Lecture 6: Optimal Monetary Policy

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Goals

- How should monetary policy be conducted?
  - Broad qualitative consensus among policymakers
    - Central banks are in charge of maintaining low and stable inflation
      - Explicit in the Federal Reserve Act of 1977
      - Explicit in the Maastricht treaty as the goal of the ECB
      - Written in the law and constitutions of countries around the world
    - Actual monetary policy assumes at least some degree of concern with stabilization of economic activity
      - Explicit in the case of the US
      - Not explicit in the case of the ECB
  - Suggests the central bank wants to minimize a “loss-function” that depends on “deviations” of output and inflation from “target”
  - Many questions
    - What is the relative weight of inflation vs. output?
    - What is the right measure of economic activity to stabilize?
    - What is the right inflation index to stabilize?
    - Should the price level or the inflation rate be stabilized?

Approach

- In this lecture we will see what the NK model has to say about optimal monetary policy
- The notion of optimality is to maximize the welfare of private agents
- This *utility-based* approach to policy design has a long tradition in public finance.
  - Examples are the optimal capital tax rate, optimal unemployment insurance, ...
- Notice our consumers do not care directly about prices. They care about leisure and consumption
- But as taxes can create deadweight losses, inflation can too. We saw two potential sources in Lecture 5
  - The aggregate markup can change, inducing too much or too little production
  - Price dispersion may change inducing allocative efficiency costs (misallocation). goods with the same marginal cost will have different prices

Loss Function

- The next few of slides provide a heuristic derivation of the Loss Function.
- I show the math only to make clear the connection between economic objects
- I will never ask you to know this math by heart in a final/qualifying exam
- Not because it’s too hard. Just because it’s pointless for you to memorize it
- I will just gloss over key steps in the lecture

- Notation:
  - U_t: $U(C_t, N_t)$.
  - U_t^n: U_t evaluated at the flexible price allocation
  - U: U_t in the steady state (no shocks whatsoever).
  - U_c and U_n: partial derivatives of U_t with respect to C, N evaluated in the steady state.
  - U_{cc}, U_{nn}: same thing for second derivatives.
  - Assume U is separable in C, N. Therefore $U_{cn} = U_{nc} = 0$.

Second-Order Approximation

- I will do a second order approximation of the utility function around U

$$U_t \approx U + U_c C \frac{C_t - C}{C} + U_n N \frac{N_t - N}{N} + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N} \right)^2$$

- For small deviations, log changes and percentage deviations are approximately equal
- use our functional form assumption that γ and φ are $-C(U_{cc}/U_c)$ and $N(U_{nn}/U_n)$, respectively and impose market clearing $C = Y$.

$$U_t \approx U + U_c C \hat{y}_t + U_n N \hat{n}_t + \frac{1-\sigma}{2} U_c \hat{y}_t^2 + \frac{1+\varphi}{2} \hat{n}_t^2$$

- collect terms and use the results on price dispersion of Lecture 5. (Full details Gali Appendix 4.A)

$$\frac{U_t - U}{U_c C} \approx -\frac{1}{2} \left(\theta var_i(\log P_{it}) + (\gamma + \varphi) \tilde{y}_t^2 \right)$$

Price Dispersion and Inflation

- Let me call $\Delta_t \equiv var_i \log P_{it}$ and let me call $\bar{P}_t = \mathbb{E}_i \log P_{it}$
- and note that due to Calvo.

$$\bar{P}_t - \bar{P}_{t-1} = \mathbb{E}_i(\log P_{it} - \bar{P}_{t-1}) = \cancelto{0}{\lambda\mathbb{E}_i(\log P_{it-1} - \bar{P}_{t-1})} + (1-\lambda)(\log P_t^* - \bar{P}_{t-1})$$

- Now $\Delta_t = var_i \log P_{it} = var_i(\log P_{it} - \bar{P}_{t-1})$ and use the definition of the variance

$$\Delta_t = \mathbb{E}_i\left[(\log P_{it} - \bar{P}_{t-1})^2\right] - (\cancelto{\bar{P}_t}{\mathbb{E}_i \log P_{it}} - \bar{P}_{t-1})^2$$

- And the Calvo property

$$\Delta_t = \lambda\mathbb{E}_i\left[(\log P_{it-1} - \bar{P}_{t-1})^2\right] + (1-\lambda)(\log P_t^* - \bar{P}_{t-1})^2 - (\bar{P}_t - \bar{P}_{t-1})^2$$

- use the first result in the side

$$\Delta_t \approx \lambda\Delta_{t-1} + \frac{\lambda}{1-\lambda}(\pi_t)^2$$

- Since $\bar{P} \approx \log P_t$. See Woodford (2003) Appendix E.2 for precise mathematical statement.

Welfare

- An affine transformation of the objective of the household

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C}$$

- Use our second order approximation:

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\theta var_i(\log P_{it}) + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

- Iterate Δ_t forward and compute its present value (Details Woodford Chapter 6 2.2).

$$\sum_{t=0}^{\infty} \beta^t \Delta_t \propto \frac{\lambda}{(1-\lambda)(1-\lambda\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$$

- Finding

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

Result 1: In the Calvo model price dispersion is very costly

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

- Key Economics
  - Households care about consumption and leisure.
  - They don't like consuming too little, or working too much vs. the efficient allocation
  - Output gap captures the differences
  - Price dispersion is a symptom of misallocation and it reduces welfare
  - Due to Calvo, inflation and price dispersion are tightly linked. This depends on Calvo!
- Back of the envelope calculations
  - Imagine utility is log in C and linear in N. Then $\gamma + \varphi = 1$.
  - Typical values for $\theta \in [4, 7]$. Let's pick $\theta = 4$
  - If $\lambda = 0.9$ and $\beta = 0.995$, then $\alpha \approx 0.01$
  - So $\theta/\alpha \approx 330$
  - Inflation is waaaaaaaay more costly than output gaps

Optimal Policy

$$\max_{\pi_t, \tilde{y}_t} \mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

- Subject to

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n)$$

- We are looking for sequences for $\left\{\hat{i}, \tilde{y}, \hat{\pi}\right\}$ such that $\mathcal{W}$ is maximized

Optimal Policy

- It is useful to separate the problem in two
  - Pick a sequence for $\{\tilde{y}, \hat{\pi}\}$ that maximizes $\mathcal{W}$ subject to the Phillips curve

$$\max_{\pi_t, \tilde{y}_t} \mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

    Subject to

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$$

  - Find a sequence for $\{\hat{i}\}$ that satisfies

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n)$$

- Solution
  - $\hat{\pi}_t = 0$, $\hat{y}_t = \hat{y}_t^n \; \forall t$ (so that $\tilde{y}_t = 0$) minimizes the objective and respects the Phillips curve
  - Pick $\hat{i}_t = \hat{r}_t^n \; \forall t$

Result 2: The Divine Coincidence

- The central bank does not want to stabilize output, it wants to stabilize the output gap!
- Divine Coincidence: The central bank can minimize output gap **and** inflation deviations simultaneously. No tradeoff.
- Notice: Central bank does not want to induce monetary policy shocks. $\hat{i}_t = \hat{r}_t^n \; \forall t$. Optimal policy 100% systematic.
- Notice: Optimal policy informationally very heavy. Central bank must now r_t^n perfectly.
- Shocks to the IS curve (demand shocks) are offset via movements of i

The Divine Coincidence: Intuition

- The flexible price allocation is optimal
- Nominal rigidities are the only constraint to reach this allocation
- If the constraint does not bind (happens with zero inflation), there is no distortion
- So neutralizing inflation implies neutralization of output gap

The Unrealistic Divine Coincidence

- If you were to ask central bankers, they would not agree that stabilizing one objective necessarily stabilizes the other.
- Rough response you would get. There are some shocks that increase inflation, and stabilizing inflation would push down output relative to the desired level
- Simple way in this model to capture that intuition: if the flexible price equilibrium is not efficient

Time-Varying Efficient Output Gap

- In our model so far, the outcome under flexible prices, $\hat{y}_t^n$, is also the efficient (first-best) outcome $\hat{y}_t^{eff}$.
- The central bank will face a trade-off and the divine coincidence will break once these are no longer the same.
- The welfare function will now penalize deviations of output from the efficient level of output $\hat{y}_t - \hat{y}_t^{eff}$. The Phillips Curve is:

$$
\begin{aligned}
\hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} \underbrace{-\kappa(\hat{y}_t^n - \hat{y}_t^{eff})}_{\equiv u_t} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + u_t
\end{aligned}
$$

- We call u_t a "cost-push shock."
  - Exogenous increase in marginal costs.

Cost Push and the Labor Wedge

- What are cost push shocks?
  - Anything that moves the labor wedge beyond sticky prices.
- Let μ_t^W be the log of a time-varying exogenous wage markup:

$$\hat{w}_t - \hat{p}_t = \hat{\mu}_t^W + \varphi \hat{n}_t + \gamma \hat{c}_t$$

- Then the Phillips curve becomes:

$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + \alpha \hat{\mu}_t^W$$

- Intuition:
  - Higher mark-ups mean higher inflation and lower output.
  - Central bank wants to offset this inefficient shock, but can only move output and inflation in the same direction.

The Planning Problem With A Tradeoff

$$\frac{1}{2}E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\vartheta(\hat{y}_{t+s}-\hat{y}^{eff}_{t+s})^2+\hat{\pi}^2_{t+s}\right]\right\}$$

subject to

$$\hat{y}_t=-\sigma E_t\{\hat{i}_t-\hat{\pi}_{t+1}\}+E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t=\kappa(\hat{y}_t-\hat{y}^{eff}_t)+\beta E_t\{\hat{\pi}_{t+1}\}+u_t$$

- First stage is optimizing objective function subject to NKPC.
  - Cost push shock increases $\hat{\pi}_t$.
  - To offset it, can push down output relative to efficient output $\hat{y}_t-\hat{y}^{eff}_t$.
  - Thus there is now a tradeoff.
  - Intuitively, monetary policy shifts aggregate demand but u_t is a shock to aggregate supply, so there is a tradeoff.

The Planning Problem: Rules vs. Discretion

- Standard quadratic loss function with linear constraints.
  - But targets depend on expectations of future policy.
  - To see this, iterate forward to get:

$$\hat{\pi}_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\kappa \left(\hat{y}_{t+s} - \hat{y}^{eff}_{t+s} \right) + u_{t+s} \right] \right\}$$

$$\hat{y}_t = E_t \left\{ \sum_{s=0}^{\infty} \left[-\sigma \left(\hat{i}_{t+s} - \hat{\pi}_{t+s+1} \right) + g_{t+s} \right] \right\}$$

- This raises issues of credibility and time consistency of policy.
  - Central bank can influence outcomes today by “promising” outcomes tomorrow.
  - But are those promises credible?
  - See Gali 5.3 for the solution to the commitment case.

The Discretionary Problem

- For today, we assume that the central bank follows a *discretionary optimal policy*.
  - Cannot make credible commitments about future actions (hopefully have time to discuss why)
  - So optimize *taking expectations of future actions as given*.
- Solve

$$\min_{\hat{\pi}_t,\hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

where

$$F_t = \frac{1}{2}E_t\left\{\sum_{s=1}^{\infty}\beta^s\left[\vartheta(\hat{y}_{t+s} - \hat{y}_{t+s}^{eff})^2 + \hat{\pi}_{t+s}^2\right]\right\}$$
$$f_t = \beta\hat{\pi}_{t+1} + u_t$$

are functions of expectations of future actions.

“Lean Against the Wind” Policy

$$\min_{\hat{\pi}_t, \hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

- The First order condition is

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta}\hat{\pi}_t$$

- “Lean Against The Wind” Policy.
  - In face of inflationary pressures from cost push shocks, *drive output below its efficient level to dampen rise in inflation*.
  - Extent to which it does so depends on:
    - κ, which determines reduced inflation per unit of output loss.
    - ϑ, the relative weight placed on output loss.
- Flip from “Old Keynesian” logic where stabilizing output at cost of inflation.

Inflation and Output Under Discretion

- Plug policy into Phillips curve:

$$\hat{\pi}_t = \frac{\vartheta\beta}{\vartheta + \kappa^2} E_t \hat{\pi}_{t+1} + \frac{\vartheta}{\vartheta + \kappa^2} u_t$$

- And iterate forward to get:

$$\hat{\pi}_t = \frac{\vartheta}{\vartheta(1 - \beta\rho_u) + \kappa^2} u_t$$

- Combine with optimality condition to get

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta(1 - \beta\rho_u) + \kappa^2} u_t$$

- So central bank lets output gap and inflation fluctuate in proportion to current value of cost push shock.
  - Intuition: Cost push increases inflation, central bank wants to smooth both so trades some inflation for output.

Interest Rate Under Discretion

- Plugging into dynamic IS:

$$\hat{y}_t = -\sigma E_t\{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t\{\hat{y}_{t+1}\} + g_t$$

obtains

$$\hat{i}_t = \hat{r}^{eff}_{t+1} + \phi_\pi \hat{\pi}_t + \frac{g_t}{\sigma}$$

where

$$\hat{r}^{e}_{t+1} = \frac{1}{\sigma} E_t\{\hat{y}^{eff}_{t+1} - \hat{y}^{eff}_t\}, \qquad \phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\sigma[\vartheta(1-\beta\rho_u) + \kappa^2]}$$

- Central bank implements the optimal outcome with what looks like an interest rate (Taylor) rule.