

Taller T.F. II

$$1.) f(t) = \cos(\omega_0 t) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$= f(\omega) = \int_{-\infty}^{\infty} \cos(\omega_0 t) e^{-i\omega t} dt =$$

$$= \int_{-\infty}^{\infty} \left(\frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2} \right) e^{-i\omega t} dt =$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left(e^{-i(\omega - \omega_0)t} + e^{-i(\omega + \omega_0)t} \right) dt =$$

$$\Rightarrow \hat{f}(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \quad \checkmark$$

$$2.) f(t) = \delta(t) = 1.$$

$$= f(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt =$$

$$= \int_{-\infty}^{\infty} \delta(t) [\cos(\omega t) - j \sin(\omega t)] dt$$

$$= \int_{-\infty}^{\infty} \delta(t) \cos(\omega t) dt - j \int_{-\infty}^{\infty} \delta(t) \sin(\omega t) dt$$

$$= \cos(0) - j \sin(0)$$

$$= 1 \quad \checkmark$$

$$3.) f(t) = e^{-at} u(t)$$

$$= F(\omega) = \int_{-\infty}^{\infty} e^{-at} \delta(\omega) d\omega =$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt =$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt =$$

$$= -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$$

$$= \boxed{\frac{1}{(a+j\omega)}}$$