210C - Part 1: Juan

Juan Herreño
UC San Diego

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1 Convex and non-convex capital adjustment costs

The exam has a total of 100 points. Please read the exam carefully.

Time is discrete. A firm is interested in maximizing its value, which consists of the present value of the stream of profits. The firm uses the Stochastic Discount Factor of the representative household Λ to discount the future. The firm takes Λ as given. Firms revenues (R) depend on firm-level productivity A (exogenous) and its current capital stock k via a production technology with decreasing returns to scale on capital ($\theta < 1$). Specifically,

$$R = Ak^{\theta}. \tag{1}$$

After production, a firm may decide to invest. Whenever a firm invests an amount of resources i, it must pay for that investment expenditure, and on top of that, incur in convex and non-convex costs of adjustment. Total investment expenditures for the firm are then

$$\text{Investment Expenditures} = i + \frac{\psi}{2}\left(\frac{i}{k}\right)^2 k + \chi \mathbb{1}_{i \neq 0}, \tag{2}$$

where i denotes investment, ψ is a parameter that dictates the size of convex adjustment costs, χ is a constant that dictates the size of non-convex adjustment costs, and $\mathbb{1}_{i\neq 0}$ is the indicator function that takes the value of 1 whenever $i \neq 0$.

The law of motion of capital is given by

$$k' = (1-\delta)k + i, \tag{3}$$

where k' denotes next period's capital stock and δ is the rate of depreciation of capital.

It will be useful to summarize the firm's problem via a value function V, that makes clear that the firm problem has a discrete choice aspect. The firm may choose to invest receiving a value function V^a after which it pays the fixed cost, or it may choose not to invest, receiving a value function V^n and no fixed cost is paid, therefore

$$V(A,k) = R + \max\left[V^n(A,K), V^a(A,k) - \chi \mathbb{1}_{i\neq 0}\right], \tag{4}$$

and V^a and V^n are Bellman equations as well.

1. **15 points:** Write the value function of the firm conditional on adjusting its capital stock (V^a). Hint: Notice that V already subtracts non-convex costs and adds R.

2. **15 points:** Write the value function of this firm conditional on non-adjustment of its capital stock and call it $V^n(A,k)$. Hint: Notice that V already includes R.

3. **20 points:** Write the problem of the firm conditional on adjustment using the value function $V^a(A,k)$. Make sure to make explicit the objective of the firm (what the firm wants to maximize or minimize), the control variables of the firm (which variables the firm chooses), and the constraints that it must respect.

4. **20 points:** State the optimality condition for investment conditional on adjustment $i^a(A,k)$, as a function of parameters of the model, state variables, and the expectation of the marginal value of one additional unit of capital to the firm.

5. **10 points:** The fixed cost χ is distributed according to a uniform distribution with support $[0,\bar{\chi}]$. Write a function called $\Omega(A,k)$ that describes the probability of adjustment of a firm with state A,k. A hint that may be useful is that for a variable x distributed uniformly with support $[a,b]$, the $P(x<c) = \frac{c-a}{b-a}$ for any $a \le c \le b$.

6. **20 points:** In maximum one short paragraph, link the mathematical objects you have derived to the economic concepts of the "intensive margin of investment" and "the extensive margin of investment".

1. **15 points:** Write the value function of the firm conditional on adjusting its capital stock (V^a). Hint: Notice that V already subtracts non-convex costs and adds R.

$$V^a(A,k) = \underbrace{-}_{\text{substract.}}\Big(\underset{\uparrow \text{ amount of investment}}{i} + \underbrace{\frac{\Psi}{2}\left(\frac{i}{k}\right)^2 k}_{\text{Convex AC}}\Big) + \underbrace{\beta\, \mathbb{E}\left[V(A',k' \mid A\right]}_{\text{continuation value}}$$

Value function cond. on choosing to invest

2. **15 points:** Write the value function of this firm conditional on non-adjustment of its capital stock and call it $V^n(A, k)$. Hint: Notice that V already includes R.

VF cond. on $\boxed{\text{not}}$ investing.

$$V^n(A, k) = \beta\, \mathbb{E}[V(A', k' \mid A]$$

If optimal not to invest:

$$V(A, k) = R + \beta\, \mathbb{E}[V(A', k' \mid A]$$

If optimal to invest:

$$V(A, k) = R \underbrace{- \left(i + \frac{\psi}{2}\left(\frac{i}{k}\right)^2 k\right) + \beta\, \mathbb{E}[V(A', k' \mid A]}_{V^a(A, k)} - \underbrace{\chi}_{\text{non-convex cost.}}$$

3. **20 points:** Write the problem of the firm conditional on adjustment using the value function $V^a(A, k)$. Make sure to make explicit the objective of the firm (what the firm wants to maximize or minimize), the control variables of the firm (which variables the firm chooses), and the constraints that it must respect.

$$\max_{i, k'} \; V^a(A, k) \quad \text{st.} \quad \underbrace{k' = (1-\delta)k + i}_{\text{LoM of capital stock}}$$

this is what they want to max.

$$V^a - x \quad \text{vs} \quad V^n$$

4. **20 points:** State the optimality condition for investment conditional on adjustment $i^a(A, k)$, as a function of parameters of the model, state variables, and the expectation of the marginal value of one additional unit of capital to the firm.

$$\max_{i,k'} \quad V^a(A,k) \quad \text{st.} \quad \underbrace{k' = (1-\delta)k + i}$$

$$\mathcal{L}(i,k') = V^a(A,k) + \lambda\left[(1-\delta)k + i - k'\right]$$

$$= -\left(i + \frac{\Psi}{2}\left(\frac{i}{k}\right)^2 k\right) + \beta\, \mathbb{E}\left[V(A',k' \mid A)\right] + \lambda\left[(1-\delta)k + i - k'\right]$$

FOC:

$$[i]: \quad -1 - \frac{\Psi \cancel{2}}{\cancel{2}} \frac{i \cancel{k}}{k^{\cancel{2}}} + \lambda = 0$$

$$\Rightarrow \boxed{\lambda = 1 + \Psi \frac{i}{k}}$$

$$[k']: \quad \beta\, \mathbb{E}\left[V_{k'}(A',k' \mid A)\right] - \lambda = 0$$

$$\boxed{\lambda = \beta\, \mathbb{E}\left[V_{k'}(A',k' \mid A)\right]}$$

Combine FOCs

$$1 + \Psi \frac{i}{k} = \Lambda\, \mathbb{E}\left[V_{k}(A', k') \mid A \right]$$

$$\boxed{\underline{i^{a}(A,k)} = \frac{\left(\Lambda\, \mathbb{E}\left[V_{k}(A', k') \mid A \right] - 1\right) \cdot k}{\Psi}}$$

5. **10 points:** The fixed cost χ is distributed according to a uniform distribution with support $[0, \bar{\chi}]$. Write a function called $\Omega(A, k)$ that describes the probability of adjustment of a firm with state A, k. A hint that may be useful is that for a variable x distributed uniformly with support $[a, b]$, the $P(x < c) = \frac{c-a}{b-a}$ for any $a \leq c \leq b$.

$\Omega(A, k)$: prob. of adjust. given that

$$\chi \sim U[0, \bar{\chi}]$$

When will the firm choose to invest?

$$V^a(A, k) - \chi \geq V^n(A, k)$$

$$\Omega(A, k) = P(V^a(A, k) - \chi \geq V^n(A, k))$$

$$= P(V^a(A, k) - V^n(A, k) \geq \chi)$$

$$\Omega(A, k) = P(\chi \leq V^a(A, k) - V^n(A, k))$$

$a = 0$
$b = \bar{\chi}$
$c = V^a - V^n$

$$\boxed{\Omega(A, k) = \frac{V^a(A, k) - V^n(A, k)}{\bar{\chi}}}$$

6. **20 points:** In maximum one short paragraph, link the mathematical objects you have derived to the economic concepts of the "intensive margin of investment" and "the extensive margin of investment".

① $V^a(A, k) \; / \; i^a$

② $\Omega(A, k)$

Qual Practice Questions

Juan Herreño
UC San Diego

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1 External Adjustment Costs

Time is discrete and firms discount the future with a discount factor β. Firms in the final-good sector produce with a production function that depends on capital $Y_t = A_t F(K_t)$, where $F(.)$ satisfies the standard assumptions of neoclassical economics. Firms buy capital, where I_t is the quantity of investment goods purchased by the firm. The price of capital goods is p_t. Firms take this price as given. The law of motion for capital is given by $K_{t+1} = (1-\delta)K_t + I_t$ where δ is the rate of depreciation of capital. The initial level of capital K_0 is given.

Capital goods are produced by a capital-producing sector. The capital-producing sector is perfectly competitive, and has access to a production technology that produces capital goods according to a production function $I_t = L_t^{1/2}$, where L is the amount of labor employed by the capital-producing firms to produce new capital goods. The wage rate these firms pay their workers is given by w. Firms take the wage rate as given.

1. Set the problem of a final-good sector firm. Your answer should describe the objective of the firm, and the constraints it faces.

2. Set the Lagrangian of the final-good sector firm and take first order conditions with respect to I_t and K_{t+1}.

3. Iterate forward the first order condition with respect to K_{t+1}. Use the transversality condition to eliminate the terminal term.

4. In **maximum** a couple of sentences, please explain the economic meaning of the first order condition with respect to I_t, and the iterated-forward first-order condition with respect to K_{t+1}.

5. The capital-producing sector is competitive and sets its price p_t equal to marginal cost in each point in time. What are the total costs of the capital-producing sector as a function of labor? as a function of output? This question asks you for two equations.

6. Derive the marginal cost function for the capital-producing firms.

7. Imagine the economy is in steady state. Use the relationship of the law of motion of capital in steady state, the first order conditons for the final-good producer, and the optimality condition of the capital producing good to find the steady state level of capital as a function of parameters and the real wage.

2 Empirical Literature on Credit Supply

1. Khwaja and Mian (2008) "*Tracing the Impact of Bank Liquidity Shocks*" runs the folowing regression

$$\Delta \log \text{Loans}_{bj} = \alpha_j + \beta \Delta \log \text{Liquidity}_b + \epsilon_{bj} \tag{1}$$

   where b indexes banks, j indexes firms, and α_j is a firm fixed-effect. Explain in one paragraph why the inclusion of firm-fixed effects is important in this literature in order to claim that $\hat{\beta}$ captures the effects of credit supply shocks.

2. May papers in this literature run *firm-level regressions* of credit on the average liquidity of their banks. Formally

$$\Delta \log \text{Loans}_j = \gamma_0 + \gamma_1 \Delta \log \bar{\text{Liquidity}}_j + \xi_j, \tag{2}$$

   where $\bar{\text{Liquidity}}_j = \sum_b \omega_{j,b,pre} \text{Liquidity}_b$, and $\omega_{j,b,pre}$ are the borrowing shares of firm j with bank b in a pre-period before any shock occurred.

   Explain in one paragraph why the literature considers important to run equation 2 on top of the information contained by equation 2.

3. Explain in one paragraph why estimates of γ_1 may overestimate or underestimate the *aggregate* effects of credit supply shocks.

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where b indexes banks, j indexes firms, and α_j is a firm fixed-effect. Explain in one paragraph why the inclusion of firm-fixed effects is important in this literature in order to claim that $\hat{\beta}$ captures the effects of credit supply shocks.

$\Delta \log \text{Loans}_{bj}$: Credit to firm j from bank b

α_j : firm fixed effect $\rightarrow$ controls for firm-level demand shocks

Source of variation: exploiting variation <u>within</u> a firm <u>across</u> banks

ϵ^D_{bj} : Credit demand shock specific to the bj pair

$$= \alpha_j + \beta \log L_b + \underbrace{\epsilon^D_{bj}}_{>0} + \epsilon_{bj}$$

2. May papers in this literature run *firm-level regressions* of credit on the average liquidity of their banks. Formally

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Explain in one paragraph why the literature considers important to run equation 2 on top of the information contained by equation ~~2~~ 1.

Eq. 1 : speaks to relative effects within a firm

↳ can't answer wheter overall debt of J increased

Eq. 2 : exploits variation across firms with ≠ levels of exposure to a Δ in liquidity

this is still a relative effect.

3. Explain in one paragraph why estimates of γ_1 may overestimate or underestimate the *aggregate* effects of credit supply shocks.

exercise

3 Asymmetric Information

There is a single time period. All the agents in the economy are risk neutral. At the beginning of the period a firm needs to incur in an expenditure equal to 1 unit of the final good. The firm manager has at her disposal $W < 1$ units of the final good, so the firm needs $1 - W$ units of financing from an outside investor. The outcome of the investment opportunity of the firm is uncertain, and yields Y units of resources, where Y is distributed uniform on a bounded support $[0, 2\gamma]$.

The outside investor has access to an outside option that yields a gross interest rate of $1 + r$. The firm also has access to the same outside option than the outside investor, with the same gross return $1 + r$. There is perfect competition among outside investors, so outside investors are happy to provide financing to any investment project that is as good as the outside option. Firms are happy to undertake the investment project if it yields a return equal to that of the outside option.

The outside investor needs to pay a verification cost K if she wants to verify the realization of Y, which is the private information of the firm.

In country A, only equity contracts exist. An equity contract specifies a share of output s that the firm needs to promise the outside investor in exchange for $1 - W$ units of financing.

In country B only defaultable debt contracts exist. A defaultable debt contract specifies a payment D that the firm should pay to the outside investor if the firm is able to $(Y > D)$. If the firm cannot pay (that is, $Y < D$), the firm defaults and the outside investor takes the realization of Y.

Country A and country B have the same distribution of entrepreneurial wealth, the same outside option, the same distribution for the outcome of the investment opportunities, and the same verification costs K.

An important parametric restriction is that $\gamma > 1 + r$.

A possibly useful hint is that the expected value of an uniformly distributed variable x in the range $[a, b]$ is equal to $(a + b)/2$ and the CDF $P(x < c) = \frac{c-a}{b-a}$ for any $a < c < b$. The conditional expectation $E(x|x < d) = (a + d)/2$ for a constant $a < d < b$.

Another possibly useful hint is that a solution to a quadratic equation $ax^2 + bx + c = 0$ is given by $x = \frac{-b+-\sqrt{b^2-4ac}}{2a}$.

1. Assume the verification cost $K = 0$. What is the shape of the equity contract, s^{PI}, where PI is short for perfect information, that guarantees the participation of the outside investor?

2. Is s^{PI} increasing or decreasing on firm's wealth W?

3. Assume the verification cost $K = 0$. What is the shape of the defaultable debt contract, D^{PI}, where PI is short for perfect information, that guarantees the participation of the outside investor?

4. Is D^{PI} increasing or decreasing on firm's wealth W?

   For the next questions you can take a shortcut. Assume that with equity contract the outside investor always verifies output, and with the defaultable debt contract the outside investor only verifies in case of default.

5. What are the expected revenues (accounting for verification costs) of the outside investor under the equity contract with $K > 0$?

6. What are the expected revenues of the outside investor (accounting for verification costs) under the defaultable debt contract with $K > 0$?

7. Find s^{AI} the share of output the firm needs to promise the outside investor when $K > 0$ such that the outside investor participates.

8. Find D^{AI} the debt payment the firm needs to promise the outside investor when $K > 0$ such that the outside investor participates. You can directly assume that competition among investors implies that the smallest positive root is the payment to the investor.

1. Assume the verification cost $K = 0$. What is the shape of the equity contract, s^{PI}, where PI is short for perfect information, that guarantees the participation of the outside investor?

Equity contract pays a fixed share of output.

Investor participates iff:

$$\underbrace{E[s^{PI} y]}_{\text{expected return}} \geq \underbrace{(1-W)(1+r)}_{OO}$$

Perfect compet. among investors:

$$E[s^{PI} y] = (1-W)(1+r)$$

$$s^{PI} E[y] = (1-W)(1+r)$$

$$s^{PI} = \frac{(1-W)(1+r)}{E[y]} = \frac{(1-W)(1+r)}{\gamma}$$

2. Is s^{PI} increasing or decreasing on firm's wealth W?

$$\frac{\partial s^{PI}}{\partial W} < 0$$

3. Assume the verification cost $K = 0$. What is the shape of the defaultable debt contract, D^{PI}, where PI is short for perfect information, that guarantees the participation of the outside investor?

4. Is D^{PI} increasing or decreasing on firm's wealth W?

Defaultable contract pay:

$$D^{PI} = \begin{cases} D^* & \text{if } y \geq D^* \quad ① \\ y & \text{if } y < D^* \quad ② \end{cases}$$

The investor participates iff:

$$\underbrace{E[D^{PI}]}_{\text{expected repayment.}} \geq (1-W)(1+r)$$

$$\underbrace{E[D^* \mid y \geq D^*] \cdot P(y \geq D^*) + E[y \mid y < D^*]\, P[y < D^*]}_{E[D^{PI}]} = 00$$

use properties of the uniform distribution to get to D^*

+

quad. formula.

For the next questions you can take a shortcut. Assume that with equity contract the outside investor always verifies output, and with the defaultable debt contract the outside investor only verifies in case of default.

5. What are the expected revenues (accounting for verification costs) of the outside investor under the equity contract with $K > 0$?

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Expected revenues with equity contract

$$E[S^{AI} \cdot y] - K = (1-W)(1+r)$$

$$E[S^{AI} y] = (1-W)(1+r) + K$$

Expected revenues with defaultable debt

$$E[D^{AI}] - K\,P[y < D^*] = (1-W)(1+r)$$

7. Find s^{AI} the share of output the firm needs to promise the outside investor when $K > 0$ such that the outside investor participates.

8. Find D^{AI} the debt payment the firm needs to promise the outside investor when $K > 0$ such that the outside investor participates. You can directly assume that competition among investors implies that the smallest positive root is the payment to the investor.

compute $E[y]$ and get expression for s^{AI}

s^{AI} vs s^{PI}