DAD 2.2 variables separables

Resolver la ED por separación de variables

1) $\frac{dy}{dx} = \operatorname{sen} 5x \Rightarrow g(x) = \operatorname{sen} 5x \quad h(y) = 1$

$dy = \operatorname{sen} 5x\, dx$

$\int dy = \int \operatorname{sen} 5x\, dx \qquad u = 5x \qquad du = 5\,dx$

$y + C_1 = \frac{1}{5} \int \operatorname{sen}(u)\, du$

$y + C_1 = \frac{1}{5} \mp \cos(u) + C_2$

$$\boxed{y = -\frac{\cos 5x}{5} + C_2 - C_1}$$

3) $dx + e^{3x} dy = 0$

$dy = -\frac{dx}{e^{3x}} \Rightarrow dy = -e^{-3x} dx \qquad u = -3x$

$du = -3\,dx$

$\int dy = -\int e^{-3x} dx$

$y + C = -\int \frac{e^{u}\, du}{-3} \Rightarrow y + C = \frac{1}{3} e^{-3x} + C_2$

$$\boxed{y = \frac{e^{-3x}}{3} - C + C_2}$$

1 SQUARE =

8) $e^{x} y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

$$e^{x} y \frac{dy}{dx} = e^{-y} + e^{-2x} e^{-y}$$

$$e^{x} y \frac{dy}{dx} = e^{-y}(1 + e^{-2x})$$

$$\frac{y\,dy}{e^{-y}} = \frac{1 + e^{-2x}}{e^{x}}\,dx$$

$$\int \frac{y}{e^{-y}}\,dy = \int \frac{1+e^{-2x}}{e^{x}}\,dx$$

$$\int y e^{y} dy = \int e^{-x} + e^{-2x-x} dx$$

$$\boxed{\int y e^{y} dy = \int e^{-x} dx + \int e^{-3x} dx}$$ todo es Por sustitución

⋮

15) $\frac{dS}{dr} = kS$

$$\int \frac{1}{S}\,dS = \int k\,dr$$

$$\ln|S| = kr + C$$

$$|S| = e^{kr+C}$$

$$\boxed{S = e^{kr+C}}$$

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17, 18 36 ⓐ 23, 24, 27, 28. 46, 47, 49

17) $\frac{dP}{dt} = P - P^2$

$\frac{dP}{P-P^2} = dt$

$\int \frac{dP}{P(1-P)} = \int dt$

$(1-P)A + BP = 1$

$\int \frac{A}{P} + \frac{B}{1-P} dP = \int dt$

$A - PA + BP = 1$

$\boxed{A=1}$ $BP - PA = 0$

$\int \frac{1dP}{P} + \int \frac{dP}{1-P} = t + C$

$B - A = 0$

$u = 1-P$

$\boxed{B=1}$

$du = -dP$

$\ln|P| - \ln|1-P| = t + C$

$\ln\left|\frac{P}{1-P}\right| = t + C$

$\boxed{\frac{P}{1-P} = e^t \cdot e^C}$

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18) $N + \frac{dN}{dt} = N t e^{t+2}$

$$1 + \frac{1}{N}\frac{dN}{dt} = t e^{t+2}$$

$$\int \frac{dN}{N} = \int t e^{t+2} dt - \int dt \qquad \begin{matrix} u = t & V = e^{t+2} \\ du = dt & dV = e^{t+2} \end{matrix}$$

$$\ln|N| = t e^{t+2} - \int e^{t+2} dt - t$$

$$\boxed{\ln|N| = t e^{t+2} - e^{t+2} + C - t}$$

36) Hallar solución para $\frac{x\,dy}{dx}$ que pase por $\quad \frac{x\,dy}{dx} = y^2 - y$

a) $(0,1)$

$$\frac{dy}{(y^2-y)\,dx} = \frac{1}{x} \Rightarrow \int \frac{dy}{y(y-1)} = \int \frac{dx}{x} \qquad Ay - A + yB = 1 \Rightarrow \boxed{A = -1}$$

$$\int \frac{A}{y} + \frac{B}{(y-1)}\,dy = \ln|x| + C \qquad y(A+B) = 0$$

$$-1 + B = 0$$

$$\boxed{+B = +1}$$

$$-\ln|y| + \ln|y-1| = \ln|x| + C$$

$$\boxed{\ln\left|\frac{y-1}{y}\right| = \ln|x| + C}$$

Para $(0,1)$:

$$\ln|0| = \ln|\cancel{0}|^{\text{indef}} + C \qquad (0,1) \text{ no corresponde a un valor de } C.$$

d) $(2, \frac{1}{4})$ $\quad \ln\left|\frac{\frac{3}{4}}{\frac{1}{4}}\right| = \ln|2| + C \Rightarrow C = \ln|3| - \ln|2| = 0 \Rightarrow \boxed{\ln\left|\frac{y-1}{y}\right| = \ln|x|}$

ln

1 SQUARE =

18) $\frac{dN}{dt} + N = Nte^{t+2}$

$\frac{dN}{dt} = N(-1 + te^{t+2})$

$\int \frac{1}{N}\, dN = \int -1 + te^{t+2}\, dt$

$u = t \qquad v = e^{t+2}$

$= -t + \int te^{t+2}$

$du = dt \qquad dv = e^{t+2}$

$= -t + te^{t+2} - \int e^{t+2}\, dt$

$\ln|N| = -t + te^{t+2} - e^{t+2} + c$

$\boxed{\ln|N| = c + e^{t+2}(t-1) - t}$

16) $\frac{dQ}{dt} = K(Q-70)$

$\int \frac{1}{Q-70}\, dQ = \int K\, dt$

$u = Q - 70$
$du = dt$

$\ln|u| = Kt + C$

$|Q-70| = e^{Kt+c}$

$Q - 70 = e^{Kt+c}$ ó $Q - 70 = -e^{t+c}$

$Q = \pm e^{Kt+c} + 70 \qquad C_1 = e^{c}$

$\boxed{Q = C_1 e^{Kt} + 70}$

1 SQUARE =

17) $\frac{dP}{dt} = P - P^2$

Fracciones parciales:

el grado del numerador < grado-denominador

$$\int \frac{1}{P-P^2}\,dP = \int dt$$

$$\int \frac{A(1-P)+BP}{P(1-P)} = t + C$$

$$\frac{A}{P} + \frac{B}{(1-P)} = \frac{1}{P(1-P)}$$

$$\int \frac{1}{P}\,dP + \int \frac{1}{1-P}\,dP = t + C$$

$$AP + B - BP = 1$$

$$AP - BP + B = 1$$

$$\ln|P| - \ln|1-P| = t + C$$

$$\boxed{B = 1}$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\ln\left|\frac{P}{1-P}\right| = t + C$$

$$P(A-B) = 0$$

$$A - 1 = 0 \Rightarrow \boxed{A = 1}$$

$$\frac{P}{1-P} = e^{t+C}$$

$$P = e^{t+C} - Pe^{t+C}$$

$$P(1+e^{t+C}) = e^{t+C}$$

$$\boxed{P = \frac{e^{t+C}}{1+e^{t+C}}}$$

1 SQUARE =

$\sqrt{a^2 - x^2}$ $\sqrt{a^2+x^2}$ $\sqrt{x^2-a^2}$

$x = a\,sen\theta$ $x = a\tan\theta$ $x = a\sec\theta$

24) $\frac{dy}{dx} = \frac{y^2-1}{x^2-1}$ $y(2) = 2$

$\int \frac{dy}{y^2-1} = \int \frac{dx}{x^2-1}$

$\int \frac{u}{u^2-1} = \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C$

$\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right| = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

$\frac{y-1}{y+1} = \frac{x-1}{x+1} + C$

$yx + y - x - 1 = xy - y + x - 1 + C$

$\cancel{yx} + y - \cancel{xy} + y = 2x$

$\boxed{y = \frac{2x}{2} + C}$

$y(2) = 2$

$2 = \frac{2(2)}{2} + C \Rightarrow \boxed{C = 0}$

$\boxed{y = x}$

25)

$$x^2 \frac{dy}{dx} = y - xy \qquad y(-1) = -1$$

$$\frac{dy}{dx} = \frac{y(1-x)}{x^2}$$

$$\ln|y| = \int \frac{1-x}{x^2}\, dx$$

$$= \int x^{-2} dx - \int \frac{dx}{x}$$

$$\ln|y| = \frac{x^{-1}}{-1} - \ln|x| + C$$

$$\ln|y| = \frac{-1}{x} - \ln|x| + C$$

$$\ln|xy| = C - \frac{1}{x}$$

$$e^{\ln|xy|} = e^{C - \frac{1}{x}}$$

$$\boxed{y = \frac{Ce^{-\frac{1}{x}}}{x}}$$

$$y(-1) = -1$$

$$-1 = \frac{Ce^{1}}{-1} \Rightarrow C = \frac{1}{e}$$

$$y = \frac{e^{-\frac{1}{x} - 1}}{x}$$

$$\boxed{y = \frac{e^{-(1+\frac{1}{x})}}{x}}$$

1 SQUARE =