Math Camp 2025 – Problem Set 6

Read the following problems carefully and justify all your work. Avoid using calculators or computers.

1. Matrix Arithmetic. Consider the following vectors and matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 3 & 2 & -4 \\ -8 & 0 & 6 \end{pmatrix} \quad \boldsymbol{D} = \begin{pmatrix} 6 & -2 \\ -1 & 3 \\ -3 & 8 \end{pmatrix} \quad \boldsymbol{a} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

Complete the following operations or give a reason why you cannot:

1. $3\boldsymbol{a} - 2\boldsymbol{b}$
2. $\|\boldsymbol{b}\|$
3. $\|\boldsymbol{a} - \boldsymbol{b}\|$
4. $\|\boldsymbol{C}\boldsymbol{a}\|$
5. $\boldsymbol{C}\boldsymbol{D}$
6. $\boldsymbol{D}\boldsymbol{C}$
7. $\boldsymbol{a} \cdot \boldsymbol{b}$
8. $\boldsymbol{a}\boldsymbol{b}^\top$
9. $\boldsymbol{a}\boldsymbol{b}^\top\boldsymbol{B} - \boldsymbol{D}\boldsymbol{C}$
10. $\boldsymbol{b}\boldsymbol{D}$
11. $\boldsymbol{A}^\top\boldsymbol{A}$
12. $\boldsymbol{b}^\top\boldsymbol{D}$
13. $\boldsymbol{B}^2$

Optional. What is a general formula for $\boldsymbol{B}^n = \underbrace{\boldsymbol{B} \cdot \ldots \cdot \boldsymbol{B}}_{n \text{ times}}$?