Risk Analysis, Vol. 12, No. 4, 1992

Bayesian Methods for Model Uncertainty Analysis with Application to Future Sea Level Rise

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Received July 1, 1991; revised April 20, 1992

This paper addresses the use of data for identifying and characterizing uncertainties in model parameters and predictions. The Bayesian Monte Carlo method is formally presented and elaborated, and applied to the analysis of the uncertainty in a predictive model for global mean sea level change. The method uses observations of output variables, made with an assumed error structure, to determine a posterior distribution of model outputs. This is used to derive a posterior distribution for the model parameters. Results demonstrate the resolution of the uncertainty that is obtained as a result of the Bayesian analysis and also indicate the key contributors to the uncertainty in the sea level rise model. While the technique is illustrated with a simple, preliminary model, the analysis provides an iterative framework for model refinement. The methodology developed in this paper provides a mechanism for the incorporation of ongoing data collection and research in decision-making for problems involving uncertain environmental change.

KEY WORDS: Bayesian methods; Monte Carlo simulation; uncertainty analysis; sea level rise; global climate.

1. INTRODUCTION

The characterization of uncertainty is critical in the application of predictive models to risk-based decision-making for environmental systems. Uncertainties arise due to (i) the limited scientific understanding of important environmental processes; (ii) the inadequacy of mathematical representations which require simplifications of physical processes and their temporal and spatial aggregation; and (iii) the limited ability to measure model parameters and inputs. A systematic analysis of these uncertainties is needed to determine their impact on model predictions and decisions that might be based upon these predictions.

Perhaps in no other area is the need for effective analysis of uncertainty more evident than in the problem of evaluating the consequences of increasing atmospheric concentrations of radiatively active gases. The major consequence of concern is global warming, with related environmental effects that include changes in local patterns of precipitation, soil moisture, forest and agricultural productivity, and a potential increase in global mean sea level.(1–5) In order to identify an optimum set of responses to sea level change, a full characterization of the uncertainties associated with the predictions of future sea level rise is essential.(6)

Significant progress has been made in the development of methods to identify and quantify the effects of uncertainty in environmental models (e.g., Refs. 7–10). A widely used approach is Monte Carlo uncertainty analysis, which has been used in a variety of applications, including probabilistic risk analysis,(11) water quality modeling,(9) air pollution modeling,(12) ecological modeling,(13) integrated multimedia modeling,(14) and as a general approach for risk and policy analysis.(7,10) In this procedure, the uncertainty in model parameters is represented by specifying a (joint) probability distribution, which is randomly sampled. The model is executed (or

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0272-4332/92/1200-0513$06.50/1

replicated) for each sample, resulting in a simulated distribution for the model output. This distribution characterizes the uncertainty in model output, given the assumed uncertainty in the model parameters, and the assumed model structure.

The selection of input distributions for a Monte Carlo analysis may be based upon an evaluation of estimates from the literature, selected experimental studies, or an analysis of available data-sets, depending on the amount and quality of information available. In practice, the final selection of an input distribution involves a subjective assessment by the analyst. This analysis is akin to choosing a prior distribution in a Bayesian evaluation, and the resulting distribution simulated for the model output may be likewise viewed as a prior distribution. In recent years, it has become recognized that observations of model output may be used to refine or update the results of a traditional Monte Carlo analysis. This approach, termed the Bayesian Monte Carlo method, forms the basis for this study.

The evolution of the Bayesian Monte Carlo method can be traced to the generalized sensitivity method of Hornberger and co-workers.[15–17] In this method, the Monte Carlo replications are divided into two sets: those with model outputs "consistent" with observed data (usually defined as falling within an acceptable range of values); and those with outputs which are inconsistent. The division allows identification of influential model parameters, and also provides the basis for an updated estimate of model uncertainty; only those replications with outputs within the acceptable range are maintained for subsequent analysis. This acceptance–rejection procedure was concurrently demonstrated by O'Neill *et al.*,[13] and has since been applied by Jaffe *et al.*,[18] and Rubin *et al.*[14] The term Bayesian Monte Carlo analysis was, to our knowledge, first used by Dilks *et al.*,[19] where the procedure for reweighting simulation results used in this study was implemented. However, a full development of the procedure within the framework of Bayes' rule was not provided in that paper. Thus, the first objective of this paper is to provide a further derivation of the Bayesian Monte Carlo method in which posterior probabilities are calculated from prior probabilities and a likelihood function for the observed model outputs. The second major objective of the paper is to illustrate the use of the method for evaluating the uncertainty associated with the predictions of future sea level rise, and the role that observed data and research can play in reducing this uncertainty.

The remainder of the paper is organized in the following manner. In the next section, we formally present the Bayesian Monte Carlo method, and then describe its application to a model for global mean sea level change due to a change in the atmospheric concentrations of radiatively active trace gases. Section 4.2. presents the results of the simulation study, and we conclude with a discussion of the results obtained and implications for policy and future work.

2. BAYESIAN METHODOLOGY FOR UNCERTAINTY ANALYSIS

The Bayesian methodology for representing and updating the probability assessment is summarized in Fig. 1. In this approach, the method can be used to evaluate parametric uncertainty with a given model structure, as well as the uncertainty associated with alternative model structures.

A particular model, or a set of models is used to predict the quantities of interest, based on model inputs and parameters. The models are deterministic, so that there is a unique correspondence between a particular set of input parameter values and the results obtained from executing a given model with that set of parameters. However, the model structures and parameters are uncertain. Prior subjective probability distributions are used to represent the uncertainty in the alternative model structures and their parameters, and the models are used in conjunction with these prior distributions to compute a prior distribution of model output (boxes 1 and 2 in Fig. 1). In the case of the Bayesian Monte Carlo method, this procedure is implemented numerically through sampling. The parameter and model output distributions are thereby approximated by probability mass functions synthesized through the sampling procedure.

Observations for the model output constitute the evidence used to update the prior distribution of model output, employing Bayes' rule. A critical step in this process is the formulation of a likelihood function, which is based upon the characteristics of the evidence-generating process. Once a posterior distribution for the output is derived, the one-to-one mapping between model parameters and model output developed in the original set of Monte Carlo replications allows inversion of the models to obtain "derived" posterior probability distributions for the alternative models and their input parameters (box 5). With the Bayesian Monte Carlo method, the posterior distributions are also probability mass functions on the original sample of simulated replications, but with updated probabilities for each replication.

The dashed box in Fig. 1 represents the use of direct information or observations of model parameters. Bayesian methods may be used to update prior distributions

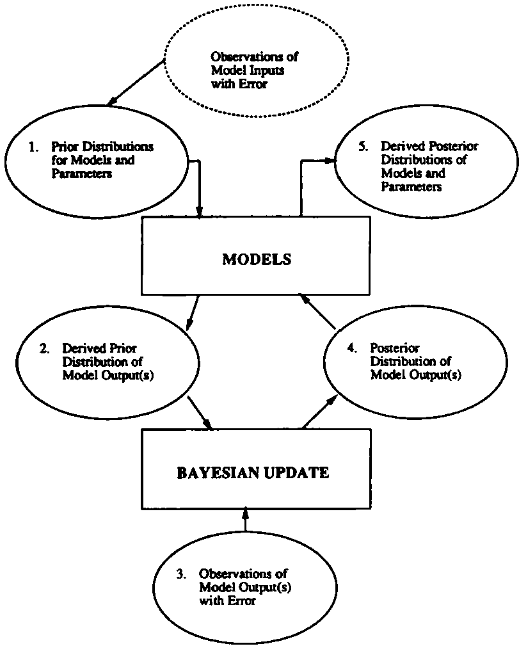

Fig. 1. Bayesian methodology for uncertainty analysis of predictive models.

of input parameters using these direct observations, resulting in posterior distributions for these inputs. However, the updates are made without the use of the model, and in the context of the formulation presented here, these posterior input distributions are treated simply as more refined priors. The use of direct information for input parameters should be considered as part of the overall uncertainty analysis and assessment of the value of alternative sources of information. However, it is not treated in the formal methodology presented herein.

We provide a general derivation of the Bayesian Monte Carlo methodology. Details regarding the formulation of a likelihood function and the actual updating procedure are provided in a subsequent section (§ 4.1.).

2.1. Bayesian Monte Carlo

We begin with a dynamic model with a vector of input parameters, $\vec{\theta}$, whose output is $M(t)$. The model parameters are uncertain, and we represent this uncertainty by a joint probability distribution function, $f_{\vec{\theta}}(\vec{\theta})$. In traditional Monte Carlo analysis, we randomly sample from this distribution, and generate a probability mass function corresponding to the original distribution. With random or proportionally stratified sampling, each sample contributes equally to the numerical approximation of the distribution. Thus, with a sample size of N, the distribution for the model parameters and the model output is approximated by a discrete probability mass function with a probability mass of $1/N$ at the sample outcome corresponding to each replication.

The key step in the development of the Bayesian Monte Carlo method is the recognition that the sampled results in a traditional Monte Carlo analysis correspond to the *prior distribution* for the model parameters and model output. Let p_i be the prior probability mass corresponding to replication i—that is, it is the probability mass for both the vector of model parameters ($\vec{\theta}_i$) and the model output (M_i). Furthermore, let us assume that we obtain evidence E regarding the model output, M, and that the error structure of the evidence gathering procedure (e.g., measurements of M) is known, allowing us to determine the likelihood function, $\mathfrak{L}(E|M)$. Then, the posterior probability mass, p_i' corresponding to replication i, is given by:

$$
\begin{aligned}
p_i' &= p_i'(M_i|E) \\
&\propto \mathfrak{L}(E|M_i)p_i(M_i) \\
&= \frac{\mathfrak{L}(E|M_i)p_i(M_i)}{\sum_{i=1}^{N} \mathfrak{L}(E|M_i)p_i(M_i)}
\end{aligned} \tag{1}
$$

Since the prior probability mass corresponding to each replication is identically $1/N$, the posterior probability mass corresponding to replication i is simply the ratio of the likelihood of the evidence given that replication, divided by the sum of the likelihood of the evidence for all N replications:

$$
p_i' = \frac{\mathfrak{L}(E|M_i)}{\sum_{i=1}^{N} \mathfrak{L}(E|M_i)} \tag{2}
$$

This posterior probability mass applies to both the vector of model parameters ($\vec{\theta}_i$), and the model output, M_i.

The method is readily extended to evaluate the prior and posterior probability of alternative model *structures* as well as input parameters. The prior probability of the model structure is given by the fraction of the replications allocated to that structure in the prior simulation. The posterior probability of the model structure is given by the sum of the likelihoods associated with that structure, divided by the sum of the likelihoods associated with all the replications. The ratio of the posterior probability of one model relative to another may be used to assess the validity of a particular model structure relative to the alternative model, conditional on the evidence.

This Bayesian procedure for model discrimination is not illustrated in this paper, however, an example is provided in Small and Escobar.[20]

The procedure described above is quite straightforward and general. However, as regards the actual implementation, the key issue concerns the formulation of an appropriate likelihood function and its numerical evaluation. This issue is discussed later in Section 4.2, in the context of the application of this method to a model for global mean sea level change.

3. FORMULATION OF THE SEA LEVEL CHANGE MODEL

Global mean sea level change is caused by a number of processes that include changes in the relative distribution of water on the land and the oceans (i.e., changes in land glacial coverage) and the thermal expansion of ocean water. An overview of the important processes and models for projecting future global mean sea level change due to global warming is provided in Chapter 9 of the recent Intergovernmental Panel on Climate Change (IPCC) report.[21] This assessment framework must provide for the estimation of future emissions of greenhouse gases and their consequent atmospheric concentrations, changes in global mean temperature, and the resulting changes in oceanic temperature, thermal expansion, and deglaciation.

A previous study by the U.S. EPA[22] used a globally averaged box-diffusion model for ocean heat transfer coupled to a surface energy balance model, in order to compute thermal expansion due to ocean warming. It has been argued that the box-diffusion model does not adequately capture the essential transport processes in the ocean, particularly the effect of local deep water formation and distributed upwelling elsewhere.[23] An alternative model is the upwelling-diffusion model, where the effect of upwelling is parameterized by an equivalent advective velocity term that modifies the diffusion equation. In this paper, we use an upwelling-diffusion model, similar to that used in a number of transient response studies as well as evaluations of thermal expansion.[24,25] The key model parameters include the temperature sensitivity (equilibrium temperature increase for doubled CO_2), the depth of the ocean mixed layer, the upwelling velocity, and the vertical diffusion coefficient for heat transfer. The input to the model consists of a trajectory of radiative forcing, in watts per meter squared, estimated from historical or projected emissions of radiatively active trace gases. A summary of the model equations is provided in the Appendix.

The computation of global mean sea level change begins with the specification of the model parameters and the model input. The model is run from the year 1900 to the year 2100.[3] The model input is a radiative forcing trajectory, which consists of a historical component (from the base year to the present) and a projected future outcome. We use the forcing history computed by Wigley[26] for this historical component. However, we recognize that there are a number of uncertainties associated with the reconstruction of a forcing history from greenhouse gas emission data, and we allow for modification of the forcing history by an additive factor to account for uncertainty. The projected forcing outcome is computed using the IPCC parameterizations as a function of the changes in the atmospheric concentrations of the radiatively active trace gases [carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and chloroflourocarbons (CFCs)]. These changes are computed using growth rates that are uncertain, and are assumed to have uniform distributions over specified ranges. The bounds for the simulated uniform distributions are specified in Table I. Figures 2a and 3a display the mean and the 5th and 95th percentiles of the historical and projected future radiative forcing trajectories, respectively.

The thermal expansion model for the ocean is solved numerically using a finite difference approximation to the advection-diffusion equation. For each layer, a typical coefficient of thermal expansion, which is a function of temperature, salinity, and pressure,[27,28] is used to find the contribution of that layer to the volume expansion. These thermal expansion coefficients are relatively insensitive to temperature changes, although they are

[3] The choice of 1900 as the base year for the model is somewhat arbitrary; however, the quality of the available sea level data improves considerably after the turn of the century.

Table I. Simulated Prior Distributions of Model Parameters and Inputs

Model parameters	Mean	5th percentile	95th percentile
Temperature sensitivity (°C)	2.5	0.689	4.289
Diffusion coefficient, cm^2/sec	1.5	0.6	2.39
Upwelling velocity, m/year	6.0	4.2	7.67
Mixed layer depth, m	100.0	55.3	144.2
Model inputs (for 1980–2100)			
CO_2 growth rate (% annually)	1.0	0.55	1.45
CH_4 growth rate (% annually)	0.5	0.05	0.95
CFC growth rate (% annually)	0.5	0.05	0.95
N_2O growth rate (% annually)	0.5	0.05	0.95

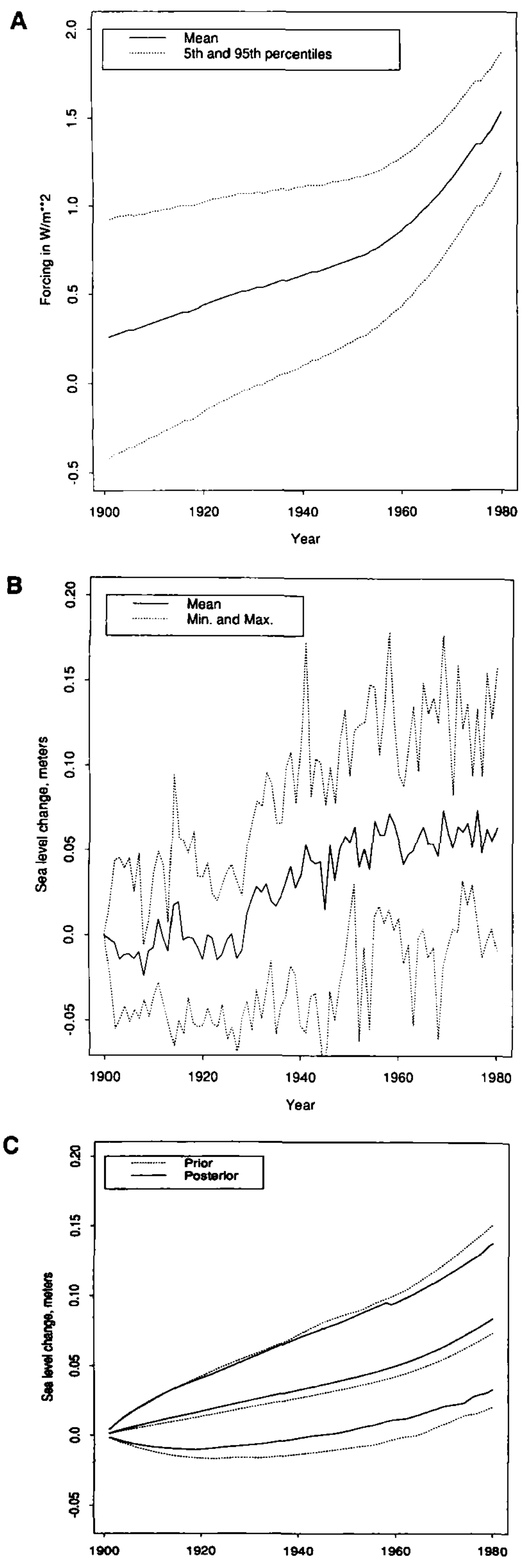

Fig. 2. Application of Bayesian Monte Carlo method to sea level simulations from 1900–1980: (a) historical radiative forcing; (b) relative sea level data from five stations; (c) prior and posterior sea level change simulations (mean, 5th and 95th percentiles shown for 200 replications).

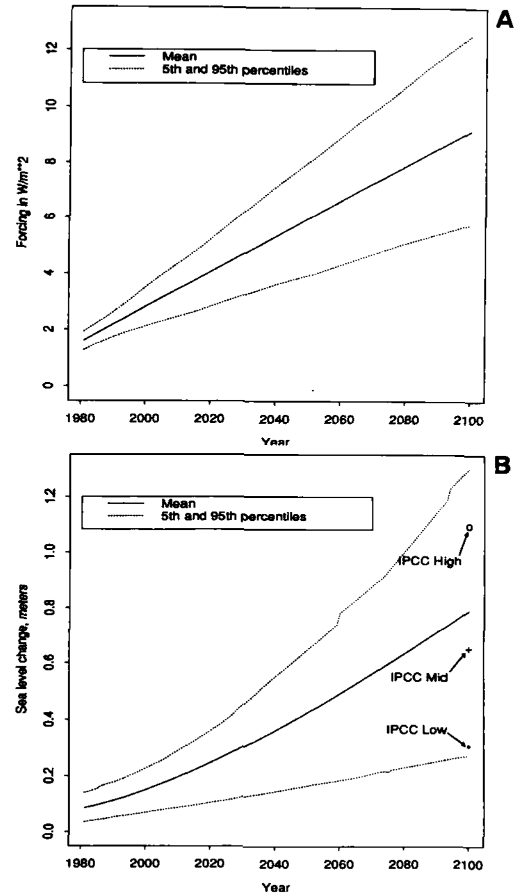

Fig. 3. Sea level projections for 1980–2100: (a) projected radiative forcing to the year 2100; (b) projection for sea level change to the year 2100.

sensitive to the reference temperature itself, as well as the assumed salinity and pressure.

While models for the thermal expansion component of global mean sea level change are relatively well-established, attempts to incorporate other factors such as deglaciation are more recent.[21,29–31] For example, the IPCC model[21] combines mass balance estimates for the continental ice-sheets along with the simple global glacier melt model of Raper *et al.*,[31] to estimate this component. However, it has been noted[21,30] that our understanding of ice dynamics and polar meteorology is still quite incomplete. This paper includes the effects of ice melt by simply multiplying the predicted thermal expansion effect by a factor of two, as was done in the previous EPA study.[22] For comparison, note that values for a similar ratio calculated from Table 9.10 of the IPCC report (p. 276),[21] range from 1.3–1.94, with a best estimate value of 1.8.

3.1. Simulation Approach

In accordance with the Bayesian Monte Carlo methodology described earlier (Section 2.1), input parameters are treated as uncertain, with specified prior distributions. Prior distributions were chosen to be uniform, based on an examination of the reported literature and observed spatial variability, and were deliberately chosen to be very wide in order to account for the range of possible conditions. This procedure attempts to account for the uncertainty inherent in a one-dimensional, lumped parameter representation of a spatially disaggregated three-dimensional system. Table I summarizes the prior distributions of the different parameters.

The Bayesian Monte Carlo approach was implemented by simulating the global sea level rise from the year 1900 to the year 2100. A Latin Hypercube Sampling program[32] was used to obtain sets of parameter vectors from the input prior distributions.

4. ANALYSIS OF SIMULATION RESULTS

In order to implement the Bayesian methodology, an appropriate likelihood function must be formulated based upon an understanding of the evidence-generating process. Specifically, Eq. (1) describes the computation of the posterior probability of a particular replication, and this requires that we evaluate the likelihood of the evidence, given that the "true value" is that given by the particular replication. In the context of sea level change, the model output consists of secular, long-term variation in global mean sea level change relative to a base year, while the evidence consists of observations of relative sea level at different tide gauge stations around the world. It is important to note that we do not observe global mean sea level directly, but rather, location-specific relative sea level. It is therefore necessary to formulate a relationship between the model output and the observations.

4.1. Analysis of Sea Level Data

There is considerable literature dealing with the analysis of local relative sea level records from tide gauge observations, and the inferences regarding global mean sea level change that may be drawn from these records.[33–36] There are several problems in identifying and quantifying a "global" signal from such data, due to the presence of a number of local and global confounding factors that affect relative sea-level. These include glacial isostatic adjustment, tectonic activity, subsidence due to sedimentary loading, an "inverted barometer effect" due to changes in regionally averaged sea level pressure, local changes in water temperature and salinity, local and nonlocal currents, and long-term wind patterns. Barnett[34] identified a number of stations from around the world that: (i) had a relatively long and continuous record; (ii) were representative of other stations in the same region; and (iii) seemed to be relatively free from the effects of many of the local confounding factors.

In the present study, we use five of the stations identified by Barnett. These are: San Francisco (U.S.), Tonoura (Japan), Sydney (Australia), Bombay (India), and Cascais (Portugal). The stations cover the major oceanic regions, with three from the Pacific Ocean (San Francisco, Sydney, Tonoura), one from the Indian Ocean (Bombay), and one from the Atlantic Ocean (Cascais). Annual mean sea level data for these stations were obtained from the archives of the Permanent Service for Mean Sea Level (U.K.), for a common period from 1900–1980. The data were normalized to be zero starting at the base year (1900). Figure 2b displays the pooled mean, minimum, and maximum for the stations over this period. For these stations, we assume that the relationship between the local relative sea level, $R_j(t)$, at a particular station j, and the global mean sea level change, $M(t)$, is given by:

$$R_j(t) = M(t) + D_j(t) \tag{3}$$

The purely local variation, D, is assumed to be linear:

$$D_j(t) = bt + \epsilon(t) \tag{4}$$

where b corresponds to the "true" local trend, and ϵ is an error process that is i.i.d. $N(0, \sigma_\epsilon)$. In addition, we assume that the particular stations chosen are unbiased relative to the global mean sea level such that the expected value of the local trend for each is zero. However, unknown local factors may still be present to result in a nonzero trend. We thus regard b as being normally distributed, with a mean of zero and a variance σ_b^2. We assume that $\sigma_b = 1.0$ mm/year, to represent the general range of uncertainty in local vs. global trends[4].

[4] For reference, recent compilations of trends in relative sea level data ($R_j(t)$) from various sources[21,37] suggest an upward trend of 0.5–3.0 mm/year, with the most likely value between 1.0 and 2.0 mm/year, consistent with station-to-station variation in this trend on the order of 1 mm/year.

4.2. Posterior Analysis

We use the relationship between the evidence and the model output described above, to compute the likelihood function appearing in Eq. (2):

$$\mathcal{L}(E|M_i(t)) = \prod_{j=1}^{NS} \mathcal{L}(R_j(t) \mid M_i(t)) \quad (5)$$

where, NS is the number of relative sea level data records being used (5 in this case), and $\mathcal{L}(R_j(t)|M_i(t))$ is the likelihood of observing the relative sea level data record $R_j(t)$ given the global mean sea level outcome M_i. Equation (5) assumes that the local component of the relative sea level data records at the different stations are independent. Equation (5) is expanded using the relationship between the data and the model output described in Eqs. (3) and (4), so that:

$$\mathcal{L}(R_j(t)|M_i(t)) = \mathcal{L}(d_{ij}(t)) \quad (6)$$

where, $d_{ij}(t) = R_j(t) - M_i(t)$. The actual observed local variation $d_{ij}(t)$ is computed for each station j, and for each model replication i. Since the distributions of both b and ϵ are assumed to be known, we can evaluate the likelihood $\mathcal{L}(d_{ij}(t))$ directly, computed as a product over all the years for which data are available. More precisely, $D_{ij}(t)$ is a linear model in b and ϵ, and the distribution of D_{ij} is determined for each year by the distributions of b and ϵ. Therefore, we can evaluate the likelihood as:

$$\mathcal{L}(d_{ij}) = \int_b \left[\prod_{t=1}^{NYEAR} f_\epsilon(\epsilon(t) = d_{ij}(t) - bt) \right] f_b(b)\, db \quad (7)$$

where $f_\epsilon(\epsilon)$ and $f_b(b)$ are the assumed normal distributions for b and ϵ, and $NYEAR$ is the number of years for which data are available. Although the $\epsilon(t)$ in Eq. (7) are independent, they are conditioned on the global mean sea level outcome $M_i(t)$, and b, which is uncertain.

It is possible to simplify the procedure considerably, by recognizing that the behavior of $d_{ij}(t)$ may be described by performing an ordinary linear regression with d_{ij} as the dependent variable and time as the independent variable, and estimating the regression parameters and their associated standard errors. The regression yields an estimate for the slope, $\hat{b}$, which may be different from zero due to two causes: (i) variation in the true value of b, given by the assumed normal distribution for b; and (ii) sampling variation in the value of $\hat{b}$ relative to b due to the finite sample size of the data-set used for the regression[5]. Thus:

$$\mathcal{L}(d_{ij}) = \int_b f(\hat{b}|b) f_b(b)\, db \quad (8)$$

To evaluate this expression, we use the distribution of b, as well as the sampling distribution of $\hat{b}$ given b. This sampling distriution is a t-distribution, however, given that there are a large number of points for the regression (up to 80 data points, corresponding to 80 years of data), we approximate this distribution by a normal distribution, with a standard deviation specified by the standard error of $\hat{b}$. Note that the regression is performed for each replication i, and for each station j, and the quantities $\hat{b}$ and $\sigma_{\hat{b}}$ are computed each time. The final expression used to compute the likelihood is:

$$\mathcal{L}(E|M_i(t)) = \prod_{j=1}^{NS} \int_{-\infty}^{+\infty} f_{\hat{b}}(\hat{b} - b) f_b(b)\, db \quad (9)$$

The integral in the above equation is computed numerically and used in the Bayesian update in Eq. (2).

Figure 2c displays the mean, 5th, and 95th percentiles for the model replications, obtained using the prior and posterior probability masses corresponding to each replication. The figure demonstrates that there has been a reduction in uncertainty obtained as a result of the updating procedure, as well as a small upward shift in the mean. Figure 3b displays the projected sea level change from 1980–2100, subsequent to the updating, for the uncertain projected radiative forcing shown in Fig. 3a. For comparison, points corresponding to the low, best, and high estimates for sea level change in the year 2100 associated with the "Business as Usual" scenario in the IPCC scientific assessment[21] are also shown. The IPCC predictions correspond closely to the range of predictions provided by the model.

The posterior probability masses for the model replications are equivalent to the posterior probability masses for the parameter vector corresponding to each replication. Summary statistics of the posterior parameter distributions are provided in Table II. We can compare the prior and posterior distributions of the model parameters to identify the parameters for which these distributions are significantly different. In the present study, temperature sensitivity is the only parameter for which there is

[5] This procedure is reasonable since we are primarily interested in the secular, long-term behavior of global mean sea level, and therefore we are looking for similar behavior in the data record.

Table II. Summary Characteristics of the Posterior Distributions of the Model Parameters

Model parameters	Mean	5^{th} percentile	95^{th} percentile
Temperature sensitivity (°C)	2.48	0.96	4.34
Diffusion coefficient, cm^2/sec	1.501	0.63	2.42
Upwelling velocity, m/year	5.98	4.223	7.816
Mixed layer depth, m	100.16	56.8	144.2

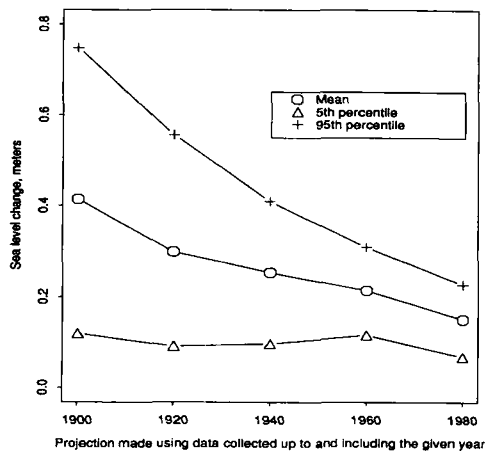

Fig. 4. Projection for sea level change in the year 2000.

a noticeable difference between the prior and posterior distribution, as is seen from a comparison of Tables I and II. This suggests that the sea level data currently available are not sufficient to impart a significant reduction in the uncertainty of the other model parameters.

4.3. Evolution of Projection Uncertainty

The Bayesian updating procedure described so far utilized the entire sea level data-set—that is, from the base year (1900) to the year 1980. In general, it is possible to repeat the procedure for different lengths of the data record. Such an analysis provides insight into the reduction in uncertainty that occurs as data are acquired over time.

We illustrate the effect of using more data in the Bayesian updating procedure in Figure 4. This figure displays the variation in the mean, 5^{th}, and 95^{th} percentile values for the global mean sea level change projected for the year 2000 as a function of the length of the data record used for the updating—that is, for 20, 40, 60, and 80 (all) years of data. While computing the projection for the year 2000, we assume that the historical radiative forcing is also known for the same period as the data. For example, with 20 years of data, the model is forced with the historical radiative forcing from 1900–1920, and in subsequent years the forcing is projected using the same uncertain growth rates for the concentrations of the greenhouse gases that were utilized in Fig. 3a). Figure 4 demonstrates the reduction in uncertainty that is obtained as a result both of using more sea-level data and having a better knowledge of the historical radiative forcing than of future radiative forcing. This provides insight into the way in which uncertainties in projecting future sea level rise (e.g., for the year 2100) may be resolved as more data are collected.

In Section 4.1, we mention that a nonzero variance for b allows for the presence of unknown local processes that produce secular behavior. However, the choice of σ_b is somewhat subjective, and is dependent upon the state of knowledge regarding the precise causes of relative sea level change at a particular station, as well as the accuracy with which we can express the relationship between the global mean sea level and the local relative sea level at that station [Eqs. (3) and (4)].[6] This variable may be affected by further research, and is therefore relevant in a value of information calculation. We performed an analysis to examine the sensitivity of the updating procedure to the choice of the variance in b. Figure 5 displays the projected global mean sea level for the year 2000, for three different choices of σ_b. As σ_b is reduced, the projections for sea level change in the year 2000 become more tightly distributed around the middle to high range of values in the distribution. Scientific research can lead to a better determination of the current value of σ_b, as well as to measurement procedures where the value of σ_b will be smaller.

5. IMPLICATIONS FOR DECISION-MAKING AND FUTURE RESEARCH

The methodology presented in this paper provides a basis for estimating the uncertainty associated with predictions of future sea level change, and determining how this uncertainty may be reduced over time with the use of climate and sea level data. The probability distributions for future sea level can then be used in risk-

[6] Imperfect knowledge of the likelihood function for observed data commonly occurs in the evaluation of environmental systems, even when there is a close match between the model predictions and the variable observed. This is all the more the case when the procedure for gathering evidence is indirect.

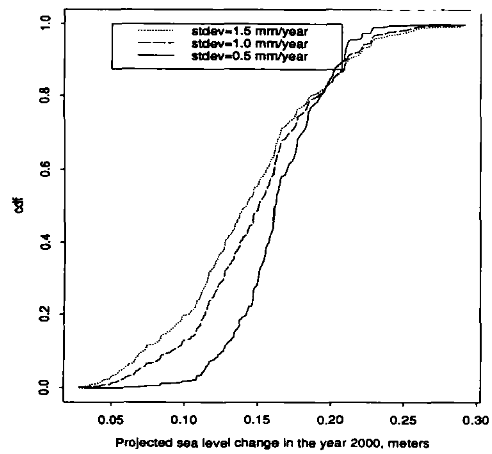

Fig. 5. Bayesian update with different standard deviations for the local trend.

based economic models for evaluating alternative responses to sea level change. From a decision-analytic perspective, the key opportunity afforded by this approach is the ability to ask "what if" questions related to various components of the assessment procedure. For example, the distribution of model predictions may be changed both by a change in the probability distributions representing the uncertainty of input parameters, as well as by a change in the likelihood function of the observed data, representing the ability to measure and estimate the global mean sea level. Changes in the probability distributions of the physical parameters of the models and better resolution of current greenhouse gas emissions can occur as a result of fundamental research and improved data collection; while changes in future emission growth rates can result from the implementation of different control strategies for reducing the emissions of these gases. Potential changes in the likelihood function of the observed data for estimated sea level change depend critically on the quality of the available sea level data. There is reason to believe that a relatively modest investment in data collection can lead to considerable reductions in this data uncertainty.[38,39] Indeed, a number of techniques already exist that may be used to better characterize crustal motions, including the use of very long baselength interferometry in conjunction with the global positioning satellite system, laser ranging or gravity measurements.[38] The methods presented in this paper provide a straightforward way of using this potential reduction in the uncertainty of global sea level estimates to provide better resolved, less uncertain projections of future sea level.

We may regard different research, data collection, and control options as ultimately affecting the uncertainty distribution for projections of future sea level. A "value of information" framework can then be employed to help prioritize future data and research needs. The Bayesian approach allows these assessments to be made sequentially, providing a mechanism for incorporating new information as it becomes available.

6. CONCLUSION

This paper may be viewed as part of an overall effort in the development of techniques for the identification and characterization of uncertainties in predictive models for environmental systems, such as the global climate system. The Bayesian Monte Carlo method is described and employed in the analysis of a predictive model for global mean sea level change. The results demonstrate the potential role of research and ongoing data collection for reducing the uncertainty of future projections. Opportunities for further work include the evaluation of alternative model structures for global climate and sea-level change using the Bayesian Monte Carlo method, and the use of the methodology for evaluating alternative land-use decisions and research priorities for coastal areas.

APPENDIX: SUMMARY OF MODEL EQUATIONS

The radiative forcing is computed from the changes in the atmospheric concentrations of the radiatively active gases using the functional forms provided in the IPCC report.[1]

The energy balance for the ocean mixed layer:

$$ch_m \frac{d\Delta T}{dt} = F(t) - \lambda \frac{\partial \Delta T_t}{\partial z}\Big|_{z=h_m} - \omega(\Delta T - T_e) \quad (10)$$

and the heat transport equation for the diffusive ocean layer:

$$\frac{\partial \Delta T_t}{\partial t} = \kappa_d \frac{\partial^2 \Delta T_t}{\partial z^2} - \omega \frac{\partial \Delta T_t}{\partial z} \quad (11)$$

where h_m is the depth of the ocean mixed layer (meters), c is the ocean heat capacity (J/Kelvin/m), κ_d is the diffusion coefficient (cm^2/sec), ω is the coefficient of the convective-upwelling term (m/year), $F(t)$ is the heat flux entering the ocean (Watts/m^2), and ΔT and ΔT_t are the changes in mixed and diffusive layer temperatures, both relative to the starting year of 1900 (°C).

Boundary conditions for the model consist of the continuity of temperature at the mixed–diffusive layer interface and setting the heat flux to zero at the bottom boundary of the diffusive layer. The initial conditions set the temperature changes to zero at $t = 0$.

ACKNOWLEDGMENTS

Initial development of the methodology in this paper was made possible with support from the U.S. EPA Environmental Research Laboratory, Athens, Georgia (CA-813713). Subsequent development was made with support from the National Science Foundation (PYI, ECE-8552772). The advice and suggestions of Greg McRae, Andrew Solow, and anonymous reviewers are gratefully acknowledged. An early version of this paper was presented at the Engineering Foundation Conference on Risk-Based Decision-making, Santa Barbara, California (October 15–20, 1989). Suggestions made by a number of attendees at this meeting helped to identify additional background references, and to further refine the analysis.

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