**Math Camp 2025 – Problem Set 2**

Read the following problems carefully and justify everything you do.

**1. Lines and Plots.**

1. Find the linear function $f(x) = ax + b$ that goes through the points $(-1, -3)$ and $(1, 1)$.

2. Say you were interested in the relationship between the amount of federal grant funds distributed by executive agencies in a jurisdiction and mean annual income. Suppose after collecting data and fitting a regression, you determined the relationship to be

$$Y = 2 + 0.5x,$$

where $Y$ is the amount of federal grants distributed in millions and $x$ is mean annual income in units of 1,000. Draw a graph showing this relationship for $x \in [0, 100]$ (it may be useful to use units of ten when labeling the axes). How much federal grant money is distributed to jurisdictions with a mean annual income of $25,000? $50,000? $100,000?

**2. Sets.**

1. Let $U = \{i \in \mathbb{N} : 0 < i < 11\}$, $A = \{1, 3, 5, 7\}$, and $B = \{i \in \mathbb{N} : 1 < i < 10\}$.

   (a) Find $A \cup B$.
   (b) Find $A \cap B$.
   (c) Depict these sets in a Venn diagram.

2. For any two sets $A$ and $B$, what if anything can we say about $B \setminus (B \setminus A)$?

   *Comment.* This can be hard. Try some examples: $B = \{1, 2, 3\}$, $A = \{1, 4\}$. Then $B = \{1, 2\}$, $A = \{3, 4\}$. Then $B = \{1, 2\}$, $A = \{2\}$. Can you spot the pattern? Can you prove it?

**3. Functions.**

1. Factor $-7\theta^2 + 21\theta - 14$.

2. Expand and simplify $(2x - 3)(5x + 7)$.

3. Factor $q^2 - 10q + 9$.

4. Factor and reduce $\dfrac{\beta - \alpha}{\alpha^2 - \beta^2}$.

5. Solve $15\delta + 45 - 5\delta = 36$.

6. Solve $0.30\Omega + 0.05 = 0.25$.

7. Solve $-4x^2 + 64 = 8x - 32$.

8. Complete the square and solve: $x^2 + 14x - 14 = 0$.

9. Complete the square and solve: $1/3y^2 + 2/3y - 16 = 0$.

   *Hint.* Get rid of the $1/3$ first.

10. Solve using the quadratic formula: $2x^2 + 5x - 7 = 0$.

11. Solve for $x$.

    (a) $x^2 = 1$
    (b) $(x - 1)(x + 2) = 0$
    (c) $3x^2 - 1 = 6x + 8$
    (d) $5 + 11x = -3x^2$
    (e) $\sqrt{4x + 13} = x + 2$
    (f) $10^{3x^2}10^x = 100$
    (g) $6x^2 - 6x - 6 = 0$
    (h) $5 + 11x = -3x^2$

12. Find the inverse of $f(x) = 5x - 2$.

13. Simplify $g(f(x))$, where $f(x) = x^2 + 2$ and $g(x) = \sqrt{x - 4}$.

14. Simplify $f(g(x))$ with the same $f$ and $g$. Is it the same as before?