Mathe Basics

Gradient:

$\boldsymbol{\nabla}\phi = \partial_i(\phi)\, g^{ij}\, e_j$

Divergenz:

$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}E^i\right)$

Rotation:

$\boldsymbol{\nabla}\times\mathbf{E} = \frac{1}{\sqrt{g}}\varepsilon^{ijk}\, e_i\, \partial_j E_k$

Laplace:

$\boldsymbol{\Delta}\phi = \frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}\, g^{ij}\, \partial_j\phi\right)$

Bogenelement:

$\mathrm{d}s = \sqrt{g_{ij}\,\mathrm{d}u^i\,\mathrm{d}u^j}$

Flächenelement:

$\mathrm{d}\mathbf{A} = \hat{\mathbf{n}}\sqrt{g}\,\mathrm{m}\,\mathrm{d}u^i\,\mathrm{d}u^j$

Volumenelement:

$\mathrm{d}V = \sqrt{g}\,\mathrm{d}u^1\,\mathrm{d}u^2\,\mathrm{d}u^3$

Bogenlänge:

$\mathrm{d}s = |v(t)|\,\mathrm{d}t$

$s = \int_0^t |v(t')|\,\mathrm{d}t'$

Tangentialvektor

$\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}s}$

Hauptnormlenvektor:

$\mathbf{n}_H = \left|\frac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}s}\right|^{-1}\frac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}s}$

Binormalenvektor:

$\mathbf{n}_B = \boldsymbol{\tau}\times\mathbf{n}_H$

Krümmungsradius:

$\rho = \left|\frac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}s}\right|^{-1}$

Variation der Konstanten:

$y(x) = \left(\int b(x)\mathrm{e}^{-A(x)}\,\mathrm{d}x\right)\mathrm{e}^{-A(x)}$

$y' = a(x)y + b(x)$

$A(x) = \int a(x)\,\mathrm{d}x$

Integrierender Faktor:

$I(y' + P(x)y = IQ(x))$

$I = \mathrm{e}^{\int P(x)\mathrm{d}x}$

Taylor:

$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

Polare Vektoren:

${v'}_i = R_{ij}v_j$

Axiale Vektoren:

${a'}_i = \det(R)R_{ij}a_j$

Satz von Stokes:

$\int_A \boldsymbol{\nabla}\times\mathbf{F}\,\mathrm{d}\mathbf{A} = \int_{\partial A}\mathbf{F}\cdot\mathrm{ds}$

Satz von Gauß:

$\int_V \boldsymbol{\nabla}\cdot\mathbf{F}\,\mathrm{d}V = \int_{\partial V}\mathbf{E}\cdot\mathrm{d}\mathbf{A}$

Impuls-Drehimpuls-Energie

2. Newtonsches Axiom:

$\dot{p} = F$

Drehmoment

$\mathbf{M} = \mathbf{r}\times\mathbf{F}$

Drehimpuls

$\mathbf{L} = \mathbf{r}\times\mathbf{p}$

$\dot{\mathbf{L}} = \mathbf{M}$

Transformierter Drehimpuls:

${L'}_i = \det(R)R_{iq}L_q$

Energie:

$E = \frac{m}{2}\dot{x}^2 + V(x)$

$\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\sqrt{\frac{2}{m}(E-V)}$

Systeme von Massenpunkten

Bewegung des Schwerpunktes:

$M\ddot{\mathbf{X}} = 0$

Gesamtdrehimpuls:

$\mathbf{L} = \sum_{i=1}^{N} m_i(\mathbf{x}_i\times\dot{\mathbf{x}}_i)$

$\mathbf{L} = M\left(\mathbf{X}\times\dot{\mathbf{X}}\right) + \Sigma m_i\left(\widetilde{\mathbf{x}}_i\times\dot{\widetilde{\mathbf{x}}}_i\right)$

$\widetilde{\mathbf{x}}_i = \mathbf{x}_i - \mathbf{X}$

Kepler

Drehimpuls:

$\mathbf{L} = \text{const.}$

$L = mr^2\dot{\varphi}$

$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}\varphi} = \frac{L}{mr^2}\frac{\mathrm{d}}{\mathrm{d}\varphi}$

Zylindersymmetrie:

$\mathbf{x} = r\, e_r$

$\dot{\mathbf{x}} = \dot{r}\, e_r + r\dot{\varphi}\, e_\varphi$

Energiesatz:

$E = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) + V(r)$

2. Keplersches Gesetz:

$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \frac{\mathbf{L}}{2m} = \text{const.}$

Zur Bahnkurve:

${r'}^2 = \frac{\dot{r}^2}{\dot{\varphi}^2} = \frac{2mr^4}{L^2}(E - U(r))$

${u'}^2 = \frac{2m}{L^2}(E - U(u^{-1}))$

Bewegungsgleichung Zentralfeld:

${u'}^2 = \frac{2mE}{L^2} - u^2 + \frac{2m\alpha}{L}u$

$2u'\, u'' = -2u\, u' + \frac{2}{p}u'$

$u'' = -u + \frac{1}{p}$

$r(\varphi) = \frac{p}{1+\varepsilon\cos\varphi}$

$\varepsilon = \sqrt{1 + \frac{2EL^2}{m\alpha^2}}$

$p = \frac{L^2}{m\alpha}$

$\dot{r} = -\frac{\dot{u}}{u^2}$

3. Keplersches Gesetz:

$T^2 = \frac{4\pi^2 m}{\alpha}a^3$

Laplace-Runge-Lenz-Vektor:

$\mathbf{Q} = \frac{1}{\alpha}(\dot{\mathbf{x}}\times\mathbf{L}) - e_r = \text{const.}$

Streuprozesse

Schwerpunktskoordinate:

$\widetilde{\mathbf{x}}_i = \mathbf{x}_i - \mathbf{X}$

$\mathbf{X} = \frac{1}{M}(m_1\mathbf{x}_1 + m_2\mathbf{x}_2)$

Laborsystem:

$\dot{\mathbf{X}} = \frac{m_1}{M}\mathbf{v}_{i,1}$

v-Transformation:

$\widetilde{\mathbf{v}}_{i,1} = \frac{m_2}{M}\mathbf{v}_{i,1}$

$\mathbf{v}_{f,1} = \widetilde{\mathbf{v}}_{f,1} + \frac{m_1}{M}\mathbf{v}_{i,1}$

θ-Transformation:

$v_{f,1}\sin\vartheta_1 = \widetilde{v}_{f,1}\sin\vartheta'$

$v_{f,1}\cos\vartheta_1 = \widetilde{v}_{f,1}\cos\vartheta' + \frac{m_1}{M}v_{i,1}$

$\tan\vartheta_1 = \frac{\sin\vartheta'}{\cos\vartheta' + m_1/m_2}$

Relative Änderung T

$\frac{T_{i,1} - T_{f,1}}{T_{i,1}} = \frac{2A}{(1+A^2)}(1 - \cos\vartheta')$

$\left\langle\frac{T_{i,1} - T_{f,1}}{T_{i,1}}\right\rangle = \frac{2A}{(1+A)^2}$

Streuquerschnitt:

$E = \frac{m}{2}v_\infty^2$

$L = bmv_\infty$

Asymptote Hyperbelast:

$\overline{\varphi} = \arccos\left(\frac{-1}{\varepsilon}\right)$

Streuwinkel:

$\sin\frac{\vartheta}{2} = \frac{1}{\varepsilon}$

Stoßparameter:

$b^2 = \left(\frac{1}{\sin^2\vartheta/2} - 1\right)\frac{\alpha^2}{m^2 v_\infty^4}$

Diff. Wirkungsquerschnitt:

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4(2E)^2}\frac{1}{\sin^4\vartheta/2}$

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{\partial b^2}{\partial\vartheta}\right|\frac{1}{2\sin\vartheta}$

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{\mathrm{d}b}{\mathrm{d}\vartheta}\right|\frac{b}{\vartheta} \qquad \vartheta \ll 1$

Mech. Ähnlichkeit / Virialsatz

Homogene Fkt:

$f(\lambda x) = \lambda^k f(x)$

$\frac{\mathrm{d}f(ax)}{\mathrm{d}(ax)}\cdot x = \frac{\partial f(ax)}{\partial a} = ka^{k-1}f(x)$

Ähnlichkeit:

$\frac{t'}{t} = \left(\frac{l'}{l}\right)^{1-k/2}$

Virialsatz:

$2\langle T\rangle = k\langle V\rangle$

Beschleunigte Bezugssysteme

Scheinkräfte:

$m\ddot{\mathbf{x}} = \mathbf{F}_E + \mathbf{F}_C + \mathbf{F}_Z + \mathbf{F}_g + \mathbf{F}_{\text{ext}}$

$\mathbf{F}_E = m[(\mathbf{a}+\mathbf{x})\times\dot{\boldsymbol{\omega}}]$

$\mathbf{F}_C = 2m(\dot{\mathbf{x}}\times\boldsymbol{\omega})$

$\mathbf{F}_Z = m[(\boldsymbol{\omega}\times\mathbf{x})\times\boldsymbol{\omega}] = m\omega^2\mathbf{y}$

$\mathbf{F}_g = m[(\boldsymbol{\omega}\times\mathbf{a})\times\boldsymbol{\omega}]$

$\mathbf{F}_{\text{ext}} = -\boldsymbol{\nabla}V$

Red. Dreikörperproblem

Periodendauer:

$\tau^2 = \frac{4\pi^2 d^3}{G(m_1+m_2)}$

BWGL:

$\ddot{x}_i \mp 2\omega\dot{x}_j - \omega^2 x_i + \frac{\partial V}{\partial x_i} = 0$

$m\ddot{\mathbf{x}}_m = -\frac{\alpha}{|\mathbf{x}_m - \mathbf{x}_M|}(\mathbf{x}_m - \mathbf{x}_M)$

Effektives Potential:

$U = V - \frac{\omega^2}{2}(x^2 + y^2)$

Starre Körper

Trägheitstensor:

$\Theta_{ij} = \int\rho\left(\delta_{ij}|\mathbf{x}|^2 - x_i x_j\right)\mathrm{d}V$

Drehimpuls:

$\mathbf{L} = \Theta\cdot\boldsymbol{\omega}$

Trägheitsmoment um Achse e

$\Theta_e = e^T\cdot\Theta\cdot e$

Satz von Steiner:

$\Theta_A = \Theta_S + ma^2$

Harm. Oszillator

Allgemein:

$\ddot{x} = -\omega^2 x - c\dot{x}$

Kriechfall: $c > 2\omega$

Schwingfall: $c < 2\omega$

Ap. Grenzfall: $c = 2\omega$