DAD 2.3 Ec. Lineales

Ec estandar

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$$\mu(x) = e^{\int P(x)dx}$$

(1) $\frac{dy}{dx} = 5y$

$\frac{dy}{dx} - 5y = 0$ (1)

$P(x) = -5$

$f(x) = 0$

$\mu(x) = e^{\int P(x)dx} = e^{-5x}$

$$\frac{d}{dx} e^{\int P(x)dx} y = e^{\int P(x)dx} f(x)$$

multiplico $\mu(x)$ * (1)

$$e^{-5x}\frac{dy}{dx} - 5e^{-5x}y = 0 \cdot \cancel{e^{-5x}}$$

$$\frac{d}{dx}(ye^{-5x}) = \left(\frac{dy}{dx}(e^{-5x}) - 5e^{-5x}y\right) * 0 = 0$$

a = b = c, so

$$\int \frac{d\, ye^{-5x}}{dx} dx = \int 0\, dx \Rightarrow e^{-5x}y = C \Rightarrow \boxed{y = Ce^{5x}}$$

definida de $(-\infty, \infty)$

1) $P(x)=?$ $f(x)=?$

2) $\frac{dy}{dx}+P(x)y=f(x)$

3) $e^{\int P(x)dx}$

4) $\frac{d}{dx}\left[ye^{\int P(x)dx}\right]y = e^{\int P(x)dx}f(x)$

5) integrar

3)

$$\frac{dy}{dx}+y=e^{3x} \Rightarrow P(x)=1 \qquad f(x)=e^{3x}$$

$$\mu(x)=e^{\int dx}=e^{x}$$

$$\frac{dy}{dx}e^{x}+ye^{x}=e^{4x}$$

$$\frac{d}{dx}e^{x}y=e^{4x}$$

$$\int e^{x}dy=\int e^{4x}dx$$

$$e^{x}y=\frac{e^{4x}}{4}+C \Rightarrow \boxed{y=\frac{e^{3x}}{4}+Ce^{-x}} \quad (-\infty,\infty)$$

$$\frac{d}{dx}e^{\int P(x)dx}y=e^{\int P(x)dx}f(x)$$

5) $\frac{dy}{dx}+3x^2y=x^2$

$$P(x)=3x^2 \qquad f(x)=x^2$$

$$\mu(x)=e^{\int 3x^2dx}=e^{x^3+c}$$

$$\frac{d}{dx}e^{x^3+c}y=e^{x^3+c}\cdot x^2$$

$$ye^{x^3+c}=\int e^{x^3+c}x^2dx \qquad \begin{array}{l}u=x^3+c\\ du=3x^2dx\end{array} \qquad (-\infty,\infty)$$

$$=\frac{1}{3}\int e^{u}du \Rightarrow y=\frac{\frac{1}{3}e^{x^3+c}+C_2}{\frac{e^{x^3+c}}{1}}=\boxed{\frac{1}{3}+C_2e^{-x^3-c}=y}$$

1 SQUARE =

2-3

7) $x^2 y' + xy = 1$

$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ $\quad P(x) = \frac{1}{x} \quad f(x) = \frac{1}{x^2}$

$e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln|x|} \boxed{= x}$

$\frac{d e^{\int P(x)dx} \cdot y}{dx} = e^{\int P(x)dx} \cdot f(x)$

$\frac{dy\,x}{dx} = \frac{1}{x}$

$\int x\,dy = \int \frac{dx}{x}$

$xy = \ln|x| + C$

$\boxed{y = \frac{\ln|x|}{x} + \frac{C}{x}}$ $\quad x \in \{(-\infty, 0) \cup (0, \infty)\}$

---

15) $y\,dx - 4(x + y^6)\,dy = 0$

$(-4x - 4y^6)dy + y\,dx = 0$

$\frac{dx\,y}{dy} - 4x - 4y^6 = 0$

$\frac{dx}{dy} - \frac{4x}{y} = 4y^5$

$f(y) = 4y^5 \quad P(y) = \frac{-4}{y}$

$e^{-4\ln|y|} = e^{\ln|y|^{-4}} = y^{-4}$

1 SQUARE =

$$\frac{d\,x\cdot y^{-4}}{dy} = y^{-4}\cdot 4y^5 \Rightarrow \int y^{-4}dx = \int 4y\,dy$$

$$x y^{-4} = \frac{4y^2}{2} + C$$

$$\boxed{x = 2y^6 + y^4 C}$$

16) $y\,dx = (ye^y - 2x)\,dy$

$$\frac{dx}{dy} = e^y - \frac{2x}{y} \Rightarrow \frac{dx}{dy} + \frac{2}{y}x = e^y$$

$P(y) = \frac{2}{y}$ $\quad f(y) = e^y$

$$e^{\int \frac{2\,dy}{y}} = e^{\ln|y|^2} = y^2$$

$$\frac{d\,y^2 x}{dy} = y^2\cdot e^y \Rightarrow \cancel{y^2}x = \cancel{y^2}\int e^y dy$$

$$\boxed{x = e^y}$$

2.2) $\frac{dP}{dt} + 2tP = P + 4t - 2 \Rightarrow \frac{dP}{dt} + P(2t-1) = 4t-2$

$$e^{\int 2t-1\,dt} = e^{t^2-t}$$

$$\frac{d\,P(e^{t^2-t})}{dt} = e^{t(t-1)}(4t-2)$$

$u = t^2 - t$

$du = 2t - 1\,dt$

$$e^{t^2-t}P = 2\int e^{t^2-t}(2t-1)\,dt$$

$$= 2\int e^u du \Rightarrow P = \frac{2(e^{t^2-t}) + C}{e^{t^2-t}} \Rightarrow \boxed{P = 2 + C\,e^{-(t^2-t)}}$$

1 SQUARE =

28) $y\frac{dx}{dy} - x = 2y^2$, $y(1)=5 \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$ $P(y) = -\frac{1}{y}$

$f(y) = 2y^2$

$e^{-\int \frac{dy}{y}} = y^{-1} = \frac{1}{y}$

$\left(\frac{1}{y}\right)\frac{dx}{dy} = \frac{1}{y}(2y^2)$

$yx = 2\int y\,dy \Rightarrow \boxed{x = \frac{y^2+C}{y}}$

$y(1)=5$: $1 = \frac{25}{5} + \frac{C}{5} \Rightarrow \boxed{C=-20} \Rightarrow x = \frac{y^2-20}{y}$

---

37) $\frac{dy}{dx} + 2y = f(x)$, $y(0)=0$ $f(x) = \begin{cases} 1, & 0 \le x \le 3 \\ 0 & x > 3 \end{cases}$

$e^{\int P(x)dx} = e^{2x}$

a) Si $0 \le x \le 3$

$\frac{dy}{dx}e^{2x} = e^{2x} \cdot 1$

$e^{2x}y = \frac{e^{2x}}{2} + C$;

$y = \frac{1}{2} + \frac{C}{e^{2x}}$

$y(0)=0$

$0 = \frac{1}{2} + C \Rightarrow \boxed{C = -\frac{1}{2}}$

$y = \frac{1}{2} - \frac{\frac{1}{2}}{\frac{e^{2x}}{1}} \Rightarrow \boxed{y = \frac{1}{2} - \frac{1}{2e^{2x}}}$ Para $0 \le x \le 3$

b) Si: $x > 3$

$\frac{dy}{dx}e^{2x} = e^{2x} \cdot 0$

$e^{2x}y = C$

$y = C_2 e^{-2x}$

c) Para C_2: Asumimos continuidad en $x=3$

$\frac{1}{2} - \frac{1}{e^6} = \frac{C_2}{e^6} \Rightarrow \boxed{C_2 = \frac{e^6}{2} - 1}$

$$\boxed{f(x) = \begin{cases} \frac{1}{2} - \frac{1}{e^{2x}}, & 0 \le x \le 3 \\ \left(\frac{e^6}{2} - 1\right)e^{-2x}, & x > 3 \end{cases}}$$

1 SQUARE =

3.1) $\frac{dy}{dx} + 2xy = f(x) \qquad y(0) = 2 \qquad f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$

$e^{\int 2x\,dx} = e^{x^2}$

Si $0 \leq x < 1$

$\frac{d\,ye^{x^2}}{dx} = e^{x^2} \cdot x$

$u = x^2$
$du = 2x\,dx$

$e^{x^2} y = \frac{1}{2} \int e^u \, du$

$\boxed{y = \frac{1}{2} + \frac{C_1}{e^{x^2}}}$

Para C_1

$2 = \frac{1}{2} + \frac{C_1}{e^0} \Rightarrow \boxed{C_1 = \frac{3}{2}}$

$\boxed{y = \frac{1}{2} + \frac{3}{2e^{x^2}}}$

Si $x \geq 1$

$e^{x^2} y = \int 0\,dx \Rightarrow \boxed{y = \frac{C_2}{e^{x^2}}}$

Para C_2.

$\lim_{x \to 1} f(x)$

$\frac{C_2}{e^1} = \frac{1}{2} + \frac{3}{2e^1} \Rightarrow C_2 = \frac{e}{2} + \frac{3}{2}$

$f(x) = \begin{cases} \frac{1}{2} + \frac{3}{2e^{x}}, & 0 \leq x < 1 \\ \frac{e+3}{2e^{x^2}}, & x \geq 1 \end{cases}$

1 SQUARE =