Data Structures and Algorithms Lab Sessions

C. Martínez

November 9, 2019

Briefing

- We use the *Jutge* (`https://jutge.org`), a virtual learning environment for computer programming
- Use your institutional email (`@est.fib.upc.edu`) and the *Racó* password to access your *Jutge* account; you should have received an email to enroll in the course EDA Curs 2019/2020 Q1
- There is a list of programming exercises associated to each lab session

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- Access lecture notes, exams, problem sets, etc. from Teaching Material. Most of the documents and other materials are available in English; you might also want to check the Catalan version of the website for additional materials
- A brief cheatsheet on STL is available from the course webpage too
- We encourage you to take the Self-Assessment Lab Test. It has 11 questions, if you have problems to answer correctly too many of these or you do not understand the “solution” check with your lab prof
- A guide to programming style is available in Catalan at the website; an English translation is also available

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The Standard Template Library

- The Standard Template Library (STL, for short) is a fundamental component of the C++ Standard Library
- STL **is not** part of the C++ language, but virtually all, except the most trivial programs, use it
- The evolution of the STL has gone hand in hand with the evolution of C++; its design and use has shaped the language itself
- There are full specifications of the STL (functional and non-functional requirements); each compiler-vendor has freedom to implement it as they want as long as all requirements are met

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The Standard Template Library

- The STL provides a rich number of general-purpose algorithms and data structures: vectors, lists, queues, sorting, ...
- STL design and implementation heavily relies on templates (compile-time genericity) as opposed to inheritance (run-time genericity)
- STL offers highly-tuned, state-of-the-art implementations for common algorithms and data structures; check if it provides a solution to your problem, don't reinvent the wheel!

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Templates

Example

```
// Pre: v.size() > 0, T has a total order <
// Post: min(v) returns the minimum element in v
template <typename T>
T minimum(const vector<T>& v) {
  T the_min = v[0];
  for (int i = 1; i < v.size(); ++i)
     if (v[i] < the_min) the_min = v[i];
  return the_min;
}
```

Templates

Example

```
vector<double> w;
...
// instatiates minimum with T = double
double m = minimum(w);
vector<string> words = read_words();
// instatiates minimum with T = string
string lexmin = minimum(words);
```

Templates

Example

```
template <typename T>
class BoundedStack {
private:
  vector<T> cont;
  int num_elems;
public:
  const int DEFAULT_MAX_ELEMS = 100;
  BoundedStack(int max_elems = DEFAULT_MAX_ELEMS) :
       cont(max_elems), num_elems(0) {};
  ...
  void push(const T& x) {
    if (num_elems < max_elems) {
      cont[num_elems] = x;
      ++num_elems;
    } else { ... }
  }
  T pop() {
    if (num_elems > 0) {
      --num_elems;
      return cont[numelems];
    } else { // error, empty stack!
    }
  }
```

Templates

Example

```
BoundedStack<double> S1(100);
double x;
while (cin >> x) S1.push(x);
...
// creates a vector of 30 bounded stacks, each of size 10
vector< BoundedStack<int> > S2(30, 10);
...
for (int i = 0; i < 30; ++i) {
  cout << "Stack " << i << ":" << endl;
  while (not S2[i].empty())
     cout << S2[i].pop() << endl;
}
// creates a bounded stack of 30 vectors, each holding 10 ints
BoundedStack< vector<int> > S2(30, 10);
...
```

Containers, Iterators, Algorithms

The three fundamental concepts in the STL are:

1. Containers: a container is a collection or set of objects where we can perform different operations such as adding new objects, removing objects, examine the objects and perform some computation on each of them, etc.
2. Iterators: an iterator is an abstraction for a pointer to an object, they allow us to access, move around and *iterate* over the objects in a container
3. Algorithms: general-purpose algorithms to make some computation or update the contents of a container, e.g., copying, finding an element that satisfies a property, sorting, mapping a function on each object of a container, etc.

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Containers, Iterators, Algorithms

You are already familiar with several STL containers:

- `vector<T>`
- `list<T>` (doubly-linked lists)
- `stack<T>`
- `queue<T>`

Containers, Iterators, Algorithms

Typical operations included in (almost) all containers:

- `size`: returns the number of objects in the collection
- `push/push_back`: adds a new element at the end of the collection
- `pop/pop_back`: removes first/last element inserted
- `empty`: returns true iff the collection contains no object
- `begin`: returns an iterator to the first object in the container
- `end`: returns a fictious iterator past-the-end of the container

Using Containers and Iterators

Example

```
#include <algorithm>
#include <list>
...
string w;
list<string> L;
while (cin >> w) L.push_back(w);
vector<string> v(L.size());
int k = 0;
for (list<string>::const_iterator it = L.begin();
     it != L.end(); ++it) {
       // ++it advances the iterator to the successor
       // *it accesses the string it points to
   v[k] = *it;
   ++k;
}
// copy(L.begin(), L.end(), v.begin()) does the same as loop above
sort(v.begin(), v.end());
L.sort(); // sort(L.begin(), L.end()) does not work
```

Generic Algorithms with Iterators

Example

```
// Pre: [beg, end) contains at least one element, comp
// Post: min(v) returns the minimum element in range
template <typename Iterator,
          typename T, typename Comparator = less<T>>
T minimum(Iterator beg, Iterator end, Comparator smaller) {
  T the_min = *beg;
  for (Iterator it = beg; it != end; ++it)
  if (smaller(*it, the_min))
     the_min = *it;
  return the_min;
}

struct Person {
  int age;
  ...
};
bool is_younger(const Person& a, const Person& b) {
  return a.age < b.age;
}
...
vector<Person> P;
Person benjamin = minimum(v.begin(), v.end(), is_younger);
...
```

Using Containers and Iterators

- The new standard C++11 introduces a new handy syntax to iterate through a container:

```
vector<string> words;
...
for (string w : words) {
  // w iterates through all the values in the
  // vector 'words'
  cout << ' ' << w;
}
```

- C++11 also introduces the keyword **auto** which will be substituted by a typename deduced by the compiler from the context

```
// auto => list<string>::const_iterator
for (auto it = L.begin(); it != L.end(); ++it) {
   v[k] = *it;
   ++k;
}
// auto => string
for (auto w : words) {
  ...
}
```

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```

Priority Queues

- A priority queue is a collection of objects which can accessed in (descending) order of priority; each element is "identified" with its priority, so we access elements from largest to smallest.
- Technically speaking `priority_queue` is a container adaptor built on top of a container with random access iterators, e.g., `vector`
- Operations:

Method	Description	Method	Description
`empty`	returns true iff PQ is empty	`size`	returns number of elems
`pop`	remove elem of largest priority	`ctor()`	creates empty PQ
`push`	add new elem	`ctor(beg, end)`	creates PQ with elems in the range [beg,end)
`top`	return most prioritary elem		

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Priority Queues

Example

```
#include <queue>
struct ChemElement {
  string symbol;
  double atomic_weight;
}
bool smaller_weight(...) { return X.atomic_weight < Y.atomic_weight; }

// new initialization syntax in C++11
vector<ChemElement> AllChemElements = { {"H", 1.008}, {"He", 4.003},
                                        ..., {"U", 238.03}};

priority_queue<ChemElement, vector<ChemElement>, smaller_weight> PT;
for (auto e : AllChemElements)
PT.push(e);

while (not PT.empty()) { // print from highest atomic weight ("U")
                         // to lowest ("H")
  ChemElement e = PT.top(); PT.pop();
  cout << e.symbol << " (" << e.atomic_weight << ")" << endl;
}
// it is significantly more efficient to fill PT with:
priority_queue<ChemElement, vector<ChemElement>,
               smaller_weight> PT(AllChemElements.begin(),
                                  AllChemElements.end());
```

Priority Queues

- Priority queues are usually implemented in C++ with heaps (to be explained in detail in theory lectures).
- Cost of the operations:

Method	Cost	Method	Cost
`empty`	$O(1)$	`size`	$O(1)$
`pop`	$O(\log n)$	`ctor()`	$O(1)$
`push`	$O(\log n)$	`ctor(beg, end)`	$O(n)$
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- Notice that n insertions (with `push`) into an initially empty priority queue have cost $O(n \log n)$; inserting n elems with `priority_queue(beg, end)` has cost $O(n)$

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Sets

- A `set<T>` is a finite set of objects in which we can efficiently add new elements, remove exisiting elements and search if a given element is or not in the set. Moreover, we can iterate through all elements of the set in ascending order.
- Operations:

Method	Description	Method	Description
`empty`	returns true iff set is empty	`size`	returns number of elems
`erase(it)`	remove element pointed to by `it`	`ctor()`	creates empty set
`insert`	add new elem	`ctor(beg, end)`	creates set with elems in the range [beg,end)
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Sets

Example

```
#include <set>
#include <cstdlib>
...
set<int> generate_random_subset(int n, int k) {
  set<int> C;
  while (C.size() != k) {
    int r = rand() \% n + 1; // r = random number in 1..n
    C.insert(r); // does nothing if r already in C
  }
}
...
set<int> lotto = generate_random_subset(49, 6);
cout << "Winning numbers:" << endl;
// prints the k selected integers in ascending order
for (int x : lotto) {
  cout << ' ' << x;
  cout << endl;
...
```

Sets

- Sets are usually implemented in C++ with some variant of balanced search trees, for instance, red-black trees (they'll be explained in theory lectures).
- Cost of the operations:

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`insert`	$O(\log n)$	`ctor(beg, end)`	$O(n \log n)$
`begin`	$O(\log n)$	`end`	$O(1)$
`find`	$O(\log n)$		

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Intermezzo: Pairs

The STL provides the convenient class `pair`, which is used by several other classes and functions in order to input or output information.

Example

```
template <typename T1, typename T2>
struct pair {
  T1 first;
  T2 second;
};

pair<int,string> p = {3, "hello"};
pair<double, int> q = make_pair(3.14, 7);
cout << "(" << p.first << "," << p.second << ")";
```

Maps

- A `map<K,V>` is a finite set of pairs ⟨ key,value ⟩ such that no two pairs have the same key, in which we can efficiently add new pairs, remove exisiting pairs, update the value associated to a key and search for a key and retrieve its associated value if present. Moreover, we can iterate through all pairs in the map in ascending order of keys.
- Operations:

Method	Description	Method	Description
`empty`	returns true iff set is empty	`size`	returns number of elems
`erase(it)`	remove pair pointed to by `it`	`ctor()`	creates empty set
`insert(p)`	add new pair p=<k,v>	`ctor(beg, end)`	creates set with elems in the range [beg,end)
`begin`	returns iterator to pair with smallest key	`end`	returns iterator past-the-end
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Maps

Example

```
#include <map>
...
map<string,int> word_freqs;
string w;

while (cin >> w)
  ++word_freqs[w];

// print the list of words in the input
// in alphabetical order and their frequencies
for (auto p : word_freqs)
cout << p.first << ": " << p.second << endl;

auto it = word_freqs.find("abracadabra");
if (it != word_freqs.end()) {
    cout << "this was a magic text!" << endl;
    word_freqs.remove(it);
}
```

Maps

- Maps, like sets, are usually implemented in C++ with some variant of balanced search trees.
- Cost of the operations:

Method	Cost	Method	Cost
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`erase(it)`	$O(\log n)$	`ctor()`	$O(1)$
`insert`	$O(\log n)$	`ctor(beg, end)`	$O(n \log n)$
`begin`	$O(\log n)$	`end`	$O(1)$
`find`	$O(\log n)$	`operator[]`	$O(\log n)$

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More on Containers

- The containers discussed here offer a very rich set of methods, we have described here just a few
- `multiset`, `multimap`: like sets and maps, but duplicities (of elements, of keys) are allowed
- The order used in sets and maps can be changed by supplying a specific parameter when instatiating the container

```
set<double, greater<double> > S;
double x;
while (cin >> x) S.insert(x);
for (auto it = S.begin(); it != S.end(); ) {
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      if (*next - *it > 0.01)
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}
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- The newest standards (2011, 2014) introduce `unordered_set` and `unordered_map` (and the multi-$*$ variants). These offer almost the same functionality as `set`s and `map`s, except that iteration in order of elements/keys is not possible, but the average cost of insertions, deletions and searches is $O(1)$.
- For instance, our example of counting word frequencies has cost $O(N \log n)$, where n is the number of distinct words in the text and N the length of the text; if we replace `map` by `unordered_map` the average cost drops down to $O(N)$, but the output comes in no particular order
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Remarks on the Programming Assignments

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- Read carefully each exercise statement
- Identify which is the most suitable container to efficiently solve the problem
- Write a draft of your solution, do not care about the syntax at this point

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Resources

- A very handy and convenient on-line reference manual for C++ in general (and the STL in particular)
  `http://www.cplusplus.com/reference`
  Other documents, tutorials, etc. can also be found at `www.cplusplus.com`
- The most authoritative reference on the STL is the book "*The C++ Standard Library 2nd ed.*" by Nicolai M. Josuttis
- Another important reference for the C++ is "*The C++ Programming Language 3rd ed.*" by Bjarne Stroustrup (creator of C++)

Divide and Conquer

- Divide and Conquer is an algorithm design technique which allows us to develop very efficient solutions to some algorithmic fudamental problems
- It is also an important principle behind the design of some efficient data structures
- Examples: binary search, quicksort, quickselect, mergesort, fast multiplication, fast matrix multiplication, FFT, ...

Divide and Conquer


```
procedure DIVIDE_AND_CONQUER(x)
    if x is simple then
        return DIRECT_SOLUTION(x)
    else
        ⟨x_1, x_2, ..., x_k⟩ := DIVIDE(x)
        for :=1 to k do
            y_i := DIVIDE_AND_CONQUER(x_i)
        end for
        return COMBINE(y_1, y_2, ..., y_k)
    end if
end procedure
```

Sometimes, when $k = 1$, the algorithm is called a *reduction algorithm*.

Divide and Conquer

- Most D&C algorithms are recursive: reason about their correctness by induction
- Generalize the problem using embeddings (cat: *immersions*)
- Obtain the specification of the embedded function either by precondition strengthening or postcondition weakening (check your Programming 2 lecture notes, or my PRO2 lecture (in Catalan))

Divide and Conquer

Example

Write an efficient recursive function that returns the position of a value x in the sorted vector v, or the position after which x should be inserted to keep v in ascending order, if x is not in v.

```
// Pre:  v is sorted in ascending order
template <typename T>
int position(const vector<T>& v, const T& x);
// Post: i = position(v, x) => -1 <= i < n and v[i] <= x < v[i+1]
```

Divide and Conquer

Example

Embedding: solve the problem in the subvector $v[left..right]$

```
// Pre: v[left..right] is sorted in ascending order
// -1 <= left <= right-1, right <= n = v.size()
// v[left] <= x < v[right]
template <typename T>
int position_rec(const vector<T>& v, const T& x, int left, int right);
// Post: i = position(v, x, left, right) =>
//      (left <= i < right and  v[i] <= x < v[i+1])
```

N.B. Use the convention that $v[-1] = -\infty$ and $v[n] = +\infty$ to define the meaning of the logical expressions above

Divide and Conquer

Example

```
template <typename T>
int position_rec(const vector<T>& v, const T& x, int left, int right) {
  if (left == right-1) return left;
  // left < right-1
  int mid = (left + right) / 2;
  // left < mid < right
  if (v[mid] <= x)
    return position_rec(v, x, mid, right);
  else
    return position_rec(v, x, left, mid);
}
```

Some Useful C++ Features

- Use function overloading to advantage

Example

```
template <tyepname T>
int ÂpositionÂ(const vector<T>& v, const T& x,
               int left, int right) {
  ...
}

template <tyepname T>
int position(const vector<T>& v, const T& x) {
  return position(v, x, -1, v.size());
}
```

Some Useful C++ Features

- Use operator overloading for cleaner and user-friendly code

Example

```
typedef vector<double> Row;
typedef vector<Row> Matrix;
Matrix operator*(const Matrix& A, const Matrix& B) {
  ...
}
ostream& operator<<(ostream& os, const Matrix& A) {
  for (int i = 0; i < A.size(); ++i) {
    for (int j = 0; j < A[i].size(); ++j)
       os << (j > 0) ? " " : "" << A[i][j];
    os << endl;
  }
  return os;
}

int main() {
  Matrix A, B;
  ...
  cout << "Result = " << A * B << endl;
}
```

- Check, for instance, `en.cppreference.com/w/cpp/language/operators` for additional info

Divide and Conquer: Jutge

P81966	Dichotomic search	Binary search
P84219	First occurrence	
P33412	Resistant dichotomic search	
P34682	Fixed points	
X82938	Search in an unimodal vector	

Divide and Conquer: Jutge

P29212	Modular exponentiation	Fast exponentiation
P61833	Powers of a matrix	
P74219	Fibonacci numbers (2)	
X39049	Powers of permutations	
P58512	Interest rates	Bisection
P80595	How many inversions	Mergesort-like

Divide and Conquer: Jutge

1. Make sure to solve at least one problem from each block
2. Subdivide the “Search in an unimodal vector” (X82938) into two subproblems: finding the *peak*, then searching for x
3. Avoid trial & error: think carefullly about your pre- and postconditions and reason about correctness; this is valid in all cases, but will be specially useful in the “Binary search” block

Divide and Conquer: Jutge

4. The efficient computation of “Fibonacci numbers” (P74219) needs fast matrix exponentiation (P61833) as a previous step. Indeed, the key to solve the problem is to find 2×2 matrix A such that

$$A^n = \begin{pmatrix} F_n & F_{n-1} \\ F_{n-1} & F_{n-2} \end{pmatrix}$$

Divide and Conquer: Jutge

5. In “Interest rates” (P58512) compute (1) the amount of money $B(d)$ that must be returned to the bank if the money is lent for d days; (2) the amount of money $R(d)$ that must be returned to Prof. Oak if the money is lent for d days. Both functions are increasing, but $R(d) - B(d)$ is decreasing and so what we want is to find the smallest $d \geq 1$ such that $R(d) - B(d) < 0$. Try $d = 1$, $d = 2$, $d = 4$, $d = 8$, $\ldots$, until a value $d = 2^k$ such that $R(d) - B(d) < 0$ is found; find the solution by bisection in the range $[2^{k-1}, 2^k]$

Divide and Conquer: Jutge

- Some of the problems in the “Divide and Coqnuer” list have been used in past Computer Exams: “Fibonacci numbers (2)”, “Powers of permutations”, “Search in an unimodal vector”
- Other D&C problems in past Computer Exams:
  - Rightmost position of insertion (P54070)
  - Bi-increasing vector (P99753)

Graphs

There are many good graph libraries around, e.g.,

- The Boost Graph Library (BGL)
- Library of Efficient Datatypes and Algorithms (LEDA)
- LEMON
- ...

Graphs

For many of the algorithms in this course we make simplifying assumptions, for instance, $V = \{0, \ldots, n-1\}$ and no vertices are added or removed. We can thus use definitions such as

```
// for undirected unweighted graphs
typedef int vertex;
typedef vector<vertex> adjacency_list; // list<vertex> is also OK
typedef vector<adjacency_list> graph;
// if {u,v} is an edge in G, then G[u] contains v and G[v] contains u

// for directed weighted graphs
typedef int vertex;
typedef pair<int,double> edge;
typedef vector<edge> adjacency_list; // list<edge> is also OK
typedef vector<adjacency_list> weighted_digraph;
// if (u,v) is an edge in G with weight w, then
// G[u] contains the pair <v,w>
```

Graphs

- The novel syntax of C++11 allows us to write loops over all vertices or all successors of a vertex u quite nicely:

```
// for all successors of u in G
for(edge e : G[u]) {
  vertex v = e.first;
  double weight = e.second;
  ...
}
```

- A few preprocessor macros can also be helpful:

```
#define target(e) (e).first
#define weight(e) (e).second

// for all successors of u in G
for(edge e : G[u]) {
    vertex v = target(e);
    double w = weight(e);
    ...
}
```

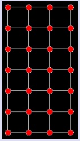
Graphs

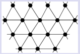

```
void dfs(const graph& G, vector<bool>& visited,
  vertex u, vertex father); // header of the recursive function

void dfs(const graph& G) {
  vector<bool> visited(G.size(), false);
  for (vertex u = 0; u < G.size(); ++u)
    if (not visited[u])
      dfs(G, visited, u, u);
}

void dfs(const graph& G, vector<bool>& visited,
         vertex u, vertex father) {
  visited[u] = true;
  // pre-visit of u
  for (vertex v : G[u]) {
    if (not visited[v])
      dfs(G, visited, v, u);
    else if (v != father) { // cycle!
         }
  }
  // post-visit of u
}
```

Grid graphs





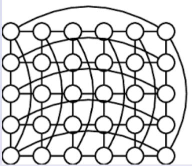
- In many problems we deal with finite subgraphs of the integer lattice on $\mathbb{Z}^2$.
- The set of vertices V is a finite subset

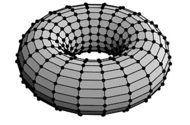
$$V \subset \{(i,j) \,|\, i \geq 0, j \geq 0\}$$

- Every element (i,j) is connected to four neighbors (up, left, down, right), to eight neighbors (as before + in diagonal), to six neighbors (hexagonal tiling), ...; E is a subset of all the connections

Grid graphs

- Sometimes the graph is circularly closed ($\rightarrow$ the grid is embedded in a cylinder or a torus in 3D)





- The notion easily generalizes to higher dimensional grids, more exotic "neighborhoods", etc.

Grid graphs

- Grid graphs are often implicitly represented with a matrix, instead of adjacency matrices or lists
- $G[i][j]$ indicates whether $(i, j) \in V$ or not
- The graph is often the induced subgraph of the lattice, induced by V
- Otherwise, $G[i][j]$ might contain a code of which edges belong to E, e.g., a 4-bit bitvector, $G[i][j].edge[k] = 1$ iff the k-th neighbor is connected to (i, j), $1 \leq k \leq 4$

Grid graphs

- DFS, BFS and all other graph algorithms are easily rewritten for grid graphs with the implicit matrix representation.
- Take for example problem P700690 Treasures in a map (1). For *maze* problems such as this, it is very convenient to assume that the graph lies in the rectangle defined by (1,1) and (n, m), and add rows 0 and $n+1$ and columns 0 and $m+1$ with *obstacles*

```
typedef vector <vector<char>> GridGraph;

GridGraph read_grid_graph(int n, int m){
  // n = nr. of rows, m = nr. of columns
  GridGraph c(n+2, vector<char>(m+2,'X'));
  for (int i = 1; i <= n; ++i)
     for (int j = 1; j <= n; ++j)
        cin >> c[i][j];

  return c;
}
```

Grid graphs


```
// DFS in a maze
bool dfs(const GridGraph& g, vector<vector<bool>>& visited,
         int i, int j) {
   visited[i][j] = true;
   if (g[i-1][j] != 'X' and not visited[i-1][j])
      dfs(g, visited, i-1, j); // up
   if (g[i][j+1] != 'X' and not visited[i][j+1])
      dfs(g, visited, i, j+1); // right
   if (g[i+1][j] != 'X' and not visited[i+1][j])
      dfs(g, visited, i+1, j); // down
   if (g[i][j-1] != 'X' and not visited[i][j-1])
      dfs(g, visited, i, j-1); // left
}
```

Priority queues and graph algorithms

- Many graph algorithms and other applications use priority queues, but they need them to support an operation to decrease the priority of an item (or to increase it).
- For example: Dijkstra's algorithm to find shortest paths, Prim's algorithm for minimum spanning trees
- One option is to implement our own PriorityQueue class—and there are alternatives to binary heaps which are extremely efficient when they have to decrease the priority of an item (e.g., Fibonacci heaps)

Priority queues and graph algorithms

- Decrease priority could be implemented as `delete` + `insert` ...but STL's `priority_queue` hasn't a method to delete a designated element
- We can use some lazy deletion scheme: when you extract an element from the queue, check if it was already processed before
- Since $V = \{0, \ldots, n-1\}$ a Boolean vector is enough (we can actually use the vector that marks visited nodes)
- If we "decrease" the priority of x by inserting (x, p'), where $p' < p$ (p is the current priority of x), then the item (x, p') will be extracted from the queue before (x, p)

Priority queues and graph algorithms

- Recall also that STL's `priority_queue` is a max-heap
- If elements are numbers, one can use changes of sign instead of using some explicit comparator
- Order among pairs is lexicographic: $\langle x, y\rangle < \langle x', y'\rangle$ iff $x < x'$, or $x = x'$ and $y < y'$
- In general we can supply a comparator function (or something equivalent):

```
#include <queue> // to use priority queues
#include <utility>
int main() {
  // a max-heap of int's ... but using
  // negative priorities it can be used as a min-heap
  priority_queue<int> P;

  // a min-heap of int's
  priority_queue<int,vector<int>,greater<int>> Q;

  // a min-heap of pairs <distance,vertex>, priority = distance
  typedef pair<double,vertex> pq_item;
  priority_queue<pq_item,vector<pq_item>,greater<pq_item>> PQ;
  ...
}
```

Priority queues and graph algorithms


```
const double INFINITY = numeric_limits<double>::max();

// Pre: no ngeative weights in G
// Post: for all u in G, D[u] = shortest distance from s to u
void Dijkstra(const weighted_graph& G, int s,
              vector<double>& D, ...) {
  priority_queue<pq_item,vector<pq_item>,greater<pq_item>> cand;
  vector<bool> visited(G.size(), false);
  for (vertex v = 0; v < G.size(); ++v) {
    cand.insert({INFINITY,v});
    D[v] = INFINITY;
  }
  cand.insert({0,s}); D[s] = 0.0;
  while(not cand.empty()) {
    pq_item p = cand.top(); cand.pop();
    double du = p.first; vertex u = p.second;
    if (not visited[u]) {
      visited[u] = true;
      for (auto e : G[u]) {
        // e is a pair <weight(u,v), v>
        vertex v = target(e);
        double dv = du + weight(e);
        if (dv < D[v]) {
          D[v] = dv;
          cand.insert({dv,v});
        }
      }
    }
  }
}
```

Priority queues and graph algorithms

- The lazy deletion strategy is a bit more expensive as the priority queue might contain spureous information (e.g., the same vertex with different distinct priorities, when only the smallest matters!)
- The number of items in `cand` with Dijkstra's original algorithm is always $\leq |V|$. But with lazy deletion, the priority queue may have up to $\approx |E|$ elements

Priority queues and graph algorithms

- The cost of Dijkstra's algorithm with a priority queue supporting `decrease_prio` in logarithmic time is $\mathcal{O}((|E| + |V|) \log |V|)$ since the visit of each edge and each vertex of the graph will trigger at most a constant number of calls to `insert/pop/decrease_prio`
- Since $\log |E| = \mathcal{O}(\log(|V|^2)) = \mathcal{O}(\log |V|)$ the asymptotic worst-case complexity of Dijkstra's algorithm with lazy deletion is the same as that of the original!