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UNIVERSITY OF ANTIOQUIA

Mass Modelling of Globular Clusters in the Milky Way

by

Juan Manuel Espejo Salcedo

Advisor:

Juan Carlos Muñoz Cuartas

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“We are just an advanced breed of monkeys on a minor planet of a very average star. But we can understand the Universe. That makes us something very special.”

Stephen Hawking

Abstract

Natural Sciences Faculty

Institute of Physics

The study of the dynamics and mass modelling of galaxies is a very complex but beautiful branch of modern Astrophysics and Cosmology. When you follow this route it is perhaps inevitable the need of studying stellar systems inside galaxies because they are inherent all along the way of the history and the formation and structure of galaxies themselves. This thesis work is intended to show our work on mass models of Globular Clusters in the Milky Way with our own data obtained in OPD observatory. By using spectra of the central region of the clusters we compute radial velocities of the stars to obtain information about the velocity dispersion profile thus obtaining information about the potential well responsible for the dynamics of those individual stars. With this information, aside the mass estimations given by the photometry results we can build mass models of the clusters looking for insight on the amount of dark matter present in this kind of structures, if dark matter is present at all...

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

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For/Dedicated to/To my...

Chapter 1

Introduction

As the oldest known subunits of our Galaxy, Globular Clusters are the primary "fossils" from its early evolution, and they may hold the key to understanding its formation. For example, the amount of dark matter that they could (or not) have may be an important clue to the understanding of the formation of the Milky Way. Despite the large amount of observational data and the advances in theoretical work, still, there is not yet a convincing model to describe the formation of these ancient structures.

Two broad types of possibilities have been considered: one is that the globular clusters were the very first condensed systems to form in the early universe, and the second is that they originated in larger star-forming systems that later merged to form the present galaxies.

The first possibility includes the suggestion of Peebles & Dicke (1968) that the globular clusters were formed by Jeans fragmentation in the early universe, this possibility is in concordance with the current scenario of galaxy formation in which galaxies are formed inside the deepest regions of the gravitational potential well provided by dark matter halos. This explanation has been received with great interest by the scientific community since it not only fits within the hierarchical scenario of structure and galaxy formation, but also may help to understand some of the open problems in the standard model, such as the abundance of low mass structures (Klypin et. al. 1999). However, evidence has been found against this scenario. For example, Odenkirchen et al. (2003) have found tidal tails surrounding globular clusters, something that is not expected if globular clusters form and reside inside extended dark matter halos.

Regarding the second possibility, Fall & Ries (1985, 1988) proposed the formation of globular clusters as a response to thermal instabilities in the hot gaseous halos of massive galaxies. The Fall & Rees hypothesis has been popular among theorists who have used

it to predict the characteristic properties of globular clusters, although it has proven difficulties to justify the assumed thermal behaviour of the cluster-forming gas clouds. This scenario presents a problem since there are observations that suggest that very low mass galaxies, not massive enough to host a hot gaseous halo, may also have their own globular clusters.

Not only the formation of these structures is puzzling, their composition is also a challenging problem because some authors say that these structures do not contain any dark matter contributions and that their gravitational stability can be explained completely with the baryonic matter inside of them. Conroy et. al. (2011) have used density profiles to argue against the presence of dark matter inside globular clusters, while Ibata et. al. (2013) have found that under general conditions it could be possible to find significant fractions of dark matter in globular clusters. Modelling the mass content of globular clusters in the galaxy would help to disentangle the mechanism driving its formation process. Mass modelling would allow to study the mass distribution in the inner region of globular clusters, determining the dominant components (Breddels M. A. et.al. 2013, Adams J. J. et. al., 2012, van den Bosch R.C.E. et. al. 2006) providing light on the problem of the origin of globular clusters.

Our aim in this project is to build our own mass model for the Globular Clusters using some data observed in OPD observatory in May 2014 so that we can be able to discuss and find new evidence on the existence or absence of dark matter in them according to these results.

With our set of data, we do the preliminary reduction and analysis of photometric and spectroscopic data, including the wavelength and flux calibration of the spectra. The photometric data will show us the mass to light ratio of the Globular Clusters thus giving us information about the baryonic mass content of the clusters. On the other hand, the spectroscopic data will give us information about the radial velocity of the stars that will provide us the statistical dispersion of velocities in the inner region of the clusters thus giving us information about the potential well in the clusters. This procedure is done using the Radial Velocity Package of IRAF called *RVSAO* which uses cross correlation techniques in the Fourier transform of templates and scientific spectra to infer the doppler broadening of the emission and/or absorption lines in our data and allows us to calculate their radial velocities.

After this analysis has been made upon all the observational data we can start the theoretical analysis including simulations of N-body systems and study of the initial conditions that will preserve the spherical symmetry of the clusters, taking into account our results on the dominant components of the clusters.

The bulk properties of GCs, with the possible exception of their innermost regions, can be modelled using the collisionless Boltzmann equation (Binney & Tremaine 1987), from which the statistical properties of the velocity distribution of stars can be derived. In particular, one can derive a formulation for the velocity dispersion tensor that, in the isotropic, non rotating case, reduces to a scalar quantity. This quantity can be determined using the appropriate Jeans equation. This calculation requires knowledge of the distribution function (DF) that determines the number of stars in a given region of space, and the gravitational potential.

The best fit for our data with the theoretical assumptions will tell us how mass is distributed in the clusters and it will allow us to conclude if there is a significant contribution of dark matter to the potential well that provides the stars the observed velocities. As mentioned before, the Mass Modelling will let us have a good insight into the problem of the formation of these structures.

Chapter 2

Theoretical Framework

Typical galaxies all around the Universe hold different structures such as stellar systems of between 10^2 and 10^6 stars which orbit their galactic core . We call these interesting systems star clusters and they are basically divided into two main types: Open Clusters and **Globular Clusters**.

- Cumulos globulares (definiciones, propiedades, formacion y evolucion, poblaciones estelares, etc)

2.1 Globular Clusters

Globular clusters are very massive stellar systems that can contain from thousands to millions of stars in a nearly spherical distribution spread over a volume of several tens to about 200 light years in diameter. These stellar systems are composed of old stars and they do not contain gas or dust.

The average star density in a Globular Cluster is about 0.4 stars per cubic parsec. In the dense center of the cluster, the star density can increase from 100 to 1000 per cubic parsec. However, even in the center of clusters, there is still plenty of space between the stars

A way of analysing globular clusters is to use Colour-Magnitude diagrams. A colour-magnitude diagram is a plot of the apparent magnitudes of the stars in a cluster against their colour indices. Globular clusters nearly all have very similar colour-magnitude diagrams

Globular clusters revolve about the nucleus of a galaxy on orbits of high eccentricity and high inclination to the galactic plane. About a third of globular clusters are concentrated

around the galactic center. A typical cluster has a period of revolution around the order of 10^8 years. A cluster spends most of its time far from the center of a galaxy, and so most of them can, and have been discovered in the spaces between galaxies.



FIGURE 2.1: Globular Cluster M15, taken by the Hubble Space Telescope with an exposition time of 900 seconds. Image by NASA

To ensure the stability of an isolated cluster, the average speed of its individual stars must not exceed the escape velocity from the cluster. If this occurred, the stars would escape into space, and the cluster would dissipate. If the stellar velocities are low enough to satisfy this condition, then the cluster is gravitationally bound, i.e. the force of gravity is strong enough to keep the member stars from escaping.

Due to clusters moving in various orbits in the Galaxy, they are bound together with gravitational forces that are stronger than the disrupting forces exerted on it by the Galaxy or other nearby stars, and this results in an added condition for the stability of a cluster. Another factor in the stability of clusters is size-the smaller and more compact the cluster, the greater its own gravitational binding force compared with the disrupting forces, and the more chance it has to survive to old age.

Because globular clusters are highly compact systems, they are consequently very stable, and so most globular clusters will probably maintain their identity almost indefinitely.

But even these clusters lose some stars, especially if they have a slow mass. This is due to there always being a few stars in a cluster that move faster than the cluster's average speed.

When a star escapes, it carries with it energy, removing this energy from the cluster as a whole. This eventually results in the cluster developing a tightly bound core surrounded by a rarefied halo of stars-much

In the dense core of a cluster, the stars in it occasionally collide, and some of the debris eventually coalesces. Predictions indicate that this dynamical evolution could lead to the development of a large Black Hole at the cluster's center. At the same time, a few stars in the outer parts of the cluster would continue to escape. The escape rate and dynamical evolution for the rich globular clusters are so slow that the clusters can easily survive for many billions of years, remaining mostly unchanged.

Proxima Centauri, and it is 4.2 light-years, or about 1.3 parsecs. Thus, if we were able to draw a sphere around the Sun with a radius of 1.3 parsecs, it would only contain 2 stars: the Sun and Proxima Centauri. If you were to draw this same sphere in the center of the globular cluster M13, it would contain approximately 10,000 stars.

The first globular cluster discovered, but then taken for a nebula, was M22 in Sagittarius, which was probably discovered by Abraham Ihle in 1665. This discovery was followed by that of southern Omega Centauri (NGC 5139) by Edmond Halley on his 1677 journey to St. Helena. This "nebula" had been known but classified as star since ancient times. Next followed the discovery of M5 in Serpens Caput by Gottfried Kirch in 1702, and that of M13 in Hercules, again by Halley, in 1714. De Chéseaux's list of (21) nebulae of 1746 contains, in addition, two new globular clusters, M71 and M4, while Jean-Dominique Maraldi discovered M15 and M2 in September of this year (1746). Guillaume Legentil possibly or probably discovered NGC 6712 in 1749. Nicholas Louis de Lacaille's catalog of (42) southern "nebula" of 1751-52 contains 8 globular clusters (among them 5 new ones), while Messier's catalog of 110 objects contains a total of 29 globulars, 20 of them new discoveries. Charles Messier was the first to resolve one globular cluster, M4, but still referred to the other 28 of these objects in his catalog as "round nebulae." Thus, in summer 1782, before William Herschel started his comprehensive deep sky survey with large telescopes, there were 34 globular clusters known. Herschel himself discovered 36 new globulars, bringing the number of known globulars to 70. He was the first to resolve virtually all of them into stars, and coined the term "globular cluster" in the discussion adjacent to his second catalog of 1000 deepsky objects (1789).

Home/News/Hubble observations cast further doubt on how globular clusters formed
 submit to reddit23Share on print Hubble observations cast further doubt on how globular

clusters formed A theory to explain how the Milky Ways mixed-generation globular clusters formed doesnt work in a nearby dwarf galaxy. By Hubble ESA, Garching, Germany — Published: Thursday, November 20, 2014 RELATED TOPICS: STARS — GLOBULAR CLUSTERS Globular clusters in Fornax Dwarf Galaxy This NASA/ESA Hubble Space Telescope image shows four globular clusters in the dwarf galaxy Fornax. New observations of the clusters show they are very similar to those found in our galaxy, the Milky Way. The finding is at odds with leading theories on how these clusters form in these theories, globular clusters should be nestled among large quantities of old stars and so the mystery of how these objects came to exist deepens. NASA/ESA/S. Larsen (Radboud University, the Netherlands) Thanks to the NASA/ESA Hubble Space Telescope, some of the most mysterious cosmic residents have just become even more puzzling. New observations of globular clusters in a small galaxy show they are very similar to those found in the Milky Way, and so must have formed in a similar way. One of the leading theories on how these clusters formed predicts that globular clusters should only be found nestled in among large quantities of old stars. But these old stars, though rife in the Milky Way, are not present in this small galaxy, and so the mystery deepens.

Globular clusters large balls of stars that orbit the centers of galaxies but can lie very far from them remain one of the biggest cosmic mysteries. They were once thought to consist of a single population of stars that all formed together. However, research has since shown that many of the Milky Way's globular clusters had far more complex formation histories and are made up of at least two distinct populations of stars.

Of these populations, around half the stars are a single generation of normal stars that were thought to form first, and the other half form a second generation of stars, which are polluted with different chemical elements. In particular, the polluted stars contain up to 50100 times more nitrogen than the first generation of stars.

The proportion of polluted stars found in the Milky Way's globular clusters is much higher than astronomers expected, suggesting that a large chunk of the first-generation star population is missing. A leading explanation for this is that the clusters once contained many more stars, but a large fraction of the first-generation stars were ejected from the cluster at some time in its past.

Before now, we didn't know whether globular clusters in smaller galaxies had multiple generations or not, but our observations show clearly that they do!

This finding means that a leading theory on how these mixed-generation globular clusters formed cannot be correct, and astronomers will have to think once more about how these mysterious objects in the Milky Way and further afield came to exist.

Radial velocity measurements have revealed that most globulars are moving in highly excentric elliptical orbits that take them far outside the Milky Way; they form a halo of roughly spherical shape which is highly concentrated to the Galactic Center, but reaches out to a distance of several 100,000 light years, much more than the dimension of the Galaxy's disk. As they don't participate in the Galaxy's disk rotation, they can have high relative velocities of several 100 km/sec with respect to our solar system; this is what shows up in the radial velocity measurements. Ninkovic (1983) has estimated excentricities of globular cluster orbits.

To determine the physical orbits of globular clusters, it is required to know their proper motions in addition to the radial velocities. Cudworth and Hanson (1993) undertook some first rough determinations of proper motions with respect to background galaxies. From these and similar measurements, Van den Bergh (1995) estimated perigalactic distances, and Dauphole et.al. (1996) calculated first approximate orbits. Much more accurate data for proper motions became only available from astrometrical data obtained with ESA's Hipparcos satellite in 1997, from which space motions (Geffert et.al. 1997) and approximate orbits (Brosche et.al. 1997) could be determined.

2.1.1 Basics

2.1.2 Formation and evolution

Efforts are under way by several groups to produce more realistic simulations that properly include the effects of star formation, but the existing results suggest that the first star formation may occur in gas that has become highly condensed at the centers of the first dark-matter halos to form; even though such systems may not closely resemble most present-day galaxies, they may still be of interest as the possible birth sites of globular clusters, as will be discussed further below. The most realistic simulations of the formation of a spiral galaxy like our own have been made by Katz (1992) and Steinmetz & Muller (1994, 1995), who have simulated the evolution of a spherical galaxy-sized piece of a standard cold dark matter universe which is arbitrarily given the appropriate amount of angular momentum. The results show an initial chaotic stage during which mergers between clumps build up a dark halo and a stellar spheroid, followed by a period during which the remaining gas organizes itself into a disk. During the initial chaotic stage several small satellites are formed by the condensation of gas in peripheral dark-matter clumps, and these satellites may survive for a few orbits before being disrupted and merged into the forming galaxy. It is plausible that such small satellites could be the birth sites of globular clusters, as in the hypothesis of Searle (1977) and Searle & Zinn (1978) that the globular clusters in the outer halo of our Galaxy were formed in

protogalactic fragments which survived for a time as independent star-forming systems before being merged to build up the halo

In several earlier reviews of globular cluster chronology, it was concluded that the globular clusters in our Galaxy are not all coeval and that age differences of several Gyr exist in at least a few well-studied cases (e.g., Larson 1990a, 1992b). This conclusion still appears to be valid, and it is supported by the most recent discussion of this subject by Chaboyer, Demarque, & Sarajedini (1996), which is based on new age estimates for 43 globular clusters

Although our Galaxy has evidently not experienced any further major accretion events capable of disrupting the disk, it is possible that minor accretion events affecting only the halo and not the disk have continued to occur (Navarro, Frenk, & White 1994); indeed, several contributors to this meeting have already noted that our Galaxy is apparently just now accreting the recently discovered Sagittarius dwarf galaxy, which contains four already known outer-halo globular clusters including Arp 2 and Ter 7

Most of the clusters whose masses are larger than $10^5 M_\odot$ have lifetimes longer than the Hubble time

An important recent advance in our understanding of star formation has been the realization that nearly all stars are formed in groups or clusters of some kind. Even in the nearby Taurus-Auriga region, which has been regarded as prototypical of isolated star formation, the T-Tauri stars are actually mostly distributed in a hierarchy of small groups of various sizes

2.1.3 Observational Properties

The HR diagram for a typical globular cluster looks very different than that of an open cluster. There are no Main Sequence stars of types OBAF, but there are many red giants. The brightest stars in a globular cluster are those at the tip of the red giant branch in the HR diagram, which explains the red appearance of the bright stars in color images of the clusters. You can also see stars populating the horizontal branch (and also why it is called the horizontal branch), the asymptotic giant branch, and even some stars that have colors and magnitudes of F stars, but far fewer than the G stars just below and to the right of them on the Main Sequence.

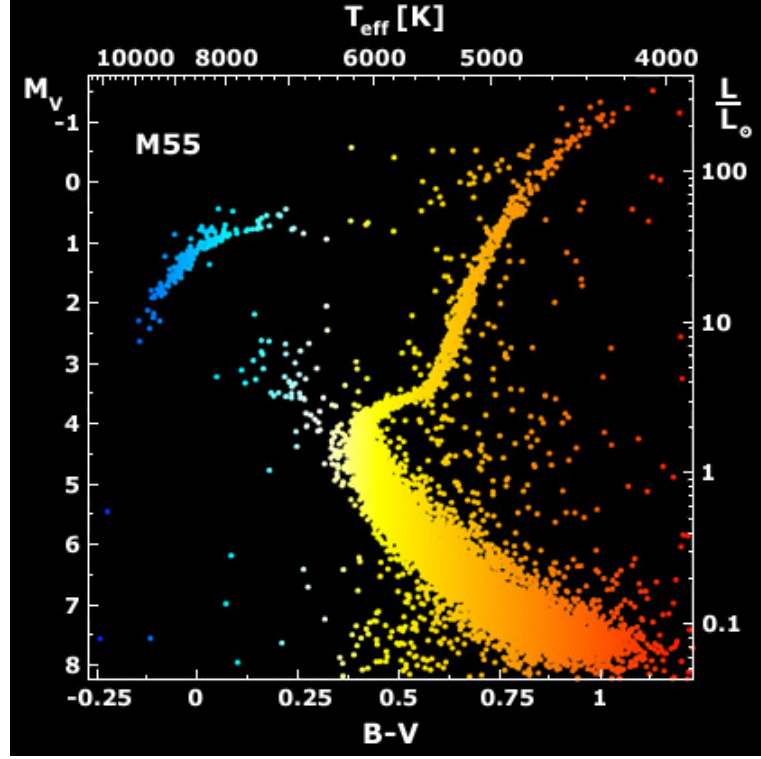


FIGURE 2.2: Color Magnitude diagram of M55. Image by NASA

(incluir poblaciones estelares)

2.2 Stellar System Dynamics

2.2.1 Potential Theory of Spherical Systems

(we focus on globular clusters)

The velocity dispersion profile of bright stars flattens when dark matter is present.

2.2.2 Collisionless Systems

$$\frac{df}{dt} = 0 \quad (2.1)$$

2.2.3 Dynamics

(Collisionless Boltzmann equation, Distribution Function, Jeans equations)

$$\frac{df}{dt} = 0 \quad (2.2)$$

$$\frac{\partial f}{\partial t} + \sum_{i=1}^3 \frac{\partial f}{\partial x_i} \dot{x}_i + \sum_{i=1}^3 \frac{\partial f}{\partial v_i} \dot{v}_i = 0 \quad (2.3)$$

$$\dot{v}_i = -\frac{\partial \phi}{\partial x_i} \quad (2.4)$$

$$\frac{\partial f}{\partial t} + \nabla f \cdot \vec{v} + \frac{\partial f}{\partial \vec{v}} \cdot \nabla \phi = 0 \quad (2.5)$$

Moment of order j of the distribution f

$$\overline{x^j} = \frac{\int x^j f dx}{\int f dx} \quad (2.6)$$

velocity dispersion tensor:

$$\sigma_{ij}^2 = \overline{v_i v_j} - \overline{v_i} \overline{v_j} \quad (2.7)$$

Anysotropy parameter:

$$\beta = 1 - \frac{(\sigma_\theta^2 + \sigma_\phi^2)}{2\sigma_r^2} \quad (2.8)$$

2.3 Scenario and Observations

there is widesprea belief that globular clusters cannot contain large amounts of dark matter, because of the Virial Theorem. This theorem relates the velocity dispersion σ at the center of the cluster to the total mass M_t and to the half-mass radius r_h of the system. For a one component globular cluster, that relation may be expressed as:

$$\langle \sigma^2 \rangle \approx 0.4 \frac{GM_t}{r_h} \quad (2.9)$$

However, if a dark component is also present, in the form of low-mass objects for example, the virial theorem must be modified to:

$$\sum_i M_t(i) \langle \sigma^2 \rangle_i = E_p \quad (2.10)$$

where the index i refers to the different stellar species, and E_p is the potential energy of the cluster.

2.4 Simulations

(N-body simulations, plummer sphere)

Chapter 3

Observations and Analysis

In order to study this problem about the dynamics of Globular Clusters in our galaxy we need scientific data that allows us to build a model that fits our observations. Under supervision of professor Juan Carlos Muñoz Cuartas and with three other undergraduate students from the University of Antioquia a trip to the OPD (Pico dos Dias Observatory) was made to Brazil in May 2014, besides the observational experience of the students, the main purpose of the trip was to get important data for this project. We needed two sets of data corresponding to spectra and photometric images of the Globular Clusters

The spectroscopic data allows us to determine the velocity dispersion profile in the inner region of globular clusters while the photometric data allows us to study the surface brightness distribution for them. We can use all of this information to infer the properties of the globular clusters' mass distribution in order to build complete dynamical models and therefore infer the amount of dark matter present in the globular clusters (if there is any).

3.1 Observational Procedures

Our stay in OPD consisted of two days in the main dome for the spectroscopic data (using the Perkin-Elmer (P&E) telescope with a 1.6m mirror and the Cassegrain Spectrograph) and four days in a smaller dome for the photometric data in the IAG telescope with a 0.6m mirror. In the following photograph, the domes of the observatory that we used for our observations:



FIGURE 3.1: OPD observatory seen from the air, the big dome was used for the spectroscopic data and the small dome at the low right part of the photo for the photometric data.

3.1.1 Spectroscopic Data

The first two days (May 14th and 15th) we took the spectroscopic data in the telescope P&E with a diameter of 1.6m. The main instrument was the Cassegrain spectrograph with a CCD Ikon-L camera and Filters BVR. The software we used was the recently installed software TCSPD which is built in a LabView environment for Windows (2010). Here's a photo of the telescope from inside the dome:



FIGURE 3.2: Perkin-Elmer telescope in the main dome in OPD used for the spectroscopic observations

We made the observations of dome flats, bias frames, comparison lamp frames, calibration stars and certain globular clusters of the milky Way organized by the best observation times using Simbad and Stellarium for the estimations of the coordinates and times respectively. We needed to keep an order of the observations to make the most of our observation time in OPD so we decided to organize our Globular Clusters in different groups or "chunks":

Chunk 1	ngc362	01:03:14.26	-70:50:55.6		18	24	periodic-6pm	12°54"
RA 0-5h								
Chunk 2	ngc104	00:24:05.00	-72:04:52.60		18	24	periodic-6pm	30°54"
RA 5-9h	ngc2808	09:12:02.00	-64:51:46.20		18,03	21,45		13°48"
	LTT 2415 (12)	05:56:24.20	-27:51:26					
	HR 1544	04:50:36.70	08:54:02					
Chunk 3	ngc3202	10:17:36.00	-46:25:00		18,14	22,49		
RA 9-15h	ngc4372	12:25:48.00	-72:40:00		17	1		18°36"
	ngc5272	13:42:12.00	-28:23:00		20,15	0,15		16°12"
	ngc5024	13:12:55.25	18:10:05.40		19,45	23,45		12°36"
	ngc4833	12:59:36.00	-70:53:00		17,3	1,31		13°30"
	ngc4590	12:39:27.98	-26:44:38.60		18,12	0,1		12°
	ngc5139	13:26:47.28	-47:28:46.10		18	2		36°18"
	ngc6752	12:25:48.00	-72:40:00		23,44	7,43		20°24"
	ngc5286	13:46:26.00	-51:22:27.30		18,19	2,18		9°6"
	LTT 3864 (12)	10:32:13.80	-35:37:42					
	HR4468	11:36:41.00	-09:48:08					
Chunk 4	ngc6205	16:41:42.00	-36:28:00		23,12	3,12		16°36"
RA 15-21	ngc6341	17:17:06.00	-43:08:00		23,48	3,48		11°12"
	ngc6362	17:31:54.99	-67:02:54		22,05	6,02		10°42"
	ngc6809	19:39:59.71	-30:57:53.10		0,12	24		19°
	ngc6397	17:40:42.00	-53:40:00		22,13	24		25°42"
	ngc6723	18:59:33.15	-36:37:56.1		23,32	24		11°
	ngc6715	18:55:03.33	-30:28:47.5		23,27	24		9°6"
	ngc6352	17:25:29.11	-48:25:19.8		21,58	24		7°06"
	ngc6541	18:08:02.00	-43:42:53.60		2,39	24		13°06"
	LTT 7987 (12)	20:10:57.10	-30:13:03					
	HR4963	13:09:57.00	-05:32:18					
Chunk 5	ngc7078	21:30:00.00	-12:10:00		2,45	24		12°18"
RA 21-0h	ngc7089	21:33:30.00	00:49:00.00		2,23	24		12°54"
	ngc7099	21:40:22.12	-23:10:47.50		2,13	24		11°
	LTT9239 (12)	22:52:40.90	-20:36:27					
	HR6634	22:41:27.40	10:49:53 AM					

FIGURE 3.3: Organized globular Clusters in groups for the proper times

Now, our set up configuration for the spectrograph was the following:

On May 14th, a diffraction grating of 900 lines per mm, a CCD IkonL and the central wavelength for the observations of 8500 Angstroms (with possibility of rotation of the slit 90, +45 and -45).we used the slit of 2.52" and obtained data for the globular clusters: NGC-5020, NGC-5272, NGC-4833, NGC-4590, NGC-5139, NGC-5286, NGC-6752, NGC-6397, NGC-6723, NGC-6715 and NGC-6541 using exposition times of 600 and 900 seconds. We also observed the calibration stars: HR-4963 and HR-4468 with 7 and 5 seconds. As it was the first day, we needed to be very careful in calibrating our instruments on order to have the objects in the right focus, we also made the rotation of the slit to use all the diffraction angles of the observations and our comparison lamps were of Ne-Ar.

On May 15th, we used the slit of 3.0", and used a central wavelength of 5500 Angstroms. This time we observed the following objects: NGC-2802, NGC-5024, NGC-4590, NGC-5139, NGC-5286, NGC-5272, NGC-6362, NGC-6397, NGC-6723, NGC-6502, NGC-6541, NGC-7078, NGC-7099, the stars HR-4468 and HR-7950 and we also observed Mars for pedagogical reasons. We used pretty much the same exposition times than the day

before, this time though, our comparison lamps were of He-Ne. All the data we took was in FITS format (Flexible Image Transfer System).

3.1.2 Photometric Data

The photometric data were acquired in the next four days (from May 16th to May 19th) in the 0.6m IAG telescope in OPD. We used the Johnson system for the different filters which were easily shifted with the given software in the control computers. Here a picture of the telescope from inside the dome:



FIGURE 3.4: IAG telescope used for the photometric data

On May 16th, we took all the calibration images, consisting of 20 bias frames with an exposition time of 0,00001 seconds; also 22, 11, 11, 20 and 10 flat frames for the B,I,R,U,V filters respectively, their exposition times differed, for U filter we took various frames of 60 and 30 seconds, for the B filter we took frames of 30 seconds each, 15s for I, 60s for R and 3s for V. We took our "focus" images to calibrate the instrument, and also various skyflats for all the filters. We targeted the following globular clusters and calibration stars in different filters: NGC5272, HR4961, NGC4590, NGC5139 AND NGC6397. The exposition time for the clusters was of 600 seconds and 2 and 4 seconds for the calibration star.

May 17th was a terrible night for observations because the sky was too cloudy and the only useful data we could get were dome flats for the filters I,R and V that we could use instead of the bad dome flats of the first day. The reduction using the flats of another day are decent but this is not the ideal situation since mechanical movements of the instrument might slightly change its configuration and therefore it probably ends up with a reduction that is not the ideal one for science purposes.

On May 18th we were more organized since we were getting familiar with the observations and therefore the data we got had little trouble in the upcoming analysis, even though the sky was clody at the end of the night. The science objects we observed were NGC5139, HR6308, NGC6723, NGC6541, NGC7078, HR7964 that were observed in the different filters. We got 20 bias frames, 14 dome flats in the vaious filters, but no skyflats.

On May 19th we observed the Clusters NGC5139, NGC4590, NGC6723, NGC6715, NGC6541, NGC6970, NGC5286, NGC639, NGC6541 and NGC6715, the calibration stars HR6386 and HR6386, 20 bias frames and flats for each filter.

3.2 First step for Anaysis

Our first goal in starting the analysis of all the relevant data was to organize all the images in order to reduce the time required to make the reductions. For every day the calibrations images, trash, calibration stars and objects were separated and they were given their correct names as they were in the headers and compared with the information sheets we filled at the time we were doing the observations. With the use of an account on the galaxy.udea.edu.co cluster, for proper and quicker analysis and safety of the data, all the files were correctly organized.

The next step was the reduction of all the images with the calibration files for each day, I started the photometric data to acquire certain skills in the use of IRAF because the reduction of the spectroscopic data was to be a little more complex and needed a deeper understanding of IRAF packages.

I started with the cluster NGC-5139 (ω Centauri) because we got lots of data for that cluster in OPD and also because ω Centauri is a well known globular cluster since it is the largest in our galaxy and we can get a lot of information from the web.

After the photometry of that cluster, the most relevant part of the reduction was to be made. The reduction and analysis of the spectroscopic data (May 14th and 15th), the methods for these reductions are quite special and are the most relevant part of the analysis because that is our most valuable information. The reduction was to be made very carefully because a good spectroscopic analysis depends upon a good reduction of the data. Just as with the photometric data, the first procedures were made for the Cluster NGC5139 to understand and master the techniques of the reduction and extractions.

3.3 Photometry

The photometry was made by the two traditional methods, PSF photometry and Aperture Photometry; even though the magnitudes calculated using both methods are quite different, the calibration constant between the two methods gave a good relation between them and made us trust the photometry results.

But first, the reduction of the data had to be done. The first step is to characterize the calibration images in order to see if there are any errors associated with the instrument or the way that the observations were made. By doing this we found that most of the flat-field images had brightness gradients in the corners and this was a problem we needed to correct because the increased value on the counts in these corners would affect the normalization of the super-flat that we would use to reduce the science data. Another systematic error that we found in all of our calibration and science frames was the presence of a strange water-looking figure at the top left corner of them, although it can be removed with the correct reduction, it obviously affected the CCD sensitivity by the time of the observations. Also, some filters showed a higher sensitivity to this systematic errors but at the end, the photometry could be made in the best data so that the dirty images don't affect our results.

In order to see how the data would be affected by the systematic errors we just mentioned, we produced a composite image using three images with the filters U,V and R and we did the same with the flats in those filters, the results are shown in the following figure:

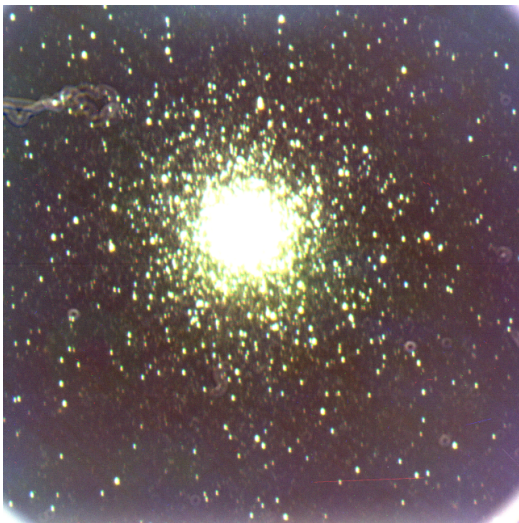


FIGURE 3.5: Composite image of NGC5139 without being previously reduced

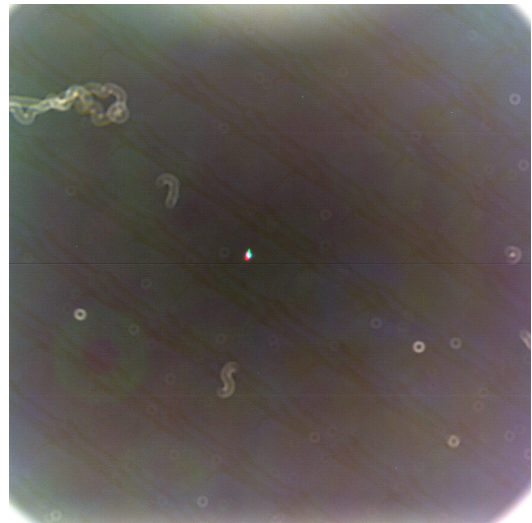


FIGURE 3.6: Composite image of the flats showing the noise that needs to be extracted

What we can infer from these images is that the flat fields and the bias frames contain the same noise that the science data thus giving us a good result in the reduction.

Once all the characterization is made we can reduce our important data using IRAF following the conventional steps consisting of:

- Building a Superbias: *Zerocombine* allows us to create the superbias using the median.
- Subtracting the Superbias to every flat and science data: We subtract the Superbias to every flat frame with no distinction on the filter, this is easily made using the task *imarith*, we also subtract them from the original science images.
- Building Superflats: It is necessary to create a Superflat frame for each filter because the response of the CCD and will be different for different wavelengths, we use *imcombine* to do this and this time we use the mode for better results.
- Divide the Superflats by the median: In order to normalize the flatfields we find the mode of each frame with *imstatistics* and then divide them by that value



FIGURE 3.7: Example of one of the Normalized Superflats for the I filter

- Reduce the science data: Finally, we divide the original images of the clusters and stars (with the bias subtracted) by the normalized Superflat to get the reduced images. This can easily be made using the task *imarith*.

3.3.1 Aperture Photometry

Now that the reduction has been made and the corrections pixel by pixel have been applied, we can proceed to do the photometry using the simplest technique, known

as Aperture Photometry which consists of adding up the pixel counts within a circle centered on each star of the cluster and subtracting the quotient of the per-pixel average value of nearby sky count divided by the number of pixels within the aperture. This will result in the raw flux value of the target object. This Aperture Photometry was done using the task *Phot*:

For stars in the NGC5139 cluster, one must choose a very small aperture of the sky because the surrounding stars contribute to the flux that needs to be extracted and they are very close to each other, as we can see in the following figure:

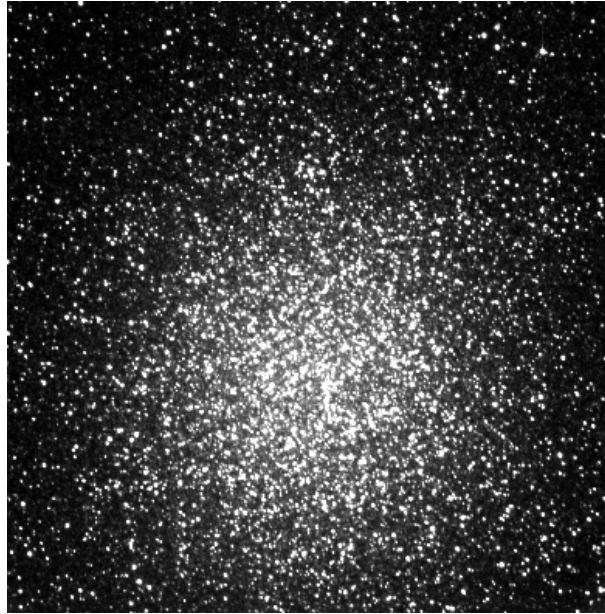


FIGURE 3.8: One of our observations of NGC5139 in the V filter with an exposition time of 480 seconds

with the task *imexamine* I find a value for the FWHM of 6.4 which I will use to set the size of the apertures to do the photometry. For the size of the aperture containing each star I chose an aperture size of four times the FWHM of the point spread function associated to the stars because it is the one that best fits the photometry and minimizes the error (calculated for some stars pressing "a" with the *imexamine* task) and for the width of the aperture I chose a value of 2.5 times the FWHM.

Another important value to take into account before editing the parameters in *phot* is the medium value of the background on the sky, in the case of this cluster, I do an average on many different places in the background of the image and find a value of sigma for the image that is equal to 53.45.

Now the photometry is done by changing some of the parameters including the readnoise and the gain. In the *fitskyparts* task inside *phot* I set the inner radius called annulus

to be 25.6 (4FWHM) and the intermediate width called dannulus of 16 (2.5FWHM). Finally, before running the task I make sure I do the photometry using various apertures because I want to see which one minimizes the errors. I set apertures to be 1FWHM, 1.5FWHM, 2FWHM, 2.5FWHM, 3FWHM, 3.5FWHM and finally run the task. The results of the magnitude found for one of the star as a function of the size of the aperture is displayed below:

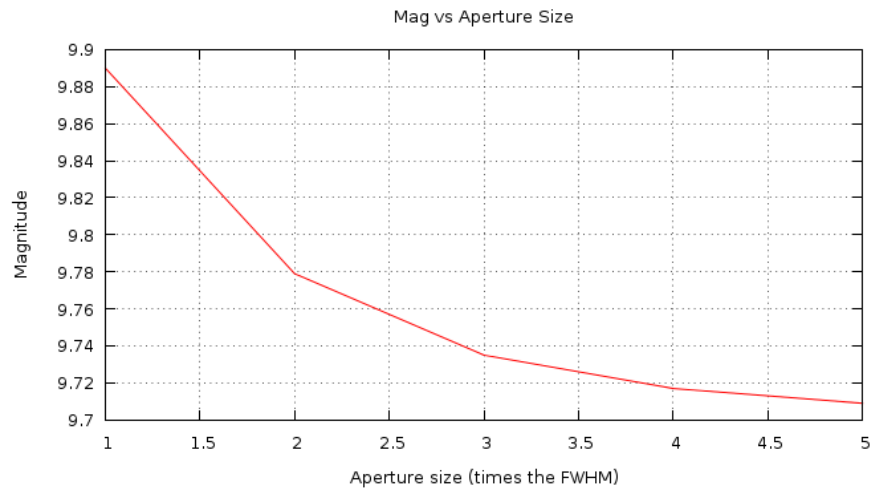


FIGURE 3.9: The magnitude decreases for bigger apertures

As we can see in the figure above, the magnitude of the chosen star in the cluster decreases for bigger apertures but so does the error because the space around the star is crowded of more stars and noise coming from the stars in the background so we infer that for crowded areas like this one the best choice is a small aperture. Although the results may not be as convincing, the use of another technique of photometry allows us to compare the results and see if the choice of a small aperture is a good way to fix or avoid the problem of the big noise of the background. For this purpose we use the technique of Point Spread Function (PSF) Photometry.

3.3.2 PSF Photometry

There exist many ways to count photons for an image taken with a CCD camera, but all of these ways obey the same principle of energy distribution in luminous objects. The point spread function for each of these objects is an assigned measure from the probabilistic distributions that approach quite well to the count of photons that one wants to do in the photometric analysis of astronomical images. The PSF photometry technique makes the most of the PSF of the objects using certain packages and tasks in a slightly different way than aperture photometry.

When doing photometry in a very crowded field, such as a globular cluster (NGC5139 in this case), where the profiles of stars overlap significantly, one must use de-blending techniques, such as point spread function (PSF) fitting, to determine the individual flux values of the overlapping sources.

This time, I made PSF photometry to some of the stars in the same cluster I started with (NGC5139), I chose bright ones that were relatively isolated to surrounding stars in order to make the estimations more accurate. First I use the same value of the PSF of some of those stars that I found doing the aperture photometry (6.4), this time the value of the sky is going to be higher because this value is going to determine the amount of stars that the task *daofind* will select to do the photometry and I'm only interested in the brighter ones, a value of 2000 would filter out many of the fainter stars and the background. This value is using the standard deviation and making an average over many values found by the command "m" in the interactive mode of *imexamine*.

After setting all the parameters of *phot*, *daopars* and *findpars*, I run *daofind* which will find the stars that match the criteria I mentioned above and will create a text file with the coordinates of those stars and will give to each star an ID.

The task *tvmark* allows me to highlight the stars of the cluster in the display mode using the text file with the coordinates, and *txdump* allows me to put explicitly the coordinates of those stars in a text file that I use for the aperture photometry to make the first guess of the PSF of the stars.

The best way to correctly select the stars that will be used for the modelling of the PSF is by using the task *pstselect* that will select the stars that are well isolated using statistical techniques.

Once the stars are selected, the next step is to use the task *psf* which matches the point spread functions of the input images, by running this task, one can visualize how the PSF is modelled for each star and accept or decline the results to be stored, the interactive mode allows us to take the decision by analysing the modelling like the one we can see in the following image of the xterm:

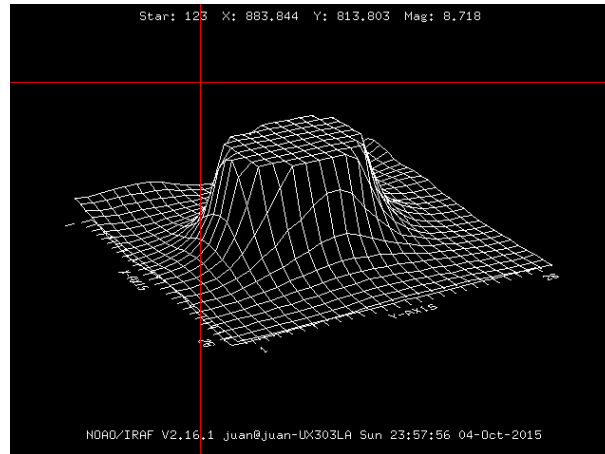


FIGURE 3.10: PSF modelling, the interactive mode allows us to accept or decline the result by pressing "a" or "d", in this case the PSF is not a soft curve with a Gaussian behaviour so it can probably be discarded

After running *psf*, there will be created an image that contains the residuals of the psf modelling which can be seen like this:

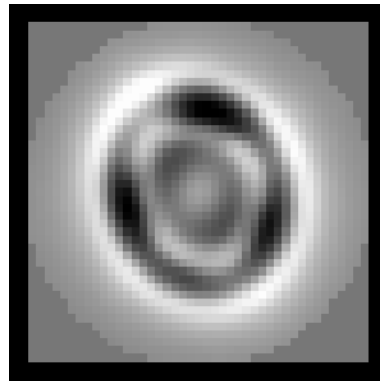


FIGURE 3.11: PSF residuals

With the task *seepsf* another image will be created but this time it will actually look like a star:

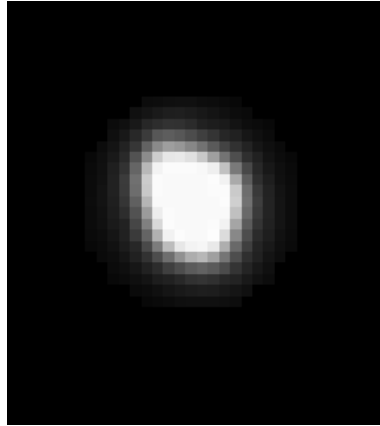


FIGURE 3.12: Seepsf converts a sampled PSF lookup table to a PSF image which can be visualized

Finally, after the modelling of the PSF was made, the task *allstar* does the photometry of the cluster using the results we just stored in the current directory. The results of the PSF photometry gave us magnitudes smaller for those stars than the results given by the aperture photometry but there is a constant difference for all the stars which we can assume to be a calibration constant between the two methods.

3.4 Spectroscopy

This is the most important part of the observational part of this project since the spectroscopic data has the most valuable information about the Globular Clusters and because our data were obtained in the largest and best telescope in OPD. Also, spectra of this kind is not easy to find in the scientific databases so we have important data to work with. The first spectroscopic procedures were also made for the ω Centauri cluster since we need to master the reduction and extraction techniques first in order to be able to start the scientific work about the mass modelling of the clusters.

3.4.1 Spectroscopic Reduction

The spectroscopic reduction is made by following some steps taking into account that we did not take any Skyflats so we need to create a response function using our dome flats. The steps are:

- First we make a Superbias combining all the bias frames and then we subtract it from all the lamp, targets and flat field frames.

- It was important to analyse the flats to see which ones are saturated, we consider that values over 65,000 counts (using *implot*) show saturated data. The ones that we could trust for May 14th were ten images called flats_0012 to flats_0021.

- The pre-superflat is made using the median given the number of images.

- We need to make a trimming in all images because there are some regions in the images that show unexpected luminosity, this is probably due to border errors in the camera or the obturator time of relaxation. The zones we decided to cut were:

[0-100] and [575 to the end]

- A critical step is the creation of a response function, this is made by collapsing the pre-superflat to one column using *blkavg*. The useful image for the creation of the Superflat is done by combining this column with *blkrep*. This gives us an image that's uniformly distributed in the dispersion axis with the following IRAF commands.

```
blkavg MasterFlat.fits[1:475,*] AvgFlatCols 475 1
```

```
blkrep AvgFlatCols AvgFlatColsMaster 475 1
```

- The pre-superflat is now divided by the response function we created (*AvgFlatColsMaster*) and this gives us the Superflat that we will use to reduce our data.

- Finally, the task we use to remove the cosmic rays is *lacos*, and it gives very accurate results, as it shows the "mask" image with the removed cosmic rays.

The vertical axis of the spectra is the dispersion axis and the horizontal axis is the spatial separation between stars. To visualize the reductions we have the following figures taken from the dirty and reduced NGC5139 spectrum (Day of observation May 14th):

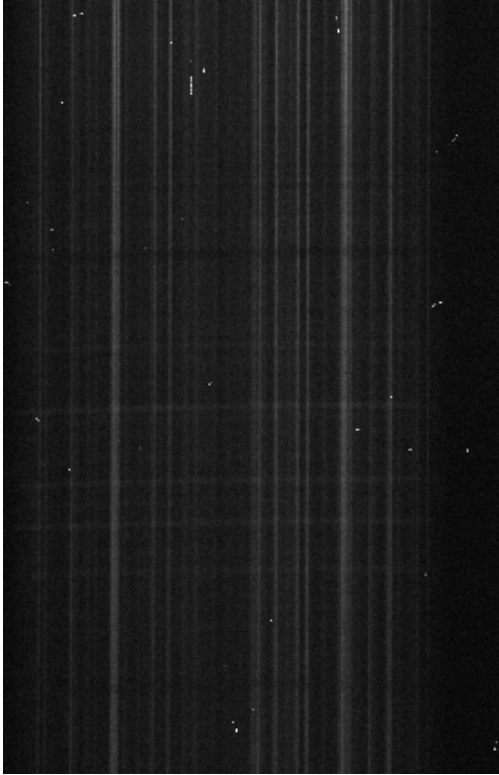


FIGURE 3.13: The dirty cluster spectrum has a small signal to noise ratio, besides border effects (such as a big gradient) that need to be trimmed and cosmic rays.

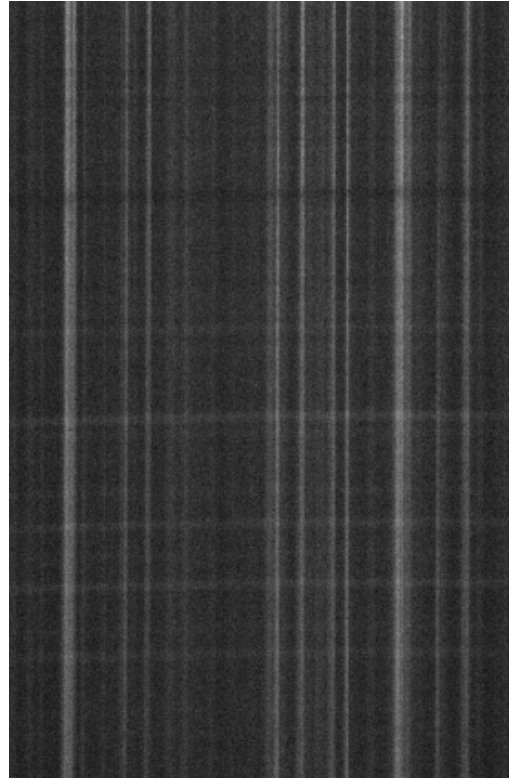


FIGURE 3.14: The clean spectrum without the bad data at the borders that needed to be trimmed, with a higher signal to noise ratio and the cosmic rays removed,

The images above are just a fraction of the whole image which is actually longer in the vertical direction, we show only a fraction for visual purposes. As we did not take skyflats, some telluric lines are visible even in the reduced spectrum but this can be solved doing a correct calibration and carefully examining the extraction of the spectra of the stars.

3.4.2 Extraction

Once the reduction is ready, we can proceed with the extraction of the spectra of the calibration stars and also the spectra of the stars in the clusters, this procedure is made with the task *apall*. First, I did the extraction of the two calibration stars for each night of observation. For May 14th our calibration stars were HR-4963 and HR-4468 and for May 15th our calibration stars were HR-4468 and HR-7950.

Taking special care of correctly choosing the background, and with the following parameter configuration:

b_number: 100

background: fit

weight: variance

saturate: 65215

rdnoise: 6

gain: 1

Interactively, one must choose very precisely the background regions to extract the spectrum and do the fitting routines with different orders until the best results are reached, the areas of the background are changed using the commands "b" (for setting the background mode) and "s" (for setting the range). A good choice for the background in the case of the calibration star is relatively easy but in the case of the extraction of the stars in the clusters one must take into account the high noise introduced by the other stars and the background so for every star one must zoom into the window using "w" and "a" between the boundaries of the range to be zoomed.

If a good choice of the background and the dispersion axis is well fit, the task runs straightforward to get the spectrum of the star. In the case of the calibration lamps, the procedure is quite similar because the extraction of their spectrum is done with *apsum*, which is very similar to *apall*.

For the calibration star HR4468, the extracted spectrum looks like this:

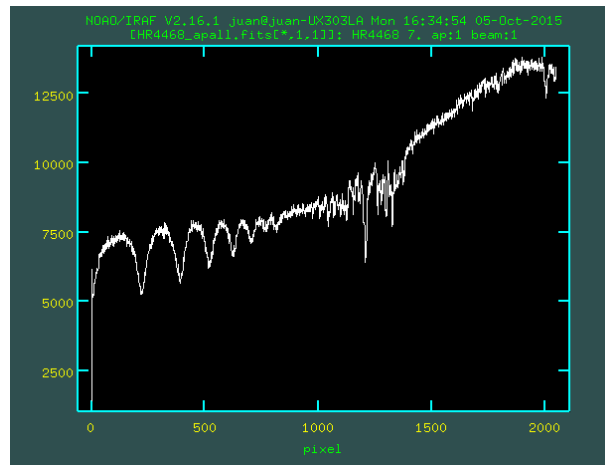


FIGURE 3.15: Extracted spectrum of HR4468, note that the dispersion axis is still in terms of pixels instead of Angstroms, this is fixed by doing the wavelength calibration, also the direction will be shifted because the strong sodium absorption lines are in the left end of the spectrum and they should be in the right end

3.4.3 Wavelength Calibration

Once the spectrum is extracted, the following step is to calibrate it in wavelength in order to make it useful for scientific analysis. The wavelength calibration is made many tasks of IRAF like *Identify*, *Refspec* and *Dispcor*. First, with *identify* I use the interactive window in IRAF to select some prominent lines in the spectrum of the calibration lamps and assign them their correct wavelength using the theoretic spectrum of the lamp. In this case our calibration lamps were Ne-Ar (for May 14th) and He-Ar (for May 15th) and OPD observatory provided us the theoretic distribution of emission lines of them. As we can see below:

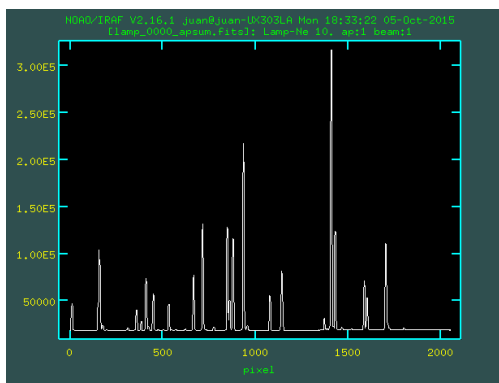


FIGURE 3.16: Emission lines of the Ne-Ar calibration lamp that need to be wavelength calibrated, the horizontal axis is in pixels and needs to be calibrated to units of wavelength

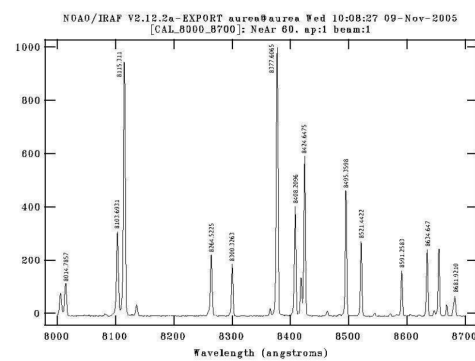


FIGURE 3.17: The theoretic emission lines of the Ne-Ar lamp provided by OPD that we use to make the wavelength calibration, note that the lines to be matched are going in the opposite direction

Running *identify* in the interactive mode and using "m" to select the larger lines and typing the wavelength, the task creates a file stored in a new folder "database" with the pixels with their corresponding values in units of Angstroms. After that, the targets (Calibration Stars and Globular Clusters) were to be calibrated with these files so it is necessary to edit their header to assign them the reference frames. It is enough to change the REFSPEC1 image header on each lamp file in order to set the wavelength calibration.

The task that actually does the calibration on wavelength for the science targets is *dispcor*, it is only necessary to run the task over all the targets with their own wavelength calibrated lamp to get the calibrated spectrum which is the useful and important file to make the analysis of the width of the lines and their redshift.

The wavelength calibrated spectrum of the star HR4468, after running *dispcor* is:

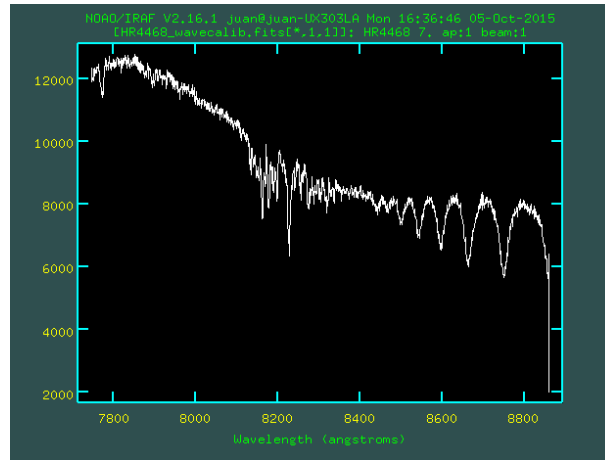


FIGURE 3.18: The spectrum has been wavelength calibrated as we can see in the horizontal axis which is in units of Angstroms. Note that the strong absorption lines are now in the right part of the spectrum, this is because in the process of making the wavelength calibration, the direction of the axis was changed.

3.4.4 Flux Calibration

This final part of the extraction and calibrations has the aim of calibrating the CCD chip response, spectrograph+telescope throughput and allow for atmospheric extinction. The result is a spectrum as observed from outside the atmosphere with an ideal uniformly sensitive detector+telescope+spectrograph. Basically, what the flux calibration does is, it takes from a tabular compilation the energy distribution of the standard star, it corrects this energy distribution for wavelength-dependent atmospheric extinction, it compares it to the energy distribution of the observed spectrum and derives from such a comparison the function that gives the response of our system for every wavelength.

The flux calibration takes place in three parts: Calibrating from the standard star, calculating the sensitivity function of the instrument, and finally, applying the calibration to the spectra. We will use the task *observatory* to determine observatory parameters, *standard* to flux calibrate each standard star, and *sensfunc* to finally determine the wavelength response and the solution will be applied to the spectra by the task *calibrate*.

In the first part, the calibration is made with one of the stars that are already included in IRAF, there are many stars so there's quite a good amount of options to choose. So the first task is the task *standard*. The observatory parameter is specified as LNA (Laboratório Nacional de Astrofísica) which is in IRAF's database.

The task *standard*

The task *standard* determines calibration pass-bands and writes them to a file called "std". The trick here is to specify the location of the the input extinction and flux calibration files. To do that, I edit the parameters of standard with the following routes:

Extinction file: onedstds\$/ctioextindt.dat

Directory containing calibration data: onedstds\$ctionewcal/

Starname in calibration list: l9239

Where I chose the Star l9239 because it has the spectral range that we use in our calibration Stars. And running the task interactively would be enough for this step.

The task *sensfunc*

The task *standard* just recorded response of each standard star so the next step is to put the results together and find a proper wavelength dependence of instrumental sensitivity and atmosphere transparency using the task *sensfunc*. It creates an image with a default name sens.0001. IRAF needs to have some general idea of atmospheric extinction before to start, so I set again extinct onedstds\$ /ctioextinct.dat.

Now, running the task interactively and taking into account that the function used to fit the instrumental response will be usually of very high order. A good idea is to use spline3 fitting (:function spline3) with some 20 pieces, i.e. (:order 20). Finally "q" exists the *sensfunc* task and writes the sens.0001 image.

The task *calibrate*

The solution to each star to be calibrated is done with the task *calibrate*. Editing the parameters of calibrate to set the appropriate extinction table: extinct onedstds\$ /ctioextinct.dat would be enough for this purpose. The task is run over all the wavelength calibrated spectra which had their airmass and other parameters appropriately set by the *eso.set* procedure. And finally it gives the flux-calibrated spectra ready for the relevant analysis concerning radial velocities.

After the flux calibration, I notice that the extremes of the spectra have irregularities given by the flux calibration procedures, but that can be cut because they don't have any relevant information.

For the star calibration I cut from 0 to 45 and from 1860 to the end, using *imcopy*:

```
imcopy flux_calib_star_fits[45:1860,*] cut_flux_calib_star.fits
```

The wavelength and flux calibrated spectrum of the star HR4468, now ready to be used for radial velocities determination looks like this:

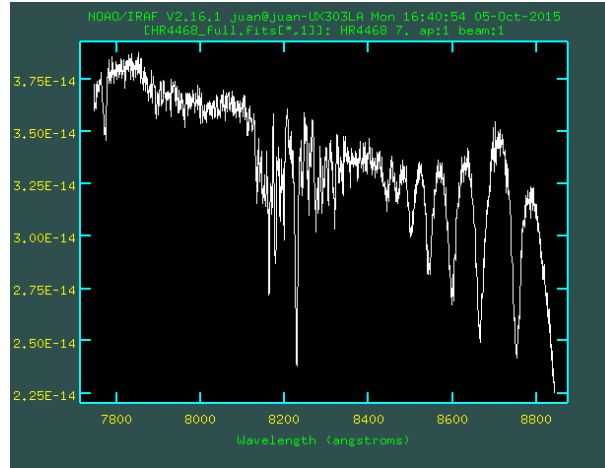


FIGURE 3.19: Flux calibrated spectrum of HR4468 calibration star

In order to normalize the spectrum, first I find the maximum value in the spectrum using minmax and then I divide the whole image by this value.

Something that can be very useful is to have the data of the spectrum in a different format so that its information can be well used and analyzed with simple programs like gnuplot, for this purpose, it is useful to create an Ascii table from the spectrum. For this purpose I need to first convert my image to a 1D image using the task *scopy* and setting `format=onedspec` (this is only necessary if the spectrum was extracted in 2-D).

Now, with the image ready in 1-D, I use the task *wspectext* to create the Ascii table like this:

```
wspectext ready_flux_star.0001.fits normal_cut_flux_star_calib.txt
```

In principle, the same procedures hold for the star in the Globular Clusters, even though it must be made more carefully because the background noise and the crowded space surrounding them affects the spectra and it might change the values of the radial velocities, an example of a wavelength and flux calibrated spectrum of one of the stars in the NGC5139 cluster is:

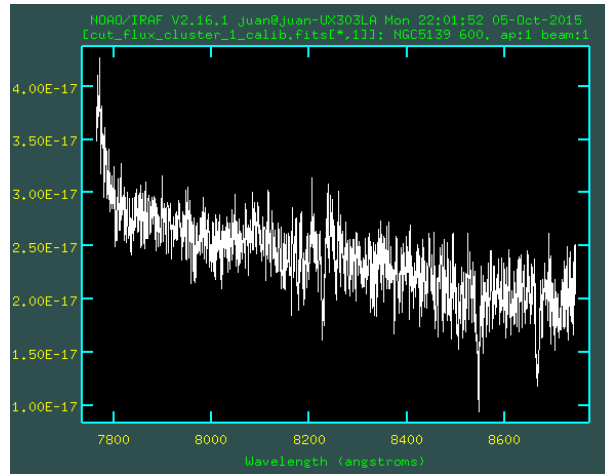


FIGURE 3.20: Spectrum of one of the most prominent stars in the NGC5139 cluster, after all the calibrations have been made

Although the noise is evidently higher than in the case of the spectrum of the calibration star, it is possible to see (even with the naked eye) two of the calcium strong absorption lines around 8543\AA and 8663\AA . After all of these procedures have been made to the important stars in the clusters and the calibration stars, the next step is to explore the best way to determine radial velocities with the use of some sophisticated tasks in IRAF such as *RVSAO*, as we discuss in the next section.

3.5 RVSAO and radial velocity determination

bvcor:

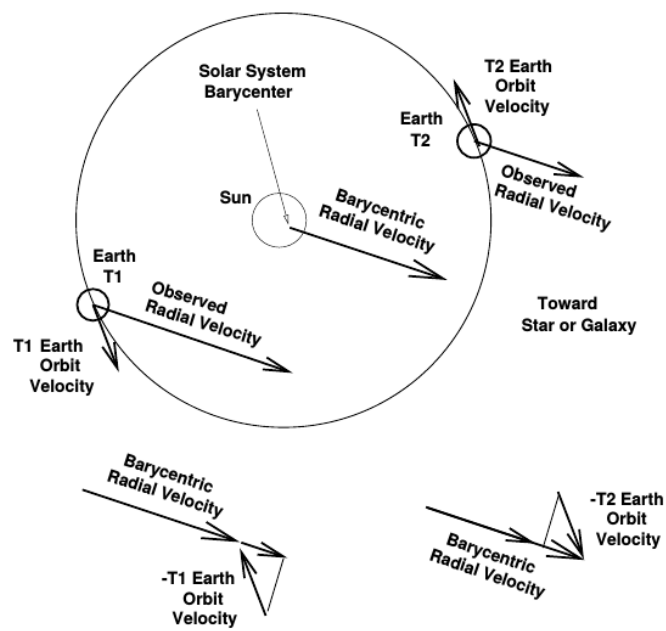


FIGURE 3.21: Correction of radial velocity observed from the earth to that which would be observed from the center of mass of the solar system is shown at two different times in the earth's orbit, taken from the RVSAO paper!!!!!!

Chapter 4

Modelling

- C

Chapter 5

Conclusions

- Cfjsdkafdsjafkl

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