Plankton Barbecue

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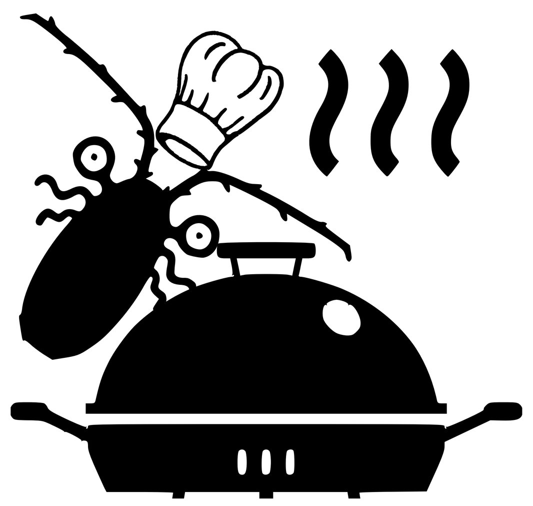

Introduction

Natural Thermal Convection
Those plumes comming out from our BBQ grill are caused by a differences on density consequence of temperature varitions.

Governing Equations

- **($\approx$) Momentum:**
$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \frac{1}{Re}\nabla^2 \omega + \beta g \frac{\partial T}{\partial x}$$
- **Stream fnc - Vorticity - Velocity:**
$$\nabla^2 \psi = -\omega, \ \ u = \frac{\partial \psi}{\partial y}, \ \ v = -\frac{\partial \psi}{\partial x}$$
- **Energy:**
$$\frac{\partial T}{\partial t} + (\overrightarrow{u} \cdot \nabla T) = \frac{1}{RePr}\nabla^2 T$$
- **(Bonus!!!) Mass Transfer:**
$$\frac{\partial c}{\partial t} + (\overrightarrow{u} \cdot \nabla c) = \frac{1}{Pe}\nabla^2 c$$

The last Eq. models a substance moving with the fluid.

Method

To solve numerically the equations:

1. Set up the initial conditions for ψ, u and v.
2. Solve the vorticity equation. (Exp FTCS).
3. Solve the Poisson equation for ψ. (Std. discr. ∇^2).
4. Compute the new velocity from ψ.
5. Solve the Energy equation for T. (Exp FTCS).
6. Solve the Mass transfer equation. (Exp FTCS).
7. Go to step 2.

BBQ Grill: Air Thermal Convection

As benchmark parameters we chose:
$$Re = 4365.30, \ Pr = 0.72, \ Pe = 0.07$$
For a rectangular cavity width dimensions: 0.65m height, and 2.6m wide, and normalized temperatures Tc=-0.5, Th=0.5.

The Figure 2 the fluid shows the normalized velocities, temperature and concentration evolution for several times, some remarkable facts are:

- Cell formations even for early times.
- Iso-thermal regions.
- Iso-concetration regions.

Cells and Spatial Resolution

The Figure 1 shows that the simulation results depend on the discretization size. In the coarse discretization only 4 cells Rayleigh-Benard cells form, compared to 6 in the fine discretization.This indicates that some discretizations cannot completely resolve the strucure of the flow.

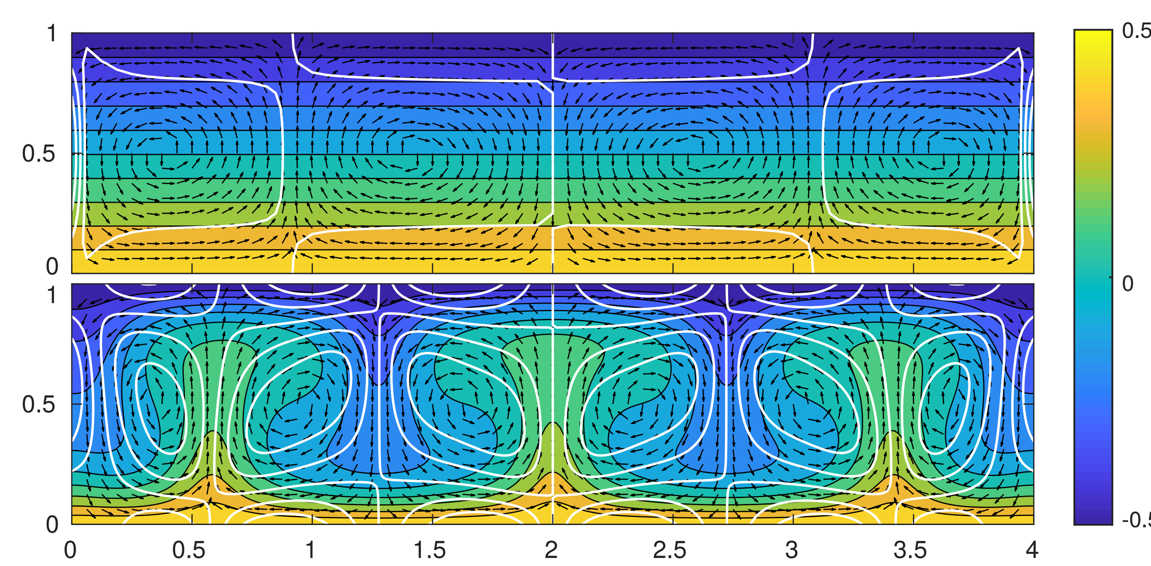

Figure 1: Cell on grids: (top) $\Delta x = 156 \times 10^{-3}, \Delta y = 62.5 \times 10^{-3}$ (bottom) $\Delta x = 39 \times 10^{-3}, \Delta y = 15.4 \times 10^{-3}$.

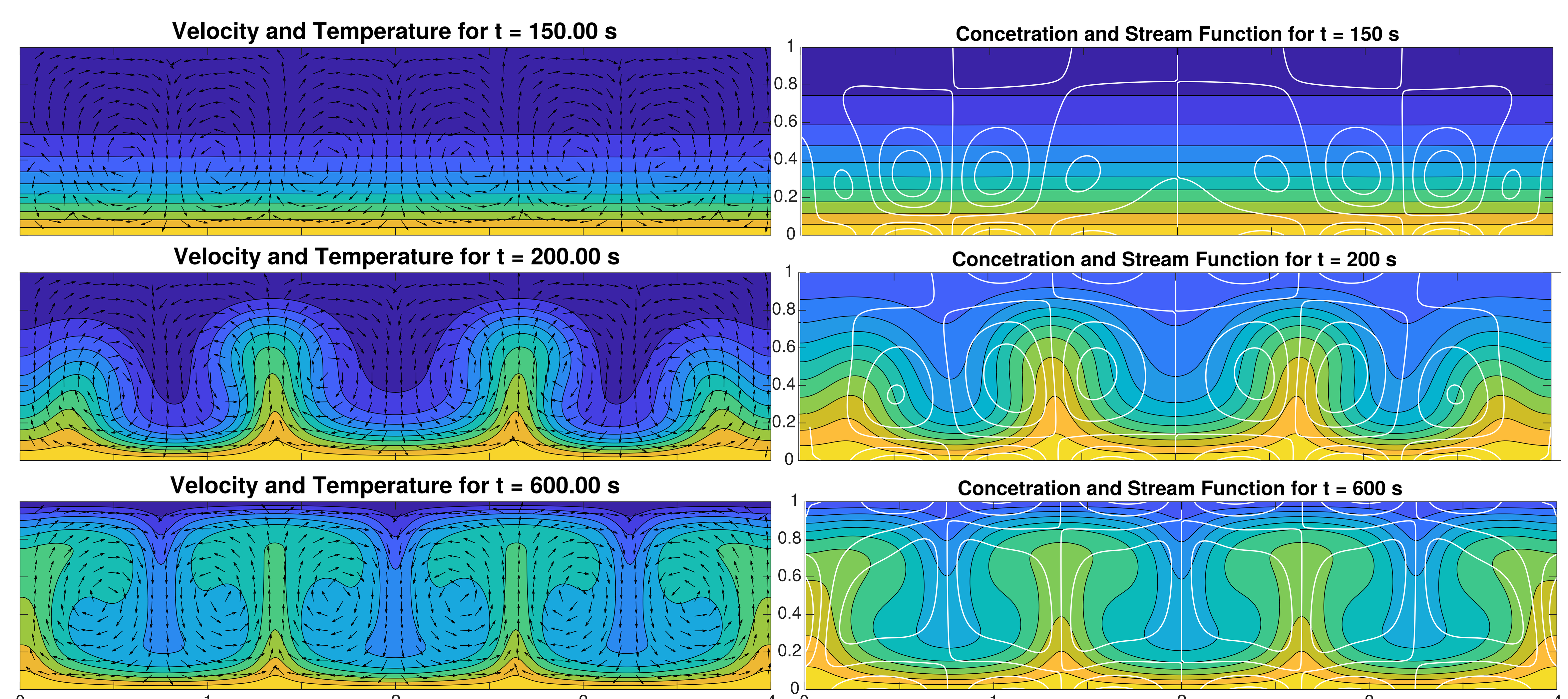

Figure 2:Cell behaviour

And the plankton?

Pythoplackton sinks in the ocean (passive swimmers), the temperature in the ocean varies upto 10C in a year, Is the convection what have this important individuals

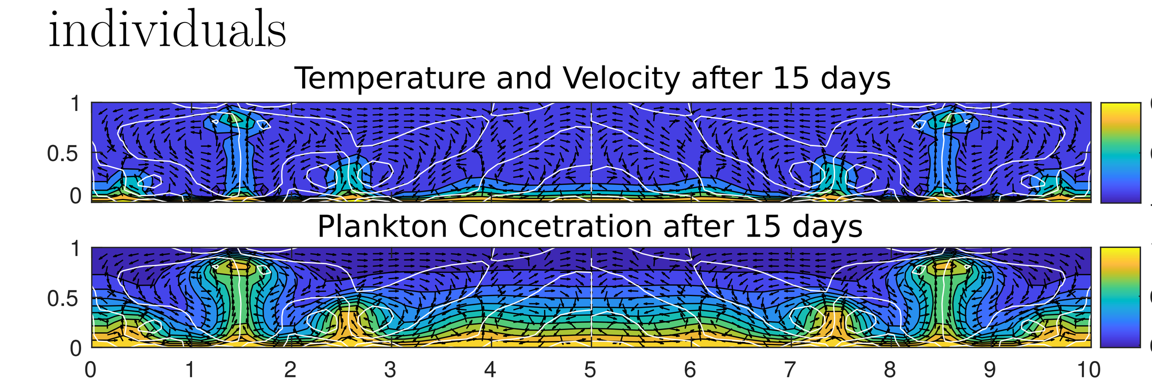

Figure 3:Cell behaviour

Final Thoughts!

Conclusions

- The number of cells depends on the geometry, aspect ratio, as well as the fluid parameters.
- The evidence of thermal cells depends on the size of the spatial discretization.
- (n.) factor $\rightarrow$ two multiplied factors give result

Pendings

- A solver using the projection method was considered, however, the stagered grid was an issue for the energy equation.
- An implentation using the second order FTBS scheme will help to deal with higher Reynolds numbers.
- The time convergence order might be improved using Crank Nicholson.

Numerical Convergence

Spatial Convergence order					
u	1.5339	v	1.3119	T	0.3145

Proof of Vieta's Formulas

The same we could do with another pattern, which state that $x_1 x_2 = \frac{c}{a}$, but proving this is going to be your task in next section.