Drag on spherical particles and steady settling velocities

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Most textbooks present results for the dependence of the drag coefficient for a smooth sphere, C_D, on the Reynolds number, Re, in the form of a curve. Such curves are difficult to read accurately especially without a fine grid. Also, the determination of the terminal settling velocity of a sphere in a fluid using this curve is an iterative process. Clift, Grace, and Weber (*Bubbles, Drops, and Particles*, Academic Press, 1978) provide correlations based on experimental data that can be used to calculate either the drag coefficient or the settling velocity directly. I have reproduced their results below for your use in solving problems. The standard definitions of the Reynolds number and the drag coefficient are given below.

$$\mathrm{Re} = \frac{d_p V \rho}{\mu}$$

$$C_D = \frac{8D}{\pi \rho d_p^2 V^2}$$

Here, d_p is the diameter of the sphere, V is its velocity, ρ is the density of the fluid, and μ is its viscosity. The symbol D stands for the drag on the sphere.

Drag Coefficient

The entire range of Reynolds numbers has been divided into 10 intervals and in each, the curve for the drag coefficient versus the Reynolds number is fitted to a suitable expression. In the results given below, $w = \log_{10} \mathrm{Re}$.

For $\mathrm{Re} \le 0.01$, $$C_D = \frac{9}{2} + \frac{24}{\mathrm{Re}}$$

For $0.01 < \mathrm{Re} \le 20$, $$C_D = \frac{24}{\mathrm{Re}}\left[1 + 0.1315\,\mathrm{Re}^{(0.82-0.05w)}\right]$$

For $20 \le \mathrm{Re} \le 260$, $$C_D = \frac{24}{\mathrm{Re}}\left[1 + 0.1935\,\mathrm{Re}^{0.6305}\right]$$

For $260 \le \mathrm{Re} \le 1.5\times 10^3$, $$\log_{10} C_D = 1.6435 - 1.1242w + 0.1558w^2$$

For $1.5\times10^3 \le \mathrm{Re} \le 1.2\times10^4$,

$$\log_{10} C_D = -2.4571 + 2.5558w - 0.9295w^2 + 0.1049w^3$$

For $1.2\times10^4 \le \mathrm{Re} \le 4.4\times10^4$, $\log_{10} C_D = -1.9181 + 0.6370w - 0.0636w^2$

For $4.4\times10^4 \le \mathrm{Re} \le 3.38\times10^5$, $\log_{10} C_D = -4.3390 + 1.5809w - 0.1546w^2$

For $3.38\times10^5 \le \mathrm{Re} \le 4\times10^5$, $C_D = 29.78 - 5.3w$

For $4\times10^5 \le \mathrm{Re} \le 10^6$, $C_D = 0.1w - 0.49$

For $10^6 < \mathrm{Re}$, $C_D = 0.19 - \dfrac{8\times10^4}{\mathrm{Re}}$

You can use the above equations to evaluate the drag coefficient when needed.

Terminal Settling Velocity

If you want to calculate the terminal settling velocity of a sphere, you will find that this velocity appears in both the drag coefficient and in the Reynolds number. Therefore, you must perform an iterative calculation to find the answer. To avoid doing this, Clift, Grace, and Weber also provide results that permit the direct calculation of the terminal settling velocity. Recall that for terminal settling, the drag on the sphere is equal to its net weight, which is the weight minus the buoyant force on the sphere. We define the Reynolds number in the same way as before, but with the understanding that it now applies with $V = V_{\text{terminal}}$. A new dimensionless group $N_D = C_D \mathrm{Re}^2$ is introduced in which the terminal settling velocity does not appear. Therefore you can calculate this group and use the equations given below to calculate the value of the Reynolds number corresponding to a given value of N_D. From the Reynolds number, you can immediately evaluate the terminal settling velocity. In the equations, $W = \log_{10} N_D$.

For $N_D \le 73;\ \mathrm{Re} \le 2.37$,

$$\mathrm{Re} = \frac{N_D}{24} - 1.7569\times10^{-4} N_D^2 + 6.9252\times10^{-7} N_D^3 - 2.3027\times10^{-10} N_D^4$$

For $73 < N_D \leq 580;\ \ 2.37 < \mathrm{Re} \leq 12.2$,

$$\log_{10} \mathrm{Re} = -1.7095 + 1.33438\,W - 0.11591 W^2$$

For $580 < N_D \leq 1.55 \times 10^7;\ 12.2 < \mathrm{Re} \leq 6.35 \times 10^3$

$$\log_{10} \mathrm{Re} = -1.81391 + 1.34671 W - 0.12427\,W^2 + 0.006344 W^3$$

For $1.55 \times 10^7 < N_D \leq 5 \times 10^{10};\ \ 6.35 \times 10^3 < \mathrm{Re} \leq 3 \times 10^5$

$$\log_{10} \mathrm{Re} = 5.33283 - 1.21728\,W + 0.19007 W^2 - 0.007005\,W^3$$