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# DYNAMIC EDGEWORTH-BERTRAND COMPETITION*

MARC DUDEY

## I. INTRODUCTION

A popular criticism of the duopoly models developed by Bertrand [1883] and Edgeworth [1897] is that their predictions are inconsistent with observed duopoly behavior. In Bertrand's model, profit-maximizing sellers with the same constant cost technology choose prices; in the only equilibrium the sellers set prices equal to marginal cost and earn zero profit. Edgeworth showed that, when capacity constraints are introduced into Bertrand's model, a pure strategy equilibrium will not generally exist.[1] Unfortunately, casual observation suggests that duopolists earn positive profits and that duopoly pricing behavior is relatively stable.[2]

This paper develops a simple, multiperiod variant of the Edgeworth model that is not susceptible to this criticism. In the model, price-setting and capacity-constrained duopolists meet consumers with unit demands and a common reservation value; by contrast with the static paradigm, the consumers come to market at different times, and the duopolists may change their prices at any time. The model has a (subgame perfect) equilibrium in pure strategies and unique equilibrium payoffs for any specification of the duopolists' capacities. Moreover, so long as at least one seller cannot supply the entire market, both sellers earn positive profit, and at least one of the duopolists can sell all the units it is able to produce.

The results are very much in the spirit of Nichol's [1935]

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1. Beckman [1965], Levitan and Shubik [1972], and Dasgupta and Maskin [1986] proved that Edgeworth's model does have Nash equilibria in mixed strategies.

2. The term "paradox" is applied to Bertrand's results by Tirole [1988] and to Edgeworth's results by Rasmusen [1989]. Friedman [1989] writes that "[w]hile the logic of Bertrand and Edgeworth is correct, the economic relevance is dubious." For related comments see Hotelling [1929], Nichol [1935], and Shapiro [1989].



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review of Edgeworth's essay. Nichol observed that, "The general results [of Edgeworth's model] do not conform with reality. Duopolists are not observed to engage in perpetual price wars." He also proposed a dynamic extension of Edgeworth's theory as a direction for further research. In his words, "According to Edgeworth, competitors consider profits only of the immediate present, and not over a period of time. The latter goal is much more sensible." The simple model examined here illustrates the virtue of Nichol's proposal.

The model may be viewed as a complement to Hotelling's [1929] well-known paper on "Stability in Competition." Hotelling used a simple game to show that static price competition between two sellers of differentiated products can yield stable prices and positive industry profits. His finding can be empirically distinguished from mine since, in Hotelling's model, positive profits are not linked with one seller's ability to sell all the units it is able to produce.[3]

The paper unfolds in the following way. The basic model is described in Section II. Section III presents nonrandom, equilibrium strategies and unique equilibrium payoffs for each pair of capacities. (The main results are stated above. An additional finding is that, with *arbitrary* capacities, equilibria may involve zero profits or below marginal cost pricing. However, such outcomes are shown to be unlikely if even one seller has some limited control over its capacity.) Section IV gives proofs of all propositions, and Section V concludes.

## II. The Basic Model

Consider the following market. Each of two capacity-constrained firms, call them Alpha and Beta, offers to sell units of an indivisible good. There are $n$ consumers, who view the units sold by Alpha and Beta as perfect substitutes and purchase at most one unit of the good each. The consumers have a common reservation value of $v$ dollars for a unit of the good, and each of them has at least $v$ dollars to spend. Each firm incurs a constant cost of $c$ dollars whenever it makes a sale, where $v > c \geq 0$.

3. In an apparent reference to Hotelling's paper, Nichol also lamented that "[e]conomists unable to accept Edgeworth's conclusions have struck out in quite different directions, and have not been encouraged to use the results of his own pioneering." Nichol's lament does not apply to my approach since capacity constraints are essential for my results.

When a consumer comes to market, she can elicit simultaneous price quotes from the firms. The consumers come to market at different times, and each consumer has only enough time to elicit one pair of price offers from the firms. The consumers can therefore be indexed so that consumer $t$ is the next to elicit offers when there are $t$ consumers who have not yet elicited offers.

I shall assume that, if a consumer obtains different price offers from the firms, he patronizes the firm charging the lower price, so long as the lower price is not greater than $v$ dollars. If he sees a pair of identical prices that are not greater than $v$ dollars, then each firm has an equal chance of attracting him. And if he sees that both firms are charging prices that are greater than $v$ dollars, he impatiently rejects both offers and does not return.

The firms seek to maximize undiscounted profits. Each firm may reset prices every time a consumer arrives, and it is assumed that, whenever a firm is sold out, it sets a price equal to $v + 1$ dollars. Each duopolist's ability to produce additional units declines by one unit every time it makes a sale.[4]

Define period $t$ to be the time period in which consumer $t$ elicits price offers, and let $p^A(t)$ and $p^B(t)$ denote dollar prices set by Alpha and Beta, respectively, in period $t$. In addition, define a firm's inventory in period $t$ to be the difference between its capacity and the number of units sold in any periods before $t$. Let $a(t)$ and $b(t)$ represent the inventories of Alpha and Beta, respectively, in period $t$. The capacities of Alpha and Beta are therefore $a(n)$ and $b(n)$, respectively. (The interesting cases are those in which at least one of the duopolists has less than $n$ units to sell, but the sum of their capacities exceeds $n$ units. Recall that, in the context of Edgeworth's static model, such a restriction on capacities would imply the nonexistence of a Nash equilibrium in pure strategies.)

When setting $p^A(t)$ and $p^B(t)$, the firms know $t$, $n$, $v$, $c$, all prices previously set by the firms and the evolution of both firms' inventories through period $t$. Notice that, if a firm knows both firms' capacities, the prices set by both firms in previous periods, and the way its own inventory has evolved, it can infer the way its rival's inventory has evolved. Thus, the theory does not depend on a firm being able to see the evolution of its rival's inventory.

The model just described may be viewed as a dynamic game

4. By contrast, Brock and Scheinkman [1985] study an infinitely repeated Edgeworth-Bertrand game in which firms face the same capacity constraint in every period.

played by the firms. A strategy for a firm in the game gives the firm's price in any period $t$ as a function of the prices set before period $t$ and the inventories held by the firm before and in period $t$. When the firms play a pair of strategies, the decision rule used by the consumers may be used to compute the expected total profit of each firm. The payoff function of each firm maps the duopolists' strategy choices into the firm's total expected profit. Subgame perfect Nash equilibrium (see Selten [1975]) is the solution concept that will be used to analyze this game.

A final remark is in order. The market just described is chosen for its simplicity, rather than its realism. It will be used to show why price instability or zero profits is not an inevitable consequence of price competition between capacity-constrained firms, even if the firms sell identical products and carry excess capacity. Nonetheless, for concreteness, the reader might wish to think about the game in terms of consultants or consulting firms that offer their services to different employers, or retailers selling a product that is temporarily in short supply (picture electronics discounters selling radios or cameras before Christmas).

## III. Results

This section presents a discussion of equilibrium strategies for arbitrary values of $a(n)$ and $b(n)$. To begin with, subsection A offers an informal analysis of the special case in which exactly one seller is unable to supply the entire market. (It is in this special case that the main results are most intuitive.) Subsections B and C present general results on equilibrium strategies and payoffs. Subsection D examines a simple extension of the model that includes limited capacity choice.

### A. An Informal, First Look at Equilibrium

Suppose that one duopolist, Alpha, has less than $n$ units to sell and that the other duopolist, Beta, has at least $n$ units to sell. In this case, it is particularly easy to identify a pair of equilibrium strategies. For example, let Alpha's strategy specify a price of $v$ dollars if $0 < a(t) < t \leq b(t)$, a price of $c$ dollars if $\min(a(t),b(t)) \geq t$, and a price of $v + 1$ dollars if $a(t) = 0$, in any period $t$. Let Beta's strategy specify a price of $v + 1$ dollars if $0 < a(t) < t \leq b(t)$, a price of $c$ dollars if $\min(a(t),b(t)) \geq t$, and a price of $v$ dollars if $a(t) = 0$ in any period $t$. (If $b(n) \geq n$, then obviously $b(t) \geq t$ in every period $t$.

For this reason, strategies do not need to be defined for states in which $b(t) < t$.)[5]

To see that such strategies form an equilibrium, observe that Alpha's strategy threatens Beta with repeated Bertrand competition unless Alpha is allowed to sell out at the monopoly price. The threat is credible (!) since equilibrium requires that both duopolists engage in finitely repeated Bertrand competition once $t = a(t)$. Given that Alpha uses this strategy, Beta may as well let its rival sell out at the monopoly price, and this is what Beta's strategy prescribes. Observe that when the firms use these strategies, they earn positive profits. More precisely, Alpha earns $(v - c)a(n)$ dollars, and Beta earns $(v - c)(n - a(n))$ dollars. It can be shown that this is the only equilibrium outcome (see Proposition 2).

Notice that the joint profits of the firms can increase when consumers elicit offers sequentially instead of simultaneously. Under the assumption that $0 < a(n) < n \leq b(n)$, sequential consumer arrival leads to a joint profit-maximizing equilibrium. However, if the consumers were to arrive simultaneously instead of sequentially, Edgeworth's static model would predict a mixed strategy Nash equilibrium. In this equilibrium the duopolists do not maximize joint profits. This is because prices associated with the mixed strategy Nash equilibrium are less than $v$ dollars with positive probability; therefore, joint duopoly profits are less than $(v - c)n$ dollars with positive probability.

Also notice that, if $n/2 < a(n) < n \leq b(n)$, Alpha will earn more than Beta, even though Alpha has less units to sell. For example, suppose that Alpha and Beta have capacities of 99 and 100 units, respectively, and that there are 100 consumers. In this case, Alpha will earn $99(v - c)$ dollars, while Beta will earn only $v - c$ dollars. This example also shows that, when no form of capacity choice is possible, small differences in capacities can lead to highly asymmetric equilibrium payoffs (the duopolist with one less unit to sell earns 99 percent of joint duopoly profits!).

### *B. Existence of Equilibrium*

If both duopolists have less than $n$ units to sell and the sum of their capacities exceeds $n$, it is not hard to see that the simple strategies described in subsection III.A do not represent an equilibrium. For instance, suppose that $a(n) \leq b(n) < n$, and let $t^* = n -$

5. A price of $v + 1$ dollars is only meant to reflect a seller's lack of interest in making the sale and the resulting desire of the consumer to look elsewhere.

$[b(n) - a(n)]$. Then there are subgames starting in period $t^*$ in which $0 < a(t^*) = b(t^*) < t^*$. It was not necessary to define the simple strategies for such subgames.

However, a backward induction argument can be used to derive equilibrium strategies for the general case.

PROPOSITION 1. For any positive selections of $a(n)$ and $b(n)$, the model has an equilibrium with the following properties. If $\min(a(t),b(t)) \geq t$, both firms set their period $t$ prices equal to $c$ dollars. If $\min(a(t),b(t)) > 0$ and $a(t) + b(t) \leq t$, both firms set their period $t$ prices equal to $v$ dollars. If $0 < a(t) < \min(b(t),t)$ and $a(t) + b(t) > t$ or if $0 = b(t) < a(t)$, Alpha sets its period $t$ price equal to $v$ dollars, and Beta sets its period $t$ price equal to $v + 1$ dollars. If $0 < b(t) < \min(a(t),t)$ and $a(t) + b(t) > t$ or if $0 = a(t) < b(t)$, Alpha sets its period $t$ price equal to $v + 1$ dollars, and Beta sets its period $t$ price equal to $v$ dollars. If $t/2 < a(t) = b(t) < t$, both firms set their period $t$ prices equal to $c + (v - c)(t - 2a(t) + 1)$ dollars.

Notice that, in the absence of any form of capacity choice, equilibrium strategies can involve some peculiar behavior. In particular, if both firms have at least $n$ units to sell, the game reduces to repeated Bertrand competition. In this case, prices are set equal to $c$ dollars, and profits are driven to zero dollars. Furthermore, below marginal cost prices appear in the first period if $(n + 1)/2 < a(n) = b(n) < n$.

The following example shows why below marginal cost pricing can occur. Suppose that both duopolists have 100 units to sell and that there are 101 consumers. Of course, if consumer 101 rejects both offers, the resulting subgame amounts to repeated Bertrand competition and yields equilibrium profits of zero dollars to both sellers. On the other hand, if consumer 101 purchases a unit from one of the firms, the firms will have 99 and 100 unsold units, respectively, in period 100. Recall from subsection III.A that the equilibrium of the resulting subgame yields $99(v - c)$ dollars to the firm making the period 101 sale and $v - c$ dollars to the other firm. Now suppose that both firms set their period 101 price equal to $c - 98(v - c)$ dollars and engage in equilibrium pricing during the last 100 periods. Both firms will then earn $v - c$ dollars. If one of the firms were to raise its period 101 price, it would earn $v - c$ dollars as the firm with 100 unsold units in period 100. If a firm were to lower its price, it would earn less than $v - c$ dollars as the firm with

99 unsold units in period 100. Thus, the 101 period game has an equilibrium in which both firms set a period 101 price of $c - 98(v - c)$ dollars. This price is less than marginal cost and may be less than zero.

Clearly, the ability of the firms to set *negative* prices depends on my assumption that a consumer will buy no more than one unit. If consumers could purchase unlimited quantities at negative prices, equilibrium prices would be nonnegative, but below marginal cost (zero) prices could still appear along the equilibrium path.[6]

## C. Equilibrium Payoffs

The next proposition shows that there are unique equilibrium payoffs associated with any positive integer selections of $a(n)$ and $b(n)$. These payoffs may be computed using the equilibrium pricing strategies described in Proposition 1.

PROPOSITION 2. For any positive selections of $a(n)$ and $b(n)$, all equilibria of the basic model yield the same payoffs to each duopolist. If $\min(a(n),b(n)) \geq n$, both firms earn zero profits in any equilibrium. If $a(n) + b(n) \leq n$, Alpha earns $(v - c)a(n)$ dollars, and Beta earns $(v - c)b(n)$ dollars in any equilibrium. If $a(n) < \min(n,b(n))$ and $a(n) + b(n) > n$, the only equilibrium outcome involves Alpha earning $(v - c)a(n)$ dollars and Beta earning $(v - c)(n - a(n))$ dollars. If $b(n) < \min(n,a(n))$ and $a(n) + b(n) > n$, then Alpha earns $(v - c)(n - b(n))$ dollars, and Beta earns $(v - c)b(n)$ dollars in any equilibrium. If $a(n) = b(n) < n$ and $a(n) + b(n) > n$, both firms earn $(v - c)(n - a(n))$ dollars in any equilibrium.

Notice that general results on equilibrium pricing are implied by Propositions 1 and 2. If $a(n) \neq b(n)$ and $\min(a(n),b(n)) < n$, or if $a(n) = b(n) \leq n/2$, then all consumers making a purchase do so at the monopoly price in any equilibrium. However, if $\min(a(n),b(n)) \geq n$, then the duopolists engage in marginal cost pricing along any

6. The results do not change if production occurs before consumers arrive (or $c = 0$) and if the following "nonnegative price constraint" is imposed. Suppose that the consumer will demand a large quantity from a duopolist setting a negative price, thereby sending the duopolist into bankruptcy. Also, assume that, if a seller sets a price of zero, the consumer will purchase one unit from the seller, even if the consumer is also making purchases at nonpositive prices from the seller's rival. It can be shown that, under these assumptions, equilibrium payoffs to the sellers are the same as in Proposition 2. The proof is similar to the proof of Proposition 2.

equilibrium path. And if $n/2 < a(n) = b(n) < n$, then, in any equilibrium, the first consumer is offered a price that is at or below marginal cost, while remaining consumers are offered the monopoly price.

### D. On Pricing at or Below Marginal Cost

The above discussion indicates that zero profit equilibria and below marginal cost pricing are, at least, theoretical possibilities. Zero profit equilibria will emerge if each firm has enough capacity to supply the entire market, and below marginal cost prices may occur if total industry capacity exceeds demand and firms have identical capacities. The message of this subsection is that these possibilities are unlikely when even one firm can exercise a degree of control over its capacity.

To begin with, consider an industry that initially consists of an incumbent monopolist with enough capacity to supply the entire market. The market consists of a number of identical consumers with unit demand functions who arrive sequentially.

Imagine that, when $n$ consumers have not yet arrived, an entrant unexpectedly appears. The entrant is capacity constrained, and competition for the $n$ remaining consumers takes the form of the model described in Section II. As before, the consumers' common reserve price equals $v$ dollars, and all production takes place at a cost of $c$ dollars per unit, where $0 \leq c < v$.

Of course, if the entrant has at least $n$ units of capacity, a zero profit equilibrium will result. However, suppose instead that the entrant can choose its capacity. If the entrant's cost of installing a unit of capacity is $k$ dollars, where $0 < k < v - c$, its optimal choice of capacity would equal $n - 1$ units (an application of Proposition 2). The final $n$ consumers would therefore yield earnings to the entrant and incumbent of $(v - c)(n - 1)$ dollars and $(v - c)$ dollars, respectively. Thus, all consumers pay the monopoly price, and the sellers avoid a zero profit equilibrium.

This story does not depend on the incumbent having unlimited capacity or on the entrant being able to select any level of capacity. To see this, consider the following modification of the model just described. Suppose that, instead of having an essentially unlimited capacity, the incumbent has $x^I$ units of capacity, where $n/2 < x^I < n$. If the entrant has the same capacity, the first price will be at or below marginal cost. But if the entrant can build up to $x_0$ units of capacity at a cost of $k$ dollars per unit, where $x_0 \leq x^I$ and $0 < k < v - c$, it would choose a capacity of $x^E = \min(x_0, x^I - 1)$ units (another application of Proposition 2). As a result, the final $n$ sales would be made at the monopoly price (the entrant would make $x^E$ sales). Again, pricing at or below marginal cost is avoided.[7]

## IV. Proofs

*Notation for the Proof of Propositions 1 and 2.* A backward induction argument will be used to study equilibrium outcomes. It will therefore be helpful to have some notation concerning subgames and subgame payoffs in equilibrium. Let $G(t,a(t),b(t))$ denote the set of all of the subgames starting in period $t$ in which the firms' inventories are $a(t)$ and $b(t)$ units, and let $S_0(t,a(t),b(t))$ be the statement: "all equilibria of all elements of $G(t,a(t),b(t))$ yield the same payoffs to the firms."

Now suppose that $G(t,a(t),b(t))$ is nonempty, all elements of $G(t,a(t),b(t))$ have equilibria, and that $S_0(t,a(t),b(t))$ holds. Let $\Pi^i(t,a(t),b(t))$, $i = A, B$, denote the equilibrium payoffs in dollars to Alpha and Beta in all elements of $G(t,a(t),b(t))$. And let $\pi^i(t,p^A(t),p^B(t),a(t),b(t))$, $i = A, B$, be Alpha and Beta's expected dollar earnings from sales in the last $t$ periods as a function of period $t$ prices and inventories in period $t$, given equilibrium pricing in the last $t - 1$ periods.

7. This model of capacity choice is based on Gelman and Salop [1983]. In the spirit of Kreps and Scheinkman [1983], one might also consider a multistage game in which duopolists simultaneously select capacities before playing the game described in Section II. If the cost of building capacity is as above, this capacity choice game has equilibria. If $n$ is odd, then one firm's capacity will equal $(n + 1)/2$ units and its rival's capacity will equal $(n - 1)/2$ units in any equilibrium. If $n$ is even, each firm's capacity will equal $n/2$ units. Along the equilibrium path, all consumers will be charged the monopoly price. Equilibrium payoffs will therefore amount to the equilibrium capacities multiplied by $v - c - k$ dollars per unit. These results continue to hold if the sellers choose capacity sequentially instead of simultaneously. The proofs follow from Proposition 2.

A result reminiscent of Gelman and Salop [1983] and Benoit and Krishna [1987] can be obtained by assuming that there is also a large fixed cost $F$ associated with building capacity, where $(v - c - k)n/2 < F < (v - c - k)(n - 1)$. The large fixed cost implies that the market is a natural monopoly. If the incumbent, Alpha, and the entrant, Beta, choose capacity sequentially, the game has a Pareto-inefficient equilibrium in which Alpha deters entry by setting capacity equal to the smallest integer greater than $F/(v - c - k)$; in this equilibrium, Alpha sells out at the monopoly price and at least one of the consumers is unable to make a purchase. Again, the proof follows from Proposition 2. (The result may be contrasted with the findings of Spence [1977] and Bulow, Geanakoplos, and Klemperer [1985].)

The relationship between $\Pi^A$, $\Pi^B$, $\pi^A$, and $\pi^B$ can be stated formally as

$$(1)\quad \pi^A(t,p^A(t),p^B(t),a(t),b(t))$$

$$= \begin{cases} p^A(t) - c + \Pi^A(t-1,a(t)-1,b(t)) & \text{if } p^A(t) < p^B(t) \text{ and } p^A(t) \le v \\ \Pi^A(t-1,a(t),b(t)-1) & \text{if } p^B(t) < p^A(t) \text{ and } p^B(t) \le v \\ [p^A(t) - c + \Pi^A(t-1,a(t)-1,b(t)) + \Pi^A(t-1,a(t),b(t)-1)]/2 & \text{if } p^A(t) = p^B(t) \le v \\ \Pi^A(t-1,a(t),b(t)) & \text{if } p^A(t) > v \text{ and } p^B(t) > v \end{cases}$$

and

$$(2)\quad \pi^B(t,p^A(t),p^B(t),a(t),b(t))$$

$$= \begin{cases} \Pi^B(t-1,a(t)-1,b(t)) & \text{if } p^A(t) < p^B(t) \text{ and } p^A(t) \le v \\ p^B(t) - c + \Pi^B(t-1,a(t),b(t)-1) & \text{if } p^B(t) < p^A(t) \text{ and } p^B(t) \le v \\ [p^B(t) - c + \Pi^B(t-1,a(t),b(t)-1) + \Pi^B(t-1,a(t)-1,b(t))]/2 & \text{if } p^A(t) = p^B(t) \le v \\ \Pi^B(t-1,a(t),b(t)) & \text{if } p^A(t) > v \text{ and } p^B(t) > v. \end{cases}$$

The proof of Propositions 1 and 2 will also use the following list of propositional functions in performing the backward induction. For any positive integer $t$, let $S_1(t)$ be the statement: "Suppose that $G(t,a(t),b(t))$ is nonempty. If, in addition, either $0 < a(t) < \min(t,b(t))$ and $a(t) + b(t) > t$ or $0 = b(t) < a(t)$, then all elements of $G(t,a(t),b(t))$ have an equilibrium in which $p^A(t) = v$ and $p^B(t) = v + 1$, $S_0(t,a(t),b(t))$ holds,

$$\Pi^A(t,a(t),b(t)) = (v - c)\min(a(t),t)$$

and

$$\Pi^B(t,a(t),b(t)) = (v - c)(t - a(t))\min(b(t),1)."$$

Let $S_2(t)$ be the statement: "Suppose that $G(t,a(t),b(t))$ is nonempty. If, in addition, either $0 < b(t) < \min(t,a(t))$ and $a(t) +$

$b(t) > t$ or $0 = a(t) < b(t)$, then all elements of $G(t,a(t),b(t))$ have an equilibrium in which $p^A(t) = v + 1$ and $p^B(t) = v$, $S_0(t,a(t),b(t))$ holds,

$$\Pi^A(t,a(t),b(t)) = (v - c)(t - b(t))\min(a(t),1)$$

and

$$\Pi^B(t,a(t),b(t)) = (v - c)\min(b(t),t).\text{''}$$

Let $S_3(t)$ be the statement: "Suppose that $G(t,a(t),b(t))$ is nonempty. If, in addition, $0 < a(t) = b(t) < t$ and $a(t) + b(t) > t$, then all elements of $G(t,a(t),b(t))$ have an equilibrium in which $p^A(t) = p^B(t) = c + (v - c)(t - 2a(t) + 1)$, $S_0(t,a(t),b(t))$ holds, and

$$\Pi^i(t,a(t),b(t)) = (v - c)(t - a(t)),\ i = A, B.\text{''}$$

Let $S_4(t)$ be the statement: "Suppose that $G(t,a(t),b(t))$ is nonempty. If, in addition, $t \leq a(t)$ and $t \leq b(t)$, then all elements of $G(t,a(t),b(t))$ have an equilibrium in which $p^A(t) = p^B(t) = c$, $S_0(t,a(t),b(t))$ holds, and

$$\Pi^i(t,a(t),b(t)) = 0,\ i = A, B.\text{''}$$

Finally, let $S_5(t)$ be the statement: "Suppose that $G(t,a(t),b(t))$ is nonempty. If, in addition, $\min(a(t),b(t)) > 0$ and $a(t) + b(t) \leq t$, then all elements of $G(t,a(t),b(t))$ have an equilibrium in which $p^A(t) = p^B(t) = v$, $S_0(t,a(t),b(t))$ holds:

$$\Pi^A(t,a(t),b(t)) = (v - c)a(t),$$

and

$$\Pi^B(t,a(t),b(t)) = (v - c)b(t).\text{''}$$

*Proof of Propositions 1 and 2.* To prove Propositions 1 and 2, it is sufficient to prove that $S_h(t)$, $h = 1, 2, 3, 4, 5$ hold for all positive integers $t$. Formal proofs of $S_4(t)$ and $S_5(t)$ are straightforward and left to the reader. Since $S_2(t)$ is a relabeling of $S_1(t)$, all that remains is to prove that $S_1(t)$ and $S_3(t)$ hold for all positive integers $t$. Leaving the easy proofs of $S_1(1)$ and $S_3(1)$ to the reader, I shall argue that, if $S_h(t)$, $h = 1, \ldots, 5$, hold for some positive value of $t$, call it $t^* - 1$, then $S_h(t^*)$ holds for $h = 1, 3$.

First, consider $S_1(t^*)$. The case in which $G(t^*,a(t^*),b(t^*))$ is nonempty and $0 = b(t^*) < a(t^*)$ is clearly trivial. So assume that $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) < \min(t^*,b(t^*))$, and $a(t^*) + b(t^*) > t^*$. In this case, expressions for $\pi^i(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$, $i = A, B$, can be derived using $S_h(t^* - 1)$, $h = 1, \ldots, 5$, (1) and (2). These expressions are given below in (3), (4), (5), (6), (7), and (8):

If $0 < a(t^*) < t^* - 1$ and $1 + a(t^*) < b(t^*)$,

$$\text{(3)} \quad \pi^A(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$

$$= \begin{cases} p^A(t^*) - c + (v - c)(a(t^*) - 1) & \text{if } p^A(t^*) \le v \text{ and } p^A(t^*) < p^B(t^*) \\ (v - c)a(t^*) & \text{if } p^B(t^*) \le v \text{ and } p^B(t^*) < p^A(t^*) \\ [p^A(t^*) - c + (v - c)(2a(t^*) - 1)]/2 & \text{if } p^A(t^*) = p^B(t^*) \le v \\ (v - c)a(t^*) & \text{if } p^A(t^*) > v \text{ and } p^B(t^*) > v \end{cases}$$

and

$$\text{(4)} \quad \pi^B(t^*,p^A(t^*),p^B(t^*),a(t^*,b(t^*))$$

$$= \begin{cases} (v - c)(t^* - a(t^*)) & \text{if } p^A(t^*) \le v \text{ and } p^A(t^*) < p^B(t^*) \\ p^B(t^*) - c + (v - c)(t^* - 1 - a(t^*)) & \text{if } p^B(t^*) \le v \text{ and } p^B(t^*) < p^A(t^*) \\ [p^B(t^*) - c + (v - c)(2t^* - 1 - 2a(t^*))]/2 & \text{if } p^A(t^*) = p^B(t^*) \le v \\ (v - c)(t^* - 1 - a(t^*)) & \text{if } p^A(t^*) > v \text{ and } p^B(t^*) > v. \end{cases}$$

If $0 < a(t^*) = t^* - 1$ and $1 + a(t^*) \le b(t^*)$, then

$$\text{(5)} \quad \pi^A(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$

$$= \begin{cases} p^A(t^*) - c + (v - c)(a(t^*) - 1) & \text{if } p^A(t^*) \le v \text{ and } p^A(t^*) < p^B(t^*) \\ 0 & \text{if } p^B(t^*) \le v \text{ and } p^B(t^*) < p^A(t^*) \\ [p^A(t^*) - c + (v - c)(a(t^*) - 1)]/2 & \text{if } p^A(t^*) = p^B(t^*) \le v \\ 0 & \text{if } p^A(t^*) > v \text{ and } p^B(t^*) > v \end{cases}$$

and

$$(6)\quad \pi^B(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$

$$= \begin{cases} (v-c)(t^*-a(t^*)) & p^A(t^*) \le v \text{ and } p^A(t^*) < p^B(t^*) \\ p^B(t^*)-c & p^B(t^*) \le v \text{ and } p^B(t^*) < p^A(t^*) \\ [\,p^B(t^*)-c & \text{if} \\ \quad +(v-c)(t^*-a(t^*))]/2 & p^A(t^*) = p^B(t^*) \le v \\ 0 & p^A(t^*) > v \text{ and } p^B(t^*) > v. \end{cases}$$

And, if $0 < a(t^*) < t^* - 1$ and $1 + a(t^*) = b(t^*)$, then

$$(7)\quad \pi^A(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$

$$= \begin{cases} p^A(t^*)-c & p^A(t^*) \le v \text{ and } p^A(t^*) < p^B(t^*) \\ \quad +(v-c)(a(t^*)-1) & \\ (v-c)(t^*-1-a(t^*)) & p^B(t^*) \le v \text{ and } p^B(t^*) < p^A(t^*) \\ [\,p^A(t^*)-c & \text{if} \\ \quad +(v-c)(t^*-2)]/2 & p^A(t^*) = p^B(t^*) \le v \\ (v-c)a(t^*) & p^A(t^*) > v \text{ and } p^B(t^*) > v \end{cases}$$

and

$$(8)\quad \pi^B(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$

$$= \begin{cases} & p^A(t^*) \le v \text{ and } p^A(t^*) \\ (v-c)(t^*-a(t^*)) & \quad < p^B(t^*) \\ p^B(t^*)-c & p^B(t^*) \le v \text{ and } p^B(t^*) \\ \quad +(v-c)(t^*-1-a(t^*)) & \text{if} \quad < p^A(t^*) \\ [\,p^B(t^*)-c & p^A(t^*) = p^B(t^*) \le v \\ \quad +(v-c)(2t^*-2a(t^*)-1)]/2 & p^A(t^*) > v \text{ and } p^B(t^*) > v. \\ (v-c)(t^*-1-a(t^*)) & \end{cases}$$

Examination of (3) and (4) reveals that, if $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) < t^* - 1$ and $1 + a(t^*) < b(t^*)$, period $t^*$ prices satisfying $p^A(t^*) = v$ and $p^B(t^*) = v + 1$ allow each firm to maximize its profit in all elements of $G(t^*,a(t^*),b(t^*))$, given the period $t^*$ price of its rival and equilibrium pricing in periods, $1, \ldots, t^* - 1$. By (5) and (6) the same conclusion is reached when $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) = t^* - 1$, and $1 + a(t^*) \le b(t^*)$. And by (7) and (8) the same conclusion is reached when $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) < t^* - 1$, and $1 + a(t^*) = b(t^*)$.

To summarize, if $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) <$

$\min(t^*,b(t^*))$, and $a(t^*) + b(t^*) > t^*$, period $t^*$ prices satisfying $p^A(t^*) = v$ and $p^B(t^*) = v + 1$ allow each firm to maximize its profits in all elements of $G(t^*,a(t^*),b(t^*))$, given the period $t^*$ price of its rival and equilibrium pricing in periods $1, \ldots, t^* - 1$. The resulting payoff to Alpha is $(v - c)a(t^*)$ dollars, and the resulting payoff to Beta is $(v - c)(t^* - a(t^*))$ dollars.

To complete the proof of $S_1(t^*)$, I shall show that there are no other equilibrium outcomes. Again, assume equilibrium pricing in periods $1, \ldots, t^* - 1$, and suppose that $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) < \min(t^*,b(t^*))$, and $a(t^*) + b(t^*) > t^*$. If $p^A(t^*) < p^B(t^*)$ and $p^A(t^*) < v$, then (3), (5), and (7) show that Alpha could increase its profits by slightly raising its price. If $p^B(t^*) \leq p^A(t^*)$ and $p^B(t^*) < v$, then (4), (6), and (8) show that Beta could increase its profits by raising its price. If $p^A(t^*) > v$ and $p^B(t^*) > v$, then (4), (6), and (8) show that Beta could increase its earnings by lowering its period $t^*$ price to $v$ dollars. If $p^B(t^*) = v$ and $p^B(t^*) \leq p^A(t^*)$, (3), (4), (5), and (7) imply that either Alpha would continue to earn $(v - c)a(t^*)$ dollars and Beta would continue to earn $(v - c)(t^* - a(t^*))$ dollars or Alpha would have no best response to $p^B(t^*) = v$ in period $t^*$. This completes the proof of $S_1(t^*)$.

The next task will be to prove $S_3(t^*)$ given $S_h(t^* - 1)$, $h = 1, \ldots, 5$. Recall that $S_3(t^*)$ assumes $G(t^*,a(t^*),b(t^*))$ is nonempty, $0 < a(t^*) = b(t^*) \leq t^* - 1$, and $a(t^*) + b(t^*) > t^*$. These assumptions together with $S_h(t^* - 1)$, $h = 1, \ldots, 5$, (1), and (2) imply that

$$(9)\quad \pi^A(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$$
$$= \begin{cases} p^A(t^*) - c + (v - c)(a(t^*) - 1) & \text{if } p^A(t^*) \leq v \text{ and } p^A(t^*) < p^B(t^*) \\ (v - c)(t^* - a(t^*)) & \phantom{\text{if }} p^B(t^*) \leq v \text{ and } p^B(t^*) < p^A(t^*) \\ (p^A(t^*) - c + (v - c)(t^* - 1))/2 & \phantom{\text{if }} p^A(t^*) = p^B(t^*) \leq v \\ (v - c)(t^* - 1 - a(t^*)) & \phantom{\text{if }} p^A(t^*) > v \text{ and } p^B(t^*) > v. \end{cases}$$

Since $S_3(t^*)$ assumes that $a(t^*) = b(t^*)$, the function $\pi^B(t^*,p^A(t^*),p^B(t^*),a(t^*),b(t^*))$ can be obtained by replacing $p^A(t^*)$ with $p^B(t^*)$ and vice versa in (9).

It follows that, given equilibrium pricing in the last $t^* - 1$ periods and given $p^B(t^*) = c + (v - c)(t^* - 2a(t^*) + 1)$, a best response for Alpha in period $t^*$ is to set $p^A(t^*) = p^B(t^*)$. This allows Alpha to earn $(v - c)(t^* - a(t^*))$ dollars. A higher value of $p^A(t^*)$

would result in Beta making the sale and Alpha earning $(v - c)(t^* - a(t^*))$ dollars. A lower value of $p^A(t^*)$ would result in Alpha making the sale and Alpha earning less than $(v - c)(t^* - a(t^*))$ dollars.

By symmetry, any subgame starting in period $t^*$ in which inventories are $0 < a(t^*) = b(t^*) < t^*$ has an equilibrium in which both firms choose a price of $c + (v - c)(t^* - 2a(t^*) + 1)$ dollars. In this equilibrium both firms earn $(v - c)(t^* - a(t^*))$ dollars.

There are no other equilibrium outcomes. Using the derived expressions for $\pi^A$ and for $\pi^B$, observe that, if either

$$v \geq p^A(t^*) > c + (v - c)(t^* - 2a(t^*) + 1)$$

or

$$v \geq p^B(t^*) > c + (v - c)(t^* - 2a(t^*) + 1),$$

then either Beta or Alpha has no best response in period $t^*$. In addition, if the minimum price seller in period $t^*$ is charging less than $c + (v - c)(t^* - 2a(t^*) + 1)$ dollars, it could increase profits by raising its price. If $p^A(t^*) > v$, Beta's best response is to set $p^B(t^*) = v$; however, Alpha has no best response in period $t^*$ when $p^B(t^*) = v$. Thus, $p^A(t^*) > v$ is inconsistent with equilibrium. Similarly, $p^B(t^*) > v$ is inconsistent with equilibrium.

Q.E.D.

## V. Conclusion

This paper has considered a dynamic model of price competition between capacity-constrained duopolists. In the finite horizon model, consumers arrive at different times, and sellers can reset prices at any time.

By contrast with the static, Edgeworthian paradigm, the model has a pure strategy equilibrium for all combinations of seller capacities. Moreover, if at least one seller cannot supply the entire market, both sellers earn positive profits. I conjecture that these results can be extended to considerably more general environments.[8]

8. See Bester [1988] for a related model in which the sellers maximize discounted profits. See Neely [1989] for a model in which the number of consumers is uncertain.

Although my analysis also yields more specific, nonrandom predictions about the price path, these results appear to be sensitive to a precise specification of institutional detail. A natural direction for further work would be to tailor the model to fit specific industries where price data are available.[9]

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