USING FREEWARE MATHEMATICAL SOFTWARE IN CALCULUS CLASSES

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ABSTRACT

Due to the Bologna Accord, the teaching of mathematics has undergone important changes. Some of the most visible modifications have been the need to complement the traditional teaching-learning process with practical, real-life cases and the possibility to reinforce the introduction and usage of key concepts through mathematical software. Nowadays, there exist many computational packages dealing with mathematics, some of the best-known being Mathematica and Matlab. However, although they are very complete and powerful, they demand the use of commercial licences, which can be a problem for some education institutions or in the cases where students desire to use the software in an unlimited number of devices or to access from several of them simultaneously.

In this contribution, we show how to apply GeoGebra and WolframAlpha to the teaching of Calculus for first-year university students. While GeoGebra is an interactive geometry, algebra, statistics, and calculus application available both as an online resource and a native application in Windows, macOS, and Linux systems, WolframAlpha is a computational knowledge engine developed by a subsidiary of Wolfram Research, the company behind Mathematica. However, unlike that product, WolframAlpha can be accessed by any individual as a web service free of charge. One of the key aspects of WolframAlpha is the possibility to use natural language and Mathematica syntax for requesting computations, which allows users to benefit from a large amount of Mathematica resources.

Being able to use GeoGebra and WolframAlpha as web services without downloading and installing software is another important advantage, as it avoids the need to have adminstrator rights to use those computational engines, which typically represents a problem in education centres where lab computers are locked so students cannot inadvertently install malware that can compromise the university's network.

As the best way to show a topic in mathematics is to provide examples, this contribution focuses on the main topics associated to a first-year Calculus class (limits, continuity, derivatives, curve interpolation and integrals), providing examples with GeoGebra and WolframAlpha for the computations and concrete examples used in actual Calculus classes.

1 Introduction

The Bologna Accord is an agreement on a common model of higher education reached on 1999 that implies the creation of a common European area of university studies. It emphasizes the creation of a European Area of Higher Education (EAHE) as a key to promote students' mobility, aiming to simplify Europe's educational qualifications and ensuring that credentials granted by an institution in one country are comparable with those earned elsewhere [1].

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Spain is one of the 48 countries currently involved in the Bologna Process. The cornerstones of such an open space are mutual recognition of degrees and other higher education qualifications, transparency (readable and comparable degrees organised in a three-cycle structure) and European cooperation in quality assurance.

Due to the Bologna Accord, the teaching of mathematics has undergone important changes, such as the need to reinforce the traditional teaching-learning process with practical, real-life cases and the possibility to introduce some key concepts by using mathematical software. Nowadays, there are many computational packages focused on mathematics, where Mathematica and Matlab are some of the best known. However, even though they are certainly very complete and powerful, they require to use commercial licences, which can be a problem for some education institution or in the cases where students desire to use the software in an unlimited number of devices or to obtain access from several of them simultaneously.

In this contribution, our goal is to show how to apply freeware mathematical software to the teaching of Calculus for first-year university students. In order to do that, GeoGebra and WolframAlpha will be used for providing actual examples used at class.

The rest of this contribution is organized as follows: Section 2 introduces U-tad, the university centre where the Calculus classes mentioned in the article are imparted. In addition to that, this section also includes details about the syllabus of the Calculus class. Section 3 presents the most relevant information about GeoGebra and WolframAlpha, while Section 4 provides several examples used at class. Finally, in Section 5 we offer some conclusions and ideas for future work.

2 U-tad

U-tad is the acronym for Centro Universitario de Tecnología y Arte Digital (Technology and Digital Art University Centre, Figure 1) [2], a private university centre founded in 2011 with a strong focus on the creation, programming, and management of digital content, products, and services. U-tad is based near Madrid, and its current academic offer includes three higher technical education courses, five undergraduate degrees and twelve postgraduate courses.

Figure 1: Centro Universitario de Tecnología y Arte Digital (U-tad).

At the Software Engineering Degree imparted at U-tad, Calculus is a first-year course focused on single variable calculus [3]. This introductory Calculus course covers the following topics:

- Concepts of function, limits and continuity.
- Differentiation rules, application to graphing, rates, approximations and extremum problems.
- Definite and indefinite integration.
- The fundamental theorem of calculus.
- Applications of integrals to geometry: area, volume and arc length.

The evaluation scheme used during last years for computing the final grade of Calculus students is the following one:

- 40% first partial exam.
- 40% second partial exam.
- 20% homework and assignments.

Homework and assignments can take two forms: traditional exercises that have to be completed using pen and paper or exercises that require the use of a computational engine. Even though in the latter case it is inevitable to use computational software, it is recommended that they also use it in traditional exercises in order to check their answers before submitting them for grading.

3 Computational engines

There are basically three possibilities regarding the usage of computational engines:

- Commercial software: Matlab, Mathematica, etc.
- Free software: GeoGebra, WolframAlpha, Maxima, SageMath, etc.
- Programming languages such as Python.

Each of these options has its benefits and disadvantages. Applications like Mathematica are very powerful, but obviously they require commercial licences and the installation of many software packages that in some applications have to be managed manually and that in any case could potentially need to allocate several gigabytes of hard drive space.

In comparison, the computational capabilities of free software are considerably lower, but for introductory subjects they may be more than enough.

Finally, programming languages like Python are very versatile and allow to perform symbolic and numeric calculations, but most first-year students are not familiar with them.

That is why, in this contribution, we are going to focus on how to apply free software to the teaching of Calculus for first-year university students.

Focusing on the freeware computational engines, even though at class students are given information about GeoGebra, WolframAlpha, Maxima and SageMath, most of them choose GeoGebra and WolframAlpha for solving the exercises, and that is the reason why in the rest of this contribution we will make reference exclusively to those two options.

3.1 GeoGebra

GeoGebra (`www.geogebra.org`) is an interactive geometry, algebra, statistics, and calculus application available both as an online resource and a native application in Windows, macOS, and Linux systems [4].

The website includes several services such as a calculator and a graphics plotter, but the most widely used option is what is called GeoGebra Classic, which puts together those individual tools.

Figure 2 shows the GeoGebra Classic interface, where it is possible to find modules for 2- and 3-dimension plotting, an input bar or the Computer Algebra System module, among others.

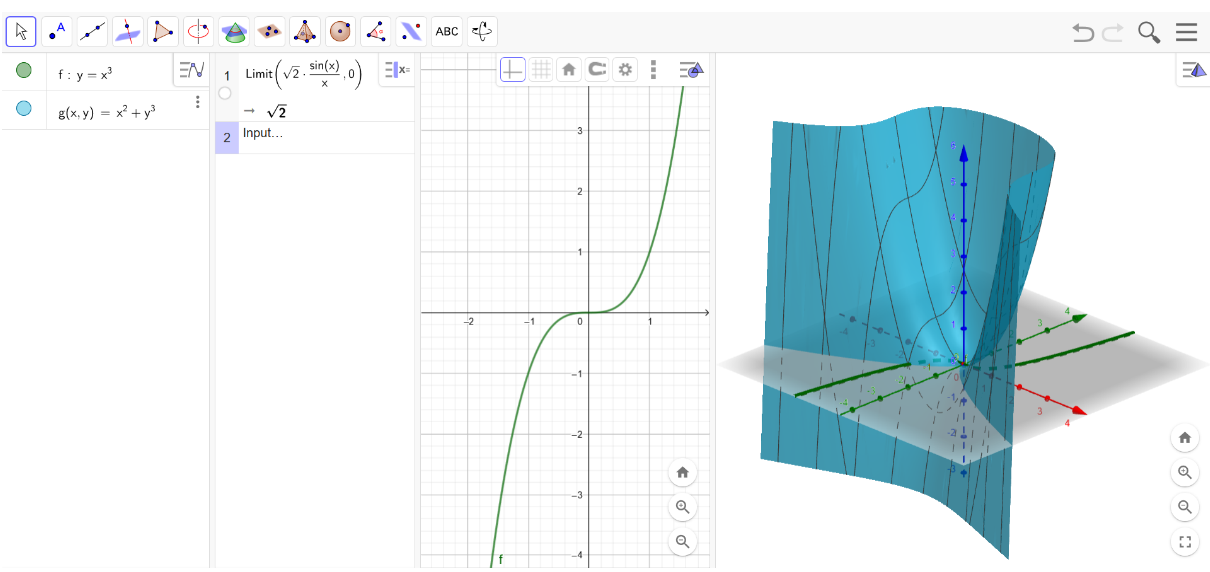

Figure 2: GeoGebra Classic screen.

GeoGebra's interface is easy to use and allows the configuration of several aspects associated to function representation, such as line width, color, and style. Those representations can be integrated into online books that can be shared with students so, for instance, they can navigate through all the examples and solutions associated to a certain topic.

3.2 WolframAlpha

WolframAlpha is a computational knowledge engine developed by a subsidiary of Wolfram Research, the company behind Mathematica [5]. Given that WolframAlpha is a reduced version of the Mathematica software, all options must be entered as text in the application's input box. However, the website provides access to many examples (see Figure 3), so students can find the right expression in a relatively short time. Obviously, the advantage of using WolframAlpha instead of Mathematica is that it can be accessed by anyone as a web service free of charge.

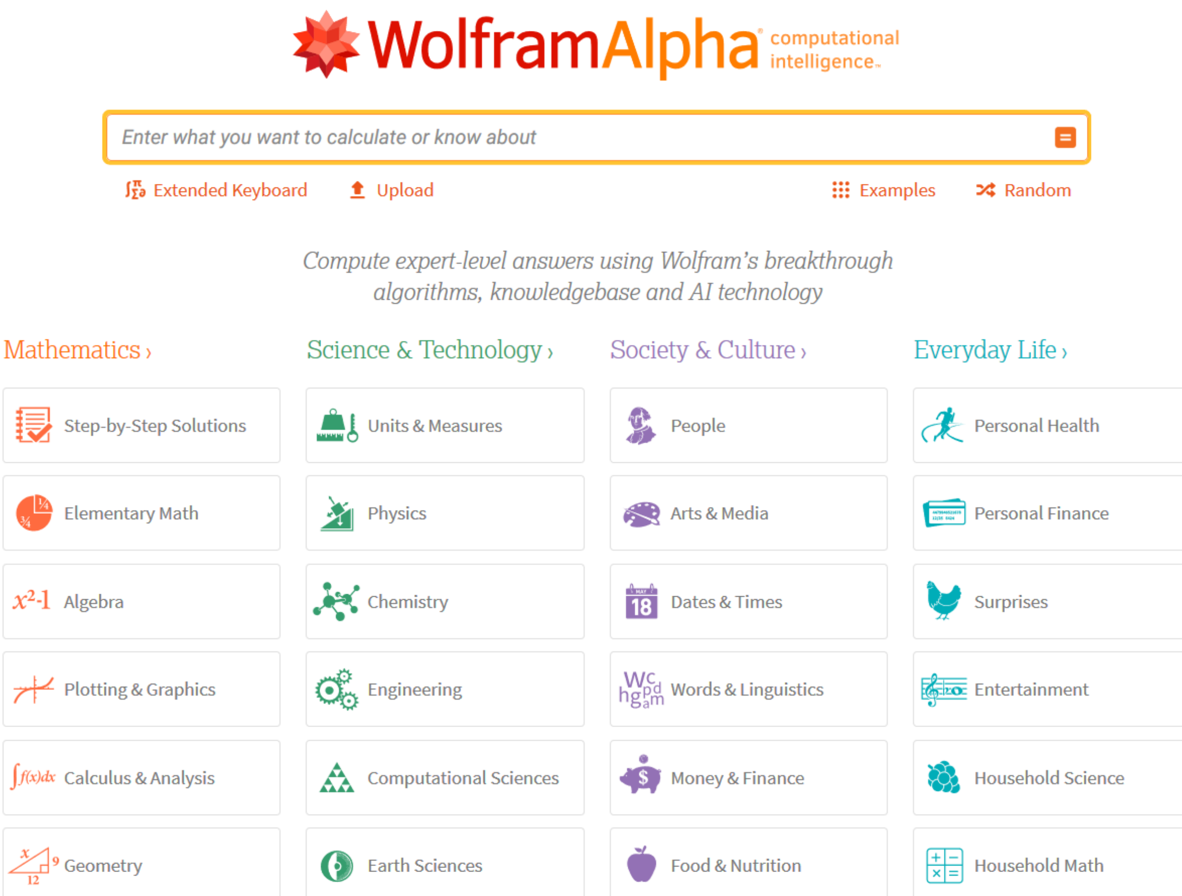

Figure 3: WolframAlpha website.

One of the key aspects of WolframAlpha is the possibility to use both natural language and Mathematica syntax for requesting computations. Figures 4 and 5 show how to generate the same calculation using the two options.

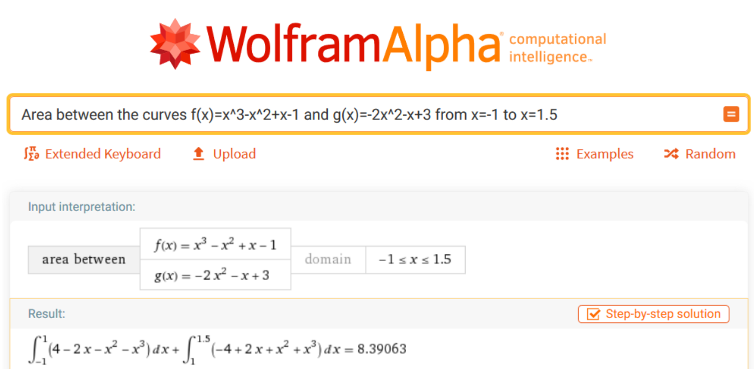

Figure 4: Example using natural language in WolframAlpha.

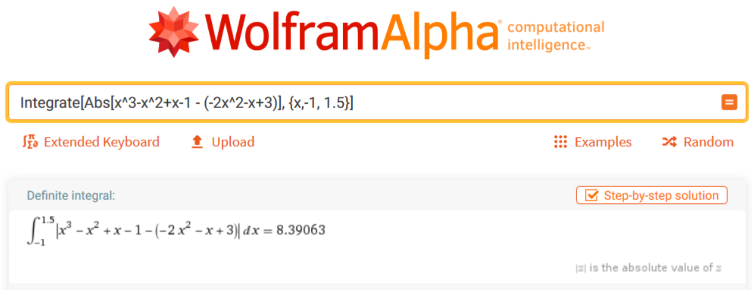

Figure 5: Example using Mathematica syntax in WolframAlpha.

4 Examples

As mentioned before, some Calculus key concepts can be reinforced or at least better understood by students when presented in a graphic way. Allowing students to replicate some model computations in other similar problems has the benefit to provide a durable link between what is taught at class and what they study at home.

Figure 6 shows an example associated to the graphic representation of a function and its asymptotes.

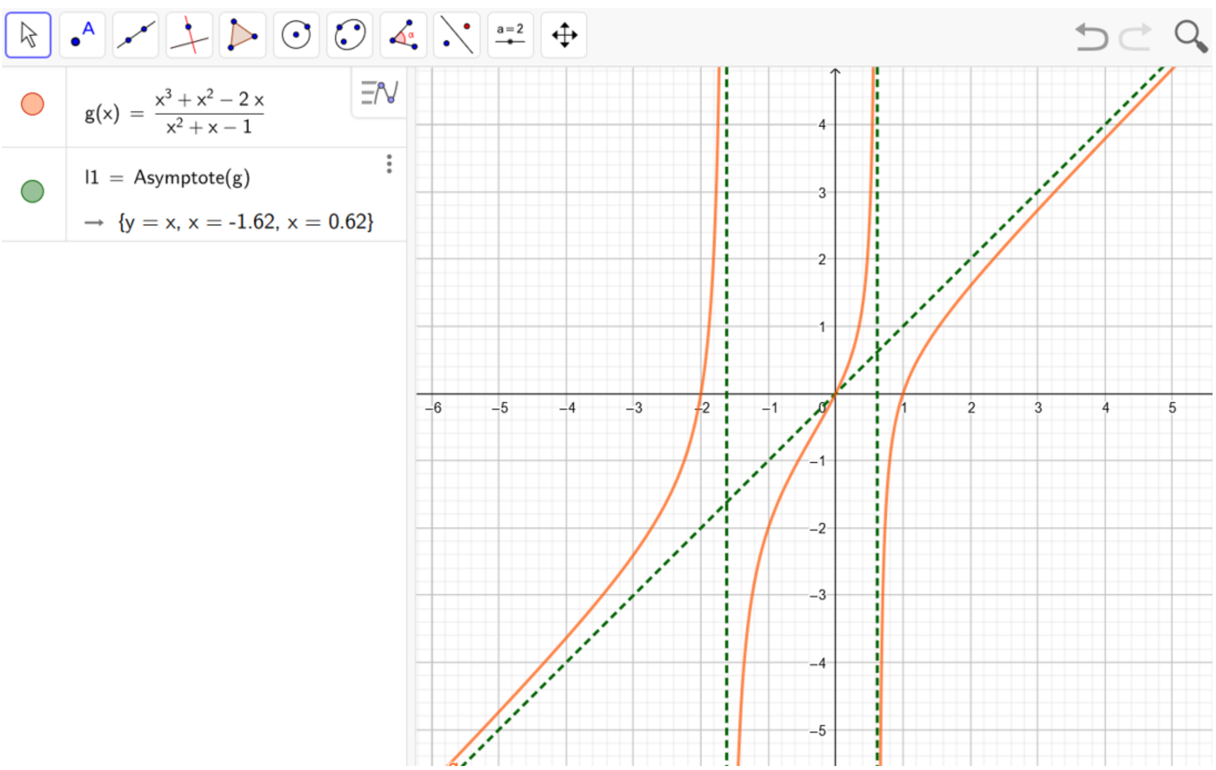

Figure 6: GeoGebra example about function representation.

If, for instance, we need to show how the Taylor polynomials work, we can include in the same solution the initial function and Taylor polynomials of different degrees, so students can realize that a higher degree implies a better approximation for real functions (see Figure 7).

Even though calculus of several variables is not included in the contents of the first-year subject taught at U-tad, in other subjects it is necessary to correctly interpret and visualize that kind of functions. In that regard, GeoGebra is a suitable option given that it allows students to rotate the image in both directions. As an example, Figure 8 shows how to represent the intersection of two surfaces.

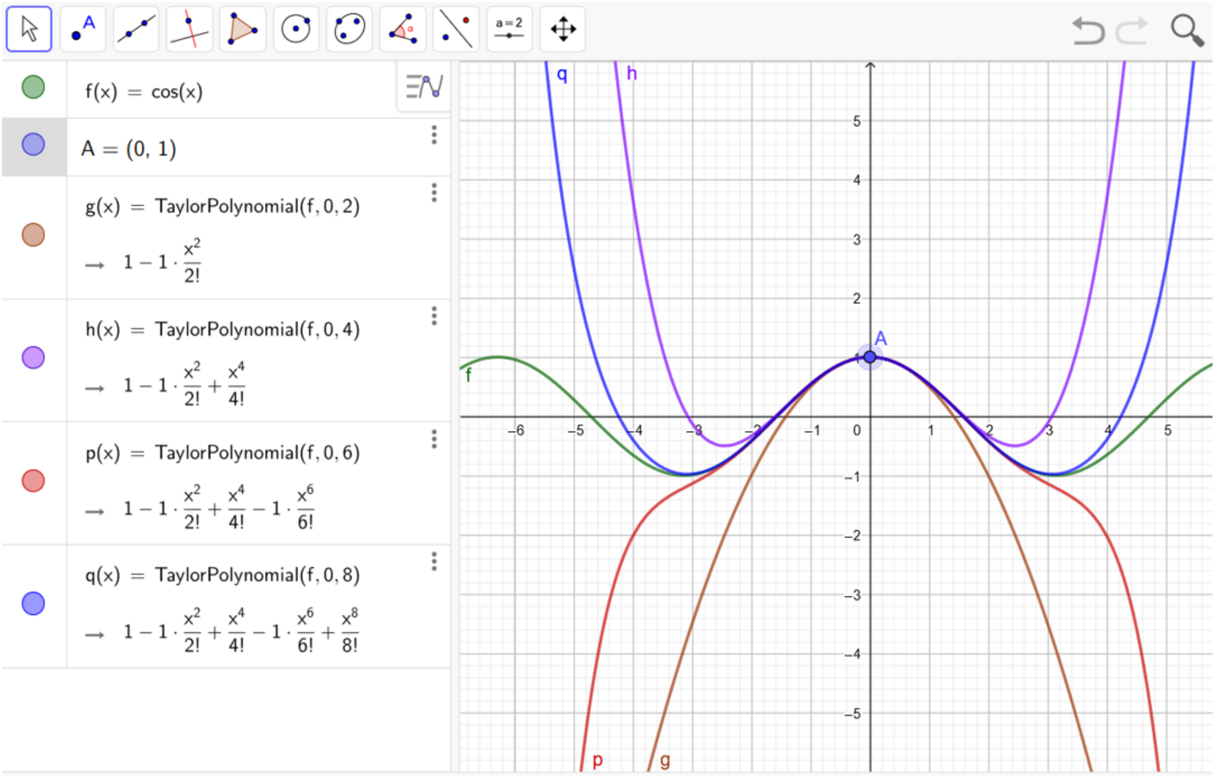

Figure 7: GeoGebra example about Taylor polynomials.

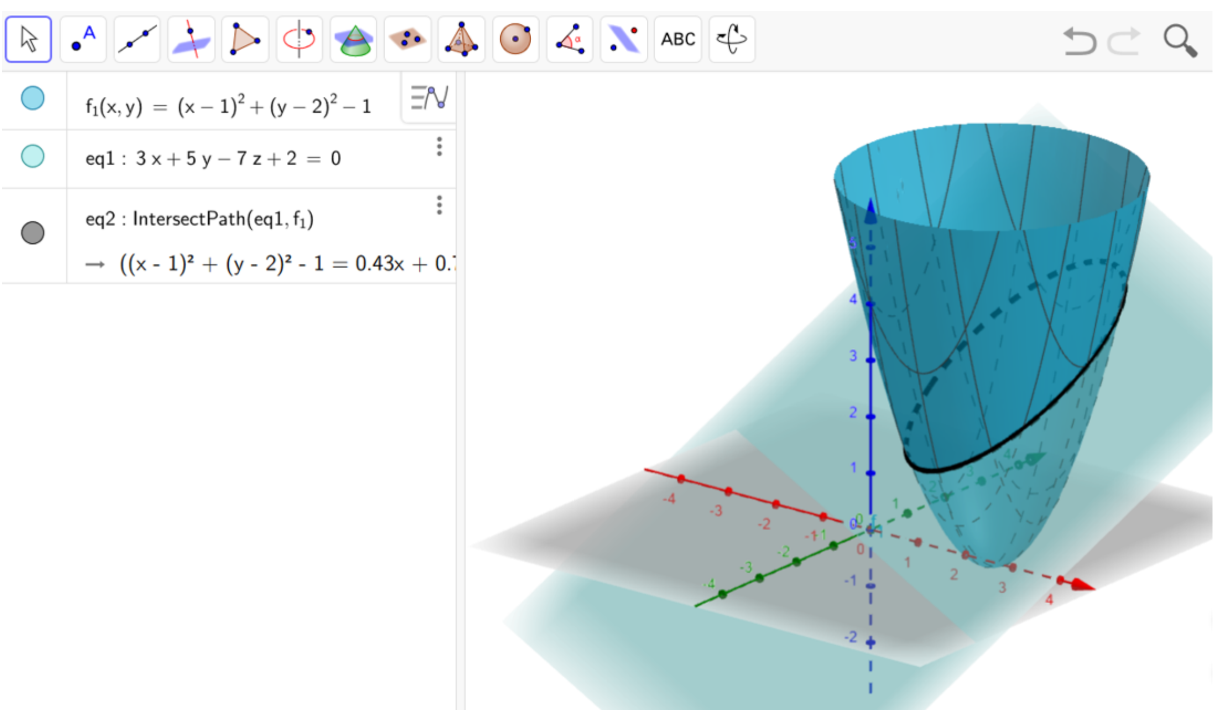

Figure 8: Intersection of two surfaces using GeoGebra.

Both engines are supported by a large number of developers who make available their work, so in both cases it is possible to access many great online demonstrations and practical examples. This feature is particularly interesting when teaching theorems and their applications, as it is a topic where many students face some difficulties. Figures 9 and 10 show how to represent Lagrange's theorem [6] and the Integral Mean Value theorem [7].

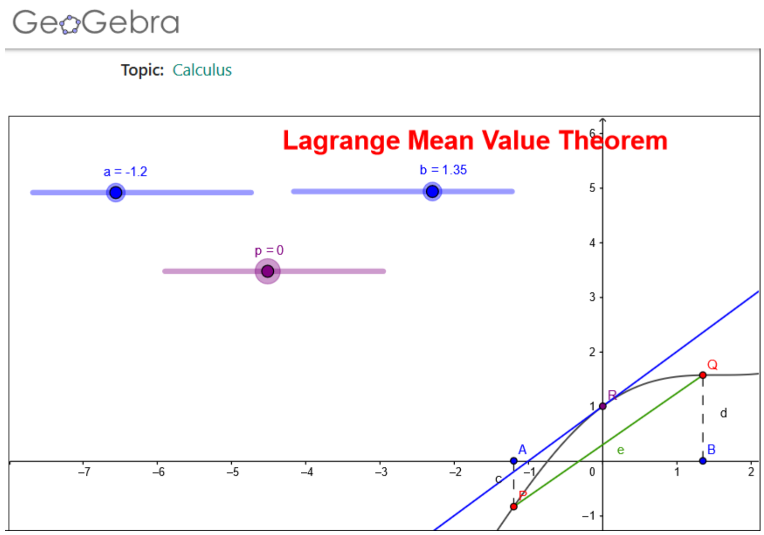

Figure 9: Lagrange Mean Value Theorem (author: Ravinder Kumar [6]).

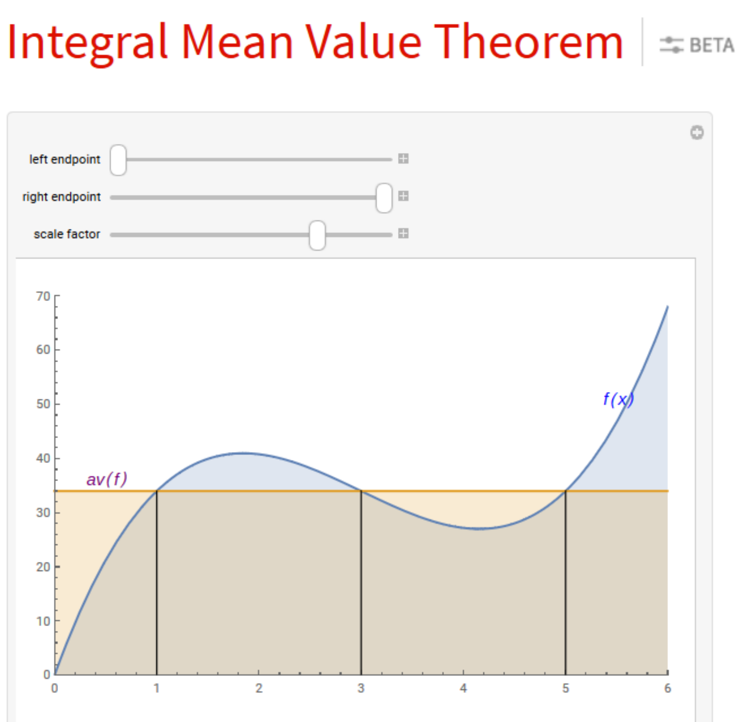

Figure 10: Integral Mean Value Theorem (author: Chris Boucher [7]).

5 Conclusions

In this contribution, we have shown how to use two of the best-known freeware mathematical software, GeoGebra and WolframAlpha, in order to enhance the comprehension of mathematical concepts associated to a first-year single variable Calculus class. Both engines allow students to grasp the key concepts seen at class and to practice problems at their leisure, resulting in better learning outcomes and grades.

Regarding the future work, it would be interesting to centralize demonstrations about all the theorems included in the subject's syllabus as a public, online book, and to extend the usage of these engines to other mathematics courses such as Complex Numbers, Differential Geometry or Ordinary Differential Equations, which will be attempted in next years.

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