Lecture 13: The Financial Accelerator

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Goal for Today

We want to capture the following narrative of booms and busts

- In good times
  - In good times it is easier for firms to access funding
  - So they can invest more
  - Expanding output, and further making easy to access funding
- In bad times
  - It is harder for firms to access debt
  - Curtailing the ability of firms to invest
  - Which reduces the capital stock in the future, and depresses production
  - Doom loop.

We will plug our Costly State Verification model into a stripped-down version of the RBC model for simplicity

Messages from CSV Models

Last lecture

- Asymmetric Information problem between borrowers and savers
- Think of borrowers as firms
- Think of lenders as banks
- Could write similar model between banks and bond-holders

This lecture

- Relax the assumption of a fixed r and fixed W
- Amplification of financial frictions with the business cycle
- Following Bernanke Gertler (1989)

Setting

Standard RBC model (As in 210A) with financial frictions

- A production function with TFP shocks, lowercase variables in per-capita terms

$$y_t = \tilde{\theta}_t f(k_t)$$

- with the non-standard assumption that $f(0) > 0$
- Investment technologies take t output and transform it into $t+1$ capital
- Two type of agents. Entrepreneurs and lenders. η share of entrepreneurs
- Heterogeneous entrepreneurs, homogeneous lenders
- OLG model: Young, old, exit. Think of an agent as a project that needs funding

Entrepreneurs and Capital Accumulation

- Only *entrepreneurs* have investment technologies.
- Entrepreneurs come in types $\omega \sim U[0, 1]$
- Input: Entrepreneur of type ω uses $x(\omega)$ units of output exactly to produce capital
- Result: κ_i units of capital, where $i = 1, .., n$ with $\kappa_j \geq \kappa_k$ whenever $j > k$, probabilities π_i.
- $\mathbb{E}\kappa_i = \kappa$
- κ uncorrelated with ω
- Whenever the economy needs more capital, more entrepreneurs will be active...
- the marginal entrepreneur will be worse than the average entrepreneur
- so the supply curve of capital will be upward sloping

Information

- realization κ_i is the private information of the entrepreneur
- Lenders have a unique auditing technology that absorbs γ units of capital
- When used, it reveals the outcome with probability 1
- Timing:
  - κ's are realized, owners announce $\tilde{\kappa}$, auditing takes place, then $\tilde{\theta}$ is realized
  - law of motion of capital

$$k_{t+1} = (\kappa - h_t\gamma)i_t$$

  - k, i, h endogenous

Savings

- OLG setting + risk neutrality wrt $t+1$ consumption
- Savings
  - Entrepreneur: $S_t^e = w_t L_t^e$
  - Lenders: $S_t = w_t L - z_y^*(r)$
- Link between savings and wage rates.
- Note: assumptions such that savings > investment, so that the marginal return on savings is r, the return on an outside option inventory holding technology

Perfect Information

Case with $\gamma = 0$.

- Indifference condition (in expectation) pins down the marginal entrepreneur $\bar{\omega}$

$$\hat{q}_{t+1}\kappa - rx(\bar{\omega}) = 0$$

- $\hat{q}_{t+1}$ is the expected price of capital tomorrow
- Everybody with $\omega < \bar{\omega}$ operates their technology
- Savings exceed investment demand by assumption (marginal return = r)

$$\eta S_t^e + (1-\eta)S_t > \int_0^{\bar{\omega}} x(\omega)d\omega$$

- $i_t = \bar{\omega}\eta$ and $k_{t+1} = \kappa i_t$, so that the capital supply curve:

$$\hat{q}_{t+1} = rx\left(\frac{k_{t+1}}{\kappa\eta}\right)/\kappa$$

Perfect Information

Case with $\gamma = 0$.

- Capital supply curve:

$$\hat{q}_{t+1} = rx\left(\frac{k_{t+1}}{\kappa\eta}\right)/\kappa$$

- Capital demand curve

$$\hat{q}_{t+1} = \theta f'(k_{t+1})$$

- The intersection of supply and demand is independent from period t variables

$$\hat{q}_{t+1} = q$$

$$k_{t+1} = k$$

- Feature: $i = \bar{\omega}\eta$, because $\bar{\omega}$ is constant due to $\hat{q}$ being constant

Asymmetric Information

Case with $\gamma > 0$

- Objective: Maximize entrepreneurs consumption

$$\max \pi_1(pc^a + (1-p)c_1) + \pi_2 c_2$$

- Subject to:
  - Participation constraint on the lenders (funding the project is better than saving in inventories)

$$\pi_1(\hat{q}\kappa_1 - (1-p)c_1 - p(c^a + \gamma\hat{q})) + \pi_2(\hat{q}\kappa_2 - c_2) \geq r(x - S^e)$$

  - IC constraint (announcing the good state is preferable to lie, and sell the excess capital in the market)

$$c_2 \geq p(0) + (1-p)(c_1 + (\kappa_2 - \kappa_1)\hat{q})$$

  - Limited liability + feasibility constraint

$$c_1 \geq 0, c^a \geq 0, 0 \leq p \leq 1$$

Two Regimes with $n = 2$ states

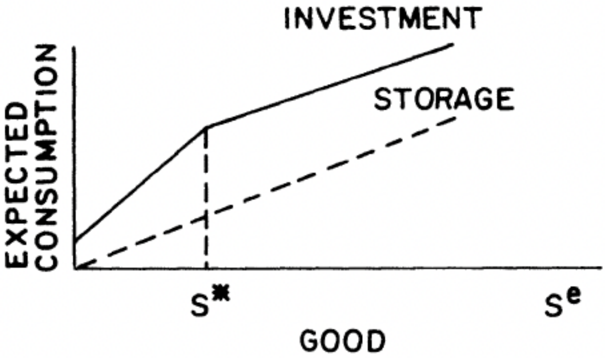
- Case 1: The entrepreneur is sufficiently wealthy (full collateralization)
  - Even if the bad state materializes $\hat{q}\kappa_1 \geq r(x(\omega) - S^e)$
  - No agency costs (distribution of states is public info)
- $\hat{q}\kappa_1 < r(x(\omega) - S^e)$ (incomplete collateralization)
  - Optimal auditing probability

$$p(\omega) = \frac{r(x(\omega) - S^e) - \hat{q}\kappa_1}{\pi_2 \hat{q}(\kappa_2 - \kappa_1) - \pi_1 \hat{q}\gamma}$$

  - (comes from setting every constraint with equality)

In the full collateralization case, outside finance is provided at rate r (no risk)

Good entrepreneur



consumption rises faster than slope *r* (dotted line) since financial costs decrease with wealth

Bad entrepreneur

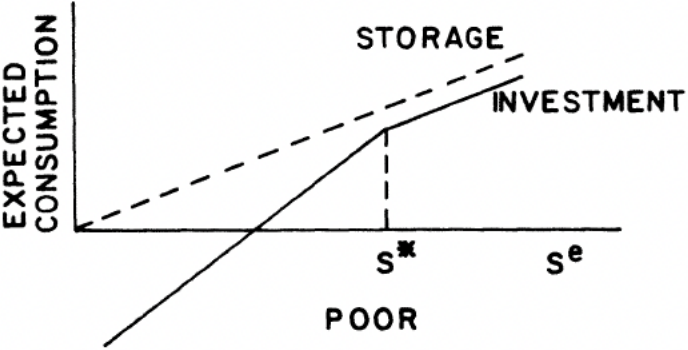
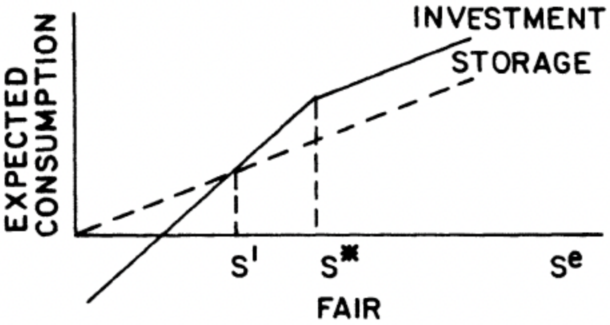


FIGURE 2

Fair entrepreneur



Convex envelope between 0 and S^*. These entrepreneurs will enter a lottery (how realistic?).
For given ω they will enter a lottery that pays S^* with probability $g(\omega) = S^e/S(\omega)^*$

Capital Supply

- Aggregate Capital tomorrow

$$k_{t+1} = \eta\left[\kappa\underline{\omega} - \pi_1\gamma\int_0^{\underline{\omega}} p(\omega)d\omega + \kappa\int_{\underline{\omega}}^{\bar{\omega}} g(\omega)d\omega\right]$$

- Can be rewritten as:

$$k_{t+1} = \eta\left[\kappa\bar{\omega} - \int_0^{\underline{\omega}} \pi_1\gamma p(\omega)d\omega - \int_{\underline{\omega}}^{\bar{\omega}} \kappa(1 - g(\omega))d\omega\right]$$

- $\underline{\omega}, \bar{\omega}, p(\omega), g(\omega)$ are all functions of q.
- This equation is the supply curve of capital (with $dk_{t+1}/d\hat{q}_{t+1} > 0$)
- Problem set question: Prove $dk_{t+1}/d\hat{q}_{t+1} > 0$

Some Properties of Capital Supply

$$k_{t+1} = \eta \left[\kappa\bar{\omega} - \int_0^{\underline{\omega}} \pi_1 \gamma p(\omega) d\omega - \int_{\underline{\omega}}^{\bar{\omega}} \kappa(1 - g(\omega)) d\omega \right]$$

- The supply schedule is to the left of the FI supply

$$k_{t+1} \leq \kappa\bar{\omega}\eta$$

- For sufficiently large $(k_{t+1}, \hat{q}_{t+1})$, the AI and FI schedules coincide
  - $p(\omega) \to 0$, and $g(\omega) \to 1$.
- $t + 1$ outcomes depend on t variables

$$p(\omega) = \frac{r(x(\omega) - S^e) - \hat{q}\kappa_1}{\pi_2 \hat{q}(\kappa_2 - \kappa_1) - \pi_1 \hat{q}\gamma}$$

Supply and Demand of Capital

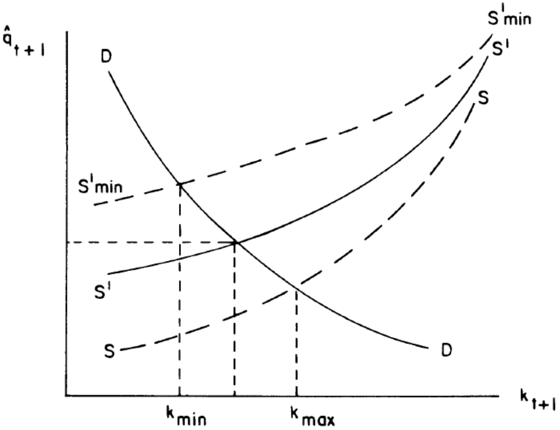


FIGURE 3

Notice the demand of capital is given by $\hat{q}_{t+1} = \theta f'(k_{t+1})$

Accelerator

- Imagine a negative shock that drives down savings (via labor income for example)
- Lower wealth increases agency problems $\rightarrow \uparrow p$
- Increases inspection costs
  - Shifts capital supply to the left
  - Financed projects become more costly to finance
  - Less projects are financed
- Lower investment, and capital tomorrow
- Less production tomorrow, and lower savings. Doom loop.

Persistent investment slumps in which financing and producing capital is costly