p2.e12.d

```
f.\ xs = \langle Max\ i: 0 \leq i < \#xs \wedge sum.\ (xs \uparrow i) = sum.\ (xs \downarrow i): i 
angle
```

caso base xs = []

```
\begin{split} f. & \mid \\ &= \{ \text{ESPEC. } f \} \\ & \langle Max \ i : 0 \leq i < \# [] \land sum. ([] \uparrow i) = sum. ([] \downarrow i) : i \rangle \\ &= \{ \text{Def de } \#, \text{aritmética, lógica} \} \\ & \langle Max \ i : False : i \rangle \\ &= \{ \text{Rango Vacío} \} \\ &-\infty \end{split}
```

caso inductivo $xs=(x \triangleright xs)$

suponiendo

```
	ext{HI } f. \, xs = \langle Max \ i : 0 \leq i < \#xs \wedge sum. \, (xs \uparrow i) = sum. \, (xs \downarrow i) : i 
angle ,
```

debemos probar:

```
\begin{split} &\frac{f.\:(x \triangleright xs)}{= \{ \text{Espec.}\:f \}} \\ & \langle Max\:i:0 \leq i < \underline{\#(x \triangleright xs)} \land sum.\: ((x \triangleright xs) \uparrow i) = sum.\: ((x \triangleright xs) \downarrow i):i \rangle \\ & = \{\: \text{def}\:\# \} \end{split}
```

```
 \langle Max \ i : \underline{0 \leq i < \#xs + 1} \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ = \{ \text{lógica} \} \\ \langle Max \ i : [i = 0 \lor 0 < i < \#xs + 1] \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ = \{ \text{Distr. de} \lor \text{con } \land \} \\ \langle Max \ i : [\underline{0 \leq i < \#xs + 1} \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i)] \lor [(i = 0 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i))] : i \rangle \\ = \{ \text{Separación de Termino} \} \\ \langle Max \ i : (\underline{i = 0 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i)} : i \rangle \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ = \{ \text{Leibniz 2} \} \\ \langle Max \ i : (\underline{i = 0 \land sum. \ ((\underline{x} \rhd xs) \uparrow 0) = sum. \ ((\underline{x} \rhd xs) \downarrow 0)] : i \rangle \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ = \{ \text{def de } \uparrow, \downarrow \} \\ \langle Max \ i : (\underline{i = 0 \land sum. \ []} = \underline{sum. \ (x \rhd xs)]} : i \rangle \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ = \{ \text{def sum} \} \\ \langle Max \ i : \underline{i = 0 \land 0 = x + sum. \ xs : i} \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land sum. \ ((x \rhd xs) \uparrow i) = sum. \ ((x \rhd xs) \downarrow i) : i \rangle \\ \end{cases}
```

Derivación por casos

Hago Inducción con Análisis de Casos

```
\begin{aligned} &\operatorname{Caso}\ 0 = x + sum.\ xs \\ &\operatorname{suponiendo}\ \operatorname{que}\ 0 = x + sum.\ xs \ \operatorname{vale}\ \operatorname{vemos} \\ &= \{\operatorname{Por}\ \operatorname{suposicion}\} \\ &\langle \mathit{Max}\ i : \underline{i = 0 \land True} : i\rangle\ \mathit{max}\ \langle \mathit{Max}\ i : 0 < i < \#xs + 1 \land sum.\ ((x \rhd xs) \uparrow i) = sum.\ ((x \rhd xs) \downarrow i) : i\rangle \\ &= \{\operatorname{neutro}\ \land\} \\ &\langle \mathit{Max}\ i : i = 0 : i\rangle\ \mathit{max}\ \langle \mathit{Max}\ i : 0 < i < \#xs + 1 \land sum.\ ((x \rhd xs) \uparrow i) = sum.\ ((x \rhd xs) \downarrow i) : i\rangle \\ &= \{\operatorname{rango}\ \operatorname{unitario}\} \\ &0\ \mathit{max}\ \langle \mathit{Max}\ i : 0 < i < \#xs + 1 \land sum.\ ((x \rhd xs) \uparrow i) = sum.\ ((x \rhd xs) \downarrow i) : i\rangle \\ &= \{\operatorname{reemplazo}\ \operatorname{de}\ \operatorname{var},\ i \leftarrow (i+1)\} \\ &0\ \mathit{max}\ \langle \mathit{Max}\ i : 0 < i + 1 < \#xs + 1 \land sum.\ ((x \rhd xs) \uparrow (i+1)) = sum.\ ((x \rhd xs) \downarrow (i+1)) : i\rangle \\ &= \{\operatorname{def}.\ \operatorname{de}\ \uparrow, \downarrow\} \end{aligned}
```

```
\begin{array}{l} 0\; max\; \langle Max\; i: \underline{0 < i+1 < \#xs+1} \wedge \underline{sum.\, (x \rhd (xs \uparrow i))} = sum.\, (xs \downarrow i): i\rangle \\ = \{\text{def sum, aritmetica}\} \\ 0\; max\; \langle Max\; i: 0 \leq i < \#xs \wedge x + sum.\, (xs \uparrow i) = sum.\, (xs \downarrow i): i\rangle \\ \text{luego no puedo continuar, debo generalizar...} \end{array}
```

$$gf.\, n.\, xs = \langle Max\,\, i: 0 \leq i < \#xs \wedge n + sum.\, (xs \uparrow i) = sum.\, (xs \downarrow i): i
angle$$

luego:

$$f. xs = gf.0. xs$$

caso base para gf xs = xs

buscamos llegar a f.xs para confirmar la igualdad

```
\begin{array}{l} gh.0.\,xs \\ = \{ \text{Especif. gf} \} \\ \langle Max\,\,i:0\leq i<\#xs \wedge \underline{0+sum.\,(xs\uparrow i)} = sum.\,(xs\downarrow i):i \rangle \\ = \{ \text{Aritmética} \} \\ \langle Max\,\,i:0\leq i<\#xs \wedge sum.\,(xs\uparrow i) = sum.\,(xs\downarrow i):i \rangle \\ = \{ \text{Especif. f} \} \\ \text{f.xs} \end{array}
```

caso inductivo para gf, $xs = (x \triangleright xs)$

suponiendo que

$$ext{HI } gf. \, n. \, xs = \langle Max \, i : 0 \leq i < \#xs \wedge n + sum. \, (xs \uparrow i) = sum. \, (xs \downarrow i) : i
angle$$

Luego vemos $\forall m, \ cuando \ vale \ gf. \ m. \ (x \triangleright xs)$ $= \{\text{Especif. gf}\}$

```
\langle Max\ i:0\leq i<\#(x\triangleright xs)\land m+sum.\ ((x\triangleright xs)\uparrow i)=sum.\ ((x\triangleright xs)\downarrow i):i
angle
= {Def. #, Aritmética}
raket{Max \ i: (i=0 \lor 0 < i < \#xs+1) \land m+sum. ((x 
darks xs) \uparrow i) = sum. ((x 
darks xs) \downarrow i): i}
= \{ Distr. de \lor con \land \}
raket{Max\ i:[i=0\land m+sum.\left((x	riangleright xs)\uparrow i
ight)=sum.\left((x	riangleright xs)\downarrow i
ight)]}\lor [0< i<\#xs+1\land m+sum.\left((x	riangleright xs)\uparrow i
ight)=sum.\left((x	riangleright xs)\downarrow i
ight)]:i}
= { Separacion de un termino }
\langle Max\ i: i=0 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i): i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i)): i 
angle
= { Leibniz 2}
\langle Max\ i: i=0 \land m+sum.\ ((x 
deta xs) \uparrow 0)=sum.\ ((x 
deta xs) \downarrow 0): i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
deta xs) \uparrow i)=sum.\ ((x 
deta xs) \downarrow i)): i 
angle
= \{ \text{Def de } \uparrow, \downarrow \}
\langle Max\ i: i=0 \land m+sum.\ []=sum.\ (x 
hd xs)): i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i)): i 
angle
= \{ Def. sum \}
\langle Max\ i: i=0 \land m+0=x+sum.\ xs: i \rangle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i)): i \rangle
No puedo continuar, debo analizar por casos para ver la igualdad
caso m + 0 = x + sum. xs
\langle Max\ i: i=0 \land m+0=x+sum.\ xs: i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i)): i 
angle
= {suposición del caso}
\langle Max\ i: i=0 \land True: i \rangle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum. ((x 
hd xs) \uparrow i) = sum. ((x 
hd xs) \downarrow i)): i \rangle
= {Elem. Neutro de \land}
\langle Max\ i: i=0: i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum. ((x 
hd xs) \uparrow i) = sum. ((x 
hd xs) \downarrow i)): i 
angle
= { Rango unitario Max}
0 \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land m + sum. ((x \triangleright xs) \uparrow i) = sum. ((x \triangleright xs) \downarrow i)) : i \rangle
= {Cambio de Variable i \leftarrow (i+1)}
0\ max\ \langle Max\ i:0\leq i<\#xs\land m+sum.\ ((x\triangleright xs)\uparrow (i+1))=sum.\ ((x\triangleright xs)\downarrow (i+1)):i
angle
= \{ \text{Def de } \uparrow, \downarrow \}
0 \ max \ \langle Max \ i : 0 \leq i < \#xs \land m + sum. \ (x \triangleright (xs \uparrow i)) = sum. \ (xs \downarrow i) : i \rangle
= {Def. sum, Asociatividad}
```

```
0\ max\ \langle Max\ i:0\leq i<\#xs\wedge (m+x)+sum.\ (xs\uparrow i)=sum.\ (xs\downarrow i):i
angle
=\{HI\}
0 max qf. (m+x). xs
caso \neg (m+0=x+sum.xs)
\langle Max\ i: i=0 \land m+0=x+sum.\ xs: i 
angle max \langle Max\ i: 0 < i < \#xs+1 \land m+sum.\ ((x 
hd xs) \uparrow i)=sum.\ ((x 
hd xs) \downarrow i)): i 
angle
= {suposición del caso}
\langle Max \ i : i = 0 \land False : i \rangle max \langle Max \ i : 0 < i < \#xs + 1 \land m + sum. \left( (x \triangleright xs) \uparrow i \right) = sum. \left( (x \triangleright xs) \downarrow i \right) : i \rangle
= {Elem. Absorbente de ^}
\langle Max\ i: False: i 
angle max \langle Max\ i: 0 < i < \#xs + 1 \land m + sum. ((x 
hd xs) \uparrow i) = sum. ((x 
hd xs) \downarrow i)): i 
angle
= { Rango Vacio Max}
-\infty \ max \ \langle Max \ i : 0 < i < \#xs + 1 \land m + sum. \ ((x \triangleright xs) \uparrow i) = sum. \ ((x \triangleright xs) \downarrow i)) : i \rangle
= {Cambio de Variable i \leftarrow (i+1)}
-\infty \ max \ \langle Max \ i:0 \leq i < \#xs \land m + sum. \ ((x 
darksim xs) \uparrow (i+1)) = sum. \ ((x 
darksim xs) \downarrow (i+1)):i 
angle
= \{ \text{Def de } \uparrow, \downarrow \}
-\infty \ max \ \langle Max \ i:0 \leq i < \#xs \land m + sum. \ (x 
dash (xs \uparrow i)) = sum. \ (xs \downarrow i):i 
angle
= {Def. sum, Asociatividad}
-\infty \ max \ \langle Max \ i : 0 \leq i < \#xs \land (m+x) + sum. \ (xs \uparrow i) = sum. \ (xs \downarrow i) : i \rangle
=\{HI\}
-\infty \ max \ gf. \ (m+x). \ xs
```

Programa Final

$$gf.\,n.\,(x riangleright xs) \doteq egin{cases} m = x + sum.\,xs
ightarrow 0\ max\ gf.\,(m+x).\,xs \ \Box \lnot (m = x + sum.\,xs)
ightarrow -\infty\ max\ gf.\,(m+x).\,xs \end{cases}$$