# Lecture 17: Taxes in the Q Theory

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# Tax Policy in the Q Theory

- Taxation of firms takes many forms
- Tax policy can, in principle, be used as countercyclical policy
- Examples of taxes/subsidies used
  - Business profit taxes
  - Dividend taxes
  - Investment tax credits: Deduct a share of investment expenditures
  - Bonus depreciation: Increases the speed at which firms can deduct investment expenditures

# Bonus depreciation

TABLE 1—REGULAR AND BONUS DEPRECIATION SCHEDULES FOR FIVE-YEAR ITEMS

| Year: | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|---|
| *Normal depreciation* | | | | | | | |
| Deductions (000s) | 200 | 320 | 192 | 115 | 115 | 58 | 1,000 |
| Tax benefit ($\tau$ = 35 percent) | 70 | 112 | 67.2 | 40.3 | 40.3 | 20.2 | 350 |
| *Bonus depreciation* (50 percent) | | | | | | | |
| Deductions (000s) | 600 | 160 | 96 | 57.5 | 57.5 | 29 | 1,000 |
| Tax benefit ($\tau$ = 35 percent) | 210 | 56 | 33.6 | 20.2 | 20.2 | 10 | 350 |

Source: Zwick and Mahon (2017)

# Why bonus depreciation matters?

Summers (1987, p. 29.5) states this most clearly: *It is only because of discounting that depreciation schedules affect investment decisions”*

- Standard PV of deductions

$$z = \sum_{j=1}^{R} \frac{D_j^m}{(1+\pi)^j (1+r)^j}$$

- Bonus depreciation

$$\lambda_t + (1 - \lambda_t) z$$

- PV of tax benefits due to bonus depreciation

$$\zeta_t = (1 - \tau_d)\tau_\pi(\lambda_t + (1 - \lambda_t) z)$$

# Naive Model

- Most of the theory you know is designed to understand non-durable goods
- Capital is durable (not only capital. Durable consumption is an important part of household expenditures)
- Standard supply and demand analysis may be arbitrarily uninformative if it ignores durability
- Let’s analyze that through an example

# Naive Model

The world lasts for one period

- A firm that operates the production technology

$$y_t = k_t^{\alpha}$$

- I am assuming the firm only uses capital for simplicity
- Firm profits are

$$\pi_t = y_t - q_t(1 - \zeta_t)k_t$$

- where $q_t$ is the relative price of capital
- Profit maximization yields the condition

$$k_t = \left(\frac{q_t(1 - \zeta_t)}{\alpha}\right)^{-1/(1-\alpha)}$$

- This is the capital demand equation.

# What is the slope of capital demand?

In this interpretation, the capital demand elasticity $\varepsilon^d = 1/(1-\alpha)$ should be between 5 and 10. Why?

- At the optimal capital demand, the share of profits to output is

$$\frac{\pi}{y} = 1 - \alpha$$

- Make sure you derive this at home.
- So if the profit share of income is between 10% to 20%, then the elasticity of capital demand is between 5 and 10.

# Capital supply and equilibrium

Assume a very simple capital supply equation

$$k_t = q_t^{\xi}$$

- Equate demand and supply and find the following equilibrium relation

$$k_t = \Theta(1 - \zeta_t)^{-\frac{\xi}{(1-\alpha)\xi+1}}$$

- For an uninteresting constant $\Theta$
- Take logs and do a first-order Taylor expansion around a point where $\zeta_t = \bar{\zeta}$

$$\log k_t \approx \log \bar{k} + \frac{\xi}{(1-\alpha)\xi + 1} \frac{1}{1 - \bar{\zeta}} (\zeta_t - \bar{\zeta})$$

# What is the value of $\xi$

Idea! Use knowledge on the profit share and the causal effects of $\zeta$ on $k$ to back out $\xi$.

- Rearrange using $\zeta = z \times \tau$

$$\Delta \log k_t \approx \frac{\xi}{(1-\alpha)\xi + 1} \frac{\tau}{1 - \bar{z}\tau} \Delta z_t$$

- Use the definition of supply and demand elasticities

$$\Delta \log k_t \approx \frac{\varepsilon^s \varepsilon^d}{\varepsilon^s + \varepsilon^d} \frac{\tau}{1 - \bar{z}\tau} \Delta z_t$$

- Zwick and Mahon ’17 (on your reading list for next class) uses $\Delta z \approx 0.05, \bar{z} \approx 0.9, \tau = 0.35, \Delta \log k_t \approx 0.17$. Therefore:

$$\varepsilon^s \approx \frac{-7\varepsilon^d}{7 - \varepsilon^d}$$

# What is the value of $\xi$

If $\alpha = 0.8$ such that $\varepsilon = 5$.

$$\varepsilon^S \approx \frac{-35}{2}$$

- A negative supply elasticity?!

If $\alpha = 0.9$ such that $\varepsilon = 10$.

$$\varepsilon^S \approx 23$$

- Very large supply elasticity?

Apparently, little disagreement about the profit share changes inference by a lot!

# Should the profit share dictate the capital demand elasticity?

- In this simple model, the shape of the production function, and nothing else, dictates the capital demand elasticity
- Sensible assumptions when firms are buying/renting non-durable inputs
- But firms are not supposed to maximize today’s profits, but the PV of profits, and buying capital today has implications over those future profits.
- Let’s see what a model with durable inputs has to say about the shape of the capital demand equation

# Firm problem with taxes

$$V_t = \max \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( F(K_{t+j})(1-\tau^{\pi})(1-\tau^{d}) - I_t p_t (1-\zeta_t) \right)$$

subject to:

$$K_{t+1} = K_t(1-\delta) + I_t$$

- Simplifying assumption: Firm internalizes the PV of the deductions at time $t$. Common in key references, e.g. Hall and Jorgenson (1967), House and Shapiro (2008)
- No labor explicitly. Easy to extend if labor market is spot, just more notation
- Firm buys capital from outside producer at price $p$. Adjustment costs occur on that capital producing firm. Referred in the literature as *external* adjustment costs

# Optimality conditions

$$q_t = p_t(1 - \zeta_t)$$

$$q_t = q_{t+1}\Lambda_{t,t+1}(1 - \delta) + \Lambda_{t,t+1}\left(F_k(K_{t+1})(1 - \tau^{\pi})(1 - \tau^{d})\right)$$

- Question: What is the effect on a change of $\zeta_t$.
- slightly simpler version of House and Shapiro (2008)
- Pedagogical exercise: imagine a transient policy
- Changes in $\zeta$ does not last for long
- Assume capital is highly durable ($\delta \approx 0$)
- And firms very forward looking ($\beta \approx 1$)

# Transitory Policy

Let me assume that The change in $\zeta$ is transitory. $\zeta_{t+1} = \bar{\zeta}$. So the economy tomorrow will be close to steady state

- $q_{t+1} \approx q^{ss}$
- $\Lambda_{t,t+1} \approx \beta$
- $\delta$ is small so that $\delta^2 \approx 0$ (For structures $\delta^2 = 0.0004$)

Transform:

$$q_t = q_{t+1}\Lambda_{t,t+1}(1-\delta) + \Lambda_{t,t+1}\left(F_k(K_{t+1})(1-\tau^\pi)(1-\tau^d)\right)$$

into:

$$q_t = q^{ss}\beta(1-\delta) + \beta\left(F_k(K_{t+1})(1-\tau^\pi)(1-\tau^d)\right)$$

# Transitory Policy

$$q_t = q^{ss}\beta(1-\delta) + \beta\left(F_k(K_{t+1})(1-\tau^{\pi})(1-\tau^{d})\right)$$

In steady state:

$$q^{ss} = \frac{\beta}{1-\beta(1-\delta)}\left(F_k(K^{ss})(1-\tau^{\pi})(1-\tau^{d})\right)$$

Divide the first equation by the second one and rearrange

$$q_t \approx q^{ss}\left(\beta(1-\delta) + (1-\beta(1-\delta))\frac{F_k(K_{t+1})}{F_k(K^{ss})}\right)$$

- For structures $1-\beta(1-\delta) = 0.052$
- $K/Y = 4$, and $I/Y = 0.18$ , so $I/K = 0.045$
- Imagine the reform had a large effect on investment. Second term still very small.

# Economics

- On one side $q_t \approx q^{ss}$
  - Capital is long lived. So the marginal benefit is dominated by stream of future MPKs into the distant future. Those do not change much with a temporary policy
- On the other side
$$q_t = p_t(1 - \zeta_t)$$
- the after-tax price of investment must be constant!
- If investment subsidies go up, the pre-tax price must go up.
- The demand for capital does not depend on $I_t$. It is horizontal.
- If capital is perfectly durable, firms are infinitely elastic on when to invest. If capital becomes cheaper today than tomorrow, I will invest today **rather than** tomorrow.
- Note this argument is about the **timing** of investment expenditures, rather than the optimal size of the firm (which is given by steady state parameter values).

# With internatl as opposed to external adjustment costs

When adjustment costs are internal, the firm is the capital producer

- The firm will internalize that the effective cost of investment rises with the size of investment
- The firm internalizes that investing more today reduces adjustment costs in the future

This is not an *either or* question. In principle firms may buy capital from outside firms with increasing marginal costs, and face installation costs

# Firm problem with taxes

$$V_t = \max \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( F(K_{t+j})(1-\tau^{\pi})(1-\tau^{d}) - I_t(1-\zeta_t) - \frac{\varphi}{2}\left(\frac{I_{t+j}}{K_{t+j}}\right)^2 K_{t+j} \right)$$

subject to:

$$K_{t+1} = K_t(1-\delta) + I_t$$

# Optimality conditions

$$\frac{I_t}{K_t} = \frac{1}{\varphi}(q_t - (1 - \zeta_t))$$

$$q_t = q_{t+1}\Lambda_{t,t+1}(1-\delta) + \Lambda_{t,t+1}\left(F_k(K_{t+1})(1-\tau^{\pi})(1-\tau^{d}) + \frac{\varphi}{2}\left(\frac{I_{t+1}}{K_{t+2}}\right)^2\right)$$

# Economics

- On one side $q_t \approx q^{ss}$
  - Capital is long lived. So the marginal benefit is dominated by stream of future MPKs into the distant future. Those do not change much with a temporary policy
- On the other side

$$\frac{I_t}{K_t} = \frac{1}{\varphi}(q_t - (1 - \zeta_t))$$

  - The relative price of investment is decreasing from $(1 - \zeta)$ to $(1 - \zeta_t)$ when $\zeta$ increases
  - It must be that firms are investing more!

$$\frac{dI/K}{d\zeta} = \frac{1}{\varphi}$$

  - Given our argument about $dq/d\zeta \approx 0$

# Economics

Notice a couple of insights

- The reaction of investment to temporary tax policy depends critically on $\varphi$
- The argument relies on a very horizontal demand curve for investment
  - The marginal value of capital depend on stream of future MPKs
  - Changes in the after-tax relative price of investment in one period will convince firms to change the timing of their investments
- $\frac{\partial I/K}{\partial q} = 1/\varphi$
  - The responsiveness of $I$ to $\zeta$ depends on how costly is to build-up capital. The slope of capital supply is given by the adjustment cost function

# Numerical Results

Table 1—Response to a Temporary Investment Subsidy

| Duration | Depreciation rate | Shadow price ($\varphi$) $\xi = 0$ | $\xi = 0.5$ | $\xi = 1$ | $\xi = 5$ | $\xi = 10$ | $\xi = 15$ | $\xi = 20$ |
|---|---|---|---|---|---|---|---|---|
| 6 months | $\delta = 0.001$ | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.996 |
| | $\delta = 0.01$ | 1.000 | 0.999 | 0.998 | 0.992 | 0.986 | 0.982 | 0.978 |
| | $\delta = 0.02$ | 1.000 | 0.998 | 0.996 | 0.986 | 0.976 | 0.969 | 0.963 |
| | $\delta = 0.05$ | 1.000 | 0.996 | 0.992 | 0.970 | 0.951 | 0.936 | 0.923 |
| | $\delta = 0.10$ | 1.000 | 0.992 | 0.985 | 0.945 | 0.911 | 0.885 | 0.864 |
| | $\delta = 0.25$ | 1.000 | 0.982 | 0.965 | 0.877 | 0.807 | 0.755 | 0.714 |
| 1 year | $\delta = 0.001$ | 1.000 | 1.000 | 0.999 | 0.997 | 0.995 | 0.993 | 0.991 |
| | $\delta = 0.01$ | 1.000 | 0.998 | 0.996 | 0.983 | 0.972 | 0.964 | 0.956 |
| | $\delta = 0.02$ | 1.000 | 0.996 | 0.993 | 0.972 | 0.954 | 0.940 | 0.928 |
| | $\delta = 0.05$ | 1.000 | 0.992 | 0.984 | 0.941 | 0.906 | 0.878 | 0.855 |
| | $\delta = 0.10$ | 1.000 | 0.985 | 0.971 | 0.896 | 0.835 | 0.790 | 0.753 |
| | $\delta = 0.25$ | 1.000 | 0.966 | 0.936 | 0.784 | 0.673 | 0.597 | 0.539 |
| 2 years | $\delta = 0.001$ | 1.000 | 0.999 | 0.999 | 0.995 | 0.990 | 0.986 | 0.983 |
| | $\delta = 0.01$ | 1.000 | 0.996 | 0.992 | 0.967 | 0.946 | 0.930 | 0.915 |
| | $\delta = 0.02$ | 1.000 | 0.992 | 0.985 | 0.946 | 0.912 | 0.886 | 0.864 |
| | $\delta = 0.05$ | 1.000 | 0.984 | 0.969 | 0.890 | 0.826 | 0.779 | 0.740 |
| | $\delta = 0.10$ | 1.000 | 0.971 | 0.946 | 0.814 | 0.715 | 0.645 | 0.591 |
| | $\delta = 0.25$ | 1.000 | 0.941 | 0.891 | 0.659 | 0.515 | 0.428 | 0.368 |
| 3 years | $\delta = 0.001$ | 1.000 | 0.999 | 0.998 | 0.992 | 0.985 | 0.980 | 0.975 |
| | $\delta = 0.01$ | 1.000 | 0.993 | 0.988 | 0.952 | 0.922 | 0.898 | 0.878 |
| | $\delta = 0.02$ | 1.000 | 0.989 | 0.979 | 0.921 | 0.873 | 0.837 | 0.807 |
| | $\delta = 0.05$ | 1.000 | 0.976 | 0.956 | 0.845 | 0.760 | 0.698 | 0.649 |
| | $\delta = 0.10$ | 1.000 | 0.959 | 0.925 | 0.749 | 0.626 | 0.545 | 0.485 |
| | $\delta = 0.25$ | 1.000 | 0.922 | 0.860 | 0.587 | 0.439 | 0.357 | 0.304 |
| Permanent | $\delta = 0.001$ | 1.000 | 0.986 | 0.972 | 0.884 | 0.806 | 0.749 | 0.704 |
| | $\delta = 0.01$ | 1.000 | 0.929 | 0.872 | 0.637 | 0.513 | 0.443 | 0.396 |
| | $\delta = 0.02$ | 1.000 | 0.908 | 0.839 | 0.578 | 0.453 | 0.387 | 0.343 |
| | $\delta = 0.05$ | 1.000 | 0.888 | 0.808 | 0.528 | 0.405 | 0.341 | 0.300 |
| | $\delta = 0.10$ | 1.000 | 0.879 | 0.794 | 0.506 | 0.384 | 0.322 | 0.282 |
| | $\delta = 0.25$ | 1.000 | 0.872 | 0.783 | 0.489 | 0.367 | 0.306 | 0.267 |

*Notes*: The table shows the equilibrium percent change in the shadow price of capital goods $\varphi$ in response to an investment subsidy of 1 percent ($d\zeta = 0.01$). Investment supply is given by equation (5). For the numerical calculations, the production function is $AK_t^{\alpha}$, $r = 0.02$, and $\alpha = 0.35$.

Source: House and Shapiro (2008). The effect on $q$ is 1 minus the effect on $\varphi$ (which in their notation is the shadow price of investment), and $\xi = (\delta\varphi)^{-1}$ in our notation.