WEEK 1: MONEY IN THE UTILITY FUNCTION

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TODAY'S AGENDA

1. RBC model with money in the utility function $\rightarrow$ Sidrauski 1967
2. Study money neutrality in this model

RBC MODEL WITH MONEY IN THE UTILITY FUNCTION

- Introduce a motive for holding money to an otherwise standard RBC model
- Households derive utility from holding money
- Services provided by money depend on their purchasing power
- so, households derive utility from the real balances they hold: $\frac{M_t}{P_t}$

HOUSEHOLDS' PROBLEM

$$\max_{\{C_t, N_t, M_t/P_t, A_{t+1}, K_{t+1}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, N_t, \frac{M_t}{P_t} \right) \right]$$

$$\begin{aligned} \text{s.t.} \qquad & P_t C_t + A_{t+1} + P_t K_{t+1} + M_t \\ & \leq W_t N_t + A_t(1 + i_{t-1}) + P_t K_t (1 + r_{t-1}) + M_{t-1} \end{aligned}$$

- A_t: nominal bonds, K_t: real bonds, M_t: money holdings
- M_t are money holdings at the end of period t, carried over to period $t+1$
- $W_t N_t$: wage income

HOUSEHOLDS' PROBLEM - OPTIMALITY CONDITIONS

- First order conditions:

$$
\begin{aligned}
C_t: &\quad \mathcal{U}_{c_t}(C_t, N_t, M_t/P_t) = \lambda_t P_t \\
N_t: &\quad -\frac{\mathcal{U}_{n_t}(C_t, N_t, M_t/P_t)}{\mathcal{U}_{c_t}(C_t, N_t, M_t/P_t)} = \frac{W_t}{P_t} \\
A_t: &\quad \lambda_t = \beta\lambda_{t+1}(1 + i_t) \\
K_t: &\quad \lambda_t = \beta\lambda_{t+1}\frac{P_{t+1}}{P_t}(1 + r_t) \\
M_t/P_t: &\quad \lambda_t = \beta\lambda_{t+1} + \mathcal{U}_{M_t/P_t}(C_t, N_t, M_t/P_t) \rightarrow \textit{NEW}
\end{aligned}
$$

- Note: MU of C and N can depend on M_t/P_t

HOUSEHOLDS' PROBLEM - OPTIMALITY CONDITIONS

- The FOCs for A_t, K_t and M_t can be re-arranged to:

$$K_t: \quad 1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}(.)}{\mathcal{U}_{c_t}(.)} (1 + r_t) \right]$$

$$A_t: \quad 1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}(.)}{\mathcal{U}_{c_t}(.)} \frac{P_t}{P_{t+1}} (1 + i_t) \right]$$

$$M_t: \quad 1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}}{\mathcal{U}_{c_t}} \frac{P_t}{P_{t+1}} \right] + \frac{\mathcal{U}_{M_t/P_t}}{\mathcal{U}_{c_t}} \rightarrow \textit{NEW}$$

- Trade-offs between assets:
  - Buy real bond and get a return of r_t or buy nominal bond and get a return of i_t minus inflation or hold money and get a return of zero(-π) plus utility benefit.
- From K_t and A_t we get the Fisher equation:

$$(1 + r_t) = \frac{1 + i_t}{\mathbb{E}_t[\pi_{t+1}]}$$

FIRM SIDE - SUPER SIMPLE

- Firms produce according to

$$Y_t = Z_t N_t^{1-\alpha}$$

- Z_t is aggregate technology that we assume exogenous
- Profit maximization taking prices and wages as given implies:

$$\frac{W_t}{P_t} = (1 - \alpha) Z_t N_t^{-\alpha}$$

- which gives us a labor demand schedule for this economy

MARKET CLEARING CONDITIONS

- All output is consumed

$$C_t = Y_t$$

- Nominal and real bonds are in zero net-supply

$$A_t = 0 \qquad\qquad K_t = 0$$

- Money supply is set exogenously by monetary authority (eg. AR(1) process)

$$M_t = M_t^s$$
$$\ln M_t^s = \rho_m \ln M_{t-1}^s + \epsilon_t^M$$

EQUILIBRIUM

- We have 9 endogenoues variables: $Y_t, C_t, N_t, A_{t+1}, K_{t+1}, W_t, P_t, r_t, i_t$ plus
- the two exogenous processes for Z_t and M_t
- Equilibrium conditions:

$$Y_t = C_t \tag{1}$$

$$\frac{W_t}{P_t} = (1-\alpha)Z_t N_t^{-\alpha} \tag{2}$$

$$Y_t = Z_t N_t^{1-\alpha} \tag{3}$$

$$-\frac{\mathcal{U}_{n_t}(C_t, N_t, M_t/P_t)}{\mathcal{U}_{c_t}(C_t, N_t, M_t/P_t)} = \frac{W_t}{P_t} \tag{4}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}(.)}}{\mathcal{U}_{c_t}(.)}(1+r_t) \right] \tag{5}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}(.)}{\mathcal{U}_{c_t}(.)} \frac{P_t}{P_{t+1}}(1+i_t) \right] \tag{6}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}}{\mathcal{U}_{c_t}} \frac{P_t}{P_{t+1}} \right] + \frac{\mathcal{U}_{M_t/P_t}}{\mathcal{U}_{c_t}} \tag{7}$$

$$A_{t+1} = 0 \tag{8}$$

$$K_{t+1} = 0 \tag{9}$$

MONEY NEUTRALITY - WHAT DETERMINES THE REAL SIDE?

- Let's look at the equations that determine the real side:

$$Y_t = C_t \tag{10}$$

$$\frac{W_t}{P_t} = (1-\alpha) Z_t N_t^{-\alpha} \tag{11}$$

$$Y_t = Z_t N_t^{1-\alpha} \tag{12}$$

$$-\frac{\mathcal{U}_{n_t}(C_t, N_t, M_t/P_t)}{\mathcal{U}_{c_t}(C_t, N_t, M_t/P_t)} = \frac{W_t}{P_t} \tag{13}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{c_{t+1}}(.)}{\mathcal{U}_{c_t}(.)} (1 + r_t) \right] \tag{14}$$

- for output, consumption, employment, real wages and the real rate
- Money Neutrality requires that neither $\mathcal{U}_{c_t}$ nor $\mathcal{U}_{n_t}$ depend on M_t/P_t
- Otherwise, nominal variables ($\frac{M_t}{P_t}(i_t)$) pop up in the equations that determine real variables

MONEY NEUTRALITY WITH MONEY IN THE UTILITY FUNCTION

1. Separable utility $\rightarrow$ money neutrality

$$\mathcal{U}\left(C_t, N_t, \frac{M_t}{P_t}\right) = \ln C_t + \chi \ln \frac{M_t}{P_t} - \xi \frac{N_t^{1+\psi}}{1+\psi}$$

2. Non-separable utility $\rightarrow$ no money neutrality

$$\mathcal{U}\left(C_t, N_t, \frac{M_t}{P_t}\right) = \frac{\left(X_t\right)^{1-\sigma} - 1}{1-\sigma} - \xi \frac{N_t^{1+\psi}}{1+\psi}$$

$$X_t = \left[(1-\theta)C_t^{1-\nu} + \theta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \qquad \nu \neq \sigma$$

- $1/\nu$ is the elasticity of substitution between C and real balances
- Are the services provided by money separable from other goods and services?

MUF GIVES US A MONEY DEMAND SCHEDULE

- Combine FOC for M_t/P_t and A_{t+1}:

$$1 = \beta\mathbb{E}_t[\frac{\mathcal{U}_{c_{t+1}}}{\mathcal{U}_{c_t}}\frac{P_t}{P_{t+1}}(1+i_t)] \quad \rightarrow (1+i_t)^{-1} = \beta\mathbb{E}_t[\frac{\mathcal{U}_{c_{t+1}}}{\mathcal{U}_{c_t}}\frac{P_t}{P_{t+1}}]$$

$$1 = \beta\mathbb{E}_t[\frac{\mathcal{U}_{c_{t+1}}}{\mathcal{U}_{c_t}}\frac{P_t}{P_{t+1}}] + \frac{\mathcal{U}_{m_t/P_t}}{\mathcal{U}_{c_t}}$$

- Combining and re-arranging gives us:

$$\frac{1}{i_t} + 1 = \frac{\mathcal{U}_{c_t}}{\mathcal{U}_{m_t/P_t}}$$

- Assume second functional form which nests both separable and non-separable cases

$$\mathcal{U}_{c_t} = (1-\theta)X_t^{\nu-\sigma}C_t^{-\nu} \qquad \mathcal{U}_{m_t/P_t} = \theta X_t^{\nu-\sigma}\left(\frac{M_t}{P_t}\right)^{-\nu}$$

MIU GIVES US A MONEY DEMAND SCHEDULE

- Plug in expressions for marginal utilities:

$$\frac{1}{i_t} + 1 = \frac{(1-\theta)X_t^{\nu-\sigma}C_t^{-\nu}}{\theta X_t^{\nu-\sigma}\left(\frac{M_t}{P_t}\right)^{-\nu}}$$

$$\frac{1}{i_t} + 1 = \frac{(1-\theta)C_t^{-\nu}}{\theta\left(\frac{M_t}{P_t}\right)^{-\nu}}$$

- Solving for $\frac{M_t}{P_t}$ gives us the money demand schedule:

$$\frac{M_t}{P_t} = \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\nu}} C_t \left(\frac{1}{i_t} + 1\right)^{\frac{1}{\nu}}$$

- The money demand schedule depends on i_t and C_t
- Money demand is increasing in consumption and decreasing in the nominal interest rate

BREAKING MONEY NEUTRALITY WITH MIU

- Inspect mechanisms through which money affects real side
- Do models with MIU yield empirically sound predictions? $\rightarrow$ briefly

HOW DO HOUSEHOLDS RESPOND TO AN EXOGENOUS INCREASE IN M?

1. **Income Effect:** before any price changes, an increase in M increases consumption
2. **Substitution Effect:** increase in M $\rightarrow$ price of money falls $(1/P_t)$ $\rightarrow$ C_t becomes relatively more expensive than money $\rightarrow$ decrease C

- What determines the overall effect?
- **Key parameter:** elasticity of substitution between M_t/P_t and C_t, $\frac{1}{\nu}$
- Empirically plausible scenario has C go up after an expansionary monetary shock ($\uparrow M_t$)
- Income effect has to dominate substitution effect $\rightarrow$ relatively low $1/\nu$

DO MODELS WITH MIU YIELD EMPIRICALLY SOUND PREDICTIONS?

1. Empirically plausible calibrations yield small real effects of monetary policy (Woodford 2003, Walsh 2010, Gali Ch.2)
2. Transmission mechanism of monetary shocks at odds with empirical evidence (see Gali Ch. 2)

   why? matching the negative comovement between i_t and Y_t, M_t requires a counterfactual path for ΔM_{t+1}

RECAP

- MIU is the simplest way to incorporate money (*alternative: CIA models*)
- Woodford 2003 discusses ways to microfound MIU
- MIU allows us to derive a money demand function
- With non-separable utility, MIU can generate money non-neutrality. However:
  - loads real effects of MP on household preferences
  - it generates small real effects of monetary policy
  - transmission mechanism at odds with empirical evidence

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