INTRO TO DYNARE

GOVERNMENT SPENDING IN THE 3-EQUATION NEW KEYNESIAN MODEL

Discussion ECON 210C - Paula Donaldson

ADD GOVERNMENT SPENDING TO THE 3-EQUATION MODEL

- The benchmark 3-equation New Keynesian (NK) model includes:
  - A forward-looking IS curve
  - A New Keynesian Phillips Curve (NKPC)
  - A Taylor rule
- We now extend this model by introducing **government spending**, g_t
- We will code the model and study how g_t affects inflation, output, and the natural interest rate

HOUSEHOLD PROBLEM AND RESOURCE CONSTRAINT

- Households choose C_t, N_t to maximize utility subject to budget constraint
- They have Dixit-Stiglitz preferences over a continuum of goods i as we saw in class.
- First-order conditions give:

$$\hat{w}_t = \gamma \hat{c}_t + \phi \hat{n}_t$$
$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1})$$

- plus demand schedule for individual varieties
- Resource constraint with government spending:

$$\hat{y}_t = (1 - S_g)\hat{c}_t + S_g \hat{g}_t$$

- where S_g is the steady-state share of government spending in output

GOVERNMENT SPENDING PROCESS

- The government purchases individual varieties according to the same CES function as households

$$G_{it} = \left(\int_0^1 g_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}$$

- Total government purchases follow an AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + u_g$$

SUPPLY SIDE

- Same as in class. Firms engage in monopolistic competition and set prices a la Calvo.
- The production function is Cobb-Douglas with labor as the only input.

$$\hat{y}_t = \hat{n}_t$$

- Marginal costs are given by:

$$\hat{mc}_t = \hat{w}_t$$

- The Phillips curve is given by:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{mc}_t$$

MODEL EQUATIONS: 8-EQUATION SYSTEM

$$(1)\ \hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1})$$

$$(3)\ \hat{w}_t = \gamma\hat{c}_t + \phi\hat{n}_t$$

$$(4)\ \hat{y}_t = \hat{n}_t$$

$$(4)\ \hat{mc}_t = \hat{w}_t$$

$$(2)\ \hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa\hat{mc}_t$$

$$(5)\ \hat{y}_t = (1 - S_g)\hat{c}_t + S_g\hat{g}_t$$

$$(6)\ \hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t$$

$$(7)\ \hat{g}_t = \rho_g\hat{g}_{t-1} + u_g$$

$$(8)\ \tilde{y}_t = \hat{y}_t - \hat{y}_t^{\text{flex}}$$

REDUCING TO A 3-EQUATION SYSTEM

- Define $\tilde{y}_t$ as the output gap
- Substitute equations to express model in terms of $\tilde{y}_t$, $\hat{\pi}_t$, and $\hat{i}_t$
- Details at the end of the slides

Reduced System:

$$\begin{aligned}
\text{Phillips Curve:} \quad & \hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa\omega\tilde{y}_t \\
\text{IS Curve:} \quad & \tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1 - S_g}{\gamma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - r_t^n) \\
\text{Monetary Policy:} \quad & \hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t \\
\text{where } r_t^n\text{:} \quad & r_t^n = -\frac{\gamma}{1 - S_g} \frac{\phi(1 - S_g)S_g}{\phi(1 - S_g) + \gamma}(\rho_g - 1)\hat{g}_t
\end{aligned}$$

- plus the exogenous government spending process

NEXT STEPS: BRING THE MODEL TO DYNARE

- Solve the 3-equation model in Dynare
- Simulate IRFs to a g_t shock
- Explore how IRFs change under:
  - Different ϕ_π (inflation response)
  - Different ϕ_y (output gap response)
- Compare inflation, output gap, interest rate, and r_t^n

DETAILS: REDUCE MODEL TO 3-EQUATION SYSTEM

- First, we substitute for consumption in the Euler equation and consumption-leisure equation using the resource constraint.

$$\frac{1}{1-S_g}\hat{y}_t - \frac{S_g}{1-S_g}\hat{g}_t = \frac{1}{1-S_g}E_t\hat{y}_{t+1} - \frac{S_g}{1-S_g}E_t\hat{g}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - E_t\pi_{t+1})$$

$$\phi\hat{y}_t = \gamma\frac{1}{1-S_g}\hat{y}_t - \gamma\frac{S_g}{1-S_g}\hat{g}_t + \hat{w}_t$$

- Substitute the real wage by its expression from the labor demand schedule:

$$\frac{\phi(1-S_g)+\gamma}{(1-S_g)}\hat{y}_t = \frac{\gamma S_g}{(1-S_g)}\hat{g}_t + \hat{mc}_t$$

- Letting the flexible price equilibrium be one with constant mark-ups we get $\hat{mc}_t = 0$, from where we can define y^{flex} as:

$$\frac{\phi(1-S_g)+\gamma}{(1-S_g)}y^{flex} = \frac{\gamma S_g}{(1-S_g)}\hat{g}_t$$

- From here we can re-express $\hat{mc}_t$ as: $\hat{mc}_t = \frac{\phi(1-S_g)+\gamma}{(1-S_g)}(\hat{y}_t - y^{flex})$

DETAILS: REDUCE MODEL TO 3-EQUATION SYSTEM - CONT.

- Denote the output gap by $\tilde{y}_t = (\hat{y}_t - y^{flex})$ and write the PC as:

$$\hat{\pi}_t = \beta E_t \pi_t \hat{+} 1 + \kappa \frac{\phi(1 - S_g) + \gamma}{(1 - S_g)} \tilde{y}_t$$

- letting $\omega = \frac{\phi(1-S_g)+\gamma}{(1-S_g)}$, we have:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \omega \tilde{y}_t$$

- Adding and substracting the output gap to the Euler Equation:

$$\tilde{y}_t - \frac{\phi(1 - S_g)S_g}{\phi(1 - S_g) + \gamma} \hat{g}_t = E_t \tilde{y}_{t+1} - E_t \frac{\phi(1 - S_g)S_g}{\phi(1 - S_g) + \gamma} g\hat{_{t+1}} - \frac{(1 - S_g)}{\gamma} (\hat{i}_t - E_t \pi\hat{_{t+1}})$$

- Letting r_t^n be the one consistent with the flexible prices equilibrium and, thus, zero output gap, we get:

$$r_t^n = \frac{\gamma}{(1 - S_g)} \frac{\phi(1 - S_g)S_g}{\phi(1 - S_g) + \gamma} (E_t g\hat{_{t+1}} - \hat{g}_t)$$

- Replace governement spending by its AR(1)

CONT.

- Finally, adding and substracting $\frac{(1-S_g)}{\gamma} r_t^n$ to:

$$\tilde{y}_t - \frac{\phi(1-S_g)S_g}{\phi(1-S_g)+\gamma}\hat{g}_t = E_t\tilde{y}_{t+1} - E_t\frac{\phi(1-S_g)S_g}{\phi(1-S_g)+\gamma}\hat{g}_{t+1} - \frac{(1-S_g)}{\gamma}(\hat{i}_t - E_t\hat{\pi}_{t+1})$$

- We get that the Dynamic IS can be expressed as:

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{(1-S_g)}{\gamma}(\hat{i}_t - E_t\hat{\pi}_{t+1} - r_t^n)$$

- We see the output gap today is a function of expected future output gaps and deviations of the real interest rate from the natural real interest rate.
- **If you spot typos or mistakes, please let me know.**