10. Exercise sheet to Experimental Physics (WS 20/21)

Names: Joshua Detrois, Leo Knapp, Juan Provencio

Group: F

Points: ___/___/___ Σ___

10.1 Exercise 1

Given:

- $m_B = 10$ kg
- $V_B = 10$ m^3
- $\rho_0 = 1.24$ kg m^{-3}
- $p_0 = 1013$ mbar
- $\rho_B = \frac{m_B}{V_B} = 1$ kg m^{-3}

We know that the density of the baloon ρ_B is constant. In order for the baloon to hover in the air, the density of the air around it has to be the same as ρ_B.

So we set:

$$\rho = \frac{p}{p_0}\rho_0 \qquad | p = p_0 e^{-g\frac{\rho_0}{p_0}h} \tag{1}$$

$$= \rho_0 e^{-g\frac{\rho_0}{p_0}h} \tag{2}$$

$$\rightarrow \rho_B = \rho_0 e^{-g\frac{\rho_0}{p_0}h} \tag{3}$$

$$\ln\left(\frac{\rho_B}{\rho_0}\right) = -g\frac{\rho_0}{p_0}h \tag{4}$$

$$h = \frac{\ln\left(\frac{\rho_B}{\rho_0}\right)p_0}{-g\rho_0} \tag{5}$$

$$\approx 1791 \text{ m} \tag{6}$$

This is a stable equilibrium point, because if the balloon were to climb just a little higher to a point where the density of air is lower than that of the balloon, the balloon would fall back to its hover point, and if the balloon were to fall down to a height where the air's density is bigger than that of the balloon, it would return to its hover position as well.

10.2 Exercise 2

Given:

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a) Since the buoy is floating, that means that the buoyant force is equal and opposite to the force of gravity. The buoyant force is

$$F_b = \rho_L g V_L \tag{7}$$

Whereas ρ_L is the density of the liquid and V_L is the displaced volume.

$$\rho_L g V_L = mg \tag{8}$$

The displaced volume of the liquid is the volume of the submerged area of the cone:

$$\begin{aligned}
V_L &= \frac{\pi r^2 h_d}{3} && |h_d = \frac{3h}{4}, r = \frac{3R}{4} && (9)\\
&= \frac{\pi 3^2 R^2 3h}{4^2 \cdot 4 \cdot 3} && && (10)\\
&= \frac{\pi 3^2 R^2 h}{4^3} && && (11)
\end{aligned}$$

If we input this into our formula we get

$$\begin{aligned}
\rho_L g \frac{\pi 3^2 R^2 h}{4^3} &= mg && |m = \rho_c V_c g && (12)\\
\rho_L g \frac{\pi 3^2 R^2 h}{4^3} &= \rho V_c g && |V_c = \frac{\pi R^2 h}{3} && (13)\\
\rho_L g \frac{\pi 3^2 R^2 h}{4^3} &= \rho_c \frac{\pi R^2 h}{3} g && && (14)\\
\rho_L \frac{3^2}{4^3} &= \frac{\rho_c}{3} && && (15)\\
\rho_c &= \frac{27 \rho_L}{64} && && (16)
\end{aligned}$$

To pull the buoy out of the water, the buoyant force helps us and gravity works against us, so we need

$$\begin{aligned}
F &= F_g - F_b && (17)\\
&= \frac{27\rho_L}{64} V(H) g - \rho_L V(h) g && (18)\\
&= \frac{27 \rho_L R^2 H}{3 \cdot 64} g - \rho_L \frac{R^2 h^2 h}{3H^2} g && (19)\\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{27H}{64} - \frac{h^3}{H^2} \right) && (20)
\end{aligned}$$

b) To determine the work done by pulling the buoy all the way up to the top we need to integrate, because fhe force changes with the height:

$$W = \int_0^{\frac{3H}{4}} \frac{\rho_L g \pi R^2}{3} \left(\frac{27H}{64} - \frac{h^3}{H^2} \right) dh \tag{21}$$

$$
\begin{aligned}
&= \frac{\rho_L g \pi R^2}{3} \int_0^{\frac{3H}{4}} \frac{27H}{64} - \frac{h^3}{H^2} dh && (22) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\int_0^{\frac{3H}{4}} \frac{27H}{64} dh - \int_0^{\frac{3H}{4}} \frac{h^3}{H^2} dh \right) && (23) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{27H}{64} h \Big|_0^{\frac{3H}{4}} - \frac{h^4}{4H^2} \Big|_0^{\frac{3H}{4}} \right) && (24) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{27H}{64} \cdot \frac{3H}{4} - \frac{(3H)^4}{4^5 H^2} \right) && (25) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{81H^2}{256} - \frac{81H^4}{1024H^2} \right) && (26) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{81H^2}{256} - \frac{81H^2}{1024} \right) && (27) \\
&= \frac{\rho_L g \pi R^2}{3} \left(\frac{243H^2}{1024} \right) && (28) \\
&= \frac{64}{25} \rho_c \frac{\pi R^2 H}{3} g \frac{243}{1024H} && (29) \\
&= \frac{9mgH}{16} && (30)
\end{aligned}
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10.3 Exercise 3

Given:

- $L = 20$ m
- $A_1 = 4000$ mm^2
- $A_2 = 50$ mm^2
- $p_A = 10$ bar

Assumed:

- $\rho = 1$ gm cm^{-3}
- $p_0 = 1$ bar

a) As per Bernoulli's Equation, we can say that

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\begin{aligned}
p_1 + \frac{\rho v_1^2}{2} + \rho g h_1 &= p_2 + \frac{\rho v_2^2}{2} + \rho g h_2 \qquad | p_1 = p_A, p_2 = p_0, h_1 = h_2 = h && (31) \\
p_A + \frac{\rho v_1^2}{2} &= p_0 + \frac{\rho v_2^2}{2} && (32)
\end{aligned}
$$

If we use the Continuity Equation, we can substitute our variable v_1 with:

$$
\begin{aligned}
A_1 v_1 &= A_2 v_2 && (33) \\
v_1 &= \frac{A_2}{A_1} v_2 && (34)
\end{aligned}
$$

So:

$$p_A + \frac{\rho}{2}\left(\frac{A_2}{A_1}v_2\right)^2 = p_0 + \frac{\rho v_2^2}{2} \tag{35}$$

$$p_A - p_0 = \frac{\rho v_2^2}{2} - \frac{\rho}{2}\left(\frac{A_2}{A_1}v_2\right)^2 \tag{36}$$

$$p_A - p_0 = \left(\frac{1}{2} - \frac{A_2^2}{A_1^2}\right)\frac{\rho}{2}v_2^2 \tag{37}$$

$$v_2 = \sqrt{\frac{p_A - p_0}{\left(\frac{1}{2} - \frac{A_2^2}{A_1^2}\right)\frac{\rho}{2}}} \tag{38}$$

$$\approx 60 \text{ m s}^{-1} \tag{39}$$

The volume flow of the water is

$$\dot{V} = v_2 A_2 \tag{40}$$

$$\approx 3 \cdot 10^{-3} \text{ m}^3 \text{ s}^{-1} \tag{41}$$

b) Given:

- $A = \pi R^2$

  $R = \sqrt{\frac{A}{\pi}}$
- $\eta = 1$ mPa s
- $L = 20$ m

We can use the Hagen-Poieuille Equation to fit our needs and determine the new pressure needed:

$$\Delta p = \frac{8\eta L\dot{V}}{\pi R^4} \tag{42}$$

$$= \frac{8\eta L\dot{V}}{\pi A_1^2} \tag{43}$$

$$= 94 \text{ Pa} \tag{44}$$