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(1). $n \equiv 0 \pmod{47}$
$3n \equiv 24 \pmod{31}$
$n \equiv 0 \pmod{5}$

se hace primero estos 2

$n = 47x$
$3n = 24 + 31y$
$\to$ $3n = 141x$
$3n = 24 + 31y$
$\to$ $141x - 31y = 24$.

$$\left[\begin{array}{c|cc} 141 & 1 & 0 \\ -31 & 0 & 1 \end{array}\right] \xrightarrow{E_1+4(2)} \left[\begin{array}{c|cc} 17 & 1 & 4 \\ -31 & 0 & 1 \end{array}\right] \xrightarrow{E_2+2(1)} \left[\begin{array}{c|cc} 17 & 1 & 4 \\ 3 & 2 & 9 \end{array}\right]$$

$$\xrightarrow{E_1-5(2)} \left[\begin{array}{c|cc} 2 & -9 & -41 \\ 3 & 2 & 9 \end{array}\right] \xrightarrow{E_2-(1)} \left[\begin{array}{c|cc} 2 & -9 & -41 \\ 1 & 11 & 50 \end{array}\right] \xrightarrow{E_1-2(1)} \left[\begin{array}{c|cc} 0 & -31 & -141 \\ 1 & 11 & 50 \end{array}\right] \xrightarrow[-1\cdot(2)]{E(1,2)} \left[\begin{array}{c|cc} 1 & 11 & 50 \\ 0 & +31 & +141 \end{array}\right]$$

$$\begin{bmatrix} 11 & 50 \\ +31 & +141 \end{bmatrix} \begin{bmatrix} 141 \\ -31 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Teorema de Bezout $\to$ $141 \cdot 11 \cdot 24 - 31 \cdot 50 \cdot 24 = 1 \cdot 24$
$141 \cdot (+31) \cdot t - 31 \cdot (+141) t = 0$

$x = 264 + 31t$
$y = 1200 + 141t$.

$3n = 24 + 31(1200 + 141t) = 37224 + 4371t$
$n = 12408 + 1457t \equiv 12408 \pmod{1457} \equiv 752 \pmod{1457}$.

$$\begin{array}{r|l} 12408 & 1457 \\ \hline 752 & 8 \end{array}$$

Ahora se hace

$n \equiv 0 \pmod{5}$
$n \equiv 752 \pmod{1457}$.
$\to$ $n = 5x$
$n = 752 + 1457y$
$\to$ $5x - 1457y = 752$.

$$\left[\begin{array}{c|cc} 5 & 1 & 0 \\ -1457 & 0 & 1 \end{array}\right] \xrightarrow{E_2+292(1)} \left[\begin{array}{c|cc} 5 & 1 & 0 \\ 3 & 292 & 1 \end{array}\right] \xrightarrow{E_1-(2)} \left[\begin{array}{c|cc} 2 & -291 & -1 \\ 3 & 292 & 1 \end{array}\right] \xrightarrow{E_2-(1)} \left[\begin{array}{c|cc} 2 & -291 & -1 \\ 1 & 583 & 2 \end{array}\right]$$

$$\xrightarrow{E_1-2(2)} \left[\begin{array}{c|cc} 0 & -1457 & -5 \\ 1 & 583 & 2 \end{array}\right] \xrightarrow[-1(2)]{E(1,2)} \left[\begin{array}{c|cc} 1 & 583 & 2 \\ 0 & 1457 & 5 \end{array}\right] \quad // \quad \begin{bmatrix} 583 & 2 \\ 1457 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -1457 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Teorema de Bezout.

$\downarrow$

$5 \cdot 583 \cdot 752 - 1457 \cdot 2 \cdot 752 = 1 \cdot 752 = 1 \cdot 752$
$5 \cdot 1457t - 1457 \cdot 5 \cdot t = 0$

$x = 438416 + 1457t$
$y = 1504 + 5t$

$$\begin{array}{r|l} 2142080 & 7285 \\ \hline 6580 & 300 \end{array}$$

$n = 5(438416 + 1457t) = 2192080 + 7285t \equiv 2192080 \pmod{7285}$
$\equiv 6580 \pmod{7285}$.

② $15^{143} \pmod{46}$ como $\text{mcd}(15,46)=1$, $15^{\varphi(46)} \equiv 1 \pmod{46}$

$\varphi(46) = \varphi(2\cdot 23) = (2-1)\cdot(23-1) = 22.$

$15\cdot 27 \equiv 45 \pmod{46}$
$15\cdot 41 \equiv 17 \pmod{46}$
$15^2 \equiv 41 \pmod{46}$
$41^2 \equiv 25 \pmod{46}$
$25^2 \equiv 27 \pmod{46}$

$15^{22} \equiv 1 \pmod{46}$ $\quad 143 \,|\, 22$, cociente $6$, resto $11$

$15^{143} = 15^{22\cdot 6+11} = (15^{22})^6 \cdot 15^{11} = 1^6 \cdot 15^{11} = 15\cdot(15^2)^5 = 15\cdot 15^{10} \equiv 15\cdot(15^2)^5 \equiv 15\cdot 41^5$

$= 15\cdot 41\cdot(41^2)^2 = 15\cdot 41\cdot 25^2 = 17\cdot 25^2 = 17\cdot 27 = 45 \pmod{46}$

③. Es $B_1 = \begin{bmatrix} 1 & 4 \\ 4 & 1 \\ 2 & 4 \end{bmatrix}$, $B_2 = \begin{bmatrix} 4 & 3 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ $\quad P_{B_1B_2} = B_2^{-1}\cdot B_1 = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 4 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}$

$B_2^{-1} = \left[\begin{array}{cc|ccc} 4 & 3 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{E_1+2(2)} \left[\begin{array}{cc|ccc} 1 & 2 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[E_3+4(1)]{E_2+4(1)} \left[\begin{array}{cc|ccc} 1 & 2 & 1 & 2 & 0 \\ 0 & 0 & 4 & 4 & 0 \\ 0 & 1 & 4 & 3 & 1 \end{array}\right] \xrightarrow{E(2,3)} \left[\begin{array}{cc|ccc} 1 & 2 & 1 & 2 & 0 \\ 0 & 1 & 4 & 3 & 1 \\ 0 & 0 & 4 & 4 & 0 \end{array}\right]$

$\xrightarrow{E_1+3(2)} \left[\begin{array}{cc|ccc} 1 & 0 & 3 & 1 & 3 \\ 0 & 1 & 4 & 3 & 1 \\ \hline 0 & 0 & 4 & 4 & 0 \end{array}\right]$ inversa por la izquierda de $B_2$

Demostrar que $B_1$ y $B_2$ son base. (Si el determinante de $B_1$ y $B_2$ es 0, son base de U).

$|B_1| = \begin{vmatrix} 1 & 4 \\ 4 & 1 \\ 2 & 4 \end{vmatrix} \underset{E_3+3(1)}{\overset{E_2+(1)}{=}} \begin{vmatrix} 1 & 4 \\ 0 & 0 \\ 0 & 1 \end{vmatrix} \overset{E_1+(3)}{=} \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$ si es base.

$|B_2| = \begin{vmatrix} 4 & 3 \\ 1 & 2 \\ 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$ si es base calcular base de $U$ ; $U = \begin{bmatrix} 1 & 2 \end{bmatrix}$

④.

$$M(f) = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \\ 3 & 3 & 2 & 2 \end{bmatrix}$$

Nucleo es anulador por la derecha de $M(f)$

La base del nucleo de $f$ ($\ker f$) $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$.

La imagen es generador por las columnas de $M(f)$. (pasamos de implícita a paramétrica.)

$$[M(f)^T \mid I] \to \left[\begin{array}{ccc|cccc} 1 & 1 & 3 & 1 & 0 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\text{filas}}{\overset{\text{red}}{\rightsquigarrow}} \left[\begin{array}{cc|ccccc} 1 & 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 & 3 \end{array}\right]$$

ec. paramétrica.

La ec paramétrica se reduce

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 4 & 3 \\ 2 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

entonces la base de la imagen es $= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 4 & 3 \\ 2 & 3 \end{bmatrix}$

(5). $V = \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 1 \\ 0 & 3 \end{bmatrix}$ ; $U = \begin{bmatrix} 2 & 0 & 2 & 3 & 2 \\ 1 & 1 & 3 & -1 & -1 \\ 2 & -1 & 3 & 2 & -1 \end{bmatrix}$

$U \cap V$ en implícitas.

U está en implícitas, calculo V en implícita.

~~V a implícita~~

$$\begin{bmatrix} 1 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\text{filas}]{\text{red}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

ec. implícita.

$$U \cap V = N\begin{pmatrix} 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 2 & 0 & 2 & 3 & 2 \\ 1 & 1 & 3 & -1 & -1 \\ 2 & -1 & 3 & 2 & -1 \end{pmatrix}$$

Esto sí.

Hay que sacar una base de V y U

Base de V (se reduce V)

$$\begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 1 \\ 0 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

como la reducción da 2 columnas pivote la base de V sería $= \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 2 & 4 \\ 3 & 1 \\ 0 & 3 \end{bmatrix}$

Base de U (se halla una ec. paramétrica) se pasa a traspuesta

$$\left[\begin{array}{ccc|ccccc} 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 2 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccccc} 1 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 2 & 1 \\ \hline 0 & 0 & 0 & 1 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 4 & 1 \end{array}\right]$$

$$\text{ec. paramétrica} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \\ 3 & 4 \\ 1 & 1 \end{bmatrix} \xrightarrow[\text{filas}]{\text{red}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(se reduce para sacar la base de U)

La base U es $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \\ 3 & 4 \\ 1 & 1 \end{bmatrix}$

Ahora se pasan las bases de V y U a implícitas.

Base de V $\rightarrow \left[\begin{array}{cc|ccccc} 1 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\text{filas}}{\overset{\text{red}}{\rightsquigarrow}} \left[\begin{array}{cc|ccccc} 1 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ \hline 0 & 0 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$

ec. implícita de la base de V

Base de U $\rightarrow \left[\begin{array}{cc|ccccc} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\text{filas}}{\overset{\text{red}}{\rightsquigarrow}} \left[\begin{array}{cc|ccccc} 1 & 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 0 & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 4 & 3 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 \end{array}\right]$

ec. implícita de la base de U

$$U \cap V = N\left(\begin{array}{ccccc} 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 4 & 3 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right)$$

⑥. $\mathbb{Z}_5$.

$$|A| = \begin{vmatrix} 4 & 2 & 2 & 2 & 1 \\ 1 & 0 & 3 & 1 & 1 \\ 1 & 3 & x+4 & 3 & 0 \\ x+4 & 3 & 4 & 1 & 3 \\ 1 & 1 & 4 & 3 & 3 \end{vmatrix} \underset{\substack{E_4+2(1) \\ E_5+2(1)}}{\overset{E_2+4(1)}{=}} \begin{vmatrix} 4 & 2 & 2 & 2 & 1 \\ 2 & 3 & 1 & 4 & 0 \\ 1 & 3 & x+4 & 3 & 0 \\ x+2 & 2 & 3 & 0 & 0 \\ 4 & 0 & 3 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 1 & 4 \\ 1 & 3 & x+4 & 3 \\ x+2 & 2 & 3 & 0 \\ 4 & 0 & 3 & 2 \end{vmatrix} \overset{E_1+1(1)}{=} \begin{vmatrix} 1 & 3 & 4 & 1 \\ 1 & 3 & x+4 & 3 \\ x+2 & 2 & 3 & 0 \\ 4 & 0 & 3 & 2 \end{vmatrix} =$$

$$\underset{E_4+3(1)}{\overset{E_2+2(1)}{=}} \begin{vmatrix} 1 & 3 & 4 & 1 \\ 3 & 4 & x+2 & 0 \\ x+2 & 2 & 3 & 0 \\ 2 & 4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 4 & x+2 \\ x+2 & 2 & 3 \\ 2 & 4 & 0 \end{vmatrix} \overset{E_2+(1)}{=} \begin{vmatrix} 3 & 4 & x+2 \\ x & 1 & x \\ 2 & 4 & 0 \end{vmatrix} = (0 + 4x\cdot(x+2) + 8x) - ((x+2)\cdot 2 + 12x + 0)$$

$$= (4x^2 + 8x + 8x) - (2x + 4 + 12x) = 4x^2 + 16x - 14x - 4 = 4x^2 + 2x - 4$$