Avellaneda-Stoikov HFT Market Making Applied to Cryptocurrency Pairs

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August 2, 2024

Contents

1 Introduction

1.1 Motivation

Motivation for this project comes from the original paper by Marco Avellaneda and Sasha Stoikov:

"High-frequency trading in a limit order book".

Market makers or specialist firms provide liquidity accross financial markets by quoting bid-ask prices, hoping to earn a profit from the spread. This is no easy task however:

First of all, a liquidity provider needs to quote a good enough price so that other participants are willing to hit/lift their bids and asks. This leads to a trade-off situation, as quoting too aggressively leads to an eroding profits if we take into account transaction costs. Pricing a wide spread on the other hand means no market participants will be willing to trade at prices too distant from the market mid-price.

Secondly, a liquidity provider will inevitably face some inventory risk. The objective of a market maker is to be as market neutral as possible. However, at any given time t, it's possible that the market maker will find a buyer but not a seller, or viceversa. This leads to the liquidity provider's book to be long/short exposed, introducing market risk.

In the aforementioned research piece, Avellaneda and Stoikov tackle these 2 factors, and develop a framework to determine optimal bid-ask prices that take into account the inventory risk.

1.2 Scope of this project

We will first build and comment the Market Making model, performing the numerical simulations the way they're described in the original paper.

We'll then test the model on different cryptocurrency pairs, trying to account for the difference in liquidity and order book dynamics, and ultimately compare the performance of the Avellaneda-Stoikov model against a symmetric spread market making model. We're interested in comparing the profit generated in both strategies, but also the risk, as measured by the ending inventory and standard deviation of profit.

1.3 Choice of Cryptocurrency pairs

We chose to test the AS model with per-second prices of BTC/USDT and XRP/USDT.

The choice of BTC/USDT is straightforward, as it is the most traded cryptocurrencies pairs, with high liquidity. We choose to test the model with XRP/USDT since it is a cryptocurrency pair with far less 24h trading volume than BTC/USDT.

2 The model

2.1 Parameters and assumptions

We'll assume all parameters of the model regarding order book statistics do not change during the trading. Neither does market volatility change during the trading session.

We'll be testing the market making model on the cryptocurrency pairs. It's important to note that while cryptocurrencies trade 24h, we make the assumption that the market maker has a fixed time horizon, and wants to liquidate his inventory at the end of his trading session.

Variable	Name
S_t	Spot or Mid-Price at time t
x	Initial agent wealth
q	Agent inventory
γ	Inventory Risk Aversion Parameter
k	Order Book Liquidity Parameter

2.2 Objective of the Market Maker

The market maker's objective is to maximize the expected exponential utility of this Profit & Loss profile at a terminal time T.

$$v(x, s, q, t) = E_t[-exp(-\gamma(x + qS_t))] \tag{1}$$

This could be rewritten to show the impact of all parameters in the value v.

$$v(x, s, q, t) = -exp(\gamma t) * exp(-\gamma qs) * exp(\frac{\gamma^2 q^2 \sigma^2 (T - t)}{2}) \tag{2}$$

2.3 Reservation Price

We defined v as the utility function of the market maker.

We assume that in the beginning of trading, the inventory of the agent is flat (i.e. q = 0). This is called "frozen inventory".

The reservation bid and ask price is a price that would make the agent indifferent between keeping its current portfolio, or adding or subtracting a stock from the portfolio.

We will call rB and rA the reservation bid and ask price, respectively.

The relation with the agent's utility function is as follows:

$$v(x - rB(s,q,t), s, q+1, t) = v(x, s, q, t) \tag{3}$$

$$v(x + rA(s,q,t), s, q-1, t) = v(x, s, q, t) \tag{4}$$

We can then combine equations (3) and (4) with equation (2)

$$v(x + r(s,q,t), s, q-1, t) = s + (\pm 1 - 2q)(\frac{\gamma^2 q^2 \sigma^2 (T-t)}{2}) \tag{5}$$

Finally, if we take the average of both price we get the final reservation price in the Avellaneda-Stoikov paper.

$$r(s,q,t) = s - q\gamma\sigma^2(T-t) \tag{6}$$

The reservation price acts as a "risk adjustment" to the market mid-price.

2.4 The effect of the fixed time horizon

Because we introduced a *(T-t)* term, our model now takes into account the time left for trading when calculating a reservation price.

As the trading session is nearing the end, the reservation price will approach the market mid-price, reducing the risk of holding the inventory too far from the desired target.

2.5 Calculating the spread

Assuming the agent can trade through limit orders around the mid-price, we can calculate the difference between the mid-price and the offered quote, notably:

$$\delta^b = s - p^b \tag{7}$$

$$\delta^a = p^a - s \tag{8}$$

Avellaneda and Stoikov arrive to the following formula for the optimal spread:

$$\delta^a + \delta^b = \gamma\sigma^2(T-t) + \frac{2}{\gamma} ln(1 + \frac{\gamma}{k}) \tag{9}$$

3 Simulating Order Arrival

In the original paper, Avellaneda and Stoikov use a Poisson distribution to model the probability the limit buy orders will be "hit" and limit sell orders will be "lifted":

3.1 Execution Probabilities

We take the bid/ask delta function from before:

$$\delta^b = s - p^b \tag{10}$$

$$\delta^a = p^a - s \tag{11}$$

The arrival of market orders that "hit/lift" the agent's buy limit orders (sell limit orders) is modeled as a Poisson process with intensity $\lambda^b(\delta^b)$ and $\lambda^a(\delta^a)$.

The logic behind using a Poisson distribution is straightforward, as the Market Maker prices more aggressively, higher are the chances the market will hit its orders. In the original paper, two functional forms of order intensities (which will be used to model lambda) are proposed: Exponential Decay and Power Law Decay.

The exponential decay function is based on the assumption that the likelihood of a limit order being executed decreases exponentially as the distance from the mid-price increases. This is formulated as:

$$\Delta p \propto \ln(Q)$$

Where Δp is the temporary market impact of a market order of size Q

$$\lambda = A \exp(-k\delta)$$

With this in mind we can calculate the probability our orders will be met:

$$Prob_{Buy} = 1 - \exp(\lambda_b(\delta_b)dt)$$

$$Prob_{Sell} = 1 - \exp(\lambda_a(\delta_a)dt)$$

4 Numerical Simulation

We first do a Numerical Simulation, with the original parameters in the paper. The evolution of the mid-price is simulated with a random walk.

Parameters are like so: $S_0 = 100$, $T = 1.0$, $\sigma = 2$, $dt = 0.005$, $q = 0$, $\gamma = 0.1$, $k = 1.5$, $A = 140$.

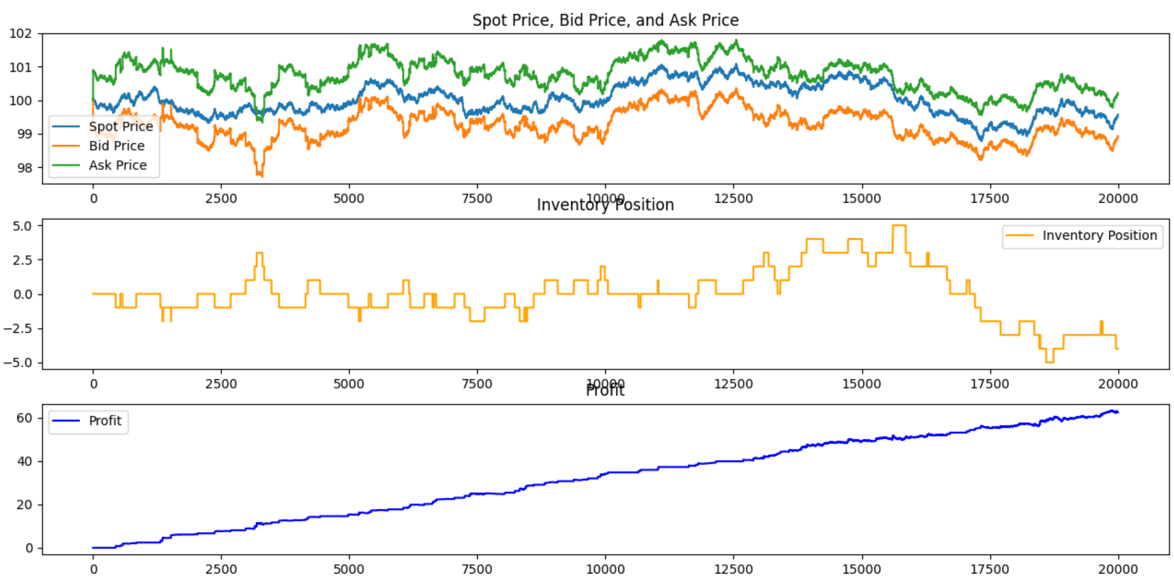


Figure 1: Numerical Simulation

Applying the Avellaneda Stoikov model to a simulated stock movement, we see that the Average inventory held is near zero.

At the beginning of trading, we see the model will quote aggressively to bring back the book to 0, as it is the example around the 3000th time step, where the model generates ask quotes below the mid price.

We also see the impact of the time horizon in the Avellaneda-Stoikov model. The quotes towards the end of trading seem a lot more ”symmetrical” compared to those in the beginning of trading. As a result of this, the inventory also becomes more volatile towards the end of trading.

5 Backtesting on Cryptocurrency Pairs

The main challenge of this section was how to properly calculate the probability that our limit orders would be hit/lifted in a cryptocurrency exchange.

The solution found to work around this problem is to take the orderbook best bid and best ask, at each time step in the backtesting, and applying a similar Poisson intensity formula as the one applied in the original paper.

At the same time, it´s necessary to adjust parameters A and K, to accommodate for the difference in liquidity and order book dynamics.

5.1 BTC/USDT

We will first test both an Avellaneda-Stoikov and a symmetric spreads market making strategy on BTC-USDT. We chose to use a lower A and a higher sigma value for the AS model. The spot price is normalized at $100 to facilitate comparisons.

Parameters are: $\sigma = 5$, $dt = 0.000042$, $\gamma = 0.1$, $k = 1.5$, $A = 80$.

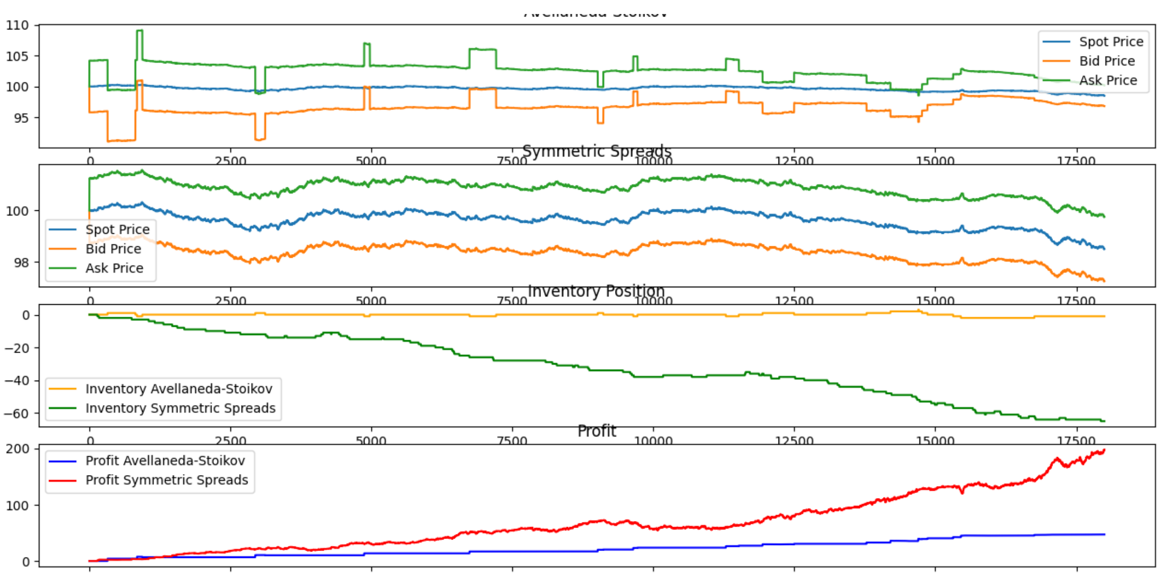
Figure 2: AS vs Symmetric - BTC/USDT

Avellaneda Stoikov	
Average Spread	$2.54
Max Spread	$3.79
Average Inventory held	-0.12
Max Inventory held (Short or Long)	4
Generated Profit	$29.91
Profit StDev	$7.96

Symmetric Spreads	
Average Spread	$0.25
Max Spread	$0.25
Average Inventory held	-17.23
Max Inventory held (Short or Long)	26
Generated Profit	$33.84
Profit StDev	$11.38

5.2 XRP/USDT

Parameters are: $\sigma = 5$, $dt = 0.000055$, $\gamma = 0.2$, $k = 0.5$, $A = 50$.



Avellaneda Stoikov	
Average Spread	$5.85
Max Spread	$8.36
Average Inventory held	-0.11
Max Inventory held (Short or Long)	3
Generated Profit	$47.16
Profit StDev	$13.49

Symmetric Spreads	
Average Spread	$2.50
Max Spread	$2.50
Average Inventory held	-31.65
Max Inventory held (Short or Long)	65
Generated Profit	$197.72
Profit StDev	$48.27

5.3 Comments on results

Upon changing the parameters of volatility sensitivity, inventory risk, and orderbook liquidity, the Avellaneda-Stoikov will become more risk adverse, quoting higher spreads and making less trades. The metrics on the evolution of inventory are inline with the numerical simulations. The model will always keep an average inventory close to 0.

A symmetric spread model will trade much more than the AS model, resulting in a higher profit. However, this does not come in free of risk, as we see, a symmetric spread model will finish trading with an heavily skewed portfolio. This situation could easily erode the profit figure, if we're dealing with an illiquid market. If the symmetrical spread market maker chooses to unload his inventory risk, it's very possible that he will struggle to find a counterparty willing to help liquidate such a big inventory.