p2.e12.d

$f.\,xs = \langle Max\ i : 0 \leq i < \#xs \wedge sum.\,(xs \uparrow i) = sum.\,(xs \downarrow i) : i\rangle$

caso base $xs = []$

$f.\,[]$
= {ESPEC. f}
$\langle Max\ i : 0 \leq i < \#[] \wedge sum.\,([] \uparrow i) = sum.\,([] \downarrow i) : i\rangle$
= {Def de $\#$, aritmética, lógica}
$\langle Max\ i : False : i\rangle$
= {Rango Vacío}
$-\infty$

caso inductivo $xs = (x \triangleright xs)$

suponiendo

HI $f.\,xs = \langle Max\ i : 0 \leq i < \#xs \wedge sum.\,(xs \uparrow i) = sum.\,(xs \downarrow i) : i\rangle$,

debemos probar:

$\underline{f.\,(x \triangleright xs)}$
= {Espec. f}
$\langle Max\ i : 0 \leq i < \underline{\#(x \triangleright xs)} \wedge sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i) : i\rangle$
= { def #}

$\langle Max\ i : \underline{0 \leq i < \#xs + 1} \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= { lógica }

$\langle Max\ i : \underline{[i = 0 \vee 0 < i < \#xs + 1] \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i)} : i \rangle$

= {Distr. de $\vee$ con $\wedge$}

$\langle Max\ i : \underline{[0 \leq i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i)]} \vee [(i = 0 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i))] : i \rangle$

= { Separación de Termino}

$\langle Max\ i : \underline{(i = 0 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i)} : i \rangle\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= {Leibniz 2}

$\langle Max\ i : (i = 0 \wedge sum.(\underline{(x \rhd xs) \uparrow 0}) = sum.(\underline{(x \rhd xs) \downarrow 0})] : i \rangle\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= {def de ↑, ↓}

$\langle Max\ i : (i = 0 \wedge \underline{sum.[]} = \underline{sum.(x \rhd xs)}] : i \rangle\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= {def sum}

$\langle Max\ i : i = 0 \wedge 0 = x + sum.xs : i \rangle\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

Derivación por casos

Hago Inducción con Análisis de Casos

Caso $0 = x + sum.xs$

suponiendo que $0 = x + sum.xs$ vale vemos

= {Por suposicion}

$\langle Max\ i : \underline{i = 0 \wedge True} : i \rangle\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= { neutro $\wedge$}

$\underline{\langle Max\ i : i = 0 : i \rangle}\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= {rango unitario}

$0\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge sum.((x \rhd xs) \uparrow i) = sum.((x \rhd xs) \downarrow i) : i \rangle$

= {reemplazo de var, $i \leftarrow (i + 1)$}

$0\ max\ \langle Max\ i : 0 < i + 1 < \#xs + 1 \wedge sum.(\underline{(x \rhd xs) \uparrow (i + 1)}) = sum.(\underline{(x \rhd xs) \downarrow (i + 1)}) : i \rangle$

= {def. de ↑, ↓}

$0\ max\ \langle Max\ i : \underline{0 < i+1 < \#xs+1} \wedge \underline{sum.(x \triangleright (xs \uparrow i))} = sum.(xs \downarrow i) : i\rangle$
= {def sum, aritmetica}
$0\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge x + sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$
luego no puedo continuar, debo generalizar...

$gf.n.xs = \langle Max\ i : 0 \leq i < \#xs \wedge n + sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$

luego:

$$f.xs = gf.0.xs$$

caso base para gf $xs = xs$

buscamos llegar a f.xs para confirmar la igualdad
$gh.0.xs$
= {Especif. gf}
$\langle Max\ i : 0 \leq i < \#xs \wedge \underline{0 + sum.(xs \uparrow i)} = sum.(xs \downarrow i) : i\rangle$
= {Aritmética}
$\langle Max\ i : 0 \leq i < \#xs \wedge sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$
= {Especif. f}
f.xs

caso inductivo para gf, $xs = (x \triangleright xs)$

suponiendo que

HI $gf.n.xs = \langle Max\ i : 0 \leq i < \#xs \wedge n + sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$

Luego vemos $\forall m,\ cuando\ vale\ gf.m.(x \triangleright xs)$
$\underline{gf.m.(x \triangleright xs)}$
= {Especif. gf}

$\langle Max\ i : \underline{0 \leq i < \#(x \triangleright xs)} \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i) : i\rangle$

= {Def. #, Aritmética}

$\langle Max\ i : \underline{(i = 0 \vee 0 < i < \#xs + 1) \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)} : i\rangle$

= { Distr. de $\vee$ con $\wedge$}

$\langle Max\ i : [i = 0 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)] \vee [0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i))] : i\rangle$

= { Separacion de un termino }

$\langle Max\ i : \underline{i = 0 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)} : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= { Leibniz 2}

$\langle Max\ i : i = 0 \wedge m + sum.\,(\underline{(x \triangleright xs) \uparrow 0}) = sum.\,(\underline{(x \triangleright xs) \downarrow 0}) : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= {Def de $\uparrow$, $\downarrow$}

$\langle Max\ i : i = 0 \wedge m + \underline{sum.\,[]} = \underline{sum.\,(x \triangleright xs)}) : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= {Def. sum}

$\langle Max\ i : i = 0 \wedge m + 0 = x + sum.\,xs : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

No puedo continuar, debo analizar por casos para ver la igualdad

caso $m + 0 = x + sum.\,xs$

$\langle Max\ i : i = 0 \wedge \underline{m + 0 = x + sum.\,xs} : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= {suposición del caso}

$\langle Max\ i : \underline{i = 0 \wedge True} : i\rangle max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= {Elem. Neutro de $\wedge$}

$\underline{\langle Max\ i : i = 0 : i\rangle} max \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= { Rango unitario Max}

$0\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.\,((x \triangleright xs) \uparrow i) = sum.\,((x \triangleright xs) \downarrow i)) : i\rangle$

= {Cambio de Variable $i \leftarrow (i + 1)$}

$0\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge m + sum.\,(\underline{(x \triangleright xs) \uparrow (i + 1)}) = sum.\,(\underline{(x \triangleright xs) \downarrow (i + 1)}) : i\rangle$

= {Def de $\uparrow$, $\downarrow$}

$0\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge m + \underline{sum.\,(x \triangleright (xs \uparrow i))} = sum.\,(xs \downarrow i) : i\rangle$

= {Def. sum, Asociatividad}

$0\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge (m+x) + sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$

= {HI}

$0\ max\ gf.(m+x).xs$

caso $\neg(m+0 = x + sum.xs)$

$\langle Max\ i : i = 0 \wedge \underline{m + 0 = x + sum.xs} : i\rangle max\langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.((x \triangleright xs) \uparrow i) = sum.((x \triangleright xs) \downarrow i)) : i\rangle$

= {suposición del caso}

$\langle Max\ i : \underline{i = 0 \wedge False} : i\rangle max\langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.((x \triangleright xs) \uparrow i) = sum.((x \triangleright xs) \downarrow i)) : i\rangle$

= {Elem. Absorbente de $\wedge$}

$\underline{\langle Max\ i : False : i\rangle} max\langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.((x \triangleright xs) \uparrow i) = sum.((x \triangleright xs) \downarrow i)) : i\rangle$

= { Rango Vacio Max}

$-\infty\ max\ \langle Max\ i : 0 < i < \#xs + 1 \wedge m + sum.((x \triangleright xs) \uparrow i) = sum.((x \triangleright xs) \downarrow i)) : i\rangle$

= {Cambio de Variable $i \leftarrow (i+1)$}

$-\infty\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge m + sum.(\underline{(x \triangleright xs) \uparrow (i+1)}) = sum.(\underline{(x \triangleright xs) \downarrow (i+1)}) : i\rangle$

= {Def de ↑, ↓}

$-\infty\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge m + \underline{sum.(x \triangleright (xs \uparrow i))} = sum.(xs \downarrow i) : i\rangle$

= {Def. sum, Asociatividad}

$-\infty\ max\ \langle Max\ i : 0 \leq i < \#xs \wedge (m+x) + sum.(xs \uparrow i) = sum.(xs \downarrow i) : i\rangle$

= {HI}

$-\infty\ max\ gf.(m+x).xs$

Programa Final

$$gf.n.(x \triangleright xs) \doteq \begin{cases} m = x + sum.xs \rightarrow 0\ max\ gf.(m+x).xs \\ \Box\neg(m = x + sum.xs) \rightarrow -\infty\ max\ gf.(m+x).xs \end{cases}$$