Particle Swarm Optimization with Gravitational Interactions for Multimodal and Unimodal Problems

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Abstract. Evolutionary computation is inspired by nature in order to formulate metaheuristics capable to optimize several kinds of problems. A family of algorithms has emerged based on this idea; e.g. genetic algorithms, evolutionary strategies, particle swarm optimization (PSO), ant colony optimization (ACO), etc. In this paper we show a population-based metaheuristic inspired on the gravitational forces produced by the interaction of the masses of a set of bodies. We explored the physics knowledge in order to find useful analogies to design an optimization metaheuristic. The proposed algorithm is capable to find the optima of unimodal and multimodal functions commonly used to benchmark evolutionary algorithms. We show that the proposed algorithm works and outperforms PSO with niches in both cases. Our algorithm does not depend on a radius parameter and does not need to use niches to solve multimodal problems. We compare with other metaheuristics respect to the mean number of evaluations needed to find the optima.

Keywords: Optimization, gravitational interactions, evolutionary computation, metaheuristic.

1 Introduction

Multimodal optimization problems deal with objective functions that commonly contain more than one global optima and several local optima. In order to find all the global optima in multimodal problems with classical methods, one typically runs a given method several times with different starting points, expecting to find all the global optima. However, these techniques do not guarantee the location of all optima. Therefore, this kind of techniques are not the best way to explore multimodal functions with complex and large search spaces. In the evolutionary computation literature exists a variety of metaheuristics challenging the typical

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problems of classical optimization. E.g. In particle swarm optimization with niches; the best particle makes a niche with all particles within a radius r, until the niche is full; it then selects the next best no niched and its closets particles to form the second niche; the process until all particles are assigned to a niche. Objective function stretching, introduced by Parsopolous [1], [7] is another algorithm whose strategy is to modify the fitness landscape in order to remove local optima and avoid the premature convergence in PSO. In a minimization problem, a possible local minimum is stretched to overcome a local maximum allowing to explore other sections of the search space identifying new solutions. GSA introduced by Rashedi [5], is a gravitational memory-less (does not include a cognitive component in the model) metaheuristic capable to find only one global optima in unimodal and multimodal problems with more than one global optima, where a heavier mass means a better solution and the gravitational constant G is used to adjust the accuracy search.

In our work we explore the properties of gravitational interactions in order to make an useful metaheuristic to find optima in unimodal and multimodal problems. In Section 2 addresses the main motivation of our work: The Newton's Law of Universal Gravitation. In Section 3 we define the Gravitational Interactions Optimization (GIO) metaheuristic for unimodal and multimodal functions. Section 4 presents to the GIO metaheuristic with differents unimodal and multimodal problems. Section 5 presents the conclusions of this work.

2 Newton's Law of Universal Gravitation

The attraction force of two particles is proportional to their masses and inversely proportional to their distance. The Law of Universal Gravitation was proposed by Isaac Newton [10]. This law is stated in Definition 1.

DEFINITION **1.** *The force between any two particles having masses m_1 and m_2, separated by a distance r, is an attraction acting along the line joining the particles and has the magnitude. Shown in Equation (1).*

$$F = G\frac{m_1 m_2}{r^2} \tag{1}$$

where G is a universal gravitational constant.

The forces between two particles with mass are an action-reaction pair. Two particles with masses m_1 and m_2 exert attracting forces F_{12} and F_{21} towards each other whose magnitudes are equal but their directions are opposed.

The gravitational constant G is an empirical physical constant involved in the computation of the gravitational attraction between particles with mass, which can be determined by the maximum deflection method [11].

$$G = 6.673 \times 10^{-11} N(m/kg)^2 \tag{2}$$

The gravitational force is extremely weak compared to other fundamental forces; e.g. the electromagnetic force is 39 orders of magnitude greater than the gravity force.

Newton's law of universal gravitation can be written in vectorial notation, which considers both: The force of the masses and the direction of each force. The vectorial notation is shown in Equation (3).

$$F_{12} = -G\frac{m_1 m_2}{|r_{12}|^2}\hat{r}_{12} \tag{3}$$

Where F_{12} is the force exerted by m_1 on m_2, G is the gravitational constant, m_1 and m_2 are the masses of the particles, $|r_{12}|$ is the euclidean distance between particles m_1 and m_2, and $\hat{r}_{12}$ is the unit vector, defined as $\frac{r_2 - r_1}{|r_2 - r_1|}$, r_1 and r_2 are the locations of particles m_1 and m_2. (See Figure 1).

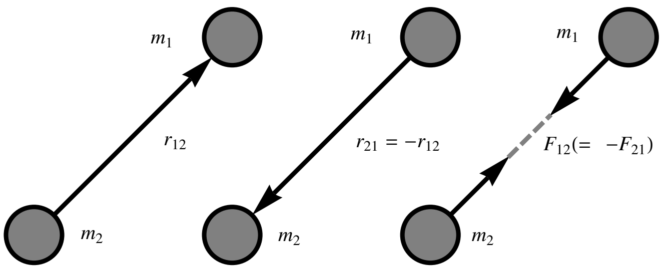


Fig. 1. The force exerted on m_2 (by m_1), F_{21}, is directed opposite to the displacement, r_{12}, of m_2 from m_1. The force exerted on m_1 (by m_2), F_{12}, is directed opposite to the displacement, r_{21}, of m_1 from m_2. $F_{21} = -F_{12}$, the forces being an action-reaction pair.

3 Gravitational Interactions Optimization

In order to find one or more optima there exists a large variety of evolutionary algorithms, e.g. genetics algorithms (GA) [4], evolutionary strategies (ES) [6], ant colony optimization (ACO) [2] , particle swarm optimization (PSO) [8], electrostatic PSO (EPSO) based on electrostatic interactions inspired upon PSO [3], etc. There exist works related to design metaheuristics that take into account the distance in order to determine the cluster membership of the particles computing and maximizing a ratio for all particles in the swarm with respect to the particle to bo updated, e.g. FER-PSO [9]. We propose a Gravitational Interaction Optimization metaheuristic (GIO) capable of solving optimization problems. The motivation of the design of this metaheuristic is to find useful properties and anolgies that can relate optimization problems with Newton's gravitational theory. In the approach presented in this paper, we abduct the interactions exhibited by a set of bodies and use them to guide the search for the global optimum in an optimization problem.

3.1 Gravitational Interactions for Unimodal Optimization

GIO is a population-based metaheuristic where a set of bodies are initially dispersed along the search space with a uniform random distribution. The fitness of bodies located on the search space are mapped as masses in a Gravitational field where the solutions are evolved. Each body stores its current position B and the best position so far B_{best} according to the fitness function. Bodies are allowed to interact in a synchronous discrete manner for a number of epochs. The body interactions follow Newton's gravitational law and move each body to a new location in such way that whole population tends to reach the global optimum (or multiple local optima for multi-modal problems).

The fitness function is a mapping that transforms a vector $X = (x_1, x_2, \ldots, x_n)$ to a scalar $f(X)$. This mapping associates the fitness value $f(X)$ to each location $X = (x_1 \cdots x_n)$ of the search space. We assign a body B to every location X in the search space where an individual of the population is found. Body B is assigned a mass, whose magnitude is a function of the fitness of its location.

Newton's law of universal gravitation describes the attraction forces that exist between two punctual bodies with masses (described in vectorial form in 3). Substituting we obtain Equation (4).

$$\boldsymbol{F_{ij}} = \frac{M\left(f(B_i)\right) \cdot M\left(f(B_j)\right)}{|B_i - B_j|^2} \hat{B_{ij}} \tag{4}$$

Where M is the mapping function that associates the fitness value f of domain $\{x : x \in \Re\}$ a mass of codomain $\{y : y \in (0, 1]\}$ for each position of the body B_i. This mapping is computed using Equation (5).

$$M(f(B_i)) = \left(\frac{f(B_i) - \mathbf{min} f(B)}{\mathbf{max} f(B_{best}) - \mathbf{min} f(B)} (1 - mapMin) + mapMin \right)^2 \tag{5}$$

Where B_i is the position of the ith body and B_j is the jth body that contributes exerting a force on the mass B_i; $|B_i - B_j|$ is the euclidean distance and $\boldsymbol{B_{ij}}$ is the unit vector between bodies B_i and B_j; $f(B_i)$ is the fitness of body B_i, $\mathbf{min} f(B)$ is the minimum fitness value of the current positions of the bodies, $\mathbf{max} f(B_{best})$ is the maximum fitness value of the best positions so far, $mapMin$ is a constant with a small positive value near zero, such that $(1 - mapMin)$ reescales the fitness value $f(B_i)$ to a mass between $[0, 1)$ values. The result is squared to emphasize the best and worst fitnesses.

One characteristic of the proposed method is the full interaction; i.e each body B_i interacts with every other body B_j through their masses. Interactions contribute to their displacement, according to the resultant force. Equation (6) computes the resultant force exerted on body B_i by the bodies B_j.

$$\boldsymbol{F_{ik}} = \sum_{j=1}^{n} \frac{M\left(f(B_i)\right) \cdot M\left(f(B_{j,best})\right)}{|B_i - B_{j,best}|^2} B_i \hat{B_{j,best}} \tag{6}$$

Where $\boldsymbol{F_{ik}}$ is a resultant force of the sum of all vector forces between $M(B_i)$ and $M(B_{j,best})$, $|B_i - B_{best,j}|$ is the Euclidean distance between the current positions of body B_i and the best position so far of the body B_j. In order to avoid numerical errors we compute the force between masses $M(B_i)$ and

$M(P_{j,best})$ only if $|B_i - B_j| \geq \times 10^{-5}$, $B_i\hat{B_{j,best}}$ is the unit vector that directs the force. In order to estimate a displacement that could enhance the solution of particle B_i, it is neccesary to solve Equation (4) for B_j. Assuming that we want to find a location of the body B_k with $M\left(f(B_k)\right) = 1$, B_k is computed using Equation (7).

$$B_k = \sqrt{\frac{M(f(B_i))}{|\boldsymbol{F_{ik}}|}}\hat{F_{ik}} \tag{7}$$

To update the position of the bodies we use equation (8) and (9).

$$V_{new} = \chi\left(V + R\cdot C\cdot B_k\right) \tag{8}$$

$$B_{t+1} = B + V_{new} \tag{9}$$

V is the current velocity of B_i, R is a random real number generated in the range of $[0, 1)$ and is multiplied by the gravitational interaction constant C, in order to expect random exploration distances with mean $\mu \approx 1$, we set $C = 2.01$, this displacement is constrained multiplying by a constant with a value of 0.86, in order to ensure the convergence. B_k is the main displacement computed by equation (7).

The complete GIO algorithm is described the Algorithms 1, 2 and 3. Algorithm 1 computes the the total force exerted by the masses $M(f(B_j))$ mass $M(f(B_i))$; in order to prevent premature convergence and division by 0, we compute only those pairs of bodies with a distance greater than ϵ. Algorithm 2 computes the velocities of the bodies, receives the bodies and computes the resultant force that attracts the mass assigned to B_i. In order to prevent a division by 0 we compute the distance only if $|Ftotal| > 0$, the new velocity is computed by Equation (8), and finally we update the velocity associated to B_i. Algorithm 3 computes the new positions B of each iteration t, the algorithm take as parameters the search range, the number of bodies *nBodies*, and the maximum number of iterations $tMax$. The algorithm computes the velocities with *computeVelocities*(*bodies*) (Algorithm 2), and updates the their positions with *updatePosition*(), which implements Equation (9), *limitPositions*() limits the positions of the bodies to the search space defined by the search range, *updateFitness*() updates the fitness according to the new positions of the bodies and finally we update the best position so far with $updateB_{best}()$.

Algorithm 1. computeFtotal(index)

$i \leftarrow index$
$Ftotal \leftarrow 0$
for $j \leftarrow 1$ to $nBodies$ **do**
  if $distance(B_i, P_j) > \epsilon$ **then**
    $Ftotal \leftarrow Ftotal + \hat{P_{ij}}M(f(B_i))M(f(P_j))/distance(B_i, P_j)^2$
  end if
end for
return $Ftotal$

Algorithm 2. computeVelocities(bodies)

for $i \leftarrow 1$ to $nBodies$ **do**
  $Ftotal \leftarrow computeFtotal(i)$
  if $|Ftotal| > 0$ **then**
    $distance \leftarrow \sqrt{M(f(B_i))}/|Ftotal|$
  else
    $distance \leftarrow 0$
  end if
  $V_{new} \leftarrow \chi(V + R\cdot C\cdot distance\cdot \hat{Ftotal})$
  $updateVelocity(B_i, V_{new})$
end for
return $Ftotal$

Algorithm 3. MainGravitationalInteraction(ranges, nBodies)

$bodies \leftarrow initializeParticles(nBodies, ranges)$
for $t \leftarrow 0$ to $maxIter$ **do**
  $computeVelocities(bodies)$
  $limitVelocity()$
  $updatePosition()$
  $limitPosition()$
  $updateFitness()$
  $updateP_{best}()$
end for

This scheme develops good results for unimodal problems. The results of the performace of the algorithm presented in this Section are presented in Section 4.

3.2 Gravitational Interactions for Multimodal Optimization

In the previous Subsection we showed the basic steps of the gravitational interactions metaheuristic. This scheme works well for unimodal problems. For multimodal problems it is necessary to add a cognitive component analogous to the one used in PSO [8]; the cognitive component is a constant that gives a weight to each body's memory. The new positions of the bodies are computed in order to find more than one optima with the Equations (10) and (11).

Adding the cognitive component to Equation (8) and using the constriction factor χ (Equation 12), makes the new Equation (10) capable to find more than one optimum in multimodal problems. The effect of this component is to make the local search more robust restricting to the bodies to local search, unless the gravitational forces of a cluster of masses overcome the force exerted by its cognitive component.

$$V_{new} = \chi\,(V + C_1\cdot R_1\cdot (B_{best} - B) + C_2\cdot R_2\cdot B_k) \tag{10}$$

$$B_{t+1} = B + V_{new} \tag{11}$$

where, analogous to PSO, C_1 and C_2 are the cognitive and the gravitational interaction constants, R_1 and R_2 are real random numbers variables in the $[0, 1)$ range and χ is the inertia constraint (Proposed by Clerk ([8])). The inertia constraint is used to avoid the bodies to explore out of the search space computed by Equation (12).

$$\chi = \frac{2\kappa}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|} \tag{12}$$

where $\phi = C_1 + C_2 > 4$, C_1 and C_2; κ is an arbitrary value in the range of $(0, 1]$. In our algorithm we set $C_1 = C_2 = 2.01$. The constriction factor in our algorithm helps to converge through the iterations. To make multimomodal Gravitational Interactions Algorithm (Algorithms 1, 2, and 3) described in the previous subsection, we replace line 8 in Algorithm 2 by Equation 10.

4 Experiments

In order to test the performance of the Gravitational Interactions Optimization algorithm for unimodal and multimodal functions, we tested both versions with some functions commonly used to measure the performance of different kinds of metaheuristics.

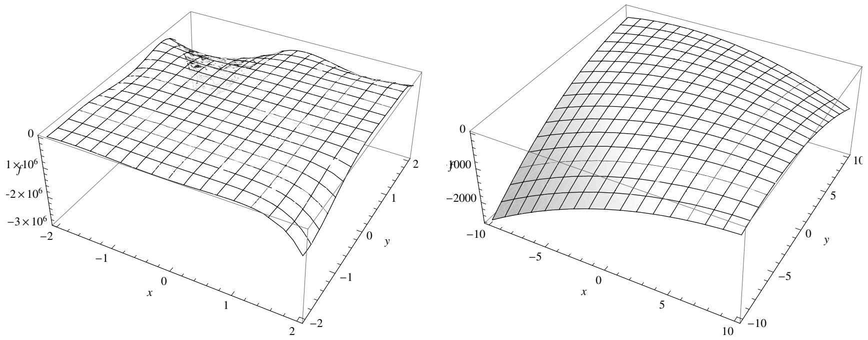
4.1 Test Functions

We show the performance of unimodal and multimodal Gravitational Interactions Optimization algorithm with 3 unimodal and 4 multimodal functions. The test functions tested are shown in the Table 1.

For unimodal optimization we used the functions in Figure 2: $U1$ is the Goldstein and Price function shown in Figure 2(a), $U2$ is the Booth function shown in Figure 2(b) and $U3$ is the 4 variable Colville Function. For multimodal optimization we used the functions of the Figure 3 $M1$ is the Branin's RCOS Function with 3 global optima (with no local optima) shown in Figure 3(a), $M2$ is the

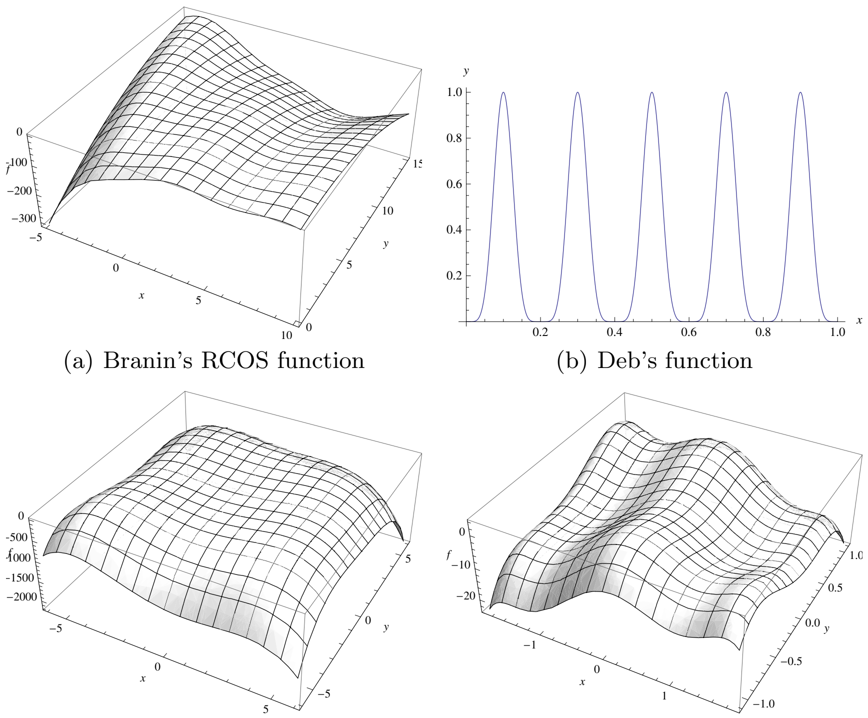
Table 1. Test functions used for our experiments

Unimodal Test Functions		
$U1$	$U1 = [1 + (1 + (x + y + 1)^2)(19 - 14x + 3y^2 + 6xy + 3y^2)] \cdot$ $[(30 + (2x - 3y)^2)(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)]$	$-2 \leq x, y \leq 2$
$U2$	$U2 = (x + 2y - 7)^2 + (2x + y - 5)^2$	$-10 \leq x, y \leq 10$
$U3$	$U3 = -1100 \cdot (w^2 - x)^2 + (w - 1)^2 + (y - 1)^2 + 90 \cdot (y^2 - z)^2 + \cdot$ $10.1 \cdot ((x - 1)^2 + (z - 1)^2) + 19.8 \cdot (x^{-1}) \cdot (z - 1)$	$-10 \leq w, x, y, z \leq 10$
Multimodal Test Functions		
$M1$	$M1 = -\left((y - \frac{5.1x^2}{4\pi^2} + \frac{5x}{\pi} - 6)^2 + 10(1 - \frac{1}{8\pi})Cos(x) + 10\right)$	$-5 \leq x \leq 10$ $0 \leq y \leq 15$
$M2$	$M2 = Sin(5\pi x)^6$	$-0 \leq x \leq 1$
$M3$	$M3 = -(x^2 + y - 11)^2 - (x + y^2 - 7)^2$	$-6 \leq x, y \leq 6$
$M4$	$M4 = -4\left((4 - 2.1x^2 + \frac{x^4}{3})x^2 + xy + (-4 + 4y^2)y^2\right)$	$-1.9 \leq x \leq 1.9$ $-1.1 \leq x \leq 1.1$



(a) Goldstein-Price function (b) Booth function

Fig. 2. Fitness landscape of two test functions with one optima used for measure the performance of Unimodal Gravitational Interactions



(a) Branin's RCOS function (b) Deb's function

(c) Himmelblau's function (d) Six-hump cammelback function

Fig. 3. Fitness landscape of multimodal test functions used in our experiments

6 global maximum univariable Deb's function shown in Figure 3(b), $M3$ is the Himmelblau's function with 4 global optima shown in Figure 3(c), $M4$ is the Six-Hump cammelback function with 2 global optima and 4 local optima shown in Figure 3(d).

4.2 Results

In our experiments we consider $\epsilon = 1 \times 10^{-3}$ to be an acceptable error to determine if the solution obtained had reached the optimum. We used 100 bodies for a maximum of 1000 iterations, we used as stop condition the inability of all the bodies to enhance their fitness memory solutions by 1×10^{-4}, or when the algorithm found all the optima. Each experiment was repeated 30 times. PSO with niches requires two extra parameters: the radius r, and the maximum number of particles on each niche $nMax$, we set $M1$ with $r = 0.5$ and $nMax = 50$, $M2$ with $r = 0.1$ and $nMax = 15$, $M3$ with $r = 0.5$ and $nMax = 30$, and $M4$ with $r = 0.5$ and $nMax = 25$.

The performance of Gravitational Interaction Optimization (GIO) is compared with Particle Swarm Optimization with niches (NPSO) in Table 2, considering the mean and the standard deviation of evaluations required to find all the global optima (column **Evaluations**) and the percentage of successes (column **Success**) to finding all the optima.

Table 2. Results of our experiments

	PSO			**GIO Unimodal**		
Functions	**Evaluations**		Success	**Evaluations**		Success
	μ	σ		μ	σ	
$U1$	1,394.44	399.22	20%	16,523.33	13,928.90	100%
$U2$	1,130.77	330.11	60%	6,057.55	3,984.54	70%
$U3$	764.00	777.75	83%	530.00	208.69	100%
	NPSO			**GIO Multimodal**		
	Evaluations			**Evaluations**		
	μ	σ		μ	σ	
$M1$	2,529.17	764.13	80%	2,803.33	972.90	100%
$M2$	276.66	81.72	100%	390.00	88.44	100%
$M3$	3,400.00	0.00	00.3%	2,323.33	288.496	100%
$M4$	1,136.67	303.41	100%	1,600.00	501.721	100%

The obtained results show that Unimodal and Multimodal Gravitational Interactions have a higher probability to converge to global optima, avoiding premature convergence that PSO and PSO with niches with a similar number of evaluations required for the functions tested.

5 Conclusions

We presented a new heuristic more reliable than PSO with no aditional parameters like the radius r and the maximum number of particles in a niche $nMax$ used in PSO with niches. In problems with high dimentions the radius r is determined by trial and error, because we can not to graph the objective function and make a visual analysis. The same algorithm is used for unimodal and multimodal cases. When used in its general form. (i.e. including the cognitive component), GIO solves both cases without the need of any a-priori information. Adding the cognitive component allow us to solve both, unimodal and multimodal optimization problems, while GSA can only solve unimodal problems. While GIO has proven to find all optima in a multimodal problem, GSA can only determine one of them.

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