Natural Convection within a Rectangular Enclosure in a Window Construction (A Numerical Study)

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Abstract

The windows are important parts of the building shield due to the indoor climate and to the heat losses from the building. Modern windows are designed with two or three panes in order to improve their thermal performance. The cavities between the panes are often hermetically sealed and filled with a gas with low thermal conductivity. A numerical study has been undertaken of the natural convection within the cavity, which plays an important role in the thermal performance of the window. Thermal radiation transfer will certainly play a major role in the heat transfer process; however, the focus here will be on the convective heat transfer.

The numerical calculations are performed on a model with two vertical parallel panes with constant surface temperatures. The cavity between the vertical panes is enclosed with insulated plates at the top and at the bottom of the model. The model is two-dimensional and the flow is assumed to be steady, laminar and incompressible. The governing equations for conservation of mass, momentum and energy are solved by the finite element code FIDAP.

The numerical results in the present study have been validated with the measurement results presented earlier by Elsherbiny [1]. Flow patterns, velocity and temperature profiles within the cavity are presented. The variation of the Nusselt number along the height of the hot pane as a function of the Rayleigh number is also investigated.

Correlation for the average Nusselt number, Nu, as a function of aspect ratio, *A*, and the Rayleigh number, *Ra*, are determined. The correlation covers the range of values for, *A*, and, *Ra*, which are of practical interest in window construction. Based on regression analysis, the following correlation is proposed, $Nu = 1 + 0.00137\, A^{-1.137}\, Ra$. In addition, for further validating of the present study, the predicted results are compared with the results obtained theoretically and numerically by others.

Keywords
Well-insulated windows, Heat transfer, Natural convection, Numerical analysis, Enclosed cavity

Introduction

In Sweden and the other Nordic countries the climate is cold several months a year. To a great part the indoor climate of a building depends on the outdoor temperature and the quality of the insulation affects heat transfer in the building, i.e. the components of the building shield. The windows are an important building component for the indoor climate, but there is a down-draught problem, as the temperature is lower on the glass than the indoor air. For the last 50 years, the construction of the windows has developed from single-glazed constructions to today's triple-glazed constructions, with enclosed spaces between the panes, see Moshfegh et al [2]. Examples of gases, usually used in these enclosed spaces, are air, argon, krypton, xenon, CO_2 and SF_6.

Natural convection in vertical and inclined spaces has been investigated both analytically and experimentally for the last 40 years. The first important theoretical analysis was done by Batchelor [3] in 1954, after that many more studies have been carried out. Over the years, the studies have focused different circumstances such as limited Rayleigh numbers, *Ra*, and aspect ratios, *A*. In 1980 Elsherbiny [1] investigated a large scale of Rayleigh numbers and aspect ratios. As computer capacity has improved during the last decade, the possibility to solve the problems numerically has increased considerably. In 1995 H. Schweiger et al [4] made numerical simulations on laminar natural convection in rectangular cavities for different Rayleigh numbers at $A = 40$.

The purpose of this paper is to analyse numerically the convection between two vertical parallel plates forming an enclosed cavity. The region, which is of interest in this paper, is window constructions with Rayleigh numbers up to 20000 and the aspect ratio from 40 to 130. This region normally covers window constructions with a space width of 10- 15 mm, a height from 400 to 1900 mm and with common gases inside the cavity such as air and argon, with outside temperatures normal for Nordic countries in the wintertime.

The numerical calculations will solve the governing equations for conservation of mass, momentum and energy. The actual model is two-dimensional and the flow is steady, laminar and incompressible. The physical model, which the governing equations will be solved on, is a rectangular cavity with an isolated top and bottom and with two vertical plates of different temperatures. The governing equations are solved by the finite element method. The results from the numerical calculations are validated with results from other investigators. The intention with these numerical calculations is to find a correlation where the Nusselt number depends on both the Rayleigh number and the aspect ratio.

Physical model

The numerical calculation is made on a model, very close to reality (see figure 1). *H* is the height of the model, constantly 640 mm. *L* varying from 4.9 mm to 32 mm and is the width of the enclosed cavity between the two vertical panes. Heat transfer between two panes of different surface temperatures in an enclosed space can be performed with the three mechanisms conduction, convection and radiation. Convection and conduction are the two mechanisms considered in this paper. The radiative heat transfer is not negligible but it is not taken into account in the present study. The physical model is intended to be an enclosed space in a window construction therefore the Rayleigh number, *Ra,* and the aspect ratio, *A,* are limited to values which are of interest for window application. Air, argon, krypton, xenon, SF_6 and CO_2 are conceivable gases to use within the cavity in window constructions. The Rayleigh number and aspect ratio of interest is up to 20000 respectively 40-130.

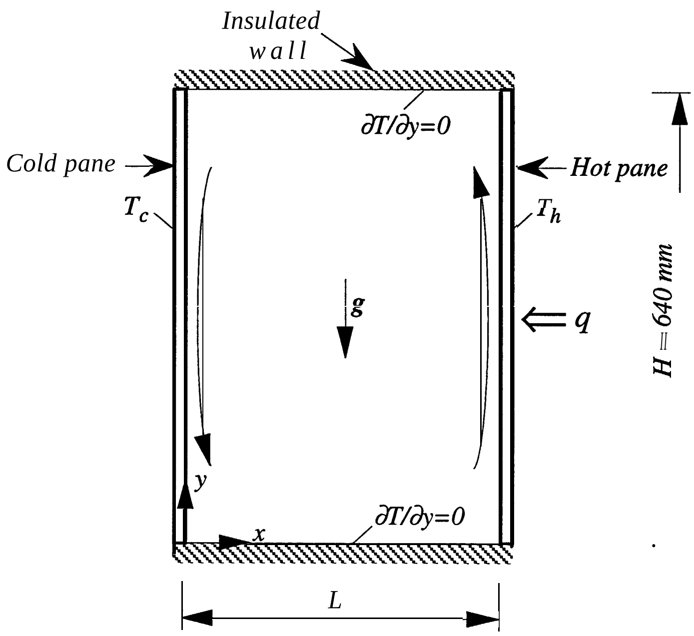


Figure 1. The physical model under consideration

Numerical calculation

The flow inside the enclosed space is assumed to be steady, laminar, incompressible and two-dimensional. Viscous dissipation is neglected and all thermo-physical properties of the fluid in the enclosed space are assumed to be constant, except for the buoyancy term of the y-momentum equation, i.e the Boussinesq approximation, see Gray et al [5]. There are no radiative processes between the surfaces. Considering the above mentioned assumptions the following governing equations are obtained.

Equation of the conservation of mass

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (1)$$

Equation of the conservation of the momentum

$$\rho(u\partial u/\partial x + v\partial u/\partial y) = -\partial p/\partial x + \mu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2) \qquad (2)$$

$$\rho(u\partial v/\partial x + v\partial v/\partial y) = -\partial p/\partial y + \mu(\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2) + \rho g\beta(T - T_{ref}) \qquad (3)$$

Equation of the conservation of energy

$$\rho c_p(u\partial T/\partial x + v\partial T/\partial y) = k(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2) \quad (4)$$

For faster convergence of the numerical solution the governing equations are solved in a dimensionless form. These following dimensionless parameters are introduced.

$$x^* = x/L, y^* = y/L, \ T^* = (T - T_{ref})/(T_h - T_c), p^* = p/p_{ref}, u^* = u/u_{ref}, v^* = v/u_{ref}$$

Where $T_{ref} = T_c, u_{ref} = \alpha/L, p_{ref} = \mu\, u_{ref}/L$. By using the proposed parameters, the dimensionless governing equations are derived as:

Conservation of mass

$$\partial u^*/\partial x^* + \partial v^*/\partial y^* = 0 \quad (5)$$

Conservation of momentum

$$1/Pr(u^*\partial u^*/\partial x^* + v^*\partial u^*/\partial y^*) = -\partial p^*/\partial x^* + \partial^2 u^*/\partial x^{*2} + \partial^2 u^*/\partial y^{*2} \quad (6)$$
$$1/Pr(u^*\partial v^*/\partial x^* + v^*\partial v^*/\partial y^*) = -\partial p^*/\partial y^* + \partial^2 v^*/\partial x^{*2} + \partial^2 v^*/\partial y^{*2} + Ra\, T^* \quad (7)$$

Conservation of energy

$$u^*\partial T^*/\partial x^* + v^*\partial T^*/\partial y^* = \partial^2 T^*/\partial x^{*2} + \partial^2 T^*/\partial y^{*2} \quad (8)$$

By comparing the governing equations with the dimensionless form of the governing equations these relationships are obtained

$$\rho = 1/Pr, c_p = Pr, \ \beta = Ra\, Pr, \ k = 1, \ \ g = 1, \ \mu = 1$$

Where

$$Ra = \beta\, g\, (T_h - T_c)\, L^3/k\, \alpha\, \upsilon, \ Pr = \upsilon/\alpha \quad (9)$$

Boundary condition in dimensionless form

On the hot pane	$u^* = v^* = 0, T^* = 1$
On the cold pane	$u^* = v^* = 0, T^* = 0$
On the insulated walls	$u^* = v^* = 0, \partial T^*/\partial y^* = 0$

The finite element code FIDAP, is used to solve the governing equations, see [6]. Non-uniform grids are used in such a way that grids points are placed at geometrically decreasing distances in the regions next to the walls, where large gradients of velocity and thermal gradients are expected. Grid refinement is applied to check the accuracy of the solution. A number of grid systems, eg. 20×100, 30×100 and 40×200 are considered and the grid system of 30×100 nodes is used for the solutions presented here for aspect ratio 40, as a trade-off between accuracy and efficiency. The demand for convergence of the solutions of temperature, pressure and velocity is when the difference between the iterations is less than 10^{-3}.

Numerical results

The results presented here are interesting for window constructions. Attention is focused on the effect of the Rayleigh number, *Ra,* based on the width of the cavity and aspect ratio of the cavity, *A,* and on the thermal behaviour of the gases within the cavity. The numerical results presented here are obtained for Rayleigh numbers, *Ra,* up to 20000, and aspect ratios, *A,* varying from 40 to 130.

The figure below shows results from a numerically calculated case where the Rayleigh number is 12000 and the aspect ratio is 20. The results, which are presented here, are the isotherms and the streamlines in the cavity; also the velocity at the top of the model are presented, see figure 2.

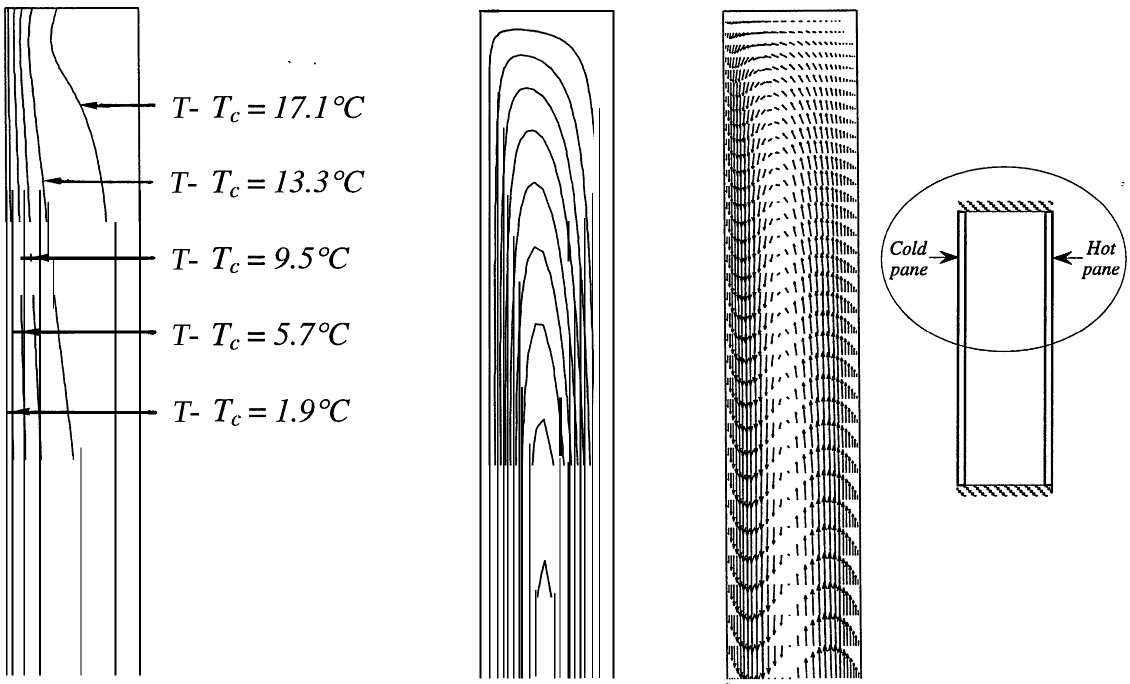


Figure 2. The predicted isotherms, streamlines and velocity for case Ra=12000 and A=20

To ensure the results from the numerical simulations of the model are reliable the results have been validated (see table 1) with measurements from Elsherbiny [1]. In addition, the predicted results are compared with the results obtained theoretically by Batchelor [3] and numerically by Schweiger et al [4] for aspect ratio 40 only, for further validation of the present numerical simulation. When Elsherbiny made his measurements he regulated the Rayleigh number,. *Ra*, by changing the density, p, with different pressure inside the enclosed space. Only calculations for atmosphere pressure were considered in the present study for numerical simulations. Therefore only measured values for aspect ratios, *A*, up to *A* = 40 were used, for higher aspect ratios in atmospheric pressure Nu = 1 which means that there is no convection in the enclosed space.

The mean Nusselt number over the whole height of the cavity is obtained by

$$Nu = \frac{1}{k \cdot \left(H/L\right) \cdot (T_h - T_c)} \int_0^H q \cdot dy \tag{10}$$

Case	*A*	*Ra*	Nusselt number,*Nu*			
			ref.[1]	ref.[3]	ref. [4]	present study
1	5	2610600	8.374	12.637*	-	8.279
2	10	343250	4.417	6.399*	-	4.367
3	20	37970	2.200	3.103*	-	2.063
4	40	5765	1.062	1.200	1.100	1.083
5	80	4000	-	1.070	-	1.038
6	120	4000	-	1.046	-	1.019

* *Pr = 0.7*

Table 1. Validation of the Nusselt number, Nu

When there is only one flow in the enclosed space, which means there is no secondary flow in the space, the Nusselt number is constant except at the top and at the bottom where the flow turns. In this regime, there is only conducting heat transfer and it is therefore called the conduction regime. The next regime is called the boundary layer regime and in this regime the heat transfer endures through the enclosed space both conducting and convection. Here the Nusselt number is not constant all over the enclosed space. The local Nusselt number decreases along the hot pane and because of that the Nusselt number must be measured all over the space. Elsherbiny measured the average Nusselt number over three areas and these three average numbers form a total average number for the whole space..

The Nusselt numbers along the height of the hot pane as a function of, *Ra*, for *A = 40* are shown in figure 3. The graph shows the numerically predicted local Nusselt numbers and the triangle stands for the measured average Nusselt numbers over the three areas. One reason for the difference between the measured and the simulated results for *Ra* = 5765 was that the value was measured over an area and then an average Nusselt number has been calculated over this area.

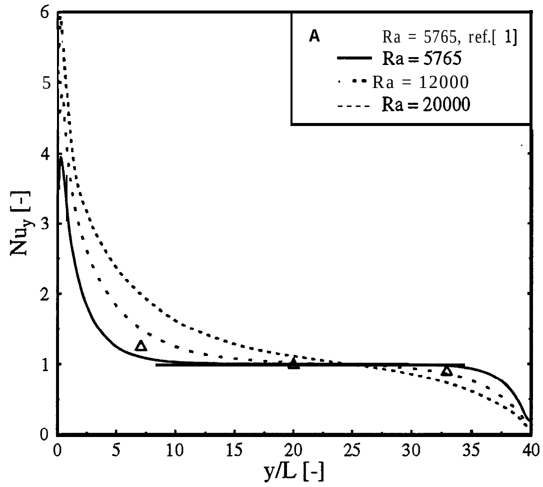
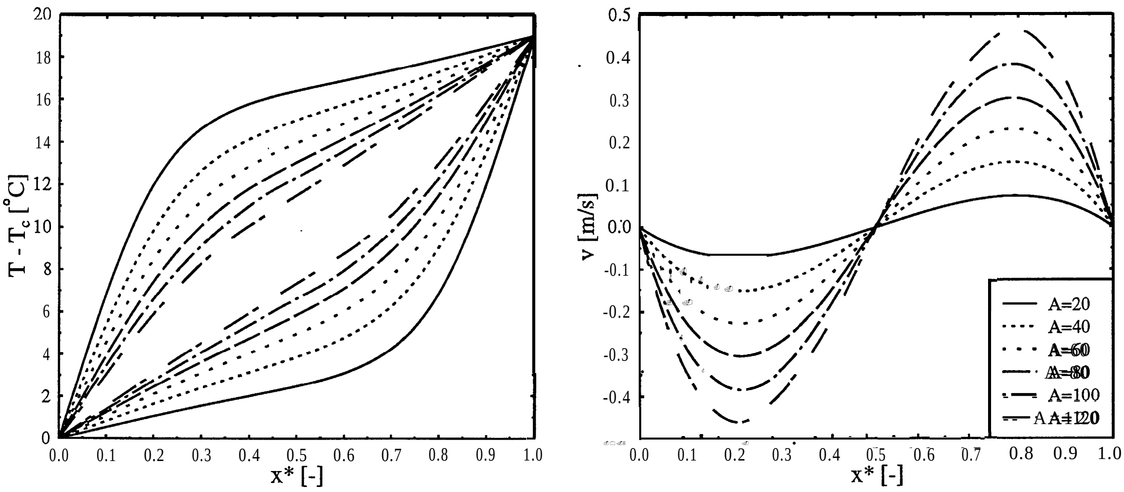


Figure 3. The local Nusselt number along the height of the heated wall

If the Rayleigh number, *Ra,* is constant and the aspect ratio varies, convection heat transfer will change within the enclosed space. When the aspect ratio decreases (the width of the enclosed space increases) the convective heat transfer increases.

Figure 4a presents temperature gradients at heights y = 30.5 mm and y = 609.5 mm. The Rayleigh number, *Ra,* is 12000. The figure shows that when the aspect ratio decreases from *A* = 120 to *A* = 20 the temperature gradient declines gradually, which shows that convection rises with the space width. The figure also shows that the temperature gradient is inversely symmetric at the two heights.

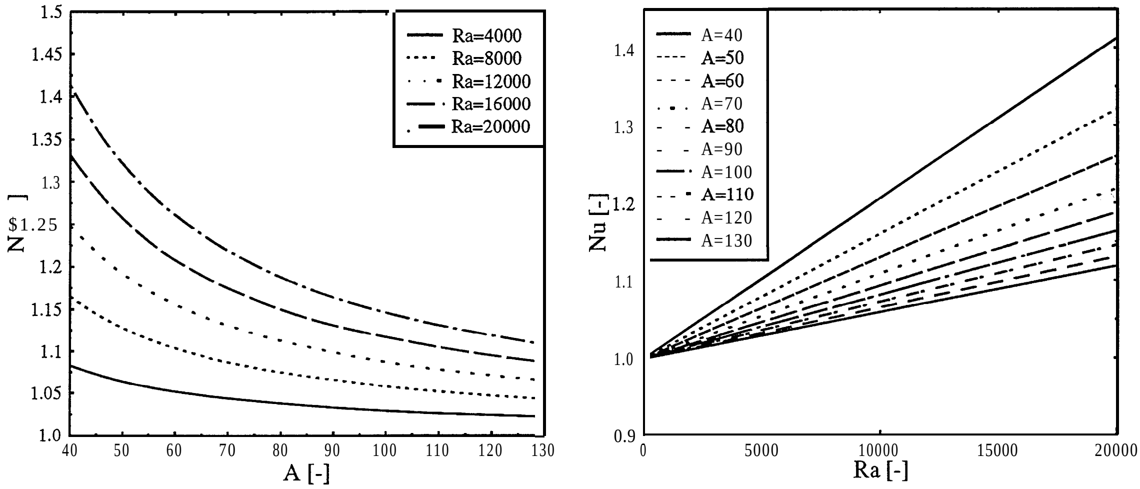


0 *a* (*b*)

Figure 4. Temperature gradient at y = 30.5 mm, y = 609.5 mm (figure 4a) and velocity profile at y = 320 mm (figure 4b) for different aspect ratios.

Figure 4b shows that when the aspect ratio, *A*, increases, the velocity, v, in the enclosed space will also increase. On the side where x^* = 1 (the hot pane) the gas rises while on the side where $x^* = 0$ (the cold pane) it falls. The velocity curves are inversely symmetric at $y^* = 0.5$. The figure 4b shows the velocity in the middle of the height (y = H/2) at a Rayleigh number of about 12000.

Figure 5 shows that there is a dependence between the Nusselt number, *Nu*, and the aspect ratio, *A*, the dependence of the Rayleigh number on the Nusselt number, Nu, is showed in the figure. Figure 5a presents the Nusselt numbers for aspect ratios from $A = 20$ to $A = 120$ at Rayleigh numbers up to $Ra = 20000$. Figure 5b displays the same results as figure 5a bu. the graph is made for Nusselt numbers against Rayleigh numbers.



(a) *(b)*

Figures 5. Variation of the Nusselt number as a function of the aspect ratio (figure 5a) and Rayleigh number (figure 5b).

The main goal of the present study is to obtain a correlation for the rate of heat transfer, Nu, as a function of *Ra* and *A*, over the range of values that are of practical interest in window applications. According to figure 5b the best-fitted expression, which represents the numerical results, has the following form:

$$Nu = 1 + C_1 \cdot A^{C_2} \cdot Ra^{C_3} \tag{11}$$

Where C_1, C_2 and C_3 are 0.00137, -1.137 and 1 .OOO respectively. The coefficients are in good agreement with those proposed by Batchelor [3].

Case	C_1	C_2	C_3	Limitations
Present study	0.00137	-1.137	1 .000	$40 \le A \le 130,\ 0 \le Ra \le 20000$
Batchelor [3]	0.00 139	- 1.000	1 .000	$A > Ra/500$

Table 2. Coefficients to equation 11

The gases suitable for window constructions have a Prandtl number, *Pr*, which varies from 0.6 to 0.8. Numerical calculations for *Pr* from 0.5 to 1.0 have been performed with constant Rayleigh numbers and the results from these calculations indicate differences smaller than 1%, which means that the Nusselt number is independent of *Pr*, when the Rayleigh number is specified.

Conclusions

An investigation was made to provide better understanding of the mechanism of convection heat transfer in a rectangular enclosure by means of numerical method. The analysis covers the range of values for aspect ratio, *A*, and the Rayleigh number, *Ra*, which are of practical interest in window construction.

For laminar natural convection within the cavity, the Nusselt number, *Nu*, according to the numerical predictions presented here is proportional to the, A^{C_2}, and Ra^{C_3} where C_2 is -1.137 and C_3 is 1 .O, respectively, for the interval $40 \leq A \leq 130$ and *Ra* up to 20000. The proposed correlation for, Nu, is unique because it covers the geometry and fluid properties as well as boundary conditions which are relevant for thermal analysis of window construction.

The predicted results are in good agreement with the results obtained theoretically and experimentally as well as numerically by other researchers.

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