affect the operation safety of the distribution network. Therefore, it is necessary to restrict the capacity of EV charging station from the viewpoint of power grid operation safety.

(i) System tidal constraint

$$\begin{cases} P_{is} = V_i \sum_{j \in i} V_j \left( G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij} \right) \\ Q_{is} = V_i \sum_{j \in i} V_j \left( G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij} \right) \end{cases} \tag{13}$$

where $P_{is}$ and $Q_{is}$ are the active and reactive power injected by node $i$; $V_i$ and $V_j$ are the voltage amplitudes of node $i$ $j$, respectively; $G_{ij}$ and $B_{ij}$ are the real and imaginary parts of the admittance between nodes $i$ and $j$, respectively; $\theta_{ij}$ is the phase angle difference between nodes $i$ and $j$.

(ii) Node voltage constraint

$$V_i^{\min} \le V_i \le V_i^{\max}, \quad i = 1, 2, \ldots, N \tag{14}$$

where $V_i$ represents the voltage amplitude of the node $i$,;$V_i^{\min}$ and $V_i^{\max}$ represent the lowest and highest voltages amplitudes allowed at note $i$ during safe operation of power grid, respectively; $N$ is the number of nodes in the distribution network.

(iii) Feeder current limit

$$\left| I_{ij} \right| \le I_{ij\max} \quad i, j = 1, 2, \ldots, N \tag{15}$$

where $I_{ij}$and $I_{ij\max}$ represent the current between the feeders $i$ and $j$ and the maximum current allowed.

## 4 Charging station services division

In this paper, the division of the service area of the charging station is mainly realised by using the weighted Voronio diagram [5]

$$V(R_i, \tau_i) = \left\{ x \in V(R_i, \tau_i) \left| \frac{d(x, R_i)}{\tau_i} \le \right| \frac{d(x, R_j)}{\tau_j}, \right. \\ \left. j = 1, 2, \ldots, n; \; j \ne i \right\} \tag{16}$$

where $R = \{R_1, R_2, \ldots, R_n\}$ is the centre point set, $x$ is any point in the plane, $d(x, R_i)$ represents the Euclidean distance between point $x$ and $R_i$, $\tau_i$ is the weight *of* $R_i$.

## 5 Case study

This paper analyses the traffic network in a certain area, preserves the main and sub_main roads with large traffic flow, abstracts the traffic network with 25 nodes and 24_nodes distribution network below as shown in Fig. 2. Use the method proposed in this paper to plan EV charging station site and capacity, and to determine the service area of these charging stations.

By investigating the distribution of 14 parking lots in this area, the number of their parking spaces is shown in Table 1. Based on the parking distribution model, the spatial distribution of the charging needs in this area is shown in Fig. 3.

In this paper, traffic demand for traffic nodes is shown in Table 2 [9].

In this paper, in the formulas (3)–(6), $w$ is 100 million yuan, $q$ is 10 million for each motor, $e$ is 3 million for the square of each motor, $ms$ is 20, $r_0$ is 0.08. The EVs' daily charging rate $p$ is 0.05. The charging period occurred at work, noon, off duty

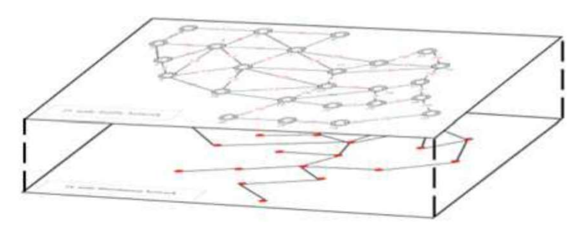

**Fig. 2** *Traffic network and distribution network*

Table 1 Calculation of parking demand in each region

| Area number | Berth quantity demand |
|---|---|
| 1 | 1303 |
| 2 | 1100 |
| 3 | 1087 |
| 4 | 1561 |
| 5 | 1371 |
| 6 | 1017 |
| 7 | 202 |
| 8 | 667 |
| 9 | 607 |
| 10 | 1957 |
| 11 | 383 |
| 12 | 624 |
| 13 | 172 |
| 14 | 548 |

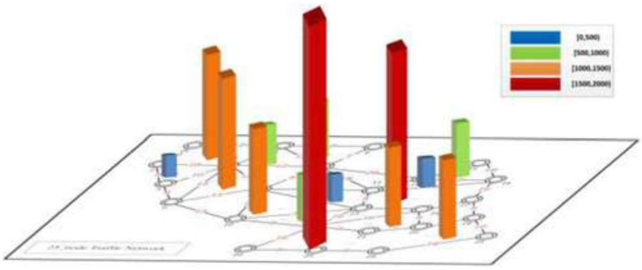

**Fig. 3** *Spatial distribution of the charging need*

**Table 2** Traffic demand for traffic nodes

| Nodes | Demand | Nodes | Demand | Nodes | Demand |
|---|---|---|---|---|---|
| 1 | 0.54 | 10 | 0.54 | 19 | 0.80 |
| 2 | 0.80 | 11 | 0.05 | 20 | 0.27 |
| 3 | 0.27 | 12 | 0.54 | 21 | 0.27 |
| 4 | 0.27 | 13 | 0.05 | 22 | 0.54 |
| 5 | 0.27 | 14 | 0.54 | 23 | 0.05 |
| 6 | 0.07 | 15 | 0.27 | 24 | 1.34 |
| 7 | 0.05 | 16 | 0.27 | 25 | 0.05 |
| 8 | 0.54 | 17 | 0.27 | | |
| 9 | 0.27 | 18 | 1.07 | | |

period, time interval $t_c$ is 6 h. The maximum queuing time expectation $W_{q\max}$ is 15 min, in the formulas (7)–(11).

In order to satisfy the maximum queuing time constrain, it is difficult to find the number of charging motors $N_{\rm cm}$ by the inverse function of formula (11). As described in the queuing M/M/S model, the condition for the formula (7)–(11) is $\rho < N_{\rm cm}$. Therefore, this paper uses the smallest integer from $\rho$ to start the cycle, until $W_{\rm q} < W_{\rm qmax}$ so as to determine the $N_{\rm cm}$ to meet the condition. By programming in MATLAB, it can be found that $W_{\rm q}$ decreases rapidly as $N_{\rm cm}$ increases, so a small number of loop operations can meet the requirements.

The number of charging station in this area is between 3 and 7 by using the method proposed in [11]. According to the model proposed in this paper, the genetic algorithm is used to solve the optimisation function. Where the population size is 50, the algebra is 40, the recombination probability is 0.8, and the mutation probability is 0.9. The calculation of the different number of charging stations is shown in Fig. 4.

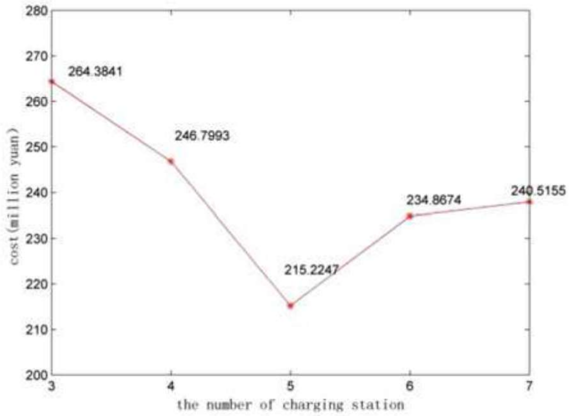


**Fig. 4** *Calculation of the different number of charging station*

**Table 3** Optimal planning result of the charging station

| Number | Charger configuration | Number of EV | Traffic nodes | Distribution network bus |
|---|---|---|---|---|
| 1 | 7 | 2091 | 4 | 3 |
| 2 | 7 | 1970 | 5 | 5 |
| 3 | 13 | 3668 | 13 | 12 |
| 4 | 13 | 809 | 12 | 19 |
| 5 | 14 | 4061 | 21 | 20 |

**Table 4** Service area and queuing waiting time expectation

| Number | The queuing waiting time expectation, min | The service area (traffic nodes) |
|---|---|---|
| 1 | 9.24 | 1,5,6,7 |
| 2 | 5.87 | 2,3,8,9 |
| 3 | 5.42 | 10,14,19 |
| 4 | 0 | 11,16,15 |
| 5 | 2.56 | 14,18,20,22,23,24,25 |

According to Fig. 4, when the number of charging station is five, the annual total cost is the least (215.2247 million yuan). The capacity and location of the charging station corresponding to the optimal solution are shown in Table 3.

The service area of EV charging station and the average queuing time are shown in Table 4.

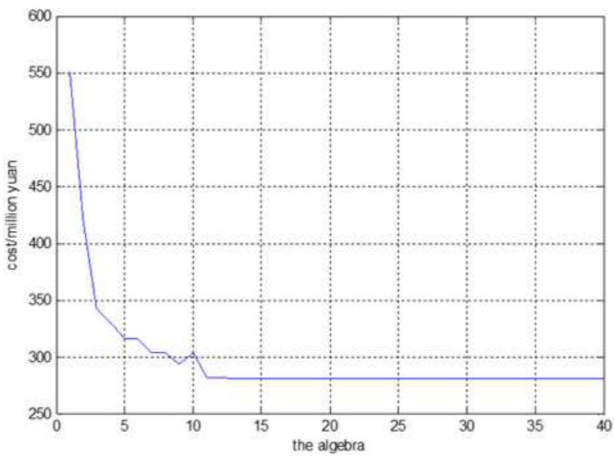


**Fig. 5** *Process of the optimal solution by genetic algorithm*

Through the genetic algorithm to solve the objective function, the process of the optimal solution is shown in Fig. 5. The figure shows that convergence is rapid and the solution is good.

## 6 Conclusion

Considering the EVs' charging requirement, economy and power grid safety, this paper proposed a model to plan the EV charging station. Firstly, determine the initial site based on the temporal and spatial distribution of EVs, and determine the maximum traffic satisfaction for the final site. Secondly, the EV charging station's charging capacity is optimised with the minimum cost as the optimisation target, with the EV user' waiting time patience limit and the distribution network running safety as the constraint condition. Finally, the division of the service area of the charging station is realised by using the weighted Voronio diagram.

As this paper on the scope of EV charging station service division only considering the weighted European distance, in the future study will divide the EV charging station service by the perspective of the actual path of EVs.


## 7 Acknowledgments

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