Lecture 1: The Classical Dichotomy

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Questions for Today

- What determines the return on nominal assets?
- What is the classical dichotomy?
- Why can the price level be indeterminate?
- What determines inflation?
- What is the Taylor Rule?

The Chicken Economy

You have worked with “real economies”. I will refer to them as “chicken economies”

- The economy starts with an initial stock of K_0 chicken
- Firms mix existing chicken K with labor L, in order to produce more chicken Y
- Newly produced chicken can be broiled and eaten C, or placed next to alive chicken I.
  $Y = C + I$ must hold
- The chicken fatality rate is δ
- The quantity of chicken available to produce chicken tomorrow is $K_{t+1} = K_t(1 - \delta) + I_t$.
- You must pay r^b chickens for each chicken you borrow
- You must pay w chickens to your workers per hour of work
- You can save in chicken, earning r^l chickens next period

There is no notion of money in the chicken economy.

Money

- Some clarifications. I will not include a monetary base, quantity of money anywhere.
- Instead we will work with a *cashless* economy
- Reasons:
  - Theory: Theories where the central bank decides on money predict too volatile inflation.
  - Estimation: Money demand equations are very unstable. Seems to just be the wrong model.
  - Real-World relevance: Paper money is less and less important
  - Policy: Central banks do not decide policy by fixing a money supply. They issue reserves to target nominal interest rates, and are willing to exchange reserves for cash one-to-one to accomodate changes in money demand.
  - Who uses cash? Mostly, criminals.

The Neutrality of Money

- We will start working with *nominal economies*. Some variables are determined in dollar terms, not in terms of chicken.
- Is abstracting from money a fundamental theoretical drawback? Does money matter?
  - The question is not: Does *wealth* matter? The answer to that is obviously yes
  - Is the choice of unit of account, and the quantity of money, all else equal, of any relevance?
- David Hume (1752) cautions:

> *If we consider any one kingdom by itself, it is evident, that the greater or less plenty of money is of no consequence; since the prices of commodities are always proportioned to the plenty of money, and a crown in Harry VII's time served the same purpose as a pound does at present.*

- Hume presents a more nuanced position later in his treatise

The Model

- Representative agent with preferences

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \log C_t\right]$$

- Can save in real bonds, and in nominal bonds:

$$P_tC_t + A_{t+1} + P_tK_{t+1} \leq P_tY_t + A_t(1+i_{t-1}) + P_tK_t(1+r_{t-1})$$

- Y_t an endowment. exogenous. stochastic. Markov.

Notice: two interest rates on two assets. Budget constraint is in dollars, not chicken.

Real Bonds Euler Equation

- Not going to go over the steps to derive FOCs. Life is too short.
- For real bonds K:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}} (1 + r_t) \right)$$

- Nothing new, your old friend the Euler equation. if you decide to save in real bonds, same trade-offs as in 210A, 210B.
- Log-linearizing the Euler equation around $C_t = C_{t+1} = C$ and $1 + r = \beta^{-1}$:

$$\mathbb{E}_t(\hat{c}_{t+1} - \hat{c}_t) = \hat{r}_t$$

The real interest rate r pins down consumption growth

The Fisher Equation

- For nominal bonds:

$$1 = \beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} (1 + i_t) \right)$$

- Euler equation for nominal bonds. Asset pricing equation $1 = \mathbb{E}(\mathrm{SDF}_{t,t+1} R_{t+1})$
- Combine with Euler for real bonds, to express as an asset pricing excess-return equation
- For nominal bonds:

$$0 = \beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \left(1 + r_t - \frac{P_t}{P_{t+1}} (1 + i_t) \right) \right)$$

- Log-linearize around a s.s. with zero inflation and constant rates to make things more evident

$$\hat{i}_t = \hat{r}_t + \mathbb{E}_t \hat{\pi}_{t+1} : \text{Fisher equation}$$

- where $\hat{\pi}_{t+1} = \hat{p}_{t+1} - \hat{p}_t$

Market Clearing and Equilibrium

- Goods market clear

$$C_t = Y_t$$

- There are no assets in net supply

$$K_t = 0$$

$$A_t = 0$$

Equilibrium definition: An equilibrium is a sequence of real variables $\{r_{t+1}, C_t, K_{t+1}\}$, and nominal variables $\{i_{t+1}, A_{t+1}, P_t\}$ starting at $t = 0$, with initial conditions A_0, K_0, and an exogenous process for Y_t, such that

- The household behaves optimally: Euler equation and Fisher equation hold
- Markets in goods and the two assets clear

Big problem: 5 equations, 6 unknowns.

Result 1: Classical Dichotomy

- The Euler equation for real bonds, the market clearing condition for goods, and for capital are a recursive system that pins down Y, C, K. No nominal variables enter the system.

$$\begin{aligned} K_{t+1} &= 0 \\ C_t &= Y_t \\ (1 + r_t)^{-1} &= \beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \right) \end{aligned}$$

- Classical Dichotomy: real variables do not depend on any nominal variable.
- A theoretical result, most evidence points against it. We will spend time breaking it.

Result 2: Price Level Indeterminacy

- Nominal holdings pinned down by $A_{t+1} = 0$
- One equation left:

$$(1 + i_t)^{-1} = \beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right)$$

- to pin down $\{i_{t+1}, P_t\}$.
- David Hume seems to be right. Dollars are just a unit of account, and agents do not suffer from money illusion. Any sequence of P as good as any other. Dollars or cents, who cares?
- How to proceed?

Pause

Let’s breathe for a second. Two results:

- Classical Dichotomy: Real equilibrium independent of nominal variables
- Price level indeterminacy: Nothing tells you the price level. Any sequence of P is ok.

These are two distinct concepts. To show it I will break the second while keeping the first.

Fiscal Policy

- Fiscal authority collects taxes and send transfers that add up to T. Purchases goods G, and receives dividends from the central bank D.
- Balanced budget

$$T_t + D_t = G_t$$

- For simplicity assume G is given, D is decided by the central bank, so the government is forced to implement T given by the budget constraint above.

Central Bank

- Central bank issues a liability. Called reserves. Stock of reserves V
- The central bank promises a nominal return i_t^V
- Budget constraint of the central bank

$$V_{t+1} = V_t(1 + i_t^V) + P_t D_t$$

- Supply of nominal assets must equal demand of nominal assets

$$A_t = V_t$$

- Note the central bank chooses both V and i^V. Central banks choose prices and quantities. Powerful.

Real Equilibrium

Real equilibrium same as before. Easy to check. Life is too short. Important: classical dichotomy still holds.

Nominal Equilibrium

- Nominal equilibrium is $\{P_t, i_t + 1\}$ such that

$$i_t = i_t^v$$

$$(1 + i_t)^{-1} = \beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right)$$

- to pin down P need to solve the following difference equation

$$r_t = i_t^v - \mathbb{E}_t \Delta p_{t+1}$$

- Fisher equation extended for the central bank picking i.

Result 3: Interest rate pegs do not work

- Imagine the central bank picks an exogenous path for i^V. wlog $i^V = 0$.
- Equilibrium then implies

$$p_t = \mathbb{E}_t p_{t+1} + r_t$$

- Cannot solve by iterating forward. No initial or terminal condition for p
- Indeterminacy of interest rate pegs: If the interest rate is exogenous, and I expect higher prices in the future, prices today jump. Any sequence of p is ok.

Result 4: Wicksellian rules

- Instead assume $i_t^V = \phi(\hat{p}_t - \hat{p}_t^*)$, for some desired price $\hat{p}^*$
- Fisher equation now
$$(\phi + 1)\hat{p}_t = \mathbb{E}_t\hat{p}_{t+1} + r_t + \phi\hat{p}_t^*$$
- Iterate forward assuming $\phi > 0$.

$$\hat{p}_t = \mathbb{E}_t \sum_{s=0}^{\infty} (1 + \phi)^{-(s+1)} (r_{t+s} + \phi\hat{p}_{t+s}^*)$$

Price level pinned down. If there are shocks to the expected path of real rates or desired targets, then the price level moves.

Result 5: Taylor Rules and the Taylor Principle

- Instead assume $i_t^v = \phi(\hat{\pi}_t - \hat{\pi}_t^*)$, for some desired inflation rate $\hat{\pi}^*$
- Fisher equation now

$$\phi\hat{\pi}_t = \mathbb{E}_t\hat{\pi}_{t+1} + r_{t+1} + \phi\hat{\pi}_t^*$$

- Iterate forward assuming $\phi > 1$.

$$\hat{\pi}_t = \mathbb{E}_t \sum_{s=0}^{\infty} (\phi)^{-(s+1)} (r_{t+s+1} + \phi\hat{\pi}_{t+s}^*)$$

Determining the inflation rate

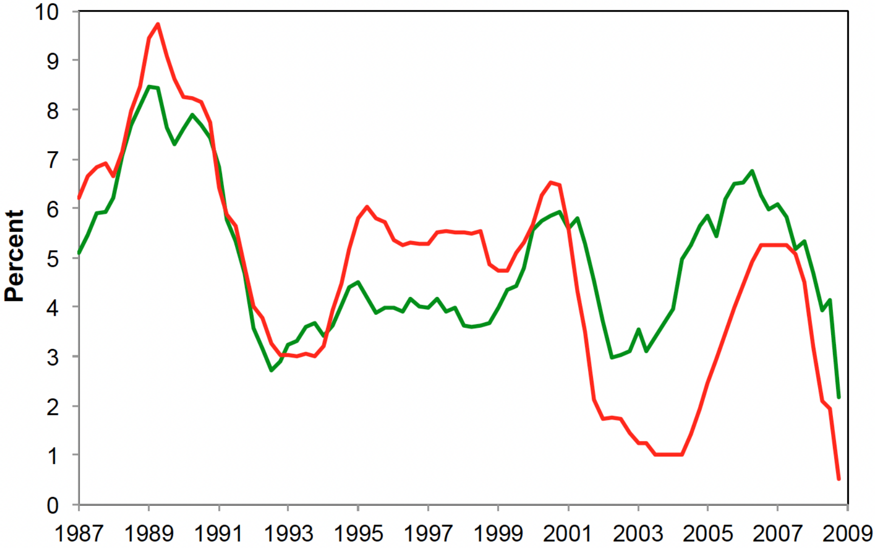
Taylor Rules and Taylor Principle

$$i_t^v = \phi(\hat{\pi}_t - \hat{\pi}_t^*), \text{ with: } \phi > 1$$

- That a Taylor rule satisfies $\phi > 1$ is called the Taylor Principle
- In words: If inflation increases, the central bank increases interest rates *more than proportionally*
- Was for years at the center of monetary policy debate:
  - Is the Taylor Principle satisfied?
  - Is the Taylor Principle close to optimal monetary policy?
  - Are observed inflationary spirals violations of the Taylor Principle?
- Note that both Wicksellian and Taylor rules required log-linearizations. They are a locally bounded equilibrium maintained under the threat that the central bank would blow up the economy. But maybe the economy can blow up? Why rule that out?

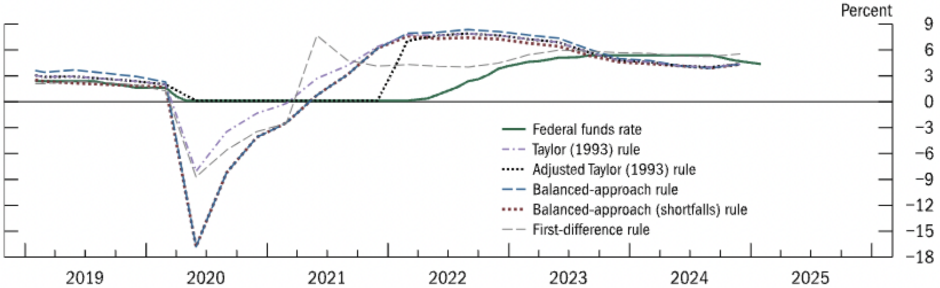
$$\uparrow \pi_t \rightarrow \uparrow i_t \rightarrow \uparrow \pi_{t+1} \rightarrow \uparrow i_{t+1} \ldots$$

Taylor Rule in the US



Taylor Rule in the US

Figure A. Historical federal funds rate prescriptions from simple policy rules



Questions for Today

- What determines the return on nominal assets? The Fisher Equation. An arbitrage condition
- What is the classical dichotomy? Monetary policy is neutral. Theoretical starting point
- Why can the price level be indeterminate? Because units are a veil. No money illusion
- What determines inflation? So far arbitrage and the ability of the government to issue liabilities and set its price
- What is the Taylor Rule: Hike interest rates aggresively when inflation occurs. Pins down inflation locally. Potential problems with inflationary explosions and the ZLB.