210C - Part 1: Juan

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1 Convex and non-convex capital adjustment costs

The exam has a total of 100 points. Please read the exam carefully.

Time is discrete. A firm is interested in maximizing its value, which consists of the present value of the stream of profits. The firm uses the Stochastic Discount Factor of the representative household Λ to discount the future. The firm takes Λ as given. Firms revenues (R) depend on firm-level productivity A (exogenous) and its current capital stock k via a production technology with decreasing returns to scale on capital ($\theta < 1$). Specifically,

$$R = Ak^{\theta}. \tag{1}$$

After production, a firm may decide to invest. Whenever a firm invests an amount of resources i, it must pay for that investment expenditure, and on top of that, incur in convex and non-convex costs of adjustment. Total investment expenditures for the firm are then

$$\text{Investment Expenditures} = i + \frac{\psi}{2}\left(\frac{i}{k}\right)^2 k + \chi \mathbb{1}_{i\neq 0}, \tag{2}$$

where i denotes investment, ψ is a parameter that dictates the size of convex adjustment costs, χ is a constant that dictates the size of non-convex adjustment costs, and $\mathbb{1}_{i\neq 0}$ is the indicator function that takes the value of 1 whenever $i \neq 0$.

The law of motion of capital is given by

$$k' = (1-\delta)k + i, \tag{3}$$

where k' denotes next period's capital stock and δ is the rate of depreciation of capital.

It will be useful to summarize the firm's problem via a value function V, that makes clear that the firm problem has a discrete choice aspect. The firm may choose to invest receiving a value function V^a after which it pays the fixed cost, or it may choose not to invest, receiving a value function V^n and no fixed cost is paid, therefore

$$V(A,k) = R + \max\left[V^n(A,K), V^a(A,k) - \chi \mathbb{1}_{i\neq 0}\right], \tag{4}$$

and V^a and V^n are Bellman equations as well.

1. **15 points:** Write the value function of the firm conditional on adjusting its capital stock (V^a). Hint: Notice that V already subtracts non-convex costs and adds R.

2. **15 points:** Write the value function of this firm conditional on non-adjustment of its capital stock and call it $V^n(A,k)$. Hint: Notice that V already includes R.

3. **20 points:** Write the problem of the firm conditional on adjustment using the value function $V^a(A,k)$. Make sure to make explicit the objective of the firm (what the firm wants to maximize or minimize), the control variables of the firm (which variables the firm chooses), and the constraints that it must respect.

4. **20 points:** State the optimality condition for investment conditional on adjustment $i^a(A,k)$, as a function of parameters of the model, state variables, and the expectation of the marginal value of one additional unit of capital to the firm.

5. **10 points:** The fixed cost χ is distributed according to a uniform distribution with support $[0,\bar{\chi}]$. Write a function called $\Omega(A,k)$ that describes the probability of adjustment of a firm with state A,k. A hint that may be useful is that for a variable x distributed uniformly with support $[a,b]$, the $P(x<c) = \frac{c-a}{b-a}$ for any $a \leq c \leq b$.

6. **20 points:** In maximum one short paragraph, link the mathematical objects you have derived to the economic concepts of the "intensive margin of investment" and "the extensive margin of investment".