# NKM - SHORT REVIEW AND PRACTICE QUESTIONS

Discussion ECON 210C - Paula Donaldson

## PLAN FOR TODAY

1. Short review of the New Keynesian Model
2. Practice question I
3. Practice question II

# THREE-EQUATION NK MODEL

The log-linearized NK model boils down to three equations:

$$\text{D-IS:} \qquad \hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$

$$\text{NKPC:} \qquad \hat{\pi}_t = \kappa \underbrace{(\hat{y}_t - \hat{y}_t^{flex})}_{\text{Output Gap}} + \beta E_t\{\hat{\pi}_{t+1}\} + u_t$$

$$\text{MR:} \qquad \hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

with

- three unknowns: $\hat{i}_t$, $\hat{y}_t$, and $\hat{\pi}_t$ ,
- productivity shocks drive the output gap $\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$,
- the monetary policy shock $v_t$.
- the cost push shock $u_t$

# THREE-EQUATION NK MODEL - DYNAMIC IS

Dynamic IS: Relates output to future expectations of output and the real interest rate

1. Solve HH block for (non-linear) Euler:

$$1 = \beta E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t\{\Lambda_{t,t+1} R_{t+1}\}$$

2. Log-linearize around zero-inflation steady state:

$$\hat{c}_t = -\frac{1}{\gamma} \left( \hat{i}_t - E_t\{\hat{\pi}_{t+1}\} \right) + E_t\{\hat{c}_{t+1}\}$$

3. Substitute with market clearing $Y = C$ and EIS $\sigma \equiv 1/\gamma$ (with iterated version):

$$\hat{y}_t = -\sigma \left( \hat{i}_t - E_t\{\hat{\pi}_{t+1}\} \right) + E_t\{\hat{y}_{t+1}\}$$

$$\Rightarrow \; \hat{y}_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} (\hat{r}_{t+s+1}) \right\}$$

# THREE-EQUATION NK MODEL - NKPC

NKPC: inflation is expectations-augmented PDV of future marginal cost / markup deviations expressed in terms of output gap

1. Log-linearized price index (Dixit-Stiglitz + Calvo pricing)

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \quad \Rightarrow \quad \hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$$

2. Log-linearized reset price from firm problem written recursively

$$P_t^* = (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} P_{t+k}^{\epsilon-1}} \frac{W_{t+s}}{A_{t+s}} \right\}$$

$$\Rightarrow \quad \hat{p}_t^* = (1-\beta\theta)(\hat{p}_t + \hat{mc}_t) + \beta\theta E_t\{\hat{p}_{t+1}^*\}$$

3. Combine and iterate for inflation in terms of marginal cost deviation:

$$\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t\{\hat{\pi}_{t+1}\}, \quad \text{where } \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

$$\Rightarrow \quad \hat{\pi}_t = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \right\}$$

# RTHREE-EQUATION NK MODEL - NKPC CONT.

4. Define marginal cost deviation in terms of output gap: output less natural (i.e., flexible price) level of output

$$\begin{aligned} \hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t &= (\gamma + \varphi)\hat{y}_t - \varphi\hat{a}_t \\ \Rightarrow \ (\gamma + \varphi)\hat{y}_t^{flex} &\equiv (1 + \varphi)\hat{a}_t \ \text{ for } \hat{mc}_t^{flex} = 0 \\ \Rightarrow \ \hat{mc}_t &= (\gamma + \varphi)(\hat{y}_t - \hat{y}_t^{flex}) \end{aligned}$$

5. Arrive at NKPC (with iterated version):

$$\begin{aligned} \hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\} \ \text{ where } \ \kappa = \lambda(\gamma + \varphi) \\ \Rightarrow \ \hat{\pi}_t &= \kappa E_t \left\{ \sum_{s=0}^{\infty} \beta^s (\hat{y}_{t+s} - \hat{y}_{t+s}^{flex}) \right\} \end{aligned}$$

# THREE-EQUATION NK MODEL - MONETARY POLICY RULE

Central banks sets the nominal interest rate according to an interest rate (Taylor) rule

1. Log-linearized monetary rule:
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

# PRACTICE QUESTIONS

# SPRING 2022 QUAL QUESTION

Consider the standard NK model with $\hat{y}_t^{flex}$ normalized to zero:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\{\hat{\pi}_{t+1}\}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + v_t$$

a. **(20pts)** Assume $v_t = \rho_v v_{t-1} + \epsilon_t^v$ with $\epsilon_t^v \sim N(0, \sigma_v^2)$. Solve for the equilibrium levels of $\hat{y}_t, \hat{\pi}_t, \hat{i}_t$, and $\hat{r}_t = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}$ as a function of $v_t$.

b. **(20pts)** Explain intuitively how a monetary policy shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 5 sentences)

c. **(15pts)** Briefly explain the identification problem in estimating the effect of monetary policy shocks on real output. (max 5 sentences)

d. **(30pts)** Briefly explain two approaches to solving the identification problem. (max 5 sentences each)

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a **(20pts)** Explain intuitively how a monetary policy shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 5 sentences)

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a **(15pts)** Briefly explain the identification problem in estimating the effect of monetary policy shocks on real output. (max 5 sentences)

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# NEW KEYNESIAN MODEL - PRACTICE QUESTION

## 1. Cost-push shocks

Consider the standard new Keynesian model

$$\hat{x}_t = E_t\hat{x}_{t+1} - E_t(\hat{i}_t - \hat{\pi}_{t+1} - \hat{r}^n_{t+1}) \tag{1}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{x}_t + u_t \tag{2}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t, \qquad \phi_\pi > 1 \tag{3}$$

(a) Interpret each of the equations (1)-(3) (max 2 sentence each).

(b) Assume $\hat{r}^n_t = 0$ and $u_t = \rho_u u_{t-1} + \epsilon^u_t$ with $\epsilon^u_t \sim N(0, \sigma^2_{\epsilon^u})$. Solve for the equilibrium levels of $\hat{x}_t$, $\hat{\pi}_t$, $\hat{i}_t$, and $\hat{r}_t = \hat{i}_t - E_t\hat{\pi}_{t+1}$ as a function of $u_t$.

(c) Explain intuitively how a supply shock affects the output gap, inflation, the nominal interest rate, and the real interest rate. (4 sentences should suffice.)

(d) Use your solution to express the loss function $L = \vartheta var(\hat{x}_t) + var(\hat{\pi}_t)$ as a function of the model parameters, where $var(\hat{x}_t)$ is the variance of the output gap and $var(\hat{\pi}_t)$ is the variance of inflation.

(e) Show that the optimal interest rate rule satisfies $\phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$.

(f) Using the optimal $\phi_\pi$, show that $\hat{x}_t = \frac{\kappa}{\vartheta(1-\beta\rho_u)}\hat{\pi}_t$.

(g) The optimal monetary policy under discretion is $\hat{x}_t = \frac{\kappa}{\vartheta}\hat{\pi}_t$. Does the optimal $\phi_\pi$ deliver a better, a worse, or the same loss? Explain intuitively. (No derivation should be necessary.)

(a) Interpret each of the equations (1)-(3)

(b) Assume $\hat{r}_t^n = 0$ and $u_t = \rho_u u_{t-1} + e_t^u$. Solver for the equilibrium levels of $\hat{x}_t, \hat{\pi}_t, \hat{i}_t$ and $\hat{r}_t$ as a function of $u_t$.

(c) Explain intuitively how a cost-push shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 4 sentences)

(d) Use your solution to express the loss function $L = \vartheta var(\hat{x}_t) + var(\hat{\pi}_t)$ as a function of the model parameters, where $var(\hat{x}_t)$ and $var(\hat{\pi}_t)$ are the variances of the output gap and inflation, respectively.

(e) Show that the optimal interest rate rule satisfies $\phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$

(f) Using the optimal $\phi_\pi$, show that $\hat{x}_t = \frac{\kappa}{\vartheta(1-\beta\rho_u)}\hat{\pi}_t$

(g) The optimal monetary policy under discretion is $\hat{x}_t = \frac{\kappa}{\vartheta}\hat{\pi}_t$.. Does the optimal $\phi_\pi$ deliver a better, a worse or the same loss? Explain intuitively. (NO derivatiosn needed)