7. Exercise sheet to Experimental Physics (WS 20/21)

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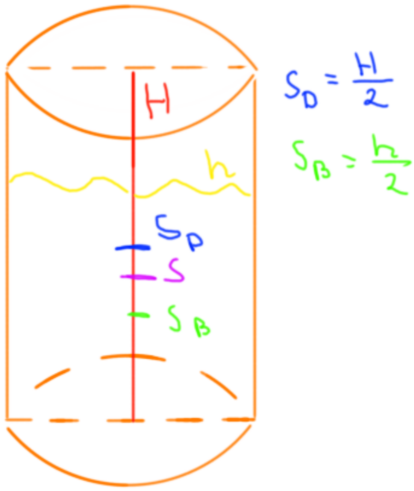
7.1 Exercise 1: Beer can

a)



Given:

- $D = 64\ mm$
  $R = 32\ mm$
- $H = 155\ mm$
- $M = 27.8\ g$

Assumed and others:

- $\rho_b = 1050\ kg\ m^{-3}$
- $S_D = \frac{H}{2}$
- $S_B = \frac{h}{2}$

Assuming a homogenous distribution of the liquid and the metal sheet in a perfect cylinder the center of mass S would be exactly along the middle vertical.

Using the formula for the center of mass in a system of two masses, we can calculate our ideal h.

$$r_s = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \qquad r_1 = \frac{H}{2},\ r_2 = \frac{h}{2}, r_s = S \tag{1}$$

$$m_1 = M,\ m_2(h) = \rho V(h) = \rho\pi r^2 h \tag{2}$$

$$S = \frac{\frac{MH}{2} + \frac{\rho\pi r^2 hh}{2}}{M + \rho\pi r^2 h} \tag{3}$$

$$S = \frac{MH + \rho\pi r^2 h^2}{2(M + \rho\pi r^2 h)} \qquad A = \rho\pi r^2 \tag{4}$$

$$= \frac{MH + Ah^2}{2(M + Ah)} \tag{5}$$

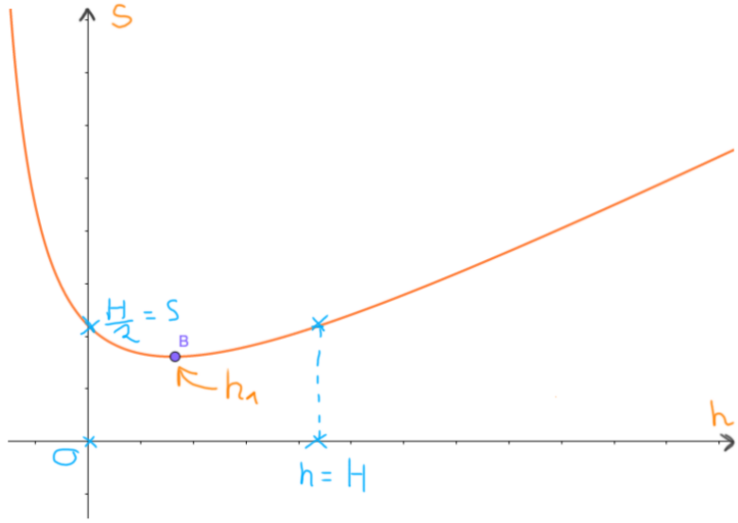
For the lowest center of mass, we can set the derivative to 0.

$$S' = 0 = \frac{[2Ah(2(M + Ah))] - [(MH + Ah^2)2A]}{[2(M + Ah)]^2} \tag{6}$$

$$= \frac{4AMh + 4(Ah)^2 - 2AMH - 2(Ah)^2}{[2(M + Ah)]^2} \tag{7}$$

$$
\begin{aligned}
&= 4AMh + 4(Ah)^2 - 2AMH - 2(Ah)^2 && (8)\\
&= 4AMh + 2A^2h^2 - 2AMH && (9)\\
&= Ah^2 + 2Mh - MH && (10)\\
&= h^2 + \frac{2M}{A}h - \frac{MH}{A} && (11)\\
h_{1,2} &= -\frac{M}{A} \pm \sqrt{\frac{M^2}{A^2} + \frac{MH}{A}} && (12)\\
h_1 &= 0.02842\ m && (13)\\
h_2 &< 0 && (14)
\end{aligned}
$$

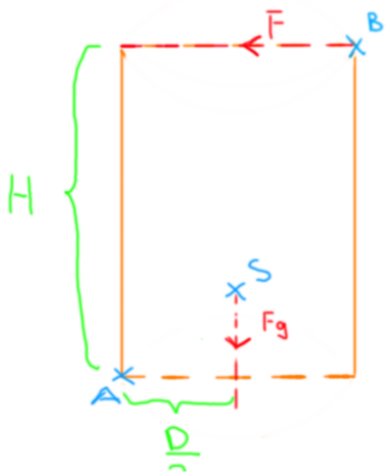
For the lowest center of mass, the beer should reach a height of 2.84 cm.



When tilting the can, a lower center of mass remains over the base of the object for a wider range of motion, making it much more stable.

b) To determine the force needed to just barely tilt the can, we can calculate when the torque exerted on the spinning point equals to 0.

At that point we have two different torques:

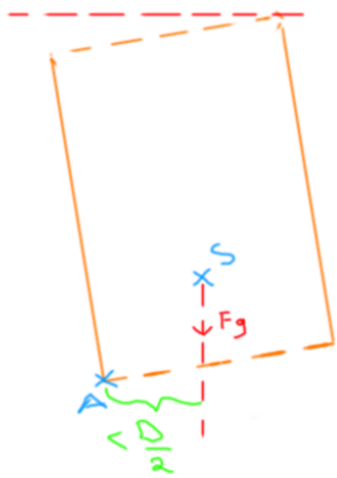


$$M_S = \frac{F_g D}{2}$$

$$M_B = FH$$

$$
\begin{aligned}
M_S + M_B &= 0\\
\frac{F_g D}{2} &= -FH
\end{aligned}
$$

$$F(h) = -\frac{F_g R}{H} \qquad F_g = (M + \rho\pi R^2 h)g$$
$$= -\frac{(M + \rho\pi R^2 h)gR}{H}$$



After we have started tilting the can, the "leverage" from point A to the line where our gravitational force is effective begins to decrease, therefore the force required to counter the torque in that direction is also smaller.

7.2 Exercise 2

a) We know that in elastic collisions, both the momentum and the energy are conserved.

$$m_1 v_1 = m_1 u_1 + m_2 u_2 \tag{15}$$

$$\frac{m_1 v_1^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2} \tag{16}$$

From that follows:

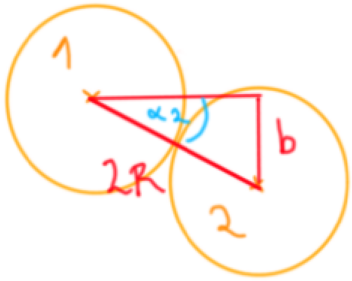
$$p_1^2 = p_1'^2 + p_2'^2 \tag{17}$$

According to the Pythagorean Theorem, this relationship implies that the two momentums p_1' and p_2' are perpendicular to each other and therefore

$$\alpha_1 + \alpha_2 = 90^\circ \tag{18}$$

b) Given:

- $v_1 = 10\ cms^{-1}$
- $b = 1.2\ cm$
- $R = 1\ cm$
- $m_1 = m_2 \equiv m$



We can solve this problem geometrically:

$$\sin \alpha_2 = \frac{b}{2R} \tag{19}$$
$$= \frac{1.2}{2} = 0.6 \tag{20}$$
$$\alpha_2 = \arcsin 0.6 \tag{21}$$
$$= 0.6435 = 36.87^\circ \tag{22}$$

And

$$\alpha_1 + \alpha_2 = 90^\circ \tag{23}$$
$$\alpha_1 = 90^\circ - 36.87^\circ \tag{24}$$
$$= 53.13^\circ \tag{25}$$

After the impact, the resting ball is pushed along the angle of collision α_2. Its velocity is

$$u_2 = v_1 \cos \alpha_2 \tag{26}$$
$$= 10\ cms^{-1} \cos 0.6435 \tag{27}$$
$$= 8\ cms^{-1} \tag{28}$$

The moving ball travels along the angle at which it was deflected α_1 with a velocity of

$$u_1 = v_1 \sin \alpha_2 \tag{29}$$
$$= 10\ cms^{-1} \sin 0.6435 \tag{30}$$
$$= 6\ cms^{-1} \tag{31}$$

c) In a center of mass (CoM) System, the initial velocity of the first sphere is

$$v_1' = v_1 - v_s = \frac{mv_1 - mv_2}{2m} \tag{32}$$
$$= \frac{mv_1}{2m} \tag{33}$$
$$= \frac{v_1}{2} \tag{34}$$

After the impact, that same sphere has a velocity of

$$u_1' = \frac{m_2 u_1 - m_2 u_2}{m_1 + m_2} \tag{35}$$
$$= \frac{m(u_1 - u_2)}{2m} \tag{36}$$
$$= \frac{u_1 - u_2}{2} \tag{37}$$
$$= -1\ cms^{-1} \tag{38}$$

respective to the CoM.

The second sphere has a velocity of

$$\begin{aligned} u_2' &= \frac{m_1 u_2 - m_1 u_1}{m_1 + m_2} && (39) \\ &= \frac{m(u_2 - u_1)}{2m} && (40) \\ &= \frac{u_2 - u_1}{2} && (41) \\ &= 1 \ cms^{-1} && (42) \end{aligned}$$

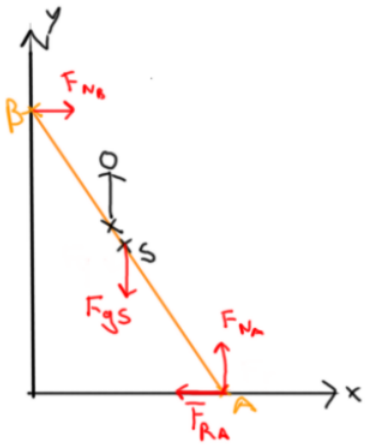
respective to the CoM.

From those values, we can get the angles α_1' and α_2'

$$\begin{aligned} \cos\alpha_1' &= \frac{u_1'}{v_1'} && (43) \\ \alpha_1' &= \arccos\frac{u_1'}{v_1'} && (44) \\ &\approx 101.54^\circ && (45) \end{aligned}$$

$$\begin{aligned} \cos\alpha_2' &= \frac{u_2'}{v_1'} && (46) \\ \alpha_2' &= \arccos\frac{u_2'}{v_1'} && (47) \\ &\approx 78.46^\circ && (48) \end{aligned}$$

7.3 Exercise 3

a) 

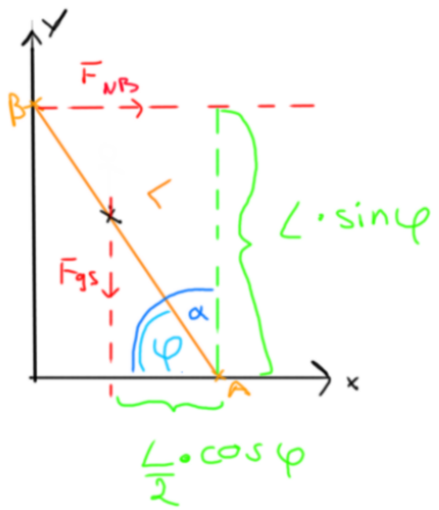
Balance of forces:

$$F_x = 0 = F_{NB} + F_R$$
$$F_y = 0 = F_{gs} + F_{NA}$$

Balance of torque:

$$M = 0 = F_{gs} \cdot r_s \cos\varphi + F_{NB} L \sin\varphi$$

b) To keep the ladder from falling over, the torque at the point at which we placed it on the ground has to equal 0.



The Torque on point A in one direction is caused by the gravitational force of the ladder F_{gs} and in the other direction by the normal force of the ladder against the wall F_{NB}. In two dimensions, we can think of it as a problem using a leverage, where the length of the leverage is the distance from point A to the effective line of the respective force.

$$
\begin{aligned}
M_{NB} &= F_{NB} L \sin\varphi & M_{gs} &= F_{gs}\frac{L}{2}\cos\varphi && (49)\\
&= mg\mu L \sin\varphi & &= \frac{mgL\cos\varphi}{2} && (50)\\
& & & && (51)
\end{aligned}
$$

$$
\begin{aligned}
M_{NB} - M_{gs} &= 0 && (52)\\
mg\mu L\sin\varphi - \frac{mgL\cos\varphi}{2} &= 0 && (53)\\
\mu\sin\varphi &= \frac{\cos\varphi}{2} && (54)\\
2\mu &= \cot\varphi && (55)\\
\varphi &= \operatorname{arccot} 2\mu && (56)
\end{aligned}
$$

c) Given:

- $m_p = 75\ kg$
- $m_L = 25\ kg$
- $L = 6\ m$
- $\mu = 0.5$

We can proceed analogous to the previous exercise, but now our center of mass has slided along the ladder to a different position.

$$
\begin{aligned}
r_{sc} &= \frac{m_1 r_1 + m_2 r_2}{m_1 + m_1} && \bigg| m_1 = m_p, m_2 = m_l, r_1 = -3, r_2 = 0 && (57)\\
&= \frac{m_p r_1}{m_p + m_l} && && (58)\\
&= -2.25\ m && && (59)
\end{aligned}
$$

The center of mass has now moved 2.25 m left of the center, so 5.25 m from the starting point. We'll call this distance l_s

$$
\begin{aligned}
F_g l_s \cos\varphi &= F_R L \sin\varphi \qquad\qquad F_R = F_N \mu, F_N = mg\cos\alpha && (60)\\
mg l_s \cos\varphi &= mg\mu L \sin\varphi && (61)\\
l_s \cos\varphi &= \mu L \sin\varphi && (62)\\
\frac{l_s}{\mu L} &= \tan\varphi && (63)\\
\varphi &= \arctan\frac{l_s}{\mu L} && (64)\\
&= 60.25^\circ && (65)
\end{aligned}
$$