Contents

4 Dynamic Force Analysis

For a kinematic chain it is important to know how forces and moments are transmitted from the input to the output, so that the links can be properly designated. The friction effects are assumed to be negligible in the force analysis presented here.

4.1 Equation of Motion for General Planar Motion

Consider a system of N particles. A particle is an object whose shape and geometrical dimensions are not significant to the investigation of its motion. An arbitrary collection of matter with total mass m can be divided into N particles, the ith particle having mass, m_i [Fig. 4.1(a)] $m = \sum_{i=1}^{N} m_i$.

A rigid body can be considered as a collection of particles in which the number of particles approaches infinity and in which the distance between any two points remains constant. As N approaches infinity, each particle is treated as a differential mass element, $m_i \to dm$, and the summation is replaced by integration over the body $m = \int_{\text{body}} dm$.

The position of the mass center of a collection of particles is defined by

$$\mathbf{r}_C = \frac{1}{m}\sum_{i=1}^{N} m_i \mathbf{r}_i, \tag{4.1}$$

where $\mathbf{r}_i = \mathbf{r}_{OP_i} = \mathbf{r}_{P_i}$ is the position vector from the origin O to the ith particle. The particle i is located at the point P_i.

As $N \to \infty$, the summation is replaced by integration over the body

$$\mathbf{r}_C = \frac{1}{m}\int_{\text{body}} \mathbf{r}\, dm, \tag{4.2}$$

where $\mathbf{r}$ is the vector from the origin O to differential element dm.

The time derivative of Eq. (4.1) gives

$$\sum_{i=1}^{N} m_i \frac{d^2\mathbf{r}_i}{dt^2} = m\frac{d^2\mathbf{r}_C}{dt^2} = m\mathbf{a}_C, \tag{4.3}$$

where $\mathbf{a}_C$ is the acceleration of the mass center. The acceleration of the mass center can be related to the external forces acting on the system. This relationship is obtained by applying Newton's laws to each of the individual particles in the system. Any such particle is acted on by two types of forces. One type is exerted by other particles that are also part of the system. Such forces are called internal forces (internal to the system). Additionally, a particle can be acted on by a force that is exerted by a particle or object not included in the system. Such a force is known as an external force (external to the system). Let $\mathbf{f}_{ij}$ be the internal force exerted on the jth particle by the ith particle. Newton's third law (action and reaction) states that the jth particle exerts a force on the ith particle of equal magnitude, and opposite direction, and collinear with the force exerted by the ith particle on the jth particle [Fig. 4.1(a)] $\mathbf{f}_{ji} = -\mathbf{f}_{ij},\ j \neq i$. Newton's second law for the ith particle must include all of the internal forces exerted by all of the other particles in the system on the ith particle, plus the sum of any external forces exerted by particles, objects outside of the system on the ith particle

$$\sum_j \mathbf{f}_{ji} + \mathbf{F}_i^{ext} = m_i \frac{d^2\mathbf{r}_i}{dt^2}, \quad j \neq i, \tag{4.4}$$

where $\mathbf{F}_i^{ext}$ is the external force on the ith particle. Equation (4.4) is written for each particle in the collection of particles. Summing the resulting equations over all of the particles from $i = 1$ to N the following relation is obtained

$$\sum_i \sum_j \mathbf{f}_{ji} + \sum_i \mathbf{F}_i^{ext} = m\mathbf{a}_C, \quad j \neq i. \tag{4.5}$$

The sum of the internal forces includes pairs of equal and opposite forces. The sum of any such pair must be zero. The sum of all of the internal forces on the collection of particles is zero (Newton's third law) $\sum\limits_i \sum\limits_j \mathbf{f}_{ji} = \mathbf{0}, \quad j \neq i$. The term $\sum\limits_i \mathbf{F}_i^{ext}$ is the sum of the external forces on the collection of particles $\sum\limits_i \mathbf{F}_i^{ext} = \mathbf{F}$. The sum of the external forces acting on a closed system equals the product of the mass and the acceleration of the mass center

$$m\,\mathbf{a}_C = \mathbf{F}. \tag{4.6}$$

Considering Fig. 4.2(b) for a rigid body and introducing the distance $\mathbf{q}$ in Eq. (4.2) gives

$$\mathbf{r}_C = \frac{1}{m} \int\limits_{\text{body}} \mathbf{r}\, dm = \frac{1}{m} \int\limits_{\text{body}} (\mathbf{r}_C + \mathbf{q})\, dm = \mathbf{r}_C + \frac{1}{m} \int\limits_{\text{body}} \mathbf{q}\, dm. \tag{4.7}$$

It results

$$\frac{1}{m}\int_{\text{body}} \mathbf{q}\, dm = \mathbf{0}, \tag{4.8}$$

that is the weighed average of the displacement vector about the mass center is zero. The equation of motion for the differential element dm is

$$\mathbf{a}\, dm = d\mathbf{F},$$

where $d\mathbf{F}$ is the total force acting on the differential element. For the entire body

$$\int_{\text{body}} \mathbf{a}\, dm = \int_{\text{body}} d\mathbf{F} = \mathbf{F}, \tag{4.9}$$

where $\mathbf{F}$ is the resultant of all forces. This resultant contains contributions only from the external forces, as the internal forces cancel each other. Introducing Eq. (4.7) into Eq. (4.9), the Newton's second law for a rigid body is obtained

$$m\, \mathbf{a}_C = \mathbf{F}$$

The derivation of the equations of motion is valid for the general motion of a rigid body. These equations are equally applicable to planar and three-dimensional motions.

Resolving the sum of the external forces into cartesian rectangular components

$$\mathbf{F} = F_x\, \mathbf{\imath} + F_y\, \mathbf{\jmath} + F_z\, \mathbf{k},$$

and the position vector of the mass center

$$\mathbf{r}_C = x_C(t)\, \mathbf{\imath} + y_C(t)\, \mathbf{\jmath} + z_C(t)\, \mathbf{k},$$

Newton's second law for the rigid body is

$$m\ddot{\mathbf{r}}_C = \mathbf{F}, \tag{4.10}$$

or

$$m\ddot{x}_C = F_x,\;\; m\ddot{y}_C = F_y,\;\; m\ddot{z}_C = F_z. \tag{4.11}$$

Angular Momentum Principle for a System of Particles

The total angular momentum of the system of N particles [Fig. 4.1(a)] about its mass center C, is the sum of the angular momenta of the particles about C

$$\mathbf{H}_C = \sum_{i=1}^{N} \mathbf{r}_{CP_i} \times m_i \mathbf{v}_i, \tag{4.12}$$

where $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt}$ is the velocity of the particle P_i.

The total angular momentum of the system about O is the sum of the angular momenta of the particles

$$\mathbf{H}_O = \sum_{i=1}^{N} \mathbf{r}_i \times m_i \mathbf{v}_i = \sum_{i=1}^{N} (\mathbf{r}_C + \mathbf{r}_{CP_i}) \times m_i \mathbf{v}_i = \mathbf{r}_C \times m\mathbf{v}_C + \mathbf{H}_C, \tag{4.13}$$

or the total angular momentum about O is the sum of the angular momentum about O due to the velocity $\mathbf{v}_C$ of the mass center of the system and the total angular momentum about the mass center.

The rate of change of the angular momentum about O equals the sum of the moments about O due to external forces and couples

$$\frac{d\mathbf{H}_O}{dt} = \sum \mathbf{M}_O. \tag{4.14}$$

Using Eqs. (4.13) and (4.14), the following result is obtained

$$\sum \mathbf{M}_O = \frac{d}{dt}(\mathbf{r}_C \times m\mathbf{v}_C + \mathbf{H}_C) = \mathbf{r}_C \times m\mathbf{a}_C + \frac{d\mathbf{H}_C}{dt}, \tag{4.15}$$

where $\mathbf{a}_C$ is the acceleration of the mass center.

With the relation

$$\sum \mathbf{M}_O = \sum \mathbf{M}_C + \mathbf{r}_C \times \mathbf{F} = \sum \mathbf{M}_C + \mathbf{r}_C \times m\mathbf{a}_C,$$

Eq. (4.15) becomes

$$\frac{d\mathbf{H}_C}{dt} = \sum \mathbf{M}_C. \tag{4.16}$$

The rate of change of the angular momentum about the mass center equals the sum of the moments about the mass center.

Equations of Motion

An arbitrary rigid body with the mass m can be divided into N particles P_i, $i = 1, 2, ..., N$. The mass of the P_i particle is m_i. Figure 4.2(a) represents the rigid body moving with general planar motion in the (x, y) plane. The origin of the cartesian reference frame is O. The mass center C of the rigid body is located in the plane of the motion, $C \in (x, y)$. Let $d_O = Oz$ be the axis through the fixed origin point O that is perpendicular to the plane of motion of the rigid body. Let $d_C = Czz$ be the parallel axis through the mass center C. The rigid body has a general planar motion and the angular velocity vector is $\boldsymbol{\omega} = \omega\mathbf{k}$. The unit vector of the $d_C = Czz$ axis is $\mathbf{k}$.
The sum of the moments about O due to external forces and couples is

$$\sum \mathbf{M}_O = \frac{d\mathbf{H}_O}{dt} = \frac{d}{dt}[(\mathbf{r}_C \times m\mathbf{v}_C) + \mathbf{H}_C]. \tag{4.17}$$

The magnitude of the angular momentum about d_C is

$$H_C = \sum_i m_i r_i^2 \omega. \tag{4.18}$$

The summation $\sum\limits_i m_i r_i^2$ or the integration over the body $\int r^2\, dm$ is defined as mass moment of inertia I_{Czz} of the body about the z-axis through C

$$I_{Czz} = \sum_i m_i r_i^2.$$

The term r_i is the perpendicular distance from d_C to the P_i particle.

The mass moment of inertia I_{Czz} is a constant property of the body and is a measure of the rotational inertia or resistance to change in angular velocity due to the radial distribution of the rigid body mass around z-axis through C.
The angular momentum of the rigid body about d_C (z-axis through C) is

$$H_C = I_{Czz}\,\omega \;\text{ or }\; \mathbf{H}_C = I_{Czz}\,\omega\,\mathbf{k} = I_{Czz}\,\boldsymbol{\omega}.$$

Substituting this expression into Eq. (4.17) gives

$$\sum \mathbf{M}_O = \frac{d}{dt}[(\mathbf{r}_C \times m\mathbf{v}_C) + I_{Czz}\boldsymbol{\omega}] = (\mathbf{r}_C \times m\mathbf{a}_C) + I_{Czz}\boldsymbol{\alpha}. \tag{4.19}$$

The rotational equation of motion for the rigid body is

$$I_{Czz}\,\boldsymbol{\alpha} = \sum \mathbf{M}_C \quad \text{or} \quad I_{Czz}\,\alpha\,\mathbf{k} = \sum M_C\,\mathbf{k}. \tag{4.20}$$

For general planar motion the angular acceleration is $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \ddot{\theta}\mathbf{k}$, where the angle θ describes the position, or orientation, of the rigid body about a fixed axis. If the rigid body is a plate moving in the plane of motion (X, Y), the mass moment of inertia of the rigid body about z-axis through C becomes the polar mass moment of inertia of the rigid body about C, $I_{Czz} = I_C$. For this case the Eq. (4.20) gives

$$I_C\,\boldsymbol{\alpha} = \sum \mathbf{M}_C. \tag{4.21}$$

A special application of Eq. (4.21) is for rotation about a fixed point. Consider the special case when the rigid body rotates about the fixed point O as shown in Fig. 4.2(b). It follows that the acceleration of the mass center is expressed as

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C - \omega^2 \mathbf{r}_C. \tag{4.22}$$

The relation between the sum of the moments of the external forces about the fixed point O and the product $I_{Czz}\,\boldsymbol{\alpha}$ is given by Eq. (4.19)

$$\sum \mathbf{M}_O = \mathbf{r}_C \times m\mathbf{a}_C + I_{Czz}\boldsymbol{\alpha}. \tag{4.23}$$

Equations (4.22) and (4.23) give

$$\begin{aligned} \sum \mathbf{M}_O \;=\; & \mathbf{r}_C \times m\,(\boldsymbol{\alpha} \times \mathbf{r}_C - \omega^2 \mathbf{r}_C) + I_{Czz}\boldsymbol{\alpha} = \\ & m\,\mathbf{r}_C \times (\boldsymbol{\alpha} \times \mathbf{r}_C) + I_{Czz}\boldsymbol{\alpha} = \\ & m\,[(\mathbf{r}_C \cdot \mathbf{r}_C)\boldsymbol{\alpha} - (\mathbf{r}_C \cdot \boldsymbol{\alpha})\mathbf{r}_C] + I_{Czz}\boldsymbol{\alpha} = \\ & m\,\mathbf{r}_C^2\boldsymbol{\alpha} + I_{Czz}\boldsymbol{\alpha} = (m\,r_C^2 + I_{Czz})\boldsymbol{\alpha}. \end{aligned} \tag{4.24}$$

According to parallel-axis theorem

$$I_{Ozz} = I_{Czz} + m\,r_C^2,$$

where I_{Ozz} denotes the mass moment of inertia of the rigid body about z-axis through O. For the special case of rotation about a fixed point O one can use the formula

$$I_{Ozz}\boldsymbol{\alpha} = \sum \mathbf{M}_O. \tag{4.25}$$

The general equations of motion for a rigid body in plane motion are (Fig. 4.3)

$$\mathbf{F} = m\,\mathbf{a}_C \quad \text{or} \quad \mathbf{F} = m\ddot{\mathbf{r}}_C, \tag{4.26}$$

$$\sum \mathbf{M}_C = I_{Czz}\,\boldsymbol{\alpha}, \tag{4.27}$$

or using the cartesian components

$$m\ddot{x}_C = \sum F_x, \;\; m\ddot{y}_C = \sum F_y, \;\; I_{Czz}\ddot{\theta} = \sum M_C. \tag{4.28}$$

Equations (4.26) and (4.27) are interpreted in two ways

1. The forces and moments are known and the equations are solved for the motion of the rigid body (direct dynamics).

2. The motion of the RB is known and the equations are solved for the force and moments (inverse dynamics).

The dynamic force analysis in this chapter is based on the known motion of the mechanism.

4.2 D'Alembert's Principle

Newton's second law can be writen as

$$\mathbf{F} + (-m\mathbf{a}_C) = \mathbf{0}, \text{ or } \mathbf{F} + \mathbf{F}_{in} = \mathbf{0}, \tag{4.29}$$

where the term $\mathbf{F}_{in} = -m\mathbf{a}_C$ is the *inertia force.* Newton's second law can be regarded as an "equilibrium" equation.

Equation (4.23) relates the total moment about a fixed point O to the acceleration of the mass center and the angular acceleration

$$\sum \mathbf{M}_O = (\mathbf{r}_C \times m\mathbf{a}_C) + I_{Czz}\boldsymbol{\alpha},$$

or

$$\sum \mathbf{M}_O + [\mathbf{r}_C \times (-m\mathbf{a}_C)] + (-I_{Czz}\boldsymbol{\alpha}) = \mathbf{0}. \tag{4.30}$$

The term $\mathbf{M}_{in} = -I_{Czz}\boldsymbol{\alpha}$ is the *inertia moment.* The sum of the moments about any point, including the moment due to the inertial force $-m\mathbf{a}$ acting at mass center and the inertial moment, equals zero.

The equations of motion for a rigid body are analogous to the equations for static equilibrium:
The sum of the forces equals zero and the sum of the moments about any point equals zero when the inertial forces and moments are taken into account. This is called *D'Alembert's principle.*

The dynamic force analysis is expressed in a form similar to static force analysis

$$\sum \mathbf{R} = \sum \mathbf{F} + \mathbf{F}_{in} = \mathbf{0}, \tag{4.31}$$

$$\sum \mathbf{T}_C = \sum \mathbf{M}_C + \mathbf{M}_{in} = \mathbf{0}, \tag{4.32}$$

where $\sum \mathbf{F}$ is the vector sum of all external forces (resultant of external force), and $\sum \mathbf{M}_C$ is the sum of all external moments about the center of mass C (resultant external moment).
For a rigid body in plane motion in the xy plane,

$$\mathbf{a}_C = \ddot{x}_C\,\imath + \ddot{y}_C\,\jmath, \quad \boldsymbol{\alpha} = \alpha\,\mathbf{k},$$

with all external forces in that plane, Eqs. (4.31) and (4.32) become

$$\sum R_x = \sum F_x + F_{in\,x} = \sum F_x + (-m\,\ddot{x}_C) = 0, \tag{4.33}$$

$$\sum R_y = \sum F_y + F_{in\,y} = \sum F_y + (-m\,\ddot{y}_C) = 0, \tag{4.34}$$

$$\sum T_C = \sum M_C + M_{in} = \sum M_C + (-I_C\,\alpha) = 0. \tag{4.35}$$

With d'Alembert's principle the moment summation can be about any arbitrary point P

$$\sum \mathbf{T}_P = \sum \mathbf{M}_P + \mathbf{M}_{in} + \mathbf{r}_{PC} \times \mathbf{F}_{in} = \mathbf{0}, \tag{4.36}$$

where

- $\sum \mathbf{M}_P$ is the sum of all external moments about P,
- $\mathbf{M}_{in}$ is the inertia moment,
- $\mathbf{F}_{in}$ is the inertia force, and
- $\mathbf{r}_{PC}$ is a vector from P to C.

The dynamic analysis problem is reduced to a static force and moment balance problem where the inertia forces and moments are treated in the same way as external forces and moments.

4.3 Free-Body Diagrams

A free-body diagram is a drawing of a part of a complete system, isolated in order to determine the forces acting on that rigid body.

The following force convention is defined: $\mathbf{F}_{ij}$ represents the force exerted by link i on link j.

Figure 4.4 shows various free-body diagrams that are considered in the analysis of a slider-crank mechanism Fig. 4.4(a).

In Fig. 4.4(b), the free body consists of the three moving links isolated from the frame 0. The forces acting on the system include an external driven force $\mathbf{F}$, and the forces transmitted from the frame at joint A, $\mathbf{F}_{01}$, and at joint C, $\mathbf{F}_{03}$. Figure 4.4(c) is a free-body diagram of the two links 1 and 2 and Fig. 4.4(d) is a free-body diagram of the two links 0 and 1. Figure 4.4(e) is a free-body diagram of crank 1 and Fig. 4.4(f) is a free-body diagram of slider 3.

The force analysis can be accomplished by examining individual links or a subsystem of links. In this way the joint forces between links as well as the required input force or moment for a given output load are computed.

4.4 Force Analysis Using Dyads

The inertia force $\mathbf{F}_{in\,j} = F_{in\,jx}\,\imath + F_{in\,jy}\,\jmath$ and the gravitational force $\mathbf{G}_j = -m_j\,g\,\jmath$ act on link j at the center of mass C_j, $j = 2, 3$. The inertia moment on link j is $\mathbf{M}_{in\,j} = M_{in\,jz}\,\mathbf{k}$.

RRR dyad

Figure 4.5(a) shows an RRR dyad with two links 2 and 3, and three pin joints, B, C, and D. First, the exterior unknown joint reaction forces are considered

$$\mathbf{F}_{12} = F_{12x}\,\imath + F_{12y}\,\jmath \quad \text{and} \quad \mathbf{F}_{43} = F_{43x}\,\imath + F_{43y}\,\jmath.$$

To determine $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$, the following equations are written:

- sum of all forces on links 2 and 3 is zero

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_2 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \imath = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{43x} = 0, \quad (4.37)$$

$$\sum \mathbf{F}^{(2\&3)} \cdot \jmath = F_{12y} + F_{in\,2y} - m_2\,g + F_{in\,3y} - m_3\,g + F_{43y} = 0. \quad (4.38)$$

- sum of moments of all forces and moments on link 2 about C is zero

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C_2} - \mathbf{r}_C) \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0}. \quad (4.39)$$

- sum of moments of all forces and moments on link 3 about C is zero

$$\sum \mathbf{M}_C^{(3)} = (\mathbf{r}_D - \mathbf{r}_C) \times \mathbf{F}_{43} + (\mathbf{r}_{C_3} - \mathbf{r}_C) \times (\mathbf{F}_{in\,3} + \mathbf{G}_3) + \mathbf{M}_{in\,3} = \mathbf{0}. \quad (4.40)$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (4.37), (4.38), (4.39), and (4.40).

The reaction force $\mathbf{F}_{32} = -\mathbf{F}_{23}$ is computed from the sum of all forces on link 2

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{32} = \mathbf{0} \quad \text{or} \quad \mathbf{F}_{32} = -\mathbf{F}_{12} - \mathbf{F}_{in\,2} - \mathbf{G}_2.$$

RRT dyad

Figure 4.5(b) shows an RRT dyad with the unknown joint reaction forces $\mathbf{F}_{12}$, $\mathbf{F}_{43}$, and $\mathbf{F}_{23} = -\mathbf{F}_{32}$. The joint reaction force $\mathbf{F}_{43}$ is perpendicular to the sliding direction $\mathbf{F}_{43} \perp \Delta$ or

$$\mathbf{F}_{43} \cdot \Delta = (F_{43x}\boldsymbol{\imath} + F_{43y}\boldsymbol{\jmath}) \cdot (\cos\theta\boldsymbol{\imath} + \sin\theta\boldsymbol{\jmath}) = 0. \tag{4.41}$$

In order to determine $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$ the following equations are written

• sum of all the forces on links 2 and 3 is zero

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \boldsymbol{\imath} = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{43x} = 0, \tag{4.42}$$

$$\sum \mathbf{F}^{(2\&3)} \cdot \boldsymbol{\jmath} = F_{12y} + F_{in\,2y} - m_2\,g + F_{in\,3y} - m_3\,g + F_{43y} = 0. \tag{4.43}$$

• sum of moments of all the forces and the moments on link 2 about C is zero

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C_2} - \mathbf{r}_C) \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0}. \tag{4.44}$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (4.41), (4.42), (4.43), and (4.44).

The reaction force components F_{32x} and F_{32y} are computed from the sum of all the forces on link 2

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{32} = \mathbf{0} \quad \text{or} \quad \mathbf{F}_{32} = -\mathbf{F}_{12} - \mathbf{F}_{in\,2} - \mathbf{G}_2.$$

RTR dyad

The unknown joint reaction forces $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$ are calculated from the relations [Fig. 4.5(c)]

• sum of all the forces on links 2 and 3 is zero

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \boldsymbol{\imath} = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{43x} = 0, \tag{4.45}$$

$$\sum \mathbf{F}^{(2\&3)} \cdot \boldsymbol{\jmath} = F_{12y} + F_{in\,2y} - m_2\,g + F_{in\,3y} - m_3\,g + F_{43y} = 0. \tag{4.46}$$

• sum of the moments of all the forces and moments on links 2 and 3 about B is zero

$$\sum \mathbf{M}_B^{(2\&3)} = (\mathbf{r}_D - \mathbf{r}_B) \times \mathbf{F}_{43} + (\mathbf{r}_{C_3} - \mathbf{r}_B) \times (\mathbf{F}_{in\,3} + \mathbf{G}_3) + \mathbf{M}_{in\,3}$$
$$+ (\mathbf{r}_{C_2} - \mathbf{r}_B) \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0}. \tag{4.47}$$

• sum of all the forces on link 2 projected onto the sliding direction $\Delta = \cos\theta\boldsymbol{\imath} + \sin\theta\boldsymbol{\jmath}$ is zero

$$\sum \mathbf{F}^{(2)} \cdot \Delta = (\mathbf{F}_{12} + \mathbf{F}_2) \cdot (\cos\theta\boldsymbol{\imath} + \sin\theta\boldsymbol{\jmath}) = 0. \tag{4.48}$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (4.45), (4.46), (4.47), and (4.48).
The force components F_{32x} and F_{32y} are computed from the sum of all the forces on link 2

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{32} = \mathbf{0} \quad \text{or} \quad \mathbf{F}_{32} = -(\mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2).$$

4.5 Force Analysis Using Contour Method

An analytical method to compute joint forces that can be applied for both planar and spatial mechanisms will be presented. The method is based on the decoupling of a closed kinematic chain and writing the dynamic equilibrium equations. The kinematic links are loaded with external forces and inertia forces and moments.

A general monocontour closed kinematic chain is considered in Fig. 4.6. The joint force between the links $i-1$ and i (joint A_i) will be determined. When these two links $i-1$ and i are separated the joint forces $\mathbf{F}_{i-1,i}$ and $\mathbf{F}_{i,i-1}$ are introduced and

$$\mathbf{F}_{i-1,i} + \mathbf{F}_{i,i-1} = \mathbf{0}. \tag{4.49}$$

Table 4.1 shows the joint forces for one degree of freedom joints. It is helpful to "mentally disconnect" the two links $(i-1)$ and i, which create joint A_i, from the rest of the mechanism. The joint at A_i will be replaced by the joint forces $\mathbf{F}_{i-1,i}$ and $\mathbf{F}_{i,i-1}$. The closed kinematic chain has been transformed into two open kinematic chains, and two paths I and II are associated. The two paths start from A_i.

For the path I (counterclockwise), starting at A_i and following I the first joint encountered is A_{i-1}. For the link $i-1$ left behind, dynamic equilibrium equations are written according to the type of the joint at A_{i-1}. Following the same path I, the next joint encountered is A_{i-2}. For the subsystem ($i-1$ and $i-2$) equilibrium conditions corresponding to the type of joint at A_{i-2} can be specified, and so on. A similar analysis is performed for the path II of the open kinematic chain. The number of equilibrium equations written is equal to the number of unknown scalars introduced by joint A_i (joint forces at this joint). For a joint, the number of equilibrium conditions is equal to the number of relative mobilities of the joint.

4.6 R-RRT (slider-crank) Mechanism

Figure 4.7(a) is a schematic diagram of a R-RRT (slider-crank) mechanism comprised of a crank 1, a connecting rod 2, and a slider 3. The mechanism shown in the figure has the dimensions: $AB = 1$ m and $BC = 1$ m. The driver link 1 makes an angle $\phi = \phi_1 = \pi/4$ rad with the horizontal axis and rotates with a constant speed of $n = 30/\pi$ rpm. The point A is selected as the origin of the xyz reference frame. The position vectors of the joints B and C are

$$\mathbf{r}_B = x_B\boldsymbol{\imath} + y_B\boldsymbol{\jmath} = \frac{\sqrt{2}}{2}\boldsymbol{\imath} + \frac{\sqrt{2}}{2}\boldsymbol{\jmath} \text{ m} \quad \text{and} \quad \mathbf{r}_C = x_C\boldsymbol{\imath} + y_C\boldsymbol{\jmath} = \sqrt{2}\boldsymbol{\imath} + 0\boldsymbol{\jmath} \text{ m}.$$

The angular velocities of links 1 and 2 are

$$\boldsymbol{\omega}_1 = \omega_1\ \mathbf{k} = 1\ \mathbf{k} \text{ rad/s} \quad \text{and} \quad \boldsymbol{\omega}_2 = \omega_2\ \mathbf{k} = -1\ \mathbf{k} \text{ rad/s}.$$

The angular accelerations of link 1 and 2 are $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$. For this particular configuration of the mechanism $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = \mathbf{0}$.
The velocity and acceleration of B are

$$\mathbf{v}_B = -\frac{\sqrt{2}}{2}\boldsymbol{\imath} + \frac{\sqrt{2}}{2}\boldsymbol{\jmath} \ \text{ m/s} \quad \text{and} \quad \mathbf{a}_B = -\frac{\sqrt{2}}{2}\boldsymbol{\imath} - \frac{\sqrt{2}}{2}\boldsymbol{\jmath} \ \text{ m/s}^2.$$

The velocity and acceleration of C are

$$\mathbf{v}_C = -\sqrt{2}\ \boldsymbol{\imath} \ \text{ m/s} \quad \text{and} \quad \mathbf{a}_C = -\sqrt{2}\ \boldsymbol{\imath} \ \text{ m/s}^2.$$

The center of mass of link 1 is C_1, the center of mass of link 2 is C_2, and the center of mass of slider 3 is C. The position vectors of the $C_i,\ i = 1, 2, 3$ are

$$\begin{aligned}
\mathbf{r}_{C_1} &= \mathbf{r}_B/2 = x_{C_1}\boldsymbol{\imath} + y_{C_1}\boldsymbol{\jmath} = \frac{\sqrt{2}}{4}\boldsymbol{\imath} + \frac{\sqrt{2}}{4}\boldsymbol{\jmath} \text{ m},\\
\mathbf{r}_{C_2} &= (\mathbf{r}_B + \mathbf{r}_C)/2 = x_{C_2}\boldsymbol{\imath} + y_{C_2}\boldsymbol{\jmath} = \frac{3\sqrt{2}}{4}\boldsymbol{\imath} + \frac{\sqrt{2}}{4}\boldsymbol{\jmath} \text{ m},\\
\mathbf{r}_{C_3} &= \mathbf{r}_C = x_{C_3}\boldsymbol{\imath} + y_{C_3}\boldsymbol{\jmath} = \sqrt{2}\ \boldsymbol{\imath} \text{ m}.
\end{aligned}$$

The acceleration vectors of the $C_i,\ i = 1, 2, 3$ are

$$\begin{aligned}
\mathbf{a}_{C_1} &= \mathbf{a}_B/2 = a_{C_{1x}}\boldsymbol{\imath} + a_{C_{1y}}\boldsymbol{\jmath} = -\frac{\sqrt{2}}{4}\boldsymbol{\imath} - \frac{\sqrt{2}}{4}\boldsymbol{\jmath} \text{ m/s}^2,\\
\mathbf{a}_{C_2} &= (\mathbf{a}_B + \mathbf{a}_C)/2 = a_{C_{2x}}\boldsymbol{\imath} + a_{C_{2y}}\boldsymbol{\jmath} = -\frac{3\sqrt{2}}{4}\boldsymbol{\imath} - \frac{\sqrt{2}}{4}\boldsymbol{\jmath} \text{ m/s}^2,\\
\mathbf{a}_{C_3} &= \mathbf{a}_C = a_{C_x}\boldsymbol{\imath} + a_{C_y}\boldsymbol{\jmath} = -\sqrt{2}\ \boldsymbol{\imath} \text{ m/s}^2.
\end{aligned}$$

The MATLAB commands for the kinematics of the mechanism (positions, velocities, and accelerations) are

```
AB = 1; BC = 1; phi = 45*(pi/180);
xA = 0; yA = 0; rA = [xA yA 0];
xB = AB*cos(phi); yB = AB*sin(phi); rB = [xB yB 0];
yC = 0; xC = xB+sqrt(BC^2-(yC-yB)^2); rC = [xC yC 0];
n = 30/pi; omega1 = [ 0 0 pi*n/30 ]; alpha1 = [0 0 0 ];
vA = [0 0 0 ]; aA = [0 0 0 ];
vB1 = vA + cross(omega1,rB); vB2 = vB1;
aB1 = aA + cross(alpha1,rB) - dot(omega1,omega1)*rB; aB2 = aB1;
omega2z = sym('omega2z','real'); vCx = sym('vCx','real');
omega2 = [ 0 0 omega2z ]; vC = [ vCx 0 0 ];
eqvC = vC - (vB2 + cross(omega2,rC-rB));
eqvCx = eqvC(1); eqvCy = eqvC(2);
solvC = solve(eqvCx,eqvCy); omega2zs=eval(solvC.omega2z);
vCxs=eval(solvC.vCx); Omega2 = [0 0 omega2zs];
vCs = [vCxs 0 0];
alpha2z = sym('alpha2z','real'); aCx = sym('aCx','real');
alpha2 = [ 0 0 alpha2z ]; aC = [aCx 0 0 ];
eqaC = aC - (aB1 + cross(alpha2,rC-rB) - dot(Omega2,Omega2)*(rC-rB));
eqaCx = eqaC(1); eqaCy = eqaC(2);
solaC = solve(eqaCx,eqaCy);
alpha2zs=eval(solaC.alpha2z); aCxs=eval(solaC.aCx);
alpha20 = [0 0 alpha2zs]; aCs = [aCxs 0 0]; alpha30 = [0 0 0];
rC1 = (rA+rB)/2; fprintf('rC1 = [ %g, %g, %g ] (m)\n', rC1);
rC2 = (rB+rC)/2; fprintf('rC2 = [ %g, %g, %g ] (m)\n', rC2);
rC3 = rC; fprintf('rC3 = [ %g, %g, %g ] (m)\n', rC3);
aC1 = aB1/2; fprintf('aC1 = [ %g, %g, %g ] (m/s^2\n', aC1 ) ;
aC2 = (aB1+aCs)/2; fprintf('aC2 = [ %g, %g, %g ] (m/s^2)\n', aC2 ) ;
aC3 = aCs; fprintf('aC3 = [ %g, %g, %g ] (m/s^2)\n', aC3 ) ;
```

The external driven force $\mathbf{F}_{ext}$ applied on link 3 is opposed to the motion of the link (opposed to $\mathbf{v}_C$). Because $\mathbf{v}_C = -\sqrt{2}\ \imath$ m/s, the external force vector will be

$$\mathbf{F}_{ext} = [-\text{Sign}(\mathbf{v}_C)]\ 100\ \imath = 100\ \imath \text{ N}.$$

The MATLAB commands for the external force on link 3 are

```
fe = 100;
Fe = -sign(vCs(1))*[fe 0 0];
```

The signum function in MATLAB is `sign(x)`. If `x` is greater than zero `sign(x)` returns 1, if `x` is zero `sign(x)` returns zero, and if if `x` is less than zero `sign(x)` returns -1.

The height of the links 1 and 2 is $h = 0.01$ m. The width of the links 3 is $w_{Slider} = 0.01$ m and the height is $h_{Slider} = 0.01$ m [Fig. 4.7(b)]. All three moving links are rectangular prisms with the depth $d = 0.001$ m. The acceleration of gravity is $g = 10$ m/s^2.

```
h = 0.01; d = 0.001; hSlider = 0.01; wSlider = 0.01; g = 10.;
```

Inertia forces and moments

Link 1

The mass of the crank 1 is

$$m_1 = \rho_1 \; AB \; h \; d,$$

where the density of the material is ρ_1. For the simplicity of calculations $m_1 = 1$ kg.

The inertia force on link 1 at C_1 is

$$\mathbf{F}_{in\,1} = -m_1 \, \mathbf{a}_{C_1} = \frac{\sqrt{2}}{4}\mathbf{\imath} + \frac{\sqrt{2}}{4}\mathbf{\jmath} \text{ N}.$$

The gravitational force on crank 1 at C_1 is

$$\mathbf{G}_1 = -m_1 \; g \; \mathbf{\jmath} = -10 \; \mathbf{\jmath} \text{ N}.$$

The total force on link 1 at the mass center C_1 is

$$\mathbf{F}_1 = \mathbf{F}_{in\,1} + \mathbf{G}_1 = \frac{\sqrt{2}}{4}\mathbf{\imath} + (\frac{\sqrt{2}}{4} - 10)\mathbf{\jmath} \text{ N}.$$

The mass moment of inertia of the link 1 is

$$I_{C_1} = m_1 \; (AB^2 + h^2)/12 = 0.0833417 \text{ kg} \cdot \text{m}^2.$$

The moment of inertia on link 1 is

$$\mathbf{M}_{in\,1} = -I_{C_1}\ \boldsymbol{\alpha}_1 = \mathbf{0}.$$

Link 2
The mass of connecting rod 2 is

$$m_2 = \rho_2\ BC\ h\ d,$$

where the density of the material of link 2 is ρ_2. For the simplicity of calculations $m_2 = 1$ kg.
The inertia force on link 2 at C_2 is

$$\mathbf{F}_{in\,2} = -m_2\,\mathbf{a}_{C_2} = \frac{3\sqrt{2}}{4}\boldsymbol{\imath} + \frac{\sqrt{2}}{4}\boldsymbol{\jmath}\ \text{N}.$$

The gravitational force on link 2 at C_2 is

$$\mathbf{G}_2 = -m_2\ g\ \boldsymbol{\jmath} = -10\ \boldsymbol{\jmath}\ \text{N}.$$

The total force on link 2 at the mass center C_2 is

$$\mathbf{F}_2 = \mathbf{F}_{in\,2} + \mathbf{G}_2 = \frac{3\sqrt{2}}{4}\boldsymbol{\imath} + (\frac{\sqrt{2}}{4} - 10)\boldsymbol{\jmath}\ \text{N}.$$

The mass moment of inertia of link 2 is

$$I_{C_2} = m_2\ (BC^2 + h^2)/12 = 0.0833417\ \text{kg}\cdot\text{m}^2.$$

The moment of inertia on link 2 is

$$\mathbf{M}_{in\,2} = -I_{C_2}\ \boldsymbol{\alpha}_2 = \mathbf{0}.$$

Link 3
The mass of the link 3 is

$$m_3 = \rho_3\ h_{Slider}\ w_{Slider}\ d,$$

where the density of the material of link 3 is ρ_3. For the simplicity of calculations $m_3 = 1$ kg.
The inertia force on link 3 at $C_3 = C$ is

$$\mathbf{F}_{in\,3} = -m_3\,\mathbf{a}_{C_3} = \sqrt{2}\ \boldsymbol{\imath}\ \text{N}.$$

The gravitational force on link 3 at $C_3 = C$ is

$$\mathbf{G}_3 = -m_3\ g\ \mathbf{j} = -10\ \mathbf{j}\ \text{N}.$$

The total force on link 3 at the mass center C is

$$\mathbf{F}_3 = \mathbf{F}_{in\,3} + \mathbf{G}_3 = \sqrt{2}\ \mathbf{i} - 10\ \mathbf{j}\ \text{N}.$$

The mass moment of inertia of slider 3 is

$$I_{C_3} = m_3\ (h^2_{Slider} + w^2_{Slider})/12 = 0.0000166667\ \text{kg} \cdot \text{m}^2.$$

The moment of inertia on slider 3 is

$$\mathbf{M}_{in\,3} = -I_{C_3}\ \boldsymbol{\alpha}_3 = \mathbf{0}.$$

The MATLAB commands for the forces and moments of inertia are

```
m1 = 1;
IC1 = m1*(AB^2+h^2)/12;
G1 = [ 0 -m1*g 0 ];
Fin1 = - m1*aC1;
Min1 = - IC1*alpha1;
m2 = 1;
IC2 = m2*(BC^2+h^2)/12;
G2 = [ 0 -m2*g 0 ];
Fin2 = - m2*aC2;
Min2 = - IC2*alpha20;
m3 = 1 ;
IC3 = m3*(hSlider^2+wSlider^2)/12;
G3 = [ 0 -m3*g 0 ];
Fin3 = - m3*aC3;
Min3 = - IC3*alpha30;
```

For a given value of the crank angle ϕ ($\phi = \pi/4$) and a known driven force $\mathbf{F}_{ext}$ find the joint reactions and the drive moment $\mathbf{M}$ on the crank.

Joint forces and drive moment

4.6.1 Newton-Euler Equations of Motion

Figure 4.7(b) shows the free-body diagrams of the crank 1, the connecting rod 2, and the slider 3. For each moving link the dynamic equilibrium equations are applied (Newton-Euler equations of motion)

$$m\ \mathbf{a}_C = \sum \mathbf{F} \quad \text{and} \quad I_C\ \boldsymbol{\alpha} = \sum \mathbf{M}_C,$$

where C is the center of mass of the link.

The force analysis starts with the link 3 because the external driven force $\mathbf{F}_{ext}$ on the slider is given.

The reaction joint force of the ground 0 on the slider 3, $\mathbf{F}_{03}$, is perpendicular on the sliding direction, x-axis: $\mathbf{F}_{03} \perp \mathbf{\imath}$ (Figure 4.8). The application point Q of the reaction force $\mathbf{F}_{03}$ is determined using Euler moment equation

$$I_{C_3}\,\boldsymbol{\alpha}_3 = \mathbf{r}_{CQ} \times \mathbf{F}_{03} \quad \text{or} \quad \mathbf{0} = \mathbf{r}_{CQ} \times \mathbf{F}_{03} \quad \Longrightarrow \quad \mathbf{r}_{CQ} = \mathbf{0} \quad \text{or} \quad C = Q.$$

It results the reaction force $\mathbf{F}_{03}$ acts at C.

For the slider 3 the vector sum of the net forces (external forces $\mathbf{F}_{ext}$, gravitational force $\mathbf{G}_3$, joint forces $\mathbf{F}_{23}$, $\mathbf{F}_{03}$) is equal to $m_3\mathbf{a}_{C_3}$ (Fig. 4.8)

$$m_3\mathbf{a}_{C_3} = \mathbf{F}_{23} + \mathbf{G}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03},$$

where $\mathbf{F}_{23} = F_{23x}\mathbf{\imath} + F_{23y}\mathbf{\jmath}$ and $\mathbf{F}_{03} = F_{03y}\mathbf{\jmath}$.
Projecting this vectorial equation onto x and y axes gives

$$\begin{aligned} m_3\, a_{C_{3x}} &= F_{23x} + F_{ext}, \\ m_3\, a_{C_{3y}} &= F_{23y} - m_3\, g + F_{03y}, \end{aligned}$$

or numerically

$$(1)(-\sqrt{2}) = F_{23x} + 100, \tag{4.50}$$

$$0 = F_{23y} - (1)(10) + F_{03y}. \tag{4.51}$$

There are two equations Eqs. (4.50) and (4.51) and three unknowns F_{03y}, F_{23x} and F_{23y} and that is why the analysis will continue with link 2.

The MATLAB commands for Newton-Euler equations for slider 3 are

```
F03 = [ 0 sym('F03y','real') 0 ];
F23 = [ sym('F23x','real') sym('F23y','real') 0 ];
eqF3 = F03+F23+Fe+G3-m3*aC3;
eqF3x = eqF3(1);
eqF3y = eqF3(2);
```

For the connecting rod 2 (Fig. 4.9), Newton equation gives

$$m_2 \mathbf{a}_{C_2} = \mathbf{F}_{32} + \mathbf{G}_2 + \mathbf{F}_{12}.$$

The previous equation can be projected on x and y axes

$$m_2\, a_{C_{2x}} = F_{32x} + F_{12x},$$
$$m_2\, a_{C_{2y}} = F_{32y} - m_2\, g + F_{12y},$$

or numerically

$$(1)(-\frac{3\sqrt{2}}{4}) = F_{32x} + F_{12x}, \tag{4.52}$$
$$(1)(-\frac{\sqrt{2}}{4}) = F_{32y} - 1(10) + F_{12y}. \tag{4.53}$$

For the link 2 a moment equation can be written with respect to C_2

$$I_{C_2}\, \boldsymbol{\alpha}_2 = \mathbf{r}_{C_2C} \times \mathbf{F}_{32} + \mathbf{r}_{C_2B} \times \mathbf{F}_{12},$$

or

$$I_{C_2}\, \alpha_2\, \mathbf{k} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_C - x_{C_2} & y_C - y_{C_2} & 0 \\ F_{32x} & F_{32y} & 0 \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_B - x_{C_2} & y_B - y_{C_2} & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix}$$

or

$$I_{C_2}\, \alpha_2 = (x_C - x_{C_2})F_{32y} - (y_C - y_{C_2})F_{32x} + (x_B - x_{C_2})F_{12y} - (y_B - y_{C_2})F_{12x},$$

or numerically

$$0 = (\sqrt{2} - \frac{3\sqrt{2}}{4})F_{32y} - (-\frac{\sqrt{2}}{4})F_{32x} + (\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{4})F_{12y} - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4})F_{12x}. \tag{4.54}$$

The MATLAB commands for Newton-Euler equations for link 2 are

```
F32 = -F23;
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqF2 = F32+F12+G2-m2*aC2;
eqF2x = eqF2(1);
eqF2y = eqF2(2);
eqM2 = cross(rB-rC2,F12)+cross(rC-rC2,F32)-IC2*alpha20;
eqM2z = eqM2(3);
```

Equations (4.50), (4.51), (4.52), (4.53), and (4.54) form a system of 5 equations with five scalar unknowns. The system can be solved using the `solve` statement.

```
sol32=solve(eqF3x,eqF3y,eqF2x,eqF2y,eqM2z);
F03ys=eval(sol32.F03y);
F23xs=eval(sol32.F23x);
F23ys=eval(sol32.F23y);
F12xs=eval(sol32.F12x);
F12ys=eval(sol32.F12y);
F03s = [ 0, F03ys, 0 ];
F23s = [ F23xs, F23ys, 0 ];
F12s = [ F12xs, F12ys, 0 ];
```

The numerical values for the joint forces for the links 3 and 2 are

$$F_{03y} = -85 - \frac{3\sqrt{2}}{2} \text{ N},$$
$$F_{23x} = -100 - \sqrt{2} \text{ N}, \quad F_{23y} = 95 + \frac{3\sqrt{2}}{2} \text{ N},$$
$$F_{12x} = -\frac{1}{4}(400 + 7\sqrt{2}) \text{ N}, \quad F_{12y} = \frac{5}{4}(84 + \sqrt{2}) \text{ N},$$

or

$$F_{03} = |\mathbf{F}_{03}| = 85 + \frac{3\sqrt{2}}{2} \text{ N},$$
$$F_{23} = |\mathbf{F}_{23}| = \sqrt{F_{23x}^2 + F_{23y}^2} = \sqrt{\frac{38063}{2} + 485\sqrt{2}} \text{ N},$$
$$F_{12} = |\mathbf{F}_{12}| = \sqrt{F_{12x}^2 + F_{12y}^2} = \frac{1}{2}\sqrt{84137 + 2450\sqrt{2}} \text{ N}.$$

For the crank 1 [Fig. 4.4], there are two vectorial equations

$$m_1 \mathbf{a}_{C_1} = \mathbf{F}_{21} + \mathbf{G}_1 + \mathbf{F}_{01},$$
$$I_{C_1} \boldsymbol{\alpha}_1 = \mathbf{r}_{C_1B} \times \mathbf{F}_{21} + \mathbf{r}_{C_1A} \times \mathbf{F}_{01} + \mathbf{M},$$

where $\mathbf{M}$, is the input (motor) moment on the crank, $\mathbf{F}_{21} = -\mathbf{F}_{12}$, and $\mathbf{F}_{01} = F_{01x}\mathbf{ı} + F_{01y}\mathbf{ȷ}$.
The above vectorial equations give three scalar equations on x, y, and z

$$m_1\, a_{C_{1x}} = F_{21x} + F_{01x},$$
$$m_1\, a_{C_{1y}} = F_{21y} - m_1\, g + F_{01y},$$
$$I_{C1}\, \alpha_1\, \mathbf{k} = \begin{vmatrix} \mathbf{ı} & \mathbf{ȷ} & \mathbf{k} \\ x_B - x_{C_1} & y_B - y_{C_1} & 0 \\ F_{21x} & F_{21y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{ı} & \mathbf{ȷ} & \mathbf{k} \\ x_A - x_{C_1} & y_A - y_{C_1} & 0 \\ F_{01x} & F_{01y} & 0 \end{vmatrix} +$$
$$+M\, \mathbf{k} = \mathbf{0},$$

or

$$m_1\, a_{C_{1x}} = F_{21x} + F_{01x},$$
$$m_1\, a_{C_{1y}} = F_{21y} - m_1\, g + F_{01y}$$
$$I_{C_1}\, \alpha_1 = (x_B - x_{C_1})F_{21y} - (y_{C_2} - y_{C_1})F_{21x} +$$
$$(x_A - x_{C_1})F_{01y} - (y_A - y_{C_1})F_{01x} + M,$$

or numerically

$$1(-\frac{\sqrt{2}}{4}) = \frac{1}{4}(400 + 7\sqrt{2}) + F_{01x}, \tag{4.55}$$
$$1(-\frac{\sqrt{2}}{4}) = -\frac{5}{4}(84 + \sqrt{2}) - 1(10) + F_{01y}, \tag{4.56}$$
$$0 = (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4})\left[-\frac{5}{4}(84 + \sqrt{2})\right] - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4})\left[\frac{1}{4}(400 + 7\sqrt{2})\right]$$
$$-\frac{\sqrt{2}}{4}F_{01y} + \frac{\sqrt{2}}{4}F_{01y} + M = 0. \tag{4.57}$$

The MATLAB commands for Newton-Euler equations for the crank 1 are

```
F01=m1*aC1-G1+F12s;
Mm=-cross(rB,-F12s)-cross(rC1,G1-m1*aC1)-IC1*alpha1;
```

Equations (4.55), (4.56) and (4.57) give

$$F_{01x} = -2(50 + \sqrt{2})\ \text{N}, \quad F_{01y} = 115 + \sqrt{2}\ \text{N},$$
$$M = 3 + 105\sqrt{2}\ \text{N} \cdot \text{m},$$

or

$$F_{01} = |\mathbf{F}_{01}| = \sqrt{F_{01x}^2 + F_{01y}^2} = \sqrt{23235 + 630\sqrt{2}}\ \text{N},$$
$$M = |\mathbf{M}| = 3 + 105\sqrt{2}\ \text{N} \cdot \text{m}.$$

Another way of calculating the moment $\mathbf{M}$ required for dynamic equilibrium is to write the moment equation of motion for link 1 about the fixed point A

$$I_A\, \alpha_1\, \mathbf{k} = \mathbf{r}_{AC_1} \times \mathbf{G}_1 + \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M} \implies \mathbf{M} = \mathbf{r}_B \times \mathbf{F}_{12} - \mathbf{r}_{C_1} \times \mathbf{G}_1,$$

where $I_A = I_{C_1} + m_1\,(AB/2)^2$. The reaction force $\mathbf{F}_{01}$ does not appear in this moment equation.

The MATLAB program using Newton-Euler equations and the results are given in Program 4.1.

4.6.2 D'Alembert's Principle

For each moving link the dynamic equilibrium equations are applied (d'Alembert's principle)

$$\sum \mathbf{F} + \mathbf{F}_{in} = \mathbf{0} \quad \text{and} \quad \sum \mathbf{M}_C + \mathbf{M}_{in} = \mathbf{0},$$

where C is the center of mass of the link. With d'Alembert's principle the moment summation can be about any arbitrary point P

$$\sum \mathbf{M}_P + \mathbf{M}_{in} + \mathbf{r}_{PC} \times \mathbf{F}_{in} = \mathbf{0}.$$

The force analysis starts with the link 3 because the external driven force $\mathbf{F}_{ext}$ is given. For the slider 3 the vector sum of the all the forces (external forces $\mathbf{F}_{ext}$, gravitational force $\mathbf{G}_3$, inertia forces $\mathbf{F}_{in\,3}$, joint forces $\mathbf{F}_{23}$, $\mathbf{F}_{03}$) is zero (Fig. 4.8)

$$\sum \mathbf{F}^{(3)} = \mathbf{F}_{23} + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03} = \mathbf{0},$$

where $\mathbf{F}_{23} = F_{23x}\mathbf{\imath} + F_{23y}\mathbf{\jmath}$ and $\mathbf{F}_{03} = F_{03y}\mathbf{\jmath}$.
Projecting this force onto x and y axes gives

$$\sum \mathbf{F}^{(3)} \cdot \mathbf{\imath} = F_{23x} + (-m_3\, a_{C3x}) + F_{ext} = 0,$$
$$\sum \mathbf{F}^{(3)} \cdot \mathbf{\jmath} = F_{23y} - m_3\, g + F_{03y} = 0,$$

or numerically

$$F_{23x} + (-1)(-\sqrt{2}) + 100 = 0, \tag{4.58}$$
$$F_{23y} - (1)(10) + F_{03y} = 0. \tag{4.59}$$

The MATLAB commands for slider 3 are

```
F03 = [ 0 sym('F03y','real') 0 ];
F23 = [ sym('F23x','real') sym('F23y','real') 0 ];
eqF3 = F03+F23+Fe+G3+Fin3;
eqF3x = eqF3(1);
eqF3y = eqF3(2);
```

For the connecting rod 2 (Fig. 4.9), the sum of the forces is equal to zero

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{32} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{12} = \mathbf{0},$$

The previous equation can be projected on x and y axes

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\imath} = F_{32x} + (-m_2\, a_{C_{2x}}) + F_{12x} = 0,$$
$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\jmath} = F_{32y} + (-m_2\, a_{C_{2y}}) - m_2\, g + F_{12y} = 0,$$

or numerically

$$F_{32x} + (-1)(-\frac{3\sqrt{2}}{4}) + F_{12x} = 0, \tag{4.60}$$
$$F_{32y} + (-1)(-\frac{\sqrt{2}}{4}) - 1(10) + F_{12y} = 0. \tag{4.61}$$

For the link 2 a moment equation can be written with respect to C_2

$$\sum \mathbf{M}^{(2)}_{C_2} = \mathbf{r}_{C_2C} \times \mathbf{F}_{32} + \mathbf{r}_{C_2B} \times \mathbf{F}_{12} + \mathbf{M}_{in\,2} = \mathbf{0}.$$

Instead of the previous one can use the sum of the moments with respect to B

$$\sum \mathbf{M}^{(2)}_{B} = \mathbf{r}_{BC} \times \mathbf{F}_{32} + \mathbf{r}_{BC_2} \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ F_{32x} & F_{32y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C_2} - x_B & y_{C_2} - y_B & 0 \\ -m_2\, a_{C_{2x}} & -m_2\, a_{C_{2y}} - m_2\, g & 0 \end{vmatrix}$$
$$-I_{C_2}\, \alpha_2\, \mathbf{k} = \mathbf{0}.$$

or

$$(x_C - x_B)F_{32y} - (y_C - y_B)F_{32x} + (x_{C_2} - x_B)(-m_2\, a_{C_{2y}} - m_2\, g)$$
$$- (y_{C_2} - y_B)(-m_2\, a_{C_{2x}}) - I_{C_2}\, \alpha_2 = 0,$$

or numerically

$$(\sqrt{2} - \frac{\sqrt{2}}{2})F_{32y} - (-\frac{\sqrt{2}}{2})F_{32x} + (\frac{3\sqrt{2}}{4} - \frac{\sqrt{2}}{2})\left[-1(-\frac{\sqrt{2}}{4}) - 1(10)\right]$$
$$- (\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2})\left[-1(-\frac{3\sqrt{2}}{4})\right] - 0 = 0. \tag{4.62}$$

For the connecting rod 2 the MATLAB commands are

```
F32 = -F23;
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqF2 = F32+F12+G2+Fin2;
eqF2x = eqF2(1);
eqF2y = eqF2(2);
eqM2 = cross(rB-rC2,F12)+cross(rC-rC2,F32)+Min2;
eqM2z = eqM2(3);
```

For the crank 1 (Fig. 4.10), there are two vectorial equations

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{in\,1} + \mathbf{G}_1 + \mathbf{F}_{01} = \mathbf{0},$$
$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_B \times \mathbf{F}_{21} + \mathbf{r}_{C_1} \times (\mathbf{F}_{in\,1} + \mathbf{G}_1) + \mathbf{M}_{in\,1} + \mathbf{M} = \mathbf{0},$$

where $M = |\mathbf{M}|$ is the magnitude of the input moment on the crank, $\mathbf{F}_{21} = -\mathbf{F}_{12}$, and $\mathbf{F}_{01} = F_{01x}\boldsymbol{\imath} + F_{01y}\boldsymbol{\jmath}$.
The above vectorial equations give three scalar equations on x, y, and z

$$\sum \mathbf{F}^{(1)} \cdot \boldsymbol{\imath} = F_{21x} + (-m_1\, a_{C_{1x}}) + F_{01x} = 0,$$
$$\sum \mathbf{F}^{(1)} \cdot \boldsymbol{\jmath} = F_{21y} + (-m_1\, a_{C_{1y}}) - m_1\, g + F_{01y} = 0,$$
$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_B & y_B & 0 \\ F_{21x} & F_{21y} & 0 \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_{C_1} & y_{C_1} & 0 \\ -m_1\, a_{C_{1x}} & -m_1\, a_{C_{1y}} - m_1\, g & 0 \end{vmatrix}$$
$$-I_{C_1}\, \alpha_1\, \mathbf{k} + M\, \mathbf{k} = \mathbf{0},$$

or

$$F_{21x} + (-m_1\, a_{C_{1x}}) + F_{01x} = 0,$$
$$F_{21y} + (-m_1\, a_{C_{1y}}) - m_1\, g + F_{01y} = 0,$$
$$x_B F_{21y} - y_B F_{21x} + x_{C_1}(-m_1\, a_{C_{1y}} - m_1\, g) - y_{C_1}(-m_1\, a_{C_{1x}}) - I_{C_1}\, \alpha_1 + M = 0,$$

or numerically

$$\frac{1}{4}(400 + 7\sqrt{2}) + \left[-1(-\frac{\sqrt{2}}{4})\right] + F_{01x} = 0, \tag{4.63}$$
$$-\frac{5}{4}(84 + \sqrt{2}) + \left[-1(-\frac{\sqrt{2}}{4})\right] - 1(10) + F_{01y} = 0, \tag{4.64}$$

$$\frac{\sqrt{2}}{2}\left[-\frac{5}{4}(84+\sqrt{2})\right]-\frac{\sqrt{2}}{2}\left[\frac{1}{4}(400+7\sqrt{2})\right]$$
$$+\frac{\sqrt{2}}{4}\left[-1(-\frac{\sqrt{2}}{4})-1(10)\right]-\frac{\sqrt{2}}{4}\left[-1(-\frac{\sqrt{2}}{4})\right]$$
$$-0+M=0. \tag{4.65}$$

For the crank 1 the MATLAB commands are

```
F01 = [ sym('F01x','real') sym('F01y','real') 0 ];
Mm = [ 0 0 sym('Mmz','real') ];
eqF1 = F01+Fin1+G1-F12;
eqF1x = eqF1(1);
eqF1y = eqF1(2);
eqM1 = cross(rB-rC1,-F12)+cross(rA-rC1,F01)+Min1+Mm;
eqM1z = eqM1(3);
```

Equations (4.58), (4.59), (4.60), (4.61), (4.62), (4.63), (4.64) and (4.65) form a system of 8 equations with eight scalar unknowns.

The MATLAB commands for solving the system of equations are

```
sol321=solve(eqF3x,eqF3y,eqF2x,eqF2y,eqM2z,eqF1x,eqF1y,eqM1z);
F03ys = eval(sol321.F03y);
F23xs = eval(sol321.F23x);
F23ys = eval(sol321.F23y);
F12xs = eval(sol321.F12x);
F12ys = eval(sol321.F12y);
F01xs = eval(sol321.F01x);
F01ys = eval(sol321.F01y);
Mmzs = eval(sol321.Mmz);
```

The following numerical solutions are obtained

$$F_{03y}=-85-\frac{3\sqrt{2}}{2}\ \text{N},$$
$$F_{23x}=-100-\sqrt{2}\ \text{N},\quad F_{23y}=95+\frac{3\sqrt{2}}{2}\ \text{N},$$
$$F_{12x}=-\frac{1}{4}(400+7\sqrt{2})\ \text{N},\quad F_{12y}=\frac{5}{4}(84+\sqrt{2})\ \text{N},$$

$$F_{01x} = -2(50 + \sqrt{2}) \text{ N}, \quad F_{01y} = 115 + \sqrt{2} \text{ N},$$
$$M = 3 + 105\sqrt{2} \text{ N} \cdot \text{m}.$$

The MATLAB program using D'Alembert principle and the results are given in Program 4.2.

4.6.3 Dyad Method

$B_R C_R C_T$ dyad
Figure 4.11 shows the dyad $B_R C_R C_T$ with the unknown joint reactions $\mathbf{F}_{12}$ and $\mathbf{F}_{03}$. The joint reaction $\mathbf{F}_{03}$ is perpendicular to the sliding direction $\mathbf{F}_{03} \perp \Delta = \imath$ or $\mathbf{F}_{03} = F_{03y}\jmath$.
The following equations are written to determine $\mathbf{F}_{12}$ and $\mathbf{F}_{03}$

• sum of all the forces on links 2 and 3 is zero

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \imath = F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{ext} = 0, \quad (4.66)$$

$$\sum \mathbf{F}^{(2\&3)} \cdot \jmath = F_{12y} + F_{in\,2y} - m_2\, g + F_{in\,3y} - m_3\, g + F_{03y} = 0. \quad (4.67)$$

• sum of moments of all the forces and moments on link 2 about C_R is zero

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C_2} - \mathbf{r}_C) \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0}. \quad (4.68)$$

The MATLAB commands for the Eqs. (4.66), (4.67), and (4.68) and for finding the unknowns are

```
F03 = [ 0 sym('F03y','real') 0 ];
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqF32 = F03+Fe+G3+Fin3+Fin2+G2+F12;
eqF32x = eqF32(1);
eqF32y = eqF32(2);
eqM2C = cross(rB-rC,F12)+cross(rC2-rC,Fin2+G2)+Min2;
eqM2Cz = eqM2C(3);
fprintf('%s = 0 (1)\n', char(vpa(eqF32x,6)));
fprintf('%s = 0 (2)\n', char(vpa(eqF32y,6)));
fprintf('%s = 0 (3)\n', char(vpa(eqM2Cz,6)));
fprintf('Eqs(1)-(3) => F03y, F12x, F12y \n');
sol32=solve(eqF32x,eqF32y,eqM2Cz);
F03ys=eval(sol32.F03y);
F12xs=eval(sol32.F12x);
F12ys=eval(sol32.F12y);
```

```
F03s = [ 0, F03ys, 0 ];
F12s = [ F12xs, F12ys, 0 ];
```

The numerical values of the joint forces are

$$F_{12x} = -102.475 \text{ N}, \quad F_{12y} = 106.768 \text{ N}, \quad \text{and} \quad F_{03y} = -87.1213 \text{ N}.$$

For the crank 1 (Fig. 4.11), there are two vectorial equations

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{in\,1} + \mathbf{G}_1 + \mathbf{F}_{01} = \mathbf{0},$$
$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_B \times \mathbf{F}_{21} + \mathbf{r}_{C_1} \times (\mathbf{F}_{in\,1} + \mathbf{G}_1) + \mathbf{M}_{in\,1} + \mathbf{M} = \mathbf{0},$$

and the MATLAB statements for finding $\mathbf{F}_{01}$ and $\mathbf{M}$ are

```
F01=-Fin1-G1+F12s;
Mm=-cross(rB,-F12s)-cross(rC1,G1+Fin1)-Min1;
```

TheMATLABprogram using the dyad method and the results are given in Program 4.3.

4.6.4 Contour Method

The diagram representing the mechanism is shown in Fig. 4.12 and has one contour, 0-1-2-3-0.

The dynamic force analysis can start with any joint.

Reaction $\mathbf{F}_{03}$

The reaction force $\mathbf{F}_{03}$ is perpendicular to the sliding direction of joint C_T ($C_{Translation}$) as shown in Fig. 4.13

$$\mathbf{F}_{03} = F_{03y}\,\mathbf{J}.$$

The application point of the unknown reaction force $\mathbf{F}_{03}$ is computed from a moment equation about C_R ($C_{Rotation}$) for link 3 (path I) (Fig. 4.13)

$$\sum \mathbf{M}_C^{(3)} = \mathbf{r}_{CP} \times \mathbf{F}_{03} = (\mathbf{r}_P - \mathbf{r}_C) \times \mathbf{F}_{03} = \mathbf{0},$$

or

$$x\,F_{05y} = 0 \quad \Rightarrow x = 0.$$

The application point of the reaction force $\mathbf{F}_{03}$ is at C ($P \equiv C$).

The magnitude of the reaction force F_{03y} is obtained from a moment equation about B_R for the links 3 and 2 (path I)

$$\sum \mathbf{M}_B^{(3\&2)} = \mathbf{r}_{BC} \times (\mathbf{F}_{03} + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext}) +$$
$$\mathbf{r}_{BC_2} \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ F_{in\,3x} + F_{ext} & F_{03y} + F_{in\,3y} - m_3\,g & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C_2} - x_B & y_{C_2} - y_B & 0 \\ F_{in\,2x} & F_{in\,2y} - m_2\,g & 0 \end{vmatrix}$$
$$+M_{in\,2}\,\mathbf{k} = \mathbf{0},$$

or numerically

$$\frac{3}{2} - \frac{5}{\sqrt{2}} + 45\sqrt{2} + \frac{F_{03y}}{\sqrt{2}} = 0.$$

The reaction F_{03y} is

$$F_{03y} = -85 - \frac{3\sqrt{2}}{2}\ \text{N}.$$

The MATLABstatements for finding $\mathbf{F}_{03}$ are

```
% Joint C_T
F03 = [ 0 sym('F03y','real') 0 ];
eqM32B = cross(rC-rB,F03+G3+Fin3+Fe)+cross(rC2-rB,Fin2+G2)+Min2;
eqM32Bz = eqM32B(3);
fprintf('%s = 0 (1)\n', char(vpa(eqM32Bz,6)));
fprintf('Eq(1) => F03y \n');
solF03=solve(eqM32Bz);
F03ys=eval(solF03);
F03s=[ 0, F03ys, 0 ];
fprintf('F03 = [ %g, %g, %g ] (N)\n', F03s);
```

Reaction $\mathbf{F}_{23}$
The pin joint at C_R, between 2 and 3, is replaced with the reaction force (Fig. 4.14)

$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\,\mathbf{\imath} + F_{23y}\,\mathbf{\jmath}.$$

For the path I, an equation for the forces projected onto the sliding direction of the joint C_T is written for link 3

$$\sum \mathbf{F}^{(3)} \cdot \mathbf{\imath} = (\mathbf{F}_{23} + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \mathbf{\imath} =$$
$$F_{23x} + F_{in\,3x} + F_{ext} = F_{23x} + 100 + \sqrt{2} = 0. \tag{4.69}$$

For the path II, shown Fig. 4.7, a moment equation about B_R is written for link 2

$$\sum \mathbf{M}_B^{(2)} = \mathbf{r}_{BC} \times (-\mathbf{F}_{23}) + \mathbf{r}_{BC_2} \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ -F_{23x} & -F_{23y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C_2} - x_B & y_{C_2} - y_B & 0 \\ F_{in\,2x} & F_{in\,2y} - m_2\,g & 0 \end{vmatrix} + M_{in\,2}\,\mathbf{k} = \mathbf{0},$$

or numerically

$$\frac{1}{2} - \frac{5\sqrt{2}}{2} - \frac{F_{23x}\sqrt{2}}{2} - \frac{F_{23y}\sqrt{2}}{2} = 0. \tag{4.70}$$

The joint force $\mathbf{F}_{23}$ is obtained from the system of Eqs. (4.69) and (4.70)

$$F_{23x} = -100 - \sqrt{2}\ \text{N} \ \text{ and } \ F_{23y} = 95 + \frac{3\sqrt{2}}{2}\ \text{N}.$$

The MATLABstatements for finding $\mathbf{F}_{23}$ are

```
% Joint C_R
F23 = [ sym('F23x','real') sym('F23y','real') 0 ];
eqF3 = F23+Fe+G3+Fin3;
eqF3x = eqF3(1);
eqM2B = cross(rC-rB,-F23)+cross(rC2-rB,Fin2+G2)+Min2;
eqM2Bz = eqM2B(3);
fprintf('%s = 0 (2)\n', char(vpa(eqF3x,6)));
fprintf('%s = 0 (3)\n', char(vpa(eqM2Bz,6)));
fprintf('Eqs(2)-(3) => F23x, F23y \n');
solF23=solve(eqF3x,eqM2Bz);
F23xs=eval(solF23.F23x);
F23ys=eval(solF23.F23y);
F23s = [ F23xs, F23ys, 0 ];
fprintf('F23 = [ %g, %g, %g ] (N)\n', F23s );
```

Reaction $\mathbf{F}_{12}$
The pin joint at B_R, between 1 and 2, is replaced with the reaction force (Fig. 4.15)

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x}\,\boldsymbol{\imath} + F_{12y}\,\boldsymbol{\jmath}.$$

For the path I, shown Fig. 4.8, a moment equation about C_R is written for link 2

$$\sum \mathbf{M}_C^{(2)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC_2} \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_B - x_C & y_B - y_C & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_{C_2} - x_C & y_{C_2} - y_C & 0 \\ F_{in\,2x} & F_{in\,2y} - m_2\,g & 0 \end{vmatrix} + M_{in\,2}\,\mathbf{k} = \mathbf{0},$$

or numerically

$$-\frac{1}{2} + \frac{5\sqrt{2}}{2} - \frac{F_{12x}\sqrt{2}}{2} - \frac{F_{12y}\sqrt{2}}{2} = 0. \tag{4.71}$$

Continuing on path I, an equation for the forces projected onto the sliding direction of the joint C_T is written for links 2 and 3

$$\sum \mathbf{F}^{(2\&3)} \cdot \boldsymbol{\imath} = (\mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \boldsymbol{\imath} =$$

$$F_{12x} + F_{in\,2x} + F_{in\,3x} + F_{ext} = F_{23x} + 100 + \sqrt{2} + \frac{3\sqrt{2}}{2} = 0. \tag{4.72}$$

The joint force $\mathbf{F}_{12}$ is obtained from the system of Eqs. (4.72) and (4.71)

$$F_{12x} = -\frac{1}{4}(400 + 7\sqrt{2})\text{ N} \text{ and } F_{12y} = \frac{5}{4}(84 + \sqrt{2})\text{ N}.$$

The MATLABstatements for finding $\mathbf{F}_{12}$ are

```
% Joint B_R
F12 = [ sym('F12x','real') sym('F12y','real') 0 ];
eqM2C = cross(rB-rC,F12)+cross(rC2-rC,Fin2+G2)+Min2;
eqM2Cz = eqM2C(3);
eqF23 = (F12+Fin2+G2+G3+Fin3+Fe);
eqF23x = eqF23(1);
fprintf('%s = 0 (4)\n', char(vpa(eqM2Cz,6)));
fprintf('%s = 0 (5)\n', char(vpa(eqF23x,6)));
fprintf('Eqs(4)-(5) => F12x, F12y \n');
solF12=solve(eqM2Cz,eqF23x);
F12xs=eval(solF12.F12x);
F12ys=eval(solF12.F12y);
F12s = [ F12xs, F12ys, 0 ];
fprintf('F12 = [ %g, %g, %g ] (N)\n', F12s );
```

Reaction $\mathbf{F}_{01}$ and equilibrium moment $\mathbf{M}$

The pin joint A_R, between 0 and 1, is replaced with the unknown reaction (Fig. 4.16)

$$\mathbf{F}_{01} = F_{01x}\,\mathbf{\imath} + F_{01y}\,\mathbf{\jmath}.$$

The unknown equilibrium moment is $\mathbf{M} = M\,\mathbf{k}$. If the path I is followed [Fig. 4.9] for the pin joint B_R, a moment equation is written for link 1

$$\sum \mathbf{M}_B^{(1)} = \mathbf{r}_{BA} \times \mathbf{F}_{01} + \mathbf{r}_{BC_1} \times (\mathbf{F}_{in\,1} + \mathbf{G}_1) + \mathbf{M}_{in\,1} + \mathbf{M} = \mathbf{0},$$

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ -x_B & -y_B & 0 \\ F_{01x} & F_{01y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C_1} - x_B & y_{C_1} - y_B & 0 \\ F_{in\,1x} & F_{in\,1y} - m_1\,g & 0 \end{vmatrix} + M\,\mathbf{k} = \mathbf{0},$$

or numerically

$$\frac{5\sqrt{2}}{2} + \frac{F_{01x}\sqrt{2}}{2} + \frac{F_{01y}\sqrt{2}}{2} + M = 0. \tag{4.73}$$

Continuing on path I the next joint encountered is the pin joint C_R, and a moment equation is written for links 1 and 2

$$\sum \mathbf{M}_C^{(1\&2)} = \mathbf{r}_{CA} \times \mathbf{F}_{01} + \mathbf{r}_{CC_1} \times (\mathbf{F}_{in\,1} + \mathbf{G}_1) + \mathbf{M}_{in\,1} + \mathbf{M}$$
$$+\mathbf{r}_{CC_2} \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ -x_C & -y_C & 0 \\ F_{01x} & F_{01y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C_1} - x_C & y_{C_1} - y_C & 0 \\ F_{in\,1x} & F_{in\,1y} - m_1\, g & 0 \end{vmatrix} + M\,\mathbf{k}$$
$$+ \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C_2} - x_C & y_{C_2} - y_C & 0 \\ F_{in\,2x} & F_{in\,2y} - m_2\, g & 0 \end{vmatrix} + M_{in\,2}\,\mathbf{k} = \mathbf{0},$$

or numerically

$$-\sqrt{2}\,F_{01y} + M - 1 + 10\sqrt{2} = 0. \tag{4.74}$$

Continuing on path I the next joint encountered is the slider joint C_T, and a force equation is written for links 1, 2, and 3

$$\sum \mathbf{F}^{(1\&2\&3)} \cdot \imath = (\mathbf{F}_{01} + \mathbf{F}_{in\,1} + \mathbf{G}_1 + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext}) \cdot \imath =$$
$$F_{01x} + F_{in\,1x} + F_{in\,2x} + F_{in\,3x} + F_{ext} = F_{12x} + 100 + \sqrt{2} + \frac{3\sqrt{2}}{2} = 0. \tag{4.75}$$

From Eqs. (4.73), (4.74), and (4.75) the components F_{01x}, F_{01y} and M are computed

$$F_{01x} = -2(50 + \sqrt{2})\text{ N} \quad \text{and} \quad F_{01y} = 115 + \sqrt{2}\text{ N},$$
$$M = 3 + 105\sqrt{2}\text{ N} \cdot \text{m}.$$

The MATLABstatements for finding $\mathbf{F}_{01}$ and $\mathbf{M}$ are

```
% Joint A_R
F01 = [ sym('F01x','real') sym('F01y','real') 0 ];
Mm = [ 0 0 sym('Mmz','real') ];
eqM1B = cross(-rB,F01)+cross(rC1-rB,Fin1+G1)+Min1+Mm;
eqM1Bz = eqM1B(3);
eqM12C=cross(-rC,F01)+cross(rC1-rC,Fin1+G1)+Min1+Mm+cross(rC2-rC,Fin2+G2)+Min2;
eqM12Cz = eqM12C(3);
```

```
eqF123 = (F01+Fin1+G1+Fin2+G2+Fin3+G3+Fe);
eqF123x = eqF123(1);
fprintf('%s = 0 (6\n', char(vpa(eqM1Bz,6)));
fprintf('%s = 0 (7)\n', char(vpa(eqM12Cz,6)));
fprintf('%s = 0 (8)\n', char(vpa(eqF123x,6)));
fprintf('Eqs(6)-(8) => F01x, F01y, Mmz \n');
solF01=solve(eqM1Bz,eqM12Cz,eqF123x);
F01xs=eval(solF01.F01x);
F01ys=eval(solF01.F01y);
Mmzs=eval(solF01.Mmz);
F01s = [ F01xs, F01ys, 0 ];
Mms = [ 0, 0, Mmzs ];
fprintf('F01 = [ %g, %g, %g ] (N)\n', F01s);
fprintf('Mm = [ %g, %g, %g ] (N m)\n', Mms);
```

TheMATLABprogram using contour method and the results are given in Program 4.4.

4.7 R-RTR-RTR Mechanism

The planar R-RTR-RTR mechanism considered is shown in Fig. 4.17. The following numerical data are given: $AC = 0.060$ m, $AE = 0.250$ m and $CD = 0.150$ m. The widths of the links 1, 3, and 5 are $AB = 0.140$ m, $DF = 0.400$ m, and respectively, $EG = 0.500$ m. The height of the links 1, 3, and 5 is $h = 0.010$ m. The width of the links 2 and 4 is $w_{Slider} = 0.050$ m, and the height is $h_{Slider} = 0.020$ m. All five moving links are rectangular prisms with the depth $d = 0.001$ m. The angular velocity of the driver link 1 is $n = 50$ rpm. The density of the material is $\rho_{Steel} = \rho = 8000$ kg/m^3. The gravitational acceleration is $g = 9.807$ m/s^2. The center of mass locations of the links $i = 1, 2, ..., 5$ are designated by $C_i(x_{C_i}, y_{C_i}, 0)$.

The external moment applied on link 5 is opposed to the motion of the link: $\mathbf{M}_{5ext} = -\text{Sign}(\omega_5)\,|M_{ext}|\,\mathbf{k}$ where $|M_{ext}| = 100$ N·m and ω_5 is the angular velocity of link 5.

Find the motor moment $\mathbf{M}_m$ required for the dynamic equilibrium and the joint reaction forces when the driver link 1 makes an angle $\phi = \dfrac{\pi}{6}$ rad with the horizontal axis.

Solution

The position vectors (in meters) of the joints were calculated at subsection 2.4

position of joint A: $\mathbf{r}_A = \mathbf{0}$;

position of joint B: $\mathbf{r}_B = x_B\,\imath + y_B\,\jmath = 0.121\,\imath + 0.070\,\jmath$;

position of joint C: $\mathbf{r}_C = y_C\,\jmath = 0.060\,\jmath$;

position of joint D: $\mathbf{r}_D = x_D\,\imath + y_D\,\jmath = -0.149\,\imath + 0.047\,\jmath$;

position of joint E: $\mathbf{r}_E = y_E\,\jmath = -0.250\,\jmath$;

position of F: $\mathbf{r}_F = x_F\,\imath + y_F\,\jmath = 0.249\,\imath + 0.080\,\jmath$; and

position of G: $\mathbf{r}_G = x_G\,\imath + y_G\,\jmath = -0.224\,\imath + 0.196\,\jmath$.

The angles of the links with the horizontal are $\phi_2 = \phi_3 = 4.715°$ and $\phi_4 = \phi_5 = 16.666°$.

The position vector of the center of mass of link 1 is

$$\mathbf{r}_{C_1} = x_{C_1}\,\imath + y_{C_1}\,\jmath = \frac{x_B}{2}\,\imath + \frac{y_B}{2}\,\jmath = 0.060\,\imath + 0.035\,\jmath \quad \text{m}.$$

The position vector of the center of mass of slider 2 is

$$\mathbf{r}_{C_2} = x_{C_2}\,\imath + y_{C_2}\,\jmath = \mathbf{r}_B.$$

The position vector of the center of mass of link 3 is

$$\mathbf{r}_{C_3} = x_{C_3}\,\boldsymbol{\imath} + y_{C_3}\,\boldsymbol{\jmath} = \frac{x_D + x_F}{2}\,\boldsymbol{\imath} + \frac{y_D + y_F}{2}\,\boldsymbol{\jmath} = 0.049\,\boldsymbol{\imath} + 0.064\,\boldsymbol{\jmath} \quad \text{m}.$$

The position vector of the center of mass of slider 4 is

$$\mathbf{r}_{C_4} = x_{C_4}\,\boldsymbol{\imath} + y_{C_4}\,\boldsymbol{\jmath} = \mathbf{r}_D.$$

The position vector of the center of mass of link 5 is

$$\mathbf{r}_{C_5} = x_{C_5}\,\boldsymbol{\imath} + y_{C_5}\,\boldsymbol{\jmath} = \frac{x_E + x_G}{2}\,\boldsymbol{\imath} + \frac{y_E + y_G}{2}\,\boldsymbol{\jmath} = -0.112\,\boldsymbol{\imath} - 0.026\,\boldsymbol{\jmath} \quad \text{m}.$$

The velocity and acceleration analysis was carried out at subsection 3.8:
acceleration of joint B: $\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = -3.323\,\boldsymbol{\imath} - 1.919\,\boldsymbol{\jmath}$ m/s^2;
acceleration of joint D: $\mathbf{a}_{D_3} = \mathbf{a}_{D_4} = 4.617\,\boldsymbol{\imath} - 1.811\,\boldsymbol{\jmath}$ m/s^2;
acceleration of joint F: $\mathbf{a}_F = -7.695\,\boldsymbol{\imath} + 3.019\,\boldsymbol{\jmath}$ m/s^2;
acceleration of joint G: $\mathbf{a}_G = 2.767\,\boldsymbol{\imath} + 0.919\,\boldsymbol{\jmath}$ m/s^2;
angular velocity of link 5: $\boldsymbol{\omega}_5 = 0.917\,\mathbf{k}$ rad/s;
angular acceleration of link 1: $\boldsymbol{\alpha}_1 = 0\,\mathbf{k}$ rad/s^2;
angular acceleration of links 2 and 3: $\boldsymbol{\alpha}_2 = \boldsymbol{\alpha}_3 = 14.568\,\mathbf{k}$ rad/s^2;
angular acceleration of links 4 and 5: $\boldsymbol{\alpha}_4 = \boldsymbol{\alpha}_5 = -5.771\,\mathbf{k}$ rad/s^2.
The acceleration vector of the center of mass of link 1 is

$$\mathbf{a}_{C_1} = \frac{\mathbf{a}_{B_1}}{2} = -1.661\,\boldsymbol{\imath} - 0.959\,\boldsymbol{\jmath} \quad \text{m/s}^2.$$

The acceleration vector of the center of mass of slider 2 is

$$\mathbf{r}_{C_2} = \mathbf{a}_{B_2} = -3.323\,\boldsymbol{\imath} - 1.919\,\boldsymbol{\jmath} \quad \text{m/s}^2.$$

The acceleration vector of the center of mass of link 3 is

$$\mathbf{a}_{C_3} = \frac{\mathbf{a}_{D_3} + \mathbf{a}_F}{2} = -1.539\,\boldsymbol{\imath} + 0.603\,\boldsymbol{\jmath} \quad \text{m/s}^2.$$

The acceleration vector of the center of mass of slider 4 is

$$\mathbf{a}_{C_4} = \mathbf{a}_{D_4} = 4.617\,\boldsymbol{\imath} - 1.811\,\boldsymbol{\jmath} \quad \text{m/s}^2.$$

The acceleration vector of the center of mass of link 5 is

$$\mathbf{a}_{C_5} = \frac{\mathbf{a}_E + \mathbf{a}_G}{2} = 1.383\,\boldsymbol{\imath} + 0.459\,\boldsymbol{\jmath} \quad \text{m/s}^2.$$

The MATLABprogram for positions, velocities, and accelerations is

```
AB = 0.14; AC = 0.06; AE = 0.25; CD = 0.15; DF=0.4; EG=0.5;
phi = 30*(pi/180);
xA = 0; yA = 0; rA = [xA yA 0];
xC = 0; yC = AC; rC = [xC yC 0];
xE = 0; yE = -AE; rE = [xE yE 0];
xB = AB*cos(phi); yB = AB*sin(phi); rB = [xB yB 0];
eqnD1 = '( xDsol - xC )^2 + ( yDsol - yC )^2 = CD^2 ';
eqnD2 = '(yB - yC)/(xB - xC) = (yDsol - yC)/(xDsol - xC)';
solD = solve(eqnD1, eqnD2, 'xDsol, yDsol');
xDpositions = eval(solD.xDsol); yDpositions = eval(solD.yDsol);
xD1 = xDpositions(1); xD2 = xDpositions(2);
yD1 = yDpositions(1); yD2 = yDpositions(2);
if (phi>=0 && phi<=pi/2)||(phi >= 3*pi/2 && phi<=2*pi)
if xD1 <= xC xD = xD1; yD=yD1; else xD = xD2; yD=yD2; end
else
if xD1 >= xC xD = xD1; yD=yD1; else xD = xD2; yD=yD2; end
end
rD = [xD yD 0];
phi2 = atan((yB-yC)/(xB-xC)); phi3 = phi2;
phi4 = atan((yD-yE)/(xD-xE))+pi; phi5 = phi4;
xF = xD + DF*cos(phi3); yF = yD + DF*sin(phi3); rF = [xF yF 0];
xG = xE + EG*cos(phi5); yG = yE + EG*sin(phi5); rG = [xG yG 0];
xC1 = xB/2; yC1 = yB/2; rC1 = [xC1 yC1 0];
rC2 = rB;
xC3 = (xD+xF)/2; yC3 = (yD+yF)/2; rC3 = [xC3 yC3 0];
rC4 = rD;
xC5 = (xE+xG)/2; yC5 = (yE+yG)/2; rC5 = [xC5 yC5 0];
n = 50.;
omega1 = [ 0 0 pi*n/30 ]; alpha1 = [0 0 0 ];
vA = [0 0 0 ]; aA = [0 0 0 ];
vB1 = vA + cross(omega1,rB); vB2 = vB1;
aB1 = aA + cross(alpha1,rB) - dot(omega1,omega1)*rB; aB2 = aB1;
omega3z=sym('omega3z','real'); alpha3z=sym('alpha3z','real');
vB32=sym('vB32','real'); aB32=sym('aB32','real');
omega3 = [ 0 0 omega3z ];
vC = [0 0 0 ];
```

```
vB3 = vC + cross(omega3,rB-rC);
vB3B2 = vB32*[ cos(phi2) sin(phi2) 0];
eqvB = vB3 - vB2 - vB3B2;
eqvBx = eqvB(1); eqvBy = eqvB(2);
solvB = solve(eqvBx,eqvBy);
omega3zs=eval(solvB.omega3z); vB32s=eval(solvB.vB32);
Omega3 = [0 0 omega3zs]; Omega2 = Omega3;
v32 = vB32s*[cos(phi2) sin(phi2) 0];
vD3 = vC + cross(Omega3,rD-rC); vD4 = vD3;
aB3B2cor = 2*cross(Omega3,v32);
alpha3 = [ 0 0 alpha3z ];
aC = [0 0 0 ];
aB3 = aC + cross(alpha3,rB-rC) - dot(Omega3,Omega3)*(rB-rC);
aB3B2 = aB32*[ cos(phi2) sin(phi2) 0];
eqaB = aB3 - aB2 - aB3B2 - aB3B2cor;
eqaBx = eqaB(1); eqaBy = eqaB(2);
solaB = solve(eqaBx,eqaBy);
alpha3zs=eval(solaB.alpha3z); aB32s=eval(solaB.aB32);
Alpha3 = [0 0 alpha3zs]; Alpha2 = Alpha3;
aD3=aC + cross(Alpha3,rD-rC) - dot(Omega3,Omega3)*(rD-rC); aD4=aD3;
omega5z=sym('omega5z','real'); alpha5z=sym('alpha5z','real');
vD54=sym('vD54','real'); aD54=sym('aD54','real');
omega5 = [ 0 0 omega5z ];
vE = [0 0 0 ];
vD5 = vE + cross(omega5,rD-rE);
vD5D4 = vD54*[ cos(phi5) sin(phi5) 0];
eqvD = vD5 - vD4 - vD5D4; eqvDx = eqvD(1); eqvDy = eqvD(2);
solvD = solve(eqvDx,eqvDy);
omega5zs=eval(solvD.omega5z); vD54s=eval(solvD.vD54);
Omega5 = [0 0 omega5zs];
v54 = vD54s*[cos(phi5) sin(phi5) 0];
Omega4 = Omega5;
aD5D4cor = 2*cross(Omega5,v54);
alpha5 = [ 0 0 alpha5z ];
aE = [0 0 0 ];
aD5 = aE + cross(alpha5,rD-rE) - dot(Omega5,Omega5)*(rD-rE);
aD5D4 = aD54*[ cos(phi5) sin(phi5) 0];
eqaD = aD5 - aD4 - aD5D4 - aD5D4cor;
```

```
eqaDx = eqaD(1); eqaDy = eqaD(2);
solaD = solve(eqaDx,eqaDy);
alpha5zs=eval(solaD.alpha5z); aD54s=eval(solaD.aD54);
Alpha5 = [0 0 alpha5zs]; Alpha4 = Alpha5;
aF = aC + cross(Alpha3,rF-rC) - dot(Omega3,Omega3)*(rF-rC);
aG = aE + cross(Alpha5,rG-rE) - dot(Omega5,Omega5)*(rG-rE);
aC1 = aB1/2;
aC2 = aB2;
aC3 = (aD3+aF)/2;
aC4 = aD3;
aC5 = (aE+aG)/2;
```

The external moment applied on link 5 is opposed to the motion of the link

$$\mathbf{M}_{5ext} = -\mathrm{Sign}(\omega_5)\,|M_{ext}|\,\mathbf{k} = -\mathrm{Sign}(0.917)\,(100)\,\mathbf{k} = -100\,\mathbf{k} \quad \mathrm{N\,m}.$$

Inertia forces and moments

Link 1

The mass of the link is

$$m_1 = \rho\ AB\ h\ d = 8000(0.14)(0.01)(0.001) = 0.0112 \quad \mathrm{kg}.$$

The inertia force of driver 1 at C_1 is

$$\mathbf{F}_{in\,1} = -m_1\,\mathbf{a}_{C_1} = -0.0112(-1.66198\boldsymbol{\imath}-0.959545\boldsymbol{\jmath}) = 0.0186142\boldsymbol{\imath}-0.0107469\boldsymbol{\jmath} \quad \mathrm{N}.$$

The gravitational force on link 1 at C_1 is

$$\mathbf{G}_1 = -m_1\ g\ \boldsymbol{\jmath} = -0.0112(9.807)\,\boldsymbol{\jmath} = -0.109838\,\boldsymbol{\jmath} \quad \mathrm{N}.$$

The mass moment of inertia of link 1 with respect to C_1 is

$$I_{C_1} = m_1\,(AB^2+h^2)/12 = 0.0112(0.14^2+0.01^2)/12 = 1.83867\times10^{-5} \quad \mathrm{kg\,m}^2.$$

The moment of inertia of driver 1 is

$$\mathbf{M}_{in\,1} = -I_{C_1}\ \boldsymbol{\alpha}_1 = \mathbf{0}.$$

To calculate the inertia force and the moment the following MATLABcommands are used

```
m1 = rho*AB*h*d;
Fin1 = -m1*aC1;
G1 = [0,-m1*g,0];
IC1 = m1*(AB^2+h^2)/12;
Min1 = -IC1*alpha1;
```

Link 2
The mass of the slider 2 is

$$m_2 = \rho \; h_{Slider} \; w_{Slider} \; d = 8000(0.02)(0.05)(0.001) = 0.008 \quad \text{kg}.$$

The inertia force of slider 2 at C_2 is

$$\mathbf{F}_{in\,2} = -m_2 \, \mathbf{a}_{C_2} = -0.008(-3.323\,\mathbf{\imath} - 1.919\,\mathbf{\jmath}) = 0.0265917\,\mathbf{\imath} + 0.0153527\,\mathbf{\jmath} \quad \text{N}.$$

The gravitational force of slider 2 at C_2 is

$$\mathbf{G}_2 = -m_2 \; g \; \mathbf{\jmath} = -0.008(9.807)\,\mathbf{\jmath} = -0.109838\,\mathbf{\jmath} \quad \text{N}.$$

The mass moment of inertia of slider 2 with respect to C_2 is

$$I_{C_2} = m_2 \, (h^2_{Slider} + w^2_{Slider})/12 = 0.008(0.02^2 + 0.05^2)/12 = 1.93333 \times 10^{-6} \quad \text{kg m}^2.$$

The moment of inertia of slider 2 is

$$\mathbf{M}_{in\,2} = -I_{C_2} \boldsymbol{\alpha}_2 = -1.93333 \times 10^{-6} \, (14.568) \, \mathbf{k} = -2.8165 \times 10^{-5} \, \mathbf{k} \quad \text{N m}.$$

TheMATLABcommands to calculate the inertia force and the moment are

```
m2 = rho*hSlider*wSlider*d;
Fin2 = -m2*aC2;
G2 = [0,-m2*g,0];
IC2 = m2*(hSlider^2+wSlider^2)/12;
Min2 = -IC2*Alpha2;
```

Link 3
The mass of the link is

$$m_3 = \rho\ FD\ h\ d = 8000(0.4)(0.01)(0.001) = 0.032 \quad \text{kg}.$$

The inertia force of link 3 is

$$\mathbf{F}_{in\,3} = -m_3\ \mathbf{a}_{C_3} = -0.032(-1.539\,\imath + 0.603\,\jmath) = 0.0492489\,\imath - 0.019326\,\jmath \quad \text{N}.$$

The gravitational force of link 3 is

$$\mathbf{G}_3 = -m_3\ g\ \jmath = -0.032(9.807)\,\jmath = -0.313824\,\jmath \quad \text{N}.$$

The mass moment of inertia is

$$I_{C_3} = m_3\ (FD^2 + h^2)/12 = 0.032(0.4^2 + 0.01^2)/12 = 0.000426933 \quad \text{kg}\,\text{m}^2.$$

The inertia moment on link 3 is

$$\mathbf{M}_{in\,3} = -I_{C_3}\boldsymbol{\alpha}_3 = -0.000426933(14.568)\,\mathbf{k} = -0.00621962\,\mathbf{k} \quad \text{N}\,\text{m}.$$

Link 4
The mass of the link is

$$m_4 = \rho\ h_{Slider}\ w_{Slider}\ d = 8000(0.02)(0.05)(0.001) = 0.008 \quad \text{kg}.$$

The inertia force is

$$\mathbf{F}_{in\,4} = -m_4\ \mathbf{a}_{C_4} = -0.008(4.617\,\imath - 1.811\,\jmath) = -0.0369367\,\imath + 0.0144946\,\jmath \quad \text{N}.$$

The gravitational force is

$$\mathbf{G}_4 = -m_4\ g\ \jmath = -0.008(9.807)\,\jmath = -0.109838\,\jmath \quad \text{N}.$$

The mass moment of inertia is

$$I_{C_4} = m_4(h^2_{Slider} + w^2_{Slider})/12 = 0.008(0.02^2 + 0.05^2)/12 = 1.93333\times 10^{-6} \quad \text{kg}\,\text{m}^2.$$

The moment of inertia is

$$\mathbf{M}_{in\,4} = -I_{C_4}\ \boldsymbol{\alpha}_4 = -1.93333 \times 10^{-6}\,(-5.771)\,\mathbf{k} = 1.11583 \times 10^{-5}\,\mathbf{k} \quad \text{N}\,\text{m}.$$

Link 5

The mass of the link is

$$m_5 = \rho\ EG\ h\ d = 8000(0.5)(0.01)(0.001) = 0.04 \quad \text{kg}.$$

The inertia force is

$$\mathbf{F}_{in\,5} = -m_5\ \mathbf{a}_{C_5} = -0.04(1.383\,\imath + 0.459\,\jmath) = -0.0553516\,\imath - 0.0183855\,\jmath \quad \text{N}.$$

The gravitational force is

$$\mathbf{G}_5 = -m_5\ g\ \jmath = -0.04(9.807)\,\jmath = -0.39228\,\jmath \quad \text{N}.$$

The mass moment of inertia is

$$I_{C_5} = m_5\ (EG^2 + h^2)/12 = 0.04(0.5^2 + 0.01^2)/12 = 0.000833667 \quad \text{kg}\,\text{m}^2.$$

The moment of inertia is

$$\mathbf{M}_{in\,5} = -I_{C_5}\ \boldsymbol{\alpha}_5 = -0.000833667\,(-5.771)\,\mathbf{k} = 0.00481155\,\mathbf{k} \quad \text{N}\,\text{m}.$$

TheMATLABcommands to calculate the inertia force and the moment for links 3, 4, and 5 are

```
m3 = rho*DF*h*d;
Fin3 = -m3*aC3;
G3 = [0,-m3*g,0];
IC3 = m3*(DF^2+h^2)/12;
Min3 = -IC3*Alpha3;
m4 = rho*hSlider*wSlider*d;
Fin4 = -m4*aC4;
G4 = [0,-m4*g,0];
IC4 = m4*(hSlider^2+wSlider^2)/12;
Min4 = -IC4*Alpha4;
m5 = rho*EG*h*d;
Fin5 = -m5*aC5;
G5 = [0,-m5*g,0];
IC5 = m5*(EG^2+h^2)/12;
Min5 = -IC5*Alpha5;
```

Joint forces and drive moment

4.7.1 Newton-Euler Equations of Motion

The force analysis starts with the link 5 because the external moment $\mathbf{M}_{5ext}$ is given. Figure 4.18(a) shows the free body diagram of the link 5. The joint reaction force of the ground 0 on the link 5 at the joint F is $\mathbf{F}_{05} = F_{05x}\mathbf{\imath}+F_{05y}\mathbf{\jmath}$. The joint reaction force of the link 4 on the link 5 is $\mathbf{F}_{45} = F_{45x}\mathbf{\imath}+F_{45y}\mathbf{\jmath}$. The application point of the force $\mathbf{F}_{45}$ is $P(x_P,\, y_P)$ and the position vector of P is $\mathbf{r}_P = x_P\mathbf{\imath} + y_P\mathbf{\jmath}$.

The symbolical six unknowns F_{05x}, F_{05y}, F_{45x}, F_{45y}, x_P, and y_P are introduced inMATLABusing the commands

```
F05x=sym('F05x','real');
F05y=sym('F05y','real');
F45x=sym('F45x','real');
F45y=sym('F45y','real');
xP=sym('xP','real');
yP=sym('yP','real');
F05=[ F05x, F05y, 0 ]; % unknown joint force of 0 on 5
F45=[ F45x, F45y, 0 ]; % unknown joint force of 4 on 5
rP=[ xP, yP, 0 ]; % unknown application point of force F45
```

The point P, the application of the force $\mathbf{F}_{45}$, is located on the direction DE, that is

$$(\mathbf{r}_D - \mathbf{r}_E) \times (\mathbf{r}_P - \mathbf{r}_E) = \mathbf{0}. \tag{4.76}$$

Equation (4.76) is written inMATLABas

```
eqP=cross(rD-rE,rP-rE);
eqPz=eqP(3);
```

The direction of the unknown joint force $\mathbf{F}_{45}$ is perpendicular to the sliding direction $\mathbf{r}_{DE}$

$$\mathbf{F}_{45} \cdot \mathbf{r}_{DE} = 0, \tag{4.77}$$

or inMATLAB

```
eqF45DE=dot(F45,rD-rE);
```

For the link 5 the vector sum of the net forces, gravitational force $\mathbf{G}_5$, joint forces $\mathbf{F}_{05}$, $\mathbf{F}_{45}$, is equal to $m_5\,\mathbf{a}_{C_5}$ [Fig. 4.18(a)]

$$m_5\,\mathbf{a}_{C_5} = \mathbf{F}_{05} + \mathbf{F}_{45} + \mathbf{G}_5,$$

or usingMATLABcommands

```
eqF5=F05+F45+G5-m5*aC5;
```

Projecting the previous vectorial onto x and y axes gives

$$m_5\,a_{C_{5x}} = F_{05x} + F_{45x}, \tag{4.78}$$

$$m_5\,a_{C_{5y}} = F_{05y} + F_{45y} - m_5\,g, \tag{4.79}$$

or using MATLAB

```
eqF5x=eqF5(1); % projection on x-axis
eqF5y=eqF5(2); % projection on y-axis
```

The vector sum of the moments that act on link 5 with respect to the center of mass C_5 is equal to $I_{C_5}\,\boldsymbol{\alpha}_5$ [Fig. 4.18(a)]

$$I_{C_5}\,\boldsymbol{\alpha}_5 = \mathbf{r}_{C_5E} \times \mathbf{F}_{05} + \mathbf{r}_{C_5P} \times \mathbf{F}_{45} + \mathbf{M}_{5ext}, \tag{4.80}$$

or in MATLAB

```
eqMC5=cross(rE-rC5,F05)+cross(rP-rC5,F45)+Me-IC5*Alpha5;
eqMC5z=eqMC5(3); % projection on z-axis
```

There are five equations Eqs. (4.76)-(4.80) and six unknowns and that is why the analysis will continue with the slider 4. The free body diagram of the slider 4 is shown in Fig. 4.18(b).

The joint reaction force of the link 3 on the slider 4 at $D = C_4$ is $\mathbf{F}_{34} = F_{34x}\mathbf{\imath} + F_{34y}\mathbf{\jmath}$ and the joint reaction force of the link 5 on the slider 4 is $\mathbf{F}_{54} = -\mathbf{F}_{45} = -F_{45x}\mathbf{\imath} - F_{45y}\mathbf{\jmath}$. TheMATLABcommands are

```
F34x=sym('F34x','real');
```

```
F34y=sym('F34y','real');
F34=[ F34x, F34y, 0 ]; % unknown joint force of 3 on 4
F54=-F45; % joint force of 5 on 4
```

For the slider 4, according to Newton's equations of motion, the vector sum of the net forces, gravitational force $\mathbf{G}_4$, joint forces $\mathbf{F}_{34}$, $\mathbf{F}_{54}$), is equal to $m_4\,\mathbf{a}_{C_4}$

$$m_4\,\mathbf{a}_{C_4} = \mathbf{F}_{34} + \mathbf{F}_{54} + \mathbf{G}_4,$$

or usingMATLABcommands

```
eqF4=F34-F45+G4-m4*aC4;
```

Projecting the previous vectorial onto x and y axes gives

$$m_4\,a_{C_{4x}} = F_{34x} + F_{54x}, \tag{4.81}$$
$$m_4\,a_{C_{4y}} = F_{34y} + F_{54y} - m_4\,g, \tag{4.82}$$

or using MATLAB

```
eqF4x=eqF4(1);
eqF4y=eqF4(2);
```

The vector sum of the moments that act on slider 4 with respect to the center of mass $D = C_4$ is equal to $I_{C_4}\,\boldsymbol{\alpha}_4$

$$I_{C_4}\,\boldsymbol{\alpha}_4 = \mathbf{r}_{C_4P} \times \mathbf{F}_{54}, \tag{4.83}$$

or in MATLAB

```
eqMC4=cross(rP-rC4,F54)-IC4*Alpha4;
eqMC4z=eqMC4(3);
```

There are eight equations Eqs. (4.76)-(4.83) with eight unknowns F_{05x}, F_{05y}, F_{45x}, F_{45y}, x_P, y_P, F_{34x}, and F_{34y}. The system is solved using MATLAB

```
sol45=solve(eqF5x,eqF5y,eqMC5z,eqF45DE,eqPz,eqF4x,eqF4y,eqMC4z);
F05xs=eval(sol45.F05x);
```

```
F05ys=eval(sol45.F05y);
F05s=[ F05xs, F05ys, 0 ];
F45xs=eval(sol45.F45x);
F45ys=eval(sol45.F45y);
F45s=[ F45xs, F45ys, 0 ];
F34xs=eval(sol45.F34x);
F34ys=eval(sol45.F34y);
F34s=[ F34xs, F34ys, 0 ];
yPs=eval(sol45.yP);
rPs=[xPs, yPs, 0];
```

The following numerical solution are obtained

$$\begin{aligned}
\mathbf{F}_{05} &= 268.165\,\imath + 135.057\,\jmath \text{ N},\\
\mathbf{F}_{45} &= -268.109\,\imath - 134.647\,\jmath \text{ N},\\
\mathbf{F}_{34} &= -268.072\,\imath - 134.583\,\jmath \text{ N}, \quad \text{and}\\
\mathbf{r}_P &= -0.149492\,\imath + 0.0476701\,\jmath \text{ m}.
\end{aligned}$$

The force analysis continues with the link 3. Figure 4.19(a) shows the free body diagram of the link 3. The joint reaction force of the link 4 on the link 3 is $\mathbf{F}_{43} = -\mathbf{F}_{34} = 268.072\imath + 134.583\jmath$ N. The joint reaction force of the ground 0 on the link 3 at the joint C is $\mathbf{F}_{03} = F_{03x}\imath + F_{03y}\jmath$. The joint reaction force of the link 2 on the link 3 is $\mathbf{F}_{423} = F_{23x}\imath + F_{23y}\jmath$. The application point of the force $\mathbf{F}_{23}$ is $Q(x_Q, y_Q)$ and the position vector of Q is $\mathbf{r}_Q = x_Q\imath + y_Q\jmath$.

The symbolical six unknowns F_{03x}, F_{03y}, F_{23x}, F_{23y}, x_Q, and y_Q are introduced inMATLABusing the commands

```
F03x=sym('F03x','real');
F03y=sym('F03y','real');
F23x=sym('F23x','real');
F23y=sym('F23y','real');
xQ=sym('xQ','real');
yQ=sym('yQ','real');
F03=[ F03x, F03y, 0 ]; % unknown joint force of 0 on 3
F23=[ F23x, F23y, 0 ]; % unknown joint force of 2 on 3
rQ=[xQ, yQ, 0]; % unknown application point of force F23
```

The point Q, the application of the force $\mathbf{F}_{23}$, is located on the direction BC, that is

$$(\mathbf{r}_B - \mathbf{r}_C) \times (\mathbf{r}_Q - \mathbf{r}_C) = \mathbf{0}. \tag{4.84}$$

Equation (4.84) is written inMATLABas

```
eqQ=cross(rB-rC,rQ-rC);
eqQz=eqQ(3);
```

The direction of the unknown joint force $\mathbf{F}_{23}$ is perpendicular to the sliding direction $\mathbf{r}_{BC}$

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0, \tag{4.85}$$

or inMATLAB

```
eqF23BC = dot(F23,rB-rC);
```

For the link 3 the vector sum of the net forces, gravitational force $\mathbf{G}_3$, joint forces $\mathbf{F}_{43}$, $\mathbf{F}_{03}$, $\mathbf{F}_{23}$, is equal to $m_3\,\mathbf{a}_{C_3}$ [Fig. 4.19(a)]

$$m_3\,\mathbf{a}_{C_3} = \mathbf{F}_{43} + \mathbf{F}_{03} + \mathbf{F}_{23} + \mathbf{G}_3,$$

or usingMATLABcommands

```
eqF3=F43+F03+F23+G3-m3*aC3;
```

Projecting the previous vectorial onto x and y axes gives

$$m_3\,a_{C_{3x}} = F_{43x} + F_{03x} + F_{23x}, \tag{4.86}$$
$$m_3\,a_{C_{3y}} = F_{43x} + F_{03y} + F_{23y} - m_3\,g, \tag{4.87}$$

or using MATLAB

```
eqF3x=eqF3(1); % projection on x-axis
eqF3y=eqF3(2); % projection on y-axis
```

The vector sum of the moments that act on link 3 with respect to the center of mass C_3 is equal to $I_{C_3}\,\boldsymbol{\alpha}_3$ [Fig. 4.19(a)]

$$I_{C_3}\,\boldsymbol{\alpha}_3 = \mathbf{r}_{C_3D} \times \mathbf{F}_{43} + \mathbf{r}_{C_3C} \times \mathbf{F}_{03} + \mathbf{r}_{C_5Q} \times \mathbf{F}_{23}, \tag{4.88}$$

or in MATLAB

```
eqMC3=cross(rD-rC3,F43)+cross(rC-rC3,F03)+cross(rQ-rC3,F23)-IC3*Alpha3;
eqMC3z=eqMC3(3); % projection on z-axis
```

There are five equations Eqs. (4.84)-(4.88) and six unknowns and that is why the analysis will continue with the slider 2. The free body diagram of the slider 2 is shown in Fig. 4.19(b).

The joint reaction force of the link 1 on the slider 2 at B is $\mathbf{F}_{12} = F_{12x}\mathbf{\imath} + F_{12y}\mathbf{\jmath}$ and the joint reaction force of the link 3 on the slider 2 is $\mathbf{F}_{32} = -\mathbf{F}_{23} = -F_{23x}\mathbf{\imath} - F_{23y}\mathbf{\jmath}$. TheMATLABcommands are

```
F12x=sym('F12x','real');
F12y=sym('F12y','real');
F12=[ F12x, F12y, 0 ]; % unknown joint force of 1 on 2
F32=-F23; % joint force of 3 on 2
```

For the slider 2 the vector sum of the net forces, gravitational force $\mathbf{G}_2$, joint forces $\mathbf{F}_{32}$, $\mathbf{F}_{12}$, is equal to $m_2\,\mathbf{a}_{C_2}$

$$m_2\,\mathbf{a}_{C_2} = \mathbf{F}_{32} + \mathbf{F}_{12} + \mathbf{G}_2,$$

or usingMATLABcommands

```
eqF2=F32+F12+G2-m2*aC2;
```

Projecting the previous vectorial onto x and y axes gives

$$m_2\,a_{C_{2x}} = F_{32x} + F_{12x}, \tag{4.89}$$

$$m_2\,a_{C_{2y}} = F_{32y} + F_{12y} - m_2\,g, \tag{4.90}$$

or using MATLAB

```
eqF2x=eqF2(1);
eqF2y=eqF2(2);
```

The vector sum of the moments that act on slider 2 with respect to the center of mass $B = C_2$ is equal to $I_{C_2}\,\boldsymbol{\alpha}_2$

$$I_{C_2}\,\boldsymbol{\alpha}_2 = \mathbf{r}_{C_2Q} \times \mathbf{F}_{32}, \tag{4.91}$$

or in MATLAB

```
eqMC2=cross(rQ-rC2,F32)-IC2*Alpha2;
eqMC2z=eqMC2(3); % projection on z-axis
```

There are eight equations Eqs. (4.84)-(4.91) with eight unknowns F_{03x}, F_{03y}, F_{23x}, F_{23y}, x_Q, y_Q, F_{12x}, and F_{12y}. The system is solved using MATLAB

```
sol23=solve(eqF3x,eqF3y,eqMC3z,eqF23BC,eqQz,eqF2x,eqF2y,eqMC2z);
F03xs=eval(sol23.F03x);
F03ys=eval(sol23.F03y);
F03s=[ F03xs, F03ys, 0 ];
F23xs=eval(sol23.F23x);
F23ys=eval(sol23.F23y);
F23s=[ F23xs, F23ys, 0 ];
F12xs=eval(sol23.F12x);
F12ys=eval(sol23.F12y);
F12s=[ F12xs, F12ys, 0 ];
xQs=eval(sol23.xQ);
yQs=eval(sol23.yQ);
rQs=[xQs, yQs, 0];
```

The following numerical solution are obtained

$$\begin{aligned}
\mathbf{F}_{03} &= -256.745\,\imath - 272.179\,\jmath \text{ N},\\
\mathbf{F}_{23} &= -11.3762\,\imath + 137.93\,\jmath \text{ N},\\
\mathbf{F}_{12} &= -11.4028\,\imath + 137.993\,\jmath \text{ N}, \quad \text{and}\\
\mathbf{r}_{Q} &= 0.121243\,\imath + 0.07\,\jmath \text{ m}.
\end{aligned}$$

The force analysis ends with the driver link 1. Figure 4.20 shows the free body diagram of the link 1. The joint reaction force of the link 2 on the link 1 is $\mathbf{F}_{21} = -\mathbf{F}_{12} = 11.4028\,\imath - 137.993\,\jmath$ N.

The joint reaction force of the ground 0 on the link 1 at the joint A is $\mathbf{F}_{01} = F_{01x}\,\imath + F_{01y}\,\jmath$. For the link 1 the vector sum of the net forces, gravitational force $\mathbf{G}_1$, joint forces $\mathbf{F}_{01}$, $\mathbf{F}_{21}$, is equal to $m_1\,\mathbf{a}_{C_1}$ (Fig. 4.20)

$$m_1\,\mathbf{a}_{C_1} = \mathbf{F}_{01} - \mathbf{F}_{12} + \mathbf{G}_1 \quad \Longrightarrow \quad \mathbf{F}_{01} = m_1\,\mathbf{a}_{C_1} + \mathbf{F}_{12} - \mathbf{G}_1,$$

or with MATLAB

```
F01=m1*aC1+F12s-G1;
```

The vector sum of the moments that act on link 1 with respect to the center of mass C_1 is equal to $I_{C_1}\,\boldsymbol{\alpha}_1$

$$I_{C_1}\,\boldsymbol{\alpha}_1 = \mathbf{r}_{C_1A} \times \mathbf{F}_{01} - \mathbf{r}_{C_1B} \times \mathbf{F}_{12} + \mathbf{M}_{mot},$$

and the equilibrium moment (motor moment) is

$$\mathbf{M}_{mot} = I_{C_1}\,\boldsymbol{\alpha}_1 - \mathbf{r}_{C_1A} \times \mathbf{F}_{01} + \mathbf{r}_{C_1B} \times \mathbf{F}_{12}.$$

InMATLABthe equilibrium moment is

```
Mm=IC1*alpha1-cross(rA-rC1,F01)+cross(rB-rC1,F12s);
```

Another way of calculating the equilibrium moment is to take the sum of the moments that act on link 1 with respect A

$$I_{C_1}\,\boldsymbol{\alpha}_1 + \mathbf{r}_{C_1} \times m_1\,\mathbf{a}_{C_1} = \mathbf{r}_{C_1} \times \mathbf{G}_1 + \mathbf{r}_B \times (-\mathbf{F}_{12}) + \mathbf{M}_{mot},$$

and the equilibrium moment is

$$\mathbf{M}_{mot} = \mathbf{r}_B \times \mathbf{F}_{12} + \mathbf{r}_{C_1} \times (m_1\,\mathbf{a}_{C_1} - \mathbf{G}_1) + I_{C_1}\,\boldsymbol{\alpha}_1,$$

or in MATLAB

```
Mm=cross(rB,F12s)+cross(rC1,m1*aC1-G1)+IC1*alpha1;
```

The joint reaction force of the ground 0 on the link 1 is $\mathbf{F}_{01} = -11.4214\,\boldsymbol{\imath}+ 138.092\,\boldsymbol{\jmath}$ N, and the equilibrium moment is $\mathbf{M}_{mot} = 17.5356\,\mathbf{k}$ N m.

4.7.2 Dyad Method

The dynamic force analysis starts with the last dyad (links 5 and 4) because the external moment $\mathbf{M}_{5ext}$ on link 5 is known.

$E_R\, D_T\, D_R$ dyad
Figure 4.21 shows the forces and the moments that act on the dyad $E_R\, D_T\, D_R$. The unknown joint reaction forces are $\mathbf{F}_{05} = F_{05x}\,\imath + F_{05y}\,\jmath$, $\mathbf{F}_{34} = F_{34x}\,\imath + F_{34y}\,\jmath$, or in MATLAB

```
F05x=sym('F05x','real');
F05y=sym('F05y','real');
F34x=sym('F34x','real');
F34y=sym('F34y','real');
F05=[ F05x, F05y, 0 ] ;
F34=[ F34x, F34y, 0 ] ;
```

The Newton equation for links 5 and 4

$$m_5\,\mathbf{a}_{C_5} + m_4\,\mathbf{a}_{C_4} = \mathbf{F}_{05} + \mathbf{G}_5 + \mathbf{G}_4 + \mathbf{F}_{34} \quad \Longrightarrow$$
$$\sum \mathbf{F}^{(5\&4)} = \mathbf{F}_{05} + \mathbf{G}_5 + \mathbf{G}_4 + \mathbf{F}_{34} - m_5\,\mathbf{a}_{C_5} - m_4\,\mathbf{a}_{C_4} = \mathbf{0}. \quad (4.92)$$

Equation (4.92) has a component on x-axis, $\sum \mathbf{F}^{(5\&4)} \cdot \imath$, a component on y-axis, $\sum \mathbf{F}^{(5\&4)} \cdot \jmath$, and the MATLABcommands are

```
eqF45=F05+G5+G4+F34-m5*aC5-m4*aC4;
eqF45x=eqF45(1); % projection on x-axis
eqF45y=eqF45(2); % projection on y-axis
```

The Euler equation of moments for links 5 and 4 about D_R gives

$$I_{C_5}\,\boldsymbol{\alpha}_5 + \mathbf{r}_{DC_5} \times m_5\,\mathbf{a}_{C_5} + I_{C_4}\,\boldsymbol{\alpha}_4 = \mathbf{r}_{DE} \times \mathbf{F}_{05} + \mathbf{r}_{DC_5} \times \mathbf{G}_5 + \mathbf{M}_{5ext} \quad \Longrightarrow$$
$$\sum \mathbf{M}_D^{(5\&4)} = (\mathbf{r}_E - \mathbf{r}_D) \times \mathbf{F}_{05} + (\mathbf{r}_{C_5} - \mathbf{r}_D) \times (\mathbf{G}_5 - m_5\,\mathbf{a}_{C_5}) + \mathbf{M}_{5ext}$$
$$-I_{C_5}\,\boldsymbol{\alpha}_5 - I_{C_4}\,\boldsymbol{\alpha}_4 = \mathbf{0}. \quad (4.93)$$

The MATLABcommands for Eq. (4.93) are

```
eqMD45=cross(rE-rD,F05)+cross(rC5-rD,G5-m5*aC5)+Me-IC5*Alpha5-IC4*Alpha4;
```

```
eqMD45z=eqMD45(3); % projection on z-axis
```

The Newton equation for link 4 projected on the sliding direction ED is

$$(m_4\,\mathbf{a}_{C_4})\cdot\mathbf{r}_{ED} = (\mathbf{F}_{34}+\mathbf{G}_4+\mathbf{F}_{54})\cdot\mathbf{r}_{ED} \quad \Longrightarrow$$
$$\sum\mathbf{F}^{(4)}\cdot\mathbf{r}_{ED} = (\mathbf{F}_{34}+\mathbf{G}_4-m_4\,\mathbf{a}_{C_4})\cdot(\mathbf{r}_D-\mathbf{r}_E)=0. \qquad (4.94)$$

The force of the link 5 on link 4 is $\mathbf{F}_{54}$ and $\mathbf{F}_{54}\cdot\mathbf{r}_{ED}=0$. The MATLABcommand for Eq. (4.94) is

```
eqF4DE=dot(F34+G4-m4*aC4,rD-rE);
```

There are four equations Eqs. (4.92)-(4.94) with four unknowns F_{05x}, F_{05y}, F_{34x}, F_{34y}. The system is solved using MATLAB

```
solDI=solve(eqF45x, eqF45y , eqMD45z, eqF4DE);
F05xs=eval(solDI.F05x);
F05ys=eval(solDI.F05y);
F34xs=eval(solDI.F34x);
F34ys=eval(solDI.F34y);
F05s=[ F05xs, F05ys, 0 ];
F34s=[ F34xs, F34ys, 0 ];
```

The force of the link 4 on link 5 is $\mathbf{F}_{45}$ is calculated from Newton equation for link 5

$$m_5\,\mathbf{a}_{C_5} = \mathbf{F}_{05}+\mathbf{G}_5+\mathbf{F}_{45} \Longrightarrow$$
$$\mathbf{F}_{45} = m_5\,\mathbf{a}_{C_5}-\mathbf{G}_5-\mathbf{F}_{05},$$

and the MATLABcommand is

```
F45=m5*aC5-G5-F05s;
```

The application point of the joint force $\mathbf{F}_{45}$ is $P(x_P,\,y_P)$. The point P is on the line ED or

$$\mathbf{r}_{ED}\times\mathbf{r}_{EP}=\mathbf{0} \quad \text{or} \quad (\mathbf{r}_D-\mathbf{r}_E)\times(\mathbf{r}_P-\mathbf{r}_E)=\mathbf{0},$$

and with MATLAB

```
eqP=cross(rD-rE,rP-rE);
eqPz=eqP(3);
```

The second equation needed to calculate x_P and y_P is the moment equation on link 4 about $D = C_4$

$$I_{C_4}\,\boldsymbol{\alpha}_4 = \mathbf{r}_{C_4P} \times (-\mathbf{F}_{45}),$$

and with MATLAB

```
eqM4=cross(rP-rC4,-F45)-IC4*Alpha4;
eqM4z=eqM4(3);
```

The coordinates x_P and y_P are calculated using theMATLABcommands

```
solP=solve(eqPz,eqM4z);
xPs=eval(solP.xP);
yPs=eval(solP.yP);
rPs=[xPs, yPs, 0];
```

$C_R\,B_T\,B_R$ dyad

Figure 4.22 shows the forces and the moments that act on the dyad $C_R\,B_T\,B_R$ (links 3 and 2). The unknown joint reaction forces are $\mathbf{F}_{03} = F_{03x}\,\imath + F_{03y}\,\jmath$, $\mathbf{F}_{12} = F_{12x}\,\imath + F_{12y}\,\jmath$, or in MATLAB

```
F03x=sym('F03x','real');
F03y=sym('F03y','real');
F12x=sym('F12x','real');
F12y=sym('F12y','real');
F03=[ F03x, F03y, 0 ];
F12=[ F12x, F12y, 0 ];
```

The joint force $\mathbf{F}_{43} = -\mathbf{F}_{34}$ was calculated from the previous dyad EDD

```
F43=-F34s;
```

The sum of all the forces that act on links 3 and 2 is

$$m_3\,\mathbf{a}_{C_3} + m_2\,\mathbf{a}_{C_2} = \mathbf{F}_{43} + \mathbf{F}_{03} + \mathbf{G}_3 + \mathbf{G}_2 + \mathbf{F}_{12} \quad \Longrightarrow$$

$$\sum \mathbf{F}^{(3\&2)} = \mathbf{F}_{43} + \mathbf{F}_{03} + \mathbf{G}_3 + \mathbf{G}_2 + \mathbf{F}_{12} - m_3\,\mathbf{a}_{C_3} - m_2\,\mathbf{a}_{C_2} = \mathbf{0}. \quad (4.95)$$

Equation (4.95) has a component on x-axis, $\sum \mathbf{F}^{(3\&2)} \cdot \mathbf{\imath}$, a component on y-axis, $\sum \mathbf{F}^{(3\&2)} \cdot \mathbf{\jmath}$, and the MATLABcommands are

```
eqF23=F43+F03+G3-m3*aC3+G2-m2*aC2+F12;
eqF23x=eqF23(1); % projection on x-axis
eqF23y=eqF23(2); % projection on y-axis
```

The sum of moments of all the forces and moments on links 3 and 2 about B_R is zero

$$\begin{aligned}
&I_{C_3}\,\boldsymbol{\alpha}_3 + \mathbf{r}_{BC_3} \times m_3\,\mathbf{a}_{C_3} + I_{C_2}\,\boldsymbol{\alpha}_2 = \mathbf{r}_{BD} \times \mathbf{F}_{43} + \mathbf{r}_{BC} \times \mathbf{F}_{03} + \mathbf{r}_{BC_3} \times \mathbf{G}_3 \quad \Longrightarrow \\
&\sum \mathbf{M}_B^{(3\&2)} = (\mathbf{r}_D - \mathbf{r}_B) \times \mathbf{F}_{43} + (\mathbf{r}_C - \mathbf{r}_B) \times \mathbf{F}_{03} + (\mathbf{r}_{C_3} - \mathbf{r}_B) \times (\mathbf{G}_3 - m_3\,\mathbf{a}_{C_3}) \\
&-I_{C_3}\,\boldsymbol{\alpha}_3 - I_{C_2}\,\boldsymbol{\alpha}_2 = \mathbf{0}. \qquad (4.96)
\end{aligned}$$

The MATLABcommands for Eq. (4.96) are

```
eqMB3=cross(rD-rB,F43)+cross(rC-rB,F03)+cross(rC3-rB,G3-m3*aC3);
eqMB2=-IC3*Alpha3-IC2*Alpha2;
eqMB23=eqMB3+eqMB2;
eqMB23z=eqMB23(3);
```

The sum of all the forces on link 2 projected on the sliding direction BC is

$$\begin{aligned}
&(m_2\,\mathbf{a}_{C_2}) \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 + \mathbf{F}_{32}) \cdot \mathbf{r}_{BC} \quad \Longrightarrow \\
&\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 - m_2\,\mathbf{a}_{C_2}) \cdot (\mathbf{r}_C - \mathbf{r}_B) = 0. \qquad (4.97)
\end{aligned}$$

The force of the link 3 on link 2 is $\mathbf{F}_{32}$ and $\mathbf{F}_{32} \cdot \mathbf{r}_{BC} = 0$. The MATLABcommand for Eq. (4.97) is

```
eqF2BC=dot(F12+G2-m2*aC2, rC-rB);
```

There are four equations Eqs. (4.95)-(4.97) with four unknowns F_{03x}, F_{03y}, F_{12x}, F_{12y}. The system is solved using MATLAB

```
solDII = solve(eqF23x, eqF23y , eqMB23z, eqF2BC);
```

```
F03xs=eval(solDII.F03x);
F03ys=eval(solDII.F03y);
F12xs=eval(solDII.F12x);
F12ys=eval(solDII.F12y);
F03s=[ F03xs, F03ys, 0 ];
F12s=[ F12xs, F12ys, 0 ];
```

The force of the link 3 on link 2 is $\mathbf{F}_{32}$ is calculated from the sum of all the forces on link 2

$$m_2\,\mathbf{a}_{C_2} = \mathbf{F}_{32} + \mathbf{G}_2 + \mathbf{F}_{12} \quad \Longrightarrow$$
$$\mathbf{F}_{32} = m_2\,\mathbf{a}_{C_2} - \mathbf{G}_2 - \mathbf{F}_{12},$$

and the MATLABcommand is

```
F32=m2*aC2-G2-F12s;
```

The application point of the joint force $\mathbf{F}_{32}$ is $Q(x_Q,\, y_Q)$. The point Q is on the line BC or

$$\mathbf{r}_{BC} \times \mathbf{r}_{QC} = \mathbf{0} \quad \text{or} \quad (\mathbf{r}_C - \mathbf{r}_B) \times (\mathbf{r}_Q - \mathbf{r}_C) = \mathbf{0},$$

and with MATLAB

```
eqQ=cross(rC-rB,rQ-rC);
eqQz=eqQ(3);
```

The second equation needed to calculate x_Q and y_Q is the sum of all the moments on link 2 about $B = C_2$

$$I_{C_2}\,\boldsymbol{\alpha}_2 = \mathbf{r}_{C_2Q} \times \mathbf{F}_{32},$$

and with MATLAB

```
eqM2=cross(rQ-rC2,F32)-IC2*Alpha2;
eqM2z=eqM2(3);
```

The coordinates x_Q and y_Q are calculated using theMATLABcommands

```
solQ=solve(eqQz,eqM2z);
xQs=eval(solQ.xQ);
yQs=eval(solQ.yQ);
rQs=[xQs, yQs, 0];
```

The joint reaction force of the ground on the link 1 and the equilibrium moment (drive moment) shown in Figure 4.20 are calculated using the procedure presented in the previous subsection.

TheMATLABprogram using the dyad method and the results are given in Program 4.6.

4.7.3 Contour Method

The contour diagram representing the mechanism is shown in Fig. 4.23. It has two contours 0-1-2-3-0 and 0-3-4-5-0.

Reaction force $\mathbf{F}_{05}$

The rotation joint E_R between the links 0 and 5 is replaced with the unknown reaction force $\mathbf{F}_{05}$ (Fig. 4.24)

$$\mathbf{F}_{05} = F_{05x}\mathbf{\imath} + F_{05y}\mathbf{\jmath}.$$

With MATLAB, the force $\mathbf{F}_{05}$ is written as

```
F05=[ sym('F05x','real'), sym('F05y','real'), 0 ];
```

Following the path I, as shown in Fig. 4.24, a force equation is written for the translation joint D_T. The projection of all forces, that act on the link 5, onto the sliding direction $\mathbf{r}_{DE}$ is zero

$$\sum \mathbf{F}^{(5)} \cdot \mathbf{r}_{DE} = (\mathbf{F}_{05} + \mathbf{G}_5 + \mathbf{F}_{in\,5}) \cdot \mathbf{r}_{DE} = 0, \tag{4.98}$$

where $\mathbf{r}_{DE} = \mathbf{r}_E - \mathbf{r}_D$.
Equation (4.98) with MATLABbecomes

```
eqER1=dot(F05+G5+Fin5,rE-rD);
```

where the command `dot(a,b)` gives the scalar product of the vectors `a` and `b`. Continuing on the path I, a moment equation is written for the rotation joint D_R

$$\sum \mathbf{M}_D^{(4\&5)} = \mathbf{r}_{DE} \times \mathbf{F}_{05} + \mathbf{r}_{DC_5} \times (\mathbf{G}_5 + \mathbf{F}_{in\,5}) + \mathbf{M}_{in\,4} + \mathbf{M}_{in\,5} + \mathbf{M}_{5ext} = \mathbf{0}, \tag{4.99}$$

where $\mathbf{r}_{DC_5} = \mathbf{r}_{C_5} - \mathbf{r}_D$.
Equation (4.99) with MATLAB gives

```
eqER2=cross(rE-rD,F05)+cross(rC5-rD,G5+Fin5)+Me+Min4+Min5;
eqER2z=eqER2(3);
```

The system of two equations is solved using MATLAB commands

```
solF05=solve(eqER1,eqER2z);
F05s=[ eval(solF05.F05x), eval(solF05.F05y), 0 ];
```

The following numerical solution is obtained

$$\mathbf{F}_{05} = 268.165\,\mathbf{\imath} + 135.057\,\mathbf{\jmath}\ \text{N}.$$

Reaction force $\mathbf{F}_{45}$

The translation joint D_T between the links 4 and 5 is replaced with the unknown reaction force $\mathbf{F}_{45}$ (Fig. 4.25)

$$\mathbf{F}_{45} = -\mathbf{F}_{54} = F_{45x}\mathbf{\imath} + F_{45y}\mathbf{\jmath}.$$

The position of the application point P of the force $\mathbf{F}_{45}$ is unknown

$$\mathbf{r}_P = x_P\mathbf{\imath} + y_P\mathbf{\jmath},$$

where x_P and y_P are the plane coordinates of the point P.
The force $\mathbf{F}_{45}$ and its point of application P with MATLAB is written as

```
F45=[ sym('F45x','real'), sym('F45y','real'), 0 ];
F54=-F45;
rP=[ sym('xP','real'), sym('yP','real'), 0 ];
```

Following the path I (Fig. 4.25), a moment equation is written for the rotation joint E_R

$$\sum \mathbf{M}_E^{(5)} = \mathbf{r}_{EP} \times \mathbf{F}_{45} + \mathbf{r}_{EC_5} \times (\mathbf{G}_5 + \mathbf{F}_{in\,5}) + \mathbf{M}_{in\,5} + \mathbf{M}_{5ext} = \mathbf{0}, \quad (4.100)$$

where $\mathbf{r}_{EP} = \mathbf{r}_P - \mathbf{r}_E$ and $\mathbf{r}_{EC_5} = \mathbf{r}_{C_5} - \mathbf{r}_E$.
One can write Eq. (4.100) using the MATLAB commands

```
eqDT1=cross(rP-rE,F45)+cross(rC5-rE,G5+Fin5)+Me+Min5;
eqDT1z=eqDT1(3);
```

Following the path II (Fig. 4.25), a moment equation is written for the rotation joint D_R

$$\sum \mathbf{M}_D^{(4)} = \mathbf{r}_{DP} \times \mathbf{F}_{54} + \mathbf{M}_{in\,4} = \mathbf{0}, \quad (4.101)$$

where $\mathbf{r}_{DP} = \mathbf{r}_P - \mathbf{r}_D$ and $\mathbf{F}_{54} = -\mathbf{F}_{45}$.
Equation (4.101) with MATLAB is

```
eqDT2=cross(rP-rD,F54)+Min4;
eqDT2z=eqDT2(3);
```

The direction of the unknown joint force $\mathbf{F}_{45}$ is perpendicular to the sliding direction $\mathbf{r}_{ED}$

$$\mathbf{F}_{45} \cdot \mathbf{r}_{ED} = 0, \tag{4.102}$$

and using MATLAB command

```
eqF45DE=dot(F45,rD-rE);
```

The application point P of the force $\mathbf{F}_{45}$ is located on the direction ED, that is

$$(\mathbf{r}_D - \mathbf{r}_E) \times (\mathbf{r}_P - \mathbf{r}_E) = \mathbf{0}. \tag{4.103}$$

One can write Eq. (4.103) using the MATLAB commands

```
eqP=cross(rD-rE,rP-rE);
eqPz=eqP(3);
```

The system of four equations is solved using the MATLAB command

```
solF45=solve(eqDT1z,eqDT2z,F45DE,eqPz);
F45s=[ eval(solF45.F45x), eval(solF45.F45y), 0 ];
rPs=[ eval(solF45.xP), eval(solF45.yP), 0 ];
```

The following numerical solutions are obtained

$$\mathbf{F}_{45} = -268.109\,\imath - 134.646\,\jmath \text{ N} \quad \text{and} \quad \mathbf{r}_P = -0.149492\,\imath + 0.0476701\,\jmath \text{ m}.$$

Reaction force $\mathbf{F}_{34}$

The rotation joint D_R between the links 3 and 4 is replaced with the unknown reaction force $\mathbf{F}_{34}$ (Fig. 4.26)

$$\mathbf{F}_{34} = -\mathbf{F}_{34} = F_{34x}\imath + F_{34y}\jmath,$$

and with MATLAB

```
F34=[ sym('F34x','real'), sym('F34y','real'), 0 ];
F43=-F34;
```

Following the path I, a force equation can be written for the translation joint D_T. The projection of all forces, that act on the link 4, onto the sliding direction ED is zero

$$\sum \mathbf{F}^{(4)} \cdot \mathbf{r}_{ED} = (\mathbf{F}_{34} + \mathbf{G}_4 + \mathbf{F}_{in\,4}) \cdot \mathbf{r}_{ED} = 0, \tag{4.104}$$

where $\mathbf{r}_{ED} = \mathbf{r}_D - \mathbf{r}_E$.
Equation (4.104) using MATLAB gives

```
eqDR1=dot(F34+G4+Fin4,rD-rE);
```

Continuing on the path I (Fig. 4.26), a moment equation is written for the rotation joint E_R

$$\sum \mathbf{M}_E^{(4\&5)} = \mathbf{r}_{EC_4} \times (\mathbf{G}_4 + \mathbf{F}_{in\,4}) + \mathbf{r}_{ED} \times \mathbf{F}_{34} + \mathbf{M}_{in\,4} + \mathbf{r}_{EC_5} \times (\mathbf{G}_5 + \mathbf{F}_{in\,5}) + \mathbf{M}_{in\,5} + \mathbf{M}_{5ext} = \mathbf{0}. \tag{4.105}$$

where $\mathbf{r}_{EC_5} = \mathbf{r}_{C_5} - \mathbf{r}_E$, and $\mathbf{r}_{EC_4} = \mathbf{r}_{C_4} - \mathbf{r}_E$.
Equation (4.105) with MATLAB becomes

```
eqDR24=cross(rC4-rE,G4+Fin4)+cross(rD-rE,F34)+Min4;
eqDR25=cross(rC5-rE,G5+Fin5)+Me+Min5;
eqDR2=eqDR24+eqDR25;
eqDR2z=eqDR2(3);
```

The system of two equations is solved using the MATLAB commands

```
solF34=solve(eqDR1,eqDR2z);
F34s=[ eval(solF34.F34x), eval(solF34.F34y), 0 ];
```

The following numerical solution is obtained

$$\mathbf{F}_{34} = -268.072\,\imath - 134.583\,\jmath \text{ N}.$$

Reaction force $\mathbf{F}_{03}$

The rotation joint C_R between the links 0 and 3 is replaced with the unknown reaction force $\mathbf{F}_{03}$ (Fig. 4.27)

$$\mathbf{F}_{03} = F_{03x}\imath + F_{03y}\jmath.$$

With MATLAB the force $\mathbf{F}_{03}$ is written as

```
F03=[ sym('F03x','real'), sym('F03y','real'), 0 ];
```

Following the path I (Fig. 4.27), a force equation is written for the translation joint B_T. The projection of all forces, that act on the link 3, onto the sliding direction CD is zero

$$\sum \mathbf{F}^{(3)} \cdot \mathbf{r}_{CD} = (\mathbf{F}_{03} + \mathbf{F}_{43} + \mathbf{G}_3 + \mathbf{F}_{in\,3}) \cdot \mathbf{r}_{CD} = 0, \tag{4.106}$$

where $\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C$.
Equation (4.106) with MATLAB command is

```
eqCR1=dot(F03-F34s+G3+Fin3,rD-rC);
```

Continuing on the path II (Fig. 4.27), a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(3\&2)} = \mathbf{r}_{BC_3} \times (\mathbf{G}_3 + \mathbf{F}_{in\,3}) + \mathbf{r}_{BC} \times \mathbf{F}_{03} + \mathbf{r}_{BD} \times \mathbf{F}_{43} + \mathbf{M}_{in\,2} + \mathbf{M}_{in\,3} = \mathbf{0}, \tag{4.107}$$

where $\mathbf{r}_{BC_3} = \mathbf{r}_{C_3} - \mathbf{r}_B$, $\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B$, and $\mathbf{r}_{BD} = \mathbf{r}_D - \mathbf{r}_B$.
With MATLAB Eq. (4.107) gives

```
eqCR2=cross(rC3-rB,G3+Fin3)+cross(rC-rB,F03)+cross(rD-rB,-F34s)+Min2+Min3;
eqCR2z=eqCR2(3);
```

To solve the system of two equations the MATLAB commands are used

```
solF03=solve(eqCR1,eqCR2z);
F03s=[ eval(solF03.F03x), eval(solF03.F03y), 0 ];
```

The following numerical solution is obtained

$$\mathbf{F}_{03} = -256.745\,\imath - 272.179\,\jmath \text{ N}.$$

Reaction force $\mathbf{F}_{23}$

The translation joint B_T between the links 2 and 3 is replaced with the unknown reaction force $\mathbf{F}_{23}$ (Fig. 4.28)

$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\,\mathbf{\imath} + F_{23y}\,\mathbf{\jmath}.$$

The position of the application point Q of the force $\mathbf{F}_{23}$ is unknown

$$\mathbf{r}_Q = x_Q\,\mathbf{\imath} + y_Q\,\mathbf{\jmath},$$

where x_Q and y_Q are the plane coordinates of the point Q.
The force $\mathbf{F}_{23}$ and its point of application Q are written in MATLAB as

```
F23=[ sym('F23x','real'), sym('F23y','real'), 0 ];
F32=-F23;
rQ=[ sym('xQ','real'), sym('yQ','real'), 0 ];
```

Following the path I (Fig. 4.28), a moment equation is written for the rotation joint C_R

$$\sum \mathbf{M}_C^{(3)} = \mathbf{r}_{CQ} \times \mathbf{F}_{23} + \mathbf{r}_{CC_3} \times (\mathbf{G}_3 + \mathbf{F}_{in\,3}) + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_{in\,3} = \mathbf{0}, \quad (4.108)$$

where $\mathbf{r}_{CQ} = \mathbf{r}_Q - \mathbf{r}_C$, $\mathbf{r}_{CC_3} = \mathbf{r}_{C_3} - \mathbf{r}_C$, and $\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C$.
Using MATLAB, Eq. (4.108) is written as

```
eqBT1=cross(rQ-rC,F23)+cross(rC3-rC,G3+Fin3)+cross(rD-rC,-F34s)+Min3;
eqBT1z=eqBT1(3);
```

Following the path II (Fig. 4.28), a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(2)} = \mathbf{r}_{BQ} \times \mathbf{F}_{32} + \mathbf{M}_{in\,2} = \mathbf{0}, \quad (4.109)$$

where $\mathbf{r}_{BQ} = \mathbf{r}_Q - \mathbf{r}_B$.
Equation (4.109) with MATLAB becomes

```
eqBT2=cross(rQ-rB,F32)+Min2;
eqBT2z=eqBT2(3);
```

The direction of the unknown joint force $\mathbf{F}_{23}$ is perpendicular to the sliding direction BC. The following relation is written

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0,$$

or with MATLAB, it is

```
eqF23BC=dot(F23,rC-rB);
```

The application point Q of the force $\mathbf{F}_{23}$ is located on the direction BC, that is

$$(\mathbf{r}_B - \mathbf{r}_C) \times (\mathbf{r}_Q - \mathbf{r}_C) = \mathbf{0}. \tag{4.110}$$

Equation (4.110) with MATLAB gives

```
eqQ=cross(rB-rC,rQ-rC);
eqQz=eqQ(3);
```

The system of four equations is solved using the MATLAB command

```
solF23=solve(eqBT1z,eqBT2z,F23BC,eqQz);
F23s=[ eval(solF23.F23x), eval(solF23.F23y), 0 ];
rQs=[ eval(solF23.xQ), eval(solF23.yQ), 0 ];
```

The following numerical solutions are obtained

$$\mathbf{F}_{23} = -11.3762\,\imath + 137.93\,\jmath \text{ N} \quad \text{and} \quad \mathbf{r}_Q = 0.121243\,\imath + 0.070\,\jmath \text{ m}.$$

Reaction force $\mathbf{F}_{12}$

The rotation joint B_R between the links 1 and 2 is replaced with the unknown reaction force $\mathbf{F}_{12}$ (Fig. 4.29)

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x}\,\imath + F_{12y}\,\jmath.$$

With MATLAB it is written as

```
F12=[ sym('F12x','real'), sym('F12y','real'), 0 ];
F21=-F12;
```

Following the path I (Fig. 4.29), a force equation is written for the translation joint B_T. The projection of all forces, that act on the link 2, onto the sliding direction BC is zero

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{G}_2 + \mathbf{F}_{in\,2}) \cdot \mathbf{r}_{BC} = 0. \quad (4.111)$$

Using MATLAB it is written as

```
eqBR1=dot(F12+G2+Fin2,rC-rB);
```

Continuing on the path I, a moment equation is written for the rotation joint C_R

$$\begin{aligned}\sum \mathbf{M}_C^{(2\&3)} &= \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC_2} \times (\mathbf{G}_2 + \mathbf{F}_{in\,2}) + \mathbf{M}_{in\,2} + \\ &\quad \mathbf{r}_{CC_3} \times (\mathbf{G}_3 + \mathbf{F}_{in\,3}) + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_{in\,3} = \mathbf{0}, \quad (4.112)\end{aligned}$$

where $\mathbf{r}_{CB} = \mathbf{r}_B - \mathbf{r}_C$, $\mathbf{r}_{CC_2} = \mathbf{r}_{C_2} - \mathbf{r}_C$, $\mathbf{r}_{CC_3} = \mathbf{r}_{C_3} - \mathbf{r}_C$, and $\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C$. Using the MATLAB, commands Eq. (4.112) gives

```
eqBR2=cross(rB-rC,F12)+cross(rC2-rC,G2+Fin2)+Min2...
+cross(rC3-rC,G3+Fin3)+cross(rD-rC,-F34s)+Min3;
eqBR2z=eqBR2(3);
```

The system of two equations is solved using the MATLAB commands

```
solF12=solve(eqBR1,eqBR2z);
F12s=[ eval(solF12.F12x), eval(solF12.F12y), 0 ];
```

and the following numerical solution is obtained

$$\mathbf{F}_{12} = -11.4028\,\imath + 137.993\,\jmath \text{ N}.$$

The motor moment $\mathbf{M}_m$

The motor moment needed for the dynamic equilibrium of the mechanism is $\mathbf{M}_m = M_m\ \mathbf{k}$ (Fig. 4.30). Following the path I (Fig. 4.30), a moment equation is written for the rotation joint A_R

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{r}_{AC_1} \times (\mathbf{G}_1 + \mathbf{F}_{in\,1}) + \mathbf{M}_{in\,1} + \mathbf{M}_m = \mathbf{0}. \quad (4.113)$$

Equation (4.113) is solved using the MATLAB command

```
M1m=-(cross(rB,-F12s)+cross(rC1,G1+Fin1)+Min1);
```

The numerical solution is

$$\mathbf{M}_m = 17.5356\ \mathbf{k} \quad \mathrm{N \cdot m}.$$

Reaction force $\mathbf{F}_{01}$

The rotation joint A_R between the links 0 and 1 is replaced with the unknown reaction force $\mathbf{F}_{01}$ (Fig. 4.31)

$$\mathbf{F}_{01} = -\mathbf{F}_{10} = F_{01x}\,\mathbf{\imath} + F_{01y}\,\mathbf{\jmath},$$

With MATLAB it is written as

```
F01=[ sym('F01x','real'), sym('F01y','real'), 0 ];
```

Following the path I (Fig. 4.31), a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(1)} = \mathbf{r}_{BA} \times \mathbf{F}_{01} + \mathbf{r}_{BC_1} \times (\mathbf{G}_1 + \mathbf{F}_{in\,1}) + \mathbf{M}_{in\,1} + \mathbf{M}_m = \mathbf{0}, \quad (4.114)$$

where $\mathbf{r}_{BA} = -\mathbf{r}_B$, and $\mathbf{r}_{BC_1} = \mathbf{r}_{C_1} - \mathbf{r}_B$.
Equation (4.114) using the MATLAB commands gives

```
eqAR1=cross(-rB,F01)+cross(rC1-rB,G1+Fin1)+Min1+M1m;
eqAR1z=eqAR1(3);
```

Continuing on the path I (Fig. 4.31), a force equation is written for the translation joint B_T. The projection of all forces, that act on the links 1 and 2, onto the sliding direction BC is zero

$$\sum \mathbf{F}^{(1\&2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{01} + \mathbf{G}_1 + \mathbf{F}_{in\,1} + \mathbf{G}_2 + \mathbf{F}_{in\,2}) \cdot \mathbf{r}_{BC} = 0, \quad (4.115)$$

or with MATLAB it is

```
eqAR2=dot(F01+G1+Fin1+G2+Fin2,rC-rB);
```

The system of two equations is solved using the MATLAB commands

```
solF01=solve(eqAR1z,eqAR2);
F01s=[ eval(solF01.F01x), eval(solF01.F01y), 0 ];
```

The following numerical solution is obtained

$$\mathbf{F}_{01} = -11.4214\,\mathbf{\imath} + 138.092\,\mathbf{\jmath} \quad \text{N}.$$

The MATLAB program for the dynamic force analysis is presented in Program 4.7.