6. Exercise sheet to Experimental Physics (WS 20/21)

Names: Joshua Detrois, Leo Knapp, Juan Provencio

Group: F

Points: ___/___/___ Σ___

6.1 Exercise 1: Movement with variable mass

Given.:

- $m_0 = 10\ t$
- $v_0 = 1\ ms^{-1}$
- $\Phi = 1.5\ ts^{-1}$
- $m(t) = m_0 + \Phi t$

a) In the following, we will try to determine the braking acceleration of the wagon after sand starts to be poured into it.

Because conservation of impulse is valid, we can write:

$$m_0 v_0 = m(t)v(t) \tag{1}$$

$$v = \frac{m_0 v_0}{m(t)} \tag{2}$$

$$v = \frac{m_0 v_0}{m_0 + \Phi t} \tag{3}$$

$$\frac{dv}{dt} = \frac{-m_0 v_0 \Phi}{(m_0 + \Phi t)^2} \tag{4}$$

$$\rightarrow a(t) = \frac{-m_0 v_0 \Phi}{(m_0 + \Phi t)^2} \tag{5}$$

Since the sand is providing no horizontal impulse to our wagon, p is constant. Therefore:

$$p = mv \tag{6}$$

$$v = \frac{p}{m} \tag{7}$$

$$v(t) = \frac{p}{m_0 + \Phi t} \tag{8}$$

With the initial conditions:

$$p = 10\ t \cdot 1\ ms^{-1} \tag{9}$$

$$= 10\ tms^{-1} \tag{10}$$

And knowing, that it takes the wagon 10 seconds to be filled with 15 tonnes of sand: we get

$$
\begin{aligned}
v(10) &= \frac{10\ tms^{-1}}{10\ t + 15\ t} && (11) \\
&= 0.4\ ms^{-1} && (12)
\end{aligned}
$$

b) Given.:

- $F = 5000\ N$
- $\Phi = -500\ ts^{-1}$
- $m_1 = 15\ t$
- $m_2 = m_0 + m_1 = 25\ t$

To justify the use of the Formula for changing mass

$$
F = m \cdot a, \tag{13}
$$

we will derive it from Newton's Second Law:

$$
\begin{aligned}
F &= \frac{dp}{dt} && (14) \\
&= \frac{d}{dt}(m(t) \cdot v(t)) && (15) \\
&= ma + \frac{dm}{dt}v && (16) \\
F\ dt &= ma\ dt + dm\ v && (17)
\end{aligned}
$$

We can justify setting $dm\ v = 0$ because dm refers to the infinitesimal difference in mass, which converges to 0, so we have

$$
\begin{aligned}
F\ dt &= ma\ dt && (18) \\
F &= ma && (19)
\end{aligned}
$$

and from that equation, we can derive

$$
\begin{aligned}
a(t) &= \frac{F}{m(t)} && (20) \\
&= \frac{F}{m_2 + \Phi t} && (21)
\end{aligned}
$$

It takes the wagon

$$m_0 = (m_0 + m_1) + \Phi t \tag{22}$$
$$10 = 25 - 0.5t \tag{23}$$
$$t = 30\ s \tag{24}$$

to be entirely depleted of sand. During this period, the Wagon was accelerated up to a velocity of

$$\int_0^t a(t')dt' = \int_0^t \frac{F}{m_2 + \Phi t'} dt' \tag{25}$$
$$= \frac{-F \ln |m_2 + \Phi t'|}{\Phi} \Bigg|_0^t \tag{26}$$
$$= \frac{-F}{\Phi} \ln |m_2 + \Phi t| - \frac{-F}{\Phi} \ln |m_2| \tag{27}$$
$$= \frac{F}{\Phi} \ln \frac{|m_2 + \Phi t|}{|m_2|} \tag{28}$$

And after 30 seconds, the velocity of the wagon is

$$v(30) \approx 9.16\ ms^{-1} \tag{29}$$

6.2 Exercise 2: Ballistics

Given.:

- $m_1 = 2\ kg$
- $L = 1\ m$
- $m_2 = 0.01\ kg$
- $v_2 = 400\ ms^{-1}$
- $\alpha = 12° = \frac{\pi}{15}$

a) The velocity of the woodblock directly after impact can be determined with support of our knowledge about conservation of energy, for we know that the total energy in the system before and after impact must be

$$E_1 + E_2 = E_1' + E_2' \tag{30}$$

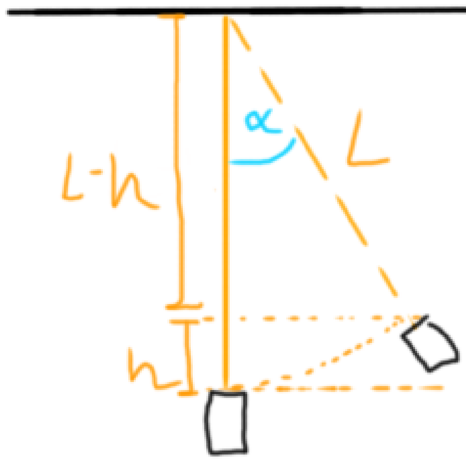
We know the total energy of the bullet to be

$$E_2 = \frac{m_2 v_2^2}{2} \tag{31}$$
$$\tag{32}$$

And the total energy of the block of wood after impact as the potential energy at its highest point.

$$E_1 = m_1 g h \tag{33}$$

We can figure out the height by using basic trigonometric operations, as shown in the sketch below:



$$\cos\alpha = \frac{L-h}{L} \tag{34}$$
$$h = L - L\cos\alpha \tag{35}$$
$$= L(1-\cos\alpha) \tag{36}$$
$$\approx 0.022\ m \tag{37}$$

We know then, that the kinetic energy at the moment of impact is equal to the potential energy at its peak, so

$$\frac{m_1 v_1'^2}{2} = m_1 g h \tag{38}$$
$$m_1 v_1'^2 = 2 m_1 g h \tag{39}$$
$$v_1'^2 = 2gh \tag{40}$$
$$v_1' = \sqrt{2gh} \tag{41}$$
$$0.657\ ms^{-1} \tag{42}$$

The only thing missing now, is to figure out the velocity of the bullet after impact, for which we can use the conservation of impulse:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{43}$$
$$m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{44}$$
$$v_2' = \frac{m_2 v_2 - m_1 v_1'}{m_2} \tag{45}$$
$$\approx 268.6\ ms^{-1} \tag{46}$$

b) Given.:

- $E_{mec} = E_{kin} + E_{pot}$

Specifically, we are intersted in the mechanical energy that gets converted into heat energy during this process, which we can formulate as such:

$$
\begin{aligned}
E_{mec} &= E_{kin_2} - (E'_{kin_1} + E'_{kin_2}) && (47)\\
&= \frac{m_2 v_2^2}{2} - \left(\frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2}\right) && (48)\\
&= 438.849 \ J && (49)
\end{aligned}
$$

c) Now, we proceed similarly but inversely to a). First we need to determine the velocity of the block after impact. If the bullet had remained stuck on the block after impact, the impulse of the block and the bullet would be

$$p_3 = (m_1 + m_2)v_3 \tag{50}$$

and due to conservation of impulse, the formula would be

$$
\begin{aligned}
p_1 + p_2 &= p_3 && (51)\\
m_1 v_1 + m_2 v_2 &= (m_1 + m_2)v_3 && (52)\\
m_2 v_2 &= (m_1 + m_2)v_3 && (53)\\
v_3 &= \frac{m_2 v_2}{m_1 + m_2} && (54)\\
&\approx 1.99 \ ms^{-1} && (55)
\end{aligned}
$$

Secondly, we need to figure out how high the block goes. Proceeding analogous to a) we know that the potential energy at the peak must be equal to the kinetic energy at the bottom, so we set

$$
\begin{aligned}
\frac{(m_1 + m_2)v_3^2}{2} &= (m_1 + m_2)gh' && (56)\\
h' &= \frac{v_3^2}{2g} && (57)\\
&= 0.202 \ m && (58)
\end{aligned}
$$

And finally:

$$
\begin{aligned}
\alpha' &= \arccos\left(\frac{L - h'}{L}\right) && (59)\\
&\approx 0.647 \approx 37.06^\circ && (60)
\end{aligned}
$$

6.3 Exercise 3: On the raft

Given.:

- m: mass of dog
- M: mass of raft
- v_H: velocity of dog
- v_F: velocity of raft

a) In the closed system of the center of gravity (CoG), we can determine the impulse of the CoG to be

$$\vec{p}_s = \sum_{i=1}^{n} m_i \vec{v}_i \tag{61}$$

Having two different masses with velocities in opposite directions, we can then say

$$\vec{p}_s = m_1 \vec{v}_1 + m_2 \vec{v}_2 \tag{62}$$
$$0 = m\vec{v}_H + M\vec{v}_F \tag{63}$$
$$-M\vec{v}_F = m\vec{v}_H \tag{64}$$

Picturing this as a two dimensional problem with the observation that the raft is moving "backwards" and the dog is moving " frontwards", we can write

$$-Mv_F = mv_H \tag{65}$$

The dog is moving with a velocity of

$$v_H = -\frac{Mv_F}{m} \tag{66}$$

and the raft with

$$v_F = -\frac{mv_H}{M} \tag{67}$$

relative to the shore. The negative signs represent the direction relative to the the other velocity.

When the dog isn't moving, then its velocity is clearly $v_H = 0$ and the one of the raft is likewise

$$v_F = \frac{m \cdot 0}{M} \tag{68}$$
$$= 0 \tag{69}$$

b) The dog comes a distance x_H closer to the shore. According to the formula for the conservation of impulse

$$-Mv_F = mv_H \tag{70}$$

and taking the velocity to be

$$v = \frac{s}{t} \tag{71}$$

we can write the equation as

$$-M\frac{x_F}{t} = m\frac{x_H}{t} \tag{72}$$

$$-Mx_F = mx_H \tag{73}$$

$$x_H = \frac{-Mx_F}{m} \tag{74}$$

c) Specifically for the given values

- $m = 11\ kg$
- $M = 111\ kg$
- $d = 3.3\ m$
- $d = x_H - x_F$ (because the distance x_F is negative relative to our coordinate system)

$d = x_H - (\frac{-mx_H}{M})$

$x_H = \frac{d}{1 + \frac{m}{M}}$

We can calculate

$$x_H = \frac{d}{1 + \frac{m}{M}} \tag{75}$$

$$\approx 3\ m \tag{76}$$

d) As a collinear collision, we can use the formula for conservation of impulse

$$
\begin{align}
mv_H + Mv_F &= mv'_h + Mv'_F \tag{77}\\
v'_H &= \frac{v_H(M-m) + 2Mv_F}{m+M} \tag{78}\\
\frac{x'_H}{t} &= \frac{\frac{x_H}{t}(M-m) + 2M\frac{x_F}{t}}{m+M} \tag{79}\\
x'_H &= \frac{x_H(M-m) + 2Mx_F}{m+M} \tag{80}\\
&\approx 3\ m \tag{81}
\end{align}
$$

e) Given.:

- $v'_H = 30\ kmh^{-1}$

  The aforementioned data is in respect to a moving frame of reference, specifically that of the raft.

  Looking for the velocity of the dog and the raft relative to the shore, we can use the following information to set up a linear system of equations (LSE):

  * $v_H = v'_H + v_F$
  * $v_F = v'_H + v_H$
  * $mv_H = Mv_F$

    Regarding these equations, it is important to remember that v_F and v_H point in opposite directions, which was taken into account in the Galilei-Transformation for our respective velocities.

  I. $\quad v_H - v_F = v'_H$
  II. $\quad mv_H + Mv_F = 0$

  After solving this LSE, we get

$$
\begin{align}
v_H &= \frac{v'_H M}{m+M} \tag{82}\\
v_F &= \frac{-v'_H m}{m+M} \tag{83}
\end{align}
$$

  And if we input the values from c), we get

$$
\begin{align}
v_H &\approx 27.3\ kmh^{-1} \text{ and} \tag{84}\\
v_F &\approx 2.7\ kmh^{-1} \text{ relative to the shore} \tag{85}
\end{align}
$$