Final Exam

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Instructions

This is an individual exam. You can consult the internet, your notes, papers, books, etc., but you must not communicate about the content of the course or the exam with anyone. You should not use AI tools to generate text to answer your questions.

Please be succinct in your answers. The written answers that we require from you will give you full marks even if you manage to deliver them in one sentence. To deter extreme wordiness, if your answer contains both correct and incorrect statements, the incorrect statements will subtract from correct statements, effectively giving you no credit for the question.

Please upload a single .pdf document with your answers. Make your .pdf readable. When we cannot decipher a piece of text or math, we will assign 0 points.

The exam has a total of 430 points.

1. **Durable Consumption in the Great Recession - The Cash-for-Clunkers Program**

   A consumer derives utility from a bundle formed by both non-durable items (think restaurant meals) denoted by C, and durable goods (think cars), denoted by D. The consumption of the bundle is denoted by M, where we call it M because we are thinking of market goods. The bundle takes a Cobb-Douglas function. Formally

$$M_t = C_t^{\alpha} D_t^{1-\alpha}. \tag{1}$$

The consumer dislikes working. Their utility then depends on their consumption of market goods, and their leisure. This utility is separable and takes a very standard form:

$$u(M_t, L_t) = \frac{M_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\psi}}{1+\psi}. \tag{2}$$

The consumer's objective is to maximize the discounted sum of their per period utility using a discount factor β. Formally

$$\max_{C_t, D_t, L_t} \sum_{t=0}^{\infty} \beta^t u(M_t, L_t), \tag{3}$$

subject to the shape of the utility function 2, the determination of the market bundle 1, and two additional constraints. The first constraint captures the law of motion of the stock of durable goods the consumer has access to as a function of the depreciation rate δ and the investment in new durables X_t. Formally,

$$D_t = (1-\delta)D_{t-1} + X_t. \tag{4}$$

Note that we are assuming that expenditures X made in the present accrue to the present stock of durables. So there is no time to build.

Moreover the consumer needs to respect their budget constraint. The consumer holds financial assets A that pay an interest rate r, and earns labor income that depends on the wage rate w. The price of non-durable consumption is normalized to 1, so every other price has the interpretation of a relative price. In particular,

$$A_{t+1} = A_t(1+r_t) + W_t L_t - P_{D,t} X_t (1-\tau_t) - C_t, \tag{5}$$

where $P_{D,t}$ is the price of the durable good in period t, and τ is a durable subsidy, think of the policy CARS (Car Allowance Rebate System), commonly known as Cash-For-Clunkers program. This program was put in place by the Obama administration during the Great Recession. You should read this short note to get some context: Click here

(a) **10 points:** State Formally the problem of the household. You should just state the optimization problem of the household, noting its objective and its constraints.

(b) **10 points:** State the Langrangian of the problem. Call λ the Lagrange multiplier with respect to the law of motion of the durable stock, and μ the Lagrange multiplier with respect to the budget constraint.

(c) **20 points:** Derive the first order conditions of the household problem.

(d) **20 points:** Iterate forward the first order condition with respect to the durable stock D_{t+1} and use the transversality condition to eliminate the terminal term.

Information on the Reform (applies starting in question e): Imagine an unexpected and transitory CARS program that is proposed, announced, and implemented, all in period t. The program gives owners of durable goods a tax incentive $\tau > 0$ if they do net purchases of durable goods $X_t > 0$. You can assume that before the program and immediately after, the tax subsidy disappears. That is $\tau_\tau = 0 \; \forall \tau \neq t$

(e) **30 points:** In **maximum** one paragraph, state the effect of having two goods (durable and non-durable consumption) in the utility function, and the presence of a financial asset on the arguments of House and Shapiro (2008) regarding the constancy of the marginal benefit of an extra unit of the durable good (capital in their case, durable consumption in this model).

Information you gathered on the reform (applies starting in question f): Imagine that now you receive more information. First you learn that consumers have log utility $u(M_t, L_t) = \log(M_t) - \frac{L_t^{1+\psi}}{1+\psi}$, which you know is the limit of the CRRA utility function when $\gamma = 1$. You can impose the limit in your first order conditions as opposed to derive them again. You have also learned that the Federal Reserve has kept interest rates constant forever. In this fictitious world, there is no distinction between the real interest rate r_t and the nominal interest rate, and you will assume that the Fed has kept, and promises to keep interest rates at a level such that $(1 + r_t)\beta = 1$ since the infinite past until eternity.

(f) **30 points:** Impose these pieces of information on your first order conditions. What can you say about the optimal sequence of non-durable consumption? What is the economic intuition? Explain in **maximum** one paragraph.

(g) **40 points:** Discuss using your equations whether the iterated-forward law of motion of λ now looks closer to the setting of House and Shapiro (2008). Explain in **a short paragraph** why the combination of log utility on M_t plus the assumption of constant rates simplifies the problem.

Information on car manufacturers (applies starting in question h): When the CEA hired you to study the CARS reform, you found out that new car prices are very unresponsive to demand. Studying the literature in Industrial Organization, you discovered that the supply curve for new cars is best described by the following relation

$$P_{D,t} = Y_t^{\xi}, \tag{6}$$

for a ξ that is positive and very small.

Moreover, the market for cars clears in this fictitious economy, so that $Y_t = X_t$.

You also measured that in this fictitious country, cars are of very high quality and depreciate very slowly. That is, δ is sufficiently small so that the assumption that λ is invariant to a transitory change in taxes holds, since the marginal benefit is dominated by future marginal utilities and because expenditures are a small fraction of the stock (both the results of δ being small).

(h) **40 points:** Use the assumptions of constant real rates, the assumption of constant λ, and the shape of the supply curve for cars, compute the semi-elasticity of the production of cars to the tax rate $\frac{d \log Y_t}{d\tau_t}$. Is this number small or large?

(i) **50 points:** After the result of the CARS program, many decades after, a new government advocates to repeat the program, again for one period. The news were not well received by the Chairman of the Federal Reserve who stated *if the new CARS program is implemented, the FOMC will increase the interest rate r_t for one period when the subsidy is implemented*. Keeping your assumption of $\gamma = 1$, but allowing for the interest rate to increase, please discuss with the help of your equations, what are the effects of an increase in the interest rate r_t on the effectiveness of the program.

(j) **50 points:** Revisit your results for an alternative case instead of a Cobb-Douglas function, M is defined by a CES function

$$M_t = \left(a_c C_t^{\frac{\eta-1}{\eta}} + a_d D_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{7}$$

where $\eta > 1$ is the elasticity of substitution between cars and restaurant meals.

Specifically, what this question asks you is to re-derive the FOCs, iterate the equations forward, and then **argue** using the equations whether the effects on output that you expect would be larger or smaller. You can assume that $u(M_t, L_t) = \log(M_t) - \frac{L_t^{1+\psi}}{1+\psi}$ and $\beta(1 + r_t) = 1 \; \forall t$. It is enough to argue with the equations instead of finding a full characterization if you prefer.

(k) **40 points:** Repeat question (h) for the case where $\delta = 1$, that is cars are fully non-durable. Are the effects of the reform larger or smaller in this case? You should maintain every other assumption of the problem as you did in question (h), but adjust the optimality conditions so they are consistent with $\delta = 1$.

2. Please refer to Problem Set two for the subset of questions I expect on each paper. Respond the questions for the following three required papers.

   (a) **30 points:** Zwick and Mahon (2017)

   (b) **30 points:** Caballero and Engel (1999)

   (c) **30 points:** Cooper and Haltiwanger (2006)