

1)

a.

$$f(t) = \begin{cases} -At - 2A & -2 \leq t < -3/2 \\ At + A & -3/2 \leq t < 0 \\ -At + A & 0 \leq t < 3/2 \\ At - 2A & 3/2 \leq t \leq 2 \end{cases}$$

$$F[f(t)] = \int_{-2}^{-3/2} (-At - 2A) e^{-i\omega t} dt + \int_{-3/2}^0 (At + A) e^{-i\omega t} dt + \int_0^{3/2} (-At + A) e^{-i\omega t} dt + \int_{3/2}^2 (At - 2A) e^{-i\omega t} dt$$

$$= -A \int_{-2}^{-3/2} t e^{-i\omega t} dt - 2A \int_{-2}^{-3/2} e^{-i\omega t} dt + A \int_{-3/2}^0 t e^{-i\omega t} dt + A \int_{-3/2}^0 e^{-i\omega t} dt - A \int_0^{3/2} t e^{-i\omega t} dt + A \int_0^{3/2} e^{-i\omega t} dt + A \int_{3/2}^2 t e^{-i\omega t} dt - 2A \int_{3/2}^2 e^{-i\omega t} dt$$

$$= -A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} \right) \right] - 2A \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} \right) + A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{-3/2}^0 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-3/2}^0 \right) \right] - A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_0^{3/2} + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_0^{3/2} \right) \right] + A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 \right) \right] - 2A \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 \right)$$

$$\begin{aligned} u &= t & du &= dt \\ dv &= e^{-i\omega t} & v &= -\frac{1}{i\omega} e^{-i\omega t} \end{aligned}$$

$$= -A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} \right) \right] - 2A \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-2}^{-3/2} \right) + A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{-3/2}^0 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-3/2}^0 \right) \right] - A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_0^{3/2} + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_0^{3/2} \right) \right] + A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 \right) \right] - 2A \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{3/2}^2 \right)$$

$$\begin{aligned}
& \frac{1}{iW} \left(-\frac{1}{iW} e^{-iWt} \Big|_{-3/2}^0 \right) + A \left(-\frac{1}{iW} e^{-iWt} \Big|_{-3/2}^0 \right) - A \left[\frac{-t}{iW} e^{-iWt} \Big|_0^{3/2} \right. \\
& + \left. \frac{1}{iW} \left(-\frac{1}{iW} e^{-iWt} \Big|_0^{3/2} \right) \right] + A \left(-\frac{1}{iW} e^{-iWt} \Big|_0^{3/2} \right) + A \left[\frac{-t}{iW} e^{-iWt} \Big|_{3/2}^2 \right. \\
& + \left. \frac{1}{iW} \left(-\frac{1}{iW} e^{-iWt} \Big|_{3/2}^2 \right) \right] - 2A \left(-\frac{1}{iW} e^{-iWt} \Big|_{3/2}^2 \right) \\
& = -A \left[\frac{3}{2iW} e^{\frac{3}{2}Wi} - \frac{2}{iW} e^{2Wi} + \frac{1}{W^2} (e^{\frac{3}{2}Wi} - e^{2Wi}) \right] + \frac{2A}{iW} (e^{\frac{3}{2}Wi} - e^{2Wi}) \\
& + A \left[\frac{-3}{2iW} e^{\frac{3}{2}Wi} + \frac{1}{W^2} (1 - e^{\frac{3}{2}Wi}) \right] - \frac{A}{iW} (1 - e^{\frac{3}{2}Wi}) - A \left[\frac{-3}{2iW} e^{-\frac{3}{2}Wi} \right. \\
& + \left. \frac{1}{W^2} (e^{-\frac{3}{2}Wi} - 1) \right] - \frac{A}{iW} (e^{-\frac{3}{2}Wi} - 1) + A \left[\frac{3}{2iW} e^{-\frac{3}{2}Wi} - \frac{2}{iW} e^{-2Wi} \right. \\
& + \left. \frac{1}{W^2} (e^{-2Wi} - e^{-\frac{3}{2}Wi}) \right] + \frac{2A}{iW} (e^{-2Wi} - e^{-\frac{3}{2}Wi})
\end{aligned}$$

b)

$$f(t) = \begin{cases} At + A & -1 \leq t < 0 \\ -\frac{A}{4}t + A & 0 \leq t \leq 4 \end{cases}$$

$$\begin{aligned}
F[F(t)] &= \int_{-1}^0 (At + A) e^{-iWt} dt + \int_0^4 \left(-\frac{A}{4}t + A\right) e^{-iWt} dt \\
&= A \int_{-1}^0 t e^{-iWt} dt + A \int_{-1}^0 e^{-iWt} dt - \frac{A}{4} \int_0^4 t e^{-iWt} dt + \\
& A \int_0^4 e^{-iWt} dt = A \left[\frac{-t}{iW} e^{-iWt} \Big|_{-1}^0 + \frac{1}{iW} \left(\frac{-1}{iW} e^{-iWt} \Big|_{-1}^0 \right) \right]
\end{aligned}$$

$$+ A \left(-\frac{1}{iW} e^{-iWt} \Big|_{-1}^0 \right) - \frac{A}{4} \left[-\frac{t}{iW} e^{-iWt} \Big|_0^4 + \frac{1}{iW} \left(-\frac{1}{iW} e^{-iWt} \Big|_0^4 \right) \right]$$

$$+ A \left(-\frac{1}{iW} e^{-iWt} \Big|_0^4 \right) = A \left[-\frac{1}{iW} e^{iW} + \frac{1}{W^2} (1 - e^{iW}) \right] +$$

$$-\frac{A}{iW} (1 - e^{Wi}) - \frac{A}{4} \left[-\frac{4}{iW} e^{-4iW} + \frac{1}{W^2} (e^{-4iW} - 1) \right] - \frac{A}{iW} (e^{-4iW} - 1)$$

c) $F(t) = \cos(t)$

$$F[F(t)] = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) e^{-iWt} dt$$

$u = \cos(t), \quad du = -\sin(t) dt$
 $dv = e^{-iWt} dt, \quad v = -\frac{1}{iW} e^{-iWt}$

$$= -\frac{\cos(t)}{iW} e^{-iWt} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{iW} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(t) e^{-iWt} dt$$

$u = \sin(t), \quad du = \cos(t) dt$
 $dv = e^{-iWt} dt, \quad v = -\frac{1}{iW} e^{-iWt}$

$$= -\frac{1}{iW} \left[-\frac{\sin(t)}{iW} e^{-iWt} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{iW} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) e^{-iWt} dt \right]$$

$$= -\frac{\sin(t)}{W^2} e^{-iWt} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{W^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) e^{-iWt} dt$$

$$F[F(t)] = \frac{-1}{W^2 - 1} \left[e^{-\frac{\pi}{2}iW} + e^{\frac{\pi}{2}iW} \right]$$

$$\begin{aligned}
 d) \quad \mathcal{F}[f(t)] &= \int_{-2}^2 \frac{A}{2} t e^{-i\omega t} dt = \frac{A}{2} \int_{-2}^2 t e^{-i\omega t} dt \\
 &= \frac{A}{2} \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_{-2}^2 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_{-2}^2 \right) \right] \\
 &= \frac{A}{2} \left[\frac{-2}{i\omega} e^{2i\omega} - \frac{2}{i\omega} e^{-2i\omega} + \frac{1}{\omega^2} (e^{-2i\omega} - e^{2i\omega}) \right]
 \end{aligned}$$

$$e) \quad f(t) = \begin{cases} \frac{A}{2} t & \text{si } 0 \leq t < 2 \\ -\frac{1}{2} A t + 2A & \text{si } 2 \leq t < 3 \end{cases}$$

$$\begin{aligned}
 \mathcal{F}[f(t)] &= \int_0^2 \frac{A}{2} t e^{-i\omega t} dt + \int_2^3 (-\frac{1}{2} A t + 2A) e^{-i\omega t} dt \\
 &= \frac{A}{2} \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_0^2 + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_0^2 \right) \right] - \frac{1}{2} A \left[\frac{-t}{i\omega} e^{-i\omega t} \Big|_2^3 \right. \\
 &\quad \left. + \frac{1}{i\omega} \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_2^3 \right) \right] + 2A \left(\frac{-1}{i\omega} e^{-i\omega t} \Big|_2^3 \right) \\
 &= \frac{A}{2} \left[\frac{-2}{i\omega} e^{2i\omega} + \frac{1}{\omega^2} (e^{-2i\omega} - 1) \right] - \frac{1}{2} A \left[\frac{2}{i\omega} e^{-2i\omega} - \frac{3}{i\omega} e^{-3i\omega} \right. \\
 &\quad \left. + \frac{1}{\omega^2} (e^{-3i\omega} - e^{-2i\omega}) \right] - \frac{2A}{i\omega} (e^{-3i\omega} - e^{-2i\omega})
 \end{aligned}$$

f) $F(t) = t^2$

$$\mathcal{F}[F(t)] = \int_0^1 t^2 e^{-i\omega t} dt$$

$$\begin{cases} u = t^2 & du = 2t dt \\ dv = e^{-i\omega t} dt & v = -\frac{1}{i\omega} e^{-i\omega t} \end{cases}$$

$$= -\frac{t^2}{i\omega} e^{-i\omega t} \Big|_0^1 + \frac{2}{i\omega} \int_0^1 t e^{-i\omega t} dt$$

$$\begin{cases} u = t & du = dt \\ dv = e^{-i\omega t} dt & v = -\frac{1}{i\omega} e^{-i\omega t} \end{cases}$$

$$= -\frac{1}{i\omega} e^{-i\omega} + \frac{2}{i\omega} \left[-\frac{t}{i\omega} e^{-i\omega t} \Big|_0^1 + \frac{1}{i\omega} \left(-\frac{1}{i\omega} e^{-i\omega t} \Big|_0^1 \right) \right]$$

$$= -\frac{1}{i\omega} e^{-i\omega} + \frac{2}{i\omega} \left[-\frac{1}{i\omega} e^{-i\omega} + \frac{1}{\omega^2} (e^{-i\omega} - 1) \right]$$