Punto 6

Parte 1:

$$Y_1 = a_0 + a_1 X_1$$
$$Y_2 = a_0 + a_1 X_2$$
$$Y_3 = a_0 + a_1 X_3$$
$$\vdots$$
$$Y_n = a_0 + a_1 X_n$$

$$a_0 + a_1 X_1 = Y_1$$
$$a_0 + a_1 X_2 = Y_2$$
$$a_0 + a_1 X_3 = Y_3$$
$$\vdots$$
$$a_0 + a_1 X_n = Y_n$$

$$\begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ 1 & X_3 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix}$$

$$UX \cdot \vec{a} = \vec{Y}$$

$$UX^T \cdot UX \cdot \vec{a}^* = UX^T \vec{Y}$$

$$UX^T = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ X_1 & X_2 & X_3 & \cdots & X_n \end{bmatrix}$$

$UX^T \cdot UX$

$$= \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

$$= \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} = UX^T \, UX$$

$UX^T \vec{y} =$

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$$

$$= \begin{bmatrix} \sum y_i \\ \sum x_i \cdot y_i \end{bmatrix}$$

$$\begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum X_i Y_i \end{bmatrix}$$

$$a_0 n + a_1 \sum X_i = \sum Y_i$$

$$a_0 \sum X_i + a_1 \sum X_i^2 = \sum X_i \cdot Y_i$$

$$a_0 n = \sum Y_i - a_1 \sum X_i$$

$$a_0 = \frac{\sum Y_i}{n} - a_1 \frac{\sum X_i}{n}$$

$$a_0 = \overline{Y} - a_1 \overline{X}$$

$$\left(\frac{\sum Y_i}{n} - a_1 \frac{\sum X_i}{n} \right) \sum X_i + a_1 \sum X_i^2 = \sum X_i Y_i$$

$$\frac{\sum Y_i \cdot \sum X_i}{n} - \frac{a_1 (\sum X_i)^2}{n} + a_1 \sum X_i^2 = \sum X_i Y_i$$

$$a_i \sum X_i^2 - \frac{a_1 (\sum X_i)^2}{n} = \sum X_i Y_i - \frac{\sum X_i \cdot \sum Y_i}{n}$$

$$a_1 \left(\sum X_i^2 - \frac{(\sum X_i)^2}{n} \right) = \sum X_i Y_1 - \frac{\sum X_i \sum Y_i}{n}$$

$$a_1 = \left(\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n} \right) \Big/ \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

Parte 2:

$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2$$

$$y_2 = a_0 + a_1 x_2 + a_2 x_2^2$$

$$y_3 = a_0 + a_1 x_3 + a_2 x_3^2$$

$$\vdots$$

$$y_n = a_0 + a_1 x_n + a_2 x_n^2$$

$$\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{pmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$$

$$Uxx^2 \vec{a} = \vec{y}$$

$$Uxx^{2T} Uxx^2 \vec{a} = Uxx^{2T} \vec{y}$$

$$Uxx^{2T} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \end{bmatrix}$$

$$Uxx^{2T} Uxx^2 = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \end{bmatrix} \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{bmatrix}$$

$$UXX^{2\,T} UXX^{2} = \begin{bmatrix} n & \sum X_i & \sum X_i^2 \\ \sum X_i & \sum X_i^2 & \sum X_i^3 \\ \sum X_i^2 & \sum X_i^3 & \sum X_i^4 \end{bmatrix}$$

$$UXX^{2\,T} \vec{Y} =$$

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i X_i \\ \sum Y_i X_i^2 \end{bmatrix}$$

$$\begin{bmatrix} n & \sum X_i & \sum X_i^2 \\ \sum X_i & \sum X_i^2 & \sum X_i^3 \\ \sum X_i^2 & \sum X_i^3 & \sum X_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i X_i \\ \sum Y_i X_i^2 \end{bmatrix}$$

① $a_0 \cdot n + a_1 \sum X_i + a_2 \sum X_i^2 = \sum Y_i$

$\sum_{i=1}^{n} a_0 + a_1 x_i + a_2 x_i^2 = \sum_{i=1}^{n} Y_i$

② $a_0 \sum X_i + a_1 \sum X_i^2 + a_2 \sum X_i^3 = \sum Y_i X_i$

$\sum a_0 X_i + a_1 X_i^2 + a_2 x_i^3 = \sum Y_i X_i$

③ $a_0 \sum X_i^2 + a_1 \sum X_i^3 + a_2 \sum X_i^4 = \sum Y_i X_i^2$

$\sum a_0 X_i^2 + a_1 X_i^3 + a_2 X_i^4 = \sum Y_i X_i^2$