# Programas de Combate à Pobreza Ótimos

Uma aplicação ao *Bolsa Família*

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February 17, 2016

# Introdução

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- Impasse entre academia e política publica:
  - Academia: Maximizar o bem-estar social
  - Policy makers: Reduzir a pobreza

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  - 30 milhões de beneficiários em Março de 2015

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  - Academia: Maximizar o bem-estar social
  - Policy makers: Reduzir a pobreza
- *Bolsa Família* é o maior programa de transferência condicional de renda do mundo
  - 30 milhões de beneficiários em Março de 2015
  - Indivíduos abaixo de renda anual de US\$ 528 (EITC abaixo de US\$ 16,000)

# O que sabemos na academia sobre redistribuição de renda?

- O imposto de renda que maximiza a função de bem-estar
  - BF é um Imposto de Renda Negativo
  - Bem-estar: soma de funções de utilidade
  - Precisamos de algumas estatísticas dos dados de renda (Saez, 2001)

# O que sabemos na academia sobre redistribuição de renda?

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  - BF é um Imposto de Renda Negativo
  - Bem-estar: soma de funções de utilidade
  - Precisamos de algumas estatísticas dos dados de renda (Saez, 2001)
- Problemas
  - Não sabemos bem como somár-las.
  - No Brasil, Imposto de renda dissociado do BF

# O que proponho

- Maximizar a renda mínima brasileira dado o orçamento do programa
  - Mais próximo do objetivo real do governo
  - O que vocês acham?

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- Maximizar a renda mínima brasileira dado o orçamento do programa
  - Mais próximo do objetivo real do governo
  - O que vocês acham?
- Estimar as estatísticas necessárias usando os dados do BF e da RAIS
  - Incentivos gerados pelo programa
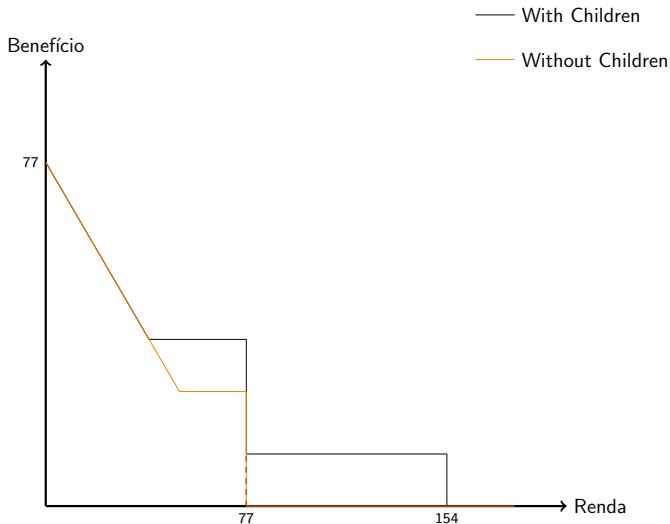  - Capacidade de fiscalização

# Domicílios com 3 membros e 1 Criança

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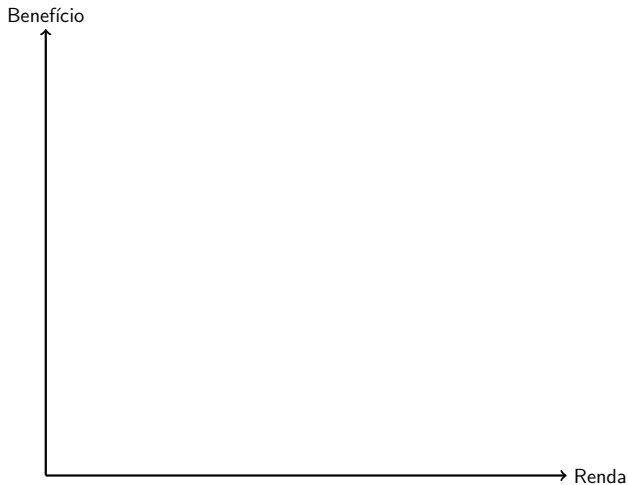
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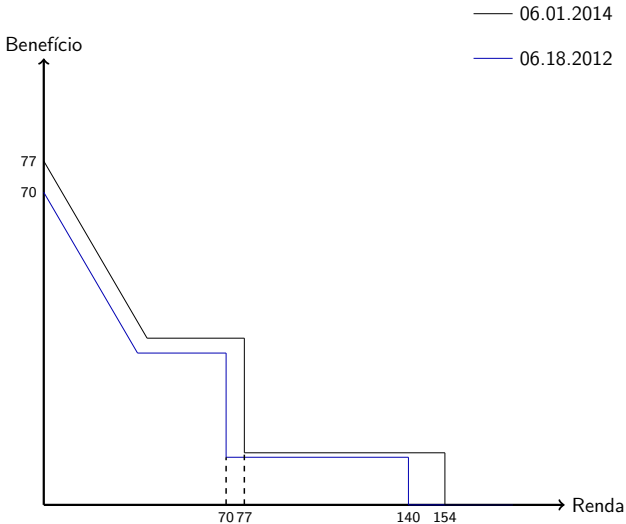
# Domicílios com 3 membros após a última reforma

# Dados

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- Cadunico para 2012-2015 (apenas 2015 hoje)
  - Renda e renda per capita domiciliar
  - Composição Familiar
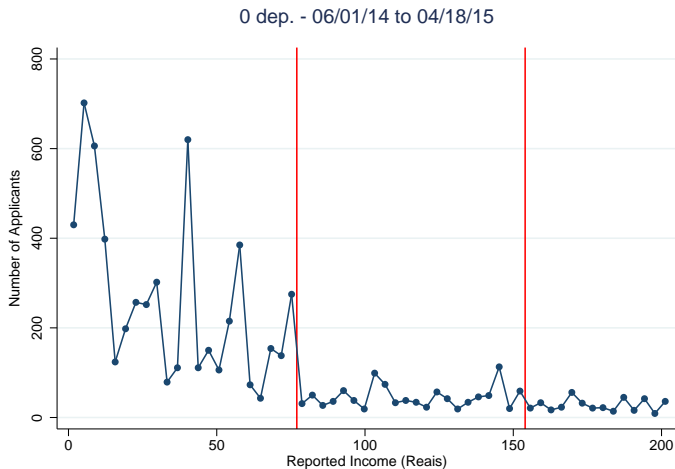  - Data da atualização

# Dados

- Cadunico para 2012-2015 (apenas 2015 hoje)
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    - Composição Familiar
    - Data da atualização
- RAIS
    - Renda reportada por firmas
    - Mês de referência
    - CPF

Table : Quem está no Mercado Formal?

|  | Só BF | Formalizados | Total |
|---|---|---|---|
| Indivíduos | 2560438 | 455495 | 3015933 |
|  | ( 84.9%) | ( 15.1%) | (100%) |
| Domicílios | 634088 | 359267 | 993355 |
|  | ( 63.8%) | ( 36.2%) | (100%) |

# Distribuição da Renda reportada (sem filhos)



0 dep. - 06/01/14 to 04/18/15

‣ Link Selected Sample

# Distribuição da Renda reportada (2 crianças)



2 dep. - 06/01/14 to 04/18/15
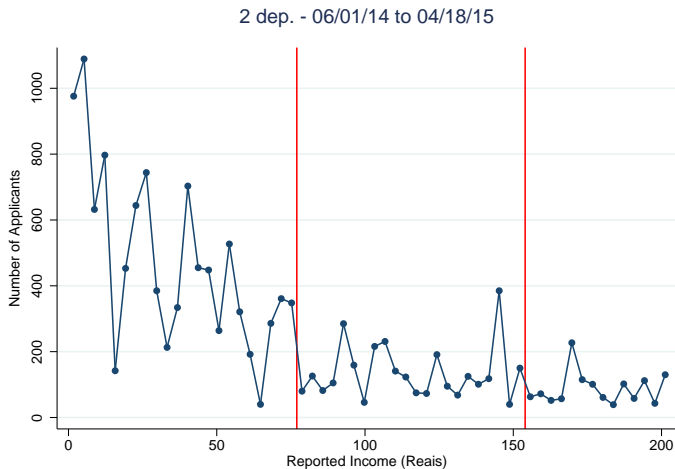
▸ Link to Selected Sample

# Distribuição da Renda formal (sem filhos)



0 dep. - 06/01/14 to 04/18/15
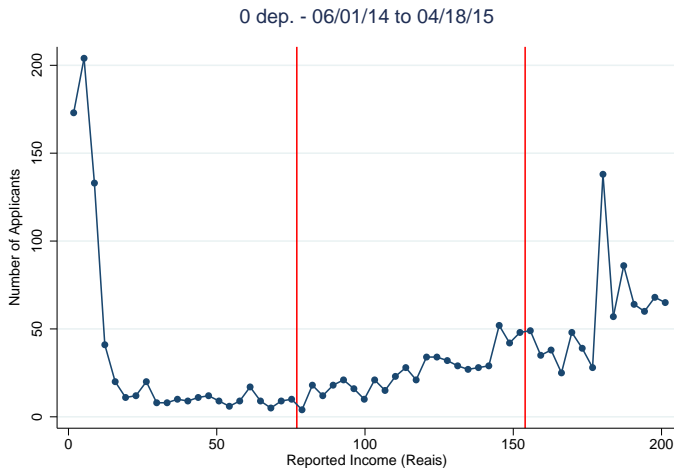
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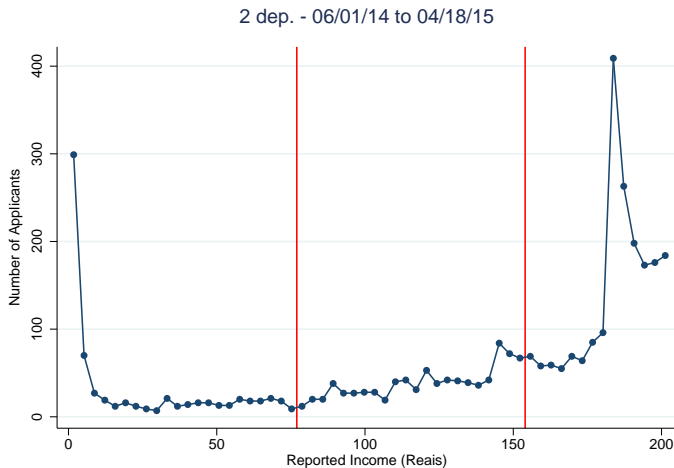


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# O que preciso nos dados

- Em qual tabela do BF cada domicílio se encaixa?
  - Número de Membros
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- Em qual tabela do BF cada domicílio se encaixa?
  - Número de Membros
  - Eligível para quais benefícios
- A frequência da renda reportada e formal per capita
  - RAIS (já tenho)
  - BPC
  - Seguro Desemprego
  - INSS

# Problemas

- Renda per capita do domicílio difere da tabela pessoa em 2% das famílias (Cadunico)
- Alguns indivíduos recebem benefícios que não deveriam (Benefícios)

# Benefício Básico para Dom. com Mais de 77 renda pc (25%!)

Table : Benefício Básico

| idHh | idInd | dta_atual_memb | rf_folha | ben. Basico |
|------|-------|----------------|----------|-------------|
| 137 | 16033997884 | 2014-08-06 | 3/1/15 | 77 |

# Benefícios

Table : Benefício de 0 a 6

| idHh | idInd | dta_atual_memb | dta_nasc_pessoa | idade (Cad) | rf_folha | idade (Folha) | benefit0to6 |
|------|-------|----------------|------------------|-------------|----------|----------------|-------------|
| 389059 | 16052941988 | 2015-02-02 | 1995-04-04 | 19 | 2/1 | 19 | 35 |

# Benefícios de 7 a 15 (muito novo)

Table : Benefício de 7 a 15

| idHh | idInd | dta_atual_memb | dta_nasc_pessoa | idade (Cadunico) | dateFolha | idade (Folha) | benefit7to15 |
|------|-------|----------------|-----------------|------------------|-----------|---------------|--------------|
| 2793 | 16324772234 | 2011-02-01 | 2008-03-12 | 2 | 3/15 | 6 | 35 |

# Benefícios de 7 a 15 (muito velho)

Table : Benefício de 7 a 15

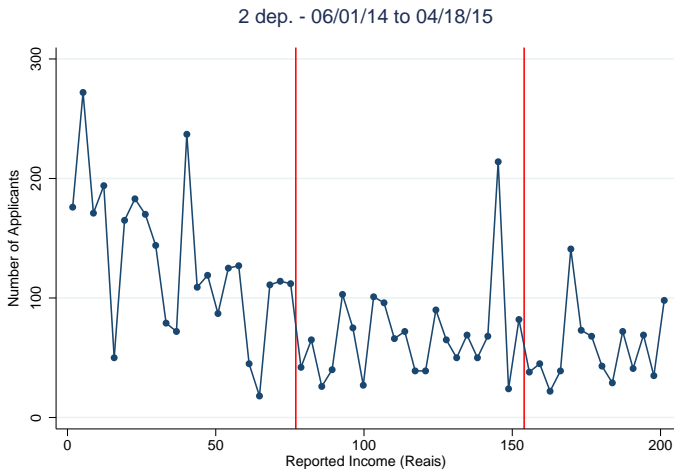| idHh | idInd | dta_nasc_pessoa | dta_atual_memb | idade (Cad) | dateFolha | idade (Folha) | benefit7to15 |
|------|-------|-----------------|----------------|-------------|-----------|---------------|--------------|
| 38833 | 16038377272 | 1999-02-04 | 2015-02-04 | 16 | 2/1/15 | 16 | 35 |

# Últimas Perguntas

- Como identificam nutrizes na base?
- Como identificam grávidas na base?
- Como identificam grávidas na base?

Muito obrigado!

# Reported Income (0 children) - Selected Sample



0 dep. - 06/01/14 to 04/18/15

▸ Back to Households with 0 Children
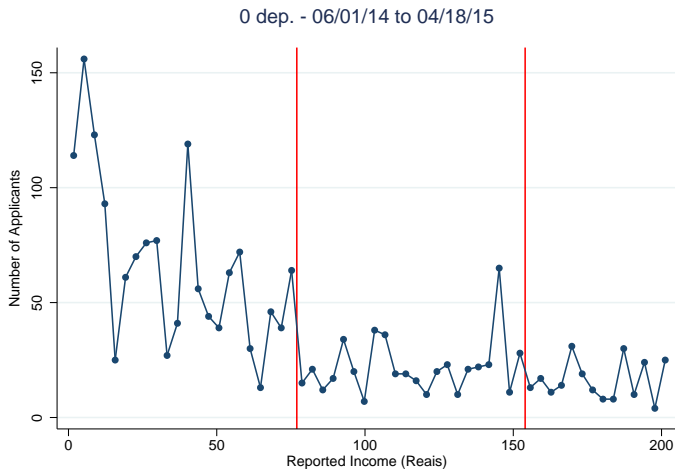
# Reported Income (2 children) - Selected Sample



2 dep. - 06/01/14 to 04/18/15

- $y_{jt}^* = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + \lambda_1 + u_{jt}$,

- $h_{ijt} = \begin{cases} 1 \text{ if } h_{ijt}^* > 0 \\ 0 \text{ otherwise,} \end{cases}$ where $h_{ijt}^* = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt}$,

- $E(h_{ijt}|X_{jt}) = Prob(h_{ijt}^* = 1|X_{jt}) = Prob\left(\epsilon_{ijt} > -f(X_{ijt})\right) = 1 - G\left(f(X_{ijt})\right)$

- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \lambda_2 + \nu_{ijt}$

- $y_{jt}^* = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + \lambda + v_{jt}$

$$\beta = \frac{\partial E(y_{jt}^*|d_{ij} = 1)}{\partial ln(c_i - c_{i-1})_t} = w_i \frac{\partial E(h_{i,j,t}|d_{ij} = 1)}{\partial ln(c_i - c_{i-1})_t}$$

$$\eta_i = \frac{1}{P(h_{ij} = 1)} \beta$$

# Theoretical Framework

$$U^E = w_{\bar{i}+\tilde{i}} + B_{\bar{i}} - p_{\bar{i}}f_{\tilde{i}} - \psi(\bar{i} + \tilde{i}, \tilde{i}, m)$$

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- $w_0 = 0 < w_1 < ... < w_I$: wages in each income level

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## Assumptions

1. *No income effect.*
2. *Expected utility*
3. *Some types cannot work*

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4. *Type $m$ reports either level 0, $i(m) - 1$ or $i(m)$:*

# Cost Minimizing Objective

- $\bar{h}_i$: Proportion of households reporting level $i$ in equilibrium
- $\tilde{h}_i$: Proportion of households producing $i$ but reporting $i - 1$.
- $\tilde{H}_i$: Proportion of households producing $i$ but reporting 0.

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- $c_i = w_i + B_i$: Consumption observed by the government
- $z$: Minimum Consumption Level.

$$\min_{\{B_i\}_{i=0}^{I}} \sum_{i=0}^{I} \{\bar{h}_i B_i - p_{i-1} f_1 \tilde{h}_i - p_0 f_i \tilde{H}_i\}$$

$$st \ c_0 \geq z \ and \ B_i \geq 0 \ \forall i.$$

## Definitions

*Reported income elasticity in the extensive margin:*

$$\bar{\eta}_i \equiv \frac{c_i - c_0}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_0)},$$

*Reported income elasticity in the intensive margin:*

$$\bar{\mathcal{E}}_i \equiv \frac{c_i - c_{i-1}}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_{i-1})}.$$

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### Proposition

Assuming that $\hat{\eta}_i^* \leq \frac{c_i^* - z}{z}$ for $i \geq v$, the cost minimizing schedule $\{B_i^*\}_{i=0}^{I}$ is:

$$B_0^* = z$$

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{\hat{\mathcal{E}}_i^*} \sum_{j=i}^{I} \left( \bar{h}_j^* + \hat{\eta}_i^* \frac{B_j^* - z}{c_j^* - z} \right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for } i = v, v+1, ..., I.$$

Where $\hat{\eta}_i^* \equiv (1 - M_{\bar{n}(i)}) h_i^* \eta_i^* + M_{\bar{n}(i)} \bar{h}_i^* \bar{\eta}_i^*$,
$\hat{\mathcal{E}}_i^* \equiv (1 - \mu_{\bar{m}(i)}) h_i^* \mathcal{E}_i^* + \mu_{\bar{m}(i)} \bar{h}_i^* \bar{\mathcal{E}}_i^*$ and $v$ is the smallest $i$ such that the $B_i^*$ implied by the second bracket is less or equal to zero.

**Problem:** Elasticities under the optimal schedule $\Rightarrow$ Non-recoverable.

### Proposition

*The cost minimizing local reform is a vector of perturbation in the benefit schedule* $\Delta B = -(C_0, .., C_I)$ *where:*

$$C_0 = \begin{cases} \bar{h}_0 - \sum_{i=1}^{v-1} \frac{B_i - B_0}{c_i - c_0} \hat{\eta}_i & \text{if } B_0 > z \\ 0 & \text{if } B_0 = z \end{cases}$$

$$C_i = \bar{h}_i + \frac{B_i - B_0}{c_i - c_0} \hat{\eta}_i + \frac{B_i - B_{i-1}}{c_i - c_{i-1}} \hat{\mathcal{E}}_i - \frac{B_{i+1} - B_i}{c_{i+1} - c_i} \hat{\mathcal{E}}_{i+1} \ 1 \le i \le v$$

$$\text{for} \quad i = 1, ..., v-1$$

$$C_i = \min\left\{ \bar{h}_i - \frac{B_0}{c_i - c_0} \hat{\eta}_i - \frac{B_{i-1}}{c_i - c_{i-1}} \hat{\mathcal{E}}_i, 0 \right\} \ \text{for } i = v, ..., I$$

*$v$: lowest level with zero benefits*

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**Here all the parameters are recoverable from the data.**

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Since there are households that cannot work $\Rightarrow B_0^* = z$

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1. $ME = dB \sum_{j=i}^{I} h_j$.
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4. $FE = -p_{i-1}f_1 de_i - p_0 \sum_{j=i}^{I} f_j dE_j$

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$$BEIM + FEIM = dk_i^I(B_i - B_{i-1}) + de_i[(B_{i-1} - B_i) - p_{i-1}f_1] =$$
$$dk_i^I(B_i - B_{i-1}) + de_i\mu_{\bar{m}(i)}(B_{i-1} - B_i) =$$
$$\left[(1 - \mu_{\bar{m}(i)})dk_i^I + \mu_{\bar{m}(i)}dh_i^I\right](B_i - B_{i-1})$$

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dk_i^I(B_i - B_{i-1}) + de_i \mu_{\bar{m}(i)}(B_{i-1} - B_i) = \\
\left[(1 - \mu_{\bar{m}(i)})dk_i^I + \mu_{\bar{m}(i)}dh_i^I\right](B_i - B_{i-1})
\end{aligned}
$$

$\bar{\eta}_i^* \leq \frac{c_i^* - z}{z}$ for $i > v$ ensures $B_{i-1}^* \geq B_i^*$ and hence $B_i^* = 0$. $\qquad\square$

$$U^E = w_{i+\tilde{i}} + B_i \underbrace{-p_i f_{\tilde{i}}}_{\text{transfer cost}} -\psi(i + \tilde{i}, \underbrace{\tilde{i}}_{\text{util. cost}}, m)$$

- $\bar{m}(i)$ indifferent between reporting $i$ and $i-1$, given real income is $i$
- $\bar{n}(i)$ indifferent between reporting $i$ and $0$, given real income is $i$
- $\mu_{\bar{m}(i)} \equiv \frac{\psi(i,1,\bar{m}) - \psi(i,0,\bar{m})}{p_i f_1 + \psi(i,1,\bar{m}) - \psi(i,0,\bar{m})}$: Share of utility cost in the int. margin
- $M_{\bar{n}(i)} \equiv \frac{\psi(i,i,\bar{n}) - \psi(i,0,\bar{n})}{p_0 f_i + \psi(i,i,\bar{n}) - \psi(i,0,\bar{n})}$: Share of utility cost in the ext. margin

### Lemma

*The wedge between the marginal benefit and marginal fine cost of misreporting (the marginal utility cost) can be written as:*
$(B_{i-1} - B_i) - p_{i-1}f_1 = (B_{i-1} - B_i)\mu_{\bar{m}(i)}$
$(B_0 - B_i) - p_0 f_i = (B_0 - B_i)M_{\bar{n}(i)}$

Proof.

$$w_i + B_{i-1} - p_{i-1}f_1 - \psi(i, 1, \bar{n}) = w_i + B_i - \psi(i, 0, \bar{n})$$
$$\Rightarrow (B_{i-1} - B_i) - p_{i-1}f_1 = \psi(i, 1, \bar{n}) - \psi(i, 0, \bar{n})$$

Multiplying the RHS by $\frac{B_{i-1}-B_i}{p_{i-1}f_1+\psi(i,1,\bar{n})-\psi(i,0,\bar{n})}$, we get the 1st relation.

$$w_i + B_0 - p_0 f_i - \psi(i, i, \bar{n}) = w_i + B_i - \psi(i, 0, \bar{n})$$
$$\Rightarrow (B_0 - B_i) - p_0 f_i = \psi(i, i, \bar{n}) - \psi(i, 0, \bar{n})$$

Multiplying the RHS by $\frac{B_0-B_i}{p_0f_i+\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}$, we get the 1st relation. $\square$

# Welfarist Objective

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## Welfarist Objective

- $\delta^m$: Welfare weight on households of type $m$
- $\tilde{i}$: hidden income so that $i + \tilde{i}$ is the real income level
- $v(m)$: Measure of households with type $m$.
- $R$: Anti-poverty program's budget
- The government solves:

$$\max_{\{B_0, B_1, \dots, B_I\}} \int_M \delta^m u^m(w_{i+\tilde{i}} + B_i, i + \tilde{i}, \tilde{i}) dv(m)$$

$$\text{subject to } \sum_i h_i B_i \le R \text{ and } B_i \ge 0 \ \forall i$$

## Proposition

*Assuming that $\eta_i^* \leq (1 - g_i^*) \frac{c_i^* - B_0^*}{B_0^*}$ for $i > v$ and that there are no income effects, the welfare maximizing schedule $\{B_i^*\}_{i=0}^I$ is:*

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{h_i^* \mathcal{E}^*_i} \sum_{j=i}^I h_j^* \left(1 - g_j^* + \frac{\eta_j^*(B_j^* - B_0^*)}{c_j^* - c_0^*}\right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I$$

$$\text{such that } \sum_{i=0}^I h_i^* B_i^* = R.$$

*Where $g_i = \frac{1}{h_i} \int_{m:i(m)=i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}} + B_i, i + \tilde{i}, \tilde{i})}{\partial c_i} dv(m)$ and $v$ is the smallest $i$ such that the $B_i^*$ implied by the second bracket is less or equal to zero.*

#### Proof.

FOC: $\int_{M_i^*} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i^*, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m) - p\left[h_i^* + \sum_{j=0}^{I} B_j^* \frac{\partial hj^*}{\partial c_i}\right] = 0$

Let $g_i = \frac{1}{ph_i} \int_{M_i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m)$

FOC becomes: $(1 - g_i)h_i^* = -\left[(B_i - B_0)\frac{\partial h_i}{\partial(c_i-c_0)}+\right.$

$(B_i - B_{i-1})\frac{\partial h_i}{\partial(c_i-c_{i-1})} - (B_{i+1} - B_i)\frac{\partial h_{i+1}}{\partial(c_{i+1}-c_i)}\Big]$

Summing over $i$, we get the first equation of the proposition.

$\eta_i^* \leq (1 - g_i^*)\frac{c_i^* - B_0^*}{B_0^*}$ for all $i > v$ guarantees that the incremental benefits are negative for these income levels. $\qquad\square$

# Why the Reported Income is the Sufficient Statistic for the Welfarist Problem?

- The Optimal Anti-Poverty Program Problem has three parts:
  1. Distorting incentives with marginal taxes:
     Workers already maximizing $\Rightarrow$ Second Order Effects
  2. Government Revenue:
     It depends on Reported Income
  3. Targeting low ability people:
     The reported income is the targeting instrument

▸ Back to Proposition

# Efficiency of Cost Minimizing Allocation

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- If the poorest cannot work, caring about his income is equivalent to caring about his utility
- Equivalent to a Rawlsian Social Planner with a budget equal to the minimum cost

Table : Income Maintenance Objectives

| Gov. cares for\ Productive | Everyone | Not Everyone |
| --- | --- | --- |
| Only Poorest | Not Efficient | Efficient |
| Below Poverty Line | Not Efficient | Not Efficient |

### Proposition

*Assuming that households respond only in the extensive margin, the optimal transfer program would be:*

$$B_0^* = z,$$
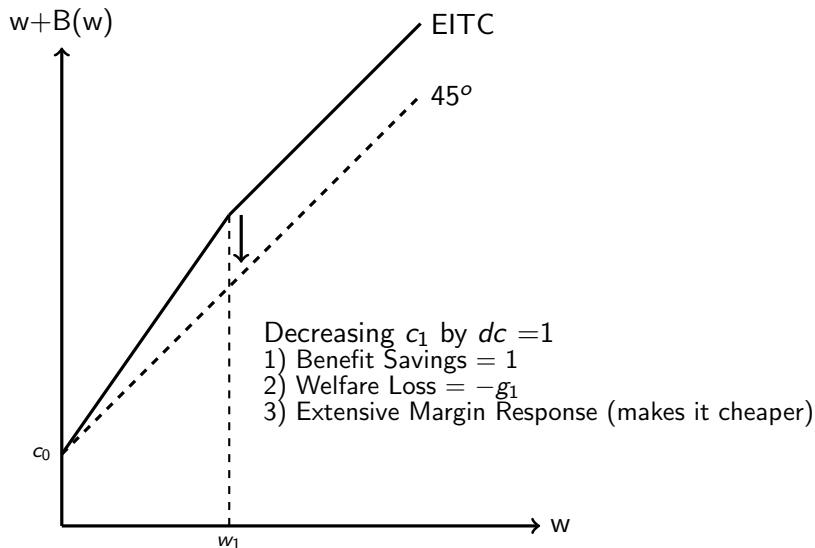$$\frac{B_i^* - B_0^*}{c_i^* - c_0^*} = \frac{1}{\eta_i^*}(g_i^* - 1),$$
$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I.$$

*Where $v$ is the smallest $i$ such that the $B_i^*$ implied by the second bracket is less or equal to zero.*
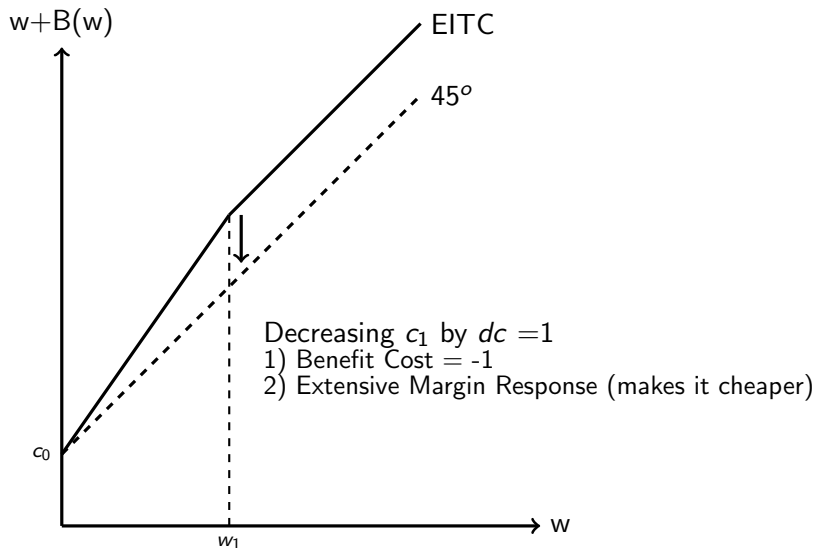
Implications

1. If $g_i^* > 1$ EITC is optimal ($B_1^* > B_0^*$)
2. EITC is never cost minimizing ($g_i^* = 0$ for all $i > 0$)

# Welcome Maximizing

# Cost Minimizing



w+B(w)

EITC

45°

Decreasing $c_1$ by $dc = 1$
1) Benefit Cost = -1
2) Extensive Margin Response (makes it cheaper)

$c_0$

$w_1$

w

### Proof.

Consider $dB_i = ... = dB_I = dB$. The change in the cost of the program due to intensive margin responses in the discrete model is:

$$(B_{i-1} - B_i)dh_i - f_1 de_i = [(1 - \mu_{\bar{m}})dk_i + \mu_{\bar{m}}dh_i](B_{i-1} - B_i) =$$
$$[(1 - \mu_{\bar{m}})\mathcal{E}_i^R k_i + \mu_{\bar{m}}\mathcal{E}_i h_i]\frac{B_i - B_{i-1}}{w_i - w_{i-1}}\frac{dB}{c_i - c_{i-1}}(w_i - w_{i-1})$$

In the continuous model, let $b_i = \frac{B_i - B_{i-1}}{w_i - w_{i-1}}$ and $f_i = \frac{p_{i-1}f_1}{w_i - w_{i-1}}$ be the marginal benefit and expected fines faced by individual with $\bar{y} = w_i$. The same perturbation $db_i = dB/(w_i - w_{i-1})$ will reduce the reported income of individuals reporting $w_i$ by $d\bar{y} = dy - d\tilde{y}$. So the tolal effect on cost is:

$$\{(1 - \mu_{\bar{m}})[wi + \tilde{y}(w_i, m)]e_i + \mu_{\bar{m}}\bar{e}_i\}\frac{db_i}{1 + b_i}h_i b_i$$

Equating the terms multiplying $(1 - \mu_{\bar{m}})$ and $\mu_{\bar{m}}$, we get the relations. $\quad\square$