Final ECON 210C exam
Suggested Solution Q1

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1. **700 points.** Regional NK Model

   There are two regions in a nation, they are called Home (H) and Foreign (F). H and F have households of equal mass who have symmetric preferences. Households in H and F consume goods produce in their own region and in the other region, although they have a degree of home bias (they prefer to consume more of the local varieties if prices are the same). Workers cannot migrate. Monopolistically-competitive firms produce differentiated varieties using a constant returns to scale technology that uses local labor to produce output of local varieties. Firms face Calvo frictions to reset their prices. A central bank sets the nation-wide monetary policy via a Taylor rule with aggregate inflation as its input. Labor markets and goods markets must clear.

   Households:

   I will write down the problem of households in H. The problem of households in F is symmetric. X_t denotes a variable X for the H region in period t. X_t^* will denote the value of the same variable X in the F region at time t.

   Households maximize the PV of the stream of utility

$$\max_{C_t,N_t,B_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

   subject to a budget constraint

$$B_{t+1} + P_t C_t \leq B_t(1+i_t) + W_t N_t + T_t \tag{2}$$

where B are bond holdings, i is the nominal interest rate W is the nominal wage, and T are lump-sum dividends, transfers and taxes.

The consumption bundle C consists of a CES bundle that aggregates between the consumption of H and F goods.

$$C_t = \left(\psi^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\psi)^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{3}$$

where η is the elasticity of substitution at the regional level of aggregation, and $\psi \in (0.5, 1)$ captures the extent of home bias. For completeness, the Consumption bundle of the F region is given by

$$C_t^* = \left(\psi^{1/\eta} \left(C_{F,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\psi)^{1/\eta} \left(C_{H,t}^*\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{4}$$

which makes clear that the consumption bundles across regions are symmetric.

The consumption bundles of households in H and F are given by another CES aggregate across a continuum of varieties in the [0,1] interval. Formally:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

$$C_{F,t} = \left(\int_0^1 C_{F,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}, \tag{6}$$

$$C_{H,t}^* = \left(\int_0^1 C_{H,t}^*(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}, \tag{7}$$

$$C^*_{F,t} = \left(\int_0^1 C^*_{F,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}. \tag{8}$$

The firm takes the prices for all of these varieties ($P_{H,t}(z)$ and $P_{F,t}(z)$) as given.

(a) State and solve the cost minimization problems (two of them) the H household faces when allocating its consumption across varieties coming from a particular origin (H or F) subject to a level of consumption bundle from that origin (that is given a prespecified level for $C_{H,t}$ or $C_{F,t}$). Hint: These problems have a lot of symmetry, by solving one you should get the other one for free.

Solution: The cost minimization problem of the H household when allocating its consumption across varieties from H is given by

$$\min_{C_{H,t}(z)} \int_0^1 P_{H,t}(z) C_{H,t}(z) dz \quad \text{s.t.} \quad C_{H,t} = \left(\int_0^1 C_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}. \tag{9}$$

The Lagrangian for this problem is given by

$$\mathcal{L} = \int_0^1 P_{H,t}(z) C_{H,t}(z) dz + \lambda_t \left(C_{H,t} - \left(\int_0^1 C_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}} \right). \tag{10}$$

The first order conditions for this problem are given by

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}(z)} = P_{H,t}(z) - \lambda C_{H,t}(z)^{-\frac{1}{\theta}} \left(\int_0^1 C_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}-1} = 0, \tag{11}$$

which implies that

$$C_{H,t}(z) = C_{H,t} \left(\frac{P_{H,t}}{\lambda_t}\right)^{-\theta} \tag{12}$$

The Lagrange multiplier λ_t can be interpreted as the marginal cost of the consumption bundle $C_{H,t}$.

The same problem can be solved for the allocation of consumption across varieties from F and the solution is symmetric. The demand schedule for $C_{F,t}(z)$ is given by

$$C_{F,t}(z) = C_{F,t}\left(\frac{P_{F,t}}{\lambda_{ft}}\right)^{-\theta} \tag{13}$$

(b) State the shape of the ideal price index for purchases of the H household for the bundles $C_{H,t}$ and $C_{F,t}$. That is, find the price indexes $P_{H,t}$ and $P_{F,t}$ that guarantee that $P_{H,t}C_{H,t} = \int_0^1 P_{H,t}(z)C_{H,t}(z)dz$, and $P_{F,t}C_{F,t} = \int_0^1 P_{F,t}(z)C_{F,t}(z)dz$, respectively. Hint: These price indexes have a lot of symmetry. By solving one of them you should get the other one for free.

Solution: Replace the expression found in the previous step for $C_{H,t}(z)$ in the definition of the CES aggregator:

$$C_{H,t} = \left(\int_0^1 \left[\left(\frac{P_{H,t}(z)}{\lambda_t}\right)^{-\theta} C_{H,t}(z)\right]^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

Re-arranging this gives:

$$\lambda_t = \left(\int_0^1 P_{H,t}(z)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$$

We want to find the expression for $P_{H,t}$ that satisfies:

$$P_{H,t}C_{H,t} = \int_0^1 P_{H,t}(z)C_{H,t}(z)di$$

Plug in the expression for $C_{H,t}(z)$ on the right-hand side:

$$P_{H,t}C_{H,t} = \int_0^1 P_{H,t}(z)\left(\frac{P_{H,t}(z)}{\lambda_t}\right)^{-\theta} C_{H,t}di$$

$$P_{H,t} = \lambda_t^{\theta}\int_0^1 P_{H,t}(z)^{1-\theta} di$$

Replace λ_t with the expression we found before:

$$P_{H,t} = \left(\int_0^1 P_{H,t}(z)^{1-\theta} di \right)^{\frac{\theta}{1-\theta}} \left(\int_0^1 P_{H,t}(z)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$
$$P_{H,t} = \left(\int_0^1 P_{H,t}(z)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

By symmetry, the price index for varieties from F is given by

$$P_{F,t} = \left(\int_0^1 P_{F,t}(z)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

(c) State and solve the cost minimization problems (one of them) the H household faces when allocating its consumption across H and F bundles subject to a prespecified level of C_t.

Solution The cost minimization problem of the H household when allocating its consumption across H and F bundles is given by

$$\min_{C_{H,t},C_{F,t}} P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \quad \text{s.t.} \quad C_t = \left(\psi^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\psi)^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{14}$$

The Lagrangian for this problem is given by

$$\mathcal{L} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + \mu_t \left(C_t - \left(\psi^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\psi)^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \right). \tag{15}$$

The first order conditions for this problem are given by

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}} = P_{H,t} - \mu_t C_{H,t}^{-\frac{1}{\eta}} \left(\psi^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\psi)^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} \psi^{\frac{1}{\eta}} = 0, \tag{16}$$

which implies that

$$C_{H,t} = \psi C_t \left(\frac{P_{H,t}}{\mu_t} \right)^{-\eta} \tag{17}$$

The Lagrange multiplier μ_t can be interpreted as the marginal cost of the consumption bundle C_t. The FOC for $C_{F,T}$ is analogous with a demand schedule given by

$$C_{F,t} = (1-\psi)C_t \left(\frac{P_{F,t}}{\mu_t}\right)^{-\eta} \tag{18}$$

The problem of the foreign household is symmetric. Its demand schedule for $C^*_{H,t}$ is given by

$$C^*_{H,t} = (1-\psi)C^*_t \left(\frac{P_{H,t}}{\mu^*_t}\right)^{-\eta} \tag{19}$$

and for $C^*_{F,t}$ is given by

$$C^*_{F,t} = \psi C^*_t \left(\frac{P_{F,t}}{\mu^*_t}\right)^{-\eta} \tag{20}$$

(d) State the shape of the ideal price index of the H household for its expenditures across H and F-produced bundles. That is, find the price index P_t that guarantee that $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$.

Solution: Replace the expression found in the previous step for $C_{H,t}$ and $C_{F,t}$ in the definition of the CES aggregator yields

$$\mu_t = \left(\psi P_{H,t}^{1-\eta} + (1-\psi)P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

We want to find the expression for P_t that satisfies:

$$P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$$

Plug in the expression for $C_{H,t}$ and $C_{F,t}$ on the right-hand side:

$$\begin{aligned}
P_tC_t &= P_{H,t}\left(\psi C_t \left(\frac{P_{H,t}}{\mu_t}\right)^{-\eta}\right) + P_{F,t}\left((1-\psi)C_t \left(\frac{P_{F,t}}{\mu_t}\right)^{-\eta}\right) \\
P_t &= \mu_t^{\eta}\left(\psi P_{H,t}^{1-\eta} + (1-\psi)P_{F,t}^{1-\eta}\right)
\end{aligned}$$

Replace μ_t with the expression we found before:

$$\begin{aligned}
P_t &= \left(\psi P_{H,t}^{1-\eta} + (1-\psi)P_{F,t}^{1-\eta}\right)^{\frac{\eta}{1-\eta}}\left(\psi P_{H,t}^{1-\eta} + (1-\psi)P_{F,t}^{1-\eta}\right) \\
P_t &= \left(\psi P_{H,t}^{1-\eta} + (1-\psi)P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}
\end{aligned}$$

(e) State the first order conditions that characterize the optimal intra and intertemporal decisions of the household. Express them as one Euler equation, and one labor supply equation.

Solution: The first order conditions of the household are given by

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t C_t^{-\gamma} - \lambda_t P_t = 0, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -\beta^t N_t^{\varphi} + \lambda_t W_t = 0, \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = \lambda_t - \beta^t \lambda_{t+1}(1 + i_{t+1}) = 0. \tag{23}$$

Combining and re-arranging yields:

$$C_t^{-\gamma} = \beta \left[C_{t+1}^{-\gamma}(1 + i_{t+1})(1 + \Pi_{t+1})^{-1} \right]$$

where Π_t is home CPI inflation. The labor supply equation is given by

$$N_{t+1}^{\varphi} = C_t^{-\gamma} \frac{W_t}{P_t}$$

(f) Log-linearize the price indexes, the resulting demand curves across varieties and bundles, the labor supply equation, and the euler equation.

Solution Log-linearize around a zero-inflation steady state whith all relative prices are equalized to one.

$$
\begin{aligned}
\hat{c}_t &= \hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_{t+1} - \hat{\pi}_{t+1}) \\
\varphi \hat{n}_t &= -\gamma \hat{c}_t + \hat{w}_t - \hat{p}_t \\
\hat{c}_{Ht} &= \hat{c}_t - \eta(\hat{p}_{Ht} - \hat{p}_t) \\
\hat{c}_{Ft} &= \hat{c}_t - \eta(\hat{p}_{Ft} - \hat{p}_t) \\
\hat{c}_{H,t}(z) &= \hat{c}_{Ht} - \theta(\hat{p}_{H,t}(z) - \hat{p}_{Ht}) \\
\hat{c}_{F,t}(z) &= \hat{c}_{Ft} - \theta(\hat{p}_{F,t}(z) - \hat{p}_{Ft}) \\
\hat{p}_t &= \psi \hat{p}_{H,t} + (1-\psi)\hat{p}_{F,t} \\
\hat{p}_{H,t} &= \int_0^1 \hat{p}_{H,t}(z) dz
\end{aligned}
$$

Firms

Firms are monopolistic competitiors. Each variety is produced by a distinct firm. Firms produce using labor using a constant return to scale technology. Specifically, the production function of a firm in region H producing a particular variety z is given by

$$Y_{H,t}(z) = L_{H,t}(z), \tag{24}$$

where $L_{H,t}(z)$ is labor demand by firm z in region H.

Firms face Calvo frictions. They can reset their prices with probability $1-\lambda$ every period.

Firms are committed to satisfy demand. In particular, the demand of a given firm comes from its local and foreign customers,

$$Y_{H,t}(z) = C_{H,t}(z) + C^*_{H,t}(z), \tag{25}$$

firms understand the structure of demand.

The problem of the firm is to maximize the NPV of its dividends using the SDF of the local household along the path where the price it sets in period t is fixed. That is:

$$\max_{P^*_{H,t}(z), Y_{H,t+k|t}(z), L_{H,t+k|t}(z)} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \lambda^k \left(P^*_{H,t}(z) Y_{H,t+k|t}(z) - W_{t+k} L_{H,t+k|t}(z) \right), \tag{26}$$

where $P^*_{H,t}(z)$ is the reset price of a given firm at time t, and $Y_{H,t+k|t}(z)$, $L_{H,t+k|t}(z)$ are the production and labor demand of a given firm in period $t+h$ given that it set a reset price in period t. Firms maximize this value function subject to demand, and their production function.

(g) Solve the problem of the firm. That is find an expression for the reset price of the firm as a function of aggregate quantities, aggregate input prices.

Solution: The firm maximizes expected profits subject to demand:

$$\max_{P^*_{H,t}(z),Y_{H,t+k|t}(z),L_{H,t+k|t}(z)} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k \left(P^*_{H,t}(z)Y_{H,t+k|t}(z) - W_{t+k}L_{H,t+k|t}(z)\right),$$

$$s.t. Y_{H,t+k|t}(z) = \left(\frac{P^*_{Ht}(z)}{P_{Ht+k}}\right)^{-\theta} (C_{Ht+k} + C^*_{Ht+k})$$

The first order condition is given by

$$\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k|t}(z)\left(P^*_{H,t} - \frac{\theta}{\theta-1}W_{t+k}\right) = 0.$$

Replacing $Y_{H,t+k|t}(z)$ with the demand schedule gives:

$$\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k \left(\frac{P^*_{Ht}(z)}{P_{Ht+k}}\right)^{-\theta} (C_{Ht+k} + C^*_{Ht+k})\left(P^*_{H,t} - \frac{\theta}{\theta-1}W_{t+k}\right) = 0.$$

where $P^*_{Ht}(z)^{-\theta}$ outside the summation cancels as it doesn't depend on k. Using $Y_{H,t+k} = C_{H,t+k} + C^*_{H,t+k}$, we can re-write the first order condition as:

$$\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k} P^{\theta}_{Ht+k} Y_{H,t+k}\left(P^*_{H,t} - \frac{\theta}{\theta-1}W_{t+k}\right) = 0.$$

Re-arranging:

$$P^*_{Ht} = \frac{\theta}{\theta-1}\frac{\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k} P^{\theta}_{Ht+k} W_{t+k}}{\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k} P^{\theta}_{Ht+k}}.$$

Expressing marginal costs in terms of the PPI index gives

$$P^*_{Ht} = \frac{\theta}{\theta-1}\frac{\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k} P^{\theta}_{Ht+k} MC_{Ht+k}}{\sum_{k=0}^{\infty} \Lambda_{t,t+k}\lambda^k Y_{H,t+k} P^{\theta-1}_{Ht+k}}.$$

where the reset price is expressed as a function of aggregate quantities and prices.

(h) Log-linearize the reset price equation, and aggregate it to find a regional Phillips curve in terms of marginal costs. That is, find an expression of the form $\hat{\pi}_{H,t} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \kappa_{mc}\hat{mc}_{Ht}$, for some slope κ_{mc}, where $\hat{\pi}_{H,t}$ is the PPI inflation of locally produced goods in region H and $\hat{mc}_{H,t}$ is the log difference of real marginal costs in region H (expressed in terms of the PPI price index in region H).

Solution: Use the expression for the SDF and re-write the FOC from previous point as:

$$\sum_{k=0}^{\infty}\beta^k\lambda^k C_{t+k}^{-\gamma}Y_{H,t+k}{P_{Ht+k}}^{\theta-1}P_{Ht}^{*} = \sum_{k=0}^{\infty}\beta^k\lambda^k C_{t+k}^{-\gamma}Y_{H,t+k}{P_{Ht+k}}^{\theta}MC_{Ht+k}$$

First, log-linearize the left hand side and use $P_{H,ss}^{*} = \frac{\theta}{\theta-1}mc_{H,ss}$

$$C_{ss}^{-\gamma}Y_{H,ss}P_{H,ss}^{\theta-1}\frac{\theta}{\theta-1}mc_{H,ss}\sum_{k=0}^{\infty}\beta^k\lambda^k\Big(-\gamma\hat{c}_{t+k}+\hat{y}_{t+k}+(\theta-1)\hat{p}_{Ht+k}+\hat{p}_{Ht}^{*}\Big)$$

Similarly, for the right hand side we have:

$$C_{ss}^{-\gamma}Y_{H,ss}P_{H,ss}^{\theta-1}\frac{\theta}{\theta-1}mc_{H,ss}\sum_{k=0}^{\infty}\beta^k\lambda^k\Big(-\gamma\hat{c}_{t+k}+\hat{y}_{t+k}+\theta\hat{p}_{Ht+k}+\hat{mc}_{Ht}\Big)$$

where marginal costs are deflated using the home PPI. Combining and canceling out terms (recall ss prices normalized to 1) yields:

$$\sum_{k=0}^{\infty}\beta^k\lambda^k\Big(\hat{p}_{Ht}^{*}-\hat{p}_{Ht+k}\Big) = \sum_{k=0}^{\infty}\beta^k\lambda^k\Big(\hat{mc}_{Ht+k}\Big)$$

Re-arranging:

$$\hat{p}^*_{Ht+k}\sum_{k=0}^{\infty}\beta^k\lambda^k = \sum_{k=0}^{\infty}\beta^k\lambda^k\Big(\hat{m}c_{Ht+k} - \hat{p}_{Ht+k}\Big)$$
$$\hat{p}_{Ht+k}\frac{1}{(1-\beta\lambda)} = \sum_{k=0}^{\infty}\beta^k\lambda^k\Big(\hat{m}c_{Ht+k} - \hat{p}_{Ht+k}\Big)$$
$$\hat{p}_{Ht+k} = (1-\beta\lambda)\sum_{k=0}^{\infty}\beta^k\lambda^k\Big(\hat{m}c_{Ht+k} - \hat{p}_{Ht+k}\Big)$$

Take out $k = 0$ of the summation:

$$\hat{p}^*_{Ht+k} = (1-\beta\lambda)[\hat{m}c_{Ht} - \hat{p}_{Ht}] + \beta\lambda E_t[\hat{p}^*_{Ht+1}]$$

Calvo pricing implies

$$\hat{p}_{Ht} = \lambda\hat{p}_{Ht-1} + (1-\lambda)\hat{p}^*_{Ht}$$

Or, in terms of PPI inflation

$$\hat{\pi}_{Ht} = (1-\lambda)(\hat{p}^*_{Ht} - \hat{p}_{Ht-1})$$

Substracting $\hat{p}_{Ht-1}$ from both sides in the expression some lines above and re-arranging yields:

$$\hat{\pi}_{Ht} = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\hat{m}c_{Ht} + \beta E_t[\hat{\pi}_{Ht+1}]$$

where $\kappa_{mc} = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$.

(i) Use the shape of the local labor supply curve to find a local New Keynesian Phillips curve. That is, find an expression of the form $\hat{\pi}_{H,t} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \kappa_c\hat{c}_t + \kappa_l\hat{l}_t + \kappa_p(\hat{p}_t - \hat{p}_{Ht})$,
where $\hat{c}$ is the log deviation of consumption, and $\hat{l}$ is the log-deviation of labo, and the last term is the relative prices of local varieties.
In doing this step, you must recognize that labor supply does not depend on PPI deflated real wages, but on CPI-deflated real wages.
Solution: Log-linearizing real marginal costs gives:

$$\hat{m}c_{Ht} = \hat{w}_t - \hat{p}_{Ht} = \varphi\hat{n}_t + \gamma\hat{c}_t + \hat{p}_t - \hat{p}_{Ht}$$

Plugging this expression in the local PC from the previous point yields:

$$\hat{\pi}_{Ht} = \beta E_t[\hat{\pi}_{Ht+1}] + \kappa_c \hat{c}_t + \kappa_n \hat{n}_t + \kappa_p(\hat{p}_t - \hat{p}_{Ht})$$

where $\kappa_c = \kappa_{mc}\gamma, \kappa_n = \kappa_{mc}\varphi, \kappa_p = \kappa_{mc}$.
Note that $\hat{c}$ is the consumption index of the home household (the counterpart of $\hat{c}^*$)

(j) Aggregate inflation is given by $\hat{\pi}_t = 0.5(\hat{\pi}_{H,t} + \hat{\pi}_{F,t})$. That is, aggregate inflation is equal to the average inflation rate across regions. Aggregate your NK regional Phillips curves into a national Phillips curve.

Solution:

$$\begin{aligned}\hat{\pi}_t = &.5\Big(\beta E_t[\hat{\pi}_{Ht+1}] + \kappa_c \hat{c}_t + \kappa_n \hat{n}_t + \kappa_p(\hat{p}_t - \hat{p}_{Ht})\Big)\\ &+ .5\Big(\beta E_t[\hat{\pi}_{Ft+1}] + \kappa_c \hat{c}^*_t + \kappa_n \hat{n}^*_t + \kappa_p(\hat{p}^*_t - \hat{p}_{Ft})\Big)\end{aligned}$$

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa_c.5[\hat{c}_t + \hat{c}^*_t] + \kappa_n.5[\hat{n}_t + \hat{n}^*_t] + \kappa_p.5[\hat{p}_t + \hat{p}^*_t] - \kappa_p.5[\hat{p}_{Ht} - \hat{p}_{Ft}]$$

Using the log-linearized expressions for $\hat{p}_t$ and $\hat{p}^*_t$ it follows that $\hat{p}_t + \hat{p}^*_t = \hat{p}_{Ht} + \hat{p}_{Ft}$. Therefore, the terms involving price indexes cancel out:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa_c.5[\hat{c}_t + \hat{c}^*_t] + \kappa_n.5[\hat{n}_t + \hat{n}^*_t]$$

Total consumption and total labor supplied are given by $\hat{C}_t = .5(\hat{c}_t + \hat{c}^*_t)$ and $\hat{N}_t = .5(\hat{n}_t + \hat{n}^*_t)$. I'm using capital letters for log-linearized aggregate quantities

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa_c \hat{C}_t + \kappa_n \hat{N}_t$$

(k) Use market clearing at the national level $\hat{Y} = \hat{N} = \hat{C}$ to re-express your national Phillips curve as a function of output only.

Solution: Using national market clearing in the previous expression we get:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa_{mc}(\varphi + \gamma)\hat{Y}_t$$

where $\hat{Y}_t$ is the log deviation of national output.

(l) Can you learn something about the national Phillips curve using the regional Phillips curves? More specifically. Are these curves related? What are their differences (if any)? Are their slopes the same? What is the economic explanation of their differences (if any) or their similarities?

Solution: The slopes of the regional and national Phillips curves are different. To see this more clearly, use market clearing to express the national NKPC in terms of employment:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa_{mc}(\varphi + \gamma)\hat{N}_t$$

where N_t is total employment in the economy. The regional NKPC is given by

$$\hat{\pi}_{Ht} = \beta E_t[\hat{\pi}_{Ht+1}] + \kappa_c \hat{c}_t + \kappa_n \hat{n}_t + \kappa_p(\hat{p}_t - \hat{p}_{Ht})$$

With separable preferences, a consumption term appears in both the national and regional Phillips curves. These capture the wealth effects on labor supply which will affect marginal costs. Such wealth effects are muted with GHH preferences (case saw in class). Because the relationship between employment and consumption is different at the aggregate and regional levels (ie. $\hat{C}_t = \hat{N}_t$ but $\hat{c}_t \neq \hat{n}_t$), the slopes of the PC are not the same. The coefficient on employment (-unemployment) in the regional Phillips curve is given by $\kappa_n^R = \kappa_{mc}\varphi$, while the coefficient on employment in the national Phillips curve is given by $\kappa_n^A = \kappa_{mc}(\varphi + \gamma)$.