Stellar Structure and Evolution Lab Report: $2M_\odot$ Star Evolution

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1. INTRODUCTION

In order to properly understand the evolution of a solar-like star from pre-main sequence to White Dwarf, we used the software MESA [8] which includes a very powerful collection of libraries and models for stellar astrophysics. We run this program for a star with a mass of $2M_\odot$ and investigate and analyze the results in the context of stellar evolution along the most relevant evolutionary stages taking into account the restrictions but also advantages provided by the program. This report includes the analysis of the various physical quantities that are relevant for a comprehensive understanding of the evolution of stars similar to our sun. We focus mainly in the quantities that we can relate to the well known Hertzsprung-Russell diagram so that we can compare our results to the theoretical framework on stellar evolution.

The outline of this report is as follows: Section 2 reviews the conducted methods and the constraints used to run the simulation. Section 3 contains the relevant figures for our $2M_\odot$ modeled star which gives us the physical insights of its evolution. Section 4 is has the detailed discussion about the physical explanation of the evolutionary stages of the star and the interpretation of the figures. Section 5 briefly summarizes the main conclusions of the assignment.

2. METHODS

For this computer lab, we made use of the software MESA (Modules for Experiments in Stellar Astrophysics). First we made a proper compilation of the libraries and file configurations which implicitly contain the information for a $2M_\odot$ star. Although most of the parameters were left as default, we changed `profile_interval` to a value of 5 and `max_num_profile_models` to a value of 1179 which were the output files that we got from the simulation. The first parameter sets the time steps frequency at which the files containing the data are written, so it was necessary to chance it because it basically sets the number of points that we will use for our plots. The latter controls the number of created files of the simulation so it's useful to have a large number of files to have a smooth behavior of the data.

3. RESULTS

In this section we present the figures corresponding to the evolution of the core in the $\log(\mathrm{T}) - \log(\rho)$ plane, the Hertzsprung Russell diagram, the convection in the pre-main sequence phase and in the main sequence phase. Their physical interpretation which will be discussed in detail in section 4.

In Figure 1 we present the $\log \mathrm{T}_c - \log \rho_c$ plane for the evolution of the core of a $2M_\odot$ star. The temperature (x axis) is given in Kelvin and the density (y axis) is given in $\mathrm{g/cm^3}$. Colored dashed lines represent limits between different regions (Radiation Pressure, Ideal Gas Pressure, NR e^- degeneracy, and ER e^- degeneracy) driven by their corresponding equations of state given in Table 1.

Zone	$\mathrm{T} \propto \rho$ relation
Radiation Pressure	$\mathrm{P_{rad}} = \frac{R}{\mu}\rho\mathrm{T}$
Ideal Gas Pressure	$\mathrm{P_{gas}} = \frac{1}{3}a\mathrm{T}^4$
NR e^- degeneracy	$\mathrm{P_{NR}} = \mathrm{K_{NR}}(\rho/\mu_e)^{5/3}$
ER e^- degeneracy	$\mathrm{P_{ER}} = \mathrm{K_{ER}}(\rho/\mu_e)^{4/3}$

Table 1: Equations of state for different zones.

The blue dashed line marks the boundary between the radiation and ideal-gas pressure dominated regions (Eqn. 1), orange for the non-relativistic degenerate electron pressure (NR) and ideal-gas pressure (Eqn. 2), black for relativistic degenerate electron pressure (ER) and ideal-gas pressure (Eqn. 3) and finally green for the ER and NR dominated regions (Eqn. 4).

The line that separates the Radiation pressure dominated zone with the Ideal gas pressure dominated zone by letting $\mathrm{P_{rad}} = \mathrm{P_{gas}}$ and putting the temperature in terms of the density. This yields:

$$\frac{\mathrm{T}}{\rho^{-1/3}} = 3.2 \times 10^7 \mu^{-1/3} \tag{1}$$

For $\mathrm{P_{gas}} = \mathrm{P_{NR}}$ we get

$$\frac{\mathrm{T}}{\rho^{2/3}} = 1.21 \times 10^5 \mu \mu_e^{-5/3} \tag{2}$$

For $\mathrm{P_{gas}} = \mathrm{P_{ER}}$

$$\frac{\mathrm{T}}{\rho^{1/3}} = 1.5 \times 10^7 \mu \mu_e^{-4/3} \tag{3}$$

And for the $\mathrm{P_{NR}} = \mathrm{P_{ER}}$ we get:

$$\rho = 9.7 \times 10^5 \mu_e \tag{4}$$

where $\mu = \frac{1}{2\mathrm{X}+\frac{3}{4}\mathrm{Y}+\frac{1}{2}\mathrm{Z}}$, $\mu_e = \frac{2}{1+\mathrm{X}}$. The values for μ and μ_e were obtained using X = 0.7,Y = 0.28 and Z = 0.02.

The black continuous line is a theoretical prediction computed using Eqn. (5) [1]. The orange dot represents the position of the Sun's core in the $\mathrm{T} - \rho$ plane [7]. The colored dots represent the time evolution of the core of our model star and the color bar represents its age.

$$\rho_c = \left(\frac{R}{CG} \frac{T_c}{\mu M^{2/3}} \right)^3 \tag{5}$$

Where $C_n = 0.36$ [1]

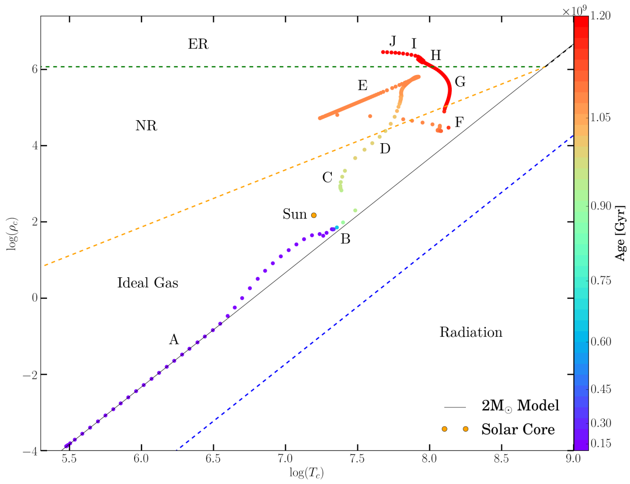

Figure 1. Evolution of the core in the $\log \mathrm{T}_c - \log \rho_c$ plane. The relevant evolutionary stages are labeled with capital letters. **A**: Pre-main Sequence, **B**: Main Sequence, **C**: First dredge-up, **D**: RGB, **E**: He burning core, **F**: He flash, **G**: AGB, **H**: Post AGB, **I**: Planetary Nebula, **J**: White Dwarf.

Figure 2 is the Hertzsprung Russell diagram and shows the time evolution of the star, the x axis is $\mathrm{T_{eff}}$ and the y axis is the Luminosity of the star. The labels indicate the evolutionary stages of the star and the color indicate its age.

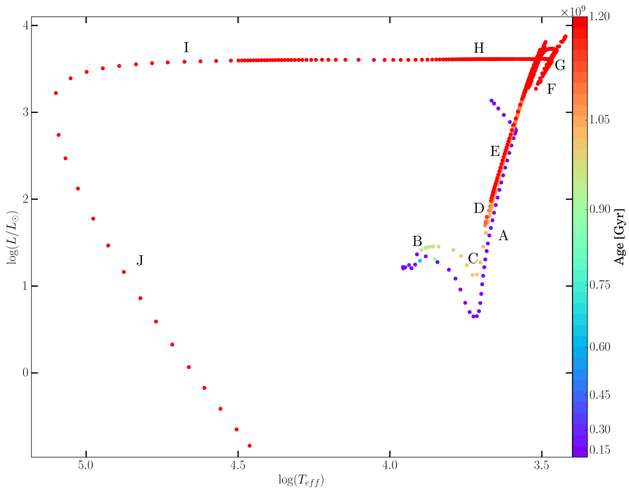

Figure 2. Hertzsprung-Russell diagram. Evolutionary stages are labeled as in Figure 1. **A**: Pre-main Sequence, **B**: Main Sequence, **C**: First dredge-up, **D**: RGB, **E**: He burning core, **F**: He flash, **G**: AGB, **H**: Post AGB, **I**: Planetary Nebula, **J**: White Dwarf.

Figure 3 shows the radial dependence of the adiabatic and radiative gradients ($\Delta_{\mathrm{ad}}, \Delta_{\mathrm{rad}}$). The green color in the background indicates the region where convection takes place.

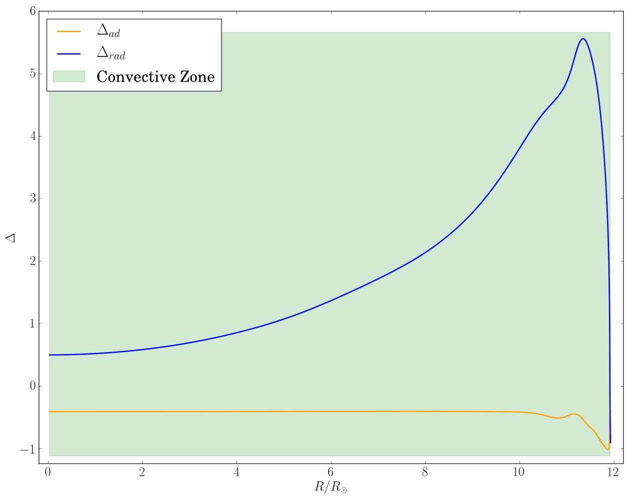

Figure 3. Radial Behavior of the adiabatic and radiative gradients in the pre-main sequence phase.

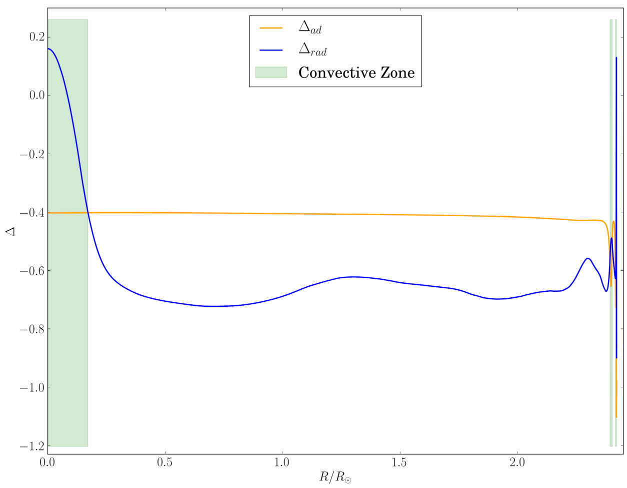

Figure 4. Radial Behavior of the adiabatic and radiative gradients in the main sequence phase.

Figure 4 ($\Delta_{\rm ad}, \Delta_{\rm rad}$ vs radius) is a similar plot as Figure 3 but for the main sequence phase of the star. Again, the colored region indicates the region where convection occurs.

4. DISCUSSION

In this section we present a detailed explanation of the evolutionary stages of the star and the physical interpretation of the figures that we introduced in the previous section.

As well known in stellar evolution theory, stars originate from molecular clouds in which some over dense regions can collapse due to gravitational instabilities if they satisfy the condition of having a mass larger than the Jeans mass. The gravitational collapse causes the central region to shine, this light is able to penetrate through the surrounding envelope and shines as a star. This configuration is known as a protostar. It will then go through various stages of evolution that we summarize chronologically in the following subsections.

4.1 Pre-main sequence

Once the protostar has reached the hydrostatic equilibrium, the pre-main sequence starts. It satisfies the virial theorem ($\mathrm{E_{tot}} = \frac{1}{2}\mathrm{E_{gr}}$) where the internal energy of the system increases merely due to the gravitational collapse and thus increasing the overall temperature. During this stage, the star remains in the ideal gas regime. As we can see in Figure 1, in the phase (A-B) both ρ and T_c increase due to the raise in the internal energy; also in Figure 2 the temperature increases at the photosphere but the luminosity decreases because of the increase in opacity. This produces a temperature gradient which transports heat through convection. Furthermore, there is a point where the luminosity starts to grow because deuterium fusion begins, this allows for a direct link between luminosity and temperature gradient.

We can see in Figure 3 that in this evolutionary stage, the opacity is large so $\nabla_{\rm rad} > \nabla_{\rm ad}$ and the star is fully convective. This is a characteristic of low temperature stars. All along this stage, the protostar shrinks and heats up until it starts fusing Hydrogen where the main sequence phase starts.

4.2 Main sequence

This stage is mainly characterized by the fusion of Hydrogen to produce Helium. As seen in Figures 1 and 2, the temperature and luminosity of the core increase because the internal energy is fed by the energy released in the thermonuclear reactions (PP chain) in the core. Temperature decreases in the photosphere because it is expanding while the core is contracting (mirror principle) which causes a decrease in L. We can see in Figure 1 that the yellow dot corresponds to the actual location of the Sun in the $\log(\mathrm{T})-\log(\rho)$ plane. The main sequence for our model star is below the Sun which makes sense because for more massive stars, the location in the plane gets closer to the radiative domination region.

As we can see in Figure 4, the outer envelope has a higher temperature which means a lower opacity so that the relation $\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}}$ implies energy transport through radiation, but the core is still convective because the opacity is high. Near the photosphere, some zones are partially ionized (small ∇_{ad}) so convection can take place even if the opacity is low. This will be specially important to understand the transition phase to red giant.

4.3 Red Giant Branch

In phase C (Subgiant), the surface convection zones extend down to the inner layers where the nuclear fusion takes place dragging out mixed material produced by the CNO cycle residuals (dredge-up). This marks the point where the star begins the red giant branch phase.

In the phase C-D, we have the red giant branch. The photosphere keeps expanding (Figure 2) while the core's density keeps increasing (Figure 1) until it reaches a point where the ideal gas solution is not longer valid and the star enters the non-relativistic regime as seen in Figure 1. Eventually, the core has exhausted its Hydrogen so the Helium ignition starts (point E, Figure 1). The outer layers start to fuse hydrogen and the core increases in mass as the shell produces more helium.

The core decreases in temperature which makes it expand, so the photosphere shrinks. After some time of Helium burning, the temperature of the core increases (although leaving the density constant) taking the star from the NR regime to the ideal gas regime. The core has a huge amount of energy because it didn't expand or contract which allows the star to have a burst of helium production known as Helium flash (Point F, Figure 1). This point marks the change of phase to the Asymptotic giant branch (AGB).

4.4 AGB

During the Asymptotic Giant Branch stage the star is burning H and He in a thin shell on top of the electron degenerate core which is mainly made of C and O. The nuclear production is mainly dominated by the He fusion into C, which burns out until it reaches the H shell causing a shell flash [1]. The shell flash happens periodically and the star shows thermal oscillation pulses (as seen in F and G in Figure 2) which increases mass loss and ejects the envelope of the star. This process makes this layers convective so elements produced by s-process can be carried out to the photosphere of the star. This is the well known dredge-up process. [3]

We can see in Figure 1 that in this phase, the star goes from the ideal gas regime to the non-relativistic regime where temperature of the core starts to decrease on its way to the ER (degenerate core) part of the diagram. The core becomes denser on its transition to a degenerate core and the outer layers expands as can be seen in Figure 2.

4.5 Post-Asymptotic giant branch

Once the AGB phase is over the core keeps on contracting increasing its density (Figure 1) while the huge stellar winds expel out the outer layers. As can be seen in point H of Figure 2 the luminosity remains constant due to the absence of thermonuclear reactions while the temperature of the photosphere increases because of the decrease in mass of the envelope. At this point the star goes from the NR region to the ER (Figure 1) where the core is denser but cooler. At the end of AGB mass loss has removed the envelope leaving behind the C and O core. The star develops intense super winds blushing away the outer layers which will form the planetary nebula. [6]

4.6 Planetary Nebula

The planetary nebula phase is characterized by a glowing shell of ionized gas ejected by the old red giant star late in its life [5]. This phase is the result of slow AGB wind and fast wind from the central star [1]. In point I of Figure 2 when the star goes out of the post-AGB phase keeps increasing the temperature of the photosphere while the luminosity is constant. However, the temperature of the core varies slightly so does its density (Figure 1). At this point all the layers are blown apart so we end up with a degenerate C,O core which is known as a White Dwarf. The luminosity of our star decreases dramatically and the star cools off (Figure 2).

4.7 White Dwarf

This is the final stage of our model star, as we can see in J of Figure 2, the outer layer has been blast away and the remnant is the core that does not have enough thermal energy to burn heavy elements so it begins to rapidly decrease its luminosity because its energy now is provided by energy stored in the interiors [1]. The star is now in the ER region where the core is electron degenerate. The rapid evolution to a faint luminosity stage comes from the emission of its stored thermal energy. In J of Figure 1, we see that the density of the core is almost constant and the variation in temperature is not significant. Nevertheless, the variation in the effective temperature is larger compared to the core's variation because there is no envelope now just the naked core of the dead star. This is the final stage of the star where it will keep on radiating energy and cooling down.

5. CONCLUSIONS

We have presented a comprehensive analysis of the evolution of a $2M_{\odot}$ star using the software MESA. As can be seen in Figure 1, our theory model predicted by Eqn. (5) fits really well the pre-main sequence, though after the main sequence the star deviated from this behavior because this equation does not take into account the mass loss of the star. Also in this figure, we can see how the Sun (orange dot) is pretty close to the main sequence of our $2M_{\odot}$ star. However (and as expected), it is slightly above because while mass increases the stars get closer to the radiation dominated part of the diagram. It can be observed how in the pre-main sequence phase (Figure 3) the star is fully convective which is the most efficient energy transport mechanism to carry on the luminosity inside the star to the photosphere. In the main-sequence where fusion of H into He is happening the picture is different because now the opacity in the outer layers is small enough to allow radiative transport (Figure 4). In Figures 1 and 2 it is evident that the star spends most of its time in the pre-main and main-sequence. After the first dredge-up the evolutionary phases of the star occur relatively rapid.

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