Math Camp 2025 – Problem Set 1

Read the following problems carefully and justify everything you do. Avoid using calculators or computers.

1. Operations. Simplify the following expressions.

1. $\dfrac{3\times 4}{3-2}+\dfrac{4+3}{7}$
2. $(3\cdot 4)/(3-2)-(4+3)/7\cdot(2+10)/3$
3. $\displaystyle\sum_{k=1}^{3}\left(9+\sqrt{9^k}\right)$
4. $\displaystyle\prod_{x=1}^{5}(2x)$
5. $\displaystyle\sum_{k=1}^{n}k$
6. $\dfrac{2g+13}{3g}+\dfrac{4g-5}{4g}$
7. $\dfrac{\dfrac{w^3z^4}{(w+1)(z-3)}}{\dfrac{(wz)^3}{(w-2)(z-3)}}$
8. $\dfrac{\prod_{i=1}^{100}2^i}{\prod_{i=2}^{100}2^i}$
9. $\displaystyle\sum_{i=1}^{N}(5^i-5^{i-1}).$

2. Exponents and Logarithms. Simplify the following expressions assuming $x, a > 0$.

1. $x^2x^5+x^4x^3$
2. $\dfrac{x^8}{(x^4)^2}$
3. $\dfrac{x^8}{(x^8)^4}$
4. $\sqrt[3]{1000}$
5. $\sqrt[6]{1000000}$
6. $\sqrt[3]{1000000}$
7. $\log_{10}(2x^35x^8)$
8. $5\log(x)-\log(x^4)$
9. $\log_4(16)$
10. $\log\left(\displaystyle\prod_{i=1}^{n}(ae^{x_i})\right)$

3. Class Questions. Go back to the questions in Lecture 1, and make sure you can answer all of them. Write down your answers to **4.4**, **5.6**, **6** and **7.5**.

4. Application. The Cobb-Douglas production function relates labor (L) and capital (K) to production (Y), such that $Y = AK^{\beta}L^{\alpha}$. (The usefulness of such functions extends beyond economics; for example, Butler (2014) utilizes a Cobb-Douglas function when studying Congressional representation.) Consider that regression equations are often specified in a form such as

$$Y=\beta_0+\beta_1x_1+\cdots+\beta_kx_k+\epsilon$$

where Y is the outcome, β_0 is the intercept, $\beta_1, \ldots, \beta_k$ are coefficients, $x_1, \ldots, x_k$ are the independent variables, and ϵ is an error term. Without worrying about the error term, manipulate the Cobb-Douglas production function so that it is in such a form, where β and α are the coefficients.

Hint. A variable in a regression may actually be a "transformed" variable; for example, for various reasons a researcher with one independent variable x_1 may choose to estimate an effect β_1 using $Y = \beta_0 + \beta_1\sqrt{x_1}$ rather than $Y = \beta_0 + \beta_1 x_1$, though you should note the coefficient's interpretation is changed.