2.6 2.7

Matrices Hermitianas y Unitarias

Matriz Hermitiana $A_{nxn} \mid A^{t} = A$

Si A es hermitiana $\forall_v \in \mathbb{C}^n$ se cumple q':

$$\langle AV, V' \rangle = \langle V, AV' \rangle$$

Si A es hermitiana, sus eigenvalues son reales

Si A es hermitiana, distintos eigenvectores con distintos eigenvalues son ortogonales

Matriz diagonal Aquella q' $(\forall_{i,i} \mid A[i][i] \neq 0)$ y el resto de valores = 0

Matriz unitaria Lo es si $A * A^{t} = I_n$

Si U es unitaria - $\forall_v \mid v \in \mathbb{C}^n$: $\langle UV, UV' \rangle = \langle V, V' \rangle$

$$|uv| = |v|$$

Si es hermitiana y Diagonal, los valores de su diagonal son sus valores propios (eigenvalues)

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Ex 2.6.1 Probar q' A es hermitiana $A = \begin{bmatrix} 7 & 6+5i \\ 6-5i & -3 \end{bmatrix}$

$$A^{\dagger} = \overline{A^{T}} = \overline{\begin{bmatrix} 7 & 6-5i \\ 6+5i & -3 \end{bmatrix}} = \begin{bmatrix} 7 & 6+5i \\ 6-5i & -3 \end{bmatrix} = A$$

Es hermitiana porque $A^{\dagger} = A$

Ex 2.6.6 Probar q' A es unitaria $A = \begin{bmatrix} \frac{1+i}{2} & \frac{i}{\sqrt{3}} & \frac{3+i}{2\sqrt{15}} \\ \frac{-1}{2} & \frac{1}{\sqrt{3}} & \frac{4+3i}{2\sqrt{15}} \\ 1/2 & -i/\sqrt{3} & -5i/2\sqrt{15} \end{bmatrix}$

$$A^{\dagger} = \begin{bmatrix} \frac{1-i}{2} & -1/2 & 1/2 \\ \frac{-i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & i/\sqrt{3} \\ 3-i/2\sqrt{15} & \frac{4-3i}{2\sqrt{15}} & 5i/2\sqrt{15} \end{bmatrix}$$

$$A^{\dagger} * A = \begin{bmatrix} \left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) & \left(\frac{i-i^2}{2\sqrt{3}}+\frac{1}{2\sqrt{3}}-\frac{i}{2\sqrt{3}}\right) & \left(\frac{3+i-3i-i^2}{4\sqrt{15}}-\frac{4+3i}{4\sqrt{15}}-\frac{5i}{4\sqrt{15}}\right) \\ \left(\frac{-i-i^2}{2\sqrt{3}}-\frac{1}{2\sqrt{3}}+\frac{i}{2\sqrt{3}}\right) & \left(\frac{-i^2}{3}+\frac{1}{3}+\frac{i^2}{3}\right) & \left(\frac{-3i-i^2+4+3i-5i^2}{2\sqrt{3}\sqrt{15}}\right) \\ \left(\frac{3+3i-i-i^2-4+3i+5i}{4\sqrt{15}}\right) & \left(\frac{3i-i^2+4-3i-5i^2}{2\sqrt{3}\sqrt{15}}\right) & \left(\frac{3^2+1^2+4^2+3^2-25i^2}{4\cdot 15}\right) \end{bmatrix}$$

$$A^{\dagger} * A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ – es unitaria porque $A^{\dagger} * A = I_n$

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Ex 2.6.5 Probar q' A es unitaria $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{+} = \begin{matrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$A * A^{+} = \begin{bmatrix} \cos^2\theta + sen^2\theta & -sen\theta\cos\theta + sen\theta\cos\theta & 0 \\ -sen\theta\cos\theta + sen\theta\cos\theta & sen^2\theta + \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

es unitaria. $A * A^{+} = I_n$

Prd tensor

Dados 2 espacios vectoriales $\mathbb{V}$ y $\mathbb{V}'$. $\mathbb{V} \otimes \mathbb{V}'$ es el conjunto

$$\{ v \otimes v' \mid v \in \mathbb{V} \ \& \ v' \in \mathbb{V}' \}$$

$$= c_0 (V_0 \otimes V_0') + c_1 (V_1 \otimes V_1') + \ldots + c_{p-1} (V_{p-1} \otimes V'_{p-1})$$

$$\sum_{i=1}^{p-1} c_i (V_i \otimes V_i') : \text{Prd tensor}$$

$$C \cdot \text{Prd tensor} = \sum_{i=0}^{p-1} (c \times c_i)(V_i \otimes V_i') \qquad \text{Es distributiva}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{bmatrix}$$

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Ex 2.7.9

$$\begin{bmatrix} 3+2i & 5-i & 2i \\ 0 & 12 & 6-3i \\ 2 & 4+4i & 9+3i \end{bmatrix} \otimes \overset{B}{\begin{bmatrix} 1 & 3+4i & 5-7i \\ 10+2i & 6 & 2+5i \\ 0 & 1 & 2+9i \end{bmatrix}}$$

$(3+2i \times B)$ $(2i \times B)$

$$= \begin{bmatrix} \begin{bmatrix} 3+2i & 9+12i+6i-8 & 15-21i+10i+14 \\ 30+6i+10i-4 & 3+2i & 6+10i+4i-10 \\ 0 & 3+2i & 6+27i+4i-18 \end{bmatrix} & \begin{bmatrix} 5-i \\ \times \\ B \end{bmatrix} & \begin{bmatrix} 2i & 6i-8 & 10i+14 \\ 20i-4 & 12i & 4i-10 \\ 0 & 2i & 4i-18 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 12 & 36+48i & 60-84i \\ 120+24i & 72 & 24+60i \\ 0 & 12 & 24+108i \end{bmatrix} & \begin{bmatrix} 0-3i \times B \end{bmatrix} \\ [2 \times B] & [(4+4i) \times B] & [(9+3i) \times B] \end{bmatrix}$$

Ex 2.7.7 $A = [2 \;\; 3]$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $(A \otimes B)^t$ & $A^t \otimes B^t$

1×2 2×2 $\Rightarrow 2 \times 4$

$$A \otimes B = \begin{bmatrix} 2 & 4 & 3 & 6 \\ 6 & 8 & 9 & 12 \end{bmatrix} \qquad A^t = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad B^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(A \otimes B)^t \begin{bmatrix} 2 & 6 \\ 4 & 8 \\ 3 & 9 \\ 6 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & 8 \\ 3 & 9 \\ 6 & 12 \end{bmatrix} = A^t \otimes B^t$$

Son iguales

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