Plankton Barbecue: Nutrient Flow due to Natural Thermal Convection

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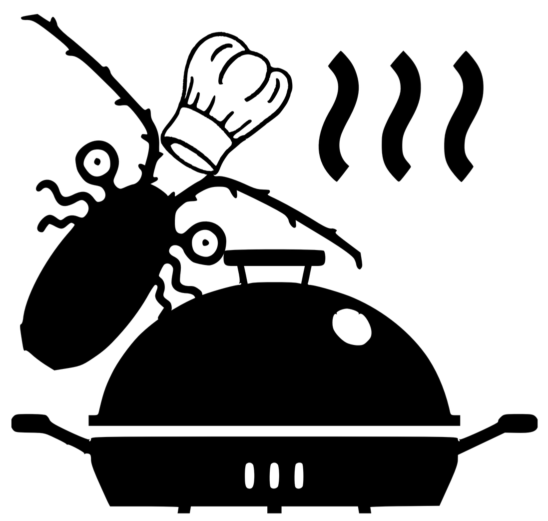

Introduction

Natural Thermal Convection

Those plumes comming out from our BBQ grill are caused by differences on density consequence of temperature variations.

Governing Equations

- **($\approx$) Momentum:**
$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \frac{1}{Re}\nabla^2 \omega + \beta g \frac{\partial T}{\partial x}$$
- **Stream fnc - Vorticity - Velocity:**
$$\nabla^2 \psi = -\omega, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$
- **Energy:**
$$\frac{\partial T}{\partial t} + (\overrightarrow{u} \cdot \nabla T) = \frac{1}{RePr}\nabla^2 T$$
- **(Bonus!!!) Mass Transfer:**
$$\frac{\partial c}{\partial t} + (\overrightarrow{u} \cdot \nabla c) = \frac{1}{Pe}\nabla^2 c$$

Where the last Eq. models a substance moving with the fluid.

Method

To solve numerically the equations ...

1. Set up the initial conditions for ψ, u and v.
2. Solve the vorticity equation. (Exp FTCS).
3. Solve the Poisson equation for ψ. (Std. discr. ∇^2).
4. Compute the new velocity from ψ.
5. Solve the energy equation for T. (Exp FTCS).
6. Solve the mass transfer equation. (Exp FTCS).
7. Go to step 2.

BBQ Grill: Air Thermal Convection

As benchmark parameters we chose:

$$Re = 4365.30, \ Pr = 0.72, \ Pe = 0.07$$

For a rectangular cavity width dimensions: 0.65m height, 2.6m wide, and normalized temperatures $Tc = 292.5°K$, $Th = 293.5°K$.

The Figure 2 shows the normalized velocities, temperature and concentration evolution for several times, some remarkable facts are:

- Buoyancy force makes rise the warmer water from the bottom.
- There are cell formations even for early times.
- Iso-thermal and Iso-concetration regions.

Cells and Spatial Resolution

The Figure 1 shows that the simulation results depend on the discretization size. In the coarser discretization only 4 cells Rayleigh-Benard cells form, compared to 6 in the finer one. This indicates that some discretizations cannot completely resolve the structure of the flow.

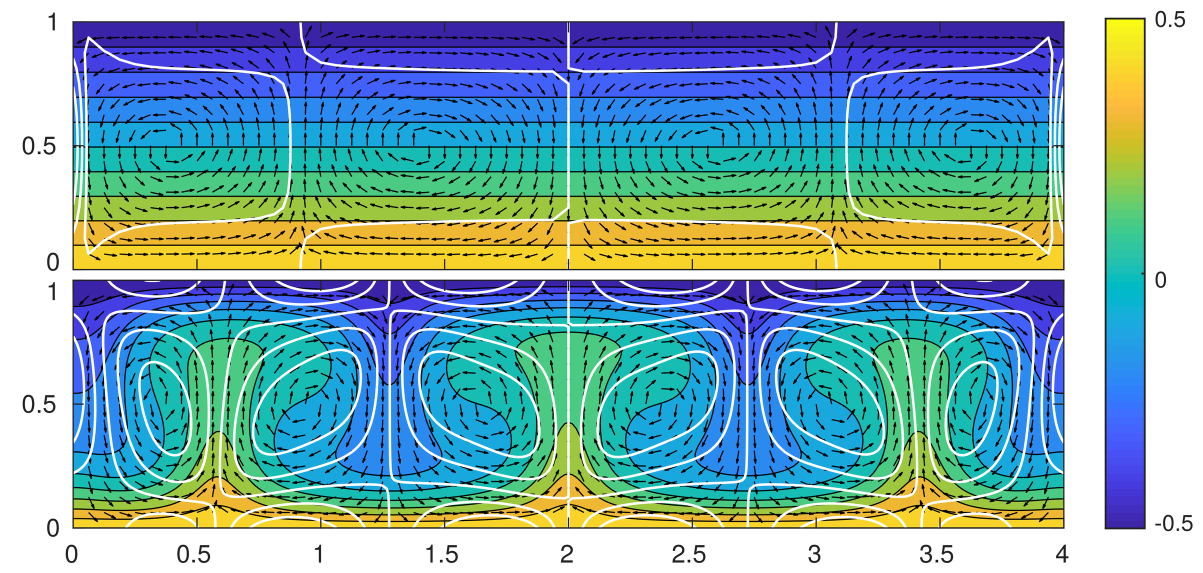

Figure 1: Cells on grids: (top) $\Delta x = 156 \times 10^{-3}$, $\Delta y = 62.5 \times 10^{-3}$ (bottom) $\Delta x = 39 \times 10^{-3}$, $\Delta y = 15.4 \times 10^{-3}$.

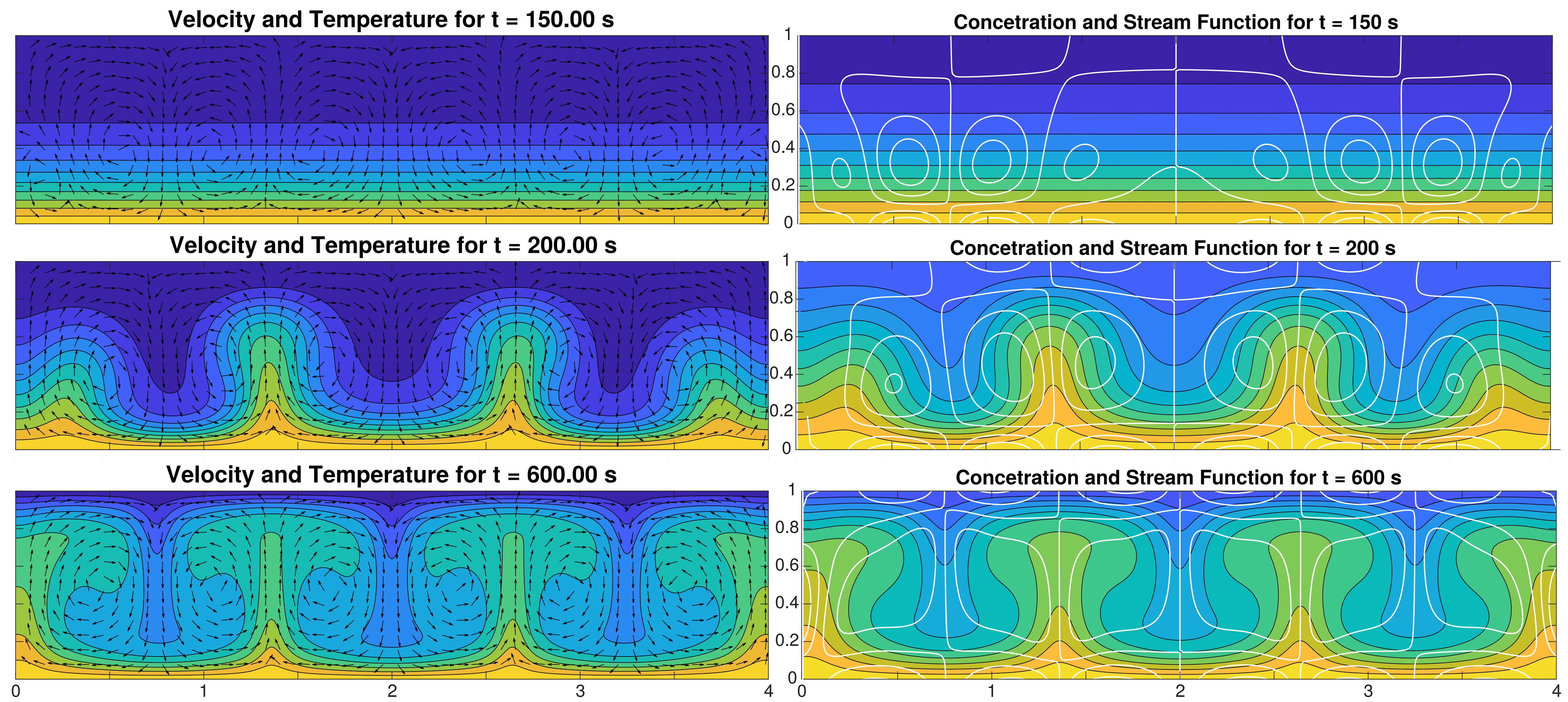

Figure 2: Air thermal convection and mass transfer normalized velocity, temperature and mass concentration for different times.

Numerical Convergence

Due to the high Reynolds number the experimental study for the convergence rates presents some challenges:

- Changes in the grid size lead to variations in the direction of rotation in the cells.
- To get a reliable reference it is necessary both very fine grid and pretty small time step.

Spatial Convergence order					
u	1.5339	v	1.3119	T	0.3145

And the plankton?

- Pythoplackton is the base of the oceanic food chain, it sinks in the oceans upto $160m$ per year (passive swimmers).
- The temperature in the ocean may vary up to $10°K$ per year.
- The buoyancy forces keep them in specific regions of ocean.

We wanted to study the effects of variations on temperature around $2°K$ in a big rectangular area in the sea (160m deep, 1.6km wide) with planktonic organisms in the bottom, getting as parameters:

$$Re = 184269, \ Pr = 7.56, \ Pe = 6965.39$$

Although the Reynolds number in enormous, some general structure of this phenomena may be preserved, e.g. the number of cells or the average concentration in specific areas.

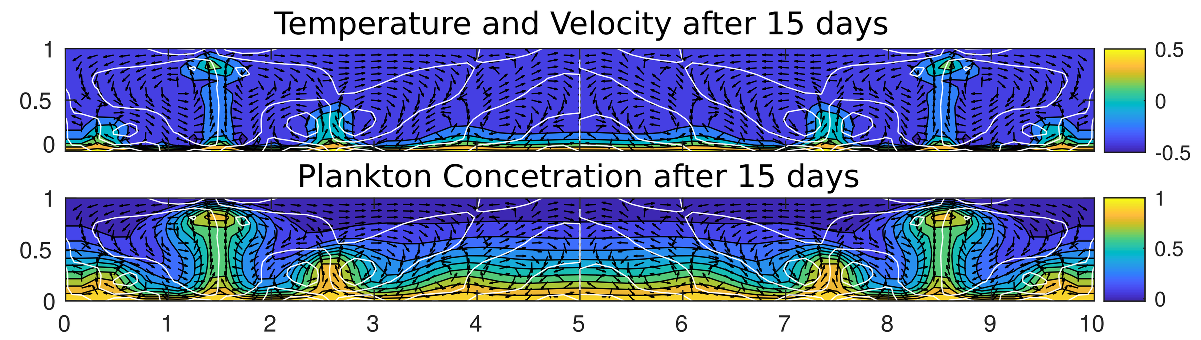

Figure 3: Thermal convection with planktonic mass transfer in the sea.

Final Thoughts!

Conclusions

- The behavior of the thermal cells depends on the size of the spatial discretization.
- Even with high Reynolds numbers the implemented method seems to show properly the general structure of the fluid.

Pendings

- A solver using the projection method was considered, however, the staggered grid was not work as expected for the energy equation.
- An implementation using the second order FTBS scheme will help to deal with higher Reynolds numbers.