4. Übungsblatt zu Experimentalphysik I (WS 20/21)

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Gruppe: F

Punkte: ___/___/___/___/___ Σ___

4.1 Aufgabe 1

Geg.:

- $\vec{r} = (x, y, z)^t$
- $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- $\hat{\vec{r}} = \frac{\vec{r}}{r}$
- $\vec{\nabla} = (\partial_x, \partial_y, \partial_z)$

z.z.:

a) $\vec{\nabla} r = \hat{\vec{r}}$

$$(\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z},)^t = \frac{(x, y, z)^t}{\sqrt{x^2 + y^2 + z^2}}$$

mit:

$$\begin{aligned}
\frac{\partial}{\partial x}(x^2 + y^2 + z^2)^{\frac{1}{2}} &= \frac{2x(x^2 + y^2 + z^2)^{\frac{-1}{2}}}{2} \\
&= \frac{x}{\sqrt{x^2 + y^2 + z^2}} \\
\frac{\partial}{\partial y}(x^2 + y^2 + z^2)^{\frac{1}{2}} &= \frac{y}{\sqrt{x^2 + y^2 + z^2}} \\
\frac{\partial}{\partial z}(x^2 + y^2 + z^2)^{\frac{1}{2}} &= \frac{z}{\sqrt{x^2 + y^2 + z^2}}
\end{aligned}$$

$$\begin{aligned}
\rightarrow (\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z})^t &= [(\frac{x}{\sqrt{x^2 + y^2 + z^2}}), (\frac{y}{\sqrt{x^2 + y^2 + z^2}}), \frac{z}{\sqrt{x^2 + y^2 + z^2}}]^t \\
&= \frac{(x, y, z)^t}{\sqrt{x^2 + y^2 + z^2}}
\end{aligned}$$

b) $\vec{\nabla} f(r) = \hat{\vec{r}} \frac{df}{dr}$

$$
\begin{aligned}
\vec{\nabla} f(r) &= \hat{\vec{r}}\frac{df}{dr} \\
&= \hat{\vec{r}}\frac{df}{dr} \\
&= (\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z})^t \frac{df}{dr} \\
&= (\frac{df}{dr}\frac{\partial r}{\partial x}, \frac{df}{dr}\frac{\partial r}{\partial y}, \frac{df}{dr}\frac{\partial r}{\partial z})^t
\end{aligned}
$$

Notation: $\frac{df}{dg}\frac{dg}{dx} = \frac{df}{dx}$

$$
\rightarrow (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}) = \vec{\nabla} f(r)
$$

c) rot $\vec{r} \equiv \vec{\nabla} \times \vec{r}$

$$
\begin{aligned}
(\nabla \times r)^i &= \epsilon^{ijk}\nabla^j r^k \\
&= (\nabla^2 r^3 - \nabla^3 r^2, \nabla^3 r^1 - \nabla^1 r^3, \nabla^1 r^2 - \nabla^2 r^1)^t
\end{aligned}
$$

mit $x = 1$, $y = 2$ und $z = 3$

$$
\begin{aligned}
&\rightarrow (\partial_y z - \partial_z y, \partial_z x - \partial_x z, \partial_x y - \partial_y x)^t \\
&= (\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}, \frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}, \frac{\partial y}{\partial x} - \frac{\partial x}{\partial y})^t
\end{aligned}
$$

d) div $\vec{r} \equiv \vec{\nabla} \cdot \vec{r}$

$$
\begin{aligned}
\vec{\nabla} \cdot \vec{r} &= \delta^{ij}\nabla^i r^j \\
&= \nabla^1 r^1 + \nabla^2 r^2 + \nabla^3 r^3 \\
&= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} \\
&= 1 + 1 + 1 = 3
\end{aligned}
$$

4.2 Aufgabe 2

a)

b) Geg.:

- Kraftfeld definiert auf $\mathbb{R}^3 \setminus (x - Achse)$
- $\vec{F} = \frac{c}{y^2 + z^2}\begin{pmatrix} 0 \\ -z \\ y \end{pmatrix}$

Der Raum ist nicht einfach zusammenhängend, deswegen gilt $\vec{F} \times \vec{\nabla} = 0$ nicht. Man muss also zeigen, ob die Bedingung:

$$\oint \vec{F} d\vec{s} = 0$$

erfüllt ist.

Als Beispiel nehmen wir eine Kurve um die Achse und projezieren die auf die y-z-Ebene:

$y = \sin\alpha$

$z = \cos\alpha$

$$\begin{aligned}\vec{F} &= \frac{c}{\sin^2\alpha + \cos^2\alpha} \begin{pmatrix} -\cos\alpha \\ \sin\alpha \end{pmatrix} \\ &= \begin{pmatrix} -c \cdot \cos\alpha \\ c \cdot \sin\alpha \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\rightarrow \oint \vec{F} d\vec{s} &= \int_0^{2\pi} \begin{pmatrix} -c \cdot \cos\alpha \\ c \cdot \sin\alpha \end{pmatrix} d\alpha \\ &= c \cdot \int_0^{2\pi} \begin{pmatrix} -\cos\alpha \\ sin\alpha \end{pmatrix} d\alpha \\ &= \\ &= \int c(-\sin\alpha, \cos\alpha)^t \cdot (-\sin\alpha, \cos\alpha)^t d\alpha \\ &= \int_0^{2\pi} c(\sin^2\alpha + \cos^2\alpha) d\alpha \\ &= \int_0^{2\pi} c = 2\pi c\end{aligned}$$

c) Geg.:

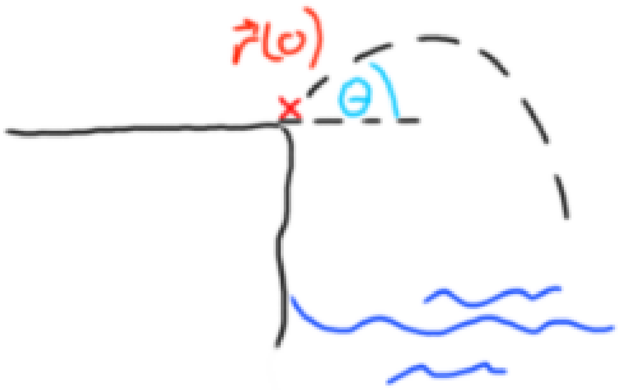
- Kraftfeld definiert auf $\mathbb{R}^3$
- $\vec{F} = \vec{a} \cdot (\vec{b} \cdot \vec{r})$
- $\vec{a}$ und $\vec{b}$ sind konstante Vektoren

Lsg.:

$$\begin{aligned}\vec{F} \times \vec{\nabla} &= 0 \\ \vec{a} \cdot (\vec{b} \cdot \vec{r}) \times \vec{\nabla} &= 0 \\ (\vec{a} \cdot \vec{b}) \cdot \vec{r} \times \vec{\nabla} &= 0 \\ 0 \cdot \vec{r} \times \vec{\nabla} &= 0\end{aligned}$$

$\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}$

4.3 Aufgabe 3



Geg.:

- $\vec{r}(0) = (0,0,0)^t$
- $|\vec{v}(0)| = v_0$
- $\vec{v}(0) = v_0(\cos\theta, 0, \sin\theta)^t$
- $\vec{a} = -k\vec{v} - g\vec{e_z}$
- $\vec{v}(t) = \int_0^t \vec{a}(t')dt'$

  $= -k\vec{v}t - g\vec{e_z}t + \vec{v}(0)$
- $\vec{r}(t) = \int_0^t \vec{v}(t')dt'$

  $= -\dfrac{k\vec{v}t}{2} - \dfrac{g\vec{e_z}t^2}{2} + \vec{v}(0) + \vec{r}(0)$

a) Ges.: $\vec{v}$

Geg.:

- $\vec{a} = \dot{\vec{v}}$
- $\vec{v} = \vec{\phi}(t)e^{-kt}$

Lsg.:

$$\begin{aligned}\dot{\vec{v}} &= \frac{d}{dt}\vec{\phi}(t)e^{-kt} \\ &= \dot{\vec{\phi}}e^{-kt} - k\vec{\phi}e^{-kt}\end{aligned}$$

$$\vec{a} = -k\vec{\phi}(t)e^{-kt} - g\vec{e_z}$$

$$\begin{aligned}
\rightarrow \vec{a} &= \dot{\vec{v}} \\
-k\vec{\phi}(t)e^{-kt} - g\vec{e_z} &= \dot{\vec{\phi}}e^{-kt} - k\vec{\phi}e^{-kt} \\
-g\vec{e_z} &= \dot{\vec{\phi}}e^{-kt} \\
\dot{\vec{\phi}} &= -ge^{kt}\vec{e_z}
\end{aligned}$$

Bestimme $\vec{\phi}$:

$$\begin{aligned}
\vec{\phi} &= \int -ge^{kt}\vec{e_z} \\
&= \frac{-ge^{kt}}{k}\vec{e_z} + \vec{C}
\end{aligned}$$

Bestimme $\vec{C}$:

$$\begin{aligned}
\begin{pmatrix} v_0 \cos\theta \\ 0 \\ v_0 \sin\theta \end{pmatrix} &= \frac{-g}{k}\vec{e_z} + \vec{C} \\
\begin{pmatrix} v_0 \cos\theta \\ 0 \\ v_0 \sin\theta \end{pmatrix} + \frac{g}{k}\vec{e_z} &= \vec{C} \\
\vec{C} &= \begin{pmatrix} v_0 \cos\theta \\ 0 \\ v_0 \sin\theta + \frac{g}{k} \end{pmatrix}
\end{aligned}$$

Setze $\vec{C}$ ein:

$$\begin{aligned}
\vec{v} &= \vec{\phi}e^{-kt} + \vec{v_0} \\
&= \begin{pmatrix} v_0 \cos\theta \\ 0 \\ \frac{-ge^{kt}}{k} + v_0 \sin\theta + \frac{g}{k} \end{pmatrix} e^{-kt} \\
&= \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ (\frac{-ge^{kt}}{k} + v_0 \sin\theta + \frac{g}{k})e^{-kt} \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ \frac{-g}{k} + v_0 \sin\theta e^{-kt} + \frac{g}{k}e^{-kt} \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt} - 1) \end{pmatrix}
\end{aligned}$$

b) Ges.: $\vec{r}$

Geg.:

- $\vec{v} = \dot{\vec{r}}$

$$\dot{\vec{r}} = \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt} - 1) \end{pmatrix}$$

$$\int \dot{\vec{r}} dt = \int \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt} - 1) \end{pmatrix} dt$$

$$\vec{r} = \begin{pmatrix} \frac{v_0 \cos\theta e^{-kt}}{-k} \\ 0 \\ \frac{v_0 \sin\theta e^{-kt}}{-k} + \frac{g e^{-kt}}{-k^2} - \frac{gt}{k} \end{pmatrix} + \vec{R}$$

$$= \begin{pmatrix} \frac{v_0 \cos\theta e^{-kt}}{-k} \\ 0 \\ \frac{gt + v_0 \sin\theta e^{-kt}}{-k} + \frac{g e^{-kt}}{-k^2} \end{pmatrix} + \vec{R}$$

Bestimme $\vec{R}$:

$$\vec{r_0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{v_0 \cos\theta e^{-k0}}{-k} \\ 0 \\ \frac{gt + v_0 \sin\theta e^{-k0}}{-k} + \frac{g e^{-k0}}{-k^2} \end{pmatrix} + \vec{R}$$

$$\vec{R} = \begin{pmatrix} \frac{v_0 \cos\theta}{k} \\ 0 \\ \frac{v_0 \sin\theta}{k} + \frac{g}{k^2} \end{pmatrix}$$

Setze $\vec{R}$ ein:

$$\vec{r} = \begin{pmatrix} \frac{v_0 \cos\theta e^{-kt}}{-k} + \frac{v_0 \cos\theta}{k} \\ 0 \\ \frac{gt + v_0 \sin\theta e^{-kt}}{-k} + \frac{g e^{-kt}}{-k^2} + \frac{v_0 \sin\theta}{k} + \frac{g}{k^2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{v_0 \cos\theta e^{-kt}}{-k} + \frac{v_0 \cos\theta}{k} \\ 0 \\ (\frac{v_0 \sin\theta}{k} + \frac{g}{k^2})(1 - e^{-kt}) - \frac{gt}{k} \end{pmatrix}$$

c) Ges.: $\lim_{t\to\infty} |\vec{v}|$

$$
\begin{aligned}
\lim_{t\to\infty} |\vec{v}| &= \sqrt{v_x^2 + v_y^2 + v_z^2} \\
&= \sqrt{(v_0 \cos\theta e^{-kt})^2 + (v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt} - 1))^2} \\
&= \sqrt{0 + (\frac{-g}{k})^2} \\
&= \sqrt{(\frac{g}{k})^2} \\
&= \frac{g}{k}
\end{aligned}
$$

Ges.: $\lim_{t\to\infty} r_x$

$$
\begin{aligned}
&\lim_{t\to\infty} \frac{v_0 \cos\theta e^{-kt}}{-k} + \frac{v_0 \cos\theta}{k} \\
&= \frac{v_0 \cos\theta}{k}
\end{aligned}
$$

d) Geg.: $T = \frac{1}{k} \ln(1 + \frac{v_0 k \sin\theta}{g})$

Bed.: Den höchsten Bahnpunkt (r_z-Richtung) findet man mit $\dot{r_z} = 0$

$$
\begin{aligned}
v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt} - 1) &= 0 \\
v_0 \sin\theta e^{-kt} + \frac{g}{k}e^{-kt} - \frac{g}{k} &= 0 \\
v_0 \sin\theta e^{-kt} + \frac{g}{k}e^{-kt} &= \frac{g}{k} \\
e^{-kt}(v_0 \sin\theta + \frac{g}{k}) &= \frac{g}{k} \\
e^{-kt} &= \frac{g}{k(v_0 \sin\theta + \frac{g}{k})} \\
e^{kt} &= \frac{k(v_0 \sin\theta + \frac{g}{k})}{g} \\
e^{kt} &= \frac{kv_0 \sin\theta}{g} + 1 \\
kt &= \ln(\frac{kv_0 \sin\theta}{g} + 1) \\
T &= \frac{1}{k} \ln(\frac{kv_0 \sin\theta}{g} + 1)
\end{aligned}
$$

e) Geg.: $H = \frac{v_0 \sin\theta}{k} - \frac{g}{k^2} \ln(1 + \frac{v_0 k \sin\theta}{g})$

Nun müssen wir unser T in die z-Koordinate der Bahnkurve einsetzen

$$
\begin{aligned}
H &= \frac{gT + v_0 \sin\theta e^{-kT}}{-k} + \frac{ge^{-kT}}{-k^2} + \frac{v_0\sin\theta}{k} + \frac{g}{k^2} \\
&= (\frac{v_0\sin\theta}{k} + \frac{g}{k^2})(1-e^{-kT}) - \frac{gT}{k} \\
&= \frac{v_0\sin\theta}{k} + \frac{g}{k^2} - (\frac{kv_0\sin\theta}{k^2} + \frac{g}{k^2})e^{-(\ln(1+\frac{kv_0\sin\theta}{g}))} - \frac{g}{k}\cdot(\frac{1}{k}\ln(\frac{1+v_0\sin\theta}{g})) \\
&= \frac{v_0\sin\theta}{k} + \frac{g}{k^2} + \frac{kv_0\sin\theta + g}{k^2}\cdot\frac{1}{1+\frac{kv_0\sin\theta}{g}} - \frac{g}{k^2}\ln(\frac{1+v_0\sin\theta}{g}) \\
&= \frac{v_0\sin\theta}{k} + \frac{g}{k^2} - \frac{kv_0\sin\theta + g}{k^2}\cdot\frac{g}{g+kv_0\sin theta} - \frac{g}{k^2}\ln(\frac{1+v_0\sin\theta}{g}) \\
&= \frac{v_0\sin\theta}{k} + \frac{g}{k^2} - \frac{g}{k^2} - \frac{g}{k^2}\ln(\frac{1+v_0\sin\theta}{g}) \\
H &= \frac{v_0\sin\theta}{k} - \frac{g}{k^2}\ln(\frac{1+v_0\sin\theta}{g})
\end{aligned}
$$

f) Geg.:

- $\vec{v}(t) = \vec{v_0} - gt\vec{e_z}$
- $\vec{r}(t) = t\vec{v_0} - \frac{gt^2}{2}\vec{e_z}$

  Für $k \to 0$
- $e^{-kt} \approx 1 - kt + \frac{k^2t^2}{2}$

Bestimme $\vec{v}$:

$$
\begin{aligned}
\lim_{k\to 0} = \vec{v}(t) &= \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ v_0 \sin\theta e^{-kt} + \frac{g}{k}(e^{-kt}-1) \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta e^{-kt} \\ 0 \\ (v_0 \sin\theta + \frac{g}{k})e^{-kt} - \frac{g}{k} \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta (1 - kt + \frac{k^2t^2}{2}) \\ 0 \\ (v_0 \sin\theta + \frac{g}{k})(1 - kt + \frac{k^2t^2}{2}) - \frac{g}{k} \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta (1 - kt) + O(k^2) \\ 0 \\ (v_0 \sin\theta + \frac{g}{k})(1 - kt) - \frac{g}{k} + O(k^2) \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta \\ 0 \\ (v_0 \sin theta + \frac{g}{k}) - ktv_0 \sin\theta - \frac{g}{k}kt - \frac{g}{k} + O(k^2) \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta \\ 0 \\ (v_0 \sin\theta + \frac{g}{k}) - gt - \frac{g}{k} + O(k^2) \end{pmatrix} \\
&= \begin{pmatrix} v_0 \cos\theta \\ 0 \\ v_0 \sin\theta - gt \end{pmatrix} \\
&= \vec{v_0} - gt\vec{e_z}
\end{aligned}
$$

Bestimme $\vec{r}$:

$$
\begin{aligned}
\lim_{k\to 0} &= \begin{pmatrix} \frac{v_0\cos\theta e^{-kt}}{-k} + \frac{v_0\cos\theta}{k} \\ 0 \\ (\frac{v_0\sin\theta}{k} + \frac{g}{k^2})(1-e^{-kt}) - \frac{gt}{k} \end{pmatrix} \\
&= \begin{pmatrix} \frac{v_0\cos\theta}{-k}(1-kt+\frac{k^2t^2}{2}) + \frac{v_0\cos\theta}{k} \\ 0 \\ (\frac{v_0\sin\theta}{k} + \frac{g}{k^2})(1-(1-kt+\frac{k^2t^2}{2})) - \frac{gt}{k} \end{pmatrix} \\
&= \begin{pmatrix} \frac{v_0\cos\theta}{-k}1 + v_0\cos\theta t + \frac{v_0\cos\theta}{k} \\ 0 \\ (\frac{v_0\sin\theta}{k} + \frac{g}{k^2})(kt-\frac{k^2t^2}{2})) - \frac{gt}{k} \end{pmatrix} \\
&= \begin{pmatrix} v_0\cos\theta t \\ 0 \\ v_0\sin\theta t + \frac{gt}{k} - \frac{v_0\sin\theta}{k}\cdot\frac{k^2t^2}{2} - \frac{gt^2}{2} - \frac{gt}{k} \end{pmatrix} \\
&= \begin{pmatrix} v_0\cos\theta t \\ 0 \\ v_0\sin\theta t - \frac{gt^2}{2} \end{pmatrix} \\
&= \vec{v_0}t - \frac{gt^2}{2}\vec{e_z}
\end{aligned}
$$

4.4 Aufgabe 4

Geg.:

- $f(x) = \frac{1}{a+bx+cx^2}$, $b^2 > 4ac$
- $\frac{1}{a+bx+cx^2} = \frac{\alpha}{x-x_1} + \frac{\beta}{x-x_2}$

Bestimme die Nullstellen:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2c}$$

Mit Linearfaktorzerlegung erhalten wir:

$$
\begin{aligned}
\frac{1}{c(x-x_1)(x-x_2)} &= \frac{\alpha}{x-x_1} + \frac{\beta}{x-x_2} \\
\to 1 &= c\alpha(x-x_2) + c\beta(x-x_1)
\end{aligned}
$$

Bestimme α mit $x = x_1$

$$
\begin{aligned}
\frac{1}{c(x-x_1)(x-x_2)} &= \frac{\alpha}{x-x_1} + \frac{\beta}{x-x_2} \\
\rightarrow 1 &= c\alpha(\frac{-b+\sqrt{b^2-4ac}}{2c} - \frac{-b-\sqrt{b^2-4ac}}{2c}) + c\beta(x_1 - x_1) \\
1 &= c\alpha(\frac{-b+\sqrt{b^2-4ac}}{2c} - \frac{-b-\sqrt{b^2-4ac}}{2c}) \\
&= \alpha(\frac{-b+\sqrt{b^2-4ac}}{2} - \frac{-b-\sqrt{b^2-4ac}}{2}) \\
&= \alpha(\frac{\sqrt{b^2-4ac}}{2} + \frac{\sqrt{b^2-4ac}}{2}) \\
&= \alpha\sqrt{b^2-4ac} \\
\rightarrow \alpha &= \frac{1}{\sqrt{b^2-4ac}}
\end{aligned}
$$

Analog für β:

$$
\beta = \frac{-1}{\sqrt{b^2-4ac}}
$$

α und β in $f(x)$ einsetzen:

$$
\begin{aligned}
f(x) &= \frac{\alpha}{x-x_1} + \frac{\beta}{x-x_2} \\
&= \frac{\frac{1}{\sqrt{b^2-4ac}}}{x - \frac{-b+\sqrt{b^2-4ac}}{2c}} + \frac{\frac{-1}{\sqrt{b^2-4ac}}}{x - \frac{-b-\sqrt{b^2-4ac}}{2c}} \\
&= \frac{1}{\sqrt{b^2-4ac}}(\frac{1}{x - \frac{-b+\sqrt{b^2-4ac}}{2c}} + \frac{1}{x - \frac{-b-\sqrt{b^2-4ac}}{2c}})
\end{aligned}
$$

Integriere:

$$
\begin{aligned}
\int f(x)dx &= \int [\frac{1}{\sqrt{b^2-4ac}}(\frac{1}{x - \frac{-b+\sqrt{b^2-4ac}}{2c}} + \frac{1}{x - \frac{-b-\sqrt{b^2-4ac}}{2c}})]dx \\
&= \frac{1}{\sqrt{b^2-4ac}}\int (\frac{1}{x - \frac{-b+\sqrt{b^2-4ac}}{2c}} + \frac{1}{x - \frac{-b-\sqrt{b^2-4ac}}{2c}})dx \\
&= \frac{1}{\sqrt{b^2-4ac}}\int \frac{1}{x - \frac{-b+\sqrt{b^2-4ac}}{2c}}dx + \frac{1}{\sqrt{b^2-4ac}}\int \frac{1}{x - \frac{-b-\sqrt{b^2-4ac}}{2c}}dx \\
&= \frac{1}{\sqrt{b^2-4ac}}[\ln(x - \frac{-b+\sqrt{b^2-4ac}}{2c}) + \ln(x - \frac{-b-\sqrt{b^2-4ac}}{2c})] + C
\end{aligned}
$$

4.5 Aufgabe 5

Geg.:

- $\dddot{x}(t) = ax(t) + b\ddot{x}(t) + c\ddot{x}(t)$

a) $\dddot{x} = ax + b\dot{x} + c\ddot{x}$

   $\dot{x} = v$

   $\dot{v} = a'$

   $\dot{a}' = ax + bv + ca'$

b) Geg.:

   - $\frac{d}{dt}\vec{r} = M\vec{r}$

$$\begin{pmatrix} \dot{x} \\ \dot{v} \\ \dot{a} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ a & 0 & 1 \\ a' & b & c \end{pmatrix} \cdot \begin{pmatrix} x \\ v \\ a \end{pmatrix}$$

   mit $M = \begin{pmatrix} 0 & 1 & 0 \\ a & 0 & 1 \\ a' & b & c \end{pmatrix}$

   und $\vec{r} = \begin{pmatrix} x \\ v \\ a \end{pmatrix}$