The Effect of Spin Upon the Rolling Motion of an Elastic Sphere on a Plane

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The motion and deformation of an elastic sphere rolling on an elastic plane are examined for the case when the sphere, in addition to its straight rolling motion, has an angular velocity of "spin" Ω about an axis normal to the plane. The action of spin is to twist the area of contact. Surface tractions resulting from this rotation are found, which demonstrate the necessity of partial slip in the area of contact. Previous investigations suggest that this slip cannot occur at the leading edge of the contact circle, so that a system of tractions is found which corresponds to zero stress at the leading point. It is shown that such a system of tractions gives rise to a transverse creep of the sphere in the direction of its rotation Ω. The magnitude of this creep is calculated for small values of Ω, when slip occurs to only a small extent. Experiments have been performed using a simple thrust bearing with plane parallel races. As the bearing rotates, the balls creep radially outward in the predicted manner. Quantitative measurements of this creep agree with the theoretical estimate over a wide range.

Introduction

The motion of a sphere which rolls without sliding upon a plane has been analyzed in a companion paper (1)[2] into three categories: (*a*) Free rolling; (*b*) rolling under tangential forces; and (*c*) rolling with spin. The motion referred to as spin is defined as a relative angular velocity between the sphere and the plane about an axis through the point of contact normal to the plane.

Spin is a recognized feature of thrust and angular-contact ball bearings and is usually held responsible for the higher friction torque and considerably shorter life of these bearings in comparison with radial bearings in which free rolling occurs. In spite of its practical importance, few attempts seem to have been made to analyze the mechanism of the motion or to determine the surface stresses which are induced. Palmgren (2) and Poritsky, Hewlett, and Coleman (3) calculate the friction torque of angular-contact bearings assuming that the relative slip between the balls and the races consists of a rigid-body rotation with the spin velocity about a normal axis through the center of the contact area. The more recent work of Mindlin (4) and Lubkin (5) suggests that it is unjustifiable to ignore tangential displacements resulting from the elasticity of the surfaces and that, at least for small spin velocities, slip might occur over only a small portion of the area of contact, the remainder consisting of a locked region in which no relative motion occurs.

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Applied Mechanics Division for presentation at the Annual Meeting, New York, N. Y., November 30–December 5, 1958, of The American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Secretary, ASME, 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1959, for publication at a later date. Discussion received after the closing date will be returned.

 Manuscript received by ASME Applied Mechanics Division, September 17, 1956. Paper No. 58—A-5.

The author was interested to observe, partly by accident, that in a simple thrust bearing, made up of two flat concentrically rotating disks with evenly spaced balls rolling between them, the balls did not move around a perfectly circular path but exhibited a creeping motion radially outward. This occurred when the velocities were sufficiently small for the centrifugal forces to be negligible. The effect is mentioned casually by Palmgren (6) in connection with his rolling experiments with tangential forces, but is dismissed as insignificant, since he states in conclusion[3] that "the ball is always, while rolling, seeking surfaces which are exactly parallel, if such exist in the path of the ball." In the analysis of rolling with spin presented in this paper an attempt is made to assess the magnitude and to explain the mechanism of this radial creep.

Statement of the Problem

In this problem the sphere is taken to roll on the plane in a straight line parallel to the axis *ox* with a steady velocity U. Contact is maintained by a force N acting in a direction normal to the plane, which gives rise of a circular area of contact of radius a given by

$$a^3 = \frac{3(1-\nu)ND}{8G} \qquad [1]$$

At the same time a steady angular velocity of spin Ω about the normal axis *oz* is maintained by an applied twisting couple M_z. In view of the observations mentioned in the introduction, tangential creep velocities ΔU and ΔV might be expected as shown in Fig. 1.

Clearly the action of the spin rotation, as the sphere rolls for-

[3] Reference (7), p. 35.

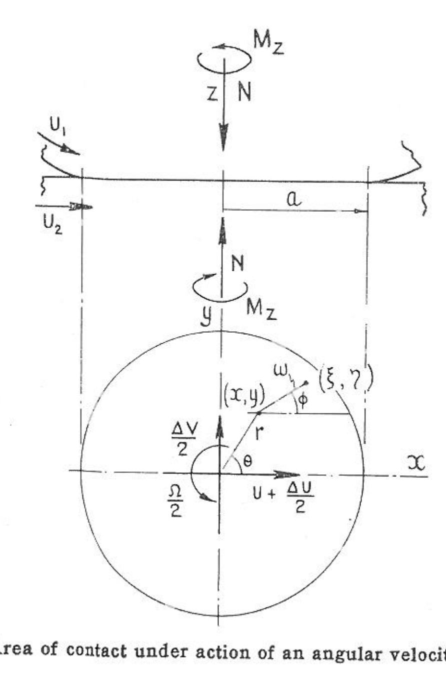

Fig. 1 Area of contact under action of an angular velocity of spin Ω

ward, is to twist the area of contact, thereby inducing tangential components of traction X and Y. It is required to find X, Y, ΔU. and ΔV and the moment M_z arising from a given angular velocity of spin.

The method of analysis follows closely that applied to the problem of rolling under tangential forces given in (1), to which reference should be made for notation, and so on. It was shown there that considerable simplification could be secured by assuming the sphere and plane to have similar elastic constants and by ignoring the warping of the contact surface, whereupon the problem reduced to one of two identical spheres rolling upon each other. The same expedient is applied to the present problem, thereby introducing several important conditions which now arise from the geometrical and elastic symmetry of the system.

Conditions From Symmetry

Reducing the problem to the contact of equal spheres with like elastic constants permits Ω to be divided equally between the surfaces, and reference to Fig. 1 indicates symmetry about the axis Ox. Therefore the equality of action and reaction gives

$$X(x, y) = -X(x, -y) \qquad [2]$$

and

$$Y(x, y) = Y(x, -y) \qquad [3]$$

$$M_z = \int_0^a \int_0^{2\pi} (Yx - Xy) r dr d\theta \qquad [4]$$

and since there is no resultant tangential force

$$\int_0^a \int_0^{2\pi} X r dr d\theta = \int_0^a \int_0^{2\pi} Y r dr d\theta = 0 \qquad [5]$$

The particle velocities at point $P(x, y)$ are now given by

$$q_x(x, y, U) = U + \frac{\Delta U}{2} - \frac{\Omega}{2} y + U \frac{\partial u}{\partial x} (x, y, \operatorname{sgn} U) \qquad [6]$$

$$q_y(x, y, U) = \frac{\Delta V}{2} + \frac{\Omega}{2} x + U \frac{\partial v}{\partial x} (x, y, \operatorname{sgn} U) \qquad [7]$$

Symmetry of the system about Ox gives

$$q_{x1}(x, -y, U) = q_{x2}(x, y, U)$$

and

$$q_{y1}(x, -y, U) = -q_{y2}(x, y, U)$$

where (x_1, y_1) and (x_2, y_2) are contacting points. The relative velocity between such points is therefore given by

$$s_x = q_x(x, y, U) - q_x(x, -y, U)$$

and

$$s_y = q_y(x, y, U) + q_y(x, -y, U)$$

which, from Expressions [6] and [7], give

$$s_x = \Delta U - \Omega y + U \frac{\partial u}{\partial x} (x, y, U) - U \frac{\partial u}{\partial x} (x, -y, U) \qquad [8]$$

$$s_y = \Delta V + \Omega x + U \frac{\partial v}{\partial x} (x, y, U) + U \frac{\partial v}{\partial x} (x, -y, U) \qquad [9]$$

As in the previous problem, we might expect the area of contact to be divided into a locked region and a region in which slip occurs. In the locked region the relative velocity between contacting points is zero, whence

$$\frac{\partial u}{\partial x} (x, -y, U) - \frac{\partial u}{\partial x} (x, y, U) = \xi_x - p \frac{y}{a} \qquad [10]$$

and

$$-\frac{\partial v}{\partial x} (x, -y, U) - \frac{\partial v}{\partial x} (x, y, U) = \xi_y + p \frac{x}{a} \qquad [11]$$

where p is a nondimensional spin factor, $\Omega a/U$.

It has not been found possible in this problem to make a simple and physically reasonable supposition for the shape of the locked region once slip has developed to any appreciable extent. The investigations are confined therefore to the assumption that negligible slip occurs over the whole circle of contact.

The method of solution follows that given in reference (1); tangential tractions are specified over the whole area of contact ($r < a$) and the "problem of the plane" solved to obtain the surface displacements, which must satisfy the no-slip conditions of Equations [10] and [11] over the area $r < a$. The boundary conditions in this case reduce to:

$$\left.\begin{array}{l} X \text{ and } Y, \text{ to be specified, consistent} \\ \text{with Conditions [2], [3], and [5]} \\ Z' = 0 \end{array}\right\} r \lessgtr a$$

and

$$X = Y = Z = 0, \qquad r > a$$

Solution for No Slip

The method employed by Mindlin (4)[4] has been used here to find the tangential surface displacements u and v due to a number of distributions of tangential tractions of similar form acting over the circle $r < a$. The surface integrals obtained by substituting these tractions into Equation [32] of reference (4), although tedious, present no real difficulties. The results of these calculations are summarized for convenience in Table 1.

Considering the data of Table 1 in the light of conditions [2], [3], [5], [10], and [11], it is apparent that the required tractions will be those giving rise to expressions for $\partial u/\partial x$ containing terms proportional to y, and for $\partial v/\partial x$ containing terms proportional to x, such tractions are

$$X_5 = \bar{X}_5 \frac{xy}{a(a^2 - r^2)^{1/2}} \qquad [12]$$

$$Y_1 = \bar{Y}_1 \frac{1}{a} (a^2 - r^2)^{1/2} \qquad [13]$$

and

$$Y_5 = \bar{Y}_6 \frac{x^2}{a(a^2 - r^2)^{1/2}} \qquad [14]$$

Each satisfies conditions [2] and [3] and, in order that there should be no resultant tangential force, it follows immediately that Y_1 and Y_5 must be added together such that

$$\bar{Y}_6 = -\bar{Y}_1 \qquad [15]$$

The displacements due to the superposition of tractions [12], [13], and [14] give

$$\frac{\partial u}{\partial x} = \frac{\bar{X}_1 \pi 3(2 - \nu)}{32Ga} \cdot y + \frac{\bar{Y}_1 \pi 2\nu}{32Ga} y - \frac{(-\bar{Y}_1)\pi\nu}{32Ga} y$$

$$\frac{\partial v}{\partial x} = \frac{\bar{X}_1 \pi 3\nu}{32Ga} x - \frac{\bar{Y}_1 \pi 2(4 - \nu)}{32Ga} x + \frac{(-\bar{Y}_1)\pi(10 - \nu)}{32Ga} x$$

which satisfy the conditions of no slip, conditions [10] and [11], provided that

$$\xi_x = \xi_y = 0 \qquad [16]$$

[4] See also "Theory of Elasticity," by A. E. H. Love, p. 242, where a solution to the problem of the plane with given surface tractions is expressed in terms of the potential functions of Boussinesq and Cerruti.

Table 1 Tangential surface displacements within the circle of contact $r < a$

Suffix	Tangential traction, $\frac{X}{\bar{X}}$	Tangential force, $\frac{T_x}{\pi a^2 \bar{X}}$	Twisting couple, $\frac{M_z}{\pi a^3 \bar{X}}$	$\frac{32Gu}{\pi a \bar{X}}$	$\frac{32G}{\pi \bar{X}}\left(\frac{\partial u}{\partial x}\right)$	$\frac{32Gv}{\pi a \bar{X}}$	$\frac{32G}{\pi \bar{X}}\left(\frac{\partial v}{\partial x}\right)$
(1)	$(a^2 - r^2)^{1/2}/a$	$2/3$	0	$\{2(2-\nu)(2a^2 - x^2 - y^2) + \nu(x^2 - y^2)\}/a^2$	$2(4-3\nu)x/a$	$2\nu xy/a^2$	$2\nu y/a$
(2)	$a/(a^2 - r^2)^{1/2}$	2	0	$8(2-\nu)$	0	0	0
(3)	$x/(a^2 - r^2)^{1/2}$	0	0	$2(4-3\nu)x/a$	$2(4-3\nu)$	$2\nu y/a$	0
(4)	$y/(a^2 - r^2)^{1/2}$	0	$-2/3$	$2(4-\nu)y/a$	0	$-2\nu x/a$	-2ν
(5)	$xy/a(a^2 - r^2)^{1/2}$	0	0	$3(2-\nu)xy/a^2$	$3(2-\nu)y/a$	$\nu\{2a^2 - 3(x^2+y^2)\}/2a^2$	$-3\nu x/a$
	$\frac{Y}{\bar{Y}}$	$\frac{T_y}{\pi a^2 \bar{Y}}$	$\frac{M_z}{\pi a^3 \bar{Y}}$	$\frac{32Gu}{\pi a \bar{Y}}$	$\frac{32G}{\pi \bar{Y}}\left(\frac{\partial u}{\partial x}\right)$	$\frac{32Gv}{\pi a \bar{Y}}$	$\frac{32G}{\pi \bar{Y}}\left(\frac{\partial v}{\partial x}\right)$
(1)	$(a^2 - r^2)^{1/2}/a$	$2/3$	0	$2\nu xy/a^2$	$2\nu y/a$	$\{2(2-\nu)(2a^2 - x^2 - y^2) - \nu(x^2 - y^2)\}/a^2$	$-2(4-\nu)x/a$
(2)	$a/(a^2 - r^2)^{1/2}$	2	0	0	0	$8(2-\nu)$	0
(3)	$x/(a^2 - r^2)^{1/2}$	0	$2/3$	$-2\nu y/a$	0	$2(4-\nu)x/a$	$2(4-\nu)$
(4)	$y/(a^2 - r^2)^{1/2}$	0	0	$-2\nu x/a$	-2ν	$2(4-3\nu)y/a$	0
(5)	$x^2/a(a^2 - r^2)^{1/2}$	$2/3$	0	$-\nu xy/a^2$	$-\nu y/a$	$\{2(4-3\nu)a^2 + (10-\nu)x^2 + (10-9\nu)y^2\}/2a^2$	$(10-\nu)x/a$

$$\bar{X}_1 = \frac{8G}{\pi} \frac{(3-\nu)}{3(3-2\nu)} p \qquad [17]$$

and

$$\bar{Y}_1 = \frac{8G}{\pi} \frac{(1-\nu)}{3(3-2\nu)} p \qquad [18]$$

The resultant tractions are therefore

$$X = \frac{8G(3-\nu)}{\pi 3(3-2\nu)} \frac{p}{a} \frac{xy}{(a^2 - r^2)^{1/2}} \qquad [19]$$

and

$$Y = \frac{8G(1-\nu)}{\pi 3(3-2\nu)} \frac{p}{a} \frac{a^2 - 2x^2 - y^2}{(a^2 - r^2)^{1/2}} \qquad [20]$$

The distribution is symmetrical about the y-axis and provides no resultant couple about the axis of spin. This is to be expected, since the stipulation of a perfectly elastic solution with no slip permits no energy dissipation as rolling proceeds.

In common with the no-slip solution for rolling under a tangential force the tractions [19] and [20] exhibit a singularity at $r = a$ which will be relieved in practice by slip.

Solution for Vanishingly Small Slip

It has been shown previously that the application of tangential tractions in both the x and y-directions to the contact surface of a rolling sphere causes slip to initiate at the trailing edge of the contact circle. In each of the cases examined, any assumption of slip between contacting points before they have passed through the locked region has been shown to contradict the law of friction. It will now be assumed that this state of affairs applies to the present problem of tangential tractions due to spin, and any tendency to slip at the point $(-a, 0)$ will be eliminated by making the tangential traction there equal to zero. This device already has been shown to give reasonable results in the case of rolling under tangential forces ($T \ll \mu N$), where a more complete solution is available for comparison [see Appendix to reference (1)].

It is possible to make the resultant traction at the point $(-a, 0)$ have zero value and still satisfy the conditions for no slip over the area of contact by adding to the no-slip solution the tractions

$$X_4 = \bar{X}_4 \frac{y}{(a^2 - r^2)^{1/2}} \qquad [21]$$

and

$$Y_3 = \bar{Y}_3 \frac{x}{(a^2 - r^2)^{1/2}} \qquad [22]$$

where

$$\bar{X}_4 = \frac{8G(3-\nu)}{\pi 3(3-2\nu)} p \quad \text{and} \quad \bar{Y}_3 = -\frac{8G(1-\nu)}{\pi 3(3-2\nu)} p$$

are chosen so that the resultant values of X and Y are zero at $(-a, 0)$. The final tractions then become

$$X = \frac{8G(3-\nu)}{\pi 3(3-2\nu)} \frac{p}{a} \frac{(a+x)y}{(a^2 - r^2)^{1/2}} \qquad [23]$$

and

$$Y = \frac{8G(1-\nu)}{\pi 3(3-2\nu)} \frac{p}{a} \frac{a^2 - 2x^2 - ax - y^2}{(a^2 - r^2)^{1/2}} \qquad [24]$$

The combined traction $|Q| = (X^2 + Y^2)^{1/2}$, which is no longer symmetrical about the y-axis, is shown in Fig. 2.

The additional tractions X_4 and Y_3 produce displacements (see Table 1) for which

$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial v}{\partial x} = \frac{\bar{X}_4 \pi(-2\nu)}{32G} + \frac{\bar{Y}_3 \pi}{32G} 2(4-\nu) = \frac{2-\nu}{3(3-2\nu)} \qquad [25]$$

This constant term, when included in the no-slip condition [11], gives rise to a transverse creep

$$\xi_y = \frac{2(2-\nu)}{3(3-2\nu)} p = \frac{2(2-\nu)}{3(3-2\nu)} \frac{\Omega a}{U} \qquad [26]$$

Hence a mechanism has been found to explain the transverse creep which is observed to accompany rolling with spin.

A second feature of the nonsymmetrical distribution of tractions [23] and [24] is that the resultant moment about the spin axis is no longer zero

$$M_z = \frac{2\pi a^3}{3} (-\bar{X}_4 + \bar{Y}_3) = -\frac{32(2-\nu)}{9(3-2\nu)} G a^3 p \qquad [27]$$

The results of this section are not exact, either for the case of no slip or when slip occurs, but may be taken to represent the limit to which the creep ξ_y and the resisting moment M_z tend when the

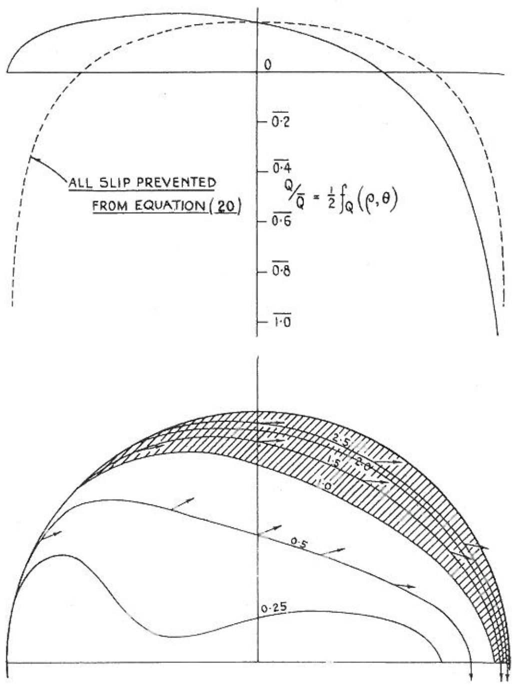

Fig. 2 Contours of resultant surface traction under action of a small angular velocity of spin (vanishingly small slip). Slip might be expected to extend over the shaded area when $pD/\mu a \simeq 1.0$.

degree of slip is small; i.e., for small values of p. The range of values of p for which Equations [26] and [27] might be expected to be valid may be deduced from the contours of traction shown in Fig. 2. It will be seen that the resultant traction

$$|Q| = \frac{8G(2-\nu)}{\pi 3(3-2\nu)}\, p\mathbf{F}_q\left(\frac{r}{a},\, \theta\right) \qquad [28]$$

reaches its maximum value, for any given radius, at $\theta \simeq \pm 60$ deg. Slip will progress radially inward from the trailing edge over the region in which $|Q| \geqq \mu Z$; i.e.

$$\frac{8G(2-\nu)}{\pi 3(3-2\nu)}\, p\mathbf{F}_q(\rho, \theta) \geqq \mu \frac{3N}{2\pi a^2}(1-\rho^2)^{1/2}$$

where $\rho = r/a$.

Substituting for a from [1] and writing $\mathbf{F}_q(\rho, \theta)/(1-\rho^2)^{1/2}$ as $\mathbf{F}_{q/z}(\rho, \theta)$, the penetration of slip is given by the condition

$$\mathbf{F}_{q/z}(\rho, \theta) \geqq \frac{3(3-2\nu)}{2(2-\nu)(1-\nu)}\left(\frac{\mu a}{pD}\right) \qquad [29]$$

Apparently, the degree of slip is governed by the nondimensional parameter $(pD/\mu a)$. The function $\mathbf{F}_{q/z}$ is sketched in Fig. 3 for $\theta = 60$ deg. Using Equation [29], the values of $(pD/\mu a)$ to produce increasing penetration of slip are deduced.[5] For example, slip penetrates to $\rho = 0.8$ at $\theta = 60$ deg when $(pD/\mu a)$ reaches the value 0.77, and to $\rho = 0.7$ when $(pD/\mu a)$ is 1.19.

We are now able to estimate, tentatively, the conditions under which the theoretically deduced transverse creep and resisting moment given by Equations [26] and [27] might be expected to apply. The results represent limiting solutions for small slip, but might be expected to provide a reasonable approximation for values of $(pD/\mu a)$ less than 1.0. Slip would then nowhere extend within $\rho = 0.75$ and would roughly cover the shaded area in Fig. 2.

[5] This calculation can only be approximate and apply to a relatively small degree of slip, since the action of slip modifies the traction within the locked region.

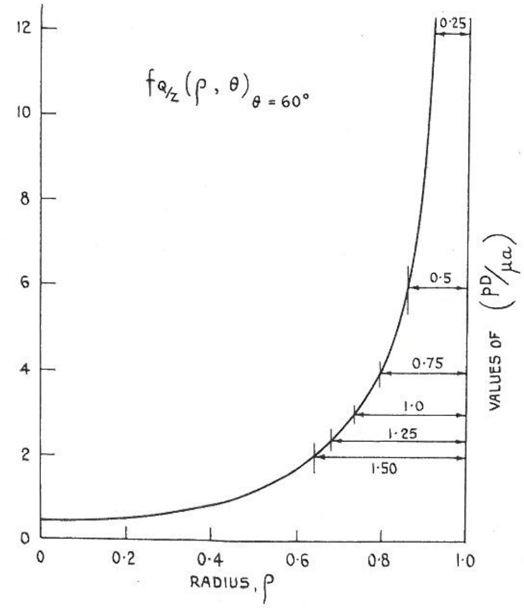

Fig. 3 Estimated progress of slip with increasing spin

Equations [26] and [27] may be rewritten in terms of the slip parameter $(pD/\mu a)$

$$\frac{\xi_y D}{\mu a} = \frac{2(2-\nu)}{3(3-2\nu)}\left(\frac{pD}{\mu a}\right) \qquad [30]$$

$$\frac{M_z D}{\mu G a^4} = -\frac{32(2-\nu)}{9(3-2\nu)}\left(\frac{pD}{\mu a}\right) \qquad [31]$$

or, more generally, to cover the values of $(pD/\mu a)$ for which slip is significant

$$\frac{\xi_y D}{\mu a} = f_1\left(\frac{pD}{\mu a}\right) \qquad [32]$$

and

$$\frac{M_z D}{\mu G a^4} = f_2\left(\frac{pD}{\mu a}\right) \qquad [33]$$

In the absence of a complete solution these functions may be determined by experiment.

Experimental

Experimental observations have been made of the transverse creep which accompanies rolling with spin. Also, with less success, an attempt has been made to measure the resisting moment about the spin axis.

A convenient arrangement in which spin occurs is provided by a simple thrust bearing. The bearing used possessed flat horizontal races, the load being transmitted through three symmetrically spaced balls. The lower race was fixed, while the upper race turned about a vertical central spindle and carried a dead load. If the upper race turns with angular velocity 2Ω the ball centers move round their track of radius R with a velocity $U = \Omega R$. It is clear that there is a relative angular velocity of 2Ω between the two races so that spin must occur between the ball and the races at one or both points of contact. If the surface conditions at both points of contact are identical, in order for the spin moments M_z acting on the ball to be equal and opposite, the ball will turn about a vertical (spin) axis with angular velocity Ω. The angular velocity of spin of the ball relative to the race at each contact is thereby $\pm\Omega$, and the rolling velocity at each contact $\pm U = \pm\Omega R$. Thus the spin factor becomes

$$p = \frac{\Omega a}{\Omega R} = \frac{a}{R} \quad \text{[34]}$$

having the same sign at each point of contact, so that the slip parameter reduces to

$$\frac{pD}{\mu a} = \frac{D}{\mu R} \quad \text{[35]}$$

As the bearing rotates the creep, transverse to the rolling direction, causes the balls to move radially outward, so that the track radius R gradually increases. This movement was measured by micrometer for one circulation of the balls around the track, and is denoted by δ. Thus for the experimental arrangement, Equations [32] and [33] become

$$\frac{\delta}{2\pi R}\,\frac{D}{\mu a} = f_1\left(\frac{D}{\mu R}\right) \quad \text{[36]}$$

$$\frac{M_z}{\mu N a} = \frac{3(1-\nu)}{8}\, f_2\left(\frac{D}{\mu R}\right) \quad \text{[37]}$$

For a given value of $(D/\mu R)$, corresponding to a given pattern of slip, Equation [36] suggests that δ is proportional to a. Variations in a were obtained by increasing the normal load. A typical set of creep measurements is shown in Fig. 4 for a series of ball diameters, which adequately demonstrate the linearity of δ with a. If the slopes of these lines are plotted in the form of Equation [36] then we should expect the results to follow a unique function of $(D/\mu R)$.

Difficulty arises in the choice of suitable values for μ. Reference to the legend accompanying Fig. 5 shows that creep measurements were made under a variety of surface conditions. Both hard and soft races were used in the "as-ground" and polished state (the balls were at all times hard and polished). The majority of the experiments were performed with lubricant present, but some measurements were made dry. To correlate these results the coefficient of sliding friction was measured in each case (see legend to Fig. 5). As would be expected, the measured coefficient of friction depended little upon surface finish and hardness. The creep measurements, on the other hand, showed a marked change as a result of polishing the races. To correlate the results into a unique function of $D/\mu R$, the effective value of μ for the polished races requires to be about half of the value for the rougher races. Even assuming that microslip over part of the area of contact is governed by Amonton's law so that we may write $Q = \mu Z$, the experiments suggest that the constant μ cannot be compared with the value associated with steady sliding.

This phenomenon had been observed by the author previously (8) during experiments on the static contact of spherical surfaces under tangential forces. It was suggested then that the infinite tractions associated with no slip might be relieved, in part, by distortion of the surface asperities rather than by microslip as assumed in the analysis. In this way a rough surface, with its more flexible asperities, would accommodate higher tractions before the onset of appreciable slip.

To avoid the difficulty of the indeterminate nature of the constant μ, the results of the creep experiments have been plotted in the form $[\delta/(2\pi a)](D/R)$ against D/R in Fig. 5. At small values of D/R the experimental results compare very favorably with the analytical solution of Equation [30]. Taking the effective value of μ to be no greater than 0.2, Equation [30] is seen to be a close fit over the range $D/(\mu R) < 1$. For large values of $D/(\mu R)$, the creep approaches a constant value whose magnitude is dependent upon the nature of the surface. When slip has progressed over a large proportion of the contact area, the magnitude

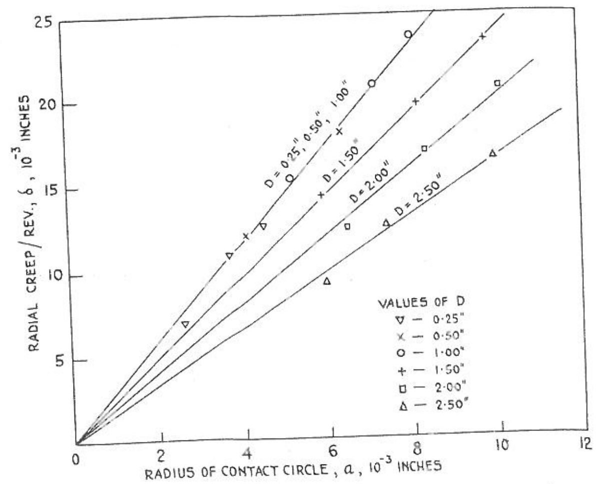

Fig. 4 Measurements of transverse creep due to spin

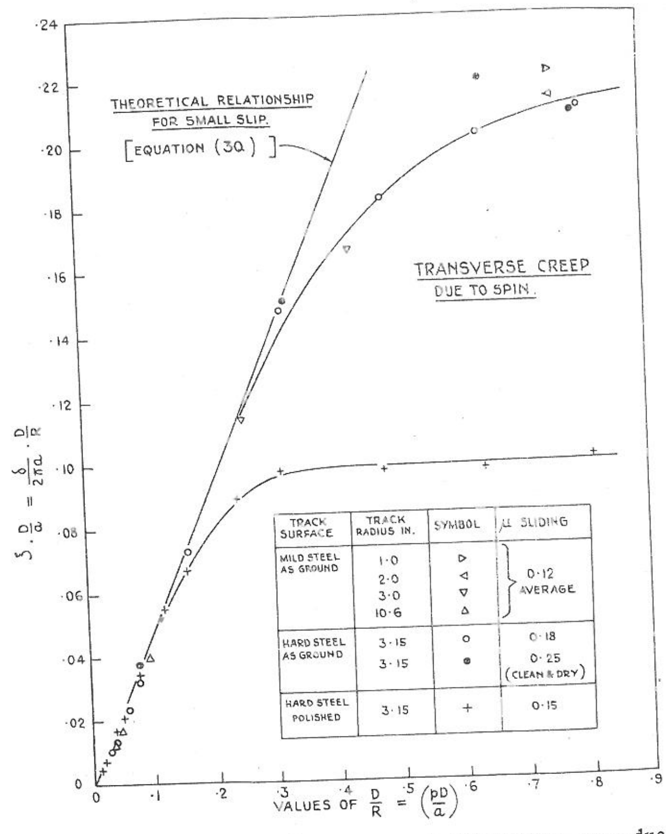

Fig. 5 Correlated results of measurements of transverse creep due to spin, compared with linear theoretical relationship

of the surface tractions, and hence the resulting creep, are no longer dependent upon the amount of spin.

An attempt was made to obtain the twisting moment function of Equation [37] experimentally by measurement of the torque-resisting rotation of the bearing. The over-all resistance was obtained by measuring the retardation of the bearing when rotating freely and slowly. In addition to the resistance due to spin, the over-all torque includes the resistance to straight free rolling—the usual rolling friction. Assuming the elastic-hysteresis theory presented by Tabor (9) to be correct, the rolling-friction force is given by

$$F = \alpha\Phi \quad \text{[38]}$$

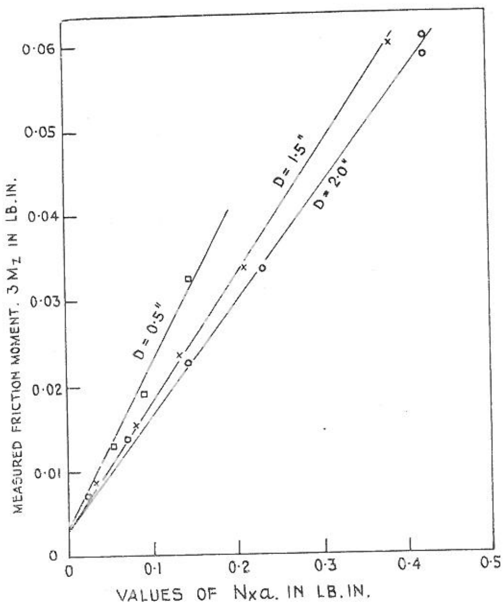

Fig. 6 Measured resisting moment during rolling with spin. This moment is made up of a component due to spin and a component due to elastic hysteresis.

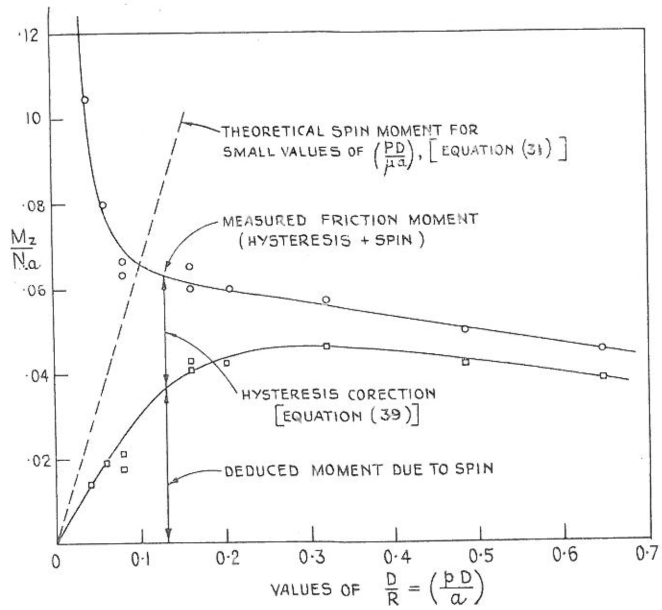

Fig. 7 Analysis of resisting moment measurements into spin and hysteresis components

where $$\Phi = 0.00105 \left(\frac{N^2}{D}\right)^{2/3} \text{ lb in.}$$

is the elastic work done per unit distance rolled for a steel ball rolling on a plane, and α is the fraction of this work dissipated by hysteresis.[6] Hence the friction moment per ball due to hysteresis is

$$(M_z)_h = FR = 0.00105 \left(\frac{N^2}{D}\right)^{2/3} \alpha R$$

Using Equation [1] this may be rewritten

$$(M_z)_h = 0.36\alpha \left(\frac{R}{D}\right) Na \qquad [39]$$

Tabor quotes a figure of 2 per cent for the value of α for hard-steel rolling surfaces. Measurements by the author (unpublished) of the hysteresis loss at the stationary contact of a hard-steel sphere and plane gave an approximately constant value of 0.4 per cent for normal loading within the elastic range. We may write therefore

$$(M_z)_h = k \left(\frac{R}{D}\right) Na$$

where k is a constant of magnitude 0.0015 to 0.0072. To the hysteresis moment must be added the moment due to spin given by Equation [37] so that (taking μ as a constant) the total resisting moment is

$$\frac{M_z}{Na} = \left(\frac{M_z}{Na}\right)_h + \left(\frac{M_z}{Na}\right)_s = k\left(\frac{R}{D}\right) + f\left(\frac{D}{R}\right) \qquad [40]$$

It may be seen that both components are functions of the ratio (D/R) and, for a constant value of this ratio, are proportional to the product $N \times a$. Typical measurements of friction torque at varying loads are shown in Fig. 6. They are approximately proportional to Na. The small, fairly constant, intercept at $Na = 0$ represents the friction of the bearing spindle.

Reference to Equation [40] shows that the hysteresis component becomes large and predominates at small values of (D/R). The slopes of Fig. 6, M/Na, are plotted against (D/R) in Fig. 7. Multiplying through by (D/R) and extrapolating back to $D/R = 0$ yields a value of $k = 0.0037$ which is consistent with the values estimated from independent measurements. Thus the hysteresis component can be subtracted from the total to leave the spin component.

It is immediately apparent that in the region of small slip (D/R small) the hysteresis component is much larger than the spin component so that no reliable comparison with the linear relationship of Equation [31] is possible. For larger values of D/R the hysteresis component becomes small and the experiments should give a reasonable measure of the resisting moment due to spin. With increasing spin velocities, slip relieves the high stresses at the trailing edge of the contact area, causing the resisting moment to depart from the linear relationship of Equation [31], and to approach a limiting value. A tendency for the moment to decrease at large values of D/R was observed.

Acknowledgments

The author gratefully acknowledges the help given by Prof. J. N. Goodier in discussion of the problem in elasticity, and by Mr. J. Walden in carefully performing many of the experiments.

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