8. Exercise sheet to Experimental Physics (WS 20/21)

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8.1 Exercise 1: Moment of inertia

a) Given:

- Density: $\rho = 1.2\ \text{gm cm}^{-3}$
- Diameter[1]: $D = 8$ cm
- Radius: $R = 4$ cm
- Center hole: $d = 1.5$ cm
- Radius of center hole: $r' = 0.75$ cm
- Thickness: $h = 0.12$ cm

The moment of inertia is:

$$\begin{aligned}
I &= \int r_\perp^2 dm && (1)\\
&= \rho \int r_\perp^2 dV && |\text{using cylinder coordinates:} \quad (2)\\
&= \rho \int_{r'}^{R} \int_0^{2\pi} \int_{\frac{-H}{2}}^{\frac{H}{2}} r^3 dr d\varphi dz && (3)\\
&= \rho 2\pi h \int_{r'}^{R} r^3 dr && (4)\\
&= \rho 2\pi h \left[\frac{r^4}{4}\right]_{r'}^{R} && (5)\\
&= 1.2\ \text{gm cm}^{-3} 2\pi 0.12\ \text{cm} 63.92\ \text{cm}^4 && (6)\\
&= 57\ \text{gm cm}^2 && (7)
\end{aligned}$$

b) In theory, we can use a very similar method to the last exercise, we only need to consider, that the radius of our cone isn't constant, so we have to use a linear function instead.

$$\begin{aligned}
I &= \rho \int_0^{r(z)} \int_0^{2\pi} \int_{\frac{-H}{2}}^{\frac{H}{2}} r^3 dr d\varphi dz && (8)\\
&= \rho 2\pi \int_{\frac{-H}{2}}^{\frac{H}{2}} \int_0^{r(z)} r^3 dr && |r(z) = \frac{R}{h} z \quad (9)
\end{aligned}$$

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[1] It's a gamecube CD

$$= \rho 2\pi \int_{\frac{-H}{2}}^{\frac{H}{2}} \int_0^{r(z)} \left(\frac{R}{h} z\right)^3 drdz \tag{10}$$

$$= \rho 2\pi \int_0^h \frac{1}{4} \left(\frac{R}{h} z\right)^4 dz \tag{11}$$

$$= \frac{\rho\pi}{2} \left(\frac{R}{h}\right)^4 h^5 \tag{12}$$

$$= \frac{\rho\pi}{2} \frac{R^4 h}{5} \tag{13}$$

$$= \frac{\rho\pi R^4 h}{10} \qquad |V = \frac{R^2 h\pi}{3} \tag{14}$$

$$= \frac{\rho R^2}{10} 3V \qquad |\rho V = m \tag{15}$$

$$= \frac{3mR^2}{10} \tag{16}$$

c) The moment of inertia of composite shapes is cumulative, so we can substract the moment of inertia that's missing from the halfsphere from the one of the cone.

$$I_{tot} = I_{cone} - I_{halfsphere} \tag{17}$$

$$= \frac{3m_c R^2}{10} - \frac{2m_h r^2}{5} \tag{18}$$

whereas r is the radius of the smaller sphere.

8.2 Exercise 2: Atwood machine

Given:

- $m_1 = 35$ kg
- $m_2 = 38$ kg
- $m_3 = 4.8$ kg
- $r = 0.3$ m
- $h = 2.5$ m

We know the potential energy at the beginning is equal to the rotational energy when the second mass is about to hit the floor.

$$E_{pot} = E_{kin} \tag{19}$$

$$(m_2 - m_1)gh = \frac{(m_1 + m_2)v^2}{2} + \frac{I\omega^2}{2} \qquad |\omega^2 = \frac{v^2}{r^2} \tag{20}$$

$$I = \frac{m_3 r^2}{2} \tag{21}$$

$$(m_2 - m_1)gh = \frac{(m_1 + m_2)v^2}{2} + \frac{m_3 v^2}{4} \tag{22}$$

$$(m_2 - m_1)gh = v^2 \left(\frac{m_1 + m_2}{2} + \frac{m_3}{4}\right) \tag{23}$$

$$v = \sqrt{\frac{(m_2 - m_1)gh}{\frac{2(m_1 + m_2) + m_3}{4}}} \tag{24}$$

$$= 1.398 \text{ m s}^{-1} \tag{25}$$

8.3 Exercise 3

Given.:

- $B = 50$ cm
- $H = 40$ cm
- $d = 5$ cm
- $\rho = 2.7$ gm cm^{-3}

  $M = 2.7$ kg
- $m = 5$ gm
- $v = 400$ m s^{-1}

To know at which angle the sign is deflected, we can use use the conservation of energy

$$E_{pot} = E_{rot} \tag{26}$$

$$Mgh_{sign} + mgh_{bullet} = \frac{I\omega^2}{2} \tag{27}$$

We can first use the conservation of angular momentum to determine the velocity of the sign after the bullt has struck it.

To do that, we first need to calculate the moment of inertia of the sign:

$$I_x^{sp} = \rho \int_V r_\perp^2 dV \tag{28}$$

$$= \rho \int_{\frac{-H}{2}}^{\frac{H}{2}} \int_{\frac{-d}{2}}^{\frac{d}{2}} \int_{\frac{-B}{2}}^{\frac{B}{2}} y^2 + z^2 dxdydz \tag{29}$$

$$= \rho \int_{\frac{-H}{2}}^{\frac{H}{2}} \int_{\frac{-d}{2}}^{\frac{d}{2}} \left[\frac{B}{2}(y^2 + z^2) + \frac{B}{2}(y^2 + z^2)\right] dydz \tag{30}$$

$$= B\rho \int_{\frac{-H}{2}}^{\frac{H}{2}} \int_{\frac{-d}{2}}^{\frac{d}{2}} y^2 + z^2 dydz \tag{31}$$

$$= B\rho \int_{\frac{-H}{2}}^{\frac{H}{2}} \left[\frac{\left(\frac{d}{2}\right)^3}{3} + \frac{dz^2}{2} - \left(\frac{\left(\frac{-d}{2}\right)^3}{3} - \frac{dz^2}{2}\right)\right] dz \tag{32}$$

$$= Bd\rho \int_{\frac{-H}{2}}^{\frac{H}{2}} \frac{d^2}{12} + z^2 dz \tag{33}$$

$$= Bd\rho\left[\frac{H}{2}\frac{d^2}{12} + \frac{\left(\frac{H}{2}\right)^3}{3} - \left(\frac{-H}{2}\frac{d^2}{12} - \frac{\left(\frac{H}{2}\right)^3}{3}\right)\right] \tag{34}$$

$$= BdH\rho\left(\frac{d^2}{12} + \frac{H^2}{12}\right) \qquad \Big| \rho = \frac{M}{BdH} \tag{35}$$

$$= \frac{M}{12}(d^2 + H^2) \tag{36}$$

Because $d << H$, we can approximate it to

$$I_x^{sp} \approx \frac{MH^2}{12} \tag{37}$$

Using Steiner's Theorem, we can determine the moment of inertia along the axis upon which the sign hangs:

$$I = M\left(\frac{H}{2}\right)^2 + \frac{M}{12}(d^2 + H^2) \tag{38}$$

$$\approx \frac{M}{4}H^2 + \frac{M}{12}H^2 \tag{39}$$

$$= \frac{MH^2}{3} \tag{40}$$

$$\approx 0.144 \text{ kg m}^2 \tag{41}$$

The moment of inertia of the bullet is simply

$$I_{bullet} = \frac{2mR^2}{5} + mr^2 \qquad \Big| r = \frac{3H}{4} \tag{42}$$

$$\approx m\left(\frac{3H}{4}\right)^2 \qquad \Big| \frac{2mR^2}{5} \text{ is negligibly small} \tag{43}$$

$$L_{bullet} + L_{sign} = L'_{bullet} + L'_{sign} \qquad \Big| L_{sign} = 0 \tag{44}$$

$$L_{bullet} = pr = mv\frac{3H}{4} \tag{45}$$

$$L'_{bullet} = I_{bullet}\omega \tag{46}$$

$$L'_{sign} = I_{sign}\omega \tag{47}$$

$$\frac{3mvH}{4} = I_{bullet}\omega + I_{sign}\omega \tag{48}$$

$$\frac{3mvH}{4} = m\left(\frac{3H}{4}\right)^2\omega + \frac{MH^2}{3}\omega \tag{49}$$

$$\frac{3mvH}{4} = \omega\left(m\left(\frac{3H}{4}\right)^2 + \frac{MH^2}{3}\right) \tag{50}$$

$$\omega = \frac{\frac{3}{4}mvH}{m\left(\frac{3H}{4}\right)^2 + \frac{MH^2}{3}} \tag{51}$$

$$= \frac{\frac{3}{4}mvH}{H^2\left(\frac{9m}{16} + \frac{Mm}{3m}\right)} \tag{52}$$

$$= \frac{\frac{3}{4}v}{H\left(\frac{9}{16} + \frac{M}{3m}\right)} \tag{53}$$

$$= \frac{3v}{H\left(\frac{9}{4} + \frac{4M}{3m}\right)} \tag{54}$$

$$\tag{55}$$

Now we can apply the conservtion of energy

$$E_{kin} = \frac{I\omega^2}{2} \tag{56}$$

$$= \frac{(I_{sign} + I_{bullet})\omega^2}{2} \tag{57}$$

$$= \frac{\left(\frac{MH^2}{3} + m\left(\frac{3H}{4}\right)^2\right) \cdot \left(\frac{3v}{H\left(\frac{9}{4} + \frac{4M}{3m}\right)}\right)^2}{2} \tag{58}$$

$$= \frac{\left(\frac{9m}{16} + \frac{M}{3}\right) 9v^2}{2\left(\frac{9}{4} + \frac{4M}{3m}\right)^2} \tag{59}$$

$$\approx 1.246 \text{ J} \tag{60}$$

And

$$E_{pot} = E_{pot_{sign}} + E_{pot_{bullet}} \tag{61}$$

$$E_{pot_{sign}} = Mgh_{sign} \qquad \text{with} \tag{62}$$

$$\cos\alpha = \frac{\frac{H}{2} - h_{sign}}{\frac{H}{2}} \tag{63}$$

$$h_{sign} = \frac{H}{2} - \frac{H\cos\alpha}{2} \tag{64}$$

and

$$E_{pot_{bullet}} = mgh_{bullet} \qquad \text{with} \tag{65}$$

$$\cos\alpha = \frac{\frac{3H}{4} - h_{bullet}}{\frac{3H}{4}} \tag{66}$$

$$h_{bullet} = \frac{3H}{4} - \frac{3H}{4}\cos\alpha \tag{67}$$

So

$$E_{pot} = E_{kin} \tag{68}$$

$$mg\left(\frac{3H}{4} - \frac{3H}{4}\cos\alpha\right) + Mg\left(\frac{H}{2} - \frac{H\cos\alpha}{2}\right) = E_{kin} \tag{69}$$

$$\frac{gH}{2}(1-\cos\alpha)\left(M + \frac{3m}{4}\right) = E_{kin} \tag{70}$$

$$1 - \cos\alpha = \frac{2E_{kin}}{gH\left(M + \frac{3m}{4}\right)} \tag{71}$$

$$\cos\alpha = 1 - \frac{2E_{kin}}{gH\left(M + \frac{3m}{4}\right)} \tag{72}$$

$$\alpha = \arccos\left(1 - \frac{2E_{kin}}{gH\left(M + \frac{3m}{4}\right)}\right) \tag{73}$$

$$= 40.09^\circ \tag{74}$$