Math Camp 2025 – Problem Set 8

Read the following problems carefully and justify all your work. Avoid using calculators or computers.

Partial derivatives.

1. Standard regression models often look something like this:

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2$$

   (a) Find the partial derivatives of y with respect to x_1 and x_2.

   (b) Interpret both.

2. Find the first-order partial derivatives with respect to x, y, and z of $f(x, y, z) = xy^2 + yz^2$.

3. Find both second-order partial derivatives of $f(x, y) = x^2 y^2$.

4. Find the second-order partial derivative of $f(x, y) = \dfrac{x}{y} + e^{xy}$.

5. Find the first- and second-order partial derivatives of $f(x, y) = \log(x + \sqrt{y})$.

6. Find the first- and second-order partial derivatives of $f(x, y) = \dfrac{x^2 + y^2}{x^3 - 4xy - y^2}$.

7. Find the second-order partial derivatives of $f(x, y) = (2x + 3y)(e^{3x} + e^{2y})$.

8. Find the second-order partial derivatives of $f(x, y, z) = x^y \log(z) - y^3 x^2 z + 2yz - x + 1$.

9. Find the gradient vector and Hessian matrix for the following functions:

   (a) $f(x, y) = x \ln(y)$,

   (b) $f(x, y) = 3x + 4y^3$,

   (c) $f(x, y, z) = xy^2 + yz^2$,

   (d) $f(x, y) = \dfrac{3}{2}x^2 - 2xy - 5x + 2y^2 - 2y$.

Multiple Integrals. Calculate:

1. $\displaystyle\iint_{[0,1]^2} (xy - x^2 - y^2)\, dxdy$.

2. $\displaystyle\iint_{[1,2]^2} (x^2 + y^2)\, dxdy$.

3. $\displaystyle\iint_{D} (x + y)\, dxdy$, where $D = \{(x, y) \in \mathbb{R} : 0 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant 2x\}$.