Solución Problemas Números Complejos

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Apuntes

- $i^2 = -1$
- $i^3 = i^2 \cdot i = -i$
- $i^4 = i^2 \cdot i^2 = 1$
- $(a+bi) \cdot (c+di) = (ac - bd + adi + bci)$
- $\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{(a+bi)\cdot(c-di)}{c^2+d^2}$

1 Ejercicio 1

Calcular parte real e imaginaria de los siguientes números complejos:
a) $z_1 = \frac{3-2i}{1+4i}$ b) $z_2 = \frac{1}{i} + \frac{1}{1+i}$ c) $z_3 = cos(i)$ d) $z_4 = sen(2+i)$

1.1 Solución

$$z_1 = \frac{3-2i}{1+4i} = \frac{3-2i}{1+4i} \cdot \frac{1-4i}{1-4i} = \frac{3-8-12i-2i}{17} = \boxed{-\frac{5}{17} - \frac{14}{17}i}$$

$$z_2 = \frac{1}{i} + \frac{1}{1+i} = \frac{1+2i}{-1+i} = \frac{1+2i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-1+2-1i-2i}{2} = \boxed{\frac{1}{2} - \frac{3}{2}i}$$

$$z_3 = cos(i) = \frac{e^{-1}+e}{2} = \frac{\frac{1}{e}+e}{2} = \boxed{\frac{1}{2e} + \frac{e}{2}}$$

$$z_4 = sen(2+i) = \frac{e^{i(2+i)} - e^{-i(2+i)}}{2i} = \frac{e^{-1+2i} - e^{1-2i}}{2i} = -\frac{i}{2}(\frac{1}{e}\cdot(cos(2)+isen(2)) - e\cdot(cos(-2)+isen(-2)))$$

$$= -\frac{i}{2}(\frac{1}{e}\cdot(cos(2)+isen(2)) - e\cdot(cos(2)-isen(2))) = \frac{1}{2e}(sen(2)-icos(2)) - \frac{e}{2}(-sen(2)-icos(2))$$

$$= \boxed{(\frac{1}{2e} + \frac{e}{2})sen(2) + i(-\frac{1}{2e} + \frac{e}{2})cos(2)}$$