Optimal Anti-Poverty Programs

An Application to the Brazilian *Bolsa Família*

Juan Rios

February 16, 2016

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  - 30 million beneficiaries as of March 2015
  - Individuals below annual income of US$ 528

The Research Question

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- In this talk:
  - What are the elasticities of reported and real income?

Related Literature

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4. Taxation in Developing Countries (Gordon and Li 2009, Pomeranz 2013, Best et al 2014, Naritomi 2015, Bachas and Soto 2015)
    - Focus here on cash-transfer programs (negative taxes).

Outline

1. Institutional Background

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2. Data
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4. Implications for the Optimal Program

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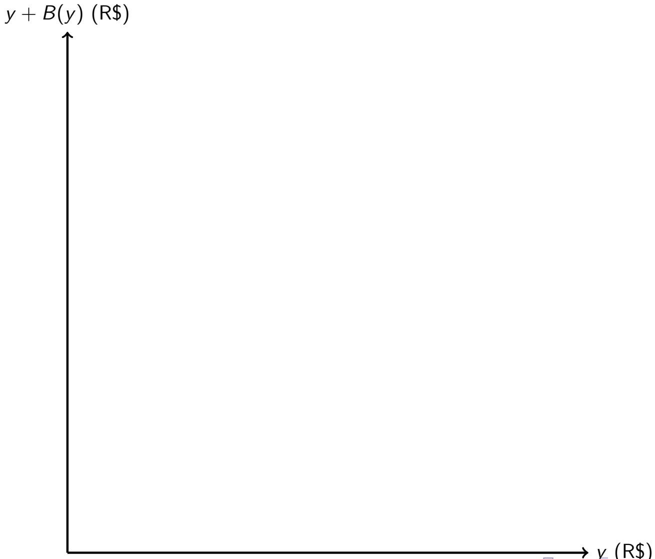
- Timing
  - Interviews on any business day
  - Updates at least once every two years
- Audits
  - Take away benefits
  - Vary with the gov. budget
  - Geographical variation

Households with 3 Members and 1 Child



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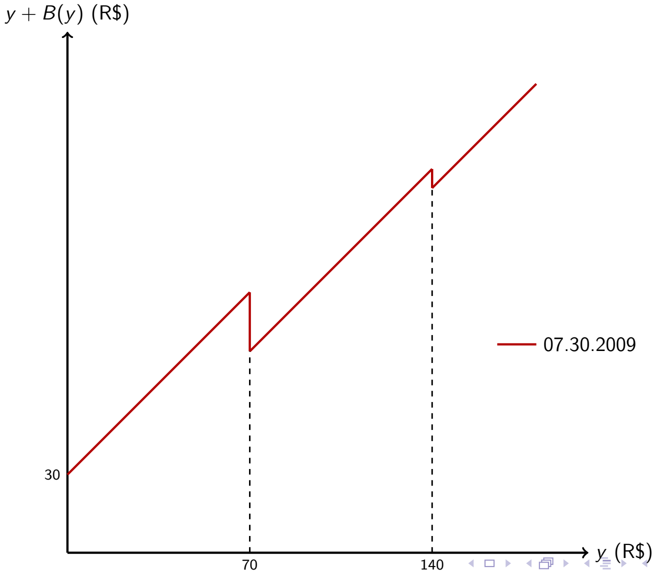

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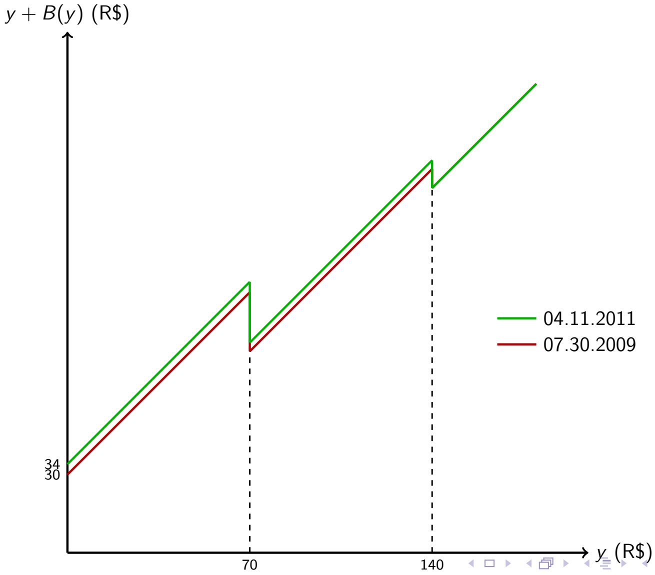

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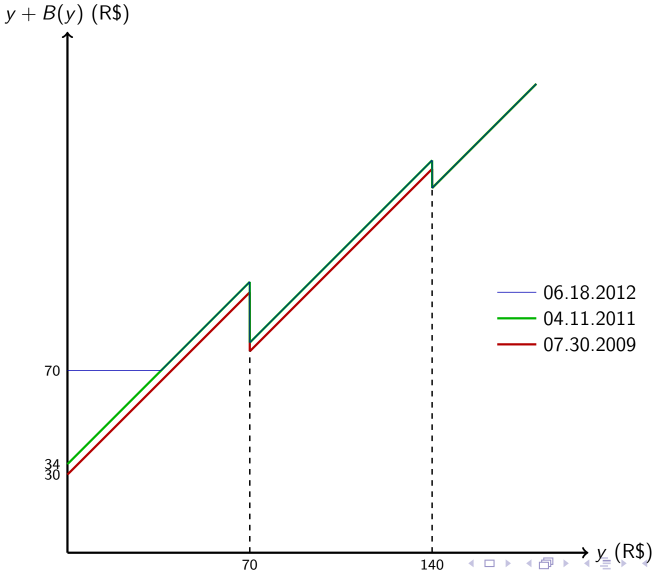

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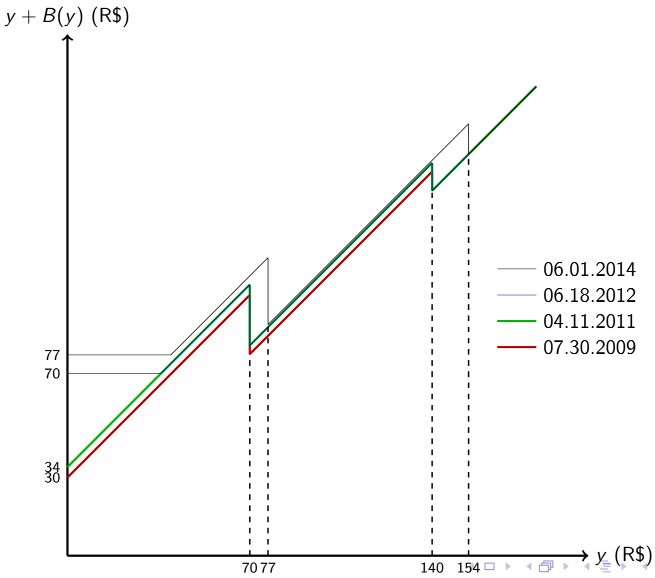
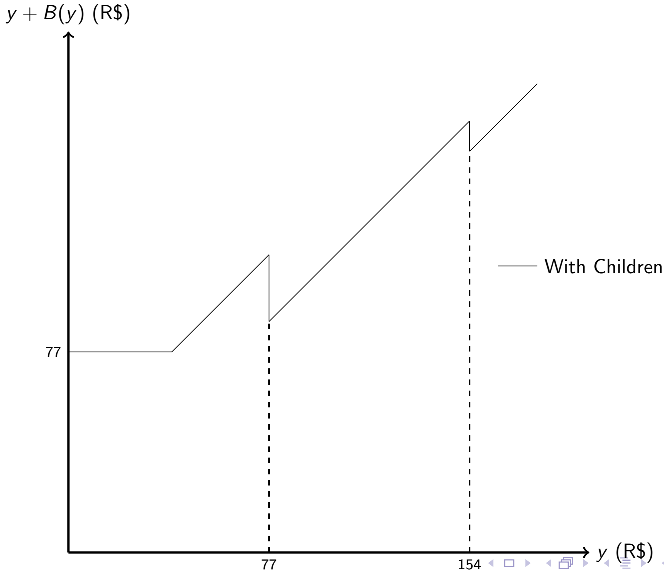


Households with 3 Members after Last Reform



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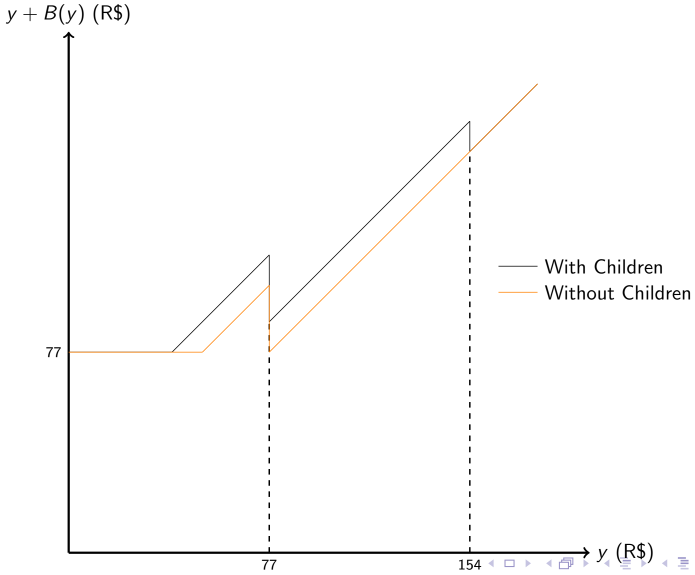

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- Bolsa Família Administrative Data for 2012-2015 (only 2015 today)
    - Panel with the self-reported income
    - Family Composition
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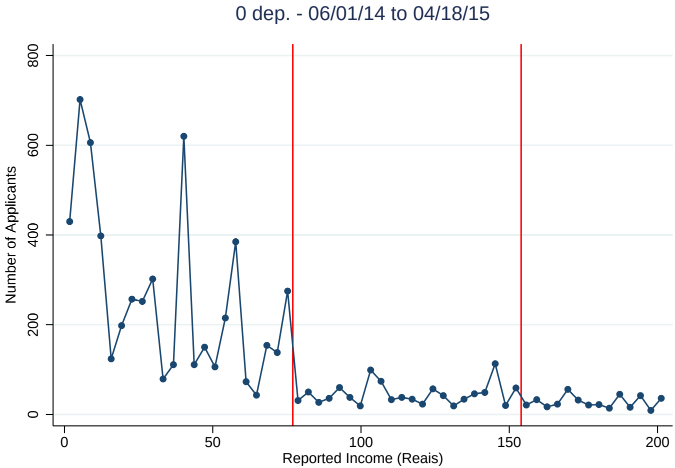
  - Panel with the self-reported income
  - Family Composition
  - Date of the Income Update
- RAIS
  - Universe of all formal employees in Brazil from 2002-2014
  - Monthly income
  - Individual level identifier

Table : Selection

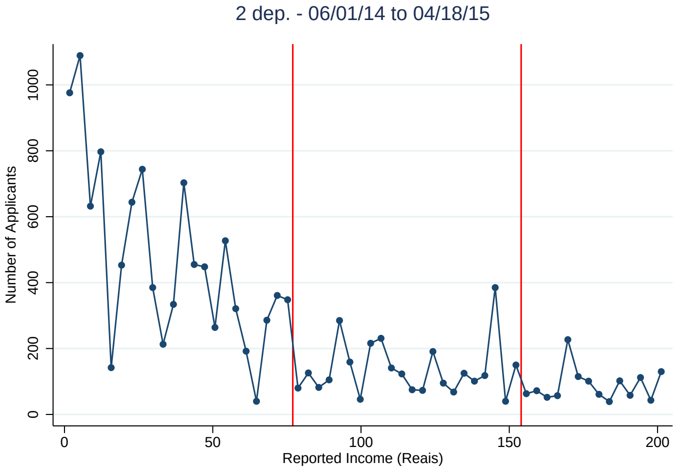
	Only BF	Formal Empl.	Total
Individuals	2560438	455495	3015933
	(84.9%)	(15.1%)	(100%)
Hhs (1 formal empl)	634088	359267	993355
	(63.8%)	(36.2%)	(100%)
Hhs (All formal empl)	746007	247348	993355
	(75.1%)	(24.9%)	(100%)

Reported Income Distribution (0 children)



Link Selected Sample

Reported Income Distribution (2 children)



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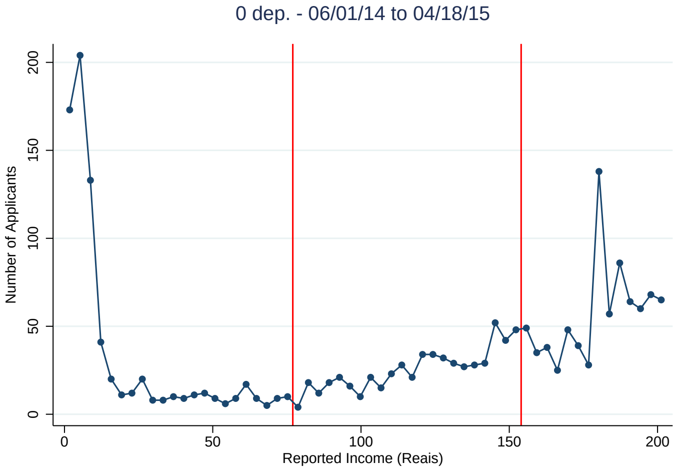
3rd Party Reported Income Distribution (0 children)



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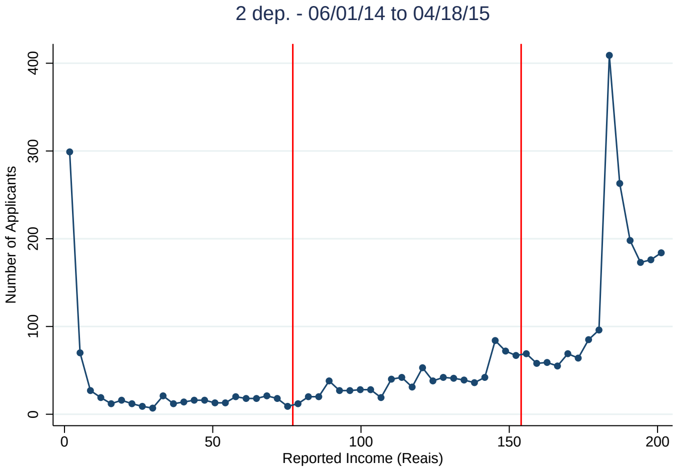
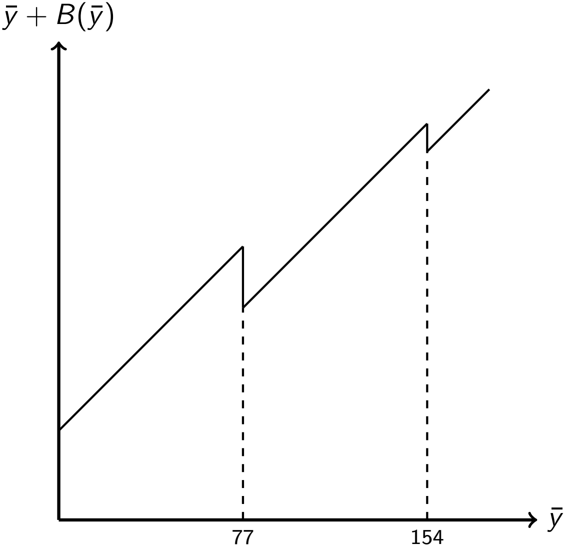


Program Schedule



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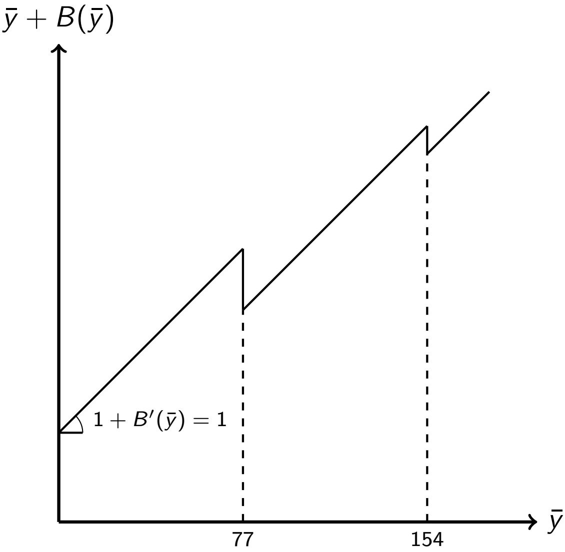

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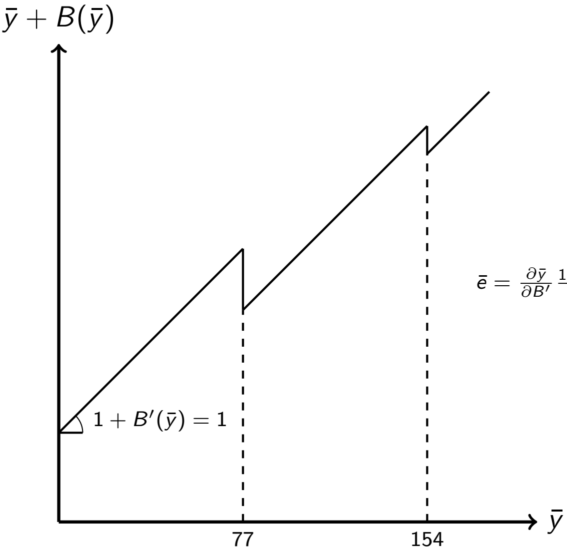
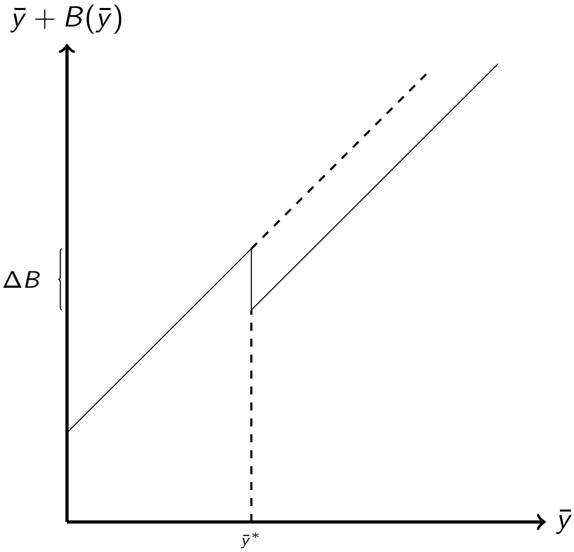


$\bar{e} = \frac{\partial \bar{y}}{\partial B'} \frac{1+B'}{\bar{y}}$ is not recoverable

Reduced-Form Bunching



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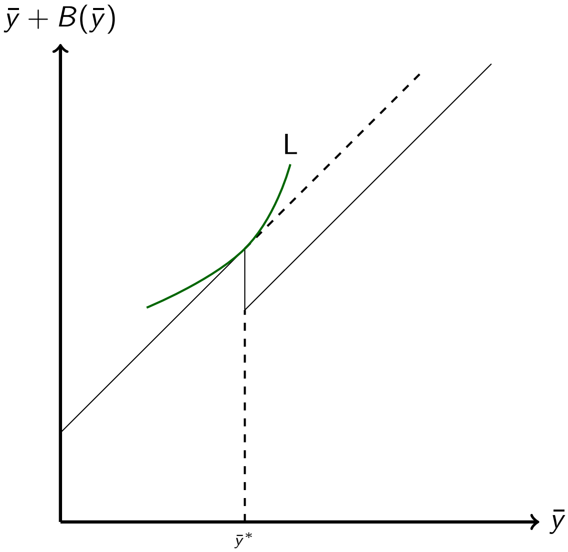

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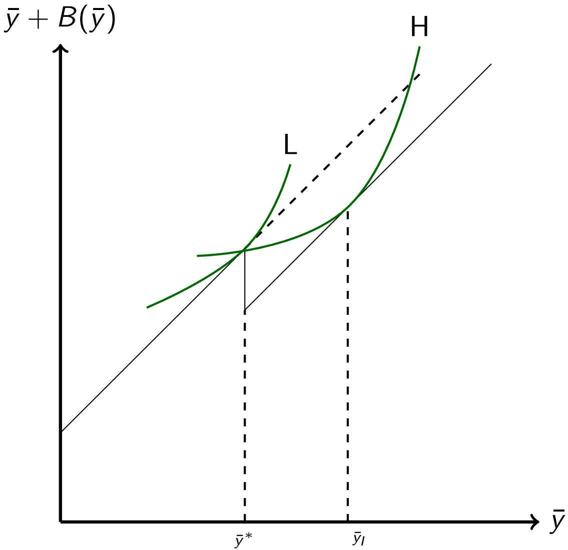

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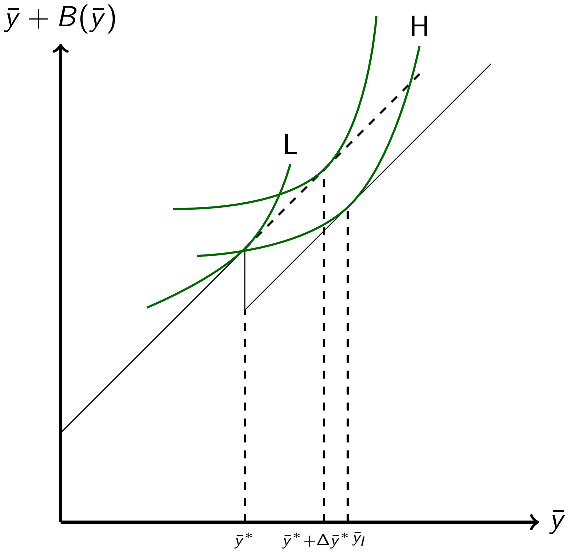

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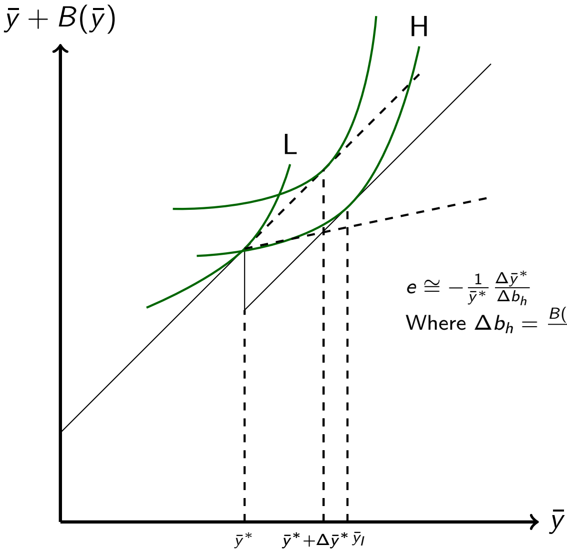

$$e \cong -\frac{1}{\bar{y}^*}\frac{\Delta\bar{y}^*}{\Delta b_h}$$

Where $\Delta b_h = \frac{B(\bar{y}^* + \Delta\bar{y}^*) - B(\bar{y}^*)}{\Delta\bar{y}^*}$

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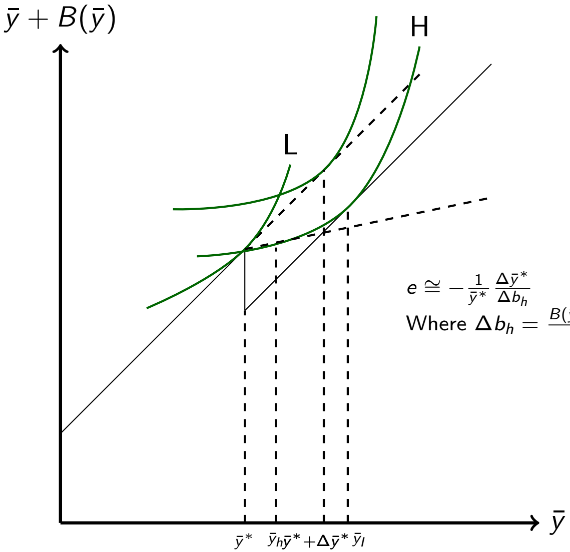

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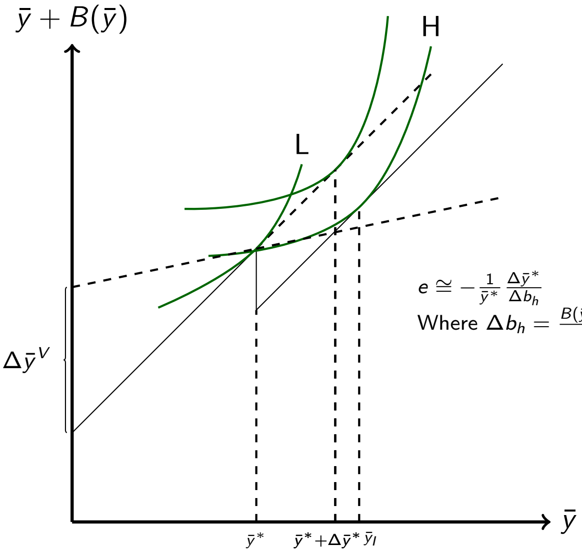

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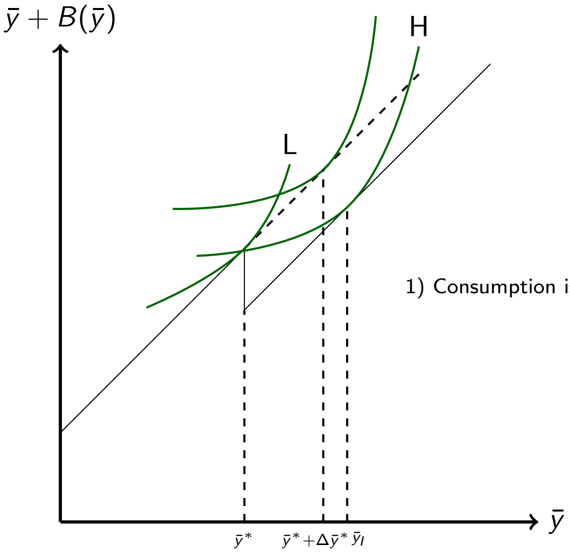

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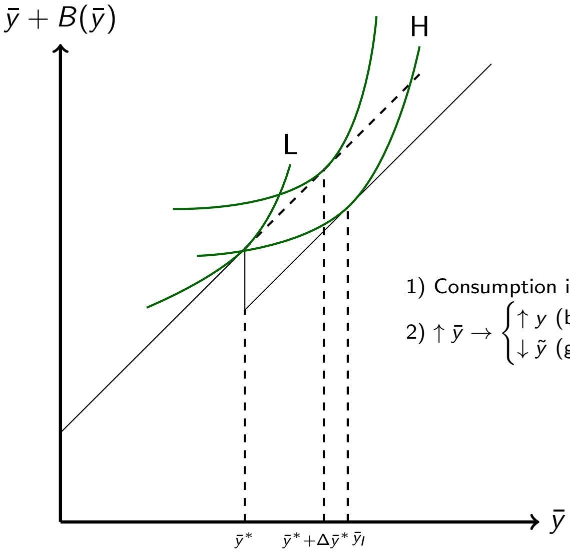

1) Consumption is actually $\bar{y} + \tilde{y} + B(\bar{y})$

2) $\uparrow \bar{y} \rightarrow \begin{cases} \uparrow y \text{ (bad)} \\ \downarrow \tilde{y} \text{ (good)} \end{cases}$

Adjusted Bunching Method

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$$U(\bar{y}, \tilde{y}; e, \tilde{e}, n) = \bar{y} + \tilde{y} + B(\bar{y}) - \frac{n}{1+1/e}\left(\frac{\bar{y}+\tilde{y}}{n}\right)^{1+1/e} - \frac{\tilde{y}^{1+1/\tilde{e}}}{1+1/\tilde{e}}$$

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- $y = \bar{y} + \tilde{y}$: Real Income
- n: Ability

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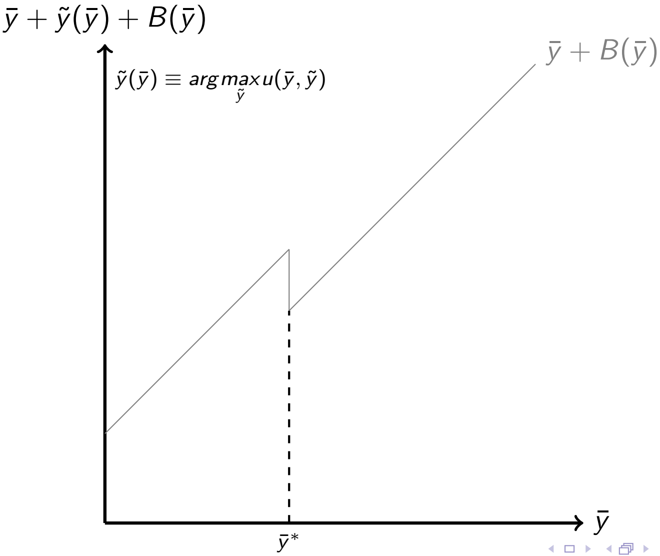
Adjusted Bunching Method (Reported Income)



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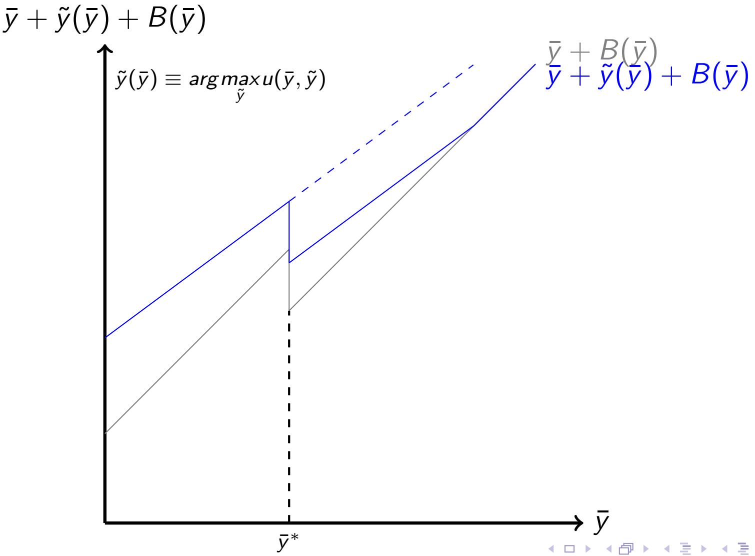

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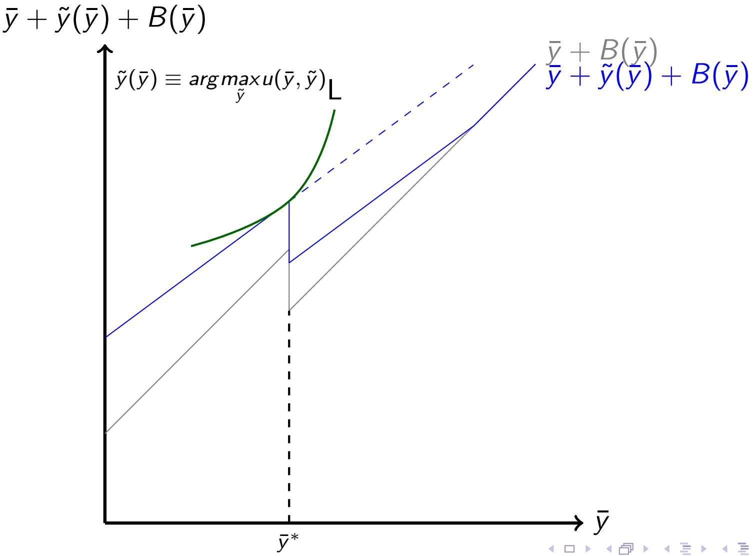

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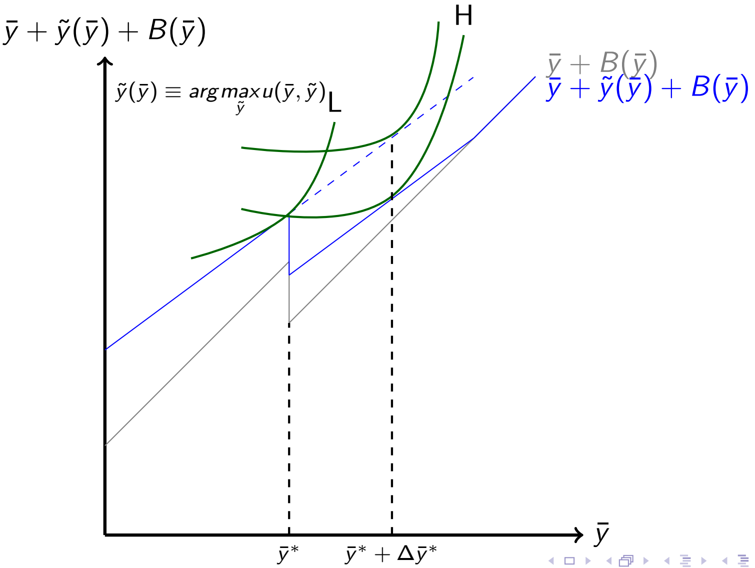

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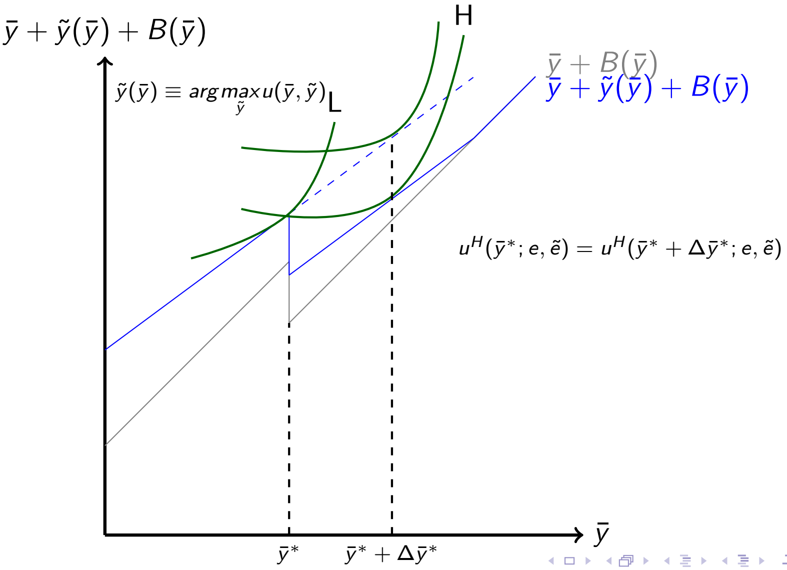

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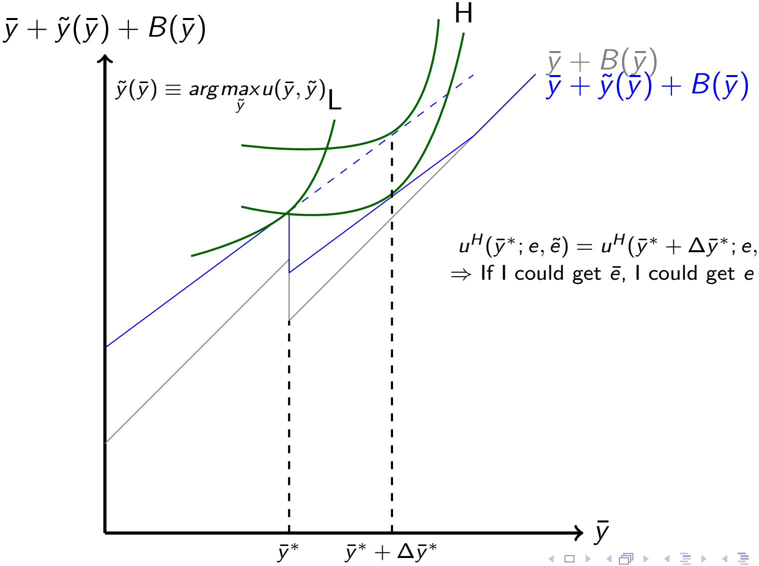
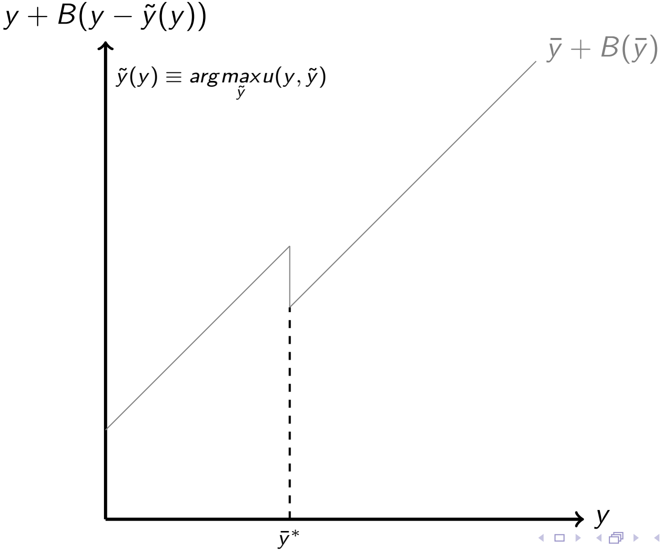


$u^H(\bar{y}^*; e, \tilde{e}) = u^H(\bar{y}^* + \Delta\bar{y}^*; e, \tilde{e})$

$\Rightarrow$ If I could get $\bar{e}$, I could get e and $\tilde{e}$

Adjusted Bunching Method (Real Income)



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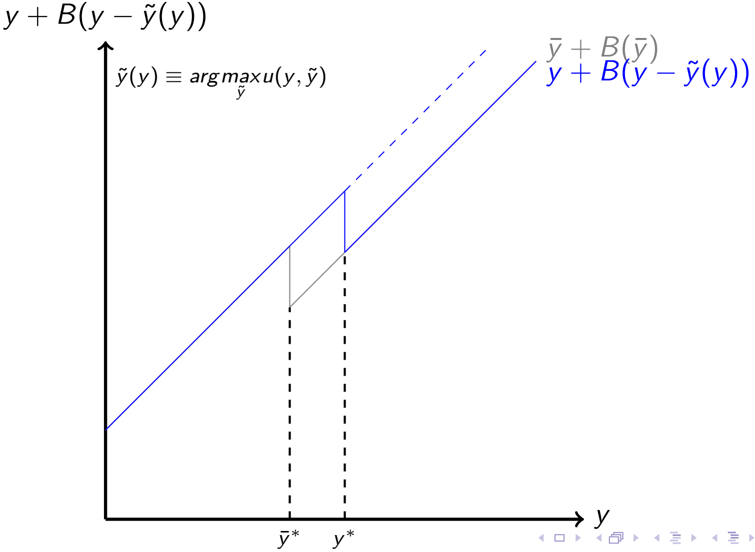

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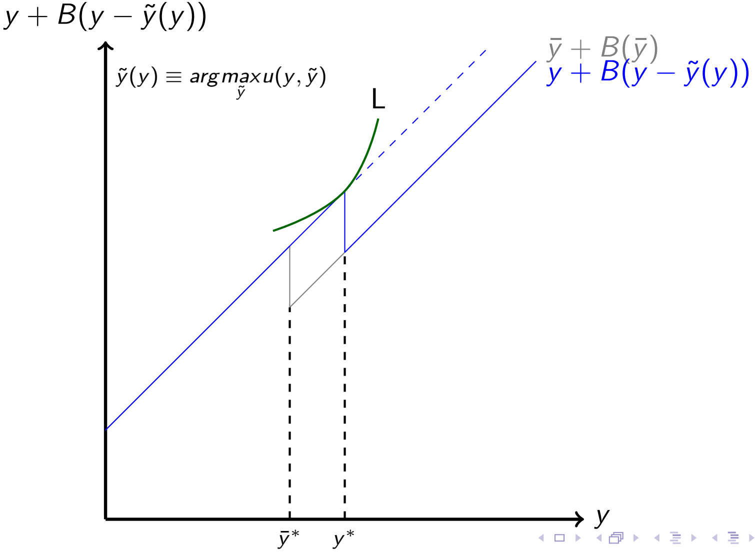

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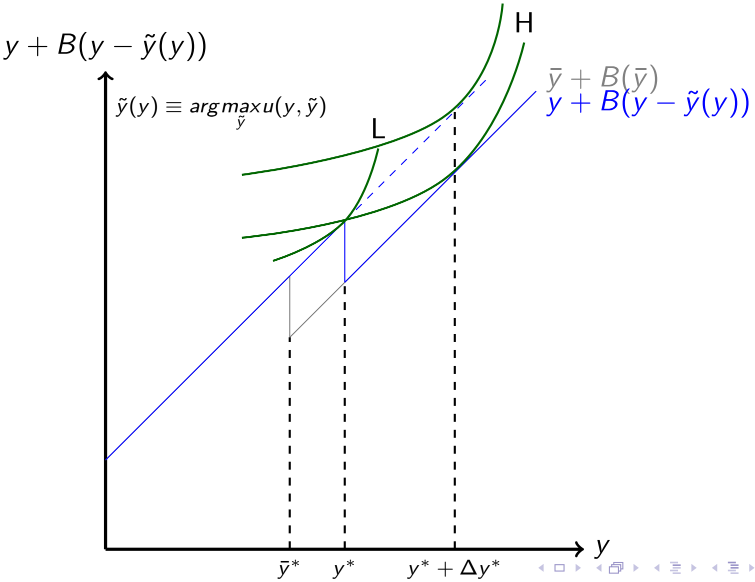

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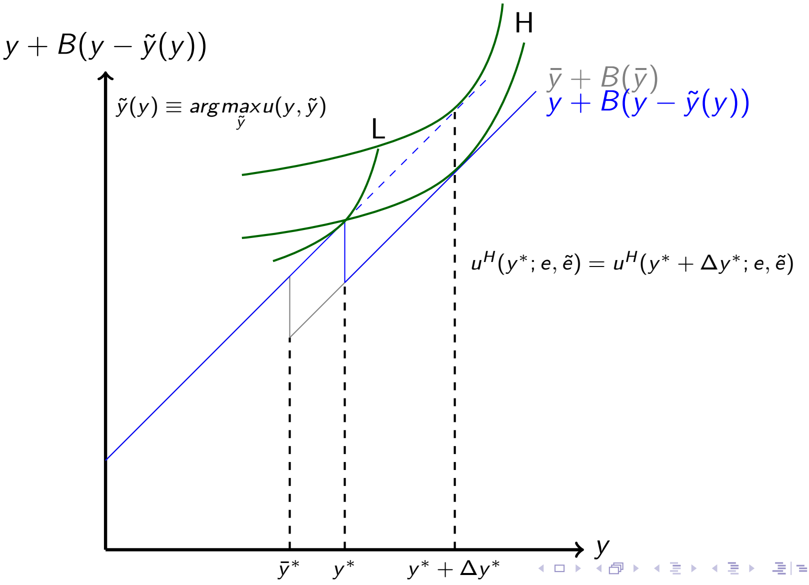

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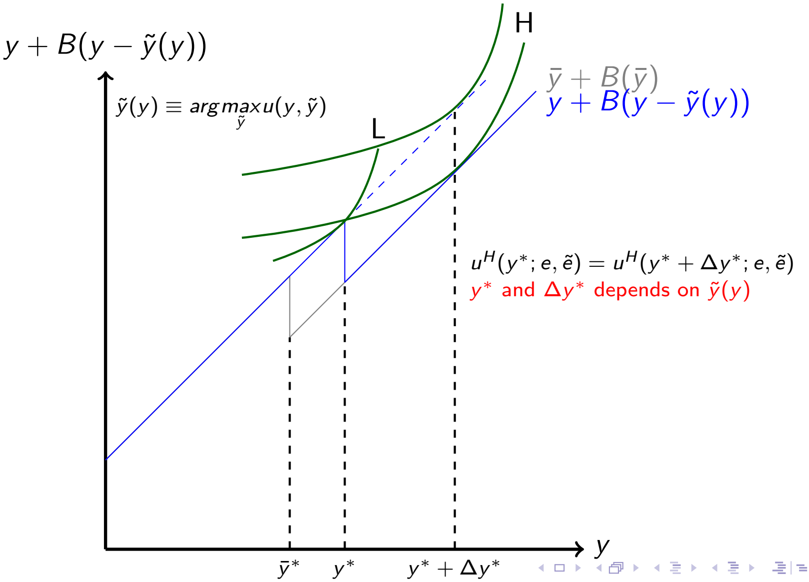
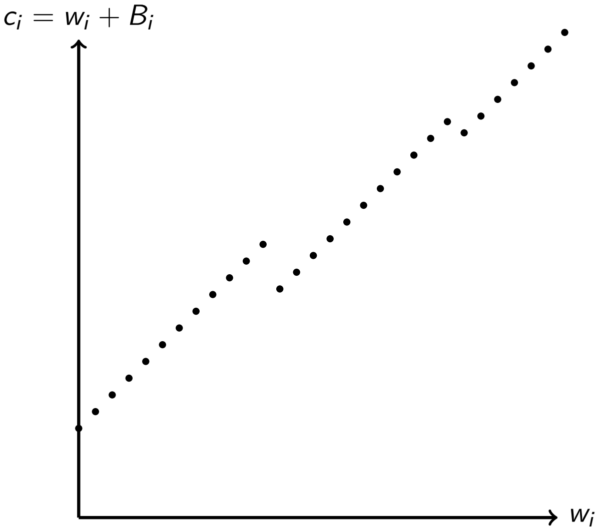


Discretized Program Schedule



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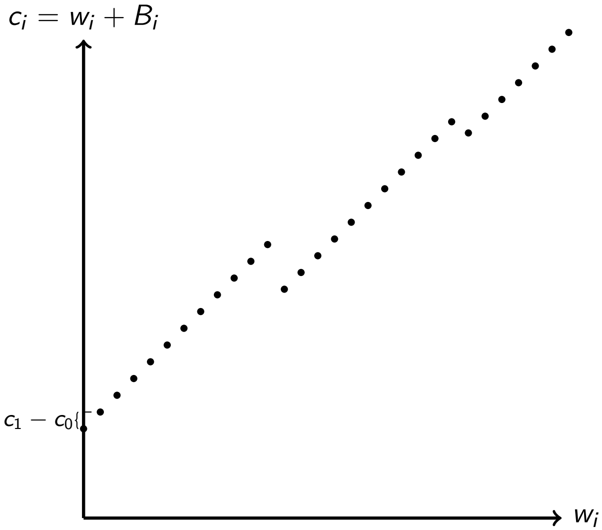

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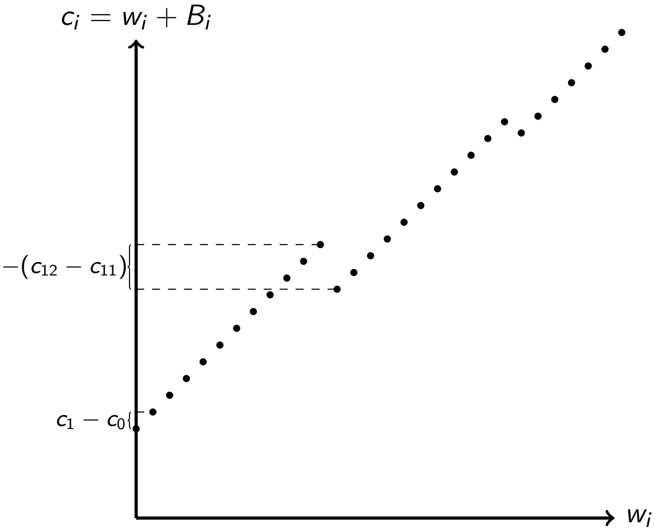

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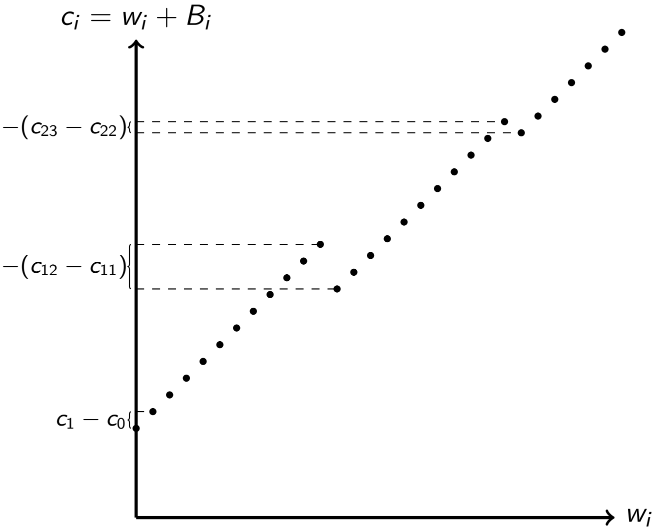

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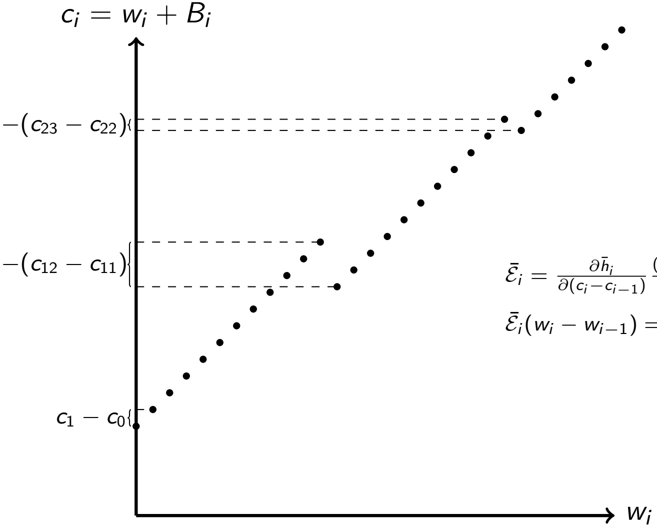

$\bar{\mathcal{E}}_i = \frac{\partial \bar{h}_i}{\partial(c_i - c_{i-1})} \frac{(c_i - c_{i-1})}{\bar{h}_i}$ is recoverable

$\bar{\mathcal{E}}_i(w_i - w_{i-1}) = \bar{e}_i w_i$

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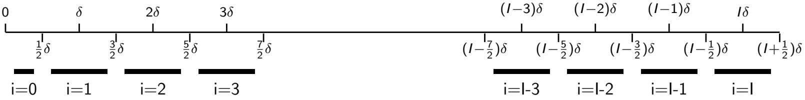

- $d_{ij} = \begin{cases} 1 & \text{if hh } j \text{ can report } 0,\ i-1 \text{ or } i, \\ 0 & \text{otherwise.} \end{cases}$ "intrinsic" income level

Empirical Strategy

- $\bar{y}_{jt}$: Income reported by household j in period t.
- $\bar{y}^*_{jt}$: Latent reported income, so that $\bar{y}_{jt} = \begin{cases} \bar{y}^*_{jt} \; if \; \bar{y}^*_{jt} > 0 \\ 0 \; if \; \bar{y}^*_{jt} \leq 0 \end{cases}$

- $d_{ij} = \begin{cases} 1 & \text{if hh } j \text{ can report } 0,\ i-1 \text{ or } i, \\ 0 & \text{otherwise.} \end{cases}$ "intrinsic" income level
- $h_{ijt} = \begin{cases} 1 & \text{hh } j \text{ reports } i \text{ in } t, \\ 0 & \text{otherwise.} \end{cases}$ "equilibrium" income level

Identifying $\eta_i = \frac{\partial h_i}{\partial (c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial (c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $\bar{y}^*_{jt} = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + \lambda_j + v_{jt}$

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- $\frac{\beta_i}{i\delta} = \mathcal{E}_i$ and $\frac{\gamma_i}{\delta} = \eta_i$

Link to Proof

Identifying $\eta_i = \frac{\partial h_i}{\partial (c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial (c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

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- $\frac{\beta_i}{i\delta} = \mathcal{E}_i$ and $\frac{\gamma_i}{\delta} = \eta_i$

Link to Proof

Identification Assumptions:

1. Reporting decisions would change in the same way with the reform across households with different household compositions.
2. Elasticities vary across income levels but not across household composition.

Optimality of the Anti-Poverty Program

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- If there are no fines $\Rightarrow$ elasticities of reported income are sufficient statistics

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- If there are fines $\Rightarrow$ elasticities of reported <u>and real</u> income are sufficient statistics

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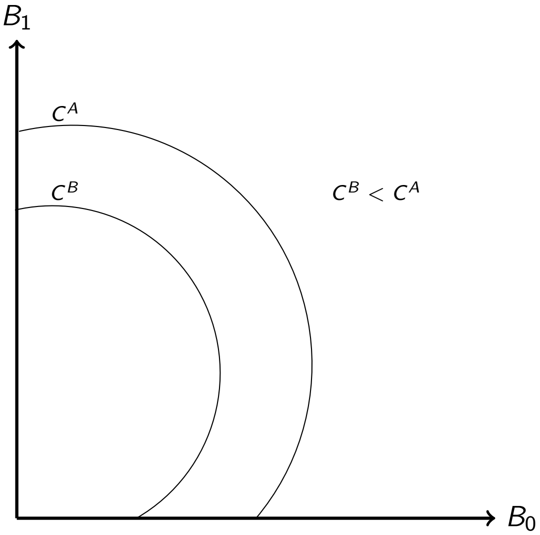
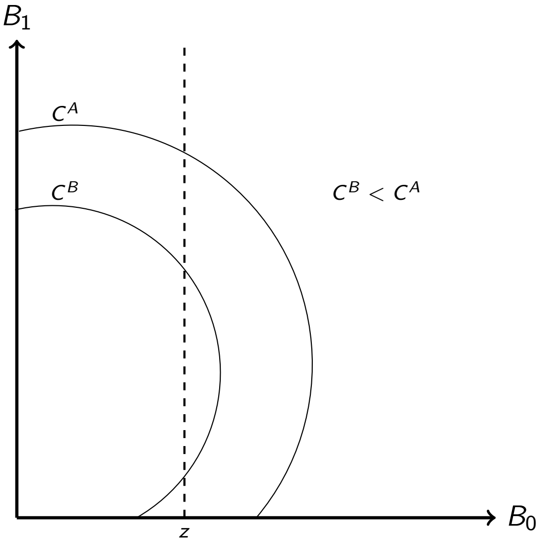
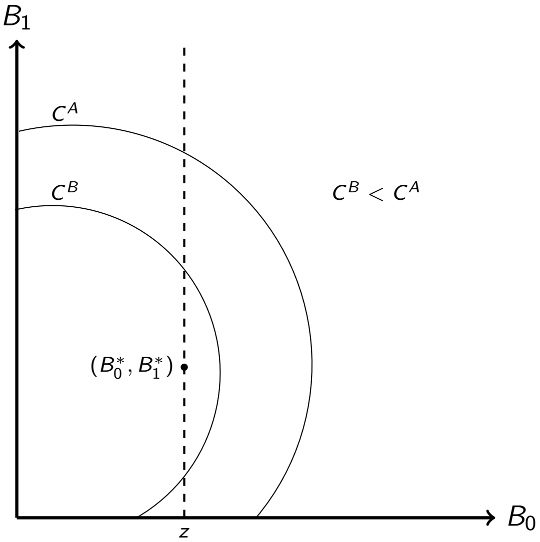
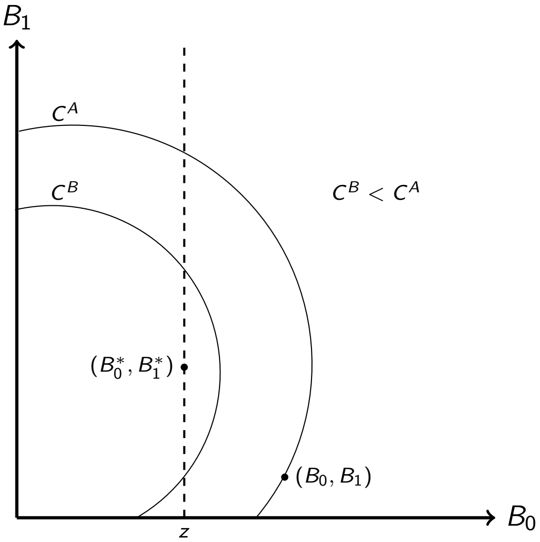
- If there are fines $\Rightarrow$ elasticities of reported and real income are sufficient statistics
- These are elasticities under the optimal schedule
- The elasticities under the observed schedule are sufficient statistics for the optimal reform

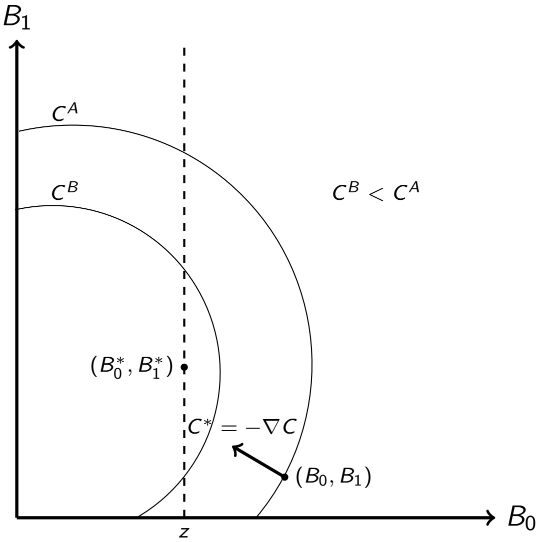
Link to Discrete Model
Link to the Government's Problem
Link to the Optimal Program
Link to the Optimal Reform

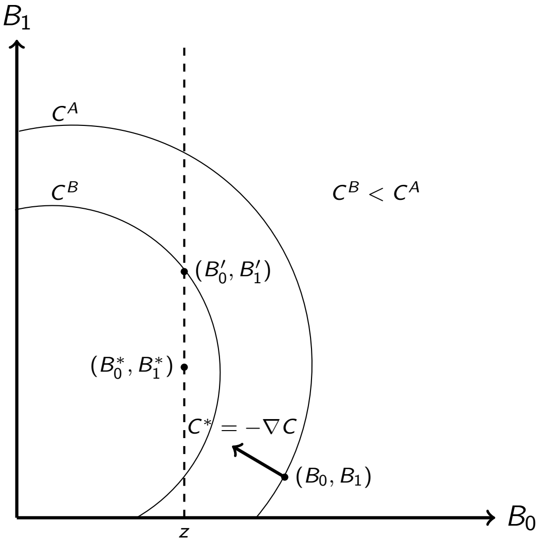

B_1
C^A
C^B
$C^B < C^A$
B_0


B_1
C^A
C^B
$C^B < C^A$
B_0
z


B_1
C^A
C^B
$C^B < C^A$
(B_0^*, B_1^*)
z
B_0


B_1
C^A
C^B
$C^B < C^A$
(B_0^*, B_1^*)
(B_0, B_1)
B_0
z


B_1
C^A
C^B
$C^B < C^A$
(B_0^*, B_1^*)
$C^* = -\nabla C$
(B_0, B_1)
z
B_0


B_1
C^A
C^B
$C^B < C^A$
(B_0', B_1')
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(B_0, B_1)
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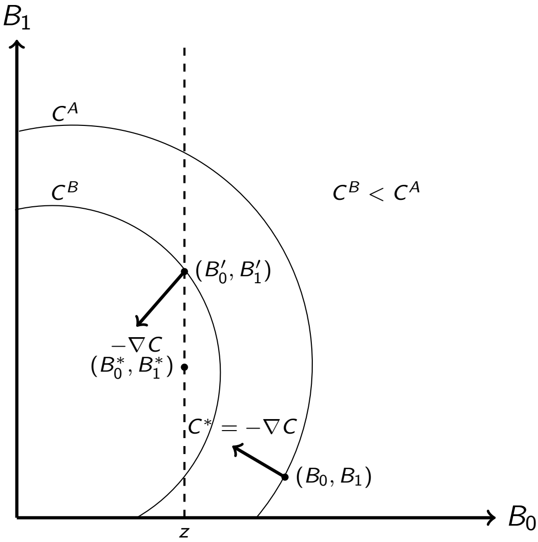
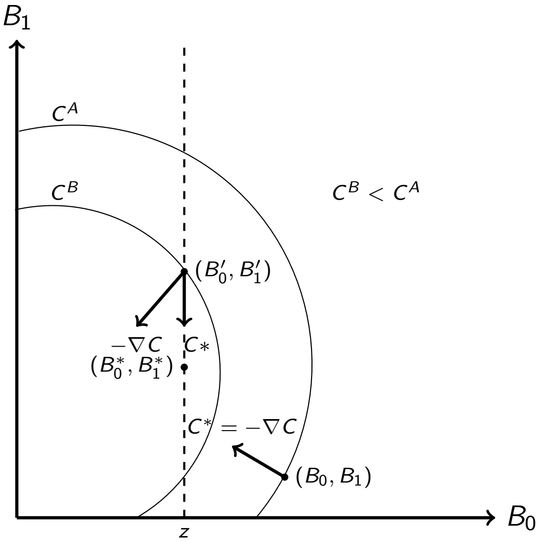


B_1
C^A
C^B
$C^B < C^A$
(B'_0, B'_1)
$-\nabla C$
(B^*_0, B^*_1)
$C^* = -\nabla C$
(B_0, B_1)
B_0
z



B_1
C^A
C^B
$C^B < C^A$
(B_0', B_1')
$-\nabla C$
$C*$
(B_0^*, B_1^*)
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z

Recap

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- Recover elasticities of reported and real income from bunching and reforms variation

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- Those elasticities are the sufficient statistics for the Optimal Anti-Poverty program

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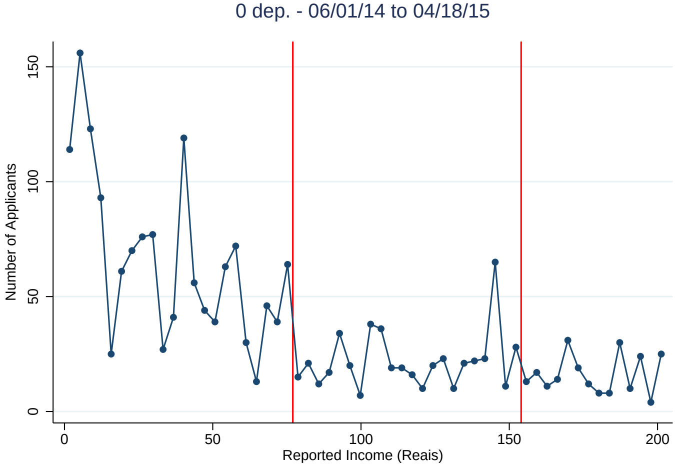
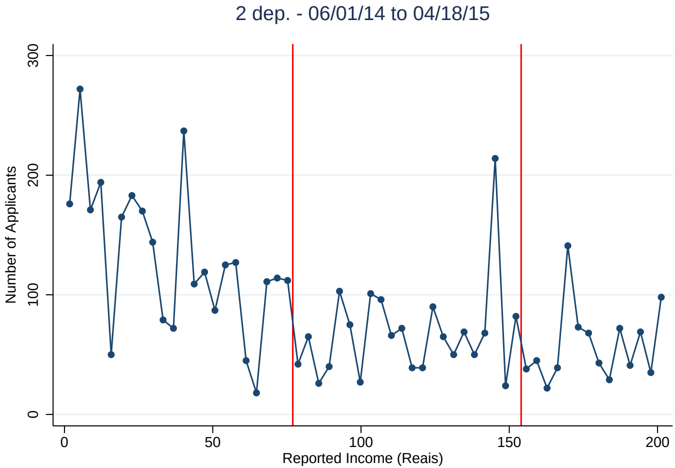
- Those elasticities are the sufficient statistics for the Optimal Anti-Poverty program
- The optimal reform can be written as a function of elasticities under the observed schedule

Reported Income (0 children) - Selected Sample



Back to Households with 0 Children

Reported Income (2 children) - Selected Sample



Back to Complete Sample

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + \lambda_1 + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$
- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \lambda_2 + \nu_{ijt}$
- $y^*_{jt} = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + \lambda + v_{jt}$

$$\beta = \frac{\partial E(y^*_{jt}|d_{ij} = 1)}{\partial ln(c_i - c_{i-1})_t} = w_i \frac{\partial E(h_{i,j,t}|d_{ij} = 1)}{\partial ln(c_i - c_{i-1})_t}$$

$$\eta_i = \frac{1}{P(h_{ij} = 1)}\beta$$

Back to Empirical

Theoretical Framework

$$U^E = w_{\bar{i}+\tilde{i}} + B_{\bar{i}} - p_{\bar{i}} f_{\tilde{i}} - \psi(\bar{i} + \tilde{i}, \tilde{i}, m)$$

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Assumptions

1. *No income effect.*
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3. *Some types cannot work*

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1. *No income effect.*
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3. *Some types cannot work*
4. *Type m reports either level* 0, $i(m) - 1$ *or* $i(m)$*:*

Cost Minimizing Objective

- $\bar{h}_i$: Proportion of households reporting level i in equilibrium
- $\tilde{h}_i$: Proportion of households producing i but reporting $i-1$.
- $\tilde{H}_i$: Proportion of households producing i but reporting 0.

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$$\min_{\{B_i\}_{i=0}^{I}} \sum_{i=0}^{I} \{\bar{h}_i B_i - p_{i-1} f_1 \tilde{h}_i - p_0 f_i \tilde{H}_i\}$$

$$st \;\; c_0 \geq z \;\; and \;\; B_i \geq 0 \; \forall i.$$

Definitions

Reported income elasticity in the extensive margin:

$$\bar{\eta}_i \equiv \frac{c_i - c_0}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_0)},$$

Reported income elasticity in the intensive margin:

$$\bar{\mathcal{E}}_i \equiv \frac{c_i - c_{i-1}}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_{i-1})}.$$

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Back to Implications

Proposition

Assuming that $\hat{\eta}_i^* \leq \frac{c_i^* - z}{z}$ *for* $i \geq v$*, the cost minimizing schedule* $\{B_i^*\}_{i=0}^{I}$ *is:*

$$B_0^* = z$$

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{\hat{\mathcal{E}}_i^*} \sum_{j=i}^{I} \left(\bar{h}_j^* + \hat{\eta}_i^* \frac{B_j^* - z}{c_j^* - z} \right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for } i = v, v+1, ..., I.$$

Where $\hat{\eta}_i^* \equiv (1 - M_{\bar{n}(i)}) h_i^* \eta_i^* + M_{\bar{n}(i)} \bar{h}_i^* \bar{\eta}_i^*$,
$\hat{\mathcal{E}}_i^* \equiv (1 - \mu_{\bar{m}(i)}) h_i^* \mathcal{E}_i^* + \mu_{\bar{m}(i)} \bar{h}_i^* \bar{\mathcal{E}}_i^*$ *and* v *is the smallest* i *such that the* B_i^* *implied by the second bracket is less or equal to zero.*

Back to Implications | Link to Proof | Link to Lemma | Link to Welf Prob | Link to Efficiency

Problem: Elasticities under the optimal schedule ⇒ Non-recoverable.

Proposition

The cost minimizing local reform is a vector of perturbation in the benefit schedule $\Delta B = -(C_0, .., C_I)$ where:

$$C_0 = \begin{cases} \bar{h}_0 - \sum_{i=1}^{v-1} \frac{B_i - B_0}{c_i - c_0} \hat{\eta}_i & \text{if } B_0 > z \\ 0 & \text{if } B_0 = z \end{cases}$$

$$C_i = \bar{h}_i + \frac{B_i - B_0}{c_i - c_0}\hat{\eta}_i + \frac{B_i - B_{i-1}}{c_i - c_{i-1}}\hat{\mathcal{E}}_i - \frac{B_{i+1} - B_i}{c_{i+1} - c_i}\hat{\mathcal{E}}_{i+1} \; 1 \leq i \leq v$$

$$\text{for } \quad i = 1, ..., v-1$$

$$C_i = min\left\{\bar{h}_i - \frac{B_0}{c_i - c_0}\hat{\eta}_i - \frac{B_{i-1}}{c_i - c_{i-1}}\hat{\mathcal{E}}_i, 0\right\} \text{ for } i = v, ..., I$$

v: lowest level with zero benefits

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Here all the parameters are recoverable from the data.

Back to Implications

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1. $ME = dB \sum_{j=i}^{I} h_j$.
2. $BEIM = dh_i^{int}(B_i - B_{i-1}) = (dk_i^{int} - de_i)(B_i - B_{i-1})$

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3. $BEEM = \sum_{j=i}^{I} dh_j^{ext}(B_j - B_0) = \sum_{j=i}^{I}(dk_j^{ext} - dE_j)(B_j - B_0)$

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At the optimum: $ME + BEIM + BEEM + FE = 0$.

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$$\begin{aligned} BEIM + FEIM = dk_i^I(B_i - B_{i-1}) + de_i[(B_{i-1} - B_i) - p_{i-1} f_1] = \\ dk_i^I(B_i - B_{i-1}) + de_i \mu_{\bar{m}(i)}(B_{i-1} - B_i) = \\ \left[(1 - \mu_{\bar{m}(i)}) dk_i^I + \mu_{\bar{m}(i)} dh_i^I\right](B_i - B_{i-1}) \end{aligned}$$

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$\bar{\eta}_i^* \leq \frac{c_i^* - z}{z}$ for $i > v$ ensures $B_{i-1}^* \geq B_i^*$ and hence $B_i^* = 0$. □

Back to Proposition

$$U^E = w_{i+\tilde{i}} + B_i \underbrace{-p_i f_{\tilde{i}}}_{transfer\ \ cost} -\psi(i+\tilde{i}, \underbrace{\tilde{i}}_{util.\ \ cost}, m)$$

- $\bar{m}(i)$ indifferent between reporting i and $i-1$, given real income is i
- $\bar{n}(i)$ indifferent between reporting i and 0, given real income is i
- $\mu_{\bar{m}(i)} \equiv \frac{\psi(i,1,\bar{m})-\psi(i,0,\bar{m})}{p_i f_1+\psi(i,1,\bar{m})-\psi(i,0,\bar{m})}$: Share of utility cost in the int. margin
- $M_{\bar{n}(i)} \equiv \frac{\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}{p_0 f_i+\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}$: Share of utility cost in the ext. margin

Lemma

The wedge between the marginal benefit and marginal fine cost of misreporting (the marginal utility cost) can be written as:

$(B_{i-1} - B_i) - p_{i-1} f_1 = (B_{i-1} - B_i)\mu_{\bar{m}(i)}$

$(B_0 - B_i) - p_0 f_i = (B_0 - B_i) M_{\bar{n}(i)}$

Proof.

$$w_i + B_{i-1} - p_{i-1}f_1 - \psi(i,1,\bar{n}) = w_i + B_i - \psi(i,0,\bar{n})$$
$$\Rightarrow (B_{i-1} - B_i) - p_{i-1}f_1 = \psi(i,1,\bar{n}) - \psi(i,0,\bar{n})$$

Multiplying the RHS by $\frac{B_{i-1}-B_i}{p_{i-1}f_1+\psi(i,1,\bar{n})-\psi(i,0,\bar{n})}$, we get the 1st relation.

$$w_i + B_0 - p_0 f_i - \psi(i,i,\bar{n}) = w_i + B_i - \psi(i,0,\bar{n})$$
$$\Rightarrow (B_0 - B_i) - p_0 f_i = \psi(i,i,\bar{n}) - \psi(i,0,\bar{n})$$

Multiplying the RHS by $\frac{B_0-B_i}{p_0 f_i+\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}$, we get the 1st relation. □

Back to Proposition

Welfarist Objective

- δ^m: Welfare weight on households of type m

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- δ^m: Welfare weight on households of type m
- $\tilde{i}$: hidden income so that $i + \tilde{i}$ is the real income level
- $v(m)$: Measure of households with type m.
- R: Anti-poverty program's budget
- The government solves:

$$\max_{\{B_0, B_1, \dots, B_I\}} \int_M \delta^m u^m(w_{i+\tilde{i}} + B_i, i + \tilde{i}, \tilde{i}) dv(m)$$

$$subject\ to\ \sum_i h_i B_i \leq R\ and\ B_i \geq 0\ \forall i$$

Proposition

Assuming that $\eta_i^* \leq (1-g_i^*)\frac{c_i^*-B_0^*}{B_0^*}$ *for* $i > v$ *and that there are no income effects, the welfare maximizing schedule* $\{B_i^*\}_{i=0}^{I}$ *is:*

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{h_i^* \mathcal{E}^*{}_i} \sum_{j=i}^{I} h_j^* \left(1 - g_j^* + \frac{\eta_j^*(B_j^* - B_0^*)}{c_j^* - c_0^*}\right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I$$

$$\text{such that } \sum_{i=0}^{I} h_i^* B_i^* = R.$$

Where $g_i = \frac{1}{h_i} \int_{m:i(m)=i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}} + B_i, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m)$ *and* v *is the smallest* i *such that the* B_i^* *implied by the second bracket is less or equal to zero.*

Proof.

FOC: $\int_{M_i^*} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i^*, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m) - p\left[h_i^* + \sum_{j=0}^{I} B_j^* \frac{\partial hj^*}{\partial c_i}\right] = 0$

Let $g_i = \frac{1}{ph_i} \int_{M_i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m)$

FOC becomes: $(1-g_i)h_i^* = -\Big[(B_i - B_0)\frac{\partial h_i}{\partial(c_i - c_0)} +$

$(B_i - B_{i-1})\frac{\partial h_i}{\partial(c_i - c_{i-1})} - (B_{i+1} - B_i)\frac{\partial h_{i+1}}{\partial(c_{i+1} - c_i)}\Big]$

Summing over i, we get the first equation of the proposition.

$\eta_i^* \leq (1-g_i^*)\frac{c_i^* - B_0^*}{B_0^*}$ for all $i > v$ guarantees that the incremental benefits are negative for these income levels. □

Why the Reported Income is the Sufficient Statistic for the Welfarist Problem?

- The Optimal Anti-Poverty Program Problem has three parts:
  1. Distorting incentives with marginal taxes:
     Workers already maximizing $\Rightarrow$ Second Order Effects
  2. Government Revenue:
     It depends on Reported Income
  3. Targeting low ability people:
     The reported income is the targeting instrument

Back to Proposition

Efficiency of Cost Minimizing Allocation

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- If the poorest cannot work, caring about his income is equivalent to caring about his utility
- Equivalent to a Rawlsian Social Planner with a budget equal to the minimum cost

Table : Income Maintenance Objectives

Gov. cares for\ Productive	Everyone	Not Everyone
Only Poorest	Not Efficient	Efficient
Below Poverty Line	Not Efficient	Not Efficient

Back to Proposition

Proposition

Assuming that households respond only in the extensive margin, the optimal transfer program would be:

$$B_0^* = z,$$

$$\frac{B_i^* - B_0^*}{c_i^* - c_0^*} = \frac{1}{\eta_i^*}(g_i^* - 1),$$

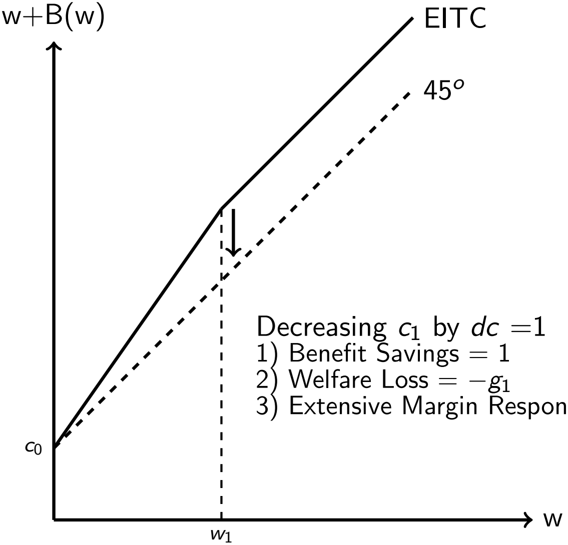
$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I.$$

Where v is the smallest i such that the B_i^* implied by the second bracket is less or equal to zero.

Implications

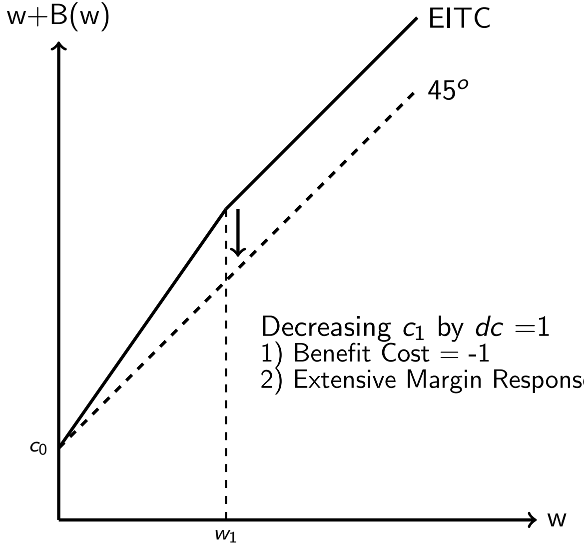
1. If $g_i^* > 1$ EITC is optimal $(B_1^* > B_0^*)$
2. EITC is never cost minimizing $(g_i^* = 0$ for all $i > 0)$

Welfare Maximizing



Decreasing c_1 by dc =1
1) Benefit Savings = 1
2) Welfare Loss = $-g_1$
3) Extensive Margin Response (makes it cheaper)

Cost Minimizing



Decreasing c_1 by dc =1
1) Benefit Cost = -1
2) Extensive Margin Response (makes it cheaper)

Proof.

Consider $dB_i = ... = dB_I = dB$. The change in the cost of the program due to intensive margin responses in the discrete model is:

$$(B_{i-1} - B_i)dh_i - f_1 de_i = [(1-\mu_{\bar{m}})dk_i + \mu_{\bar{m}} dh_i](B_{i-1} - B_i) =$$
$$[(1-\mu_{\bar{m}})\mathcal{E}_i^R k_i + \mu_{\bar{m}}\mathcal{E}_i h_i]\frac{B_i - B_{i-1}}{w_i - w_{i-1}}\frac{dB}{c_i - c_{i-1}}(w_i - w_{i-1})$$

In the continuous model, let $b_i = \frac{B_i - B_{i-1}}{w_i - w_{i-1}}$ and $f_i = \frac{p_{i-1} f_1}{w_i - w_{i-1}}$ be the marginal benefit and expected fines faced by individual with $\bar{y} = w_i$. The same perturbation $db_i = dB/(w_i - w_{i-1})$ will reduce the reported income of individuals reporting w_i by $d\bar{y} = dy - d\tilde{y}$. So the tolal effect on cost is:

$$\{(1-\mu_{\bar{m}})[wi + \tilde{y}(w_i, m)]e_i + \mu_{\bar{m}}\bar{e}_i\}\frac{db_i}{1+b_i}h_i b_i$$

Equating the terms multiplying $(1-\mu_{\bar{m}})$ and $\mu_{\bar{m}}$, we get the relations. □

Back to Discretized Schedule