WEEK 2: ROTEMBERG & WOODFORD (1997)

Paula Donaldson

ECON 210C, 2025

TODAY'S AGENDA

1. Discuss Rotemberg & Woodford (1997) $\rightarrow$ focus on sections 1-4
2. Introduction to Dynare (if we have time)

ROTEMBERG & WOODFORD (1997)

- Develops an econometric approach based on optimization behaviour to model the macroeconomy
- Provided the basis for DSGE models used in academia and central banks
- Pioneered the use of impulse response matching to estimate model parameters in DSGE models

VAR APPROACH

- Estimate a recursive VAR with 3 variables (y, π, i) in order to
  1. Estimate the monetary policy reaction function
  2. Estimate IRF of y, π, r to monetary policy shocks
- Sample: 1980:1-1995:2. Quarterly frequency
- Identification Assumptions:
  - Monetary rule well-approximated by equation 2.1
  - Monetary shock in t has no contemporaneous effect on y or π
- Monetary rule following Taylor(1993)

$$r_t = r^* + \sum_{k=1}^{n_r} \mu_k (r_{t-k} - r^*) + \sum_{k=0}^{n_\pi} \phi_k (\pi_{t-k} - \pi^*) + \sum_{k=0}^{n_y} \theta_k y_{t-k} + \epsilon_t$$

VAR APPROACH - DETAILS

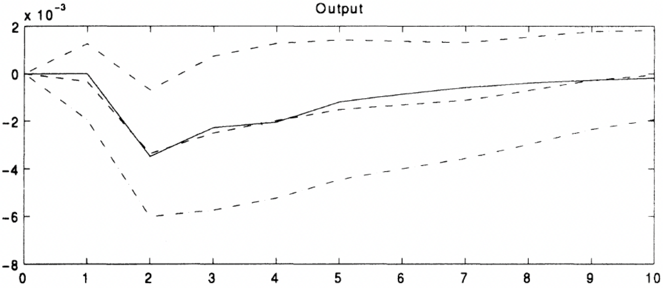
- The estimated system is:

$$\bar{Z}_t = B\bar{Z}_{t-1} + U\bar{e}_t \qquad\qquad Z_t = [r_t, \pi_{t+1}, y_{t+1}]'$$

- and $\bar{Z}_t$ is the transpose of $[Z_t', Z_{t-1}', Z_{t-2}']$
- This is a VAR with 3 lags
- Timing of variables is chosen to reflect the decision lags assumed by the authors
- i.e.: y_{t+1} and r_t share the same information set

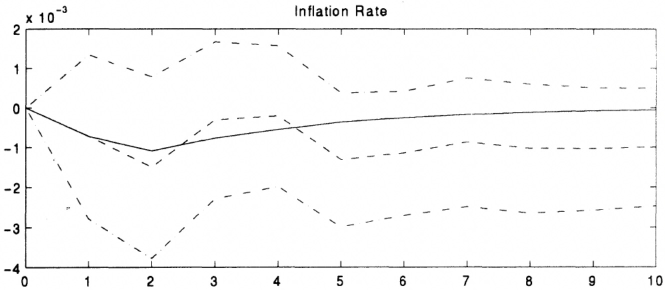
ESTIMATION RESULTS - OUTPUT

Figure 1 ESTIMATED AND THEORETICAL RESPONSES TO A MONETARY POLICY SHOCK



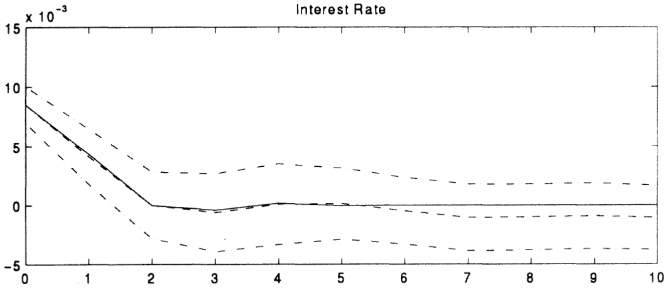
- Central dashed line indicates the point estimate.
- Response correspond to a one-standard-deviation shock.
- Solid line indicates theoretical responses (we’ll discuss those later on)

ESTIMATION RESULTS - INFLATION



- Central dashed line indicates the point estimate.
- Response correspond to a one-standard-deviation shock.
- Solid line indicates theoretical responses

ESTIMATION RESULTS - FED FUNDS RATE



- Central dashed line indicates the point estimate.
- Response correspond to a one-standard-deviation shock.
- Solid line indicates theoretical responses

ESTIMATION RESULTS - RECAP

- Interest rates increase temporarily (first two quarters)
- Output declines with a 2-quarter lag and returns to normal
- Inflation declines with a 2-quarter lag
- The authors use these findings to inform the structure of their model
- Goal: generate lagged responses of y and π to a monetary shock, within an optimization-based framework

MODEL - MAIN IDEAS

- Representative household *i* consumes a CES aggregate C_t and produces a single differentiated variety
- Supply side features monopolistic competition and sticky prices
- Monetary policy is set according to the policy rule discussed in the empirical section
- Decision lags in consumption and pricing choices

HOUSEHOLD PROBLEM

- Expected Life-time utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t^i, \xi_t) - v(y_t^i, \xi_t)]$$

- Intertemporal budget constraint

$$\mathbb{E}_t \sum_{T=t}^{\infty} \delta_{t,T} P_t C_t^i \leq \mathbb{E}_t \sum_{T=t}^{\infty} \delta_{t,T} [p_t(i) y_t^i - T_t] + A_t^i$$

- where $\delta_{t,T}$ is the SDF, T_t are lump sum taxes, A_t^i are nominal assets at the beginning of period t, ξ_t is an exogenous disturbance/preference shock
- Key assumption: consumption decisions are made two periods in advance $\rightarrow$ C_{t+2} chosen based on information available in period t
- This introduces a lag in the responsiveness of consumption to changes in r

DEMAND BLOCK

- After some manipulation, the demand block can be summarized by 3 equations

$$\hat{Y}_t = -\sigma^{-1}\mathbb{E}_{t-2}\hat{r}_t^l + \hat{G}_t \quad \longrightarrow \quad \text{IS Equation}$$

$$\hat{r}_t^l = \sum_{T=t}^{\infty}\mathbb{E}_t(\hat{R}_T - \hat{\pi}_{T+1}) \quad \longrightarrow \quad \text{Term Structure}$$

$$r_t = r^* + \sum_{k=1}^{n_r}\mu_k(r_{t-k} - r^*) + \sum_{k=0}^{n_\pi}\phi_k(\pi_{t-k} - \pi^*) + \sum_{k=0}^{n_y}\theta_k y_{t-k} + \epsilon_t \quad \longrightarrow \quad \text{MP rule}$$

SUPPLY SIDE

- Each supplier/household chooses prices taking Y_t and P_t as given
- They face the usual CES demand schedule: $y_t^i = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}$
- Real effects of MP come from assumption of sticky prices a la Calvo(1983)
- Each period a fraction $(1-\alpha)$ get to choose prices in t
  - γ of those get to charge the new price from t onwards
  - $1-\gamma$ choose the new price at t but charge it from $t+1$ onwards
  - This generates a lag in the pass-through of shocks to prices
- The price level at t is given by

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha)\gamma(p_t^1)^{1-\theta} + (1-\alpha)(1-\gamma)(p_t^2)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

OPTIMAL PRICE SETTING

- The optimal p_t^i maximizes $\mathbb{E}_{t-i}\phi_t(p)$:

$$\phi_t(p) = \sum_{j=0}^{\infty} \alpha^j \beta^j \left[\lambda_{t+j} p Y_{t+j} \left(\frac{p}{P_{t+j}}\right)^{-\theta} - v\left(Y_{t+j}\left(\frac{p}{P_{t+j}}\right)^{-\theta}; \xi_{t+j}\right)\right]$$

- The first order condition is given by

$$\mathbb{E}_{t-i}\phi_t'(p_t^i) = 0 \qquad \forall \;\; i = 1, 2$$

- After some manipulation, we get an expectations-augmented NK Phillips curve:

$$\mathbb{E}_{t-2}\hat{\pi}_t = \kappa \mathbb{E}_{t-2}(\hat{Y}_t - \hat{Y}_t^S) + \beta \mathbb{E}_{t-2}\hat{\pi}_{t+1}$$

MODEL RECAP

1. Equilibrium conditions

$$\hat{Y}_t = -\sigma^{-1}\mathbb{E}_{t-2}\hat{r}^l_t + \hat{G}_t \quad \rightarrow \quad \text{IS Equation}$$

$$\hat{r}^l_t = \sum_{T=t}^{\infty} \mathbb{E}_t(\hat{R}_T - \hat{\pi}_{T+1}) \quad \rightarrow \quad \text{Term Structure}$$

$$\mathbb{E}_{t-2}\hat{\pi}_t = \kappa\mathbb{E}_{t-2}(\hat{Y}_t - \hat{Y}^s_t) + \beta\mathbb{E}_{t-2}\hat{\pi}_{t+1} \rightarrow \text{AS equation}$$

plus monetary policy rule

2. Important model features
   a. Sticky prices $\rightarrow$ monetary policy affects real side
   b. Consumption choices set in advance $\rightarrow$ r hits output with delay
   c. Pricing lags $\rightarrow$ π takes time to respond to shocks

ESTIMATION OF MODEL PARAMETERS

- Model parameters
  - Monetary rule parameters
  - Parameters governing exogenous processes (ξ_t, G_t)
  - Structural parameters: $\alpha, \beta, \gamma, \sigma, \theta, \omega$
- Monetary rule parameters taken from VAR
- Parameters for stochastic processes $\rightarrow$ match empirical disturbances
- Structural parameters $\rightarrow$ chosen to match IRF to monetary shock from the VAR
- We will focus on this last estimation step

IRF MATCHING

- **Goal:** find the combination of structural parameters that “aligns” the model predictions after a MP shock with the IRF estimated with the VAR
- More concretely, we want to find the combination of parameters that minimizes the distance between the theoretical and empirical impulse responses:

$$\hat{\Theta} = argmin_{\Theta} [\hat{g} - g(\Theta)]' W [\hat{g} - g(\Theta)]$$

- where $g(\Theta)$ is a vector that stacks the model IRFs and $\hat{g}$ is the empirical counterpart
  - Example: we want to match the IRF of each variable up to $h = 4 \rightarrow g(\Theta)$ is a 3×5 vector
- W is a weighting matrix. Identity matrix weights all moments equally
- Alternatively, use the inverse of the variance of each empirical moment $\rightarrow$ less precise moments have a lower weight

IRF MATCHING - RESULTS

- β is calibrated to match the average real rate (standard practice)
- Two structural parameters cannot be identified from the model IRFs
  1. α = .66: calibrated using micro evidence on average price durantion (3 quarters)
  2. ω = .63 – σ: calibrated using data on labor costs
- The vector of estimated parameters is then $\Theta = [\kappa, \sigma, \Psi = \frac{1-\gamma}{\gamma\alpha}]$
- Theoretical responses are plotted as solid lines in Figure 1
- Important things to have in mind in this type of exercise:
  - How well can the model match the data?
  - Plausibility of the estimated values for structural parameters

ROTEMBERG & WOODFORD (1997) - RECAP

- Estimate empirical responses of y, π, r to monetary policy innovations
- Monetary policy innovations identified through a recursive VAR + MP rule functional form
- Build and estimate a model that does a good job of matching the empirical responses
- Not covered today but also in the paper:
  - How to recover the process of real disturbances from VAR results + model structure
  - How to construct counterfactual histories under different monetary rules
  - Welfare consequences of alternative monetary rules