Punto 11

¿Qué pasa si expandimos en series de Taylor las funciones $F(x+2h)$ y $F(x-2h)$?

$$2h \leq 1$$
$$h \leq \frac{1}{2} < 1$$
$$h < 1$$

Tenemos:

$$F(x+2h) = F(x) + 2hF'(x) + \frac{4h^2}{2}F''(x) + \frac{8h^3}{3!}F'''(x) + \cdots$$

$$F(x-2h) = F(x) - 2hF'(x) + \frac{4h^2}{2}F''(x) - \frac{8h^3}{3!}F'''(x) + \cdots$$

Haciendo $F(x+2h) + F(x-2h)$:

$$F(x+2h) + F(x-2h) =$$
$$2F(x) + \frac{8h^2}{2}F''(x) + \frac{32h^4}{24}F^{IV}(x) + \cdots$$

$$F(x+2h) + F(x-2h) = 2F(x) + 4h^2F''(x) + \frac{4}{3}h^4F^{IV}(x) + \cdots$$

Si despejamos $F''(x)$:

$$\frac{F(x+2h) + F(x-2h) - 2F(x) - \frac{4}{3}h^4F^{IV}(x) - \cdots}{4h^2} = F''(x)$$

$$F''(x) = \frac{F(x+2h) + F(x-2h) - 2F(x)}{4h^2} + \frac{-\frac{4}{3}h^4F^{IV}(x) - \cdots}{4h^2}$$

$$F''(x) = \frac{F(x+2h) - 2F(x) + F(x-2h)}{4h^2} + -\frac{1}{3}h^2F''(x) + \cdots$$

$$F''(x) = \frac{F(x+2h) - 2F(x) + F(x-2h)}{4h^2} + O(h^2)$$

Escaneado con CamScanner

$$F''(x) \approx \frac{F(x+2h) - 2F(x) + F(x-2h)}{4h^2}$$

$$F''(x_j) \approx \frac{F(x_{j+2}) - 2F(x_j) + F(x_{j-2})}{4h^2}$$

$$x_{j+n} = x_j + nh$$