Lecture 5: The New Keynesian Model Volume II

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The Three Equation Model

$$
\begin{aligned}
\hat{y}_t &= -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} \\
\hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^n) + \beta E_t\{\hat{\pi}_{t+1}\} \\
\hat{i}_t &= \phi_\pi \hat{\pi}_t + v_t
\end{aligned}
$$

- Imagine a contractionary MP shock $v_t > 0$ that increases the nominal interest rate
- Returns to saving increase, households intertemporally substitute, decreasing $\hat{c}_t$ by σ
- Market clearing implies that $\hat{y}_t$ decreases
- Since $\hat{y}_t$ decreases inflation by κ
- Households form model-consistent expectations about $\hat{\pi}_{t+1}$
- The decrease in inflation counteracts the effects on $\hat{i}$
- Changes in $E_t\{\hat{\pi}_{t+1}\}$ affect output
- and on and on and on

What is an equilibrium? A point in which all equations are consistent

The New Keynesian Model in words

- Interest rates go down. Households intertemporally substitute to consume more. Consumption demand needs to be satisfied, so firms need to produce more. To produce more, firms need to hire more labor. To hire more labor, firms need to pay higher wages. Higher wages increase the marginal cost of production of firms. Firms pass-through those costs to prices according to the slope of the Phillips curve, generating inflation.
- Key parameters:
  - How strongly households intertemporally substitute: $\sigma = 1/\gamma$
  - How much wages have to go up to accomodate more demand (Frisch, wealth effects) φ, γ
  - How much marginal costs push inflation κ.

Today

- Today we will do something very simple. We will inspect this model “under the hood”
- Is the NK model efficient?
- What are its sources of inefficiency?
- What do we lose by log-linearizing?
- What is the shape of the IRFs implied by the model?
- How do they correspond with those estimated in the data?

Inefficient Markups in Steady State

- The descentralized equilibrium is inefficient even with flexible prices
- Intuition: Firms are too small as they try to exploit their monopoly power
- “too small” means that the planner would choose larger firm sizes
- Caution. That in this model markups are inefficient does not mean that markups are always inefficient. Large class of models with entry costs, where without markups production would not take place.

Inefficient Markup in Steady State

- Descentralized economy under flexible prices

$$1 = \frac{P_{it}}{P_t} = \frac{\theta}{\theta - 1}\frac{W_t}{A_t}\frac{1}{P_t}$$

- Use the labor supply curve $\frac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\gamma}$
- And use that under flexible prices $L_t = \int_0^1 L_{it} di = \frac{Y_t}{A_t}$ to find:

$$Y_t = A_t^{\frac{\varphi+1}{\gamma+\varphi}} \chi^{-1/(\gamma+\varphi)} \left(\frac{\theta}{\theta - 1}\right)^{-1/(\gamma+\varphi)}$$

- Output is decreasing in the flexible price markup

Inefficient Markup in Steady State

- Compare with a planner that chooses allocations directly
- The planner will choose $C_{it} = C_{jt} \; \forall i, j$
- So the problem of the planner is

$$\max \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}$$

- Subject to market clearing and technologies $C_t = Y_t$, and $Y_t = A_t N_t$
- Solve that simple problem

$$Y_t = A_t^{\frac{\varphi+1}{\gamma+\varphi}} \chi^{-1/(\gamma+\varphi)}$$

- This solution guarantees that $MRS_t = MPL_t$ which given our functional forms reduces to

$$\chi N_t^{\varphi} C_t^{\gamma} = A_t$$

Inefficient Markup in Steady State

- Descentralized Equilibrium yields:

$$Y_t = A_t^{\frac{\varphi+1}{\gamma+\varphi}} \chi^{-1/(\gamma+\varphi)} \left(\frac{\theta}{\theta-1}\right)^{-1/(\gamma+\varphi)}$$

- And Planner's problem

$$Y_t = A_t^{\frac{\varphi+1}{\gamma+\varphi}} \chi^{-1/(\gamma+\varphi)}$$

- Can check descentralized equilibrium attains lower welfare. Inefficiency.

Inefficient Markup in Steady State

- Easy to correct if the planner has access to lump-sum transfers
- Idea: The issue is that firms charge prices $P_{it} = \frac{\theta}{\theta-1}\frac{W_t}{A_t}$ as opposed to $P_{it} = \frac{W_t}{A_t}$
- Imagine the planner imposes a payroll subsidy. Firms pay $W_t(1-\tau)$ for each worker. Optimal pricing yields

$$P_{it} = \frac{\theta}{\theta - 1}\frac{W_t(1-\tau)}{A_t}$$

- If planner chooses $\tau = 1/\theta$, then $\frac{\theta}{\theta-1}(1-\tau) = 1$, recovering efficiency
- Key: Access to lump-sum transfers so that households pay for the subsidy without distorting their labor supply

Inefficient Cyclical Markup

- Define the aggregate markup as the ratio of aggregate prices to aggregate marginal costs

$$\mathcal{M}_t = \frac{P_t}{MC_t} = \frac{P_t}{(1-\tau)W_t/A_t}$$

- Solve for the real wage

$$\frac{W_t}{P_t} = \frac{A_t}{(1-\tau)\mathcal{M}_t}$$

- Notice that $\frac{1}{(1-\tau)} = \frac{\theta}{\theta-1} \equiv \mathcal{M}$

$$\frac{W_t}{P_t} = A_t \frac{\mathcal{M}}{\mathcal{M}_t}$$

- Unless $\frac{\mathcal{M}}{\mathcal{M}_t} = 1$, then

$$MRS_t \neq MPL_t$$

Inefficient Cyclical Markup

$$\frac{W_t}{P_t} = A_t \frac{\mathcal{M}}{\mathcal{M}_t}$$

- Intuition. Due to price rigidity, an aggregate shock may move the average markup that firms charge
- The aggregate markup will be either too large or too small. And consequently, there will be too little or too much production
- Because this inefficiency is dynamic (it has a t subscript), it cannot be corrected with a constant tax/subsidy
- Can only be corrected with an instrument that keeps the aggregate markup constant at $\mathcal{M}$.

Inefficient Price Dispersion

- Price rigidity introduces additional inefficiencies
- Intuition. Varieties enter symmetrically in the utility function
- They all have the same marginal cost
- But due to staggered price adjustment they will have different prices
- Different goods that cost the same will have different prices
- Inducing *relative price* dispersion
- The planner does not like that.

Inefficient Price Dispersion

- Aggregate labor demand

$$L_t = \int_0^1 L_{it} di \tag{1}$$

- Use the production function and the demand curve

$$L_t = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\theta} di \tag{2}$$

- Solve for Y

$$Y_t = D_t A_t L_t$$

- for $D_t = \left(\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\theta} di \right)^{-1}$
- D as a productivity wedge on the aggregate production function

Inefficient Price Dispersion

- A second-order approximation of $\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\theta} di$ yields:

$$\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\theta} di \approx 1 + \frac{1}{2}\theta var_i(\hat{p}_{it}) > 1$$

- Proof: Gali appendix 3.3
- Price dispersion reduces aggregate output (and consumption)
- Price dispersion only shows up to the second-order
- A log-linearized model misses this crucial channel
- Key insight: Even though consumers do not care about prices directly, staggered price adjustment induces an inefficiency on aggregate consumption, with consequences on welfare

Key Economics

- Model with flexible prices:
  - Workers are on their labor supply curve.

$$\frac{W_t}{P_t} = \chi C_t^{\gamma} N_t^{\varphi} = MRS_t$$

    always holds
  - Firms are on their labor demand curves

$$\frac{W_t}{P_t} = A_t F'(N_t) = MPL_t$$

    always holds
  - therefore

$$MPL_t = MRS_t$$

Key Economics

- Model with sticky prices
  - Workers are on their labor supply curve.

$$\frac{W_t}{P_t} = \chi C_t^{\gamma} N_t^{\varphi} = MRS_t$$

    always holds
  - Firms are **not** on their labor demand curves

$$\frac{W_t}{P_t} \neq A_t F'(N_t) = MPL_t$$

    does not need to hold
  - What determines labor demand then?
  - Product demand + production function

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma(i_t - \mathbb{E}_t \hat{\pi}_{t+1})$$

  - Makes clear that the central bank is key to determine market clearing in the labor market

The natural rate of interest

- Our Euler equation $\hat{y}_t = -\sigma[\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}] + \mathbb{E}_t\hat{y}_{t+1}$
- We can rewrite in terms of the output gap. Subtract and add $\hat{y}_t^n$ and $\mathbb{E}_t\hat{y}_{t+1}^n$

$$\tilde{y}_t = -\sigma[\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}] + \mathbb{E}_t\tilde{y}_{t+1} + \mathbb{E}_t\Delta\hat{y}_{t+1}^n \tag{3}$$

- where $\tilde{y} = \hat{y} - \hat{y}^n$ is the output gap.
- Or alternatively

$$\tilde{y}_t = -\sigma[\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_t^n] + \mathbb{E}_t\tilde{y}_{t+1} \tag{4}$$

- where $\hat{r}_t^n = \frac{1}{\sigma}\mathbb{E}_t\Delta\hat{y}_{t+1}^n$ is the real interest rate that prevails in the flexible price equilibrium
- Clarifies that $\tilde{y}$ reacts to the gap in real interest rates wrt flexible price eq.
- The natural rate of interest will be important when we think of optimal policy design
- Intuitively, if the central bank can keep a real rate consistent with the natural rate, then the output gap will be zero

Length of Monetary Policy Effects

- Tempting to think the following:

  If 10% of firms adjust their prices every month, then after 10 months every firm will have adjusted their prices, and the economy will behave as if money was neutral starting in month 11, so the effects of monetary shocks will vanish pretty soon

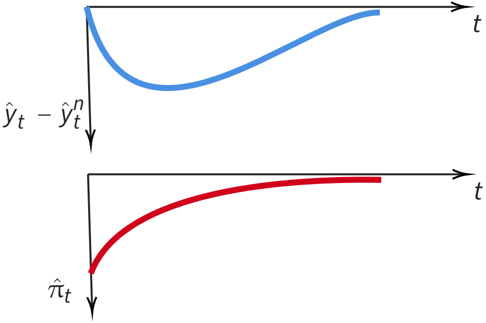
- I get migraines whenever I hear that. Two issues:
  - (1): In the Calvo model price adjustment is random, so after 10 months, 35% of firms have yet to adjust. Many price adjustments will be "wasted" on firms that have adjusted already.
  - (2): The argument assumes that adjusters will change their price to flexible price counterpart when given the chance. But that may not be the case if:
    - Firms do not want to separate too much from their competitors to not lose demand
    - Nominal frictions make it so that marginal costs (think wages) do not move much
  - These reasons are called in the literature "real rigidities" and we will talk about them extensively.
  - In terms of math. There is a difference between the effect of price rigidity α, and κ. κ and α may be substantially different

The New Keynesian Phillips Curve is front-loaded

- Inflation is the PV of future expected output gaps × the slope of the Phillips curve

$$\hat{\pi}_t = \kappa \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\hat{y}_{t+k} - \hat{y}^n_{t+k}) \tag{5}$$

- News of a sequence of $(\hat{y}_{t+k} - \hat{y}^n_{t+k}) < 0$. Inflation reaches a trough at period t



NK Model is too forward looking

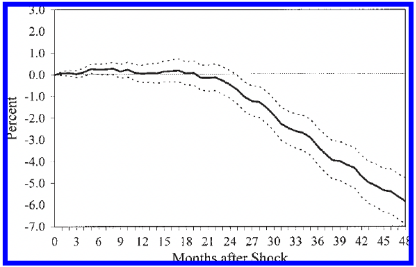


FIGURE 4. THE EFFECT OF MONETARY POLICY ON THE PRICE LEVEL

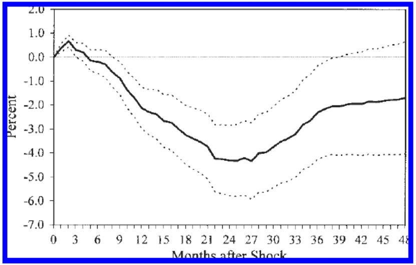


FIGURE 2. THE EFFECT OF MONETARY POLICY ON OUTPUT

IRF of the Price Level and Output. Romer and Romer (2004)

- In the NK model, the CPI decreases on impact and keeps declining
- In the data, the CPI seems to take a long time to react
- Probably a failure of the model rather than one of causal inference.

Estimating the NK model is hard

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} - \kappa(\hat{y}_t - \hat{y}_t^n) + \hat{\omega}_t \tag{6}$$

- Two parameters β, κ
- Many unobservables $\mathbb{E}_t \hat{\pi}_{t+1}$, $\hat{y}_t^n$, $\hat{\omega}_t$
- $\hat{y}_t$ may be dynamically correlated with those unobservables. Huge omitted variable bias problems
- We will spend some time thinking about this

Estimating the NK model is hard

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} \tag{7}$$

- one parameter σ. Actually two, that future output enters with coefficient of 1 is a testable implication.
- Many unobservables $\mathbb{E}_t\hat{\pi}_{t+1}, \mathbb{E}_t\hat{y}_{t+1}$