Exam suggested problems **EFT**

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Assignment: Make two examination-type questions (with answers) for a closed book exam. One should be a short question, something that can be answered in a few lines. The other, a longer question or problem, requiring calculations, maybe with several parts. Your questions don't necessarily have to be original, but they should be relevant, clearly formulated and fair. Something you would like to see in the final exam (perhaps you will!). Make sure you quote all your sources.

Exercise 1 (Short question)

Explain briefly the meaning of these main concepts discussed during the course **Effective field theory**, **Noether's Theorem**, **Goldstone's Theorem**, **Spontaneous symmetry breaking**, **Higgs Mechanism**. Also, explain why they are useful (except for the Goldstone theorem and SSB):

Solution 1:

Effective field theory: It's an approximate description of fields that approaches the fundamental description until useful (and valid) ranges and scales. It is a powerful framework to explain physical phenomena in terms of fields as carriers of interactions. ***It is useful because*** it avoids a more complicated quantum mechanical treatment without losing the most relevant features of the physics of the systems and keeping the results given by classical mechanics and general relativity.

Noether's Theorem: This theorem states that if a system has a global continuous symmetry, then there is a corresponding quantity (or current) which is conserved in time. ***It is useful because*** if we are able to see a symmetry that fulfills those conditions, then our mathematical description of the system becomes simpler to solve since it will have less degrees of freedom as the conserved quantity can act as an integral of motion for example.

Goldstone's Theorem: This theorem states that if a system has a continuous global symmetry which is broken in the ground state, then there will be massless particle called Goldstone boson associated with that broken symmetry.

Spontaneous symmetry breaking: Refers to the situation in which a physical system that has a certain global symmetry, ends up in an asymmetrical state in the ground state. The symmetry of the system doesn't hold any more in a certain range so the system is said to have a broken symmetry.

Higgs Mechanism: Consists of the coupling of the Higgs field to a field in which the SSB has created a Goldstone boson, this coupling allows the Higgs field to transfer one of its degrees of freedom to the field that will now have a massive particle. ***It is useful because*** it gives mass to particles in a physically and mathematically consistent way so that it provides a explanation of the mass of the particles in the standard model.

Exercise 2 (Problem)

Spontaneous breaking of continuous symmetries

a) Consider the Lagrangian in terms of real scalar fields:

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi_i)^2 + \frac{1}{2}\mu^2\phi_i^2\right] - \frac{\lambda}{4}\left(\phi_i^2\right)^2 \tag{1}$$

Where $i = 1, ..., N$. This Lagrangian is invariant under orthogonal transformations. For the case of $N = 2$, state the physical situation that allows you to find the vacuum expectation values and find them. What does it mean to have more than one value of the ground state for the symmetry of the system?

Solution 2a:

The physical situation that allows us to find the value of the field in the vacuum is that we need the energy to be at the minimum, so the kinetic energy given by the terms with derivatives must be zero and the potential energy must be at the minimum, that is, when $\frac{dV}{d\phi_i} = 0$.

Keeping the Einstein summation notation we have:

$$\frac{d}{d\phi_i}\left[\frac{1}{4}\mu^2\phi_i^2 - \frac{\lambda}{4}\left(\phi_i^2\right)^2\right] = 0 \tag{2}$$

$$\frac{1}{2}\mu^2\phi_{i,0} - \lambda\phi_{i,0}^3 = 0 \tag{3}$$

We are not interested in the trivial solution $\phi_i = 0$ so our vacuum expectation values are given by:

$$\frac{1}{2}\mu^2 - \lambda\phi_{i,0}^2 = 0 \qquad \to \qquad \phi_{i,0}^2 = \frac{\mu^2}{2\lambda} \tag{4}$$

As seen in the last equation, there is more than one value for the minimum of the potential in the ground state, this means that the global symmetry of the system has been broken because the system in no longer symmetrical at least in the vicinity of the ground state, this is the spontaneous symmetry breaking.

b) If we now have a complex field, the Lagrangian (symmetric under O(2))becomes:

$$\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2 \tag{5}$$

Due to the geometric representation of this potential (mexican hat), it is suggested to use polar coordinates instead of Cartesian coordinates in field space because the rotation symmetry of the potential and the periodicity of the flat direction is reflected in the variables describing the scalar fields. Use the polar coordinate transformation $\phi(x) = \rho(x)e^{i\theta(x)}$ to express the Lagrangian in the new variables around one of the minimum.

From the expression for the Lagrangian that you just found, what can you say about the mass associated with $\rho(x)$ and $\theta(x)$?

Solution 2b:

We will expand ϕ around the minimum v, that is $\phi = (v+\rho(x))e^{i\theta(x)}$

We also need to define $\phi^\dagger$ in the new variables:

$$\phi^\dagger = (v+\rho(x))e^{-i\theta(x)} \tag{6}$$

And plugging into the Lagrangian:

$$\begin{aligned}
\mathcal{L} &= \partial_\mu \left((v+\rho(x))e^{-i\theta(x)}\right)\partial^\mu \left((v+\rho(x))e^{i\theta(x)}\right) + \mu^2(v+\rho(x))\cancel{e^{-i\theta(x)}}(v+\rho(x))\cancel{e^{i\theta(x)}} \\
&- \lambda\left((v+\rho(x))\cancel{e^{-i\theta(x)}}(v+\rho(x))\cancel{e^{i\theta(x)}}\right)^2 \\
\mathcal{L} &= (\partial_\mu\rho(x) - i(v+\rho(x))\partial_\mu\theta)(\partial_\mu\rho(x) + i(v+\rho(x))\partial_\mu\theta) + \mu^2(v+\rho(x))^2 - \lambda(v+\rho(x))^4 \\
\mathcal{L} &= (\partial_\mu\rho(x))^2 + ((v+\rho(x))\partial_\mu\theta)^2 + \mu^2(v+\rho(x))^2 - \lambda(v+\rho(x))^4
\end{aligned}$$

Now, expanding all the terms, and knowing that ρ and θ are functions of x, we get the Lagrangian:

$$\begin{aligned}
\mathcal{L} &= (\partial_\mu\rho)^2 + (v\partial_\mu\theta)^2 + 2v\rho\,(\partial_\mu\theta)^2 + \rho^2\,(\partial_\mu\theta)^2 + \mu^2v^2 + 2\mu^2v\rho + \mu^2\rho^2 - \lambda v^4 \\
&- 3\lambda v^2\rho - 3\lambda v\rho^2 - \lambda\rho^4
\end{aligned}$$

The quadratic terms with constant coefficients are only these two:

$$\mu^2\rho^2 - 3\lambda v\rho^2 \tag{7}$$

From this expression we see that for $\rho(x)$ there is a quadratic term with mass $\mu^2 - 3\lambda v$ but there is not any quadratic term associated with $\theta(x)$ so it is a massless field, this means that there is a Goldstone bosson associated with this field.

References

Professor Kachelrieß, Michael ”Norwegian University of Science and Technology”, QFT lectures http://web.phys.ntnu.no/ mika/CPP/