4. Exercise sheet to Experimental Physics (WS 20/21)

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Group: F

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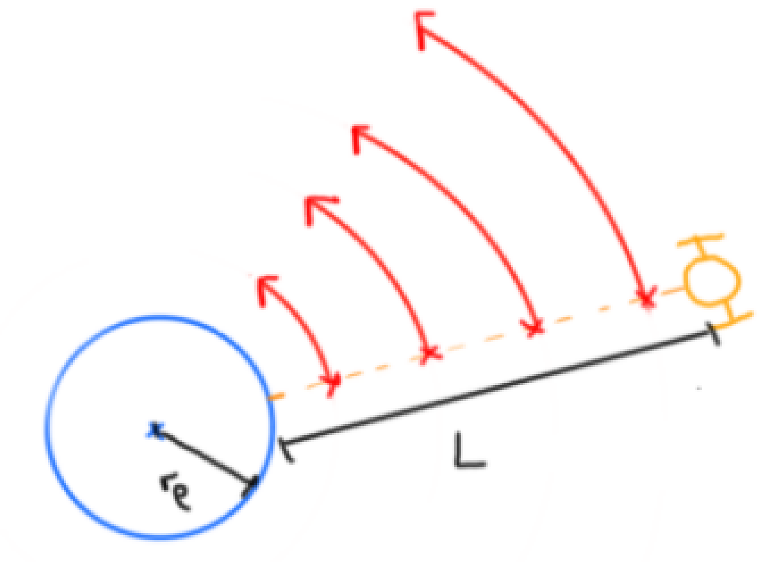
4.1 Exercise 1: Space elevator

Given:

- Earth's radius: $r_e = 6.378 \cdot 10^6 \ m$
- Earth's mass: $M_e = 5.972 \cdot 10^{24} \ kg$
- Earth's rotational velocity: $\omega = \dfrac{2\pi}{86400 \ s}$
- Earth's gravitational constant: $G = 6.67 \cdot 10^{-11} \ m^3 kg^{-1} s^{-2}$
- Homogenous mass distribution: $\lambda = \dfrac{m}{L}$
- $g = \dfrac{G \cdot M_e}{r^2}$ with $r = r_e + L$
- $F_c = m \cdot \omega^2 \cdot r$
- $F_g = m \cdot g$



Points on different parts of the rope experience different forces, points further away from the Earth experience a smaller gravitational pull towards it, but a stronger centripetal force, since the radius of their path is larger than the one of points on earth.

We need to add these individual forces and set them equal to each other, so as to find the place where the satellite at the end of the rope would be geostationary, i.e. would accelerate neither towards the Earth nor away from it.

$\rightarrow F_{c_{total}} = F_{g_{total}}$

We can determine the total amounts of both these forces by integrating the force along the Length L of the rope.

$$
\begin{aligned}
F_{g_{total}} &= \int_{r_e}^{r_e+L} \frac{mGM_e}{r^2} dr \\
&= \frac{-mGM_e}{r}\Bigg|_{r_e}^{r_e+L} \\
&= \frac{-mGM_e r_e + mGM_e(r_e+L)}{r_e^2 + r_e L} \\
&= \frac{mGM_e L}{r_e^2 + r_e L}
\end{aligned}
$$

$$
\begin{aligned}
F_{c_{total}} &= \int_{r_e}^{r_e+L} m\omega^2 r dr \\
&= \frac{m\omega^2 r^2}{2}\Bigg|_{r_e}^{r_e+L} \\
&= \frac{m\omega^2 (r_e+L)^2}{2} - \frac{m\omega^2 r_e^2}{2} \\
&= \frac{m\omega^2}{2}((r_e+L)^2 - r_e^2) \\
&= \frac{m\omega^2}{2}(2r_e L + L^2)
\end{aligned}
$$

Now we can set both of these forces equal to each other and solve for the required L:

$$
\begin{aligned}
F_{g_{total}} &= F_{c_{total}} \\
\frac{mGM_e L}{r_e^2 + r_e L} &= \frac{m\omega^2}{2}(2r_e L + L^2) \\
\frac{GM_e L}{r_e^2 + r_e L} &= \frac{\omega^2}{2}(2r_e L + L^2) \\
\frac{GM_e L}{r_e^2 + r_e L} &= \omega^2 r_e L + \frac{\omega^2 L^2}{2} \\
\frac{GM_e}{r_e^2 + r_e L} &= \omega^2 r_e + \frac{\omega^2 L}{2} \\
GM_e &= (\omega^2 r_e + \frac{\omega^2 L}{2})(r_e^2 + r_e L) \\
GM_e &= \omega^2 r_e^3 + \frac{3\omega^2 r_e^2 L}{2} + \frac{\omega^2 r_e L^2}{2} \\
0 &= \frac{\omega^2 r_e L^2}{2} + \frac{3\omega^2 r_e^2 L}{2} + \omega^2 r_e^3 - GM_e
\end{aligned}
$$

We can apply the p-q-formula if we divide the equation by $\frac{\omega^2 r_e}{2}$

$$0 = L^2 + 3r_eL + 2r_e^2 - \frac{\frac{GM_e}{\omega^2 r_e}}{2}$$

$$\rightarrow L = \frac{-3r_e}{2} \pm \sqrt{(\frac{3r_e}{2})^2 - (2r_e^2 - \frac{2GM_e}{\omega^2 r_e})}$$

We input our values and solve for L and get:

$$L = 1.438858 \cdot 10^8\ m = 143\ 886\ km$$

Compared to the distance from earth to the moon, which is around 384 000 km, we can put into perspective that this would be a very laborious project.

4.2 Exercise 2: Driving around a curve while raining

Given:

- $\mu_0 = 0.95$
- $\mu_1 = 0.5$
- $\alpha = 0°$
- $F_c = \frac{mv^2}{r}$
- $F_f = \mu F_n$
- $F_n = mg \cos \alpha$

a) The static friction, and not the sliding friction is determinant in the exchange of forces between a tire and the street, because under ideal conditions, the tire doesn't slide over the ground, which would cause a vehicle to skid.

b) Given:

   - $r = 100\ m$

   To prevent the car from rolling over or breaking loose from the curve, the centripetal force and the friction must balance each other out:

$$\begin{aligned} F_f &= F_c \\ \frac{mv^2}{r} &= \mu_0 mg \cos(0) \\ \frac{v^2}{r} &= \mu_0 g \\ v &= \sqrt{\mu_0 g r} \\ v &= \sqrt{0.95 \cdot 9.81\ ms^{-2} \cdot 100\ m} \\ v &= 30.5\ ms^{-1} = 109\ kmh^{-1} \end{aligned}$$

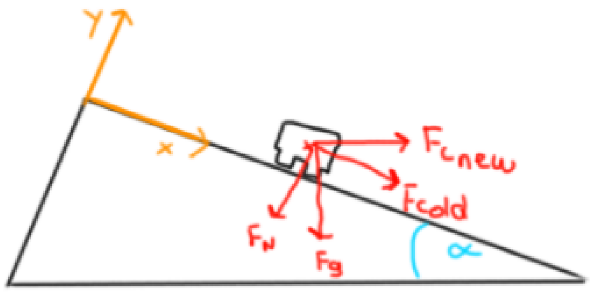
c) Over a wet street this value is reduced to

$$v = \sqrt{\mu_1 g r}$$
$$v = 22.15\ ms^{-1}$$

d) To conserve the initial velocity even under a wet street, the curve can be inclined by an angle α to counter the reduced friction coefficient.

Given:

- $\vec{F_{down}} = \begin{pmatrix} 0 \\ mg\sin\alpha \end{pmatrix}$
- $\vec{F_{c_{old}}} = \begin{pmatrix} \frac{mv^2}{r} \\ 0 \end{pmatrix}$



$$\begin{aligned}
F_{c_{new}} &= F_{down} + F_f \\
\cos\alpha F_{c_{old}} &= mg\sin\alpha + \mu_1 mg\cos\alpha \\
\frac{mv^2}{r}\cos\alpha &= g\sin\alpha + \mu_1 mg\cos\alpha \\
\frac{v^2}{r} &= g\tan\alpha + \mu_1 g \\
\frac{v^2}{gr} &= \tan\alpha + \mu_1 \\
\tan\alpha &= \frac{v^2}{gr} - \mu_1 \\
\rightarrow \alpha &= \arctan\left(\frac{v^2}{gr} - \mu_1\right)
\end{aligned}$$

After inserting the respective values we get:

$$\alpha = 24°$$

4.3 Exercise 3: Catapult

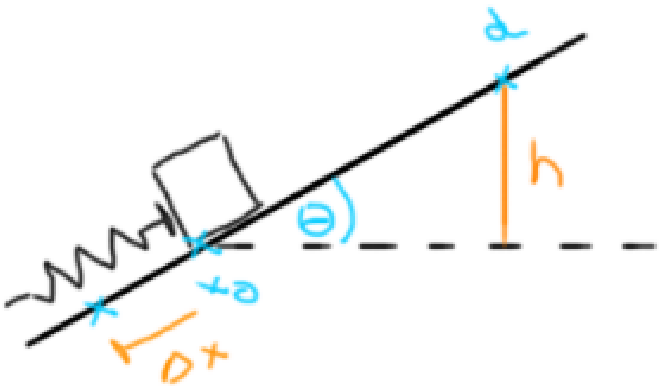
Given:

- $m = 3.2kg$
- $k = 430\ Nm^{-1}$
- $\theta = 30° = \frac{\pi}{6}$
- $\Delta x = 20\ cm = 0.2\ m$
- $F_{down} = mg\sin\theta$
- $F_{spring} = -kx^2$
- $E_{kin} = \frac{mv^2}{2}$
- $E_{pot} = mgh$
- $E_{spring} = \frac{kx^2}{2}$

a) At rest, the downhill force is equal to the force exerted by the spring:

$$\begin{aligned} F_{down} &= F_{spring} \\ mg\sin\theta &= -kx \\ \rightarrow x &= \frac{mg\sin\theta}{-k} \\ x &= \frac{3.2\ kg \cdot 9.81\ ms^{-2}\sin\frac{\pi}{6}}{-430\ Nm^{-1}} \\ x &= -0.0365\ m \end{aligned}$$

b) By pushing the spring 20 cm downwards, we are inserting a maximal potential energy into the system



The potential energy at the highest point on the plane is equal to the potential energy inserted by the loading of the spring at the beginning. By calculating the height h at which this potential energy is reached, we can measure the maximum distance d that our mass travels up the plane.

$$
\begin{aligned}
E_{spring} &= E_{pot} \\
\frac{kx^2}{2} &= mgh \\
\rightarrow h &= \frac{kx^2}{2mg} \\
h &= \frac{430\ Nm^{-1} \cdot ((0.2 + 0.0365)\ m)^2}{2 \cdot 3.2\ kg \cdot 9.81\ ms^{-2}} \\
h &= 0.3623\ m
\end{aligned}
$$

And by inserting into the formula to calculate d:

$$
\begin{aligned}
\sin\theta &= \frac{h}{d} \\
\rightarrow d &= \frac{h}{\sin\theta} \\
d &= 0.766\ m
\end{aligned}
$$

c) The kinetic energy is highest at the point where the potential energy is zero, which is at the resting position of the block.

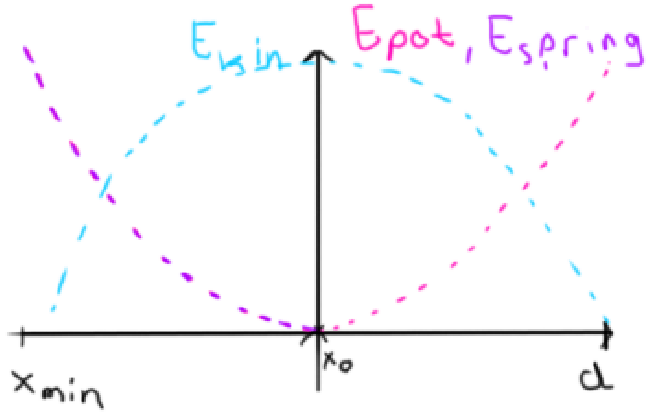
$$
\begin{aligned}
E_{kin} &= E_{pot} \\
\frac{mv^2}{2} &= mgh \\
\rightarrow v &= \sqrt{2gh}
\end{aligned}
$$

To solve this equation we need to first figure out the height of the block at its resting position:

$$
\begin{aligned}
\sin\theta &= \frac{h}{x} \\
h &= x \cdot \sin\theta \\
h &= 0.2365\ m \cdot \sin\frac{\pi}{6} \\
h &= 0.1\ m
\end{aligned}
$$

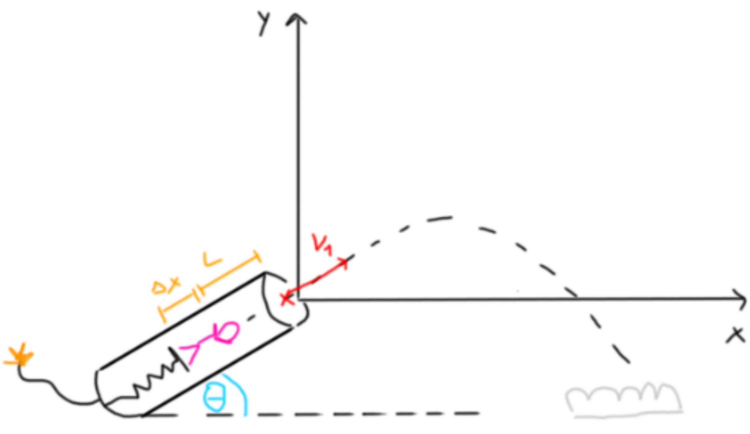
$$\rightarrow v = 2.31\ ms^{-1}$$

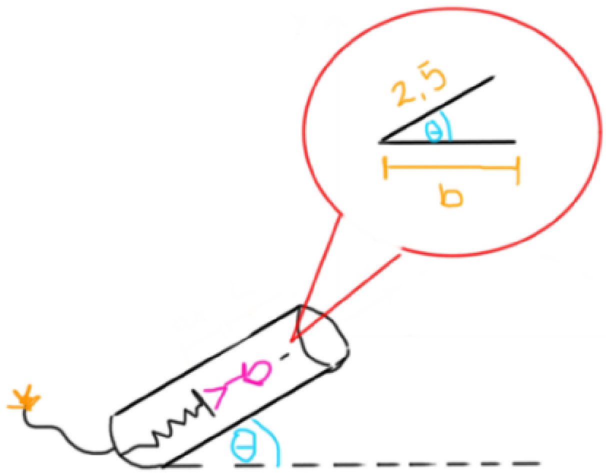
d) Sketch



e) Given:

- $L = 2.5\ m$
- $\theta = 30° = \dfrac{\pi}{6}$
- $k = 2\ kNm^{-1} = 2000\ Nm^{-1}$
- $m = 60\ kg$
- $v_1 = 10\ ms^{-1}$
- $\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta t - gt \end{pmatrix}$
- $\vec{s} = \begin{pmatrix} s_x \\ s_y \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta t \\ v_0 \sin\theta t - \dfrac{gt^2}{2} \end{pmatrix}$





$$
\begin{aligned}
\cos\theta &= \frac{b}{L} \\
L\cos\frac{\pi}{6} &= b \\
2.5\ m\frac{\sqrt{3}}{2} &= b \\
2.165\ m &= b
\end{aligned}
$$

Since the velocity in x-direction is constant, and we now know the distance the acrobat travelled in this direction, we can now determine the timeït took her to exit the cannon.

$$
\begin{aligned}
s_x &= v_0\cos\theta t \\
\rightarrow t &= \frac{2.165\ m}{v_0\cos\theta}
\end{aligned}
$$

We also know that:

$$
\begin{aligned}
|v_1| = 10\ ms^{-1} &= \sqrt{v_x^2 + v_y^2} \\
&= \sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta t - gt)^2}
\end{aligned}
$$

If we input $t = \dfrac{2.165\ m}{v_0\cos\theta}$ into this equation and solve for v_0, we get a value of:

$|v_0| = 11.5968\ ms^{-1}$

We can thus equate our maximum kinetic energy at the time of the launch with our maximum potential energy from the deformation of the spring:

$$\begin{aligned}
\frac{kx^2}{2} &= \frac{mv^2}{2} \\
x^2 &= \frac{mv^2}{k} \\
x &= \sqrt{\frac{mv^2}{k}} \\
&= \sqrt{\frac{60\ kg \cdot (11.5968\ ms^{-1})^2}{2000\ Nm^{-1}}} \\
&= 2.01\ m
\end{aligned}$$

The maximum acceleration is likewise experienced at the moment when the spring is unloaded:

$$\begin{aligned}
a &= \frac{F}{m} \\
&= \frac{-kx}{m} \\
&= \frac{-2000\ Nm^{-1} \cdot 2.01\ m}{60\ kg} \\
&= 67\ ms^{-2}
\end{aligned}$$