EFFECTS OF G IN THE NEOCLASSICAL AND NK MODELS - REVIEW

Discussion ECON 210C - Paula Donaldson

PLAN FOR TODAY

- Derive the expression for the government spending multiplier in the neoclassical model
- Discuss additional channels in the NK model
- Discuss the role of parameters in determining the size of the multiplier (Dynare)

MULTIPLIERS IN THE NEOCLASSICAL MODEL - RECAP

- The response of output to government spending is given by the following expression:

$$\frac{dY}{dG} = \frac{\eta_u}{\eta_u + \eta_v}$$

- where $-\eta_u$ and η_v are the elasticities of marginal utility of consumption and disutility of labor respectively wrt. to output

STEP 1:OPTIMALITY CONDITIONS

Household utility given by:

$$U(C) - V(N) \qquad \text{with} \qquad U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad V(N) = \frac{N^{1+\varphi}}{1+\varphi}$$

Optimality condition imply

$$\frac{V^{'}(N)}{U^{'}(C)} = \frac{W}{P} = F^{'}(N)$$

Define indirect disutility:

$$\tilde{V}(Y) = V\left(F^{-1}(Y)\right) \quad \rightarrow \quad \tilde{V}'(Y) = \frac{V'(N)}{F'(N)}$$

$\tilde{V}$ is the disutility cost of producing Y units of output. Re-write the optimality condition:

$$U'(C) = \tilde{V}'(Y) \quad \Longrightarrow \quad U'(Y-G) = \tilde{V}'(Y)$$

STEP 2: IMPLICIT DIFFERENTIATION

Starting from $U'(Y - G) = \tilde{V}'(Y)$, differentiate wrt G:

$$\frac{d}{dG}U'(Y - G) = U''(Y - G)\left(\frac{dY}{dG} - 1\right),$$
$$\frac{d}{dG}\tilde{V}'(Y) = \tilde{V}''(Y)\frac{dY}{dG}.$$

Equate and rearrange:

$$U''(C)\left(\frac{dY}{dG} - 1\right) = \tilde{V}''(Y)\frac{dY}{dG},$$
$$\left(U''(C) - \tilde{V}''(Y)\right)\frac{dY}{dG} = U''(C),$$
$$\therefore \quad \frac{dY}{dG} = \frac{U''(C)}{U''(C) - \tilde{V}''(Y)}.$$

STEP 3: ELASTICITY REPRESENTATION

Multiply and divide by $-\frac{Y}{U'(C)}$:

$$\frac{dY}{dG} = \frac{-\frac{U''(C)Y}{U'(C)}}{-\frac{U''(C)Y}{U'(C)} + \frac{\tilde{V}''(Y)Y}{U'(C)}}$$

Optimality implies $U'(C) = \tilde{V}'(Y)$, so we can substitute:

$$\frac{dY}{dG} = \frac{-\frac{U''(C)Y}{U'(C)}}{-\frac{U''(C)Y}{U'(C)} + \frac{\tilde{V}''(Y)Y}{\tilde{V}'(Y)}}$$

Define elasticities of marginal utilities: $\eta_u = -\frac{U''(C)\,Y}{U'(C)}, \quad \eta_v = -\frac{\tilde{V}''(Y)\,Y}{\tilde{V}'(Y)}$

Substitute into multiplier:

$$\frac{dY}{dG} = \frac{\eta_u}{\eta_u + \eta_v},$$

EXAMPLE

For CRRA utility:

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \qquad U'(C) = C^{-\sigma}, \qquad U''(C) = -\sigma C^{-\sigma-1}, \qquad \eta_u = \sigma(1-s_g)^{-1}.$$

For power disutility and $Y = F(N) = N$:

$$\tilde{V}(Y) = \frac{Y^{1+\varphi}}{1+\varphi}, \qquad \tilde{V}'(Y) = Y^{\varphi}, \qquad \tilde{V}''(Y) = \varphi Y^{\varphi-1}, \qquad \eta_v = \varphi.$$

Therefore:

$$\boxed{\frac{dY}{dG} = \frac{\sigma(1-s_g)^{-1}}{\sigma(1-s_g)^{-1}+\varphi} = \frac{\sigma}{\sigma+(1-s_g)\varphi}}.$$

MULTIPLIERS IN THE NEOCLASSICAL MODEL - INTUITION

- The effect of government spending on output is driven by wealth effects
    1. $\uparrow G \quad \rightarrow \quad$ expected taxes increase
    2. This creates a negative wealth effect
    3. Both consumption and leisure are *normal* goods
    4. Consumption falls
    5. Labor supply increases along an unchanged labor demand curve ($\downarrow \frac{W}{P}$)
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MULTIPLIERS IN THE NEW KEYNESIAN MODEL - RECAP

- Wealth effects are not the only channel now
- In response to a change in G, we have:
  1. Firms increase production to meet demand (only some firms can respond to the demand shock by increasing prices)
  2. This increases labor demand
  3. Inflation goes up

$$\hat{mc}_t = \hat{w}_t = \sigma\hat{c}_t + \phi\hat{n}_t$$
$$= \sigma\hat{c}_t + \phi(s_g\hat{c}_t + (1 - s_g)\hat{g}_t)$$

  4. Monetary authority responds by increasing interest rates
  5. Consumption falls as implied by Euler equation

WHAT DETERMINES THE SIZE OF THE OUTPUT RESPONSE?

- How do the following parameters impact $\frac{dY}{dG}$?
  1. ϕ_π
  2. ϕ_y
  3. $1/\sigma$ (elasticity of intertemporal substitution)
  4. θ (degree of price stickiness)
- Check our intuition in the model we coded up last week.
- Alternative: use method of undetermined coefficients to get an expression for the multiplier in the NK model.