Given a sequence of a_i for $i = 0, 1, .., n$
a_j is singular iff:

$$a_j = \frac{1}{j} \sum_{0}^{j-1} a_i$$

lets assume a_j is singular, as per the above definition. Given that, for $k > j$ for a_k to be singular again, it must hold:

$$a_k = \frac{1}{k} \left[\sum_{i=0}^{j-1} a_i + \sum_{i=j}^{k-1} a_i \right]$$

which can be split as:

$$a_k = \frac{j}{k} \left[\frac{1}{j} \sum_{i=0}^{j-1} a_i \right] + \frac{1}{k}(a_j + a_{j+1} + ... + a_{k-1})$$

or

$$a_k = (\frac{j}{k})a_j + \frac{1}{k}(a_j + a_{j+1} + ... + a_{k-1})$$

adding and substracting a_j $(k-j)/k$ times leaves:

$$a_k = (\frac{j}{k})a_j + (\frac{k-j}{k})a_j + \frac{1}{k}((a_{j+1} - a_j) + (a_{j+2} - a_j) + ... + (a_{k-1} - a_j))$$

or

$$a_k = a_j + \frac{1}{k} \sum_{i=j+1}^{k-1} (a_i - a_j)$$

Now, for a_k to be singular, the above must be met. And for it is necessary that:

$$\frac{1}{k} \sum_{i=j+1}^{k-1} (a_i - a_j) = q \ ; \ \{q \in \mathcal{Q}\}$$

where $\mathcal{Q}$ is the set of positive integers. And, of course that, as well, it happens that:

$$a_k - a_j = q$$