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# 1 Taller

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$x(t) = v(t+2) + 2 = \frac{a_0}{2} + 2 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 (t+2)) + b_n \sin(n\omega_0 (t+2))$$

$$\frac{a_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} v(t) dt = \frac{1}{6} \int_{-3}^3 v(t) dt =$$

$$2 \cdot \frac{1}{6} \int_0^3 v(t) dt = 2 \cdot \frac{1}{6} \int_0^3 (3-3t) dt = \frac{3}{3} \left( t - \frac{t^2}{2} \right) \Big|_0^3 = \frac{1}{2}$$

$$\text{Area bajo la curva} = \frac{a_0}{2} = \frac{A}{T} = \frac{2 \cdot 3/2}{6} = \boxed{\frac{1}{2}}$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} v(t) \sin(n\omega_0 t) dt = 0 \rightarrow \text{función impar} = 0$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} v(t) \cos(n\omega_0 t) dt$$

$$2 \cdot \frac{2}{6} \int_0^3 (3-3t) \cos(n\omega_0 t) dt = 2 \int_0^3 \cos(n\omega_0 t) - t \cos(n\omega_0 t) dt$$

$$= 2 \left( \frac{\sin(n\omega_0 t)}{n\omega_0} - \frac{\cos(n\omega_0 t)}{(n\omega_0)^2} - t \frac{\sin(n\omega_0 t)}{n\omega_0} \right) \Big|_0^3 =$$

$$= 2 \left( \frac{\sin(n\pi/3)}{n\pi/3} - \frac{\cos(n\pi/3)}{(n\pi/3)^2} - \frac{\sin(n\pi/3)}{n\pi/3} - 0 + \frac{1}{(n\pi/3)^2} \cdot 0 \right)$$

$$= 2 \frac{1 - \cos\left(\frac{n\pi}{3}\right)}{\left(\frac{n\pi}{3}\right)^2} = 2 \frac{2 \sin^2\left(\frac{n\pi}{6}\right)}{\left(\frac{n\pi}{3}\right)^2} =$$

$$\frac{\sin^2\left(\frac{n\pi}{6}\right)}{\left(\frac{n\pi}{6}\right)^2} = \text{sinc}^2\left(\frac{n\pi}{6}\right)$$

$$-\omega_0 = \frac{2\pi}{T} = \pi/3$$

$$-1 - \cos(2\alpha) = 2\sin^2(\alpha)$$

$$-\frac{\sin(\alpha)}{\alpha} = \text{sinc}(\alpha)$$

Representación Serie Trigonométrica de Fourier

$$v(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \text{sinc}^2\left(\frac{n\pi}{6}\right) \cos\left(\frac{n\pi}{3}t\right)$$

$$x(t) = v(t+2) + 2 = \frac{1}{2} + 2 + \sum_{n=1}^{\infty} \text{sinc}^2\left(\frac{n\pi}{6}\right) \cos\left(\frac{n\pi}{3}(t+2)\right)$$

$$= \frac{5}{2} + \sum_{n=1}^{\infty} \underbrace{\text{sinc}^2\left(\frac{n\pi}{6}\right)}_{a_n} \underbrace{\cos\left(\frac{2n\pi}{3}\right) \cos\left(\frac{n\pi}{3}t\right)}_{b_n}$$

$$- \text{sinc}^2\left(\frac{n\pi}{6}\right) \underbrace{\sin\left(\frac{2n\pi}{3}\right) \sin\left(\frac{n\pi}{3}t\right)}_{b_n}$$

$a_0/2$

Valor medio  $x(t)$

equivalencia trigonométrica -  $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$$\alpha = \frac{2n\pi}{3} \quad \beta = \frac{n\pi}{3}t$$