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Author(s): Marc Nerlove
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ADAPTIVE EXPECTATIONS AND COBWEB PHENOMENA

By Marc Nerlove*

I. Introduction

In a recent article in this *Journal*, Åkerman suggests that: "Cobweb phenomena of the traditional type involving growing disequilibrium are . . . in reality rather improbable."[1] Åkerman asserts that other writers on ". . . cobweb phenomena have operated with one unique normal supply schedule without distinguishing between short- and long-term schedules. This is the reason why they have ascribed quite exaggerated properties to the cobweb phenomena."[2] The purpose of this article is to examine Åkerman's proposition in the light of recent developments in the theory and estimation of distributed lags. Some tentative empirical results are presented which suggest that instability may exist, however "improbable" it may appear to be from a theoretical point of view. It is thus demonstrated once again that it is difficult to draw conclusions about reality from a purely theoretical model.

In his article on the cobweb theorem, Ezekiel set out to show that ". . . classical economic theory rests upon the assumption that price and production, if disturbed from their equilibrium, tend to gravitate back toward that normal. The cobweb theory demonstrates that, even under static conditions, this result will not necessarily follow."[3] The conditions under which the cobweb theorem was supposed to apply were:

"(1) where production is completely determined by the producers' response to price, under conditions of pure competition (where the producer bases plans for future production on the assumption that present prices will continue, and that

* I am indebted for helpful comments to R. J. Foote, formerly of the U. S. Agricultural Marketing Service, and to Max Steuer.

1. Gustav Åkerman, "The Cobweb Theorem: A Reconsideration," this *Journal*, LXXI (Feb. 1957), 158.

2. *Ibid.*, p. 155.

3. Mordecai Ezekiel, "The Cobweb Theorem," this *Journal*, LII (1938), 279.

his own production plans will not affect the market); (2) where the time needed for production requires at least one full period before production can be changed, once the plans are made; and (3) where the price is set by available supply."[4]

Ezekiel does not state explicitly the period to which his supply curve is supposed to refer. Buchanan has pointed out that it must be ". . . completely reversible throughout its whole length with respect to each period . . ." and it therefore has at least one characteristic of a "long-period competitive supply curve."[5] As Åkerman points out, we cannot be sure that Ezekiel had in mind any particular period, long or short.

One of the crucial conditions under which the cobweb theorem is supposed to apply — to which we return in Section III — is that ". . . the producer bases his plans for future production on the assumption that present prices will continue." If this is not the case, rigid price expectations may produce irreversible supply conditions in the short run.

Åkerman points out that Ezekiel defines supply during the consumption year as the aggregate of the year's harvest and the whole existing normal stock so that the very shortest-run supply schedule is perfectly inelastic. In his own model Åkerman treats *market* supply so that supply within the year is not perfectly inelastic. For our purposes, we assume that farmers face an excess demand curve for net additions to stocks and consumption, i.e., we deduct the schedule of excess supply from storage from the demand schedule for consumption in order to arrive at the derived demand for a particular crop at the farm level. Neglecting abandonment, we thus have a perfectly inelastic short-run supply schedule facing a derived demand schedule. Under these conditions and the other conditions outlined above, the following simple exposition of the cobweb theorem can be given:

Let q_t^D = the quantity demanded, q_t^S = the quantity supplied and P_t = the price, where all are taken for the period t. In the neighborhood of any point, say equilibrium, we can approximate the supply and demand schedules by linear functions:

$$\begin{aligned} q_t^D &= a + bP_t \\ q_t^S &= c + dP_{t-1}\,. \end{aligned} \tag{1}$$

Condition (3) above implies that $q_t^D = q_t^S$ for all t. Hence

$$P = \frac{c-a}{b} + \frac{d}{b}\,P_{t-1}\,. \tag{2}$$

4. *Ibid.*, p. 272.

5. Norman S. Buchanan, "A Reconsideration of the Cobweb Theorem," *Journal of Political Economy*, XLVII (Feb. 1939), 68–69.

The equilibrium price is one at which $P_t = P_{t-1}$. Let P_0 be the equilibrium price, then

$$P_0 = \frac{c - a}{b - d}. \tag{3}$$

If the price is initially $P \neq P_0$, will the price approach P_0, fluctuate endlessly around it, or oscillate in explosive fluctuations? Equation (2) is a first order difference equation in actual price. Its solution is

$$P_t = \frac{c - a}{b - d} + \left(\frac{d}{b}\right)^t \left[P - \frac{c - a}{b - d}\right], \tag{4}$$

where $P \neq P_0$ is the initial price.[6] It can easily be seen that only when $\left|\frac{d}{b}\right| < 1$ will the price tend toward equilibrium. Since $b < 0$ and $d > 0$, typically, only oscillations occur: they are explosive if $\frac{d}{b} < -1$; steady if $\frac{d}{b} = -1$; and damped if $-1 < \frac{d}{b} \leqslant 0$. This is essentially what the traditional cobweb theorem states.

II. Åkerman's Argument

Åkerman considers a special crop A, the demand for and supply of which have been in equilibrium for some time past. As noted earlier, he also assumes the existence of stocks, and considers the excess supply of stocks to be added to the producers' supply in arriving at the market supply. Suppose now that the demand curve for A shifts to the right. Initially, the price will rise, and, because stocks are included in supply, equilibrium of a sort will be achieved at the intersection of a less than perfectly inelastic short-run supply schedule and the new demand schedule. But, adds Åkerman, this short-run supply schedule is not the only supply schedule involved in our problem:

"A crop product A is very seldom cultivated as the sole crop of a farm. Different crops are generally combined in rotation systems. Such systems usually enhance the effectiveness of fertilizers and the possibilities of suppressing weeds, and also facilitate a more even distribution throughout the year of farm labor. A permanent enlargement of crop A, therefore, will generally be of some disadvantage and cause some additional cost. *Such cost will be undertaken only if the higher price p_a of A can be expected to be of a permanent nature* [italics mine].

6. See R. G. D. Allen, *Mathematical Economics* (New York: St. Martin's Press, 1956), chap. 6, pp. 176–208.

The extension of cultivation of A may also be attained through an extension of total land under cultivation. But such additional land will generally be of somewhat inferior quality [although not necessarily inferior, since a question of comparative rather than absolute advantage is involved] causing some additional cultivation cost and requiring a somewhat higher price p^a. The *long-term normal supply* curve for extended A production is thus of a slowly rising nature.

"In the space between the market supply curve and the long-term normal supply curve a *short-term normal supply* curve is situated. Long-term normal supply conditions will generally not be fully established until several years have elapsed after each particular price change."[7]

Thus, in the year following a shift in demand, production will expand along a "short-run normal" supply curve intermediate to the short-run and "long-run normal" supply curves.

Similarly, when price falls after production is greatly increased, production will not immediately be cut back to its former level or less than its former level, but will follow a new "short-run normal" supply curve, the elasticity of which is less than that of the "long-run normal" supply schedule but greater than that of the short-run supply schedule. Thus Åkerman's assumption of a series of ". . . different, successively arising and vanishing, moderately elastic short-term normal supply curves . . ." leads him to a graphical demonstration that cobweb fluctuations involving growing disequilibrium are improbable.[8]

In subsequent sections of this article, the difference between short- and long-run supply or demand is expressed in analytical rather than graphical form, which makes possible more definite conclusions and leads readily to empirical applications.

III. Reformulation in Terms of Adaptive Expectations

Åkerman pointedly remarks that a position along the "long-term normal" supply schedule will be taken only if the price corresponding to that position ". . . can be expected to be of a permanent nature." Later he remarks that if ". . . the price p_a has risen the farmer, generally, will not be convinced it will remain so elevated until several years have elapsed."[9] Thus it might be said that farmers react to expected normal price and that this is not generally the same as price at harvest.[1]

7. Åkerman, *op. cit.*, p. 154.
8. *Ibid.*, p. 158.
9. *Ibid.*, p. 154.
1. For some estimates of the elasticities of supply of certain agricultural commodities based on this principle see my "Estimates of the Elasticities of Supply of Selected Agricultural Commodities," *Journal of Farm Economics*, XXXVIII (May 1956), 496–509, and my "Estimates of the Elasticities of Supply of Corn, Cotton, and Wheat," unpublished Ph.D. dissertation, The Johns Hopkins University (Baltimore: 1956).

Expectations of future prices may be autonomous, induced, or divided into two components, one of which is autonomous, the other induced. Induced expectations are the result of movements in past prices.[2] Only induced price expectations are amenable to economic analysis in the present context. That a price change must remain in effect for a considerable period of time before farmers will become convinced of its permanence and take up positions along a "long-term normal" supply schedule would appear to indicate that Åkerman has in mind a particular kind of induced expectations: Farmers take past prices into account when forming their expectations of future "normal" price, but they do not give all the weight to one particular price. When current price increases, farmers may be expected to discount some of the increase, i.e., they will not believe the permanence of the entire change. Arrow and I have called such induced expectations "adaptive."[3]

Intuitive considerations of the sort which Åkerman sets forth would lead us to suppose that producers' supply depends on a great many past prices, i.e., the effect of a price change is distributed over a great many periods.[4] The distribution of the effects of a price change over many periods due to expectational rigidities may be represented by a variant of a model developed originally by Cagan.[5] Let P_t^* be expected "normal" price in period t, and let β be a constant of proportionality called the coëfficient of expectations; then the model in question is

$$(5) \qquad P_t^* - P_{t-1}^* = \beta\,[P_{t-1} - P_{t-1}^*]\,, \; 0 < \beta \leqslant 1\,.$$

In words, farmers revise their previous expectations of "normal"

2. Alain C. Enthoven and Kenneth J. Arrow, "A Theorem on Expectations and the Stability of Equilibrium," *Econometrica*, XXIV (July 1956), 288–93.

3. Kenneth J. Arrow and Marc Nerlove, "A Note on Expectations and Stability," Technical Report No. 41, Department of Economics, Stanford University, Stanford, California (March 25, 1957); to be published in revised form in *Econometrica* (April, 1958).

4. We thus have a distributed lag of an expectational nature. Such lags are discussed at length in my *Using Distributed Lags in the Analysis of Demand for Agricultural and Other Commodities*, forthcoming publication of the U. S. Department of Agriculture.

5. Phillip Cagan, "The Monetary Dynamics of Hyper-Inflation," in *Studies in the Quantity Theory of Money*, ed. M. Friedman (Chicago: University of Chicago Press, 1956). It should be noted that Cagan's model may be derived from Hicks's definition of the elasticity of expectations. See J. R. Hicks, *Value and Capital*, 2d ed., p. 205. In its logarithmic form the model may be derived by setting the elasticity of expectations equal to a constant (see Arrow and Nerlove, *op. cit.*). However, it is doubtful that Hicks actually had the model in mind.

price in each period in proportion to the difference between actual price and what was previously considered to be "normal."[6]

The model of expectation formation represented by equation (5) may be applied to the problem of instability in a single market.[7] Let

$$q_t^D = a + bP_t \tag{6}$$

be the demand equation, as before. Let the supply equation, however, be

$$q_t^S = c + dP_t^*. \tag{7}$$

Lagging equation (7) one period, substituting for q_{t-1}^S in (5), solving for P_t^*, and substituting the result in (7), we have

$$q_t^S = c\beta + d\beta P_{t-1} + (1-\beta)q_{t-1}^S. \tag{8}$$

Since we assume the market always to be in short-run equilibrium

$$q_t^D = q_t^S = q_t, \quad q_{t-1}^D = q_{t-1}^S = q_{t-1}, \ldots. \tag{9}$$

By (9), and from equation (6) lagged one period and equation (8) we have

$$\begin{aligned} q_t &= c\beta + d\beta P_{t-1} + (1-\beta)\,[a + bP_{t-1}] \\ &= (c-a)\beta + a + [(d-b)\beta + b]\,P_{t-1}. \end{aligned} \tag{10}$$

From (6), (9), and (10), therefore,

$$P_t - \left[\left(\frac{d}{b} - 1\right)\beta + 1\right] P_{t-1} = \frac{(c-a)\beta}{b}. \tag{11}$$

Equation (11) is a first order difference equation in P_t; it may be solved in terms of the initial conditions and the constant parameters a, b, c, d, and β. If P_0 is the equilibrium price, and $P \neq P_0$ is the initial price (resulting, say, from a shift in the demand curve), then the solution of (11) is

$$P_t = P_0 + (P_0 - P)\left[\left(\frac{d}{b} - 1\right)\beta + 1\right]^t. \tag{12}$$

6. Equation (5) is a first order difference equation in P_t^*:

$$P_t^* - (1-\beta)P_{t-1}^* = \beta P_{t-1}$$

Regarding P_t as a known function of time this may be solved to obtain

$$P_t^* = \beta P_{t-1} + (1-\beta)\beta P_{t-2} + (1-\beta)^2\beta P_{t-3} + \ldots\ldots.$$

Thus, if supply depends on expected "normal" price, it depends on past prices taken with a distributed lag. The distribution of lag depends on the parameter β.

7. For an application in the case of multiple markets, see Arrow and Nerlove, *op. cit.*

The second term on the right will go to zero as t increases if, and only if,

$$\left|\left(\frac{d}{b}-1\right)\beta+1\right|<1. \tag{13}$$

Thus, (13) is a necessary and sufficient condition that return to equilibrium will be achieved. (13) may be rewritten

$$1-\frac{2}{\beta}<\frac{d}{b}<1, \tag{14}$$

since $\beta > 0$ by assumption. (14) should be contrasted with

$$-1<\frac{d}{b}<1, \tag{15}$$

which is the traditional cobweb result. (14) reduces to (15) when $\beta = 1$. When $\beta = 1$, equation (5) also reduces to

$$P_t^* = P_{t-1} \tag{16}$$

so that the model given by equations (5), (6), and (7) reduces to the ordinary cobweb model in the case where the coefficient of expectations, β, is unity.

The ranges of $\frac{d}{b}$ compatible with stability are shown in Figure I for both the traditional theory of the cobweb and the model developed above. The only range compatible with stability according to the traditional theory is the cross-hatched area lying between the lines parallel to the β-axis through 1 and -1. The range compatible with stability if β is not 1 includes the doubly cross-hatched area as well. It should be noted that the permissible range widens as β falls; almost any positive elasticity of supply and negative elasticity of demand is compatible with stability when the coefficient of expectation is low.

While it can be seen from Figure I that the possibility of stability is much improved when adaptive expectations are assumed, it is by no means certain that stability will always occur. Whether or not it will, depends on the relationship among the slopes of the demand and "long-term normal" supply curves and the coefficient of expectations.[8]

8. This relation is examined empirically for corn, cotton, and wheat in Section V.

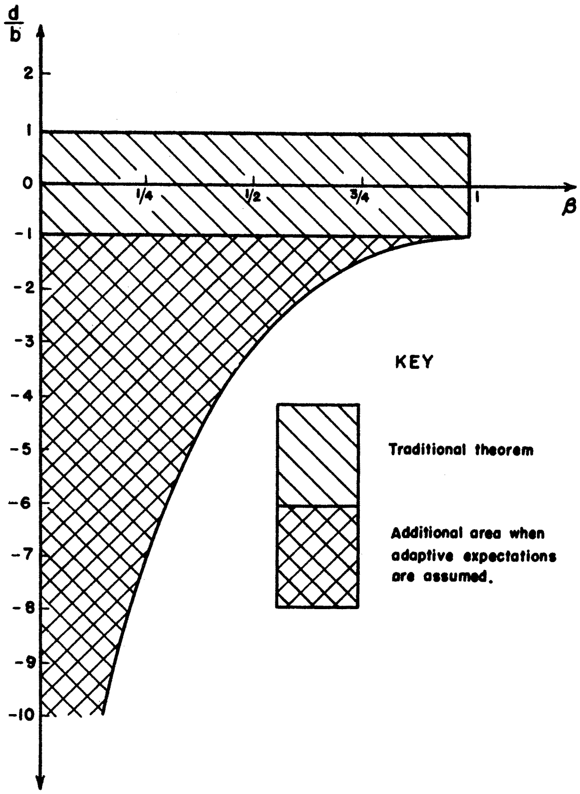

FIGURE I

Ranges of d/b Compatible with Stability under the Traditional Cobweb Theorem and under the Theorem when Adaptive Expectations are Assumed.

IV. An Alternative but Equivalent Formulation

It is possible to develop an alternative model based on the traditional Marshallian distinction between long and short runs which leads to precisely the same conclusions reached in the preceding section. The usual distinction between long-run and short-run elasticities of supply has generally been made on the basis of the elasticities of supply of certain factors to the firm.[9] The contention is that in the shortest of short runs most or all factors of production are fixed while, as time passes, successively more of these restrictions are removed. Under this interpretation of the meaning of different runs it can be shown that the short-run elasticity of supply for an individual

9. Milton Friedman, "Lectures in Price Theory," unpublished mimeographed notes (Chicago: 1951), pp. 119–24.

firm is always less than or equal to the long-run elasticity of supply. The argument hinges on the fact that the minimum cost of producing an additional unit of output cannot be less when an entrepreneur's freedom of choice is limited than when he is completely free to choose, because he always has the alternative of choosing exactly that situation to which he was previously limited.[1] Thus, from any point on a long-run supply schedule, we may think of a fan of short-run supply curves which gradually approach the long-run supply schedule, from the right above it, and from the left below it. Åkerman has pictured this situation graphically.[2]

In order to develop a mathematical model of cobweb phenomena based on the considerations discussed above, it is not necessary to reject Ezekiel's original assumption that farmers react to the price at the preceding harvest. Although it is possible to retain the assumption about expectations of long-run normal price outlined in Section III, this will not be done here. Instead, we assume that supply is a function of last period's price, P_{t-1}.

The considerations of the first paragraph of this section suggest the following simple model of the relation between current output and long-run equilibrium output; Let $\bar{q}_t^S$ be the long-run equilibrium quantity supplied, in contrast to q_t^S which is the current quantity supplied. Let $0 < \gamma_s \leqslant 1$ be a constant of proportionality which we shall call the coefficient of adjustment. The following equation, when coupled with a long-run supply curve, yields a fan of short-run supply curves through each point of the long-run curve:

$$q_t^S - q_{t-1}^S = \gamma_s\,[\bar{q}_t^S - q_{t-1}^S]\,, \quad 0 < \gamma_s \leqslant 1\,. \tag{17}$$

In words, the rate of adjustment to long-run equilibrium is proportional to the difference between current output and the long-run equilibrium output.[3]

Let the demand curve be equation (6) as before, but now let the long-run supply curve be

$$\bar{q}_t^S = c + dP_{t-1}\,. \tag{18}$$

That is, suppose farmers base their long-term plans on price the preceding harvest, but that they cannot or do not realize these

1. *Ibid.*, p. 122.
2. Åkerman, *op. cit.*, Figure I.
3. The characteristics of this model are discussed more fully in *Distributed Lags and Demand Analysis*, *op. cit.*, and in "Estimates of the Elasticities of Supply of Corn, Cotton, and Wheat," *op. cit.*, pp. 59–67. I have used it in quite a different context in my "A Note on the Long-Run Automobile Demand," *Journal of Marketing*, XXII (July 1957), 57–64.

plans in the short run. We assume market equilibrium in both the long and short runs; hence,

$$\text{(19)} \qquad \bar{q}_t^S = \bar{q}_t^D = \bar{q}_t \quad \text{and}$$
$$q_t^S = q_t^D = q_t.$$

Since no difference between short- and long-run demand is assumed, the current and long-run quantities demanded are one and the same. By a procedure similar to that used in the preceding section, we have from equations (6) and (17)–(19):

$$\text{(20)} \qquad P_t - \left[\left(\frac{d}{b} - 1\right)\gamma_s + 1\right]P_{t-1} = \frac{(c-a)\gamma_s}{b}.$$

Equation (20) is precisely the difference equation (11) with γ_s in place of β. It therefore has the same solution, and the necessary and sufficient conditions for a convergent cobweb are

$$\text{(21)} \qquad 1 - \frac{2}{\gamma_s} < \frac{d}{b} < 1.$$

The model expressed by equations (6), (17), and (18) reduces to the ordinary cobweb model when $\gamma_s = 1$.

V. Some Empirical Results

In previous sections we have developed two models of cobweb phenomena which, although based on different considerations, lead to the same necessary and sufficient condition for stability. In this section we present empirical estimates of the long-run elasticities of supply of corn, cotton, and wheat based on models of the types discussed above. These are combined with existing estimates of the elasticities of demand to yield measures of the ratio d/b for the three commodities.[4] These ratios are then compared with the lower limit $1 - 2/\beta$ or $1 - 2/\gamma_s$.

Previous estimates of the elasticities of demand for corn, cotton, and wheat at the farm level indicate that they range from a low of about $-.1$ for wheat to about $-.3$ for cotton and a high of $-.6$ for corn.[5]

Estimates of the elasticities of supply of corn, cotton, and wheat have previously been derived on the same general assumptions as

4. In the neighborhood of equilibrium, the ratio of the elasticity of supply to the elasticity of demand crudely measures the ratio of the two slopes.

5. See Richard J. Foote, John W. Klein, and Malcolm Clough, *The Demand and Price Structure for Corn and Total Feed Concentrates*, U. S. Department of Agriculture Technical Bulletin No. 1061 (1952), pp. 39–41; Frank Lowenstein and Martin S. Simon, "Analyses of Factors that Affect Mill Consumption of Cotton in the United States," *Agricultural Economics Research*, VI (1954), 101–10; and Kenneth W. Meinken, "The Demand and Price Structure for Wheat," U. S. Department of Agriculture Technical Bulletin No. 1136 (1955), pp. 42–43.

those on which the models of sections III and IV are based.[6] The estimates of the elasticities of supply and β or γ_s have been derived by two methods: one I have called the "iterative" method, and the other the "noniterative." For an explanation of the two methods the reader is referred to the sources of the estimates. The period of estimation was 1909–32. The estimates obtained by the iterative method range from a low of 0.4 for corn to 1.2 for wheat and a high of 4.5 for cotton; those obtained by the noniterative range from a low of 0.2 for corn to 0.7 for cotton and a high of 0.9 for wheat. As the two sets of estimates differ considerably, it is well to regard the results obtained with either set as essentially illustrative.

As a by-product of the estimates of the long-run elasticities of supply described in the preceding paragraph, estimates of the coefficients of expectations, β, or the coefficient of adjustment, γ_s, were derived. The two models presented above suggest precisely the same form of regression, so that what is called β in one regression may equally well be interpreted as γ_s. Which interpretation we use makes little difference in examining the results for stability. The iterative estimates of β or γ_s range from a low of 0.04 for cotton to 0.25 for corn and a high of 0.4 for wheat; the noniterative estimates are 0.4 for cotton and 0.5 for wheat and corn.

Table I gives the ratios of the elasticity of supply to the elasticity of demand for cotton, wheat, and corn for the two methods of estimation (columns (3) and (4)). Columns (5) and (6) present the

TABLE I

COTTON, WHEAT, AND CORN: STABILITY OF AN EQUILIBRIUM OF DEMAND AND SUPPLY FOR VARIOUS ESTIMATES OF THE ELASTICITY OF SUPPLY AND THE COEFFICIENTS OF EXPECTATIONS OR ADJUSTMENT

Crop	Assumed Elasticity of Demand[1]	Estimated Elasticity of Supply ÷ Assumed Elasticity of Demand[2]		Estimated $(1-2/\beta)$ or $(1-2/\gamma_S)$[2]	
		Iterative Estimate of the Elasticity of Supply	Noniterative Estimate of the Elasticity of Supply	Iterative Estimate	Noniterative Estimate
(1)	(2)	(3)	(4)	(5)	(6)
Cotton	−.3	−15.1*	−2.2*	−49.0	−3.9
Wheat	−.1	−11.8	−9.3	− 4.4	−2.8
Corn	−.6	− .7*	− .3*	− 7.0	−2.7

* Stable equilibrium indicated.
1. Sources cited in footnote 5, p. 236.
2. Sources cited in footnote 6 below.

6. See "Estimates of the Elasticities of Supply of Selected Agricultural Commodities," *op. cit.*, and "Estimates of the Elasticities of Supply of Corn, Cotton, and Wheat," *op. cit.*, pp. 247–96.

values of $1-2/\beta$ or $1-2/\gamma_s$ for the two methods and the three crops. A comparison of one of the columns (3) or (4) with its corresponding member of the two columns (5) and (6) indicates the nature of the equilibrium between demand and supply. If the ratio in column (3) or (4) is less in absolute value than the corresponding ratio in column (5) or (6), then the equilibrium is stable. If it is greater in absolute value, the equilibrium is unstable. If the two are equal, the equilibrium is neutral in the sense that perpetual but nonexplosive fluctuations occur if the equilibrium is disturbed. Ratios indicating a stable equilibrium are marked with an asterisk. Only wheat shows indication that its equilibrium is unstable, and this holds regardless of which method we use to estimate the elasticity of supply. The other crops, cotton and corn, appear to have stable equilibriums of demand and supply.

It may seem somewhat difficult to believe that the equilibrium of demand and supply is actually unstable in the case of wheat. We should, however, remember that instability may exist only within a certain range of prices; while the demand for wheat may, in fact, be highly inelastic in the range of prices which prevailed during the period used in estimation, it is probable that at a lower range of prices it is highly elastic. At a somewhat lower range of prices, wheat would be used for feed and would compete strongly with corn. An elasticity of demand for wheat of only $-.3$ or $-.35$ would suffice to turn an indication of instability into one of stability; at higher prices the elasticity of supply may be less; hence, instability may occur only within a relatively narrow range around equilibrium.

VI. More Complicated Models

It is possible to develop more complex models of cobweb phenomena by techniques similar to those used in preceding sections. As an indication of how to proceed, one such model is presented in this section; it is not, however, fully analyzed.

Let $\bar{q}_t^D$ be the long-run equilibrium quantity demanded and q_t^D be the current quantity. Let $\bar{q}_t^S$ and q_t^S be, as before, the long-run equilibrium quantity supplied and the current quantity supplied, respectively. Our model is specified by six equations: The long-run demand equation

$$\bar{q}_t^D = a + bP_t \; ; \tag{22}$$

the demand adjustment equation

$$q_t^D - q_{t-1}^D = \gamma_D \, [\bar{q}_t^D - q_{t-1}^D] \, , \quad 0 < \gamma_D \leqslant 1 \; ;[7] \tag{23}$$

7. Equations (22) and (23) taken together imply that the current quantity demanded depends on the price taken with a distributed lag. Under the present

the long-run supply equation

$$(24) \qquad \bar{q}_t^S = c + dP_{t-1} ;$$

the supply adjustment equation

$$(25) \qquad q_t^S - q_{t-1}^S = \gamma_s [\bar{q}_t^S - q_{t-1}^S] , \quad 0 < \gamma_s \leqslant 1 ;$$

and, finally, two market equilibrium conditions, short-run

$$(26) \qquad q_t^D = q_t^S = q_t ,$$

and long-run

$$(27) \qquad \bar{q}_t^D = \bar{q}_t^S = \bar{q}_t .$$

A second-order difference equation may be derived from equations (22)–(27) as follows: From (22), (23), and (26) and (27) we have

$$(28) \qquad q_t = a\gamma_D + b\gamma_D P_t + (1 - \gamma_D)q_{t-1} ,$$

and from (24), (25), (26), and (27) we have

$$(29) \qquad q_t = c\gamma_s + d\gamma_s P_{t-1} + (1 - \gamma_s)q_{t-1} .$$

$$(30) \qquad b\gamma_D P_t - D\gamma_s P_{t-1} + a\gamma_D - c\gamma_s = (\gamma_D - \gamma_s)q_{t-1} ;$$

eliminating q_{t-1}

$$(31) \; a\gamma_D(1 - \gamma_s) - c\gamma_s(1 - \gamma_D) + b\gamma_d(1 - \gamma_s)P_t - d\gamma_s(1 - \gamma_D)P_{t-1} = (\gamma_D - \gamma_s)q_t .$$

Lagging (31) by one period and substituting the result in (30), we have a second-order difference equation.

$$(32) \; P_t - \left[\left(\frac{d}{b\gamma_D} - 1\right)\gamma_s + 1\right]P_{t-1} + \frac{d}{b}\frac{\gamma_s}{\gamma_D}(1 - \gamma_D)P_{t-2} = \frac{(c-a)\gamma_s}{b} .$$

This difference equation reduces to equation (20) in section IV when $\gamma_D = 1$. It may be solved but its solution is not presented here.

Other more complicated models involving stocks and/or expectations could be constructed, but the example presented above should suffice to indicate the possibilities.

interpretation this means that the long-run elasticity of demand differs from and is greater than the short-run elasticity. That this should be so becomes reasonable when we consider that our demand includes an excess supply from stocks.

VII. Conclusions

It is possible to construct more sophisticated models of cobweb phenomena than the model which was first presented by Ezekiel in the pages of this *Journal*. In particular, mathematical models may be constructed which distinguish between long- and short-run elasticities of supply and which incorporate both a long-run supply curve and its associated short-run supply curves into the analytical framework. These models are more suggestive of, and more amenable to, empirical application than Åkerman's recent diagrammatic extension of the cobweb theorem.

The models presented in this article show that, while the range of possible instability is lessened when account is taken of the distinction between long- and short-run supply schedules, the possibility still exists. The illustrative empirical results which are presented suggest that wheat, in the absence of price support programs, may be characterized by such instability, at least in the neighborhood of equilibrium.

The methods outlined here may be used to construct further models of cobweb phenomena which are easily adapted to fit complex, individual situations which we may wish to investigate.

Marc Nerlove.

U. S. Agricultural Marketing Service