

CMP-3004

Computer Organization

Spring 2023

Homework 1

Please solve the following exercises. If you are using pen and paper, take a picture of your work and upload all your work as a single pdf file to GitHub and submit the link to D2L.

1. Transform the following numbers from one base to another.

Decimal to binary

$$\begin{array}{r} 10 \\ 5 \\ 2 \end{array} \begin{array}{l} /_2 = 5 \\ /_2 = 2 \\ /_2 = 1 \end{array} \quad \begin{array}{c} \text{con} \\ \text{con} \\ \text{con} \end{array} \quad \begin{array}{l} 0 \\ 1 \\ 0 \end{array}$$

1010₂//

1369

$$\begin{array}{r} 1869 \\ 684 \\ 342 \\ 171 \\ 85 \\ 42 \\ 21 \\ 10 \\ 5 \\ 2 \end{array} \begin{array}{l} /_2 = 684 \\ /_2 = 342 \\ /_2 = 171 \\ /_2 = 85 \\ /_2 = 42 \\ /_2 = 21 \\ /_2 = 10 \\ /_2 = 5 \\ /_2 = 2 \\ /_2 = 1 \end{array} \quad \begin{array}{c} \text{con} \\ \text{con} \end{array} \quad \begin{array}{l} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{array}$$

10101011001₂//

9234876

9234876	12	=	4617438	Con
4617438	12	=	2308719	Con
2308719	12	=	1154359	Con
1154359	12	=	577179	Con
577179	12	=	288589	Con
288589	12	=	144294	Con
144294	12	=	72147	Con
72147	12	=	36073	Con
36073	12	=	18036	Con
18036	12	=	9018	Con
9018	12	=	4509	Con
4509	12	=	2284	Con
2284	12	=	1127	Con
1127	12	=	563	Con
563	12	=	281	Con
281	12	=	140	Con
140	12	=	70	Con
70	12	=	35	Con
35	12	=	17	Con
17	12	=	8	Con
8	12	=	4	Con
4	12	=	2	Con
2	12	=	1	Con

49263749

49263749	1	2	=	24631874	con	1
24631874	1	2	=	12315937	con	0
12315937	1	2	=	6157968	con	1
6157968	1	2	=	307894	con	0
307894	1	2	=	1539492	con	0
			=	7969246	con	0
			=	384873	con	0
			=	192436	con	1
			=	96218	con	0
			=	48109	con	0
			=	24054	con	1
			=	12027	con	0
			=	6013	con	1
			=	3006	con	1
			=	1503	con	0
			=	751	con	1
			=	375	con	1
			=	187	con	1
			=	93	con	1
			=	46	con	1
			=	23	con	0
			=	11	con	1
			=	5	con	1
			=	2	con	1
			=	1	con	0

Decimal to binary using 2's complement

Use the minimum number of bits required to express the number

-20

$$+ 20 = 010100$$

$$\begin{array}{r} 101011 \\ + 000001 \\ \hline 101100 \end{array}$$

-1025

+ 1025 → 01 000600001

$$\begin{array}{r}
 \cancel{1} 0 1 1 1 1 1 1 1 1 0 \\
 + \underline{0 0 0 0 0 0 0 0 0 0 0} 1 \\
 \hline
 1 0 1 1 1 1 1 1 1 1
 \end{array}$$

-3925

$$+ 3925 \rightarrow 0111101010101$$

↪ 1 0000 10101010

10000000000001
10000 101010 11

-104596

$$+ 104596 \quad - \quad 011001100010010100$$

6100110011101101011

$$\begin{array}{r} + 00000000000000000000 \\ \hline 100110011101101100 \end{array}$$

Unsigned binary to hex

Use the long and the short methods

1100111101010110011011101101100000101001

Short

$C F S 6 6 E D 8 . 2^9$ $_{16,1}$

long

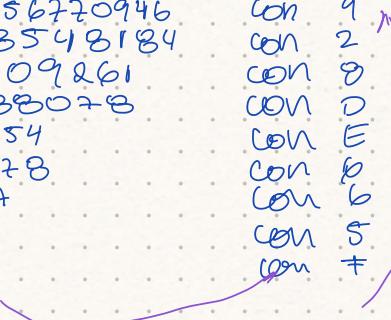
Binary \rightarrow Decimal \rightarrow hexa

$$1 \times 2^{31} + 1 \times 2^{38} + 1 \times 2^{35} + 1 \times 2^{34} + 1 \times 2^{33} + 1 \times 2^{32} + 1 \times 2^{30} + 1 \times 2^{28} + 1 \times 2^{26} + 1 \times 2^{25} + 1 \times 2^{22} + 1 \times 2^{21} + 1 \times 2^{19} = 890508335145$$

890508335145 116

$$\begin{aligned} &= 55656770946 \\ &= 3478548184 \\ &= 217409261 \\ &= 13588078 \\ &= 849254 \\ &= 53078 \\ &= 3317 \\ &= 207 \\ &= C \end{aligned}$$

con 9
con 2
con 8
con D
con E
con B
con 6
con 5
con F



100001111000111000111000111110011

8 + 8 0 3 8 D 3 F 3

Short

8 + 8E 3B + 3F 3

Long

$$1 \times 2^{39} + 1 \times 2^{34} + 1 \times 2^{33} + 1 \times 2^{32} + 1 \times 2^{31} + 1 \times 2^{22} + 1 \times 2^{26} + 1 \times 2^{25} + 1 \times 2^4 + 1 \times 2^{20} + 1 \times 2^9 + \\ 1 \times 2^{15} + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0 =$$

582206678003

$$582206678003 / 16 = 36387917375 \text{ rem } 3$$

$$= 227417415 \quad \text{rem } 5$$

$$= 142136195 \quad \text{rem } 3$$

$$= 8883512 \quad \text{rem } 5$$

$$= 555219 \quad \text{rem } 8$$

$$= 34701 \quad \text{rem } 3$$

$$= 268 \quad \text{rem } 5$$

$$= 135 \quad \text{rem } 0$$

$$= 8 \quad \text{rem } 7$$

101011010101100011001010100101010101010101010

A D S C S S 4 A A A

Short

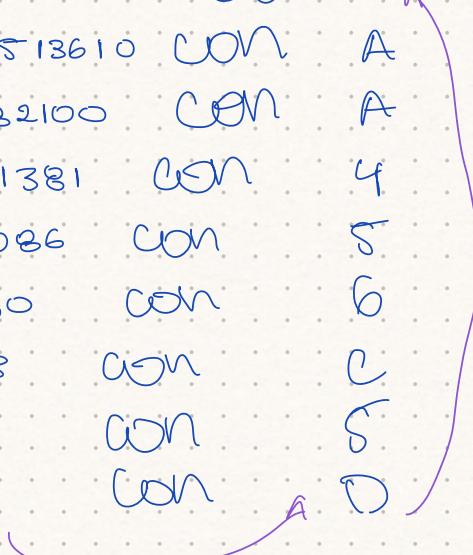
$AD^5C6S^4 AAA_{16}$

Long

$$1x2^{39} + 1x2^{37} + 1x2^{35} + 1x2^{34} + 1x2^{32} + 1x2^{30} + 1x2^{28} + 1x2^{27} + 1x2^{26} + 1x2^{22} + 1x2^{21} \\ + 1x2^{18} + 1x2^{16} + 1x2^{14} + 1x2^{12} + 1x2^9 + 1x2^7 + 1x2^5 + 1x2^3 + 1x2^1 =$$

$744579484330_{10},$

$$\begin{aligned} 744579484330_{16} &= 465362170 \text{ CON } A \\ &= 2908513610 \text{ CON } A \\ &= 181782100 \text{ CON } A \\ &= 11361381 \text{ CON } 4 \\ &= 710086 \text{ CON } 5 \\ &= 44380 \text{ CON } 6 \\ &= 2773 \text{ CON } C \\ &= 173 \text{ CON } S \\ &= A \text{ CON } D \end{aligned}$$



1010001010101010101010101111111000000

A Z A A A A B F C O

Short

A₂ AAAABFCO

longs

$$1 \times 2^{89} + 1 \times 2^{87} + 1 \times 2^{83} + 1 \times 2^{31} + 1 \times 2^{29} + 1 \times 2^{27} + 1 \times 2^{25} + 1 \times 2^{23} + 1 \times 2^{21} + 1 \times 2^{19} + 1 \times 2^{17} + 1 \times 2^{15} + \\ 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 = 698648018880,$$

$$698648018880 \div 16 = 43665501180 \text{ CON } O$$

$$= 2729093823 \text{ CON } F$$

$$= 170568363 \text{ CON } F$$

$$= 10660522 \text{ CON } B$$

$$= 666282 \text{ CON } A$$

$$= 41642 \text{ CON } A$$

$$= 2602 \text{ CON } A$$

$$= 162 \text{ CON } A$$

$$= 8 \text{ CON } 2$$



Signed binary to octal

Use the long and short methods

1111100000111100000001110101011
7 7 0 1 7 4 0 1 6 5 3

Short

$$- 77017401653_{10,11}$$

long

$$\begin{aligned} & 1 \times 2^{32} + 1 \times 2^{31} + 1 \times 2^{30} + 1 \times 2^{29} + 1 \times 2^{28} + 0 \times 2^{27} + 1 \times 2^{26} + 1 \times 2^{19} + 1 \times 2^{18} + \\ & 1 \times 2^{17} + 1 \times 2^{16} + 1 \times 2^{15} + 1 \times 2^{14} + 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^1 + 1 \times 2^0 = \\ & 8459781035_{10,11} \end{aligned}$$

$$8459781035_10 = 1057472629_{10}$$

$$= 132184078_{10}$$

$$= 16523009_{10}$$

$$= 2065376_{10}$$

$$= 258172_{10}$$

$$= 32271_{10}$$

$$= 4033_{10}$$

$$= 504_{10}$$

$$= 63_{10}$$

$$= 7_{10}$$

$$= 1_{10}$$

$$= 0_{10}$$

$$= 7_{10}$$

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$$\begin{array}{r} 1057472629 \\ \times 8 \\ \hline 8459781035 \end{array}$$

+
010101010101111111111111111111110000000
2 5 2 5 7 7 7 7 6 0 0

shot

+ 252577776008,

Loris

$$1x2^{31} + 1x2^{30} + 1x2^{29} + 1x2^{28} + 1x2^{27} + 1x2^{26} + 1x2^{25} + 1x2^{24} + 1x2^{19} + 1x2^{18} + \\ 1x2^{17} + 1x2^{16} + 1x2^{15} + 1x2^{14} + 1x2^{13} + 1x2^{12} + 1x2^{11} + 1x2^{10} = 2869709504_{10_4}$$

$$286470950418 = 358080688 \text{ con } 0 \text{ r} 5$$

$$= 44761086 \text{ con } 0$$

$$= 5595135 \text{ con } 8$$

$$= 699391 \quad \text{con} \neq$$

= 87423 con 7

$$= 10927 \text{ cm}^2$$

$$= 1365 \text{ car } t$$

= 170 con s

$$= 21 \quad \cos 2$$

= 2 con 5

(Handwritten note: A horizontal line with arrows at both ends, spanning the width of the page.)

0011100011100000011111111000001010101
1 6 1 6 0 1 7 7 4 0 5 2

Short

-161601774052₈

Cone

$$1 \times 2^{33} + 1 \times 2^{32} + 1 \times 2^{31} + 1 \times 2^{27} + 1 \times 2^{26} + 1 \times 2^{25} + 1 \times 2^{18} + 1 \times 2^{17} + 1 \times 2^{16} + 1 \times 2^{15} \\ + 1 \times 2^{14} + 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^{11} + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^1 = 15267789842_{10},$$

$$15267788842148 = 1908473605 \text{ cdh } 2$$

= 238559200 con 5

= 2981.9900 0.00

$$= 372 + 487$$

$$= 465935 \quad \text{an } +$$

$$= 58241 \quad \text{on } 7$$

$$= 72^\circ 0$$

$$= 910 \text{ cm } 0$$

$\equiv 113$ con 6

$$= 14 \quad \text{won} \quad 0$$

$=$ | $\cos \theta$

.....

001010101010100000101010101011111000
1 2 5 2 4 6 5 2 5 3 7 0

Short

-125240525370

long

$$1 \times 2^{33} + 1 \times 2^3 + 1 \times 2^9 + 1 \times 2^{24} + 1 \times 2^{25} + 1 \times 2^{23} + 1 \times 2^{17} + 1 \times 2^{15} + 1 \times 2^{13} + \\ 1 \times 2^9 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 = 11450624760_{10}$$

$$\begin{aligned} 11450624760_{10} &= 1431328045 \quad \text{CON 0} \\ &= 178916011 \quad \text{CON 2} \\ &= 22364501 \quad \text{CON 3} \\ &= 2795562 \quad \text{CON 5} \\ &= 349445 \quad \text{CON 2} \\ &= 43680 \quad \text{CON 5} \\ &= 5460 \quad \text{CON 0} \\ &= 682 \quad \text{CON 4} \\ &= 85 \quad \text{CON 2} \\ &= 10 \quad \text{CON 5} \\ &= 1 \quad \text{CON 2} \\ &= \end{aligned}$$

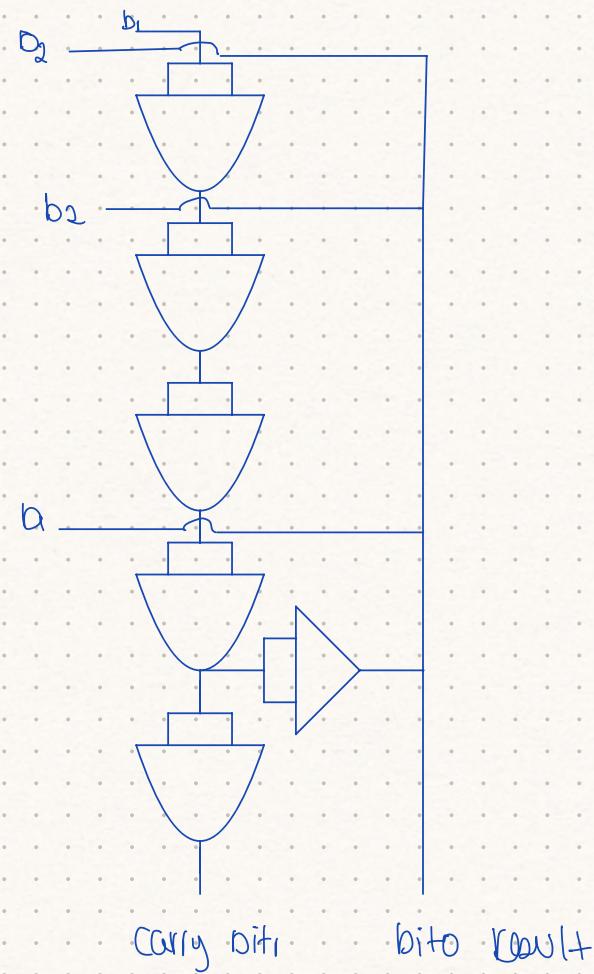
↗

2. Boolean circuits

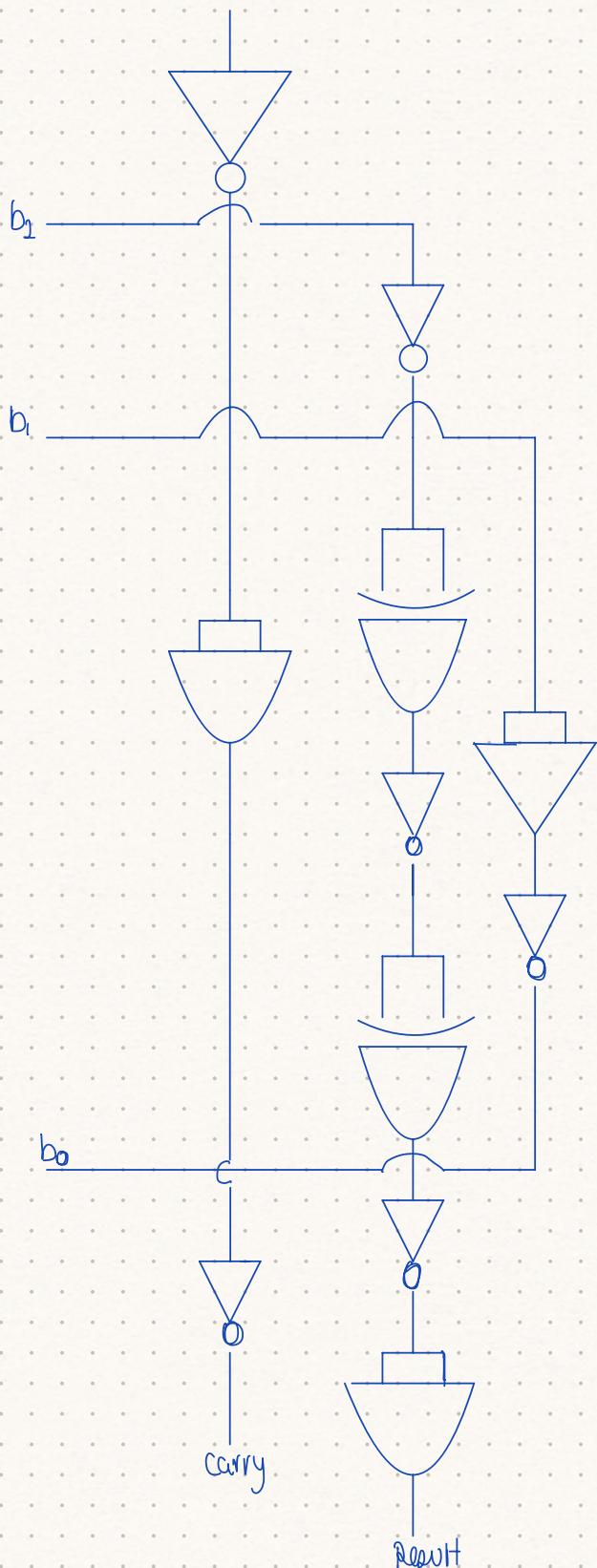
Draw the Boolean circuit and make the truth table for the following:

Multiplication of two binary numbers of length 2 bits

b ₁	b ₂	Carry	bit ₁	bit ₂	R
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	0	0	1	1
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	1	1	0	2



Two's complement for a binary number of length 3 bits



b_2	b_1	b_0	carry	Result
0	0	0	0	000
0	0	1	1	111
0	1	0	1	110
0	1	1	1	101
1	0	0	0	000
1	0	1	0	111
1	1	0	0	110
1	1	1	0	101

3. Do the following multiplications in binary

Use the minimum number of bits required

-5×8

$$\begin{array}{r}
 1011 \\
 0000 \quad 0 \rightarrow 0000 \quad 0100 \quad 0 \rightarrow 0000 \quad 0010 \quad 0 \rightarrow 0000 \quad 0001 \quad 0 \rightarrow 0101 \quad 0001 \quad 0 \rightarrow 0010 \quad 1000 \quad //1 \\
 0100 \qquad 0011
 \end{array}$$

$$\begin{array}{r}
 PD = PD - MD \\
 \overbrace{}^{1010} \\
 + \overbrace{}^1 \\
 \hline 1011
 \end{array}$$

$11 \times (-10)$

$$\begin{array}{r}
 01011 \\
 00000 \quad 0 \rightarrow 00000 \quad 01011 \quad 0 \rightarrow 10101 \quad 01011 \quad 0 \rightarrow 11010 \quad 010111 \quad 1 \rightarrow 11101 \quad 01010 \quad 1 \rightarrow 01000 \quad 01010 \quad 1 \rightarrow 00010 \\
 00101 \qquad 00100 \qquad 00100
 \end{array}$$

$$PD = PD - MD$$

$$\begin{array}{r}
 01011 \rightarrow 10100 \\
 + \overbrace{}^1 \\
 \hline 10101 \\
 + \overbrace{}^{00000} \\
 \hline 10101
 \end{array}$$

$$PD = PD + MD$$

$$\begin{array}{r}
 11101 \\
 + \overbrace{}^{01011} \\
 \hline 10100
 \end{array}$$

$$\begin{array}{r}
 01011 \\
 00101 \quad 0 \rightarrow 11001 \quad 00101 \quad 0 \rightarrow 11100 \quad 10010 \quad //1 \\
 00001
 \end{array}$$

$$PD = PD - MD$$

$$\begin{array}{r}
 00100 \\
 + \overbrace{}^{10101} \\
 \hline 11001
 \end{array}$$

2×3

$$\begin{array}{r}
 010 \\
 000 \quad 011 \quad 0 \rightarrow 110 \quad 010 \\
 011 \quad 010
 \end{array}
 \begin{array}{r}
 010 \\
 111 \quad 001 \quad 1 \rightarrow 111 \quad 100 \quad 1 \\
 001 \quad 001
 \end{array}
 \begin{array}{r}
 010 \\
 100 \quad 1 \rightarrow 001 \\
 001
 \end{array}
 \begin{array}{r}
 010 \\
 000 \quad 110 \quad // \quad 0 \\
 000
 \end{array}$$

$$DD = PD - MD$$

$$\begin{array}{r}
 010 \rightarrow 101 \\
 + 1 \\
 \hline
 110 \\
 + 000 \\
 \hline
 110
 \end{array}$$

$$PD = PD + MD$$

$$\begin{array}{r}
 111 \\
 + 010 \\
 \hline
 1001
 \end{array}$$

$(-4) \times (-8)$

$$\begin{array}{r}
 \downarrow \quad \downarrow \\
 0100 \quad 1000 \\
 \downarrow \quad \downarrow \\
 1011 \quad 0111 \\
 + 1 \quad + 1 \\
 \hline
 1100 \quad 1000
 \end{array}
 \begin{array}{r}
 1100 \\
 0000 \quad 1000 \quad 0 \rightarrow 0000 \\
 0100 \quad 0011
 \end{array}
 \begin{array}{r}
 1100 \\
 0000 \quad 0 \rightarrow 0000 \\
 0010 \quad 0001
 \end{array}
 \begin{array}{r}
 1100 \\
 0000 \quad 0001 \quad 0
 \end{array}$$

$PD = PD - MD$

$$\begin{array}{r}
 0000 \\
 + 0100 \\
 \hline
 0100
 \end{array}$$

$$\begin{array}{r}
 1100 \\
 \rightarrow 0100 \quad 0001 \quad 0 \rightarrow 0010 \quad 0000 \quad // \quad 1 \\
 0001
 \end{array}$$