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0.1 Solution

0.1.1 1.1

We have

$$E_1(u) = \|u - v\|^2 + \lambda \|\nabla u\|^2 = \sum_{i=1}^{3} \|(K_i * u - v_i)\|^2$$

With this minimization problem, we can found that the solution is:

$$\hat{u} = \frac{\sum_i \overline{\widehat{K_i(w)}} \hat{v}_i}{\sum_i |\overline{\widehat{K_i(w)}}|^2}$$

This is the implementation in the function `resoud_quad_fourier` called by `minimisation_quadratique`.

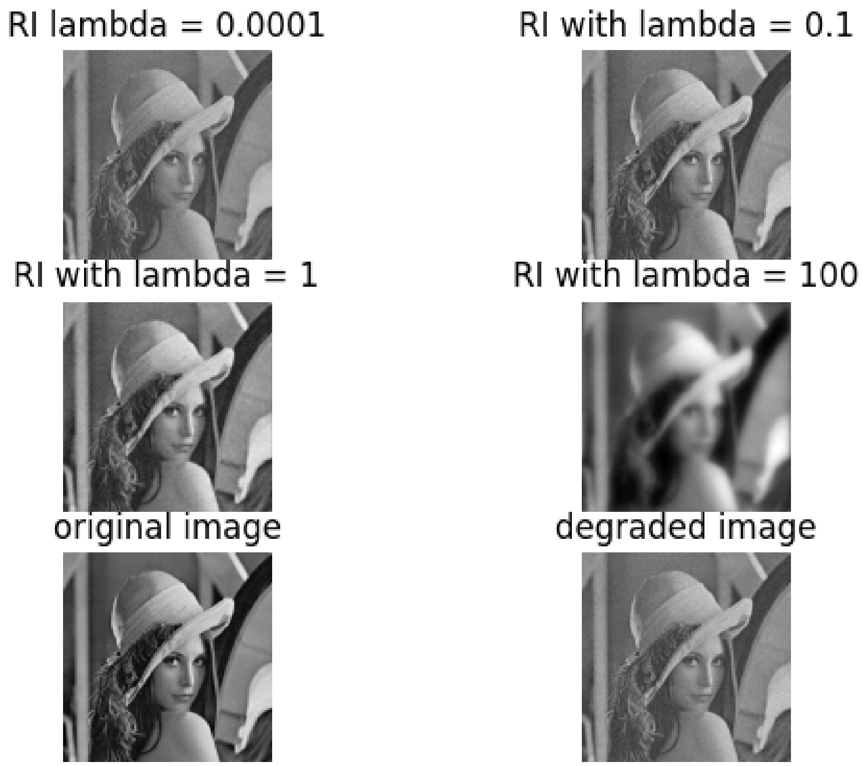
0.1.2 1.2

When the λ is so small, the function does not do anything. In contrast, when the λ is so large, the function makes the image so blurry.

When we change the value of λ, we variate the importance to the regularization term with respect to the data attachment term.

[ ]:
```
im = imread('lena.tif')
imb = degrade_image(im,25)

#Restaurations
lambda1 = 0.0001
lambda2 = 0.1
lambda3 = 1
lambda4 = 100

restau1 = minimisation_quadratique(imb,lambda1)
restau2 = minimisation_quadratique(imb,lambda2)
restau3 = minimisation_quadratique(imb,lambda3)
restau4 = minimisation_quadratique(imb,lambda4)
```

```python
#show restored images in 3 columns and 2 rows
#Hide axes


plt.subplot(3,2,1)
plt.imshow(restau1,cmap='gray')
plt.axis('off')
plt.title('RI lambda = 0.0001')
plt.subplot(3,2,2)
plt.imshow(restau2,cmap='gray')
plt.axis('off')
plt.title('RI with lambda = 0.1')
plt.subplot(3,2,3)
plt.imshow(restau3,cmap='gray')
plt.axis('off')
plt.title('RI with lambda = 1')
plt.subplot(3,2,4)
plt.imshow(restau4,cmap='gray')
plt.axis('off')
plt.title('RI with lambda = 100')
plt.subplot(3,2,5)
plt.imshow(im,cmap='gray')
plt.axis('off')
plt.title('original image')
plt.subplot(3,2,6)
plt.imshow(imb,cmap='gray')
plt.axis('off')
plt.title('degraded image')
plt.show()
```



0.1.3 1.3

We found that the $\lambda = 0.3308$ is the λ that respects the theorical data attachment norm

[ ]:
```
im = imread('lena.tif')
imb = degrade_image(im,5)

goal = norm2(imb - im)

lmin = 0.001
lmax = 1

min_restauration = minimisation_quadratique(imb,lmin)
max_restauration = minimisation_quadratique(imb,lmax)

min_norm = norm2(imb - min_restauration)
max_norm = norm2(imb - max_restauration)

for i in range(100):
    l = (lmin + lmax)/2
    restauration = minimisation_quadratique(imb,l)
    norm = norm2(imb - restauration)
```

```
    if norm > goal:
        lmax = l
        max_norm = norm
        max_restauration = restauration
    else:
        lmin = l
        min_norm = norm
        min_restauration = restauration

print("lambda = ", l)
```

```
lambda =  0.330324850846221
```

0.1.4 1.4

We found that the best λ is $\lambda = 0.1122$ In any case less than lambda of the data attachment

```
[ ]: vk = np.arange(-1,0,0.05)

best_error = norm2(imb)+10
best_lambda = 0
for k in vk:
    lamb = 10**k
    restau = minimisation_quadratique(imb,lamb)
    erreur = norm2(im-restau)

    if erreur < best_error:
        best_error = erreur
        best_lambda = lamb

print("best lambda = ", best_lambda)
```

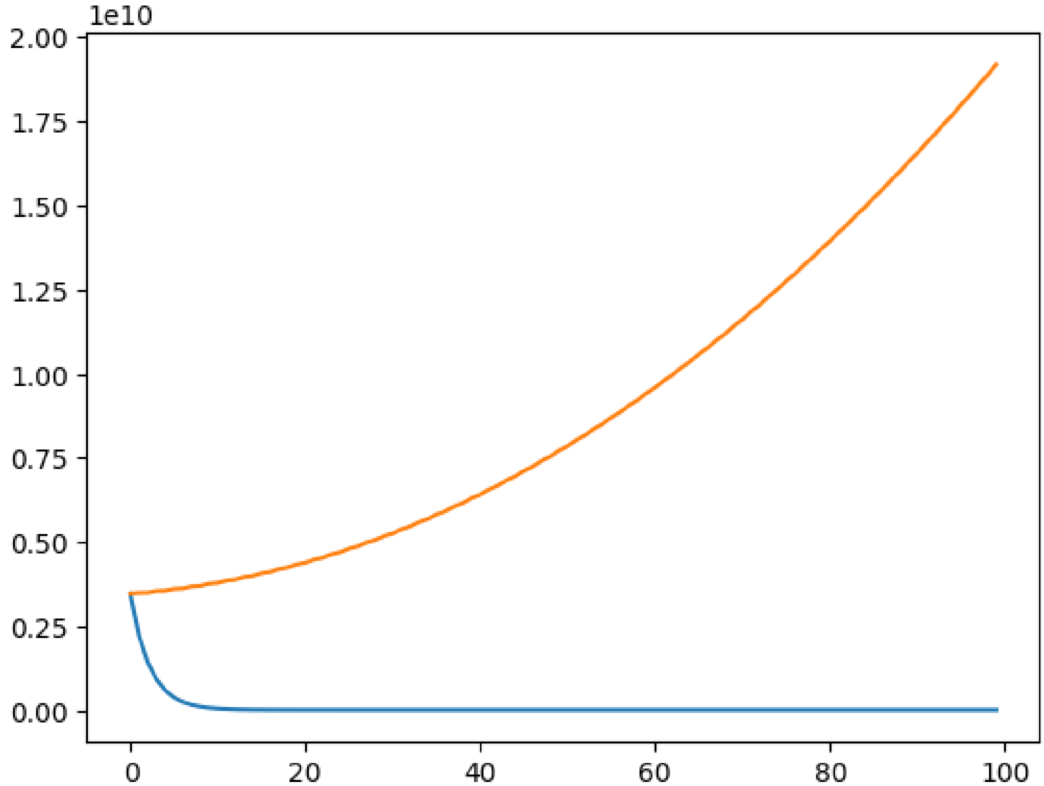
```
best lambda =  0.11220184543019636
```

0.1.5 2.1

It is observed that a gradient descent with a non-constant step performs poorly with total variation. Sometimes, the energy even increases after decreasing

```
[ ]: im = imread('lena.tif')
imb = degrade_image(im,25)

(u,energ)=minimise_TV_gradient(imb,1,0.1,100)  #step = 0.1
(u,energ2)=minimise_TV_gradient(imb,1,1,100) #step = 1

plt.plot(energ)
plt.plot(energ2)
plt.show()
```

```
[<matplotlib.lines.Line2D at 0x1f55fb8de10>]
```



0.1.6 2.2

The Chambolle's method and minimize_TV_gradient both minimize an energy corresponding to the non-periodic gradient. To obtain equivalent energies, it would be necessary to perform 1000 gradient descent steps with a step size of 0.01.

[ ]:
```
u1,en1=minimise_TV_gradient(imb, 40, 0.1, 100)
e2grad = E2_nonperiodique(u1,imb,40)
print("E2 = ", e2grad)

u_chambolle = vartotale_Chambolle(imb,40, itmax=30)
e2chambolle = E2_nonperiodique(u_chambolle,imb,40)
print("E2 = ", e2chambolle)

print(e2chambolle / e2grad)
```

```
E2 =  264395118.145648
E2 =  203226196.25747216
0.7686457968014387
```

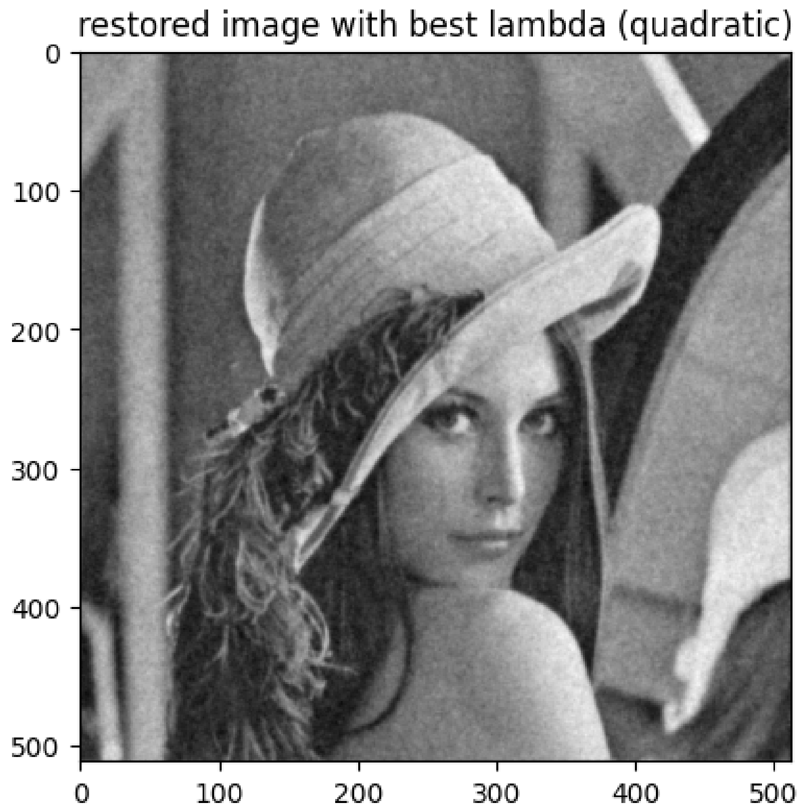
0.1.7 3

error_tv is smaller than error_quad, and as a result, the obtained image is significantly better in terms of quality. The edges are better preserved in the restauration_tv image.

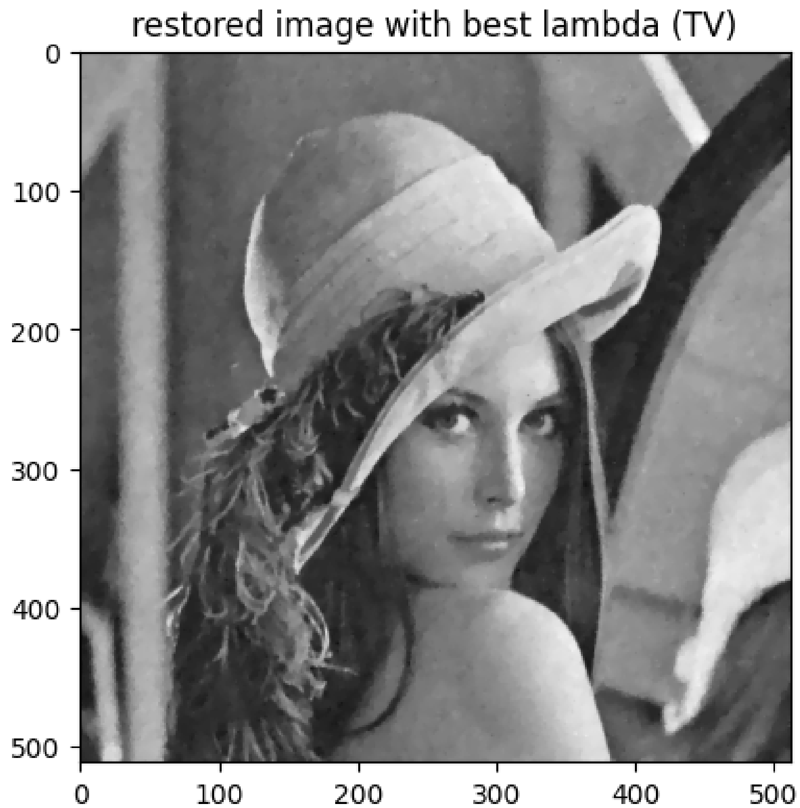
[ ]:
```
im = imread('lena.tif')
imb = degrade_image(im,25)

vk = np.arange(-1,1,0.05)
best_error = norm2(imb)+10

for k in vk:
    lamb = 10**k
    restau = minimisation_quadratique(imb,lamb)
    erreur = norm2(im-restau)

    if erreur < best_error:
        best_error = erreur
        best_lambda = lamb

print("best lambda = ", best_lambda)

best_lambda_quad = best_lambda
restauration_quad = minimisation_quadratique(imb,best_lambda_quad)
error_quad = norm2(im - restauration_quad)

#show restored image
plt.imshow(restauration_quad,cmap='gray')
plt.title('restored image with best lambda (quadratic)')
plt.show()

#Total Variation
vk = np.arange(1.39,1.8,0.02)
best_error = norm2(imb)+10

for k in vk:
    lamb = 10**k
    restau = vartotale_Chambolle(imb,lamb, itmax=30)
    erreur = norm2(im-restau)

    if erreur < best_error:
        best_error = erreur
        best_lambda = lamb

print("best lambda = ", best_lambda)

best_lambda_tv = best_lambda
restauration_tv = vartotale_Chambolle(imb,best_lambda_tv)
```

```
error_tv = norm2(im - restauration_tv)

#show restored image
plt.imshow(restauration_tv,cmap='gray')
plt.title('restored image with best lambda (TV)')
plt.show()
```

best lambda = 1.1220184543019658



best lambda = 44.668359215096324


restored image with best lambda (TV)
0
100
200
300
400
500
0
100
200
300
400
500