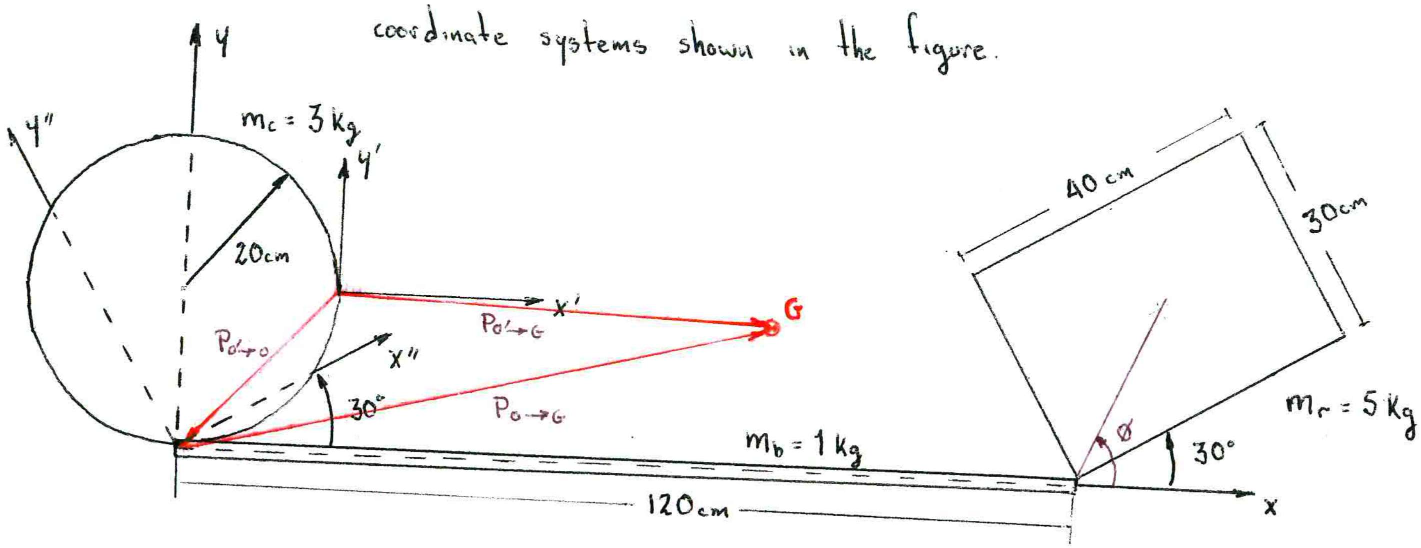
Example 4.6 Determine the location of the centre of mass of the following rigid body with respect to the three coordinate systems shown in the figure.



$$\emptyset = \arctan\left(\frac{30}{40}\right) + 30 = 66.87^\circ$$

Frame O

$$x_G = \frac{\sum_{i=1}^{3} m_i x_{Gi}}{\sum_{i=1}^{3} m_i} = \frac{1(60) + 3(0) + 5(120 + 25\cos(66.87))}{1 + 3 + 5} = 78.79 \text{ cm}$$

$$y_G = \frac{\sum_{i=1}^{3} m_i y_{Gi}}{\sum_{i=1}^{3} m_i} = \frac{1(0) + 3(20) + 5(25\sin(66.87))}{9} = 19.44 \text{ cm}$$

Frame O'

$$x_G = \frac{1(40) + 3(-20) + 5(100 + 25\cos(66.87))}{9} = 58.79 \text{ cm}$$

$$y_G = \frac{1(-20) + 3(0) + 5(25\sin(66.87) - 20)}{9} = -0.56 \text{ cm}$$

or using the solution found with frame O.

$$\bar{P}_{o' \to G} = \bar{P}_{o' \to o} + \bar{P}_{o \to G} = \begin{bmatrix} -20 \\ -20 \end{bmatrix} + \begin{bmatrix} 78.79 \\ 19.44 \end{bmatrix} = \begin{bmatrix} 58.79 \\ -0.56 \end{bmatrix} \begin{matrix} \to x_G \\ \to y_G \end{matrix}$$

Frame O''

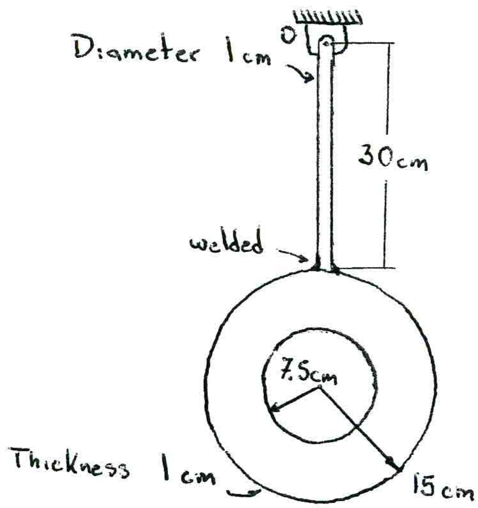
$$x_G = \frac{1(60\cos(-30)) + 3(20\cos(60)) + 5(120\cos(-30) + 20)}{9} = 77.95 \text{ cm}$$

$$y_G = \frac{1(60\sin(-30)) + 3(20\sin(60)) + 5(120\sin(-30) + 15)}{9} = -22.56 \text{ cm}$$

or using the solution found with frame O

$${}^{o''}_{o}R = \begin{bmatrix} \cos(30) & \sin(30) \\ -\sin(30) & \cos(30) \end{bmatrix} \qquad {}^{o''}P_{o \to G} = {}^{o''}_{o}R\, {}^{o}\bar{P}_{o \to G} = \begin{bmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 78.79 \\ 19.44 \end{bmatrix} = \begin{bmatrix} 77.95 \\ -22.56 \end{bmatrix} \begin{matrix} \to x_G \\ \to y_G \end{matrix}$$

Example 4.7 The pendulum shown in the figure below is suspended from point O. It consists of a slender rod connected to a disk with a hole in the middle. Both elements are made of the same material, $\rho = 8000\ kg/m^3$. Determine the required torque by the motor attached at O to achieve an angular acceleration of $\alpha = 50\ rad/s^2$.



slender-bar

$$m_b = \rho_b V_b = 8000\left(\pi (0.005)^2 (0.3)\right) = 0.1885\ Kg$$

$$I_{b_o} = \frac{1}{12} m l^2 + m\left(x_{G_b}^2 + y_{G_b}^2\right)$$

$$= \frac{1}{12}(0.1885)(0.3)^2 + (0.1885)(0.15)^2 = 0.00565\ Kg\,m^2$$

disk

$$m_d = \rho_d V_d = 8000\left(\pi (0.15)^2 (0.01)\right) = 5.655\ Kg$$

$$I_{d_o} = \frac{1}{2} m_d r_d^2 + m_d\left(x_{G_d}^2 + y_{G_d}^2\right)$$

$$= \frac{1}{2}(5.655)(0.15)^2 + 5.655\,(0.45)^2 = 1.21\ Kg\cdot m^2$$

Hole

$$m_h = \rho_h V_h = 8000\left(\pi (0.075)^2 (0.01)\right) = 1.41\ Kg$$

$$I_{h_o} = \frac{1}{2}(1.41)(0.075)^2 + 1.41\,(0.45)^2 = 0.29\ Kg\cdot m^2$$

The total inertia is

$$I_o = I_{b_o} + I_{d_o} - I_{h_o} = 0.924\ Kg\cdot m^2$$

$$\tau_M = M_o = I_o \alpha = (0.924)(50) = 46.2\ N\cdot m$$