Taller

Axiomas de la probabilidad.

P_1 y P_2 son medidas de probabilidad.

$a_1 + a_2 = 1$

$a_1, a_2 \in R^+$

$P(A) = a_1 P_1(A) + a_2 P_2(A)$

1. $P(A) \geq 0$:

$P_1(A) \geq 0$ y $P_2(A) \geq 0$

$a_1 P_1(A) \geq 0$ $a_2 P_2(A) \geq 0$

$a_1 P_1(A) + a_2 P_2(A) \geq 0$

$P(A) \geq 0$

2. $P(\Omega) = 1$

$P(\Omega) = a_1 P_1(\Omega) + a_2 P_2(\Omega)$

$P(\Omega) = a_1 + a_2$

$P(\Omega) = 1$

$P(\emptyset) = 0$

$P(\emptyset) = a_1 P_1(\emptyset) + a_2 P_2(\emptyset)$

$P(\emptyset) = a_1 . 0 + a_2 . 0$

$P(\emptyset) = 0$

3. $A_i \cap A_j = \emptyset \quad i \neq j$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = a_1 P_1\left(\bigcup_{n=1}^{\infty} A_n\right) + a_2 P_2\left(\bigcup_{n=1}^{\infty} A_n\right)$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = a_1 \sum_{n=1}^{\infty} P_1(A_n) + a_2 \sum_{n=1}^{\infty} P_2(A_n)$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} a_1 P_1(A_n) + a_2 P_2(A_n)$$

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$$

P es una medida de probabilidad.

Punto 3:

a. $P(\emptyset) = 0$

Supongamos el espacio medible $\mathfrak{F}$:

$$(\emptyset, \Omega, A_1, A_2, \dots, A_n)$$

$P(\emptyset \cup A_1)$

"A_1 y $\emptyset$ no comparten elementos.

$$A_1 \cap \emptyset = \emptyset$$

$$P(\emptyset \cup A_1) = P(A_1) + P(\emptyset)$$

$$P(A_1) = P(A_1) + P(\emptyset)$$

$$P(A_1) - P(A_1) = P(\emptyset)$$

$$P(\emptyset) = 0$$

$$\overset{\emptyset}{\bigcirc} \cup \overset{A_1}{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}} = \overset{A_1}{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}}$$

b. $P(A^c) = 1 - P(A)$

$$P(A^c) = 1 - P(A)$$

$$P(A^c) + P(A) = 1$$

$$P(A^c) + P(A) = P(\Omega)$$

$$\Omega = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}$$

$$A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad A^c = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$A \cup A^c = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}$$

$$A \cup A^c = \Omega$$

$$P(\Omega) = 1$$

$$P(A^c \cup A) = 1$$

$$P(A^c) + P(A) = 1$$

$$P(A^c) = 1 - P(A)$$

F. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Es el caso de la unión de conjuntos que comparten elementos entre sí.

Cuando

$P(A_i \cup A_j)$ $\quad A_i \cap A_j = \emptyset$

$i \neq j$

$P(A_i \cup A_j) = P(A_i) + P(A_j) - \cancel{P(A_i \cap A_j)}^{\emptyset}$

Ahora, supongamos:

$A = A' \cup C$

$B = B' \cup C$

$A \cap B = C$

$A \cup B = A' \cup B' \cup C$

Ejemplo:

$A = (a_1, a_2, a_3, c_1, c_2, c_3)$

$B = (b_1, b_2, b_3, c_1, c_2, c_3)$

$A \cup B = (a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3)$

$$P(A \cup B) = P((A' \cup C) \cup (B' \cup C))$$

$$= P(A' \cup B' \cup C)$$

$$= P(A') + P(B') + P(C)$$

$$P(A \cup B) = P(A') + P(B') + P(C)$$

$$P(A \cup B) + P(C) = [P(A') + P(C)] + [P(B') + P(C)]$$

$$P(A \cup B) + P(C) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(C)$$

$$P(A \cup B) = P(A + P(B) - P(A \cap B)$$

Técnicas de conteo.

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r r r   b b b   v v v

r r r b	r b r b
r r r v	r v r v
b b b r	b v b v
b b b v	r b v r
v v v r	r b v b
v v v b	r b v v.

— 12 Formas cuando el orden no importa