ED sustitución

Ver método de solución de Buitrago

Homogenea: Llegas a vars separables

lo es si: $t^n f(x,y) = f(t^n x, t^n y)$

si es homogenea:

$u = \frac{y}{x}$ ó $v = \frac{x}{y}$ → para $\frac{dx}{dy}$ si $u = \frac{y}{x}$

Determinar solución de $(x+y)\,dx = (x-y)\,dy$

$$u + x\frac{du}{dx} = \frac{1+u}{1-u}$$

Ec Bernoulli, Llegas a lineal siempre

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$ → no es lineal

$n \notin \{0, 1\}$

Cuidado truco: $u = y^{1-n}$ y dividir entre y^n la ED

$u = y^{-1} = \frac{1}{y}$ $y = u^{-1}$

EDO convertible a separables

$$\frac{dy}{dx} = Ax + By + C$$

La sustitucion debe ser linea

$$u = Ax + By + C$$

$$\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$$

$u = y - 2x + 3$

$$\frac{du}{dx} = \frac{dy}{dx} - 2 \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + 2$$

$$\frac{du}{dx} + 2 = 2 + \sqrt{u}$$

$$\frac{du}{dx} = \sqrt{x}$$

1 SQUARE =

$u = \frac{x}{y}$ $\quad v = \frac{y}{x}$ $\quad y = ux$

2-5

5) $(y^2 + yx)\,dx - x^2\,dy = 0$

$y = ux$

$(u^2x^2 + ux^2)\,dx - x^2(u\,dx + x\,du) = 0$

$u^2x^2dx + \cancel{ux^2dx} - \cancel{x^2u\,dx} - x^3du = 0$

$u^2x^2dx = x^3du$

$$\boxed{\frac{dx}{x} = \frac{du}{u^2}}$$

y ajó resolves

10) $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$, $\quad x > 0$

$y = ux$

$dy = u\,dx + x\,du$

$xu + x^2du = ux + \sqrt{x^2 - x^2u^2}$

$\cancel{xu} + x^2du = \cancel{ux} + x\sqrt{1 - u^2}$

$x\frac{du}{dx} = \sqrt{1 - u^2} \Rightarrow \frac{du}{\sqrt{1-u^2}} = \frac{dx}{x}$

~~$u\,dx = u + \sqrt{1-u^2}$~~

$\arcsin(u) = \ln|x| + C$ $\qquad u = \frac{y}{x}$

$\arcsin\left(\frac{y}{x}\right) = \ln|x| + C$

1 SQUARE =

13) $\left(x + ye^{y/x}\right)dx - xe^{y/x}dy = 0$ $\qquad y(1) = 0$

$u = \frac{y}{x} \qquad y = ux \qquad dy = u\,dx + x\,du$

$\left(x + uxe^{u}\right)dx - xe^{u}(u\,dx + x\,du) = 0$

$x\,dx + \cancel{xue^{u}dx} - \cancel{xe^{u}u\,dx} - x^2e^{u}du = 0$

$x\,dx - x^2e^{u}du = 0$

$dx = xe^{u}du$

$\boxed{\frac{dx}{x} = e^{u}du}$ y ajá

$\ln|x| = e^{u} + C$

$\boxed{\ln|x| = e^{y/x} + C}$

Evaluando $y(1) = 0$

$\ln|1| = e^{0/1} + C$

$0 = 1 + C \Rightarrow C = -1$

$\boxed{\ln|x| = e^{y/x} - 1}$

1 SQUARE = ____________

16) $\frac{dy}{dx} - y = e^x y^2$

$y^{-2}\frac{dy}{dx} - \frac{1}{y} = e^x \qquad u = y^{-1} \Rightarrow \frac{du}{dy} = \frac{-1}{y^2} \Rightarrow \frac{dy}{du} = y^2$

$-y^{-2}\left(\frac{\frac{y^2}{du}}{\frac{dx}{1}}\right) - u = e^x \Rightarrow \frac{du}{dx} + u = e^x \Rightarrow P(x) = +1 \quad Q(x) = e^x$

$e^{\int +1dx} = e^{+x}$

$\frac{d\,e^{-x}u}{du} = e^{-x}\cdot e^x$

$e^{-x}u = \int e^{2x}dx$

$\boxed{u = \frac{\frac{1}{2}e^{2x} + C}{e^{-x}}} \Rightarrow \frac{1}{y} = \frac{-\frac{e^{2x-x}}{2} + ce^{-x}}{1}$

$\boxed{y = \frac{1}{-\frac{e^x}{2} + ce^{-x}}}$

1 SQUARE =

18) $x\frac{dy}{dx} - (1+x)y = xy^2$

$\frac{dy}{dx} - \frac{y}{x} - y = y^2 \Rightarrow \frac{dy}{dx} - y\left(1+\frac{1}{x}\right) = y^2 \Rightarrow y^{-2}\frac{dy}{dx} - y^{-1}\left(1+\frac{1}{x}\right) = 1$

$-y^{-2}\frac{y^2 du}{dx} - u\left(1+\frac{1}{x}\right) = 1 \Rightarrow \frac{du}{dx} + u\left(1+\frac{1}{x}\right) = -1$

$u = \frac{1}{y}$

$P(x) = 1 + \frac{1}{x} \quad Q(x) = -1$

$\frac{du}{dy} = -\frac{1}{y^2}$

$e^{\int 1+\frac{1}{x}dx} = e^{x+\ln|x|} = e^x \cdot x$

$dy = -y^2 du$

$\frac{d\, x e^x u}{dx} = -xe^x$

$\cancel{x}\cancel{e^x}du = -\cancel{x}\cancel{e^x}dx$

en el libro integran sin simplificar, por qué no es válida la simplificación?

$u = -x + C$

$\frac{1}{y} = -x + C \Rightarrow \boxed{y = \frac{1}{-x+C}}$

21)

$$x^2 \cdot \frac{dy}{dx} - 2xy = 3y^4 \qquad y(1) = \frac{1}{2}$$

$$y^{-4}\frac{dy}{dx} - \frac{2}{x\,y^3} = \frac{3}{x^2} \qquad u = y^{-3} \Rightarrow \frac{du}{dy} = \frac{-3}{y^4} \Rightarrow dy = -3y^4 du$$

$$-3\frac{du}{dx} - \frac{2}{x}u = \frac{3}{x^2} \Rightarrow \frac{du}{dx} + \frac{2}{3x}u = \frac{-1}{x^2} \qquad P(x) = \frac{2}{3x} \qquad Q(x) = \frac{-1}{x^2}$$

$$e^{\frac{2}{3}\int \frac{1}{x}dx} = e^{\ln|x|^{2/3}} = x^{2/3}$$

$\frac{2}{3} - 2 =$

$$\frac{d\,x^{2/3}u}{du} = \frac{-x^{2/3}}{x^2}$$

$$x^{2/3}du = -x^{-4/3}dx \qquad -\frac{4}{3} + 1 = -\frac{1}{3}$$

$$x^{2/3}u = -\frac{x^{-4/3+1}}{-4/3+1} + C$$

$$x^{2/3}u = \frac{\cancel{-}3x^{-1/3} + C}{\cancel{-}1} \Rightarrow u = \frac{3x^{-1/3} + C}{x^{2/3}} \Rightarrow \frac{1}{y^3} = \frac{3x^{-1/3} + C}{x^{2/3}}$$

$$y = \sqrt{\frac{3}{x} + \frac{C}{\sqrt[3]{x^2}}}$$

La cagué en el despeje de dy.....

1 SQUARE =

2.5 solución ED Por sustitución.

$u = \frac{y}{x}$ $\quad V = \frac{x}{y}$

2.5

3) $x\,dx + (y-2x)\,dy = 0$

$y = ux \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx} \Rightarrow \boxed{dy = u\,dx + x\,du}$

$x\,dx + (ux - 2x)(u\,dx + x\,du) = 0$

$x\,dx + [(u-2)x](u\,dx + x\,du) = 0$

$x\,dx + xu(u-2)\,dx + x^2(u-2)\,du = 0$

$x[1 + u(u-2)]\,dx + (u-2)x^2\,du = 0$

$x(u-1)^2\,dx + (u-2)x^2\,du = 0$

$\frac{u-2}{(u-1)^2}\,du = -\frac{x\,dx}{x^2}$

$\vdots$

1 SQUARE =

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22) $y^{1/2}\dfrac{dy}{dx} + y^{3/2} = 1$ $y(0) = 4$

$\sqrt{y}\dfrac{dy}{dx} + y\sqrt{y} = 1$ $u = \sqrt{y}$ $\dfrac{du}{dy} = \dfrac{1}{2\sqrt{y}}$

$dy = 2\sqrt{y}\,du$

$u\,2u\dfrac{du}{dx} + u^2 \cdot u = 1$

$dy = 2u\,du$

$2u^2\dfrac{du}{dx} + u^3 = 1$

$2u^2 du = dx(1-u^3)$

$\int \dfrac{2u^2}{1-u^3}du = \ln|x|$ $z = 1-u^3$

$dz = -2u^2 du$

$-\int \dfrac{dz}{z} = \ln|x|$

$\frac{1}{2}' =$

$-\ln|z| = \ln|x|$

$-\ln|1-(\sqrt{y})^3| = \ln|x| + C$

$\boxed{C = \ln|x(1-y\sqrt{y})|}$

1 SQUARE = ____