Math Camp 2025 – Problem Set 5

Read the following problems carefully and justify all your work. Avoid using calculators or computers.

1. Indefinite Integrals. Find the following indefinite integrals.

1. $\int (3x^3 + 2x^2 - e^x)\, dx$
2. $\int \frac{2x}{x^2}\, dx$
3. $\int \frac{1}{x^2}\, dx$
4. $\int 2x(x^2 - 64)^2\, dx$
5. $\int \frac{1}{x \log(x)}\, dx$
6. $\int (\exp(5x^3)x^2 - x + 2)\, dx$
7. $\int (10 - x)^{10}\, dx$

2. Definite and Improper Integrals. Calculate the following integrals.

1. $\int_4^5 2x\, dx$
2. $\int_{e^{\sqrt{2}}}^{e^2} \frac{\log(x)}{x}\, dx$
3. $\int_{-\infty}^0 e^x\, dx$
4. $\int_2^{+\infty} \frac{2x - 1}{(x^2 - x)^2}\, dx$
5. $\int_1^9 2y^5\, dy$
6. $\int_{-1}^0 (3x^2 - 1)\, dx$
7. $\int_{-1}^1 (14 + x^2)\, dx$

8. $\int_1^{-1} (14 + x^2)\, dx$

9. $\int_1^2 \frac{1}{x}\, dx$

10. $\int_1^2 \frac{1}{x^2}\, dx$

3. Integration by parts. Calculate the following integrals.

1. $\int \frac{\log(x)}{x^3}\, dx$

2. $\int x^2 e^x\, dx$

3. $\int_1^e x \log(x)\, dx$

4. $\int \frac{x^3}{(x^2 + 7)^2}\, dx$

5. $\int (\log(x))^2\, dx$