MACHINE LEARNING LAB - TUTORIAL 6

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1. DATA PRE-PROCESSING

In [96]:
```
import pandas as pd
import numpy as np
from numpy import random
import math
import matplotlib.pyplot as plt
from sklearn import linear_model
from sklearn.linear_model import Ridge, SGDRegressor, Lasso, Linear
Regression
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split, GridSearchCV,
cross_val_score
from sklearn.metrics import classification_report
from mpl_toolkits.mplot3d import Axes3D
```

D1 DATA

In [97]:
```
x = np.random.normal(loc =1, scale =0.05, size=(100, 1))
psi = np.random.normal(loc =1, scale =0.05, size=(100, 1))
y = 1.3*x**2 + 4.8*x + 8 + psi
# Printing small range of data to improve visualization of results
print('X:',x[0:5])
print('\n')
print('y:',y[0:5])
```

```
X: [[1.03479641]
 [0.99369751]
 [0.95686033]
 [0.9356232 ]
 [0.96222382]]


y: [[15.32950796]
 [15.03569433]
 [14.72574598]
 [14.60940609]
 [14.83175488]]
```

D2 DATA

In [98]:
```
missing_values = ['-','na','Nan','nan','n/a','?']
#red_wine = pd.read_csv('/winequality-red.csv', sep=';', na_values
= missing_values)
D2 = pd.read_csv("winequality-red.csv", sep=';', na_values = missin
g_values)
D2.head()
```

Out[98]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	al
0	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	
1	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	
2	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	
3	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	
4	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	

Check for missing or incongruent values

In [99]:
```
# Check for missing or incongruent values
check = D2.empty
print('checking missing values:',check)
print('Sum of errors:',D2.isnull().sum())
```

```
checking missing values: False
Sum of errors: fixed acidity           0
volatile acidity        0
citric acid             0
residual sugar          0
chlorides               0
free sulfur dioxide     0
total sulfur dioxide    0
density                 0
pH                      0
sulphates               0
alcohol                 0
quality                 0
dtype: int64
```

Normalization of the data

In [100]:
```
def normalize(dataset):
    dataNorm=((dataset-dataset.min())/(dataset.max()-dataset.min())
)
    dataNorm["quality"]=dataset["quality"]
    return dataNorm
D2 = normalize(D2)
D2.head()
```

Out[100]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH
0	0.247788	0.397260	0.00	0.068493	0.106845	0.140845	0.098940	0.567548	0.606299
1	0.283186	0.520548	0.00	0.116438	0.143573	0.338028	0.215548	0.494126	0.362205
2	0.283186	0.438356	0.04	0.095890	0.133556	0.197183	0.169611	0.508811	0.409449
3	0.584071	0.109589	0.56	0.068493	0.105175	0.225352	0.190813	0.582232	0.330709
4	0.247788	0.397260	0.00	0.068493	0.106845	0.140845	0.098940	0.567548	0.606299

2. GLMs

Task A: pick 3 hyperparameters and learn each model (OLS, Ridge, LASSO)

In [101]:
```
np.random.seed(0)
D2_train = D2.sample(frac=0.8)
D2_test = D2.drop(D2_train.index)
```

Task B: pick 3 hyperparameters and learn each model (OLS, Ridge, LASSO)

Since in the documentation it is said explicitly there is no need to create a column of ones for

$$B_0$$

because the code already does that, the step is avoided.

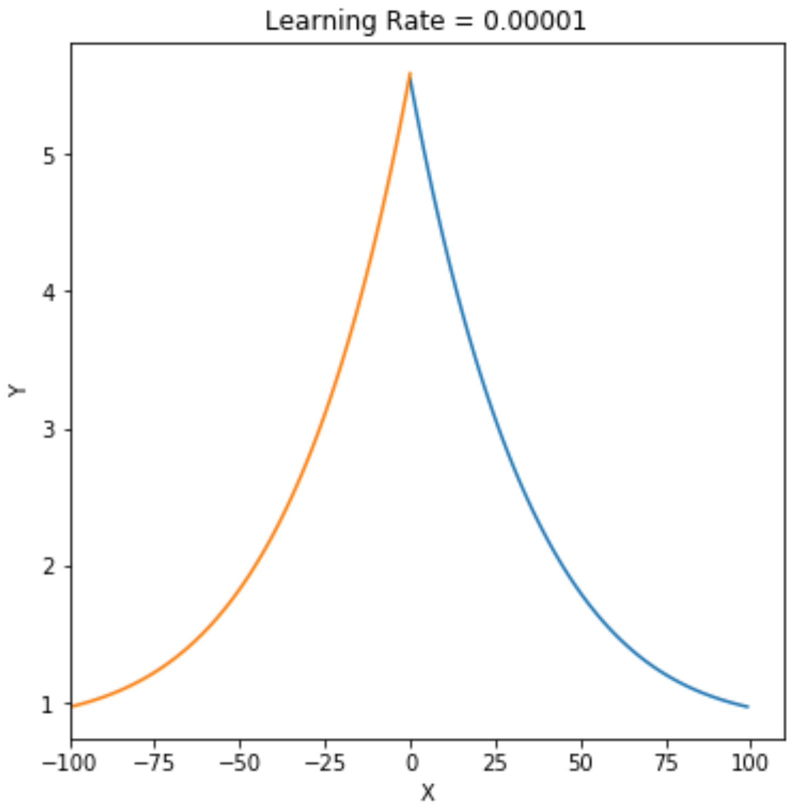
In [102]:
```
Y = D2_train['quality'].values
X = D2_train.drop(['quality'], axis=1).values
n = X.shape[1]
beta = np.zeros(n)
beta = np.reshape(beta, (len(beta),1))
Y_test = D2_test['quality'].values
Y_test = np.reshape(Y_test, (len(Y_test),1))
X_test = D2_test.drop(['quality'], axis=1).values
```

Ordinary Least Squares

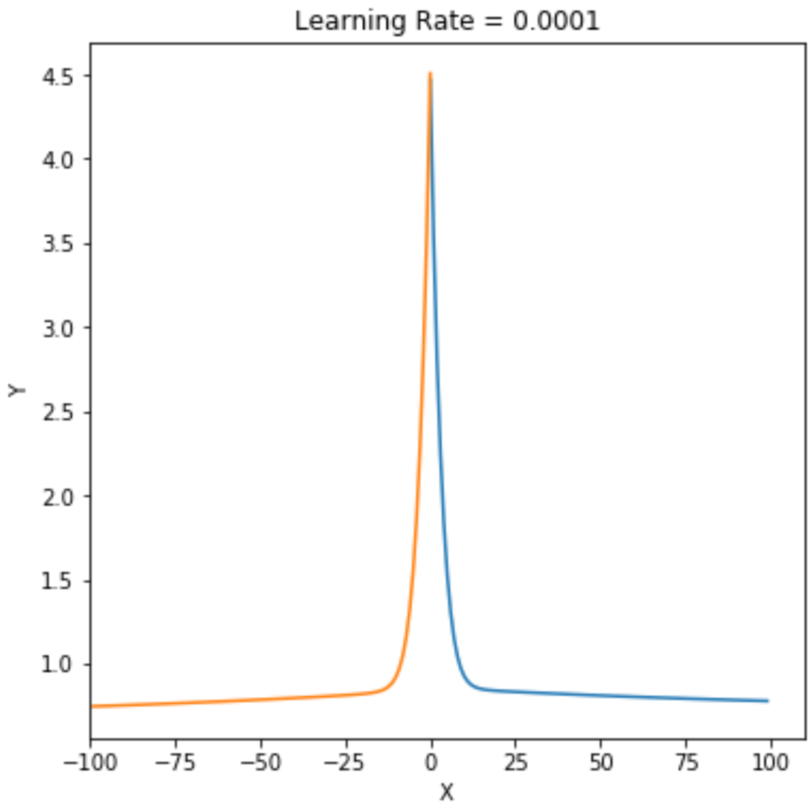
In [103]:
```
def RMSE_function(y, y_hat):
    error = np.sqrt(np.sum((y - y_hat)**2)/y.shape[0])
    return error

def SGD_regression(X, y, X_test, y_test, u):
    total_train = 0
    total_test = 0
    train_RMSE = []
    test_RMSE = []
    sets_ols = linear_model.SGDRegressor(eta0 = u, warm_start=True,
penalty=None, learning_rate='constant')
    for i in range(100):
        sets_it = sets_ols.partial_fit(X, Y)
        betas = sets_it.coef_
        y_hat = sets_it.predict(X)
        y_hat_test = sets_it.predict(X_test)
        total_train = np.sqrt(mean_squared_error(Y, y_hat))
        total_test = np.sqrt(mean_squared_error(Y_test, y_hat_test)
)
        train_RMSE.append(total_train)
        test_RMSE.append(total_test)

    return betas, total_train, total_test, train_RMSE, test_RMSE, t
otal_test
```

In [104]:
```
a, b, c, d, e, g= SGD_regression(X, Y, X_test, Y_test, 0.00001)
a, b, c, r, io, iu= SGD_regression(X, Y, X_test, Y_test, 0.0001)
a, b, c, p, u, cv= SGD_regression(X, Y, X_test, Y_test, 0.001)
print('RMSE test:', g)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), d)
plt.plot(rango*-1,e)
plt.title('Learning Rate = 0.00001')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()
print('RMSE test:', iu)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), r)
plt.plot(rango*-1,io)
plt.title('Learning Rate = 0.0001')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()
print('RMSE test:', cv)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), p)
plt.plot(rango*-1,u)
plt.title('Learning Rate = 0.001')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()
```

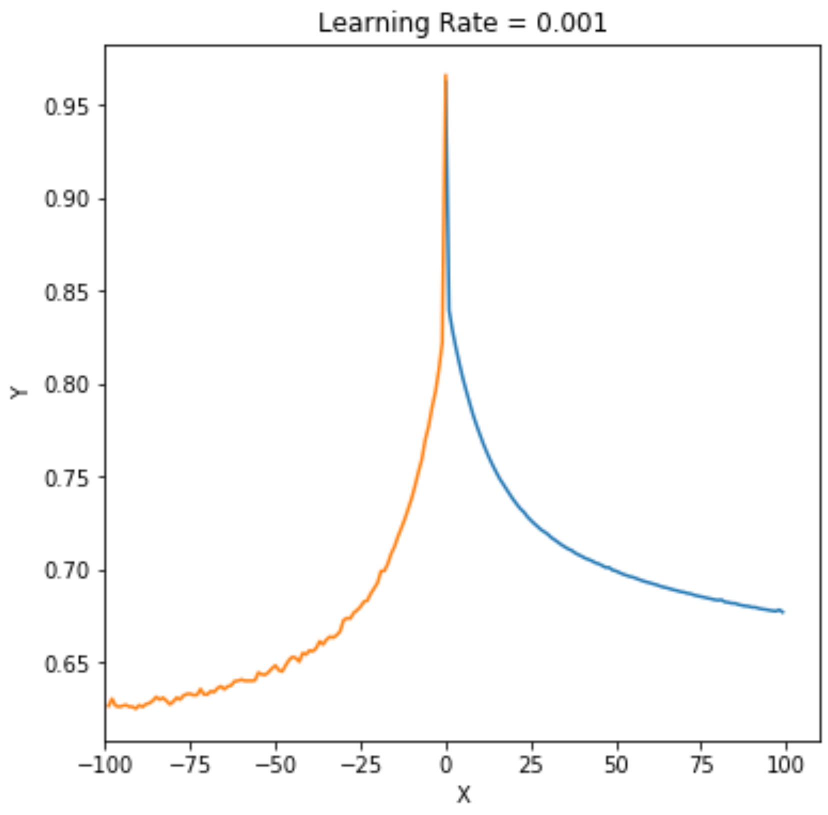
```
RMSE test: 0.9745611696325205
```



```
RMSE test: 0.7457940250429301
```



```
RMSE test: 0.6267548401888524
```

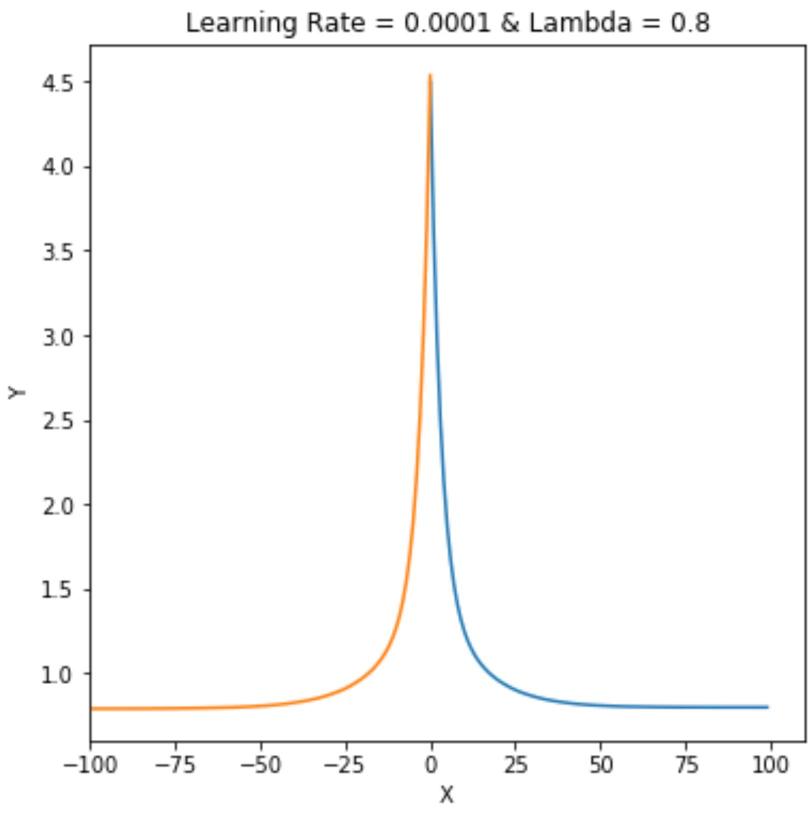


RIDGE regression

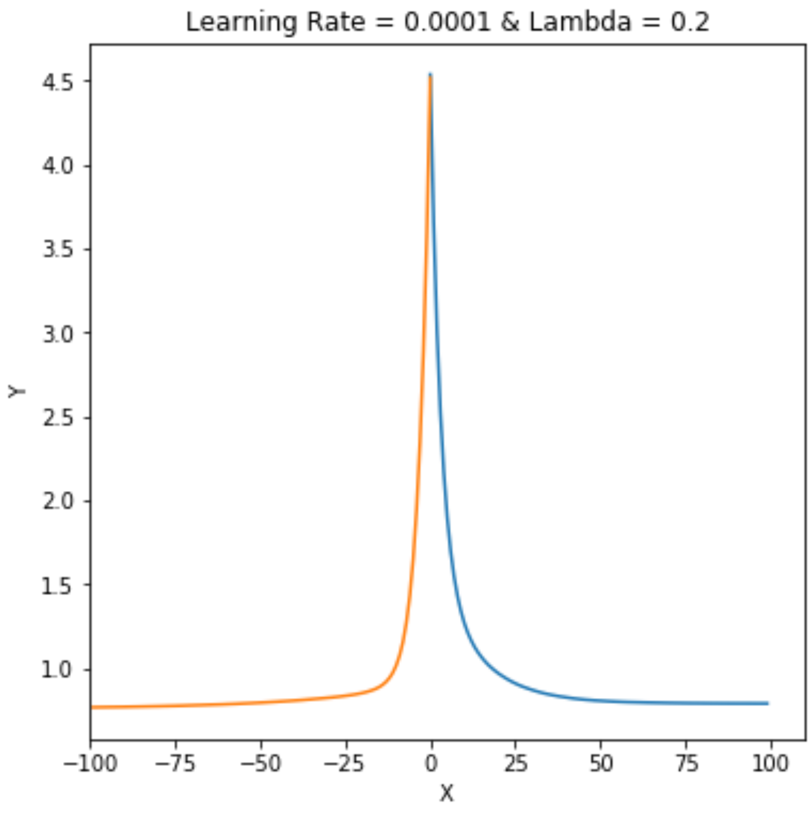
In [105]:
```
def RMSE_function(y, y_hat):
  error = np.sqrt(np.sum((y - y_hat)**2)/y.shape[0])
  return error

def SGD_regression_ridge(X, y, X_test, y_test, u, parameter):
    total_train = 0
    total_test = 0
    train_RMSE = []
    test_RMSE = []
    sets_ridge = linear_model.SGDRegressor(penalty='L2', alpha = pa
rameter, eta0 = u, warm_start=True, learning_rate='constant')
    for i in range(100):
        sets_it = sets_ridge.partial_fit(X, Y)
        betas = sets_it.coef_
        y_hat = sets_it.predict(X)
        y_hat_test = sets_it.predict(X_test)
        total_train = np.sqrt(mean_squared_error(Y, y_hat))
        total_test = np.sqrt(mean_squared_error(Y_test, y_hat_test)
)
        train_RMSE.append(total_train)
        test_RMSE.append(total_test)

    return betas, total_train, total_test, train_RMSE, test_RMSE, t
otal_test

    return betas, total_train, total_test, train_RMSE, test_RMSE
```

In [106]:
```
a, b, c, d, e, g = SGD_regression_ridge(X, Y, X_test, Y_test, 0.000
1,0.8)
a, b, c, r, io, hj = SGD_regression_ridge(X, Y, X_test, Y_test, 0.0
001, 0.2)
a, b, c, p, u, lk = SGD_regression_ridge(X, Y, X_test, Y_test, 0.00
01, 0.5)
print('RMSE test:', g)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), d)
plt.plot(rango*-1,e)
plt.title('Learning Rate = 0.0001 & Lambda = 0.8')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()

print('RMSE test:', hj)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), e)
plt.plot(rango*-1,io)
plt.title('Learning Rate = 0.0001 & Lambda = 0.2')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()

print('RMSE test:', lk)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), p)
plt.plot(rango*-1,u)
plt.title('Learning Rate = 0.0001 & Lambda = 0.5')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()
```

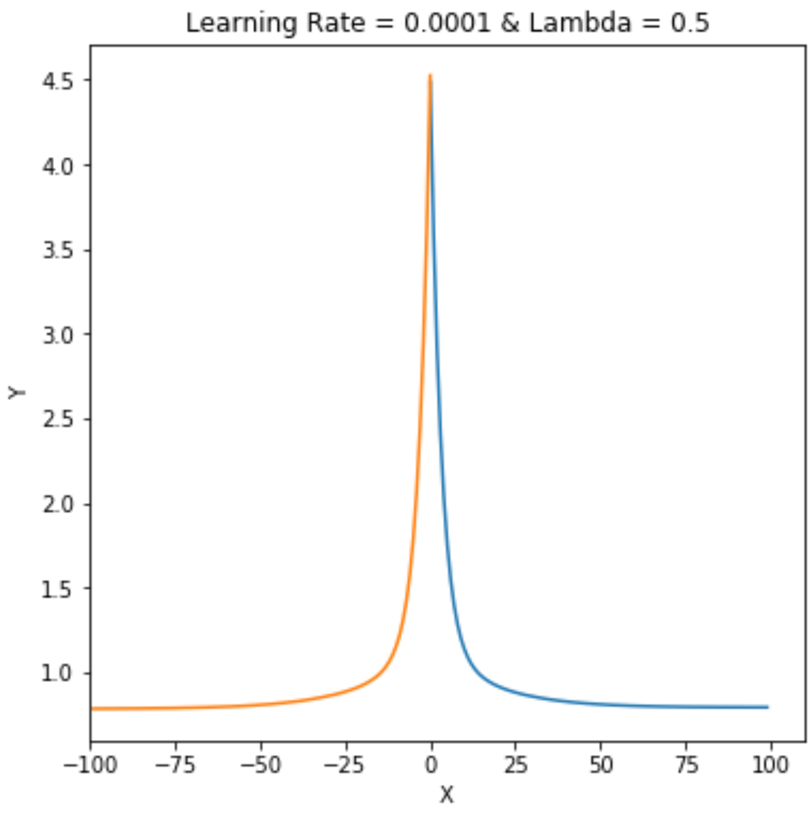
```
RMSE test: 0.7889343172866163
```



```
RMSE test: 0.7659424576252672
```



```
RMSE test: 0.7828331405997372
```

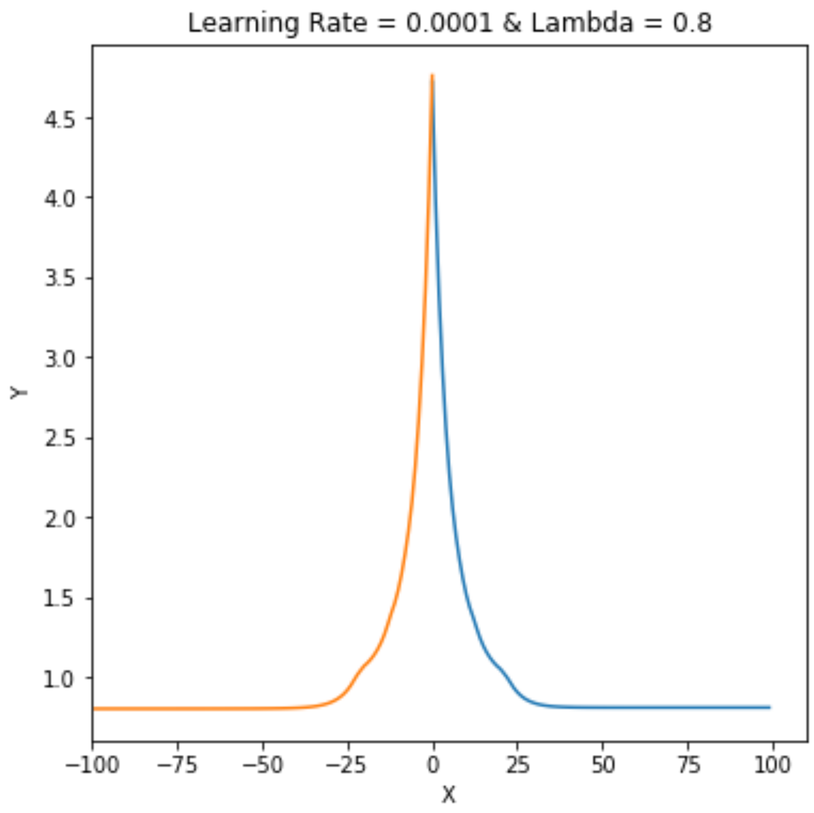

Learning Rate = 0.0001 & Lambda = 0.5
Y
X
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
−100
−75
−50
−25
0
25
50
75
100

LASSO

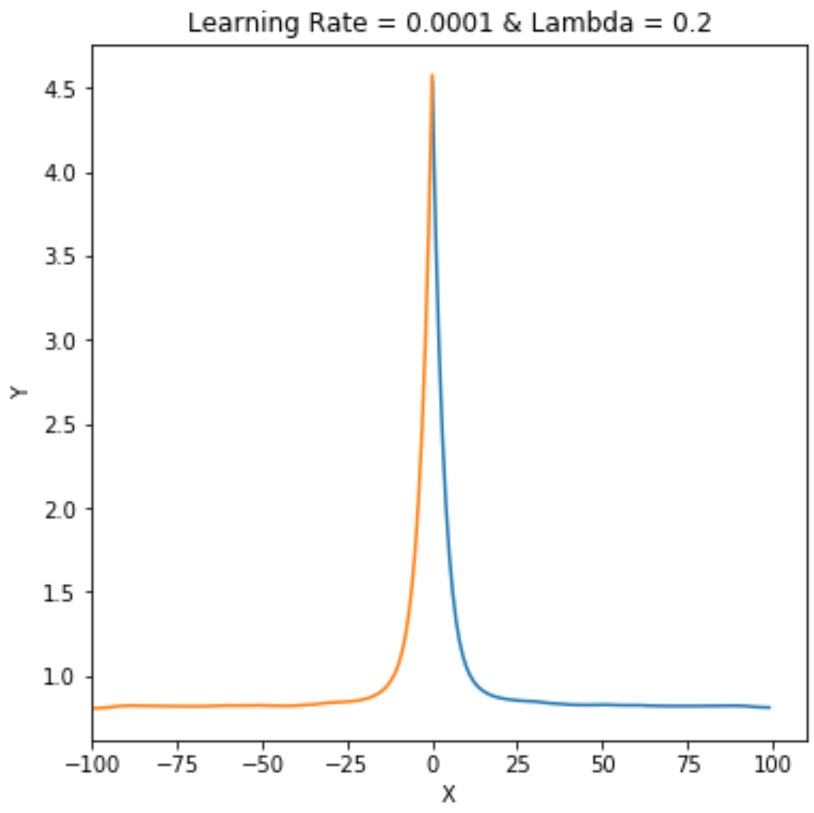
In [107]:
```
def RMSE_function(y, y_hat):
  error = np.sqrt(np.sum((y - y_hat)**2)/y.shape[0])
  return error

def SGD_regression_lasso(X, y, X_test, y_test, u, parameter):
    total_train = 0
    total_test = 0
    train_RMSE = []
    test_RMSE = []
    sets_lasso = linear_model.SGDRegressor(tol=1e-3, penalty='L1',
alpha = parameter, eta0 = u,  warm_start=True, learning_rate='const
ant')
    for i in range(100):
        sets_it = sets_lasso.partial_fit(X, Y)
        betas = sets_it.coef_
        y_hat = sets_it.predict(X)
        y_hat_test = sets_it.predict(X_test)
        #total_train = RMSE_function(Y, y_hat)
        total_train = np.sqrt(mean_squared_error(Y, y_hat))
        #total_test = RMSE_function(Y_test, y_hat_test)
        total_test = np.sqrt(mean_squared_error(Y_test, y_hat_test)
)
        train_RMSE.append(total_train)
        test_RMSE.append(total_test)

    return betas, total_train, total_test, train_RMSE, test_RMSE, t
otal_test
```

In [108]:
```
a, b, c, d, e, ae = SGD_regression_lasso(X, Y, X_test, Y_test, 0.0001, 0.8)
a, b, c, r, io, yu = SGD_regression_lasso(X, Y, X_test, Y_test, 0.0001, 0.2)
a, b, c, p, u, po = SGD_regression_lasso(X, Y, X_test, Y_test, 0.0001, 0.5)

print('RMSE test:', ae)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), d)
plt.plot(rango*-1,e)
plt.title('Learning Rate = 0.0001 & Lambda = 0.8')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()

print('RMSE test:', yu)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), r)
plt.plot(rango*-1,io)
plt.title('Learning Rate = 0.0001 & Lambda = 0.2')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()

print('RMSE test:', po)
fig= plt.figure(figsize=(6,6))
rango = np.array(range(100))
plt.plot(range(100), p)
plt.plot(rango*-1,u)
plt.title('Learning Rate = 0.0001 & Lambda = 0.5')
plt.xlabel('X')
plt.ylabel('Y')
plt.xlim(-100, 110)
plt.show()
```

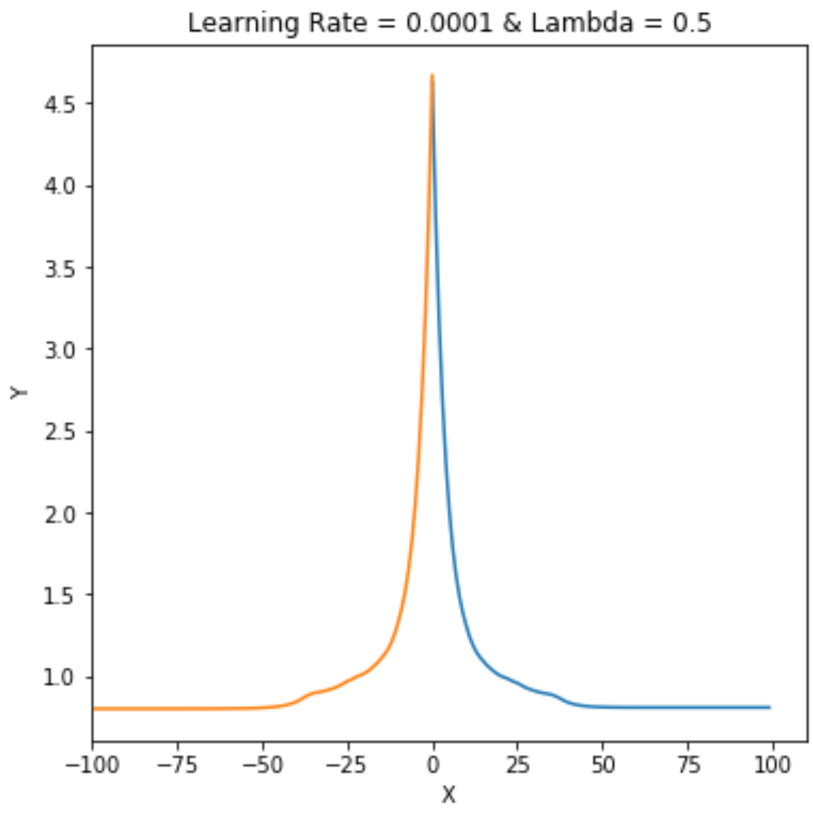
```
RMSE test: 0.8012122384001842
```



```
RMSE test: 0.8061146186993237
```



```
RMSE test: 0.8012338952186141
```



Task C: Tune the hyperparameters using GridSearchCV and plot the results.

OLS

In [109]:
```
model = SGDRegressor(warm_start=True, random_state=3)
param_grid = {
    'penalty': [None],
    'learning_rate': ['constant'],
    'eta0' : [0.0001, 0.00001, 0.001],
    'max_iter': [1000],
}
sets = GridSearchCV(model, param_grid, cv=5)
sets.fit(X, Y)
print("Best score: " + str(sets.best_score_))
```

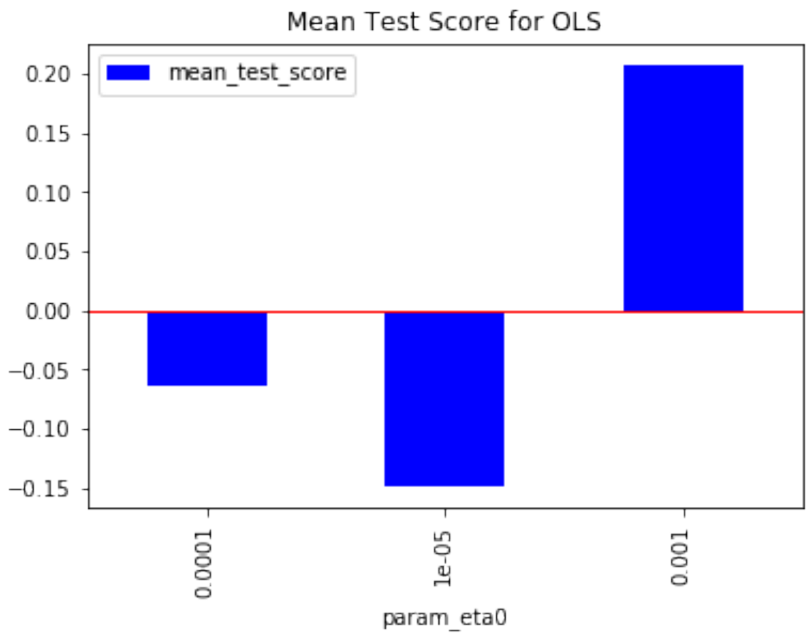
```
Best score: 0.20781459001958902
```

In [110]:
```
df = pd.DataFrame(sets.cv_results_)
df.sort_values('rank_test_score') # showing the results from the be
st to the worst.
```

Out[110]:

	mean_fit_time	std_fit_time	mean_score_time	std_score_time	param_eta0	param_learnii
2	0.002096	0.000081	0.000361	0.000040	0.001	c
0	0.002683	0.000166	0.000649	0.000130	0.0001	c
1	0.008948	0.000922	0.000495	0.000104	1e-05	c

In [111]:
```
df.plot(kind='bar',x='param_eta0', y='mean_test_score',color='blue'
)
plt.axhline(linewidth=1, color='red')
plt.title('Mean Test Score for OLS')
plt.show()
```



RIDGE

In [112]:
```
model = SGDRegressor(warm_start=True, random_state=3)
param_grid = {
    'penalty': ['L2'],
    'learning_rate': ['constant'],
    'eta0' : [0.001],
    'alpha':[0.001, 0.0001, 0.01],
    'max_iter': [1000],
}
sets_ridge = GridSearchCV(model, param_grid, cv=5)
sets_ridge.fit(X, Y)
print("Best score: " + str(sets.best_score_))
```

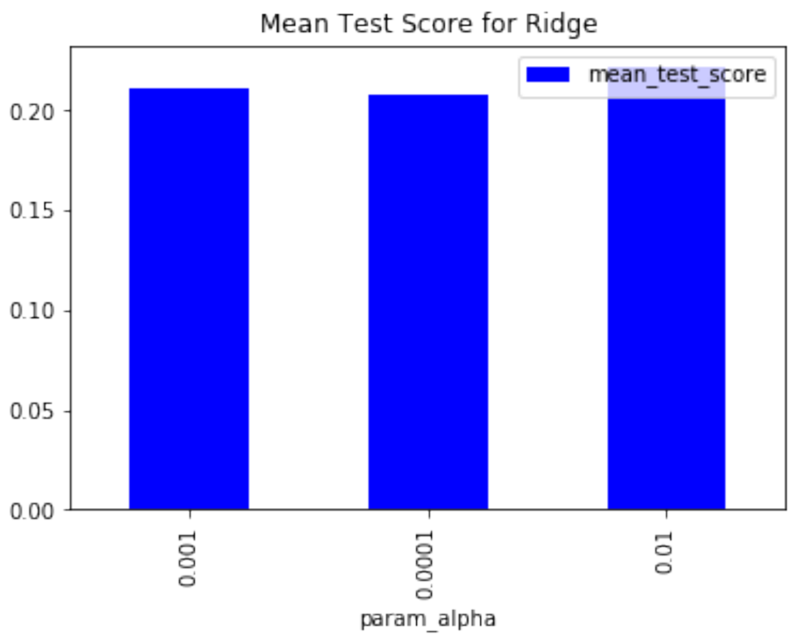
```
Best score: 0.20781459001958902
```

In [113]:
```
df_ridge = pd.DataFrame(sets_ridge.cv_results_)
df_ridge.sort_values('rank_test_score') # showing the results from
the best to the worst.
```

Out[113]:

	mean_fit_time	std_fit_time	mean_score_time	std_score_time	param_alpha	param_eta0
2	0.002644	0.000548	0.000383	0.000059	0.01	0.001
0	0.003487	0.000381	0.000573	0.000056	0.001	0.001
1	0.002658	0.000194	0.000526	0.000205	0.0001	0.001

In [114]:
```
df_ridge.plot(kind='bar',x='param_alpha', y='mean_test_score',color
='blue')
plt.axhline(linewidth=1, color='red')
plt.title('Mean Test Score for Ridge')
plt.show()
```



LASSO

In [115]:
```
model = SGDRegressor(warm_start=True, random_state=3)
param_grid = {
    'penalty': ['L1'],
    'learning_rate': ['constant'],
    'eta0' : [0.001],
    'alpha':[0.001, 0.0001, 0.01],
    'max_iter': [1000],
}
sets_lasso = GridSearchCV(model, param_grid, cv=5)
sets_lasso.fit(X, Y)
print("Best score: " + str(sets.best_score_))
```

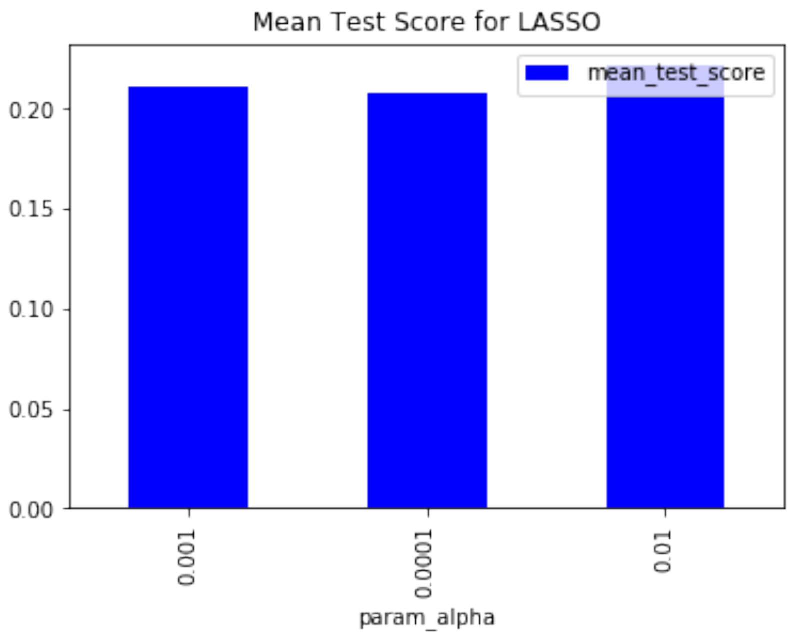
```
Best score: 0.20781459001958902
```

In [116]:
```
df_lasso = pd.DataFrame(sets_ridge.cv_results_)
df_lasso.sort_values('rank_test_score') # showing the results from
the best to the worst.
```

Out[116]:

	mean_fit_time	std_fit_time	mean_score_time	std_score_time	param_alpha	param_eta0
2	0.002644	0.000548	0.000383	0.000059	0.01	0.001
0	0.003487	0.000381	0.000573	0.000056	0.001	0.001
1	0.002658	0.000194	0.000526	0.000205	0.0001	0.001

In [117]:
```
df_ridge.plot(kind='bar',x='param_alpha', y='mean_test_score',color
='blue')
plt.axhline(linewidth=1, color='red')
plt.title('Mean Test Score for LASSO')
plt.show()
```

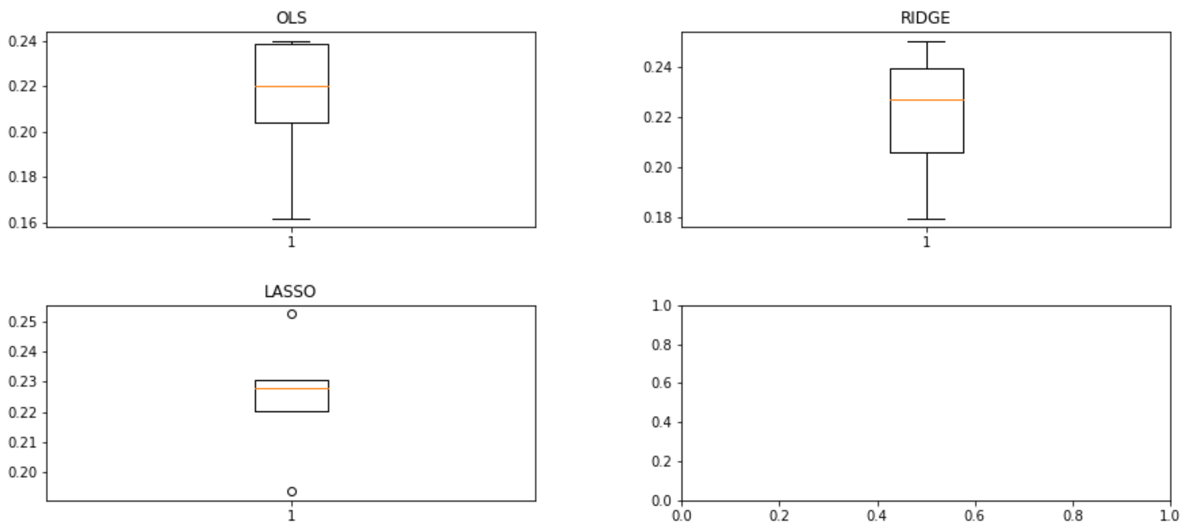


Task D: Evaluate the optimal hyperparameter on each model

In [118]:
```
sets_ols = linear_model.SGDRegressor(eta0 = 0.001, warm_start=True,
penalty=None, learning_rate='constant')
sets_ridge = linear_model.SGDRegressor(penalty='L2', alpha = 0.01,
eta0 = 0.001, warm_start=True, learning_rate='constant')
sets_lasso = linear_model.SGDRegressor(tol=1e-3, penalty='L1', alph
a = 0.01, eta0 = 0.001,  warm_start=True, learning_rate='constant')
print('Cross Validation OLS:',cross_val_score(sets_ols, X, Y, cv=5)
)
print('\n')
print('Cross Validation Ridge:',cross_val_score(sets_ridge, X, Y, c
v=5))
print('\n')
print('Cross Validation Lasso:',cross_val_score(sets_lasso, X, Y, c
v=5))
print('\n')
```

```
Cross Validation OLS: [0.24546509 0.21394921 0.23375992 0.16332196
0.18647486]


Cross Validation Ridge: [0.25020776 0.23021281 0.2341357  0.189195
71 0.213593  ]


Cross Validation Lasso: [0.23327785 0.20772981 0.21929566 0.179428
72 0.19319631]
```

In [119]:
```
fig, axs = plt.subplots(2, 2, figsize=(6, 6))

axs[0, 0].boxplot(cross_val_score(sets_ols, X, Y, cv=5))
axs[0, 0].set_title('OLS')

axs[0, 1].boxplot(cross_val_score(sets_ridge, X, Y, cv=5))
axs[0, 1].set_title('RIDGE')

axs[1, 0].boxplot(cross_val_score(sets_ridge, X, Y, cv=5))
axs[1, 0].set_title('LASSO')


fig.subplots_adjust(left=0.08, right=2, bottom=0.10, top=0.9, hspac
e=0.4, wspace=0.3)
```



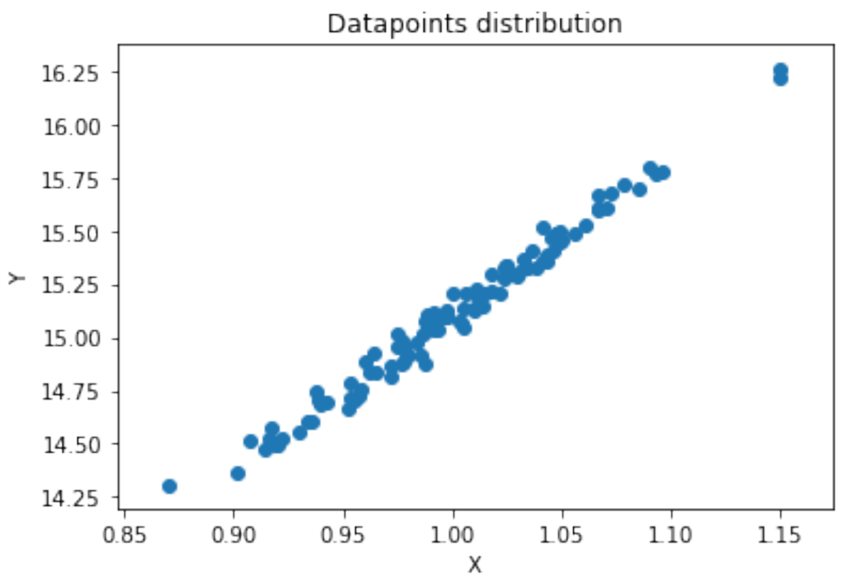
Observations

1. OLS is returning lower values considering the different hyperparameter analyzed in the early steps.
2. **This conclusions may vary at the moment of running the code because of randomness.**
3. LASSO has a better distribution of the values: it contains all values near to the median. In addition, most of the data is located near to the 25 percentile. It has consistency through all datapoints.
4. OLS on the contrary, most of the scores are far away from the minimum. On the other hand, it has a good inner distribution: most of the values are located equally above or below the median: It means that in some way there is underfitting for several datapoints. The data would be not generalizing well.
5. Ridge experienced a different distribution: values close to the max but not far away from the min. In addition, the values are distributed with a majority located near to the 25 percentile.
6. Although both OLS and Ridge has minimum values (around 16) in my opinion is not good generalization by having such a gap between values.

3. POLYNOMIAL REGRESSION

Task A: Prediction with high degree of polynomials

In [120]:
```
plt.scatter(x,y)
plt.title('Datapoints distribution')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```

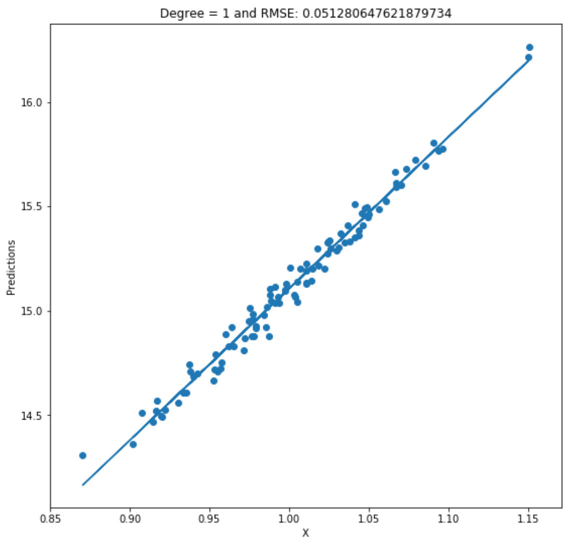


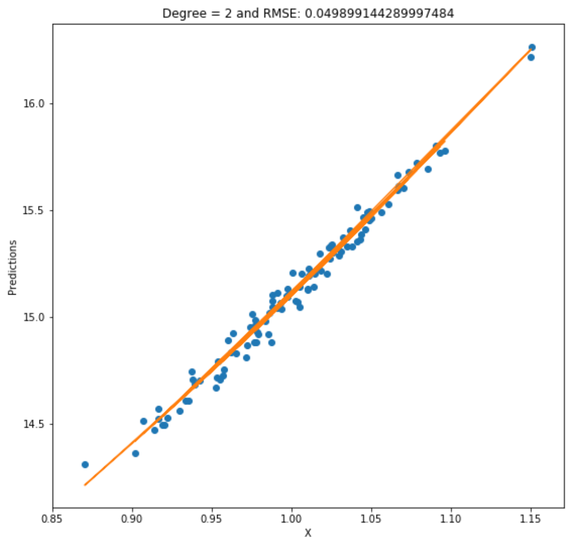
In [121]:
```
RMSE = []
def polynomial_regression(x, y, degree):
    polynomial_features= PolynomialFeatures(degree)
    x_poly = polynomial_features.fit_transform(x)

    linear_Regression = LinearRegression().fit(x_poly, y)

    y_hat = linear_Regression.predict(x_poly)

    error = RMSE_function(y, y_hat)  # function used in the previou
s exercise
    return error, y_hat, degree
```

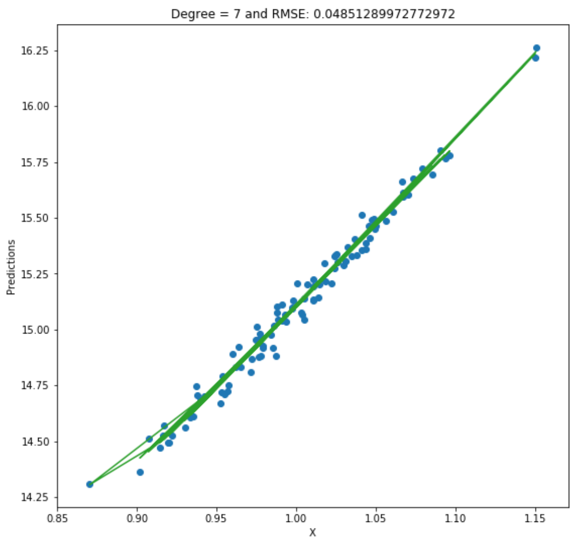
In [122]:
```
degress = [1, 2, 7, 10, 16, 100]
for i in range(len(degress)):
    RMSE, a, c = polynomial_regression(x, y, degress[i])
    print('Degree:', degress[i], 'RMSE',RMSE)
```

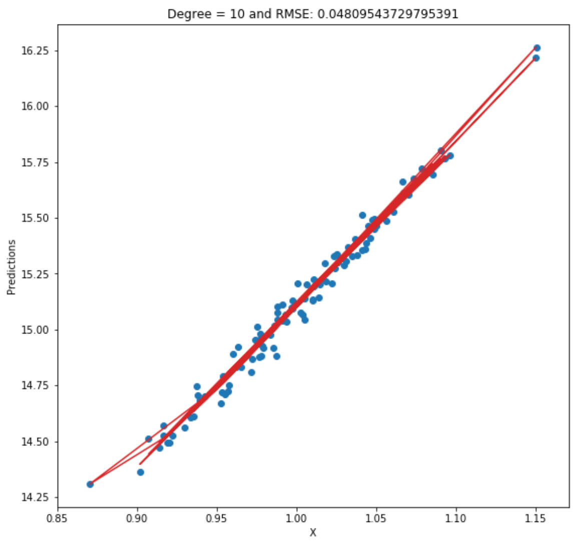
```
Degree: 1 RMSE 0.051280647621879734
Degree: 2 RMSE 0.049899144289997484
Degree: 7 RMSE 0.04851289972772972
Degree: 10 RMSE 0.04809543729795391
Degree: 16 RMSE 0.04807467280869197
Degree: 100 RMSE 0.04756108753378391
```

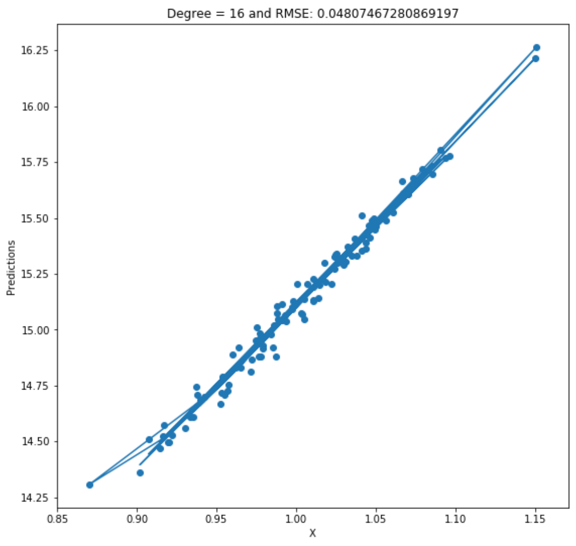
In [123]:
```python
a, b, c = polynomial_regression(x, y, 1)
d, e, t = polynomial_regression(x, y, 2)
h, i, p = polynomial_regression(x, y, 7)
k, l, ñ = polynomial_regression(x, y, 10)
n, o, s = polynomial_regression(x, y, 16)
q, r, v = polynomial_regression(x, y, 100)
fig, axs = plt.subplots(3, 2,figsize=(20,30))
axs[0, 0].scatter(x, y)
axs[0, 0].plot(x, b)
axs[0, 0].set_title('Degree = {} and RMSE: {}'.format(c,a))
axs[0, 1].scatter(x, y)
axs[0, 1].plot(x, e, 'tab:orange')
axs[0, 1].set_title('Degree = {} and RMSE: {}'.format(t,d))
axs[1, 0].scatter(x, y)
axs[1, 0].plot(x, i, 'tab:green')
axs[1, 0].set_title('Degree = {} and RMSE: {}'.format(p,h))
axs[1, 1].scatter(x, y)
axs[1, 1].plot(x, l, 'tab:red')
axs[1, 1].set_title('Degree = {} and RMSE: {}'.format(ñ,k))
axs[2, 0].scatter(x, y)
axs[2, 0].plot(x, o, 'tab:blue')
axs[2, 0].set_title('Degree = {} and RMSE: {}'.format(s,n))
axs[2, 1].scatter(x, y)
axs[2, 1].plot(x, r, 'tab:gray')
axs[2, 1].set_title('Degree = {} and RMSE: {}'.format(v,q))

for ax in axs.flat:
    ax.set(xlabel='X', ylabel='Predictions')
```

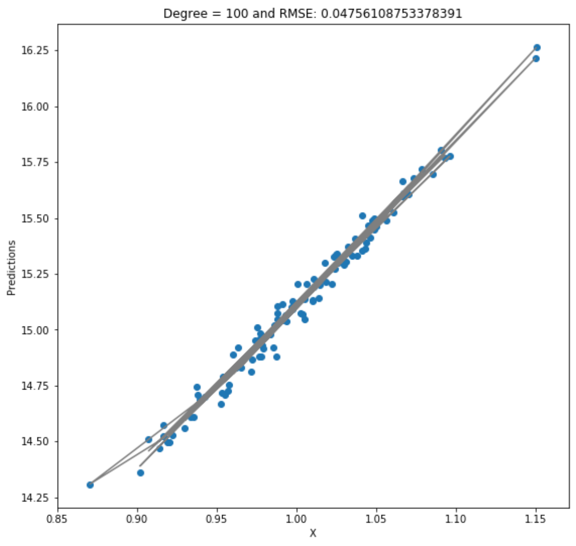

Degree = 1 and RMSE: 0.051280647621879734
Predictions
X


Degree = 2 and RMSE: 0.049899144289997484
Predictions
X


Degree = 7 and RMSE: 0.04851289972772972
Predictions
X


Degree = 10 and RMSE: 0.04809543729795391
Predictions
X


Degree = 16 and RMSE: 0.04807467280869197
Predictions
X


Degree = 100 and RMSE: 0.04756108753378391
Predictions
X

Observations

1. By plotting for the first time the datapoints it is possible to appreciate that they follow a **linear distribution.**

2. Degree 2 graph shows that it is still possible to represent all the datapoints in a more **accurate** way (It reduces the distance for the far-away datapoints slightly). Moreover, there are no big distances from the mean to the datapoints, therefore no overfitting exists.

3. Unfortunately, from degree 7 and on the predictions are incurring in **overfitting**. Since the data follows a linear distribution it is not required an improvement to understand the relationship between the variables. The more we increase the degree, the more overfitting the result will be, the more datapoints the line will cross.

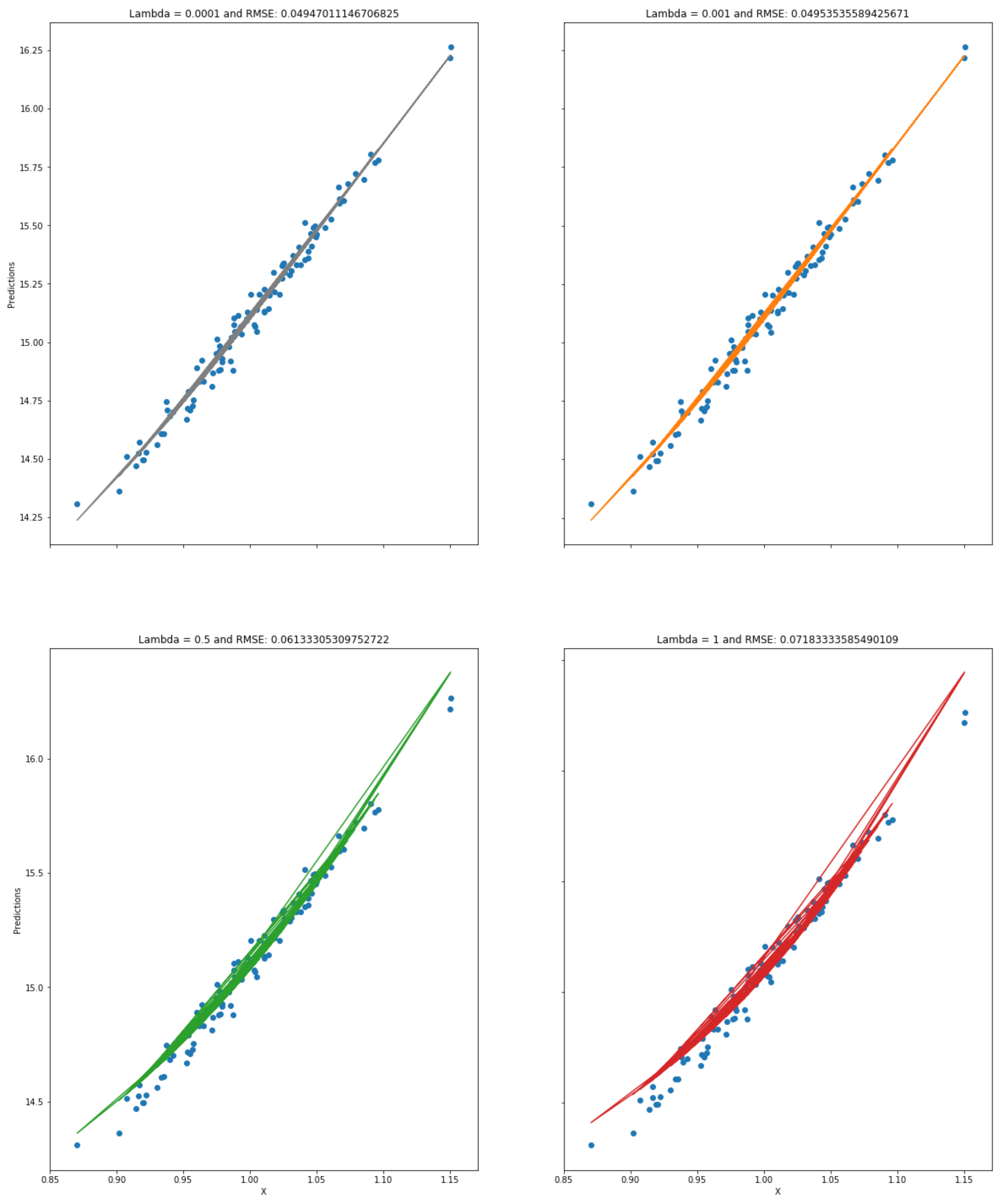
Task B: Effect of Regularization

In [124]:
```
lambdas = [0, 0.000001, 0.01, 1]

def polynomial_regression(x, y, lambdas):
    polynomial_features= PolynomialFeatures(degree=10)
    x_poly = polynomial_features.fit_transform(x)

    ridge = linear_model.Ridge(lambdas).fit(x_poly, y)

    y_hat = ridge.predict(x_poly)

    error = RMSE_function(y, y_hat)  # function used in the previou
s exercise
    return error, y_hat, lambdas
```

In [125]:
```
for i in range(len(lambdas)):
    RMSE, a, c = polynomial_regression(x, y, lambdas[i])
    print('Lambdas:', lambdas[i], 'RMSE',RMSE)
    #plt.scatter(x, y)
    #plt.plot(x, a, color='red')
    #plt.show()
```

```
Lambdas: 0 RMSE 0.04808399335670202
Lambdas: 1e-06 RMSE 0.04877067368353639
Lambdas: 0.01 RMSE 0.04961681748077817
Lambdas: 1 RMSE 0.07183333585490109
```

In [126]:
```
a, b, c = polynomial_regression(x, y, 0.0001)
d, e, t = polynomial_regression(x, y, 0.001)
h, i, p = polynomial_regression(x, y, 0.5)
k, l, ñ = polynomial_regression(x, y, 1)
fig, axs = plt.subplots(2, 2,figsize=(20,25))
axs[0, 0].scatter(x, y)
axs[0, 0].plot(x, b, 'tab:grey')
axs[0, 0].set_title('Lambda = {} and RMSE: {}'.format(c,a))
axs[0, 1].scatter(x, y)
axs[0, 1].plot(x, e, 'tab:orange')
axs[0, 1].set_title('Lambda = {} and RMSE: {}'.format(t,d))
axs[1, 0].scatter(x, y)
axs[1, 0].plot(x, i, 'tab:green')
axs[1, 0].set_title('Lambda = {} and RMSE: {}'.format(p,h))
axs[1, 1].scatter(x, y)
axs[1, 1].plot(x, l, 'tab:red')
axs[1, 1].set_title('Lambda = {} and RMSE: {}'.format(ñ,k))

for ax in axs.flat:
    ax.set(xlabel='X', ylabel='Predictions')

# Hide x labels and tick labels for top plots and y ticks for right
plots.
for ax in axs.flat:
    ax.label_outer()
```


Lambda = 0.0001 and RMSE: 0.04947011146706825
Lambda = 0.001 and RMSE: 0.04953535589425671
Lambda = 0.5 and RMSE: 0.06133305309752722
Lambda = 1 and RMSE: 0.07183333585490109
Predictions
X
16.25
16.00
15.75
15.50
15.25
15.00
14.75
14.50
14.25
16.0
15.5
15.0
14.5
0.85
0.90
0.95
1.00
1.05
1.10
1.15

Observations

Reminder: The degree was settled to 10

1. First, at simple sight it is possible to visualize how the parametrization reduces surprisingly the overfitting of the graphs. Considering a degree of 10 plotting an illustration that follows an almost linear line is a remarkable effective.
   For a clear identification please check graph 4 of previous plot and graphs 1 and 2 of current plot.

2. Moreover, selecting the ideal parameter plays a major role: The degree of the graph suggest a high variance outcome and the objective is to find a trade-off to lower the variance without affecting the bias. If a big value of regularization is chosen the model will end with an underfitting outcome as it is possible to appreciate in graphs 3 and 4.

3. In addition, by playing along with the degrees, if degree 2 is chosen: the higher the value of the regularization, the more the curve will tend to 0(horizontal line - slope of 0).

4. COORDINATE DESCENT

In [127]:
```
Y = D2_train['quality'].values
Y = np.reshape(y, (len(y),1))
X = D2_train.drop(['quality'], axis=1).values
column_one = np.ones((X.shape[0],1))
X = np.concatenate((column_one, X), axis = 1)
n = X.shape[1]
y_test = D2_test['quality'].values
y_test = np.reshape(y_test, (len(y_test),1))
y_test = np.reshape(y_test, (len(y_test),1))
X_test = D2_test.drop(['quality'], axis=1).values
column_ones = np.ones((X_test.shape[0],1))
X_test = np.concatenate((column_ones, X_test), axis = 1)
```

In [128]:
```
Y = D2_train['quality'].values
Y = np.reshape(Y, (len(Y),1))
X = D2_train.drop(['quality'], axis=1).values
column_one = np.ones((X.shape[0],1)) # Adding Bias
X = np.concatenate((column_one, X), axis = 1)
n = X.shape[1]
beta = np.zeros(n)
beta = np.reshape(beta, (len(beta),1))
Y_test = D2_test['quality'].values
Y_test = np.reshape(Y_test, (len(Y_test),1))
X_test = D2_test.drop(['quality'], axis=1).values
```

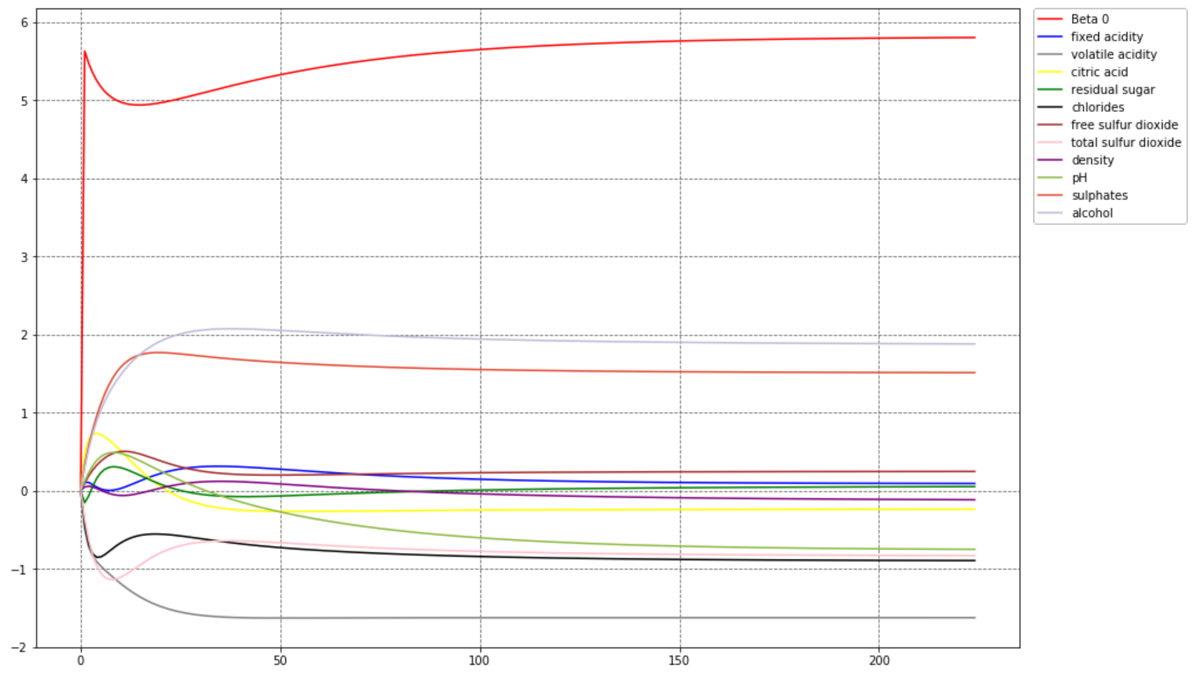
TASK A: Coordinate Descent

In [129]:
```
def function(X, y, beta):
    function = np.dot((y - X@beta).T,(y - X@beta))
    return function


def coordinate_descent(X, y, beta):
    max_iters = 0
    for _ in range(300):
        max_iters += 1
        for i in range(len(X.T)):
            X_n = X.T[i]
            X_n = np.reshape(X_n, (len(X_n), 1))
            betas_xi = np.delete(beta, i, axis=0)
            X_m = np.delete(X.T, i, axis=0)
            betas_new = ((y - X_m.T@betas_xi).T@X_n / np.dot(X_n.T, 
X_n))
            beta[i] = betas_new
        historico.append(beta.copy())
        if function(X, y, historico[-2]) - function(X, y, historico
[-1]) < 0.0001:
            return beta, historico, max_iters
    raise Exception('Not converged in maximum number of iterations'
)
```

In [130]:
```
beta = np.zeros(X.shape[1])
beta = np.reshape(beta, (len(beta),1))
betas_array= []
betas_a= []
historico = []
historico.append(beta.copy())

betas, historico, iterations = coordinate_descent(X, Y, beta)
historico = np.array(historico)
print('Final Betas','\n', betas)
print('\n')
print('betas history','\n', historico[0:5].T) # printing the first
5 betas measured.
print('\n')
print('Total iterations until convergence', iterations)
```

```
Final Betas
 [[ 5.80094413]
 [ 0.09243506]
 [-1.62564049]
 [-0.23818641]
 [ 0.05207733]
 [-0.89514217]
 [ 0.24597495]
 [-0.83127508]
 [-0.11648133]
 [-0.75122626]
 [ 1.51067164]
 [ 1.87871669]]


betas history
 [[[ 0.          5.62783425  5.47087537  5.3477753   5.25047055]
  [ 0.          0.10585026  0.10612064  0.07690617  0.04581601]
  [ 0.         -0.49999961 -0.7222497  -0.83621955 -0.90808512]
  [ 0.          0.47282886  0.65759617  0.72235528  0.73379007]
  [ 0.         -0.15440341 -0.06740735  0.05528999  0.1569014 ]
  [ 0.         -0.42115356 -0.68131491 -0.80977985 -0.85337636]
  [ 0.          0.09801607  0.18882331  0.26346984  0.32480438]
  [ 0.         -0.3606864  -0.63108157 -0.82681556 -0.96313546]
  [ 0.          0.04797302  0.0575558   0.0475311   0.02817183]
  [ 0.          0.11875201  0.22931689  0.31890291  0.38631287]
  [ 0.          0.33519777  0.57844992  0.77941519  0.95335613]
  [ 0.          0.29223508  0.53305511  0.72879147  0.88978586]]]


Total iterations until convergence 224
```

In [131]:
```
fig= plt.figure(figsize=(15,10))
plt.plot(range(iterations+1), historico[:,0], 'r', color="red", lab
el='Beta 0')
plt.plot(range(iterations+1), historico[:,1], 'r', color="blue", la
bel='fixed acidity')
plt.plot(range(iterations+1), historico[:,2], 'r', color="gray", la
bel='volatile acidity')
plt.plot(range(iterations+1), historico[:,3], 'r', color="yellow",
label='citric acid')
plt.plot(range(iterations+1), historico[:,4], 'r', color="green", l
abel='residual sugar')
plt.plot(range(iterations+1), historico[:,5], 'r', color="black", l
abel='chlorides')
plt.plot(range(iterations+1), historico[:,6], 'r', color="brown", l
abel='free sulfur dioxide')
plt.plot(range(iterations+1), historico[:,7], 'r', color="pink", la
bel='total sulfur dioxide')
plt.plot(range(iterations+1), historico[:,8], 'r', color="purple",
label='density')
plt.plot(range(iterations+1), historico[:,9], 'r', color="#8EBA42",
label='pH')
plt.plot(range(iterations+1), historico[:,10], 'r', color="#E24A33"
, label='sulphates')
plt.plot(range(iterations+1), historico[:,11], 'r', color="#bfbbd9"
, label='alcohol')
plt.legend(bbox_to_anchor=(1.17, 1),loc = 'upper right', borderaxes
pad=0)
plt.grid(b=True, which='major', color='#666666', linestyle='--')
plt.show()
```



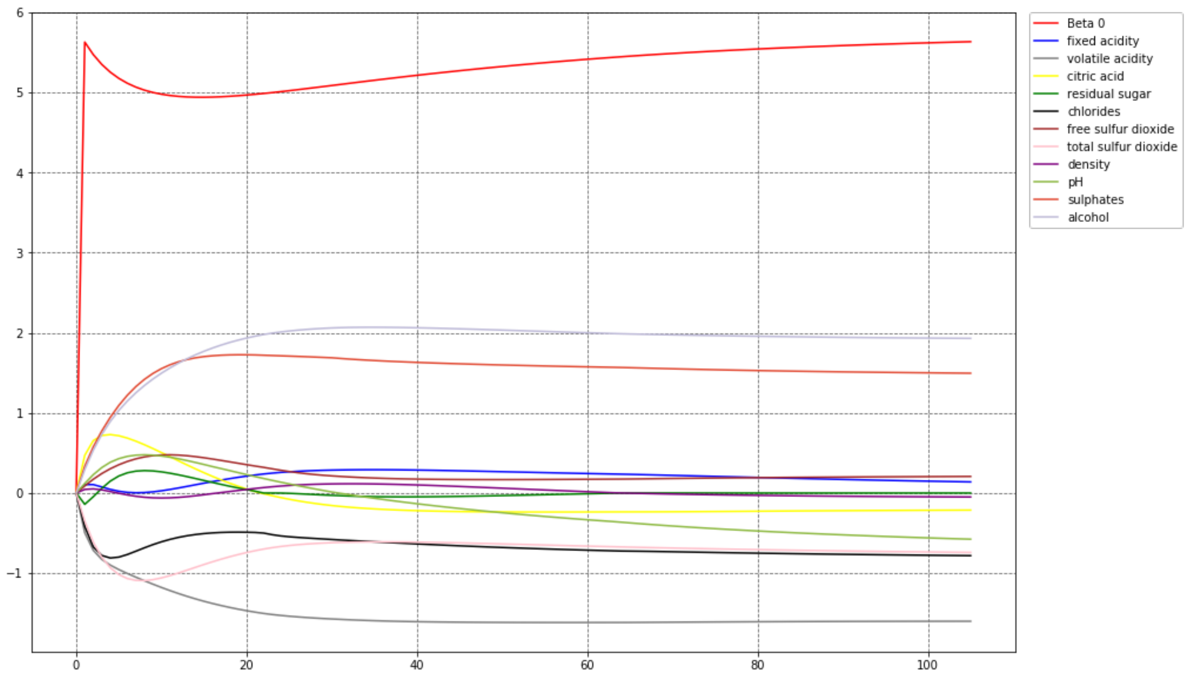
TASK B: Coordinate Descent with Lasso Regularization

In [132]:
```
def function_Lasso(X, y, beta, X_n):
    function = (np.dot((y - X@beta).T,(y - X@beta)) + (1*(1/2))) /
np.dot(X_n.T, X_n)
    return function

def coordinate_descent_Lasso(X, y, beta, parameter):
    max_iters = 0
    for j in range(300):
        max_iters += 1
        for i in range(len(X.T)):
            X_n = X.T[i]
            X_n = np.reshape(X_n, (len(X_n), 1))
            betas_xi = np.delete(beta, i, axis=0)
            X_m = np.delete(X.T, i, axis=0)
            betas = ((y - X_m.T@betas_xi).T@X_n / np.dot(X_n.T, X_n
))
            error = (parameter*(1/2) / np.dot(X_n.T, X_n))
            if betas > error:
                betas_new = betas - error
            elif abs(betas) <= error:
                betas_new = 0
            elif betas < error:
                betas_new =  betas + error
            beta[i] = betas_new
        historico.append(beta.copy())
        if function_Lasso(X, y, historico[-2], X_n) - function_Lass
o(X, y, historico[-1], X_n) < 0.0001:
            return beta, historico, max_iters

    raise Exception('Not converged in maximum number of iterations'
)
```

In [133]:
```
beta = np.zeros(X.shape[1])
beta = np.reshape(beta, (len(beta),1))

historico = []
historico.append(beta.copy())

beta_final, historic, iterations = coordinate_descent_Lasso(X, Y, b
eta, 0.6)
historic = np.array(historic)
print('Final Betas','\n', beta_final)
print('\n')
print('betas history', '\n', historic[0:5].T) # Printing the first
5 Betas stored.
print('\n')
print('Iterations until convergence:', iterations)
```

```
Final Betas
 [[ 5.63212064]
 [ 0.13838133]
 [-1.60047085]
 [-0.21362217]
 [ 0.        ]
 [-0.78199962]
 [ 0.20648598]
 [-0.74153202]
 [-0.04952224]
 [-0.57657098]
 [ 1.49496807]
 [ 1.92952639]]


betas history
 [[[ 0.          5.62759969  5.47117768  5.34861429  5.25167105]
  [ 0.          0.10465319  0.10462813  0.07548638  0.04486241]
  [ 0.         -0.49560818 -0.71564219 -0.82842294 -0.89996624]
  [ 0.          0.47004831  0.65367988  0.71791389  0.72925071]
  [ 0.         -0.14140257 -0.0467186   0.05786083  0.1481485 ]
  [ 0.         -0.417571   -0.67211093 -0.77944448 -0.81141428]
  [ 0.          0.08837081  0.17121932  0.2427143   0.30266026]
  [ 0.         -0.35028575 -0.6128587  -0.80205753 -0.93276594]
  [ 0.          0.04469102  0.05150984  0.0391745   0.01814753]
  [ 0.          0.11832645  0.22835052  0.31722062  0.38376762]
  [ 0.          0.33335898  0.57525485  0.77384781  0.94498947]
  [ 0.          0.29284219  0.5339932   0.73105593  0.89363334]]]


Iterations until convergence: 105
```

In [134]:
```
fig= plt.figure(figsize=(15,10))
plt.plot(range(iterations+1), historic[:,0], 'r', color="red", labe
l='Beta 0')
plt.plot(range(iterations+1), historic[:,1], 'r', color="blue", lab
el='fixed acidity')
plt.plot(range(iterations+1), historic[:,2], 'r', color="gray", lab
el='volatile acidity')
plt.plot(range(iterations+1), historic[:,3], 'r', color="yellow", l
abel='citric acid')
plt.plot(range(iterations+1), historic[:,4], 'r', color="green", la
bel='residual sugar')
plt.plot(range(iterations+1), historic[:,5], 'r', color="black", la
bel='chlorides')
plt.plot(range(iterations+1), historic[:,6], 'r', color="brown", la
bel='free sulfur dioxide')
plt.plot(range(iterations+1), historic[:,7], 'r', color="pink", lab
el='total sulfur dioxide')
plt.plot(range(iterations+1), historic[:,8], 'r', color="purple", l
abel='density')
plt.plot(range(iterations+1), historic[:,9], 'r', color="#8EBA42",
label='pH')
plt.plot(range(iterations+1), historic[:,10], 'r', color="#E24A33",
label='sulphates')
plt.plot(range(iterations+1), historic[:,11], 'r', color="#bfbbd9",
label='alcohol')
plt.legend(bbox_to_anchor=(1.17, 1),loc = 'upper right', borderaxes
pad=0)
plt.grid(b=True, which='major', color='#666666', linestyle='--')
plt.show()
```



Observations

1. It is interesting to analyze the behavior of the bias: It starts with a value of 1, but in order to model the data it grows up to help the model reach convergence.

2. It is possible to appreciate that the model with regularization reaches convergence faster than otherwise.

3. Another interesting fact is the curvature of the lines in the first iterations: the curvature of the model with regularization identify faster the optimal by finding the ideal trade-off. Moreover, it is possible to appreciate the sparsity of the betas, which if the parameter is increasing all coefficients will tend to zero. After increasing lambda it is possible to identify the features with large signal, aka. influential ones.

4. The values of the betas are lower in the model with regularization considering the penalization formula.