Lecture 20: Lumpy Investment in GE

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Outline of this lecture

- In this lecture plug in our heterogeneous firm blocks into a GE model
- With the purpose of:
  - In what way general equilibrium adjustment changes the implications of partial equilibrium models?
  - Are microeconomic frictions relevant to understand macro effects of (let's say) tax policy?

Generic Setting

Firms have a DRS production function

$$y = e^z e^a k^\alpha n^\gamma$$

a captures idiosyncratic productivity (iid across firms)

$$a_{it} = \rho_a a_{t-1} + \epsilon_{it}\sigma_a.$$

z captures aggregate productivity

$$z_t = \rho_z z_{t-1} + \xi_t \sigma_z.$$

Firms discount period τ future profits with the household stochastic discount factor $\Lambda_{t,t+\tau}$

Generic Setting

$$V(k,a,\chi,\mathcal{S}) = \max_n \left[e^z e^a k^\alpha n^\gamma - w(\mathcal{S})n\right] + \max\left[V^n(k,a,\chi,\mathcal{S}), V^a(k,a,\chi,\mathcal{S}) - \chi w(\mathcal{S})\right]$$

The value function conditional on non-adjustment is given by:

$$V^n(k,a,\chi,\mathcal{S}) = \mathbb{E}(\Lambda(\mathcal{S},\mathcal{S}')V(k',a',\chi',\mathcal{S}')|a,\mathcal{S}),$$

subject to

$$k' = k(1-\delta)$$

The value function conditional on adjustment is given by:

$$V^a(k,a,\chi',\mathcal{S}) = \max_i -i - \phi\left(\frac{i}{k}\right)^2 k + \mathbb{E}((\Lambda(\mathcal{S},\mathcal{S}')V(k',a',\chi',\mathcal{S}')|a,\mathcal{S}),$$

subject to

$$k' = k(1-\delta) + i$$

.

Generic setting

In the background there is a representative household that supplies labor, and consumes.

- There is a labor supply function in the background
- The Stochastic Discount Factor will capture household preferences for consumption smoothing

The state space

- The state space is given by $(k, a, \mathcal{S})$
- In particular, $(\mathcal{S})$ captures the aggregate state space. What is in it?
- Easy part: It includes z. $\mathcal{S} = \mathcal{H}(\Gamma, z)$.
- What is in Γ ?

What is the state?

- Generic answer: All variables that affect firm's value must be determined by the state.
- $\mathcal{S} = \mathcal{H}(\Gamma, z)$.
- Γ is the distribution of firms over their individual state-space (a, k, χ)
- Why is Γ a relevant object for these firms?
- To make intertemporal choices, firms must forecast the future. Two aspects
  - They need to forecast tomorrow's wage rate. $w(\mathcal{S}')$
  - They need to forecast tomorrow's SDF $\Lambda(S', S'')$.
  - With Γ it is possible to forecast future labor demand and supply, and back out the market-clearing wage
  - It is not possible to forecast w without knowing Γ in general.

The Curse of Dimensionality

- $\mathcal{S}$ creates an important computational challenge
- Γ is an infinite-dimensional object $\Gamma(a, k, \chi)$. Joint distribution on $\mathbb{R} \times \mathbb{R}^{+} \times \mathbb{R}^{+}$
- We could discretize the state-space. Imagine $k_n = 100$, $a_n = 2$, and χ iid. Γ adds 200 state variables
- The mass of firms in each of the (k, a) bins.
- Still impossible to implement in modern computers
- Approximations as outlined in the Menu Cost lecture

Thomas 2002

- Research Question: Does lumpy adjustment of capital affect aggregate investment dynamics in an otherwise standard general equilibrium business cycle model?
- Remember: RBC model close to linear
- Lumpy investment model had a state-dependent response of investment
- Conclusion
  - No!
  - "Lumpy investment appears largely irrelevant for equilibrium business cycle analysis"
  - Quantitative dynamics of macro aggregates are virtually indistinguishable from a frictionless RBC model

Thomas 2002

- Productivity $A_t = X_t z_t$
- Trend growth in productivity $X_t = X_{t-1}\Psi_A$
- stochastic aggregate productivity $z_t = \rho z_{t-1} + \xi_t$
- Standard household behavior $U(C_t, L_t)$.

Thomas 2002

- Thomas (2002) simplifies the generic framework we detailed in a number of ways
- Worth detailing a couple of them
  - Firms face non-convex costs but do not face convex costs
  - Firms face aggregate productivity shocks but do not face idiosyncratic productivity shocks

Thomas 2002

Cross-sectional distribution of plants.

- Remember, in principle need to track the mass of firms in the state-space
- but, symmetry (no convex costs, no idiosyncratic shocks) implies that all firms that adjust at time t, choose the same $k_{0,t+1}$
- Can keep track of the capital across *vintages* rather than firms

$$k_{0,t+1} = (1-\delta)k_{j,t} + i_{j,t}$$

$$k_{j+1,t+1} = (1-\delta)k_{j,t}$$

Thomas 2002

- As in Caballero and Engel (1999), adjustment hazards are increasing in capital imbalances
- Assumption $\chi \sim U(0, B)$
- There exists a cutoff for the fixed cost $\bar{\chi}_j$ such that firms with vintage j adjust iff $\chi < \bar{\chi}_j$
- The probability of adjustment for firms with vintage j is noted by $\alpha_{j,t}$
- There exists J sufficienty large such that $\alpha_{J,t} = 1$
- Clever: Transformed a problem with infinitely many state variables, into a problem with J states.

Thomas 2002

- The problem of individual firms is not smooth
- Planner's problem is smooth: summarized by α.
- Possible to linearize
- Finite number of vintages $\rightarrow$ finite number of states.

Thomas 2002

TABLE 2
STATIONARY PLANT DISTRIBUTION

	Time-since-Adjustment Group					
	0	1	2	3	4	5
Adjustment fraction: α_j	.059	.197	.377	.576	.782	1.000
Population density: θ_j	.293	.276	.221	.138	.059	.013

NOTE.—For each group, j, α_j represents the fraction of the group's members investing in each period along the balanced growth path; θ_j is the fraction of all plants that are members of group j.

Thomas 2002

TABLE 3
SUMMARY OF MODELS

Name	Description
Benchmark neoclassical	No adjustment costs
State-dependent adjustment	Nonconvex adjustment costs; endogenously varying adjustment rates
Constant adjustment	Nonconvex adjustment costs; adjustment rates fixed at table 2 values
Partial adjustment	Convex adjustment costs

Source: Thomas (2002) — constant adjustment case designed to answer following question: How much is lost in terms of short-run aggregate dynamics when we assume adjustment rates are constant over the cycle?

Thomas 2002

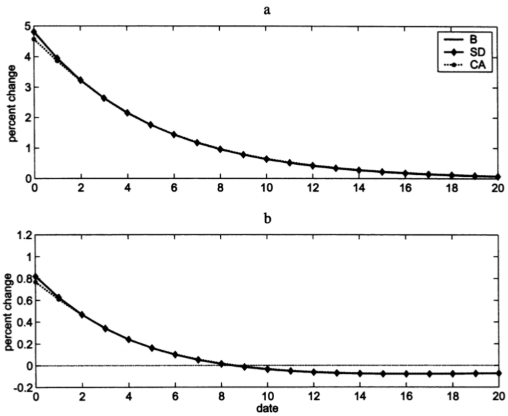

FIG. 2.—Impulse responses for aggregate quantities: percentage deviations from (growth-deflated) steady state in response to a 1 percent rise in aggregate productivity. *a*, Investment; *b*, employment. B: benchmark; SD: state-dependent adjustment; CA: constant adjustment.

Khan and Thomas 2008

- Some debate on whether the results were the outcome of the very specific modeling choices
- Khan and Thomas (2008) find pretty much the same results in a model that looks closer to what we wrote in this lecture note

$$U(c, L) = \log c + \varphi L$$

Khan and Thomas 2008

TABLE VII

ROLE OF FIXED COSTS IN PLANT-LEVEL INVESTMENTS

	Inaction	Positive Spike	Negative Spike	Positive Invest.	Negative Invest.
Establishment data[a]	*0.081*	*0.186*	*0.018*	*0.815*	*0.104*
Models with plant-specific TFP shocks					
1. Frictionless model	0.032	0.204	0.028	0.611	0.356
2. Lumpy investment model	0.073	0.185	0.010	0.752	0.175

[a]Data are from Cooper and Haltiwanger (2006). Inaction, $|i/k| < 0.01$; positive spike, $i/k > 0.20$; negative spike, $i/k < -0.20$; positive investment, $i/k \geq 0.01$; negative investment, $i/k \leq -0.01$.

Khan and Thomas 2008

TABLE IV

AGGREGATE BUSINESS CYCLE MOMENTS

	Output	TFP[a]	Hours	Consump.	Invest.	Capital
A. Standard deviations relative to output[b]						
GE frictionless	(2.277)	0.602	0.645	0.429	3.562	0.494
GE lumpy	(2.264)	0.605	0.639	0.433	3.539	0.492
B. Contemporaneous correlations with output						
GE frictionless		1.000	0.955	0.895	0.976	0.034
GE lumpy		1.000	0.956	0.900	0.976	0.034

[a] Total factor productivity.

[b] The logarithm of each series is Hodrick–Prescott-filtered using a weight of 100. The output column of panel A reports percent standard deviations of output in parentheses.

Rough intuition

- α returns to scale (CRS: $\alpha = 1$).

$$\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -\frac{1}{1-\alpha}\frac{1}{\delta}\frac{1+r_t}{r_t+\delta}$$

- When $\alpha \to 1$, the investment of unconstrained firms is infinitely elastic
- Small changes in aggregate prices induce infinite responses

Winberry 2021

$$\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -\frac{1}{1-\alpha}\frac{1}{\delta}\frac{1+r_t}{r_t+\delta}$$

- Under a reasonable calibration:
- $\alpha = 0.7, \delta = 0.025, r_t = 0.01$: $\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -3,847$
- r_t is an equilibrium outcome, so much depends on how r_t behaves.
- The standard model has very strong strategic substitutability
- That others do not adjust induces higher incentives to adjust
- Mediated by the response of the real interest rate to aggregate shocks

Winberry 2021

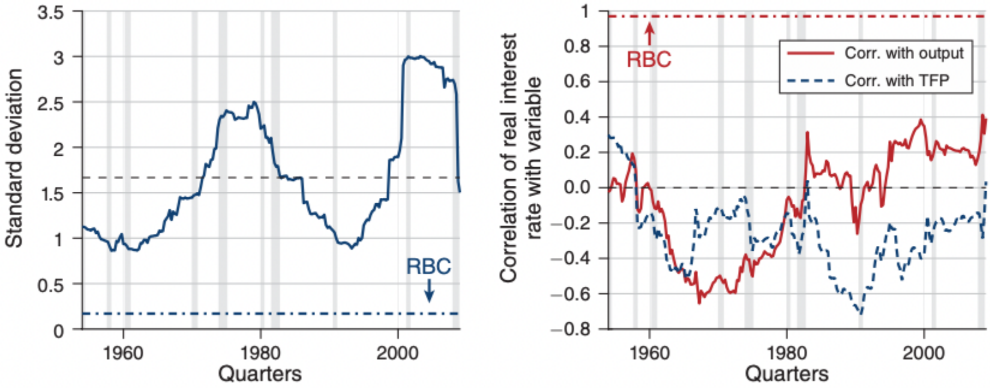

Figure 1. Stability of Cyclical Dynamics of Risk-Free Rate

Winberry 2021

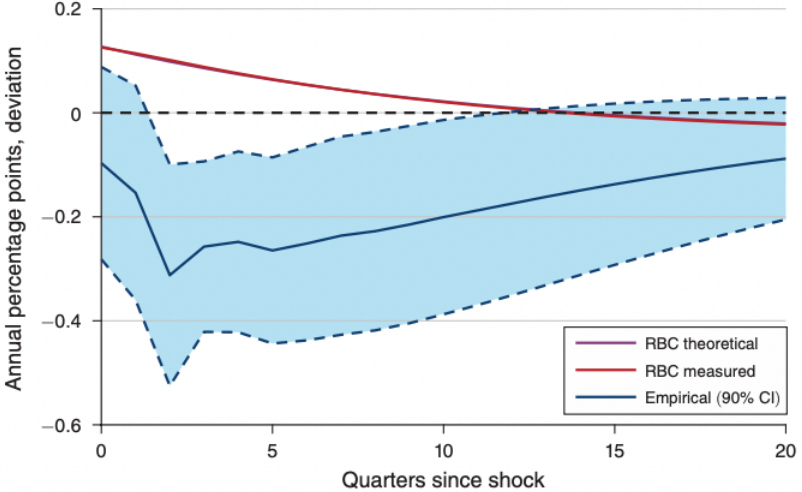

Figure 2. Impulse Response of the Real Interest Rate to TFP Shock

Habits in Consumption

- Fix the dynamics of r by changing optimal consumption decisions

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \log\left(C_t - \chi \frac{N_t^{1+\xi}}{1+\xi} - X_t\right)$$

$$X_t = \lambda \hat{C}_t$$

$$\hat{C}_t = C_t - \chi \frac{N_t^{1+\xi}}{1+\xi}$$

Winberry 2021

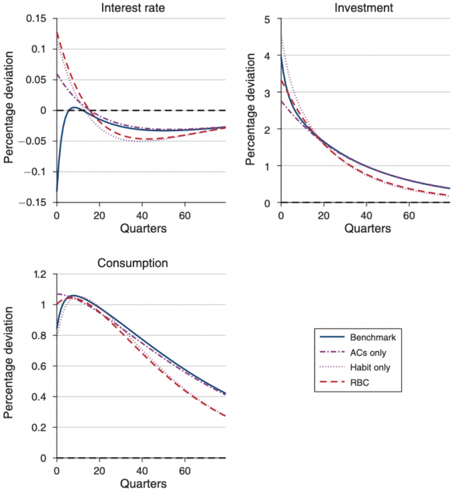

Figure 3. Identification of Habit Formation and Adjustment Costs

Winberry 2021

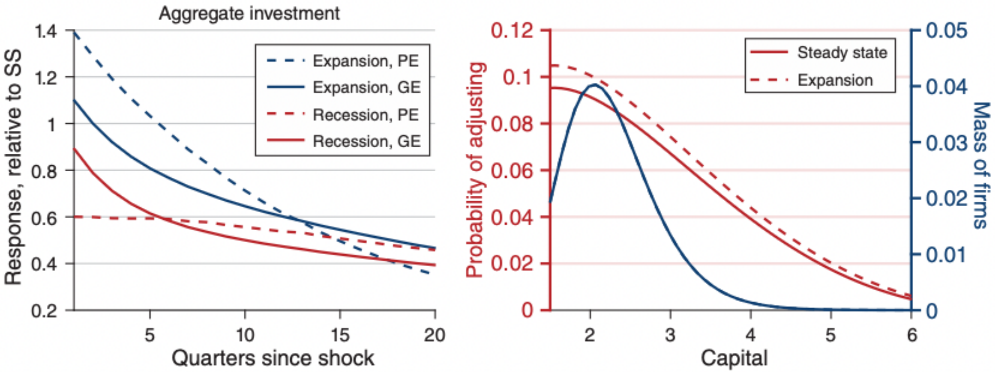

Figure 5. Procyclical Impulse Responses of Aggregate Investment

Koby and Wolf 2022

- The PE semi-elasticity of investment to interest rates in Khan and Thomas (2008) = 500%
- In Winberry (2021) is roughly 7%
- Meaning: In Khan and Thomas interest rates need to increase 71 times less to accomodate the same change in investment demand
- How to choose?
- Zwick and Mahon cross-sectional responses imply a semi-elasticity close to Winberry's
- 100 times smaller than that of Khan and Thomas (2008)

Koby and Wolf 2022

- Why is Zwick and Mahon (2017) evidence relevant?
- Because it answers precisely the question of:

  After differencing-out general equilibrium responses, how much more investment occurs at a firm that faces a smaller cost of capital relative to a firm that has a higher cost of capital?
- Note that Zwick and Mahon (2017) do not answer the macro question
- But their evidence let us to distinguish across macro models

Messages

- GE effects can undo lessons in PE
- But there is not a single way to aggregate
- Cross-sectional evidence can be useful to distinghish across models

Policy Messages

- Tax policy is less effective in recessions
- More firms are likely to make an extensive margin investment in expansions than in recessions