Math Camp 2025 – Problem Set 7

Read the following problems carefully and justify all your work. Avoid using calculators or computers.

Let

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \qquad \boldsymbol{D} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad \boldsymbol{E} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 8 \end{pmatrix}$$

1. For each of them, identify whether the matrix is: square, symmetric, triangular, identity, $\mathbf{0}$, or none of the above.
2. Calculate $\text{tr}(\boldsymbol{A})$.
3. Calculate $5(\text{tr}(\boldsymbol{B}) + \text{tr}(\boldsymbol{E}))$.
4. Calculate $\det(\boldsymbol{C}), \det(\boldsymbol{D}), \det(\boldsymbol{E})$.

Now consider the following matrices:

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 5 \\ 1 & -2 & -1 \\ 5 & -1 & 2 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 6 & 3 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{D} = \begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix} \qquad \boldsymbol{E} = \begin{pmatrix} 0 & 1 & 2 \\ 5 & 1 & -1 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

1. Is $\boldsymbol{E}^\top \boldsymbol{E}$ square? symmetric? triangular?
2. Find the trace of $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$.

Invert the following matrices or give a reason why you cannot:

1. $\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$
2. $\begin{pmatrix} -1 & 3 \\ -2 & 6 \end{pmatrix}$
3. $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$
4. $\begin{pmatrix} 11 & 3 & 5 \\ 3 & 2 & 19 \\ 0 & 0 & 0 \end{pmatrix}$
5. $\begin{pmatrix} 3 & 8 & 6 \\ 0 & -3 & -5 \\ -9 & 0 & 4 \end{pmatrix}$

Find all the solutions to the following systems of linear equations:

1. $\begin{cases} -3x + 5y + 5z = -43 \\ x - 4y - 2z = 31 \\ 3x - 4z = 7 \end{cases}$

2. $\begin{cases} x - 2y - z = -15 \\ -x - y + z = -6 \\ x - 6y - z = -43 \end{cases}$