Optimal Anti-Poverty Programs

An Application to the Brazilian *Bolsa Família*

Juan Rios

January 30, 2016

Motivation

- Cash transfer programs are common (Brazil, Chile, Mexico etc)
- *Bolsa Família* is the largest Conditional Cash Transfer program in the world
  - 30 million beneficiaries as of March 2015
  - Individuals below annual income of US$ 528

The Research Question

- What is the benefit schedule that minimizes the cost of the program given a minimum consumption level?
    - Labor supply responses
    - Mis-reporting responses
- In this talk:
    - What are the elasticities of reported and real income?

Link to Literature

Outline

Outline

Outline

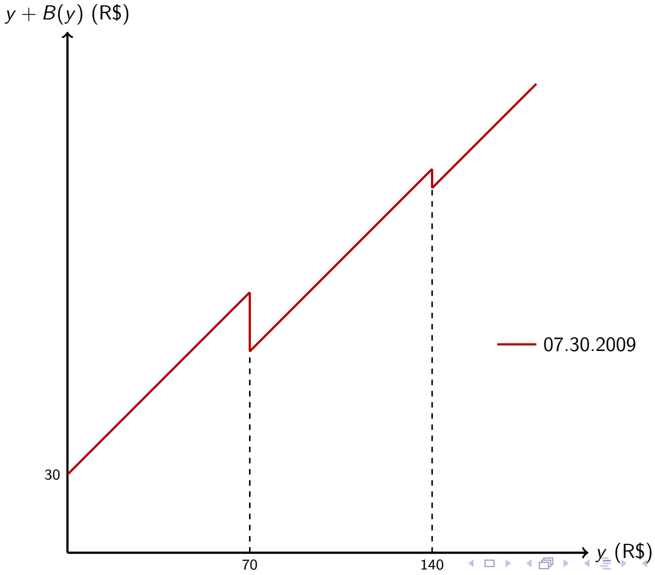
1. Institutional Background
2. Data
3. Elasticities Estimation

Outline

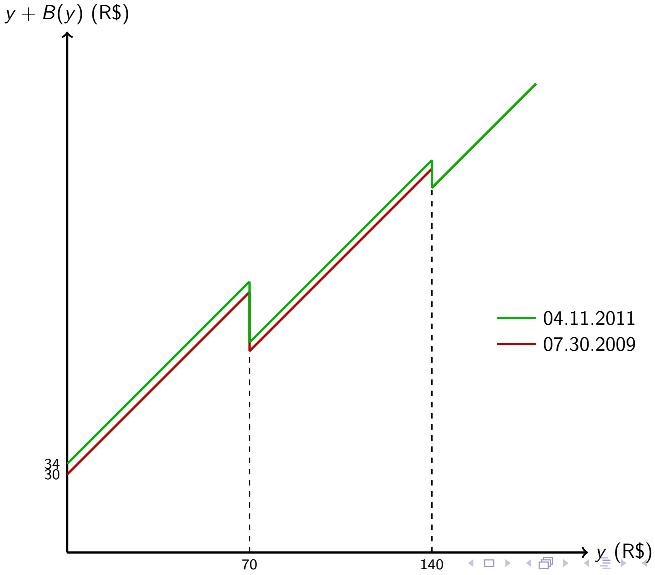
The *Bolsa Família* Program

- Benefits based on:
  - Household income per capita
  - Household Composition
- Information is collected in program's offices
  - Assets, demographics and income are self-reported to interviewers
  - Interviewers may adjust the reported income
  - Computer calculates the per capita income
- Timing
  - Interviews on any business day
  - Updates at least once every two years
- Audits
  - Take away benefits
  - Vary with the gov. budget
  - Geographical variation

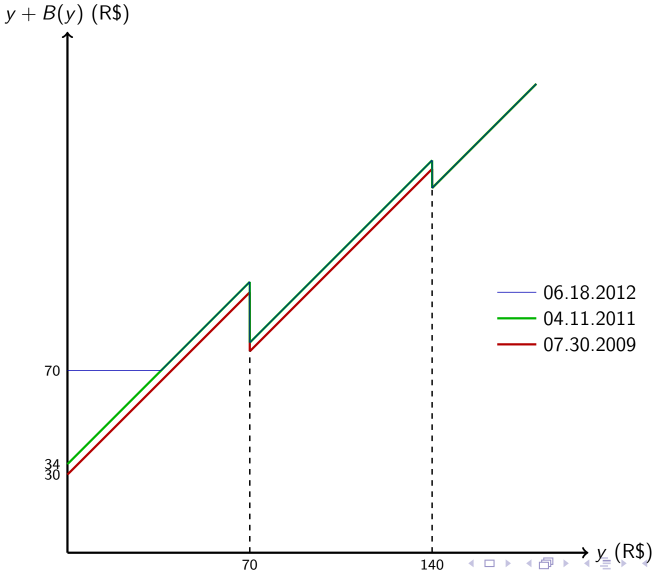
Households with 3 Members and 1 Child



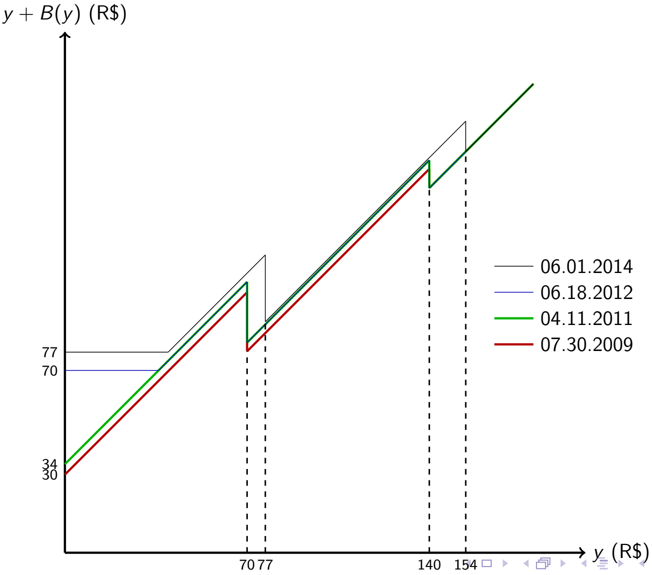
Households with 3 Members and 1 Child



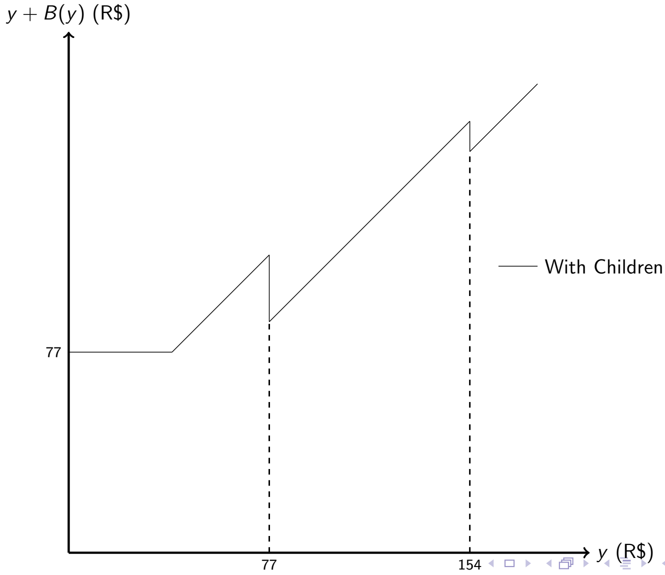
Households with 3 Members and 1 Child



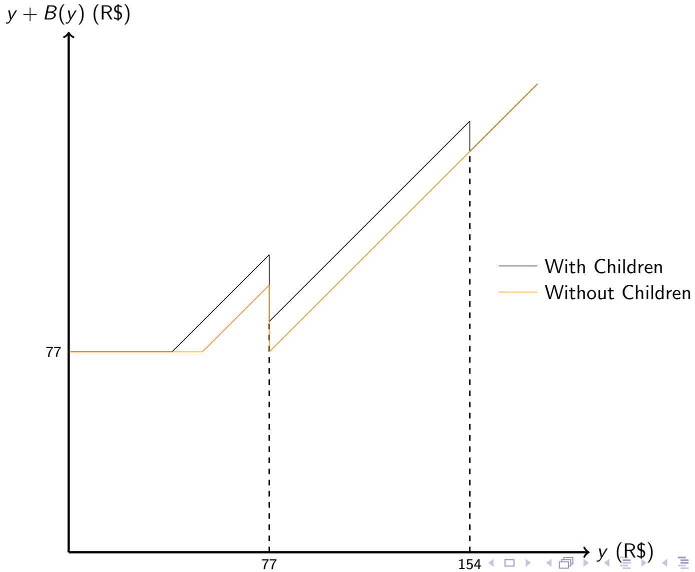
Households with 3 Members and 1 Child



Households with 3 Members after Last Reform



Households with 3 Members after Last Reform



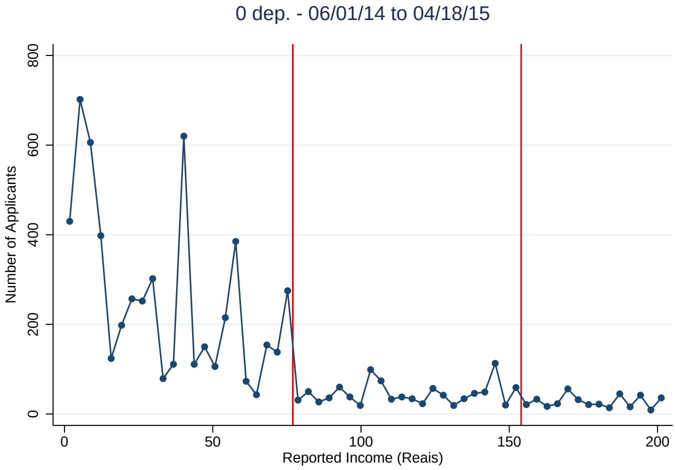
Data

- Bolsa Família Administrative Data for 2011-2015 (only 2015 today)
  - Panel with the self-reported income
  - Family Composition
  - Date of the Income Update
- RAIS
  - Universe of all formal employees in Brazil from 2002-2014
  - Monthly income
  - Individual level identifier

Table : Selection

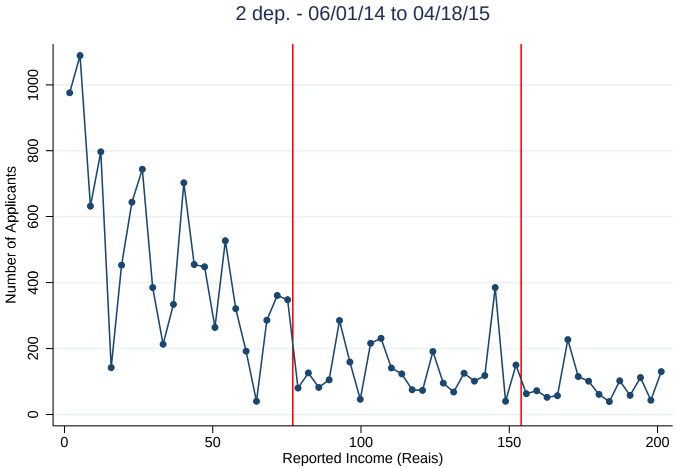
	Only BF	Formal Empl.	Total
Individuals	2560438	455495	3015933
	(84.9%)	(15.1%)	(100%)
Hhs (1 formal empl)	634088	359267	993355
	(63.8%)	(36.2%)	(100%)
Hhs (All formal empl)	746007	247348	993355
	(75.1%)	(24.9%)	(100%)

Reported Income Distribution (0 children)



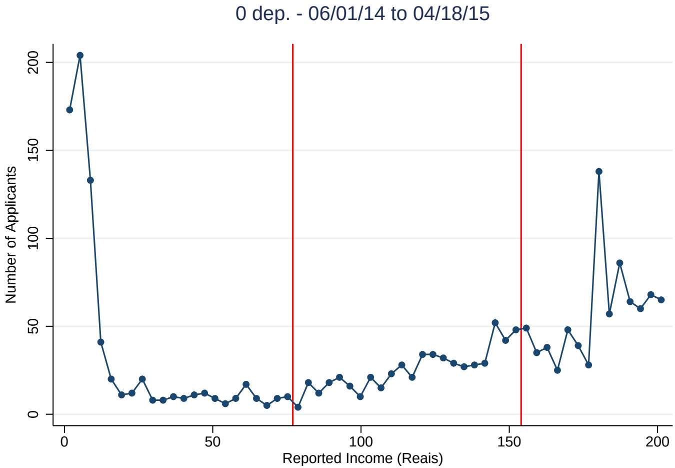
Link Selected Sample

Reported Income Distribution (2 children)

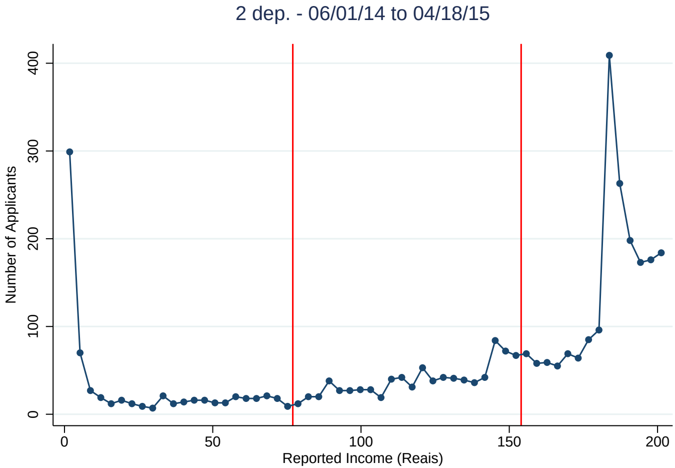


Link to Selected Sample

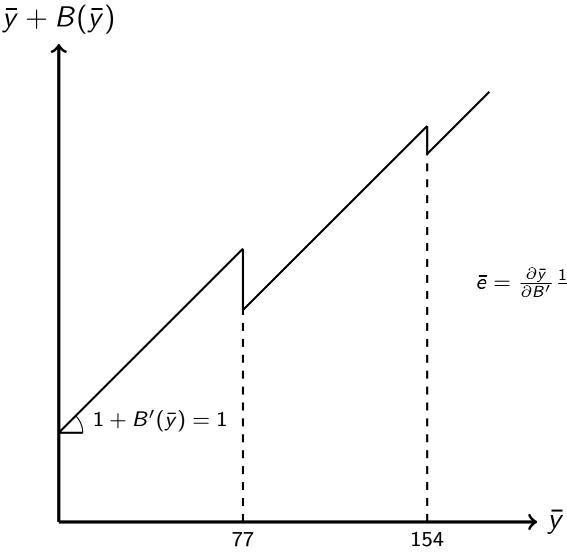
3rd Party Reported Income Distribution (0 children)



3rd Party Reported Income Distribution (2 children)



Program Schedule



$\bar{e} = \frac{\partial \bar{y}}{\partial B'} \frac{1+B'}{\bar{y}}$ is not recoverable

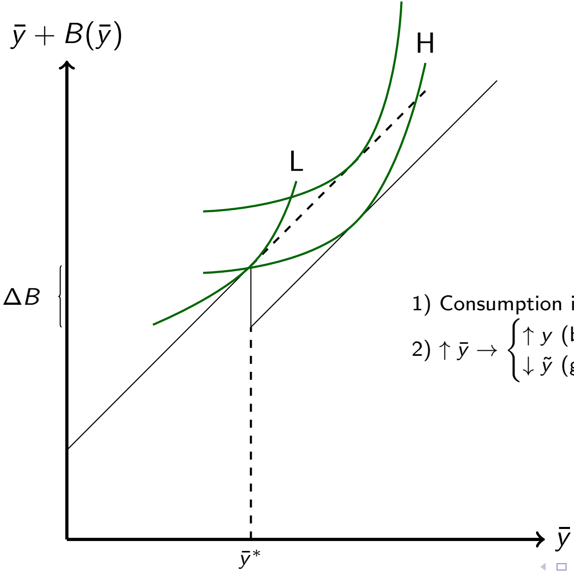
Adjusted Bunching Method

$$U(\bar{y}, \tilde{y}; e, \tilde{e}, n) = \bar{y} + \tilde{y} + B(\bar{y}) - \frac{n}{1+1/e}\left(\frac{\bar{y}+\tilde{y}}{n}\right)^{1+1/e} - \frac{\tilde{y}^{1+1/\tilde{e}}}{1+1/\tilde{e}}$$

pause

- $\bar{y}$: Reported Income
- $\tilde{y}$: Hidden Income
- $y = \bar{y} + \tilde{y}$: Real Income
- n: Ability
- $e = \frac{1+B'}{y}\frac{\partial y}{\partial B'}$: Elasticity of Real Income
- $\tilde{e} = \frac{1}{\tilde{y}}\frac{\partial \tilde{y}}{\partial B'}$: Hidden Income Response
- $\bar{e} = \frac{1+B'}{\bar{y}}\frac{\partial \bar{y}}{\partial B'} = \frac{1+B'}{\bar{y}}\left(\frac{y}{1+B'}e - \tilde{y}\tilde{e}\right)$: Elasticity of Reported Income

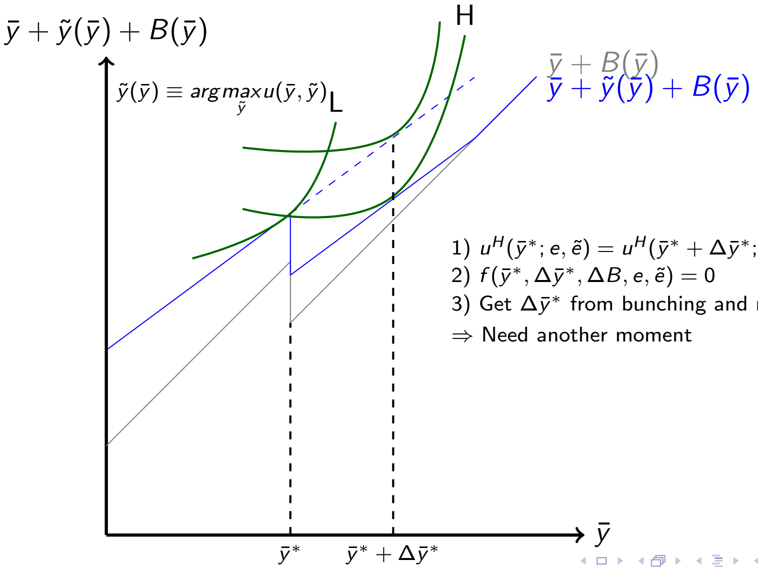
Usual Bunching



1) Consumption is actually $\bar{y} + \tilde{y} + B(\bar{y})$

2) $\uparrow \bar{y} \rightarrow \begin{cases} \uparrow y \text{ (bad)} \\ \downarrow \tilde{y} \text{ (good)} \end{cases}$

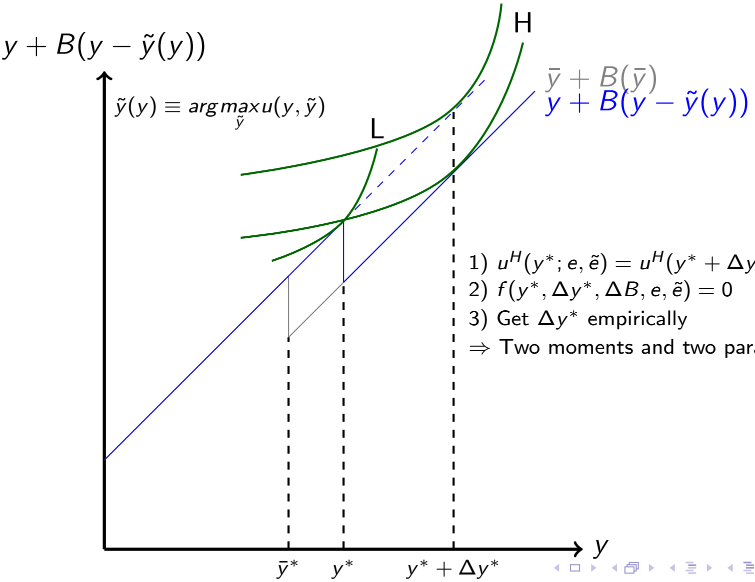
Adjusted Bunching Method (Reported Income)



1) $u^H(\bar{y}^*; e, \tilde{e}) = u^H(\bar{y}^* + \Delta\bar{y}^*; e, \tilde{e})$
2) $f(\bar{y}^*, \Delta\bar{y}^*, \Delta B, e, \tilde{e}) = 0$
3) Get $\Delta\bar{y}^*$ from bunching and missing mass

$\Rightarrow$ Need another moment

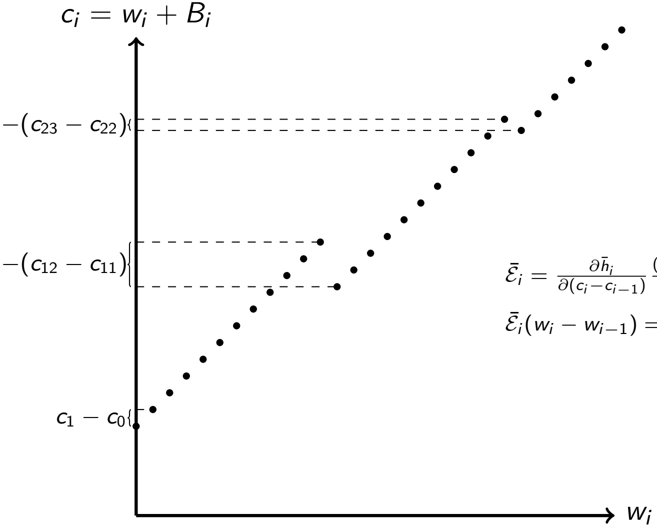
Adjusted Bunching Method (Real Income)



1) $u^H(y^*; e, \tilde{e}) = u^H(y^* + \Delta y^*; e, \tilde{e})$
2) $f(y^*, \Delta y^*, \Delta B, e, \tilde{e}) = 0$
3) Get Δy^* empirically

$\Rightarrow$ Two moments and two parameters

Discretized Program Schedule



$\bar{\mathcal{E}}_i = \frac{\partial \bar{h}_i}{\partial (c_i - c_{i-1})} \frac{(c_i - c_{i-1})}{\bar{h}_i}$ is recoverable

$\bar{\mathcal{E}}_i (w_i - w_{i-1}) = \bar{e}_i w_i$

Empirical Strategy

- y_{jt}: Income reported by household j in period t.

Empirical Strategy

- y_{jt}: Income reported by household j in period t.
- y^*_{jt}: Latent income, so that $y_{jt} = \begin{cases} y^*_{jt} \ \text{if} \ y^*_{jt} > 0 \\ 0 \ \text{if} \ y^*_{jt} \leq 0 \end{cases}$

Empirical Strategy

- y_{jt}: Income reported by household j in period t.
- y^*_{jt}: Latent income, so that $y_{jt} = \begin{cases} y^*_{jt} \text{ } if \text{ } y^*_{jt} > 0 \\ 0 \text{ } if \text{ } y^*_{jt} \leq 0 \end{cases}$
- $\Delta y_j = y_{jt_1} - y_{jt_0}$

Empirical Strategy

- y_{jt}: Income reported by household j in period t.
- y^*_{jt}: Latent income, so that $y_{jt} = \begin{cases} y^*_{jt} \ if \ y^*_{jt} > 0 \\ 0 \ if \ y^*_{jt} \leq 0 \end{cases}$
- $\Delta y_j = y_{jt_1} - y_{jt_0}$
- $\delta \equiv med(\Delta y_j)$: Income bracket width

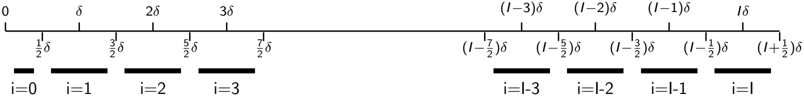
Empirical Strategy

- y_{jt}: Income reported by household j in period t.
- y_{jt}^*: Latent income, so that $y_{jt} = \begin{cases} y_{jt}^* \ if \ y_{jt}^* > 0 \\ 0 \ if \ y_{jt}^* \leq 0 \end{cases}$
- $\Delta y_j = y_{jt_1} - y_{jt_0}$
- $\delta \equiv med(\Delta y_j)$: Income bracket width

- $d_{ij} = \begin{cases} 1 & \text{if hh } j \text{ can report } 0,\ i-1 \text{ or } i, \\ 0 & \text{otherwise.} \end{cases}$ "intrinsic" occupation

Empirical Strategy

- y_{jt}: Income reported by household j in period t.
- y^*_{jt}: Latent income, so that $y_{jt} = \begin{cases} y^*_{jt} \ \ if \ \ y^*_{jt} > 0 \\ 0 \ \ if \ \ y^*_{jt} \leq 0 \end{cases}$
- $\Delta y_j = y_{jt_1} - y_{jt_0}$
- $\delta \equiv med(\Delta y_j)$: Income bracket width



- $d_{ij} = \begin{cases} 1 & \text{if hh } j \text{ can report } 0,\ i-1 \text{ or } i, \\ 0 & \text{otherwise.} \end{cases}$ "intrinsic" occupation
- $h_{ijt} = \begin{cases} 1 & \text{hh } j \text{ reports } i \text{ in } t, \\ 0 & \text{otherwise.} \end{cases}$ "equilibrium" occupation

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt}$,
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt}$,

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$

Identifying $\eta_i = \frac{\partial h_i}{\partial (c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial (c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where \;\; h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$
- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \nu_{ijt}$

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$
- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \nu_{ijt}$
- $y^*_{jt} = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + v_{jt}$

Identifying $\eta_i = \frac{\partial h_i}{\partial (c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial (c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$
- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \nu_{ijt}$
- $y^*_{jt} = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + v_{jt}$
- d_{ij} defined empirically as a function of hh's composition, education, location and housing infrastructure. Link to Intrinsic

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

- $y^*_{jt} = \sum_{i=1}^{I} d_{ij}(w_i * h_{i,j,t} + w_{i-1} * h_{i-1,j,t}) + u_{jt},$
- $h_{ijt} = \begin{cases} 1 \text{ if } h^*_{ijt} > 0 \\ 0 \text{ otherwise,} \end{cases} \quad where\ h^*_{ijt} = f(\underbrace{(c_i - c_0), (c_i - c_{i-1})}_{X_{jt}}) + \epsilon_{ijt},$
- $E(h_{ijt}|X_{jt}) = Prob(h^*_{ijt} = 1|X_{jt}) = Prob\Big(\epsilon_{ijt} > -f(X_{ijt})\Big) = 1 - G\Big(f(X_{ijt})\Big)$
- $\Rightarrow h_{ijt} = 1 - G(f(X_{ijt})) + \nu_{ijt}$
- $y^*_{jt} = \sum_{i=1}^{I} \beta_i d_{ij} ln(c_i - c_{i-1})_{jt} + \sum_{i=1}^{I} \gamma_i d_{ij} ln(c_i - c_0)_{jt} + v_{jt}$
- d_{ij} defined empirically as a function of hh's composition, education, location and housing infrastructure. Link to Intrinsic
- $\frac{\beta_i}{w_i} = \eta_i$ and $\frac{\gamma_i}{\delta} = \eta_i$

Identifying $\eta_i = \frac{\partial h_i}{\partial(c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial(c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

Identification Assumption: Labor supply and Mis-reporting decisions are not correlated with unobservables that changed with the program schedule.

Identifying $\eta_i = \frac{\partial h_i}{\partial (c_i - c_0)} \frac{c_i - c_0}{h_i}$ and $\mathcal{E}_i = \frac{\partial h_i}{\partial (c_i - c_{i-1})} \frac{c_i - c_{i-1}}{h_i}$

Identification Assumption: Labor supply and Mis-reporting decisions are not correlated with unobservables that changed with the program schedule.

- $v_{jt} = u_{jt} + \sum_{i=1}^{I} (\beta_i + \gamma_i) \nu_{ijt} * d_{ij}$

Tobit Assumption: $u_{jt} \sim N(0,1)$

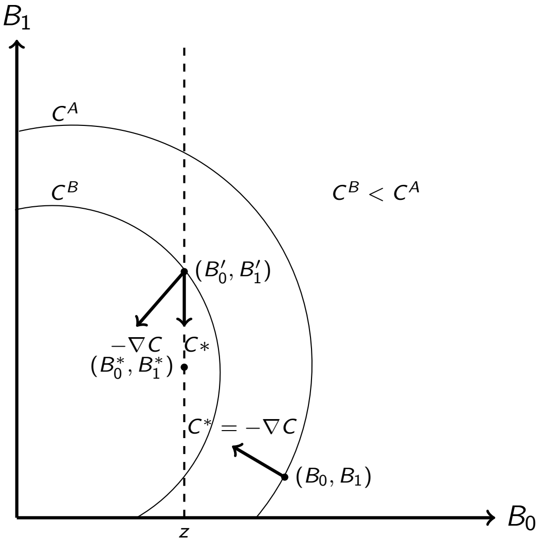
- $L(B) = \prod_{j=1}^{N} \prod_{t=1}^{T} \Big\{ \phi\left(y_{jt} - X_{jt}B + (\beta_i + \gamma_i) G(X_{jt})\right) [1 - G(X_{jt})]$
$\phi\left(y_{jt} - X_{jt}B - (\beta_i + \gamma_i)[1 - G(X_{jt})]\right) G(X_{jt}) \Big\}$

Link to Timing

Optimality of the Anti-Poverty Program

- If there are no fines $\Rightarrow$ elasticities of reported income are sufficient statistics
- If there are fines $\Rightarrow$ elasticities of reported <u>and real</u> income are sufficient statistics
- These are elasticities <u>under the optimal schedule</u>
- The elasticities <u>under the observed schedule</u> are sufficient statistics for the <u>optimal reform</u>

▸ Link to Discrete Model ▸ Link to the Government's Problem ▸ Link to the Optimal Program
▸ Link to the Optimal Reform


B_1
C^A
C^B
$C^B < C^A$
(B_0', B_1')
$-\nabla C$
$C*$
(B_0^*, B_1^*)
$C^* = -\nabla C$
(B_0, B_1)
B_0
z

Recap

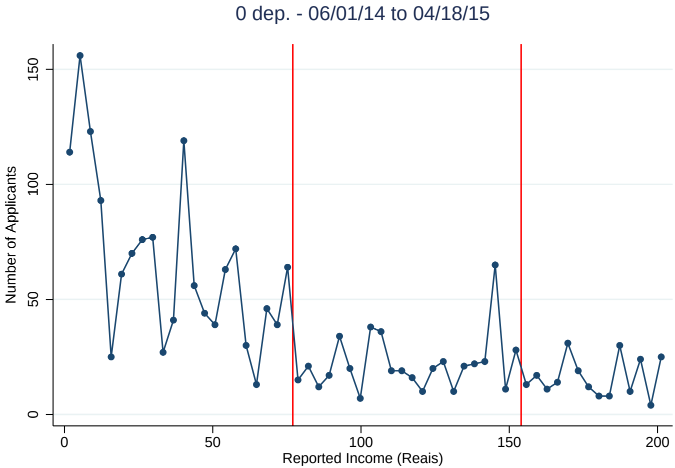
- Recover elasticities of reported and real income from bunching in both distributions
- Those elasticities are the sufficient statistics for the Optimal Anti-Poverty program
- The optimal reform can be written as a function of elasticities under the observed schedule

Related Literature

1. Taxable Income Elasticity Estimation
   - Disentangle real responses from mis-reporting responses
   - Correction for the bunching estimation
2. Optimal Income Maintenance Programs (Kanbur and Stern 1987, Kanbur et al 1994, Besley and Coate 1992 and 1995, Kleven and Kopczuk 2011)
   - Bring this discussion to the data
   - Incorporate extensive margin responses
3. Modern Optimal Tax (Saez 2001 and 2002, Rotschild and Scheuer 2013 and 2015, and Lockwood 2015, Huang and Rios 2015)
   - Framework more relevant for developing countries
   - Optimal reform as a function of elasticities under the observed schedule.
4. Taxation in Developing Countries (Gordon and Li 2009, Best et al 2014, Pomeranz 2013, Naritomi 2015, Bachas and Soto 2015,)
   - Focus here on cash-transfer programs (negative taxes).

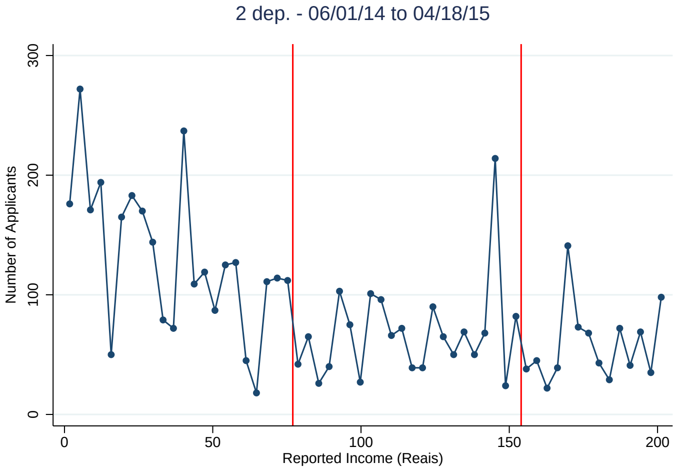
Back to Research Question

Reported Income (0 children) - Selected Sample



▸ Back to Households with 0 Children

Reported Income (2 children) - Selected Sample



Back to Complete Sample

Theoretical Framework

$$U^E = w_{\bar{i}+\tilde{i}} + B_{\bar{i}} - p_{\bar{i}} f_{\tilde{i}} - \psi(\bar{i} + \tilde{i}, \tilde{i}, m)$$

- $\bar{i}$: reported income level
- $\tilde{i}$: hidden income level $\Rightarrow \bar{i} + \tilde{i}$ real income level
- $w_0 = 0 < w_1 < ... < w_I$: wages in each income level
- $B_0, B_1, ..., B_I$: Benefits for each reported level
- $p_{\bar{i}}$: probability of being audited if reports $\bar{i}$
- $f_{\tilde{i}}$: fine of hiding $\tilde{i}$
- $\psi(\cdot, \cdot, m)$: Labor supply and misreporting costs of types m.

Assumptions

1. *No income effect.*
2. *Expected utility*
3. *Some types cannot work*
4. *Type m reports either level* 0, $i(m) - 1$ *or* $i(m)$*:*

Cost Minimizing Objective

- $\bar{h}_i$: Proportion of households reporting level i in equilibrium
- $\tilde{h}_i$: Proportion of households producing i but reporting $i-1$.
- $\tilde{H}_i$: Proportion of households producing i but reporting 0.
- $c_i = w_i + B_i$: Consumption observed by the government
- z: Minimum Consumption Level.

$$\min_{\{B_i\}_{i=0}^{I}} \sum_{i=0}^{I} \{\bar{h}_i B_i - p_{i-1} f_1 \tilde{h}_i - p_0 f_i \tilde{H}_i\}$$

$$st \ c_0 \geq z \ and \ B_i \geq 0 \ \forall i.$$

Definitions

Reported income elasticity in the extensive margin:

$$\bar{\eta}_i \equiv \frac{c_i - c_0}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_0)},$$

Reported income elasticity in the intensive margin:

$$\bar{\mathcal{E}}_i \equiv \frac{c_i - c_{i-1}}{\bar{h}_i} \frac{\partial \bar{h}_i}{\partial (c_i - c_{i-1})}.$$

h_i: Proportion of households producing i.

Definitions

Real income elasticity in the extensive margin:

$$\eta_i \equiv \frac{c_i - c_0}{h_i} \frac{\partial h_i}{\partial (c_i - c_0)},$$

Real income elasticity in the intensive margin:

$$\mathcal{E}_i \equiv \frac{c_i - c_{i-1}}{h_i} \frac{\partial h_i}{\partial (c_i - c_{i-1})}.$$

Back to Implications

Proposition

Assuming that $\hat{\eta}_i^* \leq \frac{c_i^* - z}{z}$ *for* $i \geq v$*, the cost minimizing schedule* $\{B_i^*\}_{i=0}^{I}$ *is:*

$$B_0^* = z$$

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{\hat{\mathcal{E}}_i^*} \sum_{j=i}^{I} \left(\bar{h}_j^* + \hat{\eta}_i^* \frac{B_j^* - z}{c_j^* - z} \right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for } i = v, v+1, ..., I.$$

Where $\hat{\eta}_i^* \equiv (1 - M_{\bar{n}(i)}) h_i^* \eta_i^* + M_{\bar{n}(i)} \bar{h}_i^* \bar{\eta}_i^*$,
$\hat{\mathcal{E}}_i^* \equiv (1 - \mu_{\bar{m}(i)}) h_i^* \mathcal{E}_i^* + \mu_{\bar{m}(i)} \bar{h}_i^* \bar{\mathcal{E}}_i^*$ *and* v *is the smallest* i *such that the* B_i^* *implied by the second bracket is less or equal to zero.*

Back to Implications | Link to Proof | Link to Lemma | Link to Welf Prob | Link to Efficiency

Problem: Elasticities under the optimal schedule $\Rightarrow$ Non-recoverable.

Proposition

The cost minimizing local reform is a vector of perturbation in the benefit schedule $\Delta B = -(C_0, .., C_I)$ where:

$$C_0 = \begin{cases} \bar{h}_0 - \sum_{i=1}^{v-1} \frac{B_i - B_0}{c_i - c_0} \hat{\eta}_i & \text{if } B_0 > z \\ 0 & \text{if } B_0 = z \end{cases}$$

$$C_i = \bar{h}_i + \frac{B_i - B_0}{c_i - c_0} \hat{\eta}_i + \frac{B_i - B_{i-1}}{c_i - c_{i-1}} \hat{\mathcal{E}}_i - \frac{B_{i+1} - B_i}{c_{i+1} - c_i} \hat{\mathcal{E}}_{i+1} \; 1 \le i \le v$$

$$\text{for} \quad i = 1, ..., v-1$$

$$C_i = min\left\{ \bar{h}_i - \frac{B_0}{c_i - c_0} \hat{\eta}_i - \frac{B_{i-1}}{c_i - c_{i-1}} \hat{\mathcal{E}}_i, 0 \right\} \; \text{for } i = v, ..., I$$

v: lowest level with zero benefits

Here all the parameters are recoverable from the data.

Back to Implications

Proof.

Since there are households that cannot work $\Rightarrow B_0^* = z$

Consider the perturbation at the optimum $dB_i = dB_{i+1} = ... = dB_I = dB$.

1. $ME = dB \sum_{j=i}^{I} h_j$.
2. $BEIM = dh_i^{int}(B_i - B_{i-1}) = (dk_i^{int} - de_i)(B_i - B_{i-1})$
3. $BEEM = \sum_{j=i}^{I} dh_j^{ext}(B_j - B_0) = \sum_{j=i}^{I}(dk_j^{ext} - dE_j)(B_j - B_0)$
4. $FE = -p_{i-1} f_1 de_i - p_0 \sum_{j=i}^{I} f_j dE_j$

At the optimum: $ME + BEIM + BEEM + FE = 0$.

$$\begin{aligned} BEIM + FEIM = dk_i^I(B_i - B_{i-1}) + de_i[(B_{i-1} - B_i) - p_{i-1}f_1] = \\ dk_i^I(B_i - B_{i-1}) + de_i \mu_{\bar{m}(i)}(B_{i-1} - B_i) = \\ \left[(1 - \mu_{\bar{m}(i)})dk_i^I + \mu_{\bar{m}(i)} dh_i^I\right](B_i - B_{i-1}) \end{aligned}$$

$\bar{\eta}_i^* \leq \frac{c_i^* - z}{z}$ for $i > v$ ensures $B_{i-1}^* \geq B_i^*$ and hence $B_i^* = 0$. □

Back to Proposition

$$U^E = w_{i+\tilde{i}} + B_i \underbrace{-p_i f_{\tilde{i}}}_{transfer\ \ cost} - \psi(i+\tilde{i}, \underbrace{\tilde{i}}_{util.\ cost}, m)$$

- $\bar{m}(i)$ indifferent between reporting i and $i-1$, given real income is i
- $\bar{n}(i)$ indifferent between reporting i and 0, given real income is i
- $\mu_{\bar{m}(i)} \equiv \frac{\psi(i,1,\bar{m})-\psi(i,0,\bar{m})}{p_i f_1+\psi(i,1,\bar{m})-\psi(i,0,\bar{m})}$: Share of utility cost in the int. margin
- $M_{\bar{n}(i)} \equiv \frac{\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}{p_0 f_i+\psi(i,i,\bar{n})-\psi(i,0,\bar{n})}$: Share of utility cost in the ext. margin

Lemma

The wedge between the marginal benefit and marginal fine cost of misreporting (the marginal utility cost) can be written as:

$(B_{i-1} - B_i) - p_{i-1} f_1 = (B_{i-1} - B_i)\mu_{\bar{m}(i)}$

$(B_0 - B_i) - p_0 f_i = (B_0 - B_i) M_{\bar{n}(i)}$

Proof.

$$w_i + B_{i-1} - p_{i-1} f_1 - \psi(i, 1, \bar{n}) = w_i + B_i - \psi(i, 0, \bar{n})$$
$$\Rightarrow (B_{i-1} - B_i) - p_{i-1} f_1 = \psi(i, 1, \bar{n}) - \psi(i, 0, \bar{n})$$

Multiplying the RHS by $\frac{B_{i-1} - B_i}{p_{i-1} f_1 + \psi(i,1,\bar{n}) - \psi(i,0,\bar{n})}$, we get the 1st relation.

$$w_i + B_0 - p_0 f_i - \psi(i, i, \bar{n}) = w_i + B_i - \psi(i, 0, \bar{n})$$
$$\Rightarrow (B_0 - B_i) - p_0 f_i = \psi(i, i, \bar{n}) - \psi(i, 0, \bar{n})$$

Multiplying the RHS by $\frac{B_0 - B_i}{p_0 f_i + \psi(i,i,\bar{n}) - \psi(i,0,\bar{n})}$, we get the 1st relation. □

Back to Proposition

Welfarist Objective

- δ^m: Welfare weight on households of type m
- $\tilde{i}$: hidden income so that $i + \tilde{i}$ is the real income level
- $v(m)$: Measure of households with type m.
- R: Anti-poverty program's budget
- The government solves:

$$\max_{\{B_0, B_1, \ldots, B_I\}} \int_M \delta^m u^m(w_{i+\tilde{i}} + B_i, i + \tilde{i}, \tilde{i}) dv(m)$$

$$\text{subject to } \sum_i h_i B_i \leq R \text{ and } B_i \geq 0 \; \forall i$$

Proposition

Assuming that $\eta_i^* \leq (1-g_i^*)\frac{c_i^*-B_0^*}{B_0^*}$ *for* $i > v$ *and that there are no income effects, the welfare maximizing schedule* $\{B_i^*\}_{i=0}^{I}$ *is:*

$$\frac{B_i^* - B_{i-1}^*}{c_i^* - c_{i-1}^*} = -\frac{1}{h_i^* \mathcal{E}^*{}_i} \sum_{j=i}^{I} h_j^* \left(1 - g_j^* + \frac{\eta_j^*(B_j^* - B_0^*)}{c_j^* - c_0^*}\right) \text{ for } i = 1, ..., v-1$$

$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I$$

$$\text{such that } \sum_{i=0}^{I} h_i^* B_i^* = R.$$

Where $g_i = \frac{1}{h_i}\int_{m:i(m)=i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m)$ *and* v *is the smallest* i *such that the* B_i^* *implied by the second bracket is less or equal to zero.*

Proof.

FOC: $\int_{M_i^*} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i^*, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m) - p\left[h_i^* + \sum_{j=0}^{I} B_j^* \frac{\partial hj^*}{\partial c_i}\right] = 0$

Let $g_i = \frac{1}{ph_i} \int_{M_i} \delta^m \frac{\partial u^m(w_{i+\tilde{i}}+B_i, i+\tilde{i}, \tilde{i})}{\partial c_i} dv(m)$

FOC becomes: $(1-g_i)h_i^* = -\Big[(B_i - B_0)\frac{\partial h_i}{\partial(c_i - c_0)} +$
$(B_i - B_{i-1})\frac{\partial h_i}{\partial(c_i - c_{i-1})} - (B_{i+1} - B_i)\frac{\partial h_{i+1}}{\partial(c_{i+1} - c_i)}\Big]$

Summing over i, we get the first equation of the proposition.

$\eta_i^* \leq (1-g_i^*)\frac{c_i^* - B_0^*}{B_0^*}$ for all $i > v$ guarantees that the incremental benefits are negative for these income levels. $\square$

Why the Reported Income is the Sufficient Statistic for the Welfarist Problem?

- The Optimal Anti-Poverty Program Problem has three parts:
  1. Distorting incentives with marginal taxes:
     Workers already maximizing $\Rightarrow$ Second Order Effects
  2. Government Revenue:
     It depends on Reported Income
  3. Targeting low ability people:
     The reported income is the targeting instrument

Back to Proposition

Efficiency of Cost Minimizing Allocation

- The objective function is concerned with income and not welfare
- If the poorest cannot work, caring about his income is equivalent to caring about his utility
- Equivalent to a Rawlsian Social Planner with a budget equal to the minimum cost

Table : Income Maintenance Objectives

Gov. cares for\ Productive	Everyone	Not Everyone
Only Poorest	Not Efficient	Efficient
Below Poverty Line	Not Efficient	Not Efficient

Back to Proposition

Proposition

Assuming that households respond only in the extensive margin, the optimal transfer program would be:

$$B_0^* = z,$$

$$\frac{B_i^* - B_0^*}{c_i^* - c_0^*} = \frac{1}{\eta_i^*}(g_i^* - 1),$$

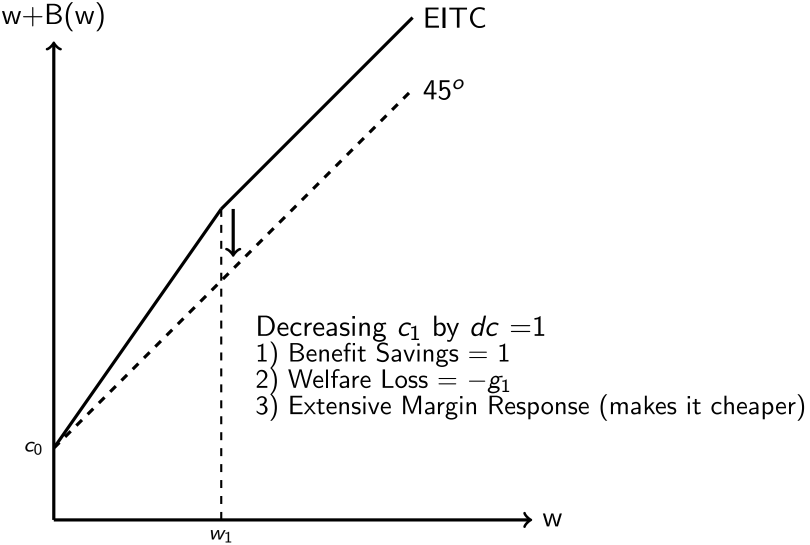
$$B_i^* = 0 \text{ for all } i = v, v+1, ..., I.$$

Where v is the smallest i such that the B_i^* implied by the second bracket is less or equal to zero.

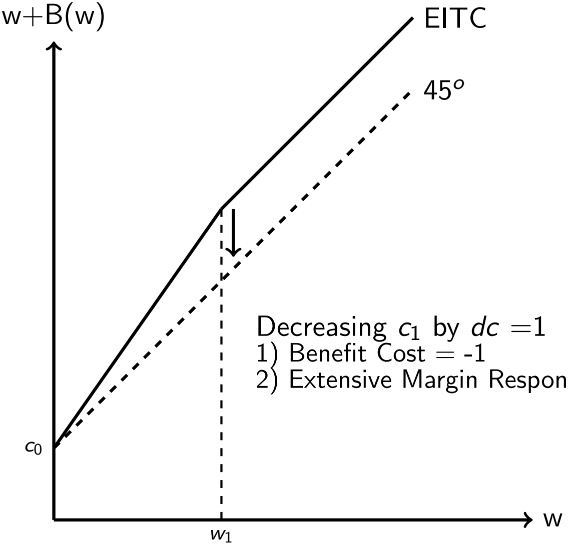
Implications

1. If $g_i^* > 1$ EITC is optimal $(B_1^* > B_0^*)$
2. EITC is never cost minimizing $(g_i^* = 0 \text{ for all } i > 0)$

Welfare Maximizing



Cost Minimizing



Decreasing c_1 by dc =1
1) Benefit Cost = -1
2) Extensive Margin Response (makes it cheaper)

Proof.

Consider $dB_i = ... = dB_I = dB$. The change in the cost of the program due to intensive margin responses in the discrete model is:

$$(B_{i-1} - B_i)dh_i - f_1 de_i = [(1-\mu_{\bar{m}})dk_i + \mu_{\bar{m}} dh_i](B_{i-1} - B_i) =$$
$$[(1-\mu_{\bar{m}})\mathcal{E}_i^R k_i + \mu_{\bar{m}}\mathcal{E}_i h_i]\frac{B_i - B_{i-1}}{w_i - w_{i-1}}\frac{dB}{c_i - c_{i-1}}(w_i - w_{i-1})$$

In the continuous model, let $b_i = \frac{B_i - B_{i-1}}{w_i - w_{i-1}}$ and $f_i = \frac{p_{i-1} f_1}{w_i - w_{i-1}}$ be the marginal benefit and expected fines faced by individual with $\bar{y} = w_i$. The same perturbation $db_i = dB/(w_i - w_{i-1})$ will reduce the reported income of individuals reporting w_i by $d\bar{y} = dy - d\tilde{y}$. So the tolal effect on cost is:

$$\{(1-\mu_{\bar{m}})[wi + \tilde{y}(w_i, m)]e_i + \mu_{\bar{m}}\bar{e}_i\}\frac{db_i}{1+b_i}h_i b_i$$

Equating the terms multiplying $(1-\mu_{\bar{m}})$ and $\mu_{\bar{m}}$, we get the relations. □

Back to Discretized Schedule