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Punto 1. Solucion

El Sistema masa, resorte y amortiguadores se puede modelar a partir de la Conservacion de las fuerzas

$$F_S(t) + F_F(t) + F_I(t) = F_E(t)$$

donde: $F_S(t) = ky(t)$, $F_F(t) = c\frac{dy(t)}{dt}$ y $F_I = \frac{m\,d^2y(t)}{dt^2}$

Por Consiguiente

$$m\frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + ky(t) = F_E(t) = x(t)$$

Aplicando la transformacion de laplace

$$\mathcal{L}\left\{\frac{d^n x(t)}{dt}\right\} = s^n X(s) \quad \text{tenemos que}$$

$$ms^2 y(s) + csy(s) + ky(s) = X(s)$$

y

$$H(s) = \frac{y(s)}{X(s)} = \frac{1}{ms^2 + cs + k}$$

Funcion de transferencia sistema masa, resorte y amortiguador

ahora, para el cto electrico presentado, hallamos la respectiva funcion de transferencia

LVK malla int

$$-V_i(t) + L\frac{d}{di}i_1(t) + \frac{1}{C}\int_0^t (i_1(t) - i_2(t))\,dt = 0$$

Utilizando las impedancias transformadas, obtenemos

$$V_i(s) = LsI_1(s) + I_1(s) + (I_1(s) - I_2(s))\frac{1}{cs} \quad (1)$$

ahora hallamos LVK malla $i_2(t)$

$$i_2(t)R + \frac{1}{c}\int_0^t (i_2(t) - I_1(t))\,dt = 0$$

donde $V_o(t) = i_2(t)R$

Utilizando las impedancias obtenemos.

$$I^2(s)R + (I_2(s) - I_1(s))\frac{1}{Cs} = 0$$

Despejando $I_1(s)$, se obtiene

$$\frac{I_1(s)}{Cs} = \frac{I_2(s)}{Cs} + I_2(s)R$$

$$I_1(s) = \frac{I_2(s)\,Cs + I_2(s)\,RCs}{Cs}$$

$$I_1(s) = I_2(s)\,(1 + CRs)$$

Remplazando (2) en (1)

$$Vi(s) = Ls\,I_2(s)(1 + CRs) + (I_2(s)(1+CRs) - I_2(s))\frac{1}{Cs}$$

$$Vi(s) = Ls\,I_2(s) + CRLs^2 I_2(s) + (I_2(s) + CRs\,I_2(s) - I_2(s))\frac{1}{Cs}$$

$$Vi(s) = Ls\,I_2(s) + CRLs^2 I^2(s) + R I_2(s)$$

$$Vi(s) = I_2(s)\,[CRLs^2 + Ls + R]$$

$$\frac{I_2(s)}{Vi(s)} = \frac{1}{CRLs^2 + Ls + R}$$

Remplazando $I_2(s) = \frac{Vo(s)}{R}$ obtenemos:

$$\frac{Vo(s)}{R\,Vi(s)} = \frac{1}{CRLs^2 + Ls + R}$$

$$\frac{Vo(s)}{Vi(s)} = \frac{R}{CRLs^2 + Ls + R}$$

$$\frac{Vo(s)}{Vi(s)} = \frac{1}{CLs^2 + \frac{L}{R}s + 1}$$

Funcion transferencia cto electrico

Equivalencia cto. electrico en Pendulo elastico

Cto electrico	Pendulo elastico
CL	m
L/R	C
1	K

Entonces:

$$H(s) = \frac{1}{LCs^2 + L/R\, s + 1}$$

Su equivalente en Paralelo es:

$$H(s) = \frac{1}{ms^2 + cs + k} = \frac{1/m}{\left(s^2 + \frac{cs}{m} + \frac{k}{m}\right)}$$

Hallando la forma Canonica de segundo orden

- Comparando

$$s^2 + 2\xi w_n s + w_n^2 = s^2 + \frac{cs}{m} + \frac{k}{m}$$

- igualando Coeficiente

$1 = 1 \longrightarrow$ Coef s^2

$2\xi w_n = \frac{c}{m} \longrightarrow$ Coef s

$w_n^2 = \frac{k}{m} \longrightarrow$ Coef independiente

- Hallando frecuencia natural no amortiguada

$$w_n = \sqrt{\frac{k}{m}}$$

- Hallando factor de amortiguamiento

$$2\xi\sqrt{\frac{k}{m}} = \frac{c}{m}$$

$$\xi = \frac{c}{2m\sqrt{\frac{k}{m}}}$$

- Hallando la ganancia K

$$K w_n^2 = \frac{1}{m}$$

$$K = \frac{1}{m w_n^2}$$

$$K = \frac{1}{m\,\frac{k}{m}}$$

$$\boxed{K = \frac{1}{k}}$$

Finalmente la Forma Canonica de segundo orden es:

$$H(s) = K \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2}$$

$$H(s) = \frac{1}{K} \frac{K/m}{s^2 + 2\left(\frac{c}{2m\sqrt{\frac{K}{m}}}\right)\sqrt{K/m}\, s + \frac{K}{m}}$$

$$H(s) = \frac{1/m}{s^2 + \frac{c}{m} s + \frac{K}{m}}$$

$$H(s): \frac{1}{m\left(s^2 + \frac{c}{m} s + \frac{K}{m}\right)}$$

- Hallando la frecuencia natural amortiguada

$$\omega d = \omega_n \sqrt{1 - \xi^2}$$

$$\omega d = \left(\sqrt{\frac{K}{m}}\right)\left(\sqrt{1 - \left(\frac{c}{2m\left(\frac{c}{2m\sqrt{\frac{K}{m}}}\right)}\right)^2}\right)$$

$$\boxed{\omega d = \frac{\sqrt{\frac{K}{m}}\sqrt{4Km - c^2}}{2\sqrt{Km}}}$$

- El tiempo de levantamiento y tiempo pico se hace por
  - Hallando el tiempo de establecimiento

$$ts = \frac{3}{\xi \omega_n}$$

$$ts = \frac{3}{\left(\frac{c}{2m\sqrt{\frac{K}{m}}}\right)\sqrt{K/m}}$$

$$\boxed{ts = \frac{6m}{s}}$$

Funcion de transferencia para masa resorte amortiguado lazo cerrado

- Podemos representar la funcion de transferencia de un sistema de lazo cerrado de la sgte manera

$$H_{LC} = \frac{H(s)}{1 + A(s)H(s)}$$

En este caso:

$$H(s) = \frac{1}{ms^2 + cs + k} \longrightarrow$$ "Funcion de tranferencia lazo abierto"

$$A(s) = 1$$

- Calculamos $H_{LC}(s)$

$$H_{LC}(s) = \frac{\frac{1}{ms^2 + cs + k}}{1 + \left(\frac{1}{ms^2 + cs + k}\right) \cdot 1} = \frac{\frac{1}{ms^2 + cs + k}}{\frac{ms^2 + cs + k + 1}{ms^2 + cs + k}}$$

$$H_{LC}(s) = \frac{ms^2 + cs + k}{(ms^2 + cs + k)\cdot(ms^2 + cs + k + 1)}$$

$$H_{LC}(s) = \frac{1}{ms^2 + cs + k + 1} \quad ó \quad \frac{1/m}{s^2 + \frac{c}{m}s + \left(\frac{k+1}{m}\right)}$$

- Hallando la forma canonica de segundo orden
  - Comparando

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + \frac{c}{m}\cdot s + \left(\frac{k+1}{m}\right)$$

- Igualando coeficiente

$$1 = 1 \longrightarrow \text{Coef } s^2$$

$$2\xi\cdot\omega_n = \frac{c}{m} \longrightarrow \text{Coef } s$$

$$\omega_n^2 = \frac{k+1}{m} \longrightarrow \text{Coef independiente}$$

Hallando frecuencia natural no amortiguada

$$\omega_n = \sqrt{\frac{k+1}{m}}$$

• Hallando Factor amortiguamiento

$$\xi = \frac{c}{2m\omega_n} = \boxed{\frac{c}{2m\sqrt{\frac{k+1}{m}}}}$$

• Hallando la ganancia

$$K\omega_n^2 = \frac{1}{m}$$

$$K = \frac{1}{m\,\omega_n^2}$$

$$K = \frac{1}{m\left(\sqrt{\frac{k+1}{m}}\right)^2}$$

$$\boxed{K = \frac{1}{k+1}}$$

• Entonces la forma canonica de segundo orden es:

$$H_{LC}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$H_{LC}(s) = \frac{\frac{k+1}{m}}{s^2 + 2\left(\frac{c}{2m\sqrt{\frac{k+1}{m}}}\right)\sqrt{\frac{k+1}{m}}\cdot s + \frac{k+1}{m}}$$

$$\boxed{H_{LC}(s) = \frac{1}{m\left(s^2 + \frac{c}{m}s + \frac{k+1}{m}\right)}}$$

• Hallando frecuencia natural amortiguado

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = \sqrt{\frac{k+1}{m}}\sqrt{1-\left(\frac{c}{2m\sqrt{(k+1)/m}}\right)^2}$$

$$\boxed{\omega_d = \frac{\sqrt{\frac{k+1}{m}}\sqrt{4km+4m-c^2}}{2\sqrt{m(k+1)}}}$$

- Hallando el tiempo de establecimiento

$$t_s = \frac{3}{\xi \omega_n} = \frac{3}{\dfrac{c}{2m\sqrt{\dfrac{k+1}{m}}}\sqrt{\dfrac{k+1}{m}}} = \boxed{\frac{6m}{c}}$$

Espectros de cada etapa. (generado Punto #2)

1) $Am(t)\cos(2\pi f_0 t + \Theta_0)$

$$A\cdot m(t)\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right) = A\left(\frac{m(t)e^{j2\pi f_0 t}}{2}\right) + \left(\frac{m(t)e^{-j2\pi f_0 t}}{2}\right)$$

Con $F\{x(t)\cdot e^{\pm j\omega_0 t}\} = X(\omega \pm \omega_0)$

$$\frac{A}{2} M((\omega - 2\pi f_0) + (\omega + 2\pi f_0))$$

2) $\cos(2\pi f_0 t + \Theta_0)$ Con $\Theta_0 = 0$

$$\cos(2\pi f_0 t) = \left\{\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right\} = F\left\{\frac{e^{j2\pi f_0 t}}{2}\right\} + F\left\{\frac{e^{-j2\pi f_0 t}}{2}\right\}$$

Con $F\{e^{\pm j\omega_0 t}\} = 2\pi\delta(\omega \mp \omega_0)$

$F(\omega) = \pi\delta(\omega - 2\pi f_0) + \pi\delta(\omega + 2\pi f_0) \longrightarrow$ Mixer (1x2)

$$A_1 m(t)\cos^2(2\pi f_0 t + \Theta_0) = \frac{A_1 m(t)}{2} + \frac{A_1 m(t)}{2}\cos(4\pi f_0 t + 2\Theta_0)$$

$$F(\omega) = \frac{AM(\omega)}{2} + \frac{A}{2} m(t) * \left(\frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2}\right)$$

$$F(\omega) = \frac{AM(\omega)}{2} + \frac{A}{2}\left(\frac{m(t)e^{j4\pi f_0 t}}{2}\right) + \left(\frac{m(t)e^{-j4\pi f_0 t}}{2}\right)$$

Con $F\{x(t)e^{\pm j\omega_0 t}\} = X(\omega \pm \omega_0)$

$F(\omega) = \frac{AM(\omega)}{2} + \frac{A}{4} M((\omega - 4\pi f_0) + (\omega + 4\pi f_0)) \longrightarrow$ lowpass

$$\frac{A_1}{2} m(t)$$

$F(\omega) = \frac{AM(\omega)}{2} \longrightarrow$ scale amplitude by $\frac{2}{A_1}$

$$\frac{A_1}{2} m(t)\cdot\frac{2}{A_1} = m(t)$$

$$F(m(t)) = M(\omega)$$