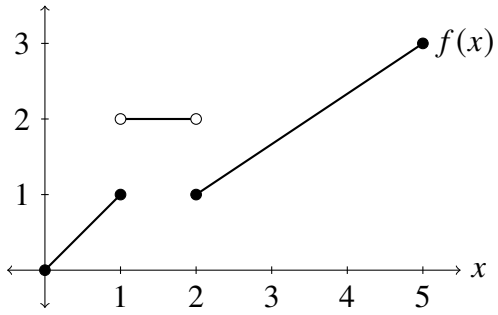
Math Camp 2025 – Problem Set 3

Read the following problems carefully and justify everything you do.

1. Limits. Find the following limits or show that they don't exist.

1. $\lim_{x\to+\infty} \frac{e}{x}$.
2. $\lim_{x\to-\infty} \frac{e}{x}$.
3. $\lim_{x\to 3} \frac{x}{x^3-27}$.
4. $\lim_{x\to 3} \frac{x-3}{x^3-27}$.
5. $\lim_{x\to\infty} \frac{x+1}{2x}$.
6. $\lim_{x\to\infty} \left(\frac{1}{2}\right)^x$.
7. $\lim_{x\to\infty} \frac{3x^3+2x^2-x+3}{4x^4+3x^3+2x^2+x+4}$.
8. $\lim_{x\to 0} \frac{1}{x^2}$.
9. (a) $\lim_{x\to 1} f(x)$.
   (b) $\lim_{x\to 2} f(x)$.
   (c) $\lim_{x\to 5} f(x)$.

   where $f : [0,5] \to \mathbb{R}$ is given by the following graph.



2. Continuity.

1. Consider $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x & \text{if } x \geqslant 1. \end{cases}$$

   Is it continuous?

2. The following functions are defined for all $x \in \mathbb{R}$ except for one point x_0. Find x_0 and determine if they can be defined at x_0 so that they are continuous on $\mathbb{R}$.

   (a) $f(x) = \dfrac{x-3}{x^3-27}$.

   (b) $f(x) = \dfrac{1}{x}$.

   (c) $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x & \text{if } x > 1. \end{cases}$.

   (d) $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } x > -1. \end{cases}$.