9. Exercise sheet to Experimental Physics (WS 20/21)

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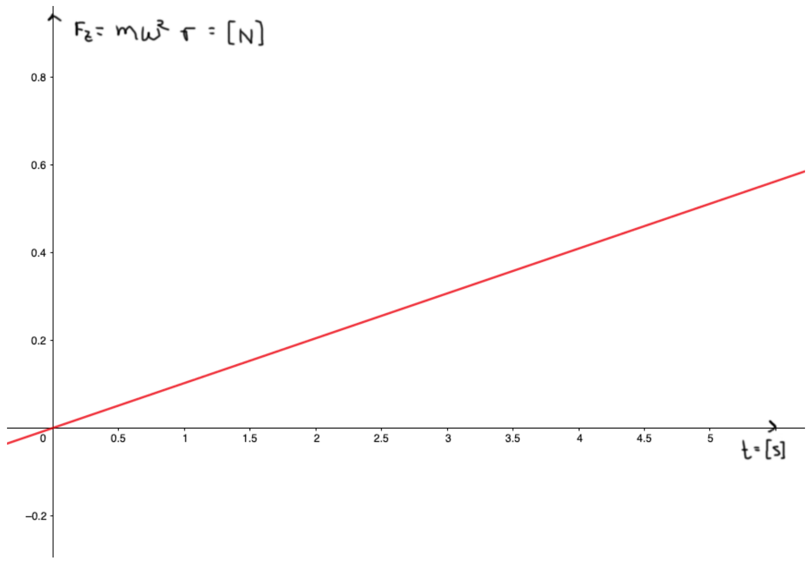
Points: ___/___/___ Σ___

9.1 Exercise 1

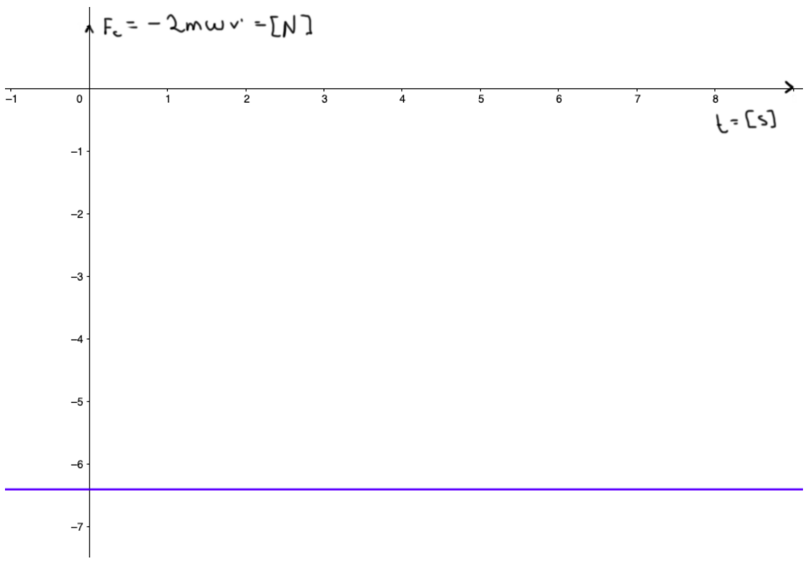
Given:

- $R = 5$ m
- $\omega = 0.2$ rad s^{-1} ≈ 0.032 s^{-1}
- $m = 100$ kg
- $v' = 1$ m s^{-1}
- $I = [0 \text{ s}, 5 \text{ s}]$

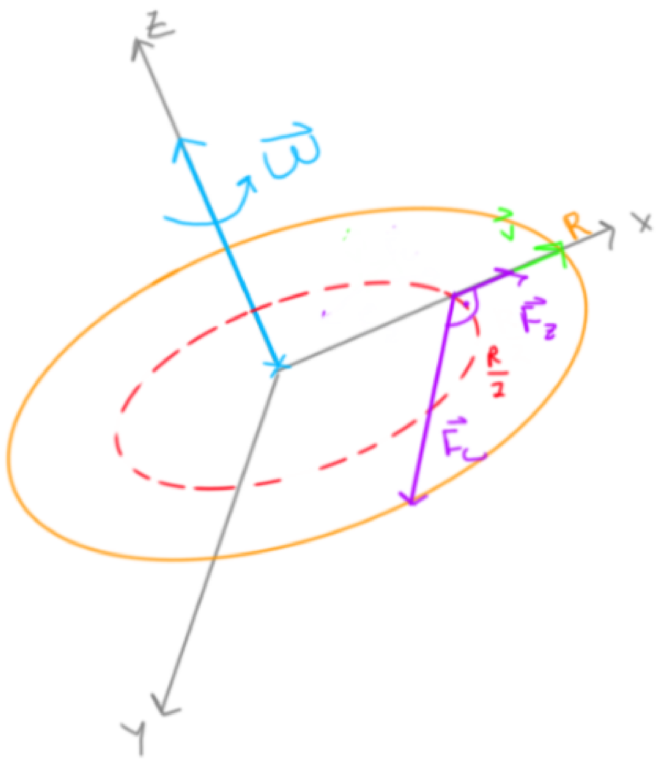
a) Sketch:



b) Sketch:



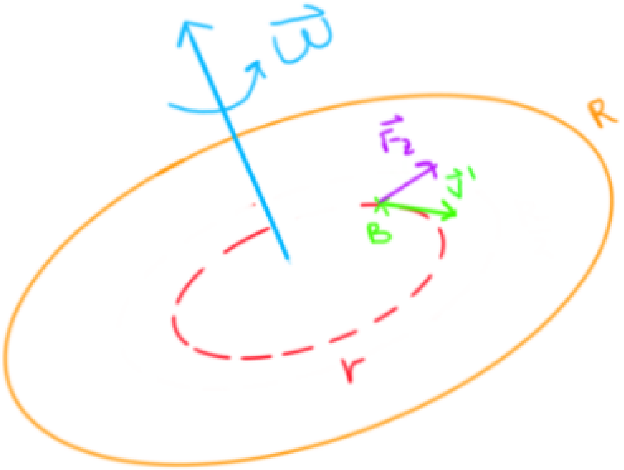
c) Sketch:



Given:

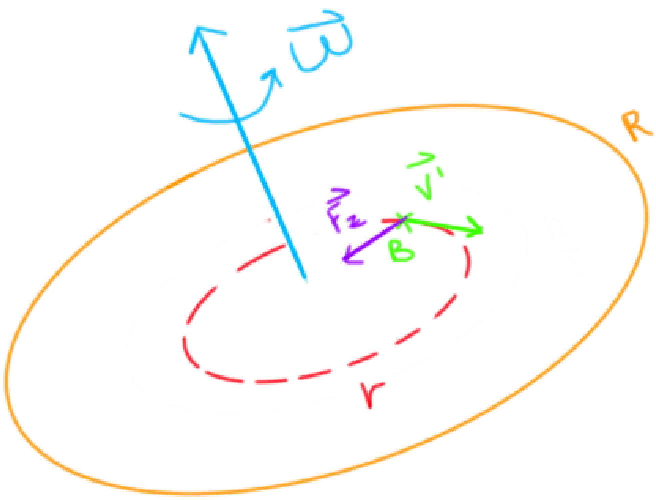
- $m = 50$ kg
- $r = 2$ m
- $v' = 0.4$ m s^{-1}

d) Sketch:



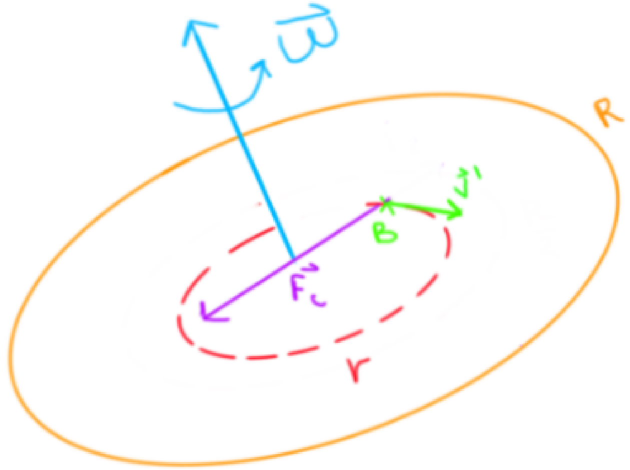
e)

$$
\begin{aligned}
F_z &= \frac{mv'^2}{r} && (1) \\
&= 4 \text{ N} && (2)
\end{aligned}
$$



f) Since we know the angle between the velocity and the angular velocity, we can use the formula:

$$
\begin{aligned}
\vec{F}_C &= -2m\omega v' && (3) \\
&= -8 \text{ N} && (4)
\end{aligned}
$$



g) From B's point of view, there's a total force of -4 N pulling him towards the center, since a part of the Coriolis Force counters the centrifugal force. However, for an observer outside of that frame of reference, it appears that Person B is walking with an angular velocity of $w' = \dfrac{v'}{r} = 0.2\ \mathrm{s}^{-1}$ against the angular velocity of the spinning system, so it would appear that B is stationary and there are no forces acting upon them.

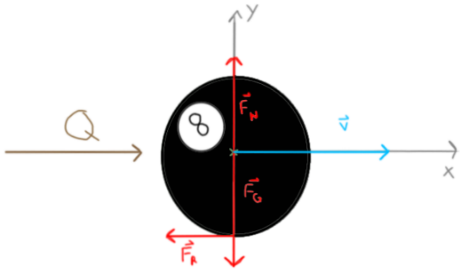
9.2 Exercise 2

Given.:

- $\mu > 0$

To solve this problem, we can use the fact that the angular momentum is preserved to solve this problem.

But first let's take a look at the forces involved here:



The vertical forces cancel each other out, so we know that

$$\sum F_y = 0 \tag{5}$$
$$F_g + F_N = 0 \tag{6}$$
$$F_g = -F_N \tag{7}$$
$$\tag{8}$$

But there is a force acting against the movement of the ball, namely the friction against the table:

$$\sum F_x \neq 0 \tag{9}$$
$$ma = F_R \tag{10}$$
$$ma = \mu F_N \tag{11}$$
$$ma = -\mu mg \tag{12}$$

We can say that both at the beginning after the ball's been hit and at the end when it comes to a constant velocity wihout gliding, the angular momentum must be equal, so we use the formula:

$$L_0 = L_1 \tag{13}$$
$$rp = I\omega \tag{14}$$
$$rmv_0 = \frac{Iv_1}{r} \tag{15}$$

So that this condition is valid, we have to be able to use

$$\omega = \frac{v}{r} \tag{16}$$

which only happens when the ball isn't gliding anymore. Therefore, we need to calculate the time it takes the ball to stop gliding:

We know:

$$v = v_0 + at \qquad |a = -\mu g \tag{17}$$
$$v = v_0 - \mu gt \tag{18}$$

We also know that the sum of the torque is

$$\sum M = I\alpha \qquad |M = Fh \tag{19}$$
$$F = \mu mgr \tag{20}$$

$$I = \frac{2mr^2}{5} \tag{21}$$

$$\begin{aligned}
\mu mgr &= I\alpha && (22)\\
\mu mgr &= \frac{2mr^2}{5}\alpha && (23)\\
\mu g &= \frac{2r}{5}\alpha && (24)\\
\alpha &= \frac{5\mu g}{2r} && (25)
\end{aligned}$$

In the next step, we can use this value for a to calculate the angular velocity of the ball, which we know to be

$$\begin{aligned}
\omega &= \omega_0 + \alpha t && |\omega_0 = 0 && (26)\\
\omega &= \frac{5\mu g}{2r}t && && (27)
\end{aligned}$$

Now we can determine when the previously mentioned condition is valid:

$$\begin{aligned}
\omega &= \frac{v}{r} && (28)\\
\frac{5\mu g}{2r}t &= \frac{v_0 - \mu gt}{r} && (29)\\
\mu g\left(\frac{5}{2} + 1\right)t &= v_o && (30)\\
t &= \frac{v_0}{\mu g\left(\frac{5}{2} + 1\right)} && (31)
\end{aligned}$$

If we enter this time into our velocity, we get

$$\begin{aligned}
v_1 &= v_0 - at && (32)\\
&= v_0 - \mu g\frac{v_0}{\mu g\left(\frac{5}{2} + 1\right)} && (33)\\
&= v_0 - \frac{2v_0}{7} && (34)\\
&= \frac{5v_0}{7} && (35)
\end{aligned}$$

9.3 Exercise 3

a) Given:

- $M = -\kappa_D\varphi$
- $\kappa_D = \frac{\pi GR^4}{2L}$

- $T = 2\pi\sqrt{\frac{L}{g}}$

We know that

$$M = I\alpha \tag{36}$$
$$= I\ddot{\varphi} \tag{37}$$

So we get the equation of motion in relation to the angle φ:

$$I\ddot{\varphi} = -\kappa_D\varphi \tag{38}$$
$$\ddot{\varphi} = -\frac{\kappa_D\varphi}{I} \tag{39}$$

For the period length, we can use

$$L\ddot{\varphi} = g\varphi \tag{40}$$
$$\ddot{\varphi} = \frac{g\varphi}{L} \tag{41}$$

If we input these into our formula for the period length, we get

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{42}$$
$$= 2\pi\sqrt{\frac{I}{\kappa_D}} \tag{43}$$

b) Given:

- $d = 2$ mm

  $R = 1$ mm
- $L = 2$ m
- $D = 10$ cm

  $r = 5$ cm[1]
- $h = 7$ cm
- $\rho_{Fe} = 7.85\ \text{cm}^{-3}$
- $T = 1.86$ s
- $m = \rho_{Fe}\pi r^2 h$

[1] the choice of lowercase r is for convenience, since the exercise sheet already defined the Radius of the wire to be capital R

To calculate the shear modulus, we can substitute our value κ_D in the equation for the period length we just derived and solve for G

$$T = 2\pi\sqrt{\frac{I}{\kappa_D}} \tag{44}$$
$$= 2\pi\sqrt{\frac{I}{\frac{\pi G R^4}{2L}}} \tag{45}$$
$$\frac{T}{2\pi} = \sqrt{\frac{I}{\frac{\pi G R^4}{2L}}} \tag{46}$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{I}{\frac{\pi G R^4}{2L}} \tag{47}$$
$$\left(\frac{T}{2\pi}\right)^2 \frac{1}{2L} = \frac{I}{\pi G R^4} \tag{48}$$
$$\left(\frac{T}{2\pi}\right)^2 \frac{\pi R^4}{2LI} = \frac{1}{G} \tag{49}$$
$$G = \left(\frac{2\pi}{T}\right)^2 \frac{2LI}{\pi R^4} \tag{50}$$

By resubstituting the moment of inertia of the spinning cylinder $I = \frac{mr^2}{2}$, we get

$$G = \left(\frac{2\pi}{T}\right)^2 \frac{2Lmr^2}{2\pi R^4} \tag{51}$$
$$= \left(\frac{2\pi}{T}\right)^2 \frac{L\rho_{Fe}\pi r^2 h r^2}{\pi R^4} \tag{52}$$
$$= \left(\frac{2\pi}{T}\right)^2 \frac{L\rho_{Fe} r^4 h}{R^4} \tag{53}$$
$$= 7.84 \cdot 10^6 \mathrm{Pa} \tag{54}$$

c) Given:

- $D = 10$ cm

  $R = 5$ cm
- $L = 10$ cm
- $\omega = 50\pi\ \mathrm{s}^{-1}$

For a rotating motion the power is

$$P = M\omega \tag{55}$$

and the torque given in the exercise is

$$M = -\kappa_D \varphi \tag{56}$$

Therefore, the angle is

$$\begin{align}
\varphi &= \frac{-M}{\kappa_D} \tag{57}\\
&= \frac{-P}{\omega \kappa_D} \tag{58}\\
&= \frac{-P}{\omega \frac{\pi G R^4}{2L}} \tag{59}\\
&= \frac{-P 2L}{\omega \pi G R^4} \tag{60}\\
&\approx 2.4 \text{ rad} \approx 137^\circ \tag{61}
\end{align}$$