Lecture 10: Real Rigidities

Juan Herreño
UCSD

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Terminology

- Three distinct but similarly worded terms
  - Nominal stickiness: It is difficult, or costly to adjust individual **nominal prices**
  - Real rigidities: Responsiveness of **real** prices to a changes in **real** economic activity
  - Nominal rigidities: Reactions of **real** economic activity to changes in **nominal** variables
- Result I: Nominal stickiness + real rigidities → large nominal rigidities
- Result II: Nominal stickiness + no real rigidities → small nominal rigidities
- Result III: No nominal stickiness + real rigidities → no nominal rigidities

Economics

- Costs of price adjustment are certainly present in the data:
  - Physical costs of changing prices
  - Organizational costs of deciding new prices
  - Informational costs of doing so
- However, these costs sound rather “small”.
- Can “small” price adjustment costs generate large aggregate fluctuations?
- Can the price of asparagus explain the Great Depression?

Small menu costs and large inaction

- The economy is characterized by a symmetric steady state

$$P_{it} = P_t$$

- In steady state, the money supply satisfies:

$$M_t = P_t Y_t = 1. \tag{1}$$

- Just a normalization, remember that in the absence of nominal rigidities, nominal variables do not matter.

Small menu costs and large inaction

Assume a profit function exclusive of menu costs of the form

$$X_{it} = X\left(p_{it} - p_t, m_t\right) \tag{2}$$

- In words: The demand curve depends on Y and on relative prices. I will not impose CES.
- Profits are equal to $Profits_{it} = X_{it} - D_{it}k$
- D_{it} is a dummy that takes the value of 1 if the firm resets its price
- k is a menu cost

Price flexibility is a Nash equilibrium if whenever every firm is fully adjusting, firm i decides to fully adjust as well. (in that case, adjusting your price is a best response)

Small menu costs and large inaction

Remember: maintained assumption that every firm adjusts

$$X_{it} = X\left(p_{it} - p_t, m_t\right) \tag{3}$$

- If firm *i* adjusts, then

$$X_{it} = X\left(0, m_t\right)$$

- If firm *i* does not adjust, then

$$X_{it} = X\left(p_{it}^{*} - p_t, m_t\right)$$

- The firm will not adjust its price iff

$$X\left(0, m_t\right) - X\left(p_{it}^{*} - p_t, m_t\right) \leq k$$

Small menu costs and large inaction

- Let me do a second-order optimization of $X\left(p_{it}^{*}-p_t,m_t\right)$ around $p_{it}^{*}=p_t$

$$X\left(p_{it}^{*}-p_t,m_t\right)\approx X\left(0,m_t\right)+X_p\left(0,m_t\right)\left(p_{it}^{*}-p_t\right)+\frac{1}{2}X_{pp}\left(0,m_t\right)\left(p_{it}^{*}-p_t\right)^2$$

- Use the result in the last slide

$$-X_p\left(0,m_t\right)\left(p_{it}^{*}-p_t\right)-\frac{1}{2}X_{pp}\left(0,m_t\right)\left(p_{it}^{*}-p_t\right)^2\leq k$$

- Notice that firm optimality in the steady state implies that $X_p(0,.)=0$.
- Therefore

$$-\frac{1}{2}X_{pp}\left(0,m_t\right)\left(p_{it}^{*}-p_t\right)^2\leq k$$

- Firms will not adjust if this inequality is satisfied

Small menu costs and large inaction

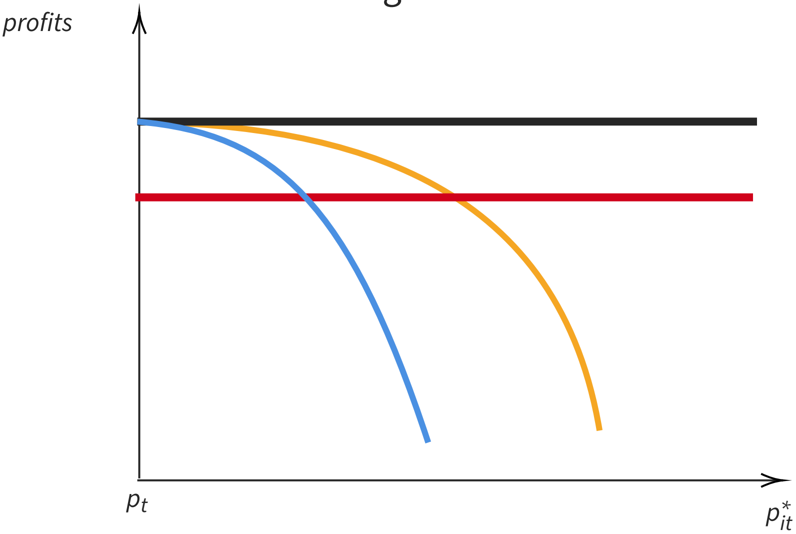
Price flexibility is not a Nash equilibrium iff

$$-\frac{1}{2}X_{pp}\left(0, m_t\right)\left(p_{it}^* - p_t\right)^2 \leq k$$

- There exists a small enough price change (these are log changes)
- so that second-order menu costs induce price rigidity

This was Mankiw (1985), by the way.

Small menu costs and large inaction



Note: The first derivative of the profit funciton at $p^*_{it} = p_t$ is zero. The curvature of the profit function determines the range of inaction

Small menu costs and large inaction

Conclusion

- The losses from price deviations are second order
- Therefore, small menu costs can rule out price flexibility.

Real Rigiditiies

Question: Is an equilibrium where all prices are fixed a Nash Equilibrium?

- Again, a best response idea: If no one is adjusting, do I want to adjust?
- Note: the premise implies that nominal price adjustment (by firm *i*) is a real price adjustment (since the price index is fixed).
- Note: The premise implies that the reaction of real prices is to real aggregate demand (since the price index is fixed).
- Therefore, real rigidities are about the response of real prices to real aggregate demand.

Real Rigiditiies

- Consider shocks to nominal demand
- Profits without any shock are $X(0, 0)$.
- Profits with a shock and without adjustment are $X(0, m_t)$.
- Profits with a shock and own adjustment are $X(p^*_{it}(m_t) - p_t, m_t)$
- Therefore, price rigidity is a Nash equilibrium if

$$X(p^*_{it}(m_t) - p_t, m_t) - X(0, m_t) \leq k$$

Real Rigiditiies

- Optimality requires

$$X_p(p^*_{it}(m_t) - p_t, m_t) = 0$$

- Take derivatives

$$X_{pp}\frac{dp^*_{it}(m_t)}{dm_t} + X_{pm} = 0$$

- Therefore

$$\frac{dp^*_{it}(m_t)}{dm_t} = -\frac{X_{pm}}{X_{pp}}$$

This is our definition of real rigidities. How real prices change after changes in real demand. Call this ρ, and say there are a lot of real rigidities if ρ is small.

Real Rigidities

$$X(p^*_{it}(m_t) - p_t, m_t) - X(0, m_t) \leq k$$

- Take a second-order approximation
- And use $\frac{dp^*_{it}(m_t)}{dm_t} = -\frac{X_{pm}}{X_{pp}}$
- You will find

$$X(p^*_{it}(m_t) - p_t, m_t) - X(0, m_t) \approx -\frac{X^2_{pm}}{2X_{pp}} m^2_t$$

- So Price rigidity is an equilibrium as long as

$$X(p^*_{it}(m_t) - p_t, m_t) - X(0, m_t) \approx -\frac{X^2_{pm}}{2X_{pp}} m^2_t \leq k$$

- Solve for the absolute value shock m_t where price rigidity is a Nash equilibrium

$$m^* = \left(\frac{2k}{\rho X_{pm}} \right)$$

Real Rigidities

$$m^* = \left(\frac{2k}{\rho X_{pm}}\right)$$

- Note: If there are no price rigidities ($k = 0$), nominal rigidities are zero.
- Note: If real rigidities increase (π smaller), nominal rigidities increase.
- Note: These results are very generic

This was Ball and Romer (1990), by the way

Real Rigidities in the NK Model

- Back of the envelope calculation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\beta\lambda)(1-\theta)}{\lambda}(\gamma + \varphi)\tilde{y}_t$$

- If the frequency of price changes is 0.1 per month, a good calibration for $\frac{(1-\beta\lambda)(1-\lambda))}{\lambda} = 0.1$ quarterly
- Using log utility and Frisch of 1 (roughly mid point between micro and macro labor supply elasticities). Then a good number for $\frac{(1-\beta\lambda)(1-\lambda))}{\lambda}(\gamma + \varphi) = 0.2$.
- Compare with the slope in Hazell, Herreño, Nakamura, Steinsson (2022) in your reading list (= 0.008 after adjusting between *u* and *y*)
- Orders of magnitude smaller

The calibrated model implies a steep supply curve. Lack of real rigidities in the model.

Two types of real rigidities

- Two types of rigidities (related to the second and cross-derivatives in the earlier examples)
  - Micro RR. If my marginal cost changes and I can adjust my price freely, by how much do I adjust it?
  - Macro RR: If aggregate real demand changes, by how much real marginal costs adjust?
- Note that micro RR linked to the PE pass-through of MC shocks under flexible prices
- Note that macro real rigidities are connected to the elasticity of aggregate marginal costs

Real Rigidities in the NK Model

More general version of the Calvo model:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_{mc} \tilde{mc}_t$$
$$\tilde{y}_t = \Omega \tilde{mc}_t$$
$$\kappa_{mc} = \varphi \omega$$

- One implicit result in Gali Gertler

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \varphi \omega \Omega \tilde{y}_t$$

- where
- $\varphi = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$ is the effect of staggered Calvo prices. Price rigidity
- $\omega = \frac{\partial p^*_{it}}{\partial mc_t}$ the desired pass-through of marginal costs to prices in the flexible price equilibrium: micro real rigidities
- $\Omega = \frac{\partial mc_t}{\partial \tilde{y}_t}$ is the elasticity of marginal costs when demand changes. Macro real rigidities

Real Rigidities in the NK Model

- $\omega = \frac{\partial p_{it}^*}{\partial mc_t}$ the desired pass-through of marginal costs to prices in the flexible price equilibrium
- In our textbook model $\omega = 1$. Firms have full price through of marginal costs in the flexible price equilibrium

$$\log P_{it} = \log \mu + \log MC_t$$

- But that is not necessary. Imagine instead that demand is not isoelastic (away from CES)

Micro real rigidities

I will introduce now a model with micro real rigidities and re-derive portions of the Calvo model.

Micro real rigidities

Final good producer: Intermediate input varieties assembled into final good using Kimball aggregator:

$$\int_0^1 \Upsilon\left(\frac{y_{it}}{Y_t}\right) di = 1$$

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- Demand function: $\frac{y_{it}}{Y_t} = \Upsilon'^{-1}\left(\frac{p_{it}}{\mathcal{P}_t}\right)$
- Price elasticity of demand: $\frac{\partial \log y_{it}}{\partial \log p_{it}} = -\theta\left(\frac{y_{it}}{Y_t}\right)$

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- $P_t^Y = \int_0^1 p_{it} \frac{y_{it}}{Y_t} di$ ideal price index, $\mathcal{P}_t = \frac{P_t^Y}{D_t}$ subs. price index, $D_t = \int_0^1 \Upsilon'\left(\frac{y_{it}}{Y_t}\right) \frac{y_{it}}{Y_t} di$ "demand index"

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Intermediate input firms:

- Calvo pricing: can reset price with probability 1-λ
- $y_{it} = e^{z_i} l_{it}$

Notations: $s_{it} = \frac{p_{it} y_{it}}{P_t^Y Y_t}$ and $\mathbb{E}_s[X_{it}] = \int_0^1 s_{it} X_{it} di$

Micro real rigidities

Problem: $$\max_{p_{it}} \mathbb{E}_t \left[\sum_{s=0}^{+\infty} \lambda^s \Lambda_{t,t+s} \left[p_{it} y_{it+s} - w_{t+s} e^{-z_i} y_{it+s} \right] \right] \text{ s.t.: } y_{it+s} = \Upsilon'^{-1} \left(\frac{p_{it}}{\mathcal{P}_{t+s}} \right) Y_{t+s}$$

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Optimal reset price:

$$\hat{p}_{it|t}^{new} = (1-\beta\lambda)\mathbb{E}_t \left[\sum_{s=0}^{+\infty} (\beta\lambda)^s \left(\hat{\mu}_{it+s|t}^f + \hat{mc}_{it+s|t} \right) \right] \quad \text{with } \mu_{it}^f = \frac{\theta_{it}}{\theta_{it}-1} \text{ flexible price markup}$$

Micro real rigidities

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$$\hat{\mu}^f_{it+s|t} = -\Gamma_i \theta_i (\hat{p}^{new}_{it|t} - \hat{\mathcal{P}}_{t+s}) \text{ with } \Gamma_i = \frac{\partial \log \mu^f_i}{\partial \log \frac{y_i}{Y}} \text{ markup elasticity}$$

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$$\rho_i = \frac{1}{1+\Gamma_i\theta_i} \text{ flexible price passthrough}$$

Micro real rigidities

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Marginal cost based Phillips curve slope:

$$\frac{\partial \hat{\pi}_t}{\partial(\hat{mc}_t - \hat{P}^Y_t)} = \underbrace{\varphi \mathbb{E}_\lambda[\rho_i]}_{\kappa_{mc}} \text{ with } \varphi \equiv \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$$

Micro real rigidities

Problem: $$\max_{p_{it}} \mathbb{E}_t \left[\sum_{s=0}^{+\infty} \lambda^s \Lambda_{t,t+s} \left[p_{it} y_{it+s} - w_{t+s} e^{-z_i} y_{it+s} \right] \right] \text{ s.t.: } y_{it+s} = \Upsilon'^{-1} \left(\frac{p_{it}}{\mathcal{P}_{t+s}} \right) Y_{t+s}$$

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Micro real rigidities

Marginal cost based Phillips curve slope:

$$\frac{\partial \hat{\pi}_t}{\partial(\hat{mc}_t - \hat{P}_t^Y)} = \underbrace{\varphi \mathbb{E}_s[\rho_i]}_{\kappa_{mc}} \text{ with } \varphi \equiv \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$$

- Notice that if markup elasticities are positive, then pass-throughs are < 1.
- In this case, micro real rigidities dampen the reaction of inflation to marginal costs.
- CES a special case where the markup elasticity $= 0$ so passthroughs $= 1$.

Macro Real Rigidities

- Many sources of macro real rigidities
  - Roundabout production functions
  - Sticky wages
- Generically, any economic mechanism that induces marginal costs to move by less

Macro Real Rigidities

- Imagine a production structure

$$y_{it} = l_{it}^{\phi} x_{it}^{1-\phi}$$

- with x being materials
- Assume that materials are a CES bundle of the product of every other firm
- Therefore the marginal cost of production of the firm is

$$mc_{it} = \left(\frac{W_t}{\phi}\right)^{\phi} \left(\frac{P_t}{1-\phi}\right)^{1-\phi}$$

  - If prices are sticky, then P is sluggish. Marginal costs move by less. We are using products as inputs.
  - If wages are sticky, then W is sluggish. Marginal costs move by less.

  If marginal costs move by less after a change in demand, then prices need to react by less as well.

Evidence?

Table 1: Strategic complementarities: baseline estimates

	OLS		IV		
Dep. var.: Δp_{it}	(1)	(2)	(3)	(4)	(5)
Δmc_{it}	0.348^{***}	0.348^{***}	0.588^{***}	0.650^{***}	0.616^{***}
	(0.040)	(0.041)	(0.094)	(0.112)	(0.103)
Δp_{-it}	0.400^{***}	0.321^{***}	0.549^{***}	0.484^{***}	
	(0.079)	(0.095)	(0.097)	(0.118)	
# obs.	64,823	64,823	64,823	64,823	64,823
Year F.E.	yes	yes	yes	yes	yes
Industry F.E.	no	yes	no	yes	yes
H_0: $\psi + \gamma = 1$	0.747	0.669	1.137	1.133	yes
[p-value]	[0.00]	[0.00]	[0.05]	[0.16]	
Overid J-test χ^2			2.41	0.74	1.44
[p-value]			[0.30]	[0.69]	[0.70]
Weak IV F-test			199.1	154.6	156.3

Source: Amiti, Itskhoki, Konings (2019). Pass-through of 0.64

Evidence?

Dependent variable: $\Delta \ln P^{Y}_{i,t}$

Estimator	OLS	OLS	OLS	OLS	2SLS
Instruments	-	-	-	-	WID, Shea, $\Delta e_{i,t}$
	(1)	(2)	(3)	(4)	(5)
$\Delta \ln Y_{i,t}$	-0.06	0.09	0.13	0.17	0.24
	(0.09)	(0.02)	(0.02)	(0.02)	(0.09)
$\Delta \ln Q_{i,t}$			-0.16	-0.11	-0.16
			(0.03)	(0.03)	(0.08)
$\Delta \ln \mathrm{UVC}_{i,t}$		0.90	0.90	0.89	0.89
		(0.02)	(0.02)	(0.03)	(0.03)
R-squared	0.004	0.869	0.876	0.910	0.908
Fixed Effects	no	no	no	yes	yes
First stage and instrument diagnostics					
F main effect					17.37
Hansen J (p-value)					0.538

Notes: The estimates are based on equation (16). Driscoll-Kraay standard errors are reported in parentheses. Fixed effects include industry fixed effects, time fixed effects, and time fixed effects interacted with industries' lagged foreign sales share ($\sum_{j \in \mathcal{J}^F} s_{j,i,t-1}$). First stage estimates for specification (5) are reported in Appendix Table D1.

Source: Boehm, Pandalai-Nayar (2022). Marginal cost elasticity of 0.24.

New back of the envelope

- Don’t put too much weight on these. Combines data sources, countries, methods, time aggregation.
- Just want to illustrate the importance.
- remember the slope $\kappa = \varphi\omega\Omega$
- Using the textbook calibration: $\kappa \approx 0.2$.
- Including real rigidities: $\kappa \approx 0.1 \times 0.64 \times 0.24 \approx 0.015$