Bayesian Modeling
Homework Assignment 3

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Task 1: Choosing and Describing a Reflective Measurement Model

(a) Model Choice:

- Given that our latent variable / the latent trait that we want to estimate is "voting propensity", which is a continuous latent trait, and that the indicators we will be using to estimate it are a series of binary indicators that illustrate whether voters voted (y/n) in specific elections, we use an **ITEM RESPONSE Model**, which is used for continuous concepts and discrete indicators.
- Further, we choose a **"2 PL" (Birnbaum) IRT Model** over a "1PL" (Rasch) Model for the fact that the 2PL model introduces a second parameter, 'slope' (discrimination) that varies across each indicator.
  ↪ In this ~~data~~ data, we could foreseeably find that certain elections are more informative in helping us determine someone's propensity to vote (for example: everyone votes in a 2012 general election, giving us less information than a 2010 midterm), so we give each indicator the possibility to vary in both slope and intercept.

(b) Formal Model Notation

- For i indicators $(1, 2, \ldots I)$ (elections) and j voters $(1, 2, \ldots J)$:
- Likelihood: $y_{ij} \sim \text{Binomial}(1, \gamma)$

$$\gamma = \text{logit}^{-1}(\beta_i(\eta_j - \alpha_i))$$

- With priors:

$$\alpha_i \sim \text{Normal}(\mu_\alpha, \sigma_\alpha)$$

$$\beta_i \sim \text{LogNormal}(\emptyset, \sigma_\beta)$$

$$\eta_j \sim \text{Normal}(\emptyset, 1)$$

$$\mu_\alpha \sim \text{Students T}(3, \emptyset, 1)$$

$$\sigma_\alpha \sim \text{Students T}(3, \emptyset, 1)$$

$$\sigma_\beta \sim \text{Students T}(3, \emptyset, 1)$$

(c) Interpretation of Core Parameters

y_{ij} is the probability of individual [j] getting a value of 1 on indicator [i]. In other words, the probability of voter [j] voting in election [i], given the voter's latent propensity to vote.

α_i is the indicator-specific intercept (difficulty), which tells us how strong your propensity for voting had to be in order for you to have voted in election [i]

β_i is the indicator-specific slope (discrimination), which tells us how good of a measure of a voter's propensity to vote is the election in question [i]. How good is this specific election at distinguishing voters.

η_j is the subject-specific latent trait (ability), which tells us how vote-prone (how likely to exercise their right to vote) a specific voter is.