

Math Camp 2025 – Problem Set 4

Read the following problems carefully and justify all your work. Avoid using calculators or computers.

1. Extrema. Find the local maxima and minima of the following functions.

1. $f(x) = \frac{1}{3}x^3$

2. $f(x) = \frac{x}{e^x}$

3. $f(x) = \frac{x^2 - 1}{x - 1}$

4. $f(x) = x^2(x - 1)$

5. $f(x) = e^{2x} + 3e^{-4x}$

6. $f(x) = xe^{2x}$

7. $f(x) = \log((3x - 1)^2)$

8. $f(x) = \frac{5^x}{5}$

9. $f(x) = (1 + x^2)^3$

10. $f(x) = h(g(x))$, where $h(x) = \log(x)$ and $g(x) = x^2$

2. Concavity. For each function above, identify the intervals on which the function is convex and those on which it is concave.

3. L'Hôpital's Rule. Compute the following limits.

1. $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

2. $\lim_{x \rightarrow 0} \frac{8^x - 4^x}{x^3 - x^2 - x}$

3. $\lim_{x \rightarrow +\infty} \frac{e^{2x}}{2x + 144}$

4. $\lim_{x \rightarrow +\infty} \frac{2 + \log(x)}{x^2 + 3}$

5. $\lim_{x \rightarrow 0} (x \log(x) - x)$

4. Applied Problem. Political scientists often employ rational choice theory to study politics. Political actors such as legislators are assumed to have goals, and to choose actions designed to achieve them. This is operationalized by defining an actor's *utility functions* and *feasible actions*, and determining which feasible action maximizes her utility.

For example, say a legislator i 's utility function u was defined by $u_i(c) = v - c^2$, where v is the legislator's vote share in an election, and c is the portion of her wealth the legislator spent on the campaign. That is, the legislator gains utility from gaining votes, but loses utility from spending her wealth to get them. Now say vote share was determined entirely by campaign spending such that $v = c$; what level of campaign spending maximizes the legislator's utility?