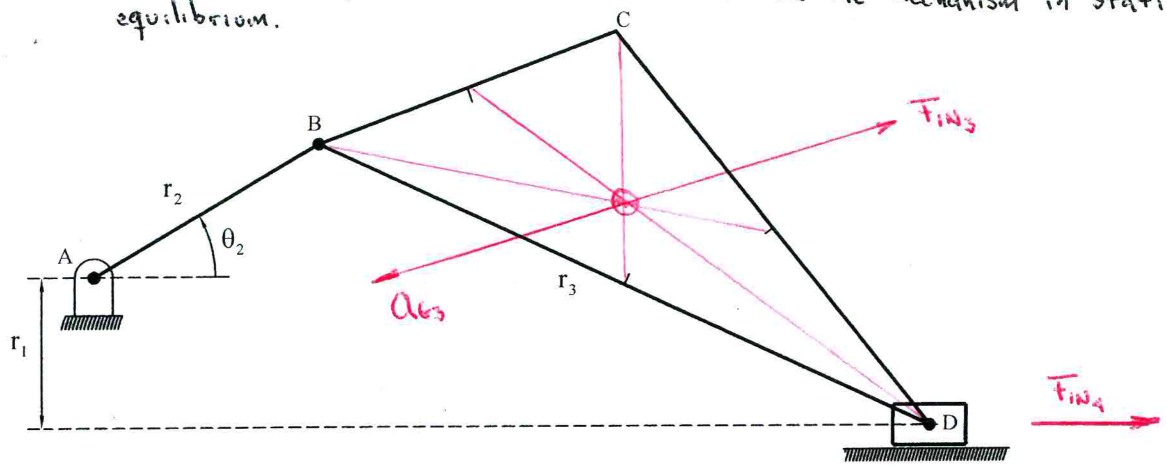
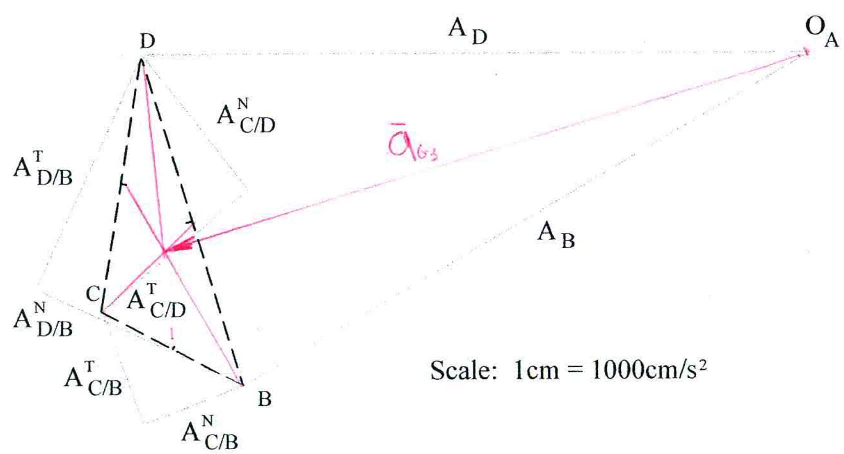
Example 4.8 Consider the slider-crank mechanism of examples 3.6 (Velocity), 3.8 (Acceleration), and 3.11 (Analytical), where $r_1 = 2cm$, $r_2 = 3.5cm$, $r_3 = 9cm$, $\theta_2 = 30°$, and $\dot{\theta}_2 = 50\ rad/s$ (constant). The acceleration polygon is provided and $\bar{a}_D = 8960\ cm/s^2$ and $\ddot{\theta}_3 = 378\ rad/s^2$ are calculated. Assume the inertia caused by link 2 very small, while $m_3 = 300gr$, $m_4 = 150gr$, and $I_{G3} = 2.5 \times 10^{-3}\ Kg \cdot m^2$, are the properties of links 3 and 4. Determine the torque required at link 2 to maintain the mechanism in static equilibrium.





Scale: 1cm = 1000cm/s²

$a_D = 8.9\ cm \equiv 8900\ cm/s^2$

$\bar{a}_{G3} = 9\ cm$
$= 9000\ cm/s^2$

$\ddot{\theta}_3 = 378\ rad/s^2$

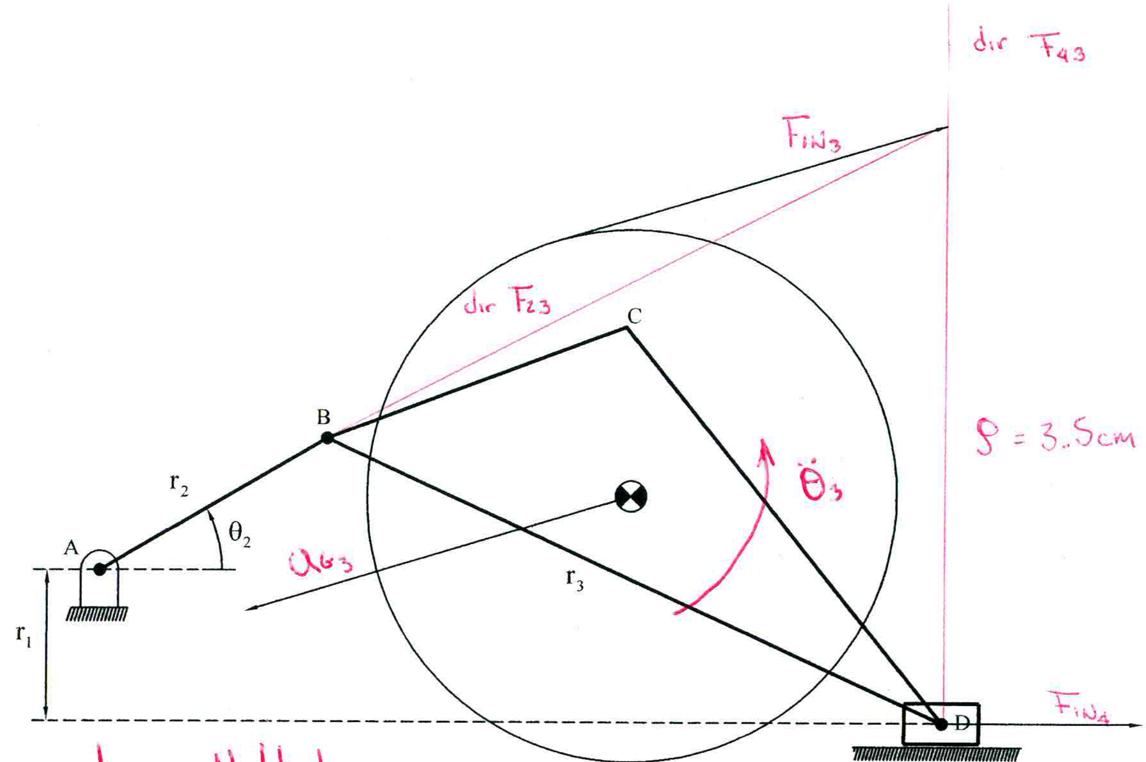
$$\bar{F}_{IN3} = -m_3 \bar{a}_{G3} = -0.3(90) = 27\ N$$

$$M_{IN3} = -I_{G3} |\ddot{\theta}_3| = -2.5 \times 10^{-3} (378) = 0.945\ N \cdot m$$

$$\delta = \frac{|M_{IN}|}{|F_{IN}|} = \frac{0.945}{27} = 0.035m \equiv 3.5cm$$

$$\bar{F}_{IN4} = -m_4 \bar{a}_{G4} = -0.15(-8900) = 13.35\ N$$

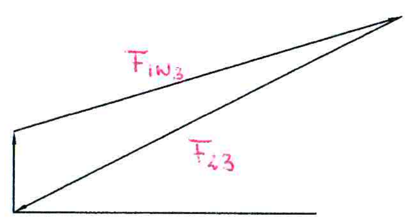
Using the Inertia Circle



Superposition Method

Only F_{IN_3}

FBD link 3



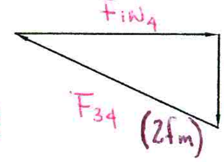
Scale: 1cm = 5 N

$F_{IN_3} = 27N \equiv 5.4N$

Only F_{IN_4}

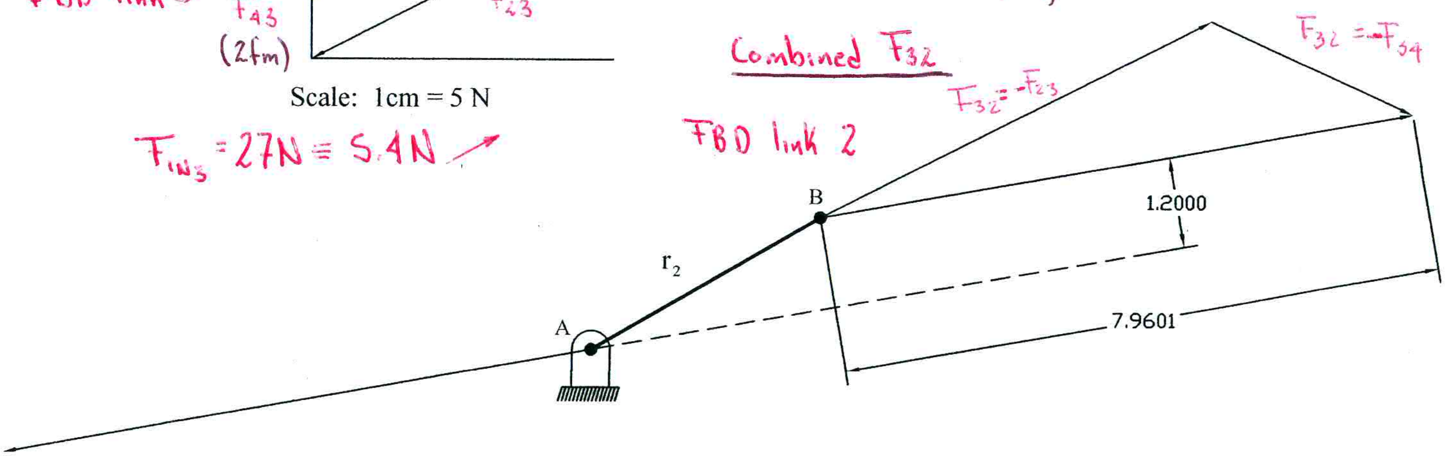
$F_{IN_4} = 13.35N$
$= 2.7cm$

FBD link 4



Combined F_{32}

FBD link 2

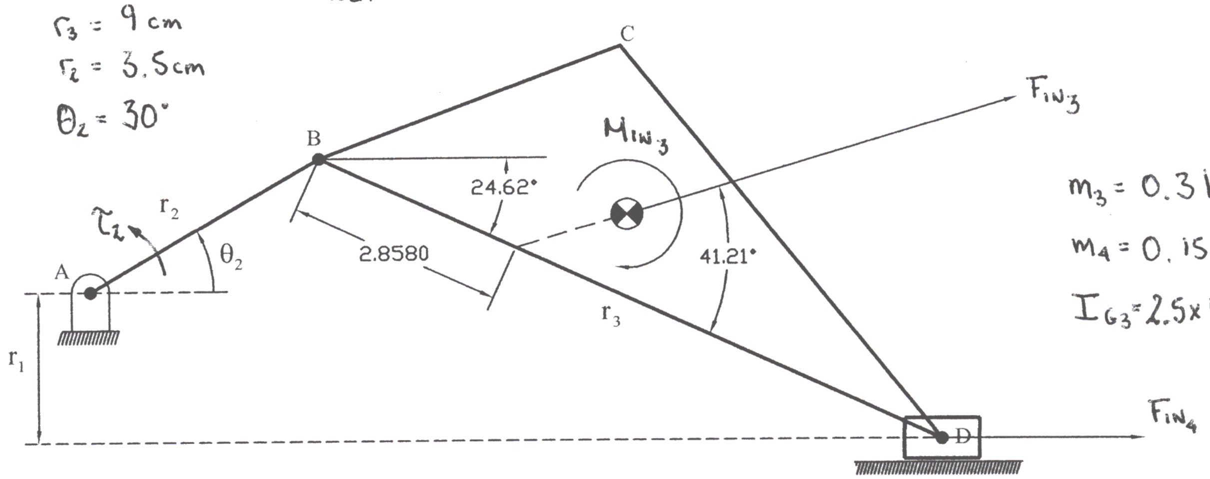


$\tau = F_{32} \cdot d = 39.8 \cdot (0.012) = 0.4776 \text{ N}\cdot\text{m}$

Example 4.9 Consider the slider-crank mechanism of example 4.8

Determine the required torque τ_2 using the analytical superposition method.

$r_3 = 9$ cm
$r_2 = 3.5$ cm
$\theta_2 = 30°$



$m_3 = 0.3$ Kg
$m_4 = 0.15$ Kg
$I_{G_3} = 2.5 \times 10^{-3}$ Kg·m²

From the acceleration analysis, we know $\bar{a}_{G_3} = 90\ m/s^2 \swarrow$, $\bar{a}_D = 89\ m/s^2 \leftarrow$, and $\ddot{\theta}_3 = 378\ rad/s^2$ CCW. Thus, the equations of motion are:

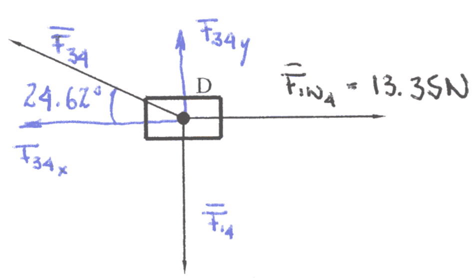
$$\bar{F}_{IN_3} = -m_3\,\bar{a}_{G_3} = -0.3(-90) = 27\text{ N} \nearrow$$

$$M_{IN_3} = -I_{G_3}\,\ddot{\theta}_3 = -2.5\times10^{-3}(378) = 0.945\text{ N·m}$$

$$\bar{F}_{IN_4} = -m_4\,\bar{a}_{G_4} = -0.15(-89) = 13.35\text{ N} \longrightarrow$$

Free-body diagram of Link 4

Link 4

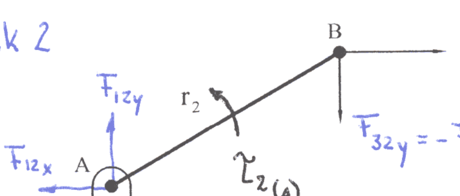


$\sum F_x = 0$ $\quad F_{34x} = -13.35$ N $\longleftarrow$

$F_{34y} = F_{34x}\tan(24.62) = 6.12$ N $\uparrow$

Link 3 is a two force member

Link 2



$F_{32x} = -F_{34x} = 13.35$ N $\longrightarrow$

$F_{32y} = -F_{34y} = 6.12$ N $\downarrow$

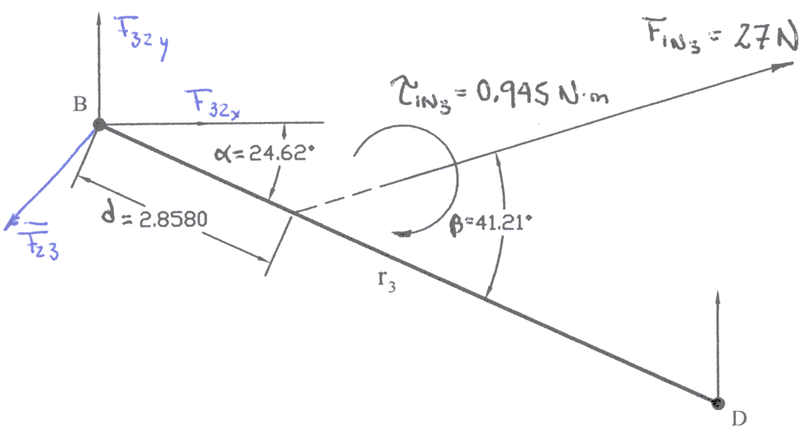
$\sum M_A = 0$

$\tau_2 - F_{32y}\,r_2\cos\theta_2 - F_{32x}\,r_2\sin\theta_2 = 0$

$\tau_{2(4)} = 6.12(0.035)\cos(30) + 13.35(0.035)\sin(30)$

$\boxed{\tau_{2(4)} = 0.419\text{ N·m}}$

Free-body diagram of Link 3



Link 4 is a two force member

Link 3

$$\sum M_B = 0$$

$$-\tau_{IN_3} + F_{IN_3}\, d \sin\beta + F_{43}\, r_3 \cos\alpha = 0$$

$$F_{43} = \frac{\tau_{IN_3} - F_{IN_3}\, d \sin\beta}{r_3 \cos\alpha}$$

$$= \frac{0.945 - 27(0.0286)\sin(41.21)}{(0.09)\cos(24.62)}$$

$$= 5.332 \text{ N} \uparrow$$

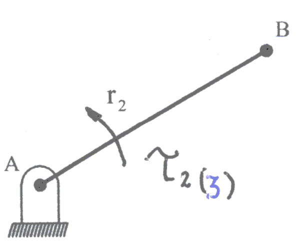
$$\sum F_y = 0$$

$$F_{23y} + F_{43y} + F_{IN_3} \sin(\beta - \alpha) = F_{23y} + 0.5332 + 27\sin(41.21 - 24.62) = 0$$

$$\therefore\ F_{23y} = -13.04 \text{ N}$$

$$\sum F_x = 0 \qquad F_{23x} + F_{IN_3}\cos(\beta - \alpha) = 0 \implies F_{23x} = -27\cos(41.21 - 24.62)$$

$$= -25.876 \text{ N}$$



Link 2

$$\sum M_A = 0 \qquad \tau_{2(3)} + F_{32y}\, r_2 \cos\theta_2 - F_{32x}\, r_2 \sin\theta_2 = 0$$

$$\tau_{2(3)} = 25.876(0.035)\sin(30) - 13.04(0.035)\cos(30)$$

$$\boxed{\tau_{2(3)} = 0.0575 \text{ N·m}}$$

TOTAL TORQUE

$$\tau_2 = \tau_{2(4)} + \tau_{2(3)} = 0.419 + 0.0575 = 0.477 \text{ N·m}$$

$$\boxed{\tau_2 = 0.477 \text{ N·m}}$$