Lecture 11: Financial Frictions in the Long Run

Juan Herreño
UCSD

May 7, 2025

Motivation

- Large dispersion in MPKs in developing countries (Hshieh and Klenow, 2009)
  - Capital does not flow to the most productive hands
- Large income differences across countries
  - Capital does not flow from rich to poor countries
- Work in development suggests a large role for financial market imperfections (Banerjee and Newman, 1993; Piketty, 1997; Aghion and Bolton, 1997; Banerjee and Duflo, 2005; ...)

Will follow Moll (2014)

Simple Framework

- Start with a static model, firms have Cobb-Douglas production functions with CRS

$$y_j = \left(z_j k_j\right)^{\alpha} l_j^{1-\alpha}$$

- Financial constraint: May rent capital up to a multiple of their wealth

$$k_j < \lambda a_j$$

- $\lambda \geq 1$ the extent of financial frictions. First best: $\lambda \to \infty$
- Firms sell homogeneous goods, rent labor and capital in competitive markets

$$\Pi_j = y_j - wl_j - (r + \delta)k_j$$

- Distribution of a exogenous for now
- r and w are endogenous objects, of course

Simple Framework

- Optimal labor demand is very simple

$$l_j = z_j k_j \left(\frac{w}{1-\alpha} \right)^{-1/\alpha}$$

- Replace on the profit function

$$\Pi_j = (z_j \pi - r - \delta) k_j, \text{ with: } \pi = \alpha \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)/\alpha}$$

- Profits are linear in k. Corner solution: Either $k_j = \lambda a_j$ or $k_j = 0$.
- The marginal entrant has a productivity $\underline{z}$ such that

$$\Pi^*(\underline{z}) = 0$$

- Threshold is independent of a and defined by $\underline{z}\pi = r + \delta$

Intuition

- Efficiency calls for allocating production to high MPK firms
- Efficiency-improving to reallocate capital from low z to high z firms
- Financial market imperfections prevent that to happen
- Firms of wealthy owners with low z may be larger than firms of poor owners with high z

Simple Framework

- Preview
  - We can aggregate this economy from the bottom-up

$$Y = ZK^{\alpha}L^{1-\alpha}$$

  - Financial frictions aggregate to endogenous TFP losses
  - Intuition: In the first best, only entrepreneurs with the max z operate their technology (CRS)
  - Away from the first best, there is negative selection of the marginal entrepreneur

Steps for Aggregation

- Define the share of wealth held by type z

$$\omega(z) = \frac{\int_0^\infty ag(a,z)da}{\int adG(a,z)}$$

- And the CDF of wealth held by entrepreneurs of productivity less than z

$$\Omega(z) = \int_0^z \omega(x)dx$$

- Market clearing condition in the capital market implies that

$$\int adG(a,z) = \int k(a,z)dG(a,z)$$

Steps for Aggregation

$$\int a dG(a,z) = \int k(a,z) dG(a,z)$$

- Active entrepreneurs demand λa_j, non-active entrepreneurs demand 0 capital. Implying that (homework)
$$1 = \lambda(1 - \Omega(\underline{z}))$$
- Given wealth shares (exogenous for now), pins down $\underline{z}$ as a function of λ.
- If λ is lower, then $\underline{z}$ lower as well:
  - Share of wealth owned by non-active entrepreneurs decreases
- The marginal entrant is less productive!

Steps for Aggregation

- Aggregate production is the integral of individual production

$$Y = \int_0^{\bar{z}} \int_0^{\infty} y(a,z)g(a,z)dadz$$

- Can be written (become familiar with the math!)

$$Y = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda K \int_{\underline{z}}^{\bar{z}} z\omega(z)dz$$

- Not a production function. Y depends on input prices (w).

Steps for Aggregation

$$Y = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda K \int_{\underline{z}}^{\infty} z\omega(z)dz$$

- Use mkt clearing for labor (inelastic supply L)

$$L = \int \int l(a,z)dG(a,z)$$

- Similar math than before (check labor demand eqin slide 4)

$$L = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} \lambda K \int_{\underline{z}}^{\bar{z}} z\omega(z)dz$$

- solve for $\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}$ and plug back into aggregate output

$$Y = \lambda^{\alpha} K^{\alpha} L^{1-\alpha} \left(\int_{\underline{z}}^{\bar{z}} z\omega(z)dz\right)^{\alpha}$$

Steps for Aggregation

$$Y = \lambda^{\alpha} K^{\alpha} L^{1-\alpha} \left(\int_{\underline{z}}^{\infty} z\omega(z)dz \right)^{\alpha}$$

- We are done. Improve the interpretation. Remember mkt clearing of K

$$1 = \lambda(1 - \Omega(\underline{z}))$$

- And by laws of conditional expectations

$$\mathbb{E}_{\omega}\left[z|z > \underline{z}\right] = \frac{\left(\int_{\underline{z}}^{\infty} z\omega(z)dz\right)}{1 - \Omega(\underline{z})}$$

- Is the (wealth share-weighted) average productivity among active entrepreneurs

$$Y = ZK^{\alpha}L^{1-\alpha}$$

- with $Z = \mathbb{E}_{\omega}\left[z|z > \underline{z}\right]^{\alpha}$

Lessons from the static model

$$Y = ZK^{\alpha}L^{1-\alpha}$$

$$Z = \mathbb{E}_{\omega}\left[z|z > \underline{z}\right]^{\alpha}$$

- We recovered your neoclassical growth model from 210A
- But with an endogenous TFP
- Starting from a continuum of heterogeneous firms with inequality
- TFP depends on the distribution of technical productivity in the population
- But also on allocative efficiency: are talented folks able to access the means of production?
- Under financial frictions, it matters who (z) owns the wealth a.
- Not in the first best. The financial market takes care of dictating the flow of funds

Why does not capital flow to poorer countries?

- Remember the free entry condition

$$\underline{z}\,\alpha\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} = r+\delta$$

- Use this useful intermediate step we derived

$$Y = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda K \int_{\underline{z}}^{\infty} z\omega(z)dz$$

- Along with mkt clearing in K

$$r+\delta = \alpha \frac{\underline{z}}{\mathbb{E}_{\omega}\left[z \middle| z > \underline{z}\right]} Z K^{\alpha-1} L^{1-\alpha}$$

- Note that $\frac{\underline{z}}{\mathbb{E}_{\omega}\left[z|z>\underline{z}\right]} \in [0,1]$

Why does not capital flow to poorer countries?

$$r + \delta = \alpha \left(\frac{\underline{z}}{\mathbb{E}_\omega \left[z | z > \underline{z} \right]} \right) Z K^{\alpha-1} L^{1-\alpha}$$

- If we forget the parenthesis, exactly the condition $MPK = r + \delta$ you know
- Under financial frictions, unproductive entrepreneurs operate their technologies
- The marginal entrepreneur is less productive than the average active entrepreneur
- Which *lowers* the return on capital
- Financial frictions distort the incentives to capital accumulation!

Lessons from the Static Model

- Effect of financial frictions?
  - Productive entrepreneurs are constrained
  - Less productive entrepreneurs become active
  - For the same level of wealth, there is lower allocative efficiency
- TFP is endogenous
- Marginal entrepreneur is less productive than the average active entrepreneur
- Potentially arbitrarily large TFP losses due to financial frictions
- If λ is small enough in developing countries, we could explain income differences across countries

Conceptual Problem of the static model

- Take the problem of a productive entrepreneur $z > \underline{z}$
- Earning profits equal to $(z\pi - r - \delta)k_j > 0$
- Profits are linear in k
- Borrowing constraint $k < \lambda a$
- Incentives to *self-finance* (increase a)
- Static models do not allow for that
- Incentives depend on the *persistence* of z
- Dynamic effects potentially different to static effects

Conceptual Problem of the static model

- Dynamic effects potentially different to static effects
- Lesson: Questions that involve a time dimension (growth) often require theories that take time seriously

Putting dots in the model

- We are going to allow people to save (everything should have a (t) but I'll save notation whenever it's obvious)

$$\dot{a} = ra + f(z, k, l) - wl - (r + \delta)k - c$$

- Preferences over consumption

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c(t) dt$$

- Due to log utility we get an amazing result. The law of motion for wealth

$$\dot{a} = (\lambda \max\{z\pi - r - \delta, 0\} + r)a - c$$

- Can be expressed as

$$\dot{a} = s(z)a \text{ where } s(z) = \lambda \max\{z\pi - r - \delta, 0\} + r - \rho$$

- Intuition: with log utility, the MPC out of changes in wealth is equal to the discount rate

The importance of dynamics

- $s(z)$ increasing in z. More productive entrepreneurs increase their wealth shares
- Consider a fully persistent productivity process $z(t) = z$

$$\lim_{t\to\infty} \omega(z,t) = \begin{cases} 1 & \text{if} \quad z = \max\{z\} \\ 0 & \text{if} \quad z < \max\{z\} \end{cases}$$

- and TFP given by $Z = \mathbb{E}_{\omega}\left[z|z > \underline{z}\right]^{\alpha} = \max\{z\}^{\alpha}$
- For any value of λ!
- Self-finance undoes the losses from TFP in this case.

The importance of dynamics

- Now consider iid productivity shocks

$$g_t(a, z) = \varphi_t(a)\psi(z)$$

- Wealth and productivity are independent, and

$$\omega(z, t) = \psi(z)$$

- Large TFP losses
- The paper shows these results formally

The importance of dynamics

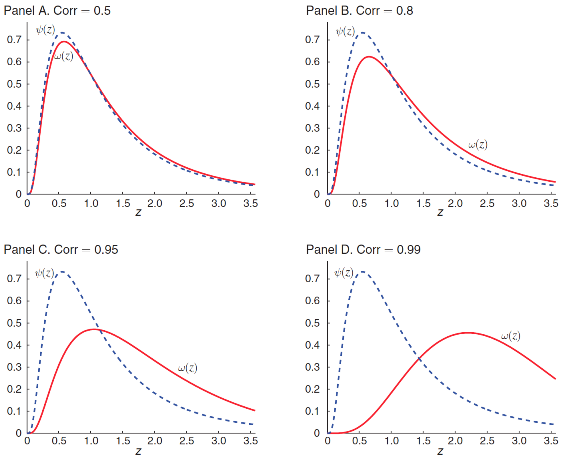

FIGURE 1. WEALTH SHARES AND AUTOCORRELATION

Notes: The dashed lines are the productivity distribution $\psi(z)$ from (27). The solid lines are the wealth shares $\omega(z)$: i.e., the solution to (22) for the stochastic process (26). As persistence θ (equivalently autocorrelation) increases, wealth becomes more concentrated with high-productivity entrepreneurs.

The importance of dynamics

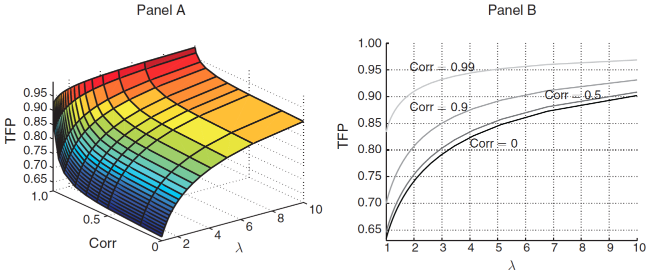

FIGURE 2. TFP AND AUTOCORRELATION

Notes: Panel B displays a cross-section of the three-dimensional graph in panel A. Again, note the sensitivity in the range corr $= 0.75$ to corr $= 1$. Parameters are $\alpha = 1/3$, $\rho = \delta = 0.05$, $\sigma\sqrt{-\log(0.85)} = 0.56$, and I vary corr $= \exp(-(1/\theta))$.

Transitional Dynamics

- So far, effects comparing steady states
- But after a reform that changes λ, how long should it take for economies to reach new steady state?
- insight: If z is persistent, it can take a long time
- intuition: self-financing takes time

The importance of dynamics

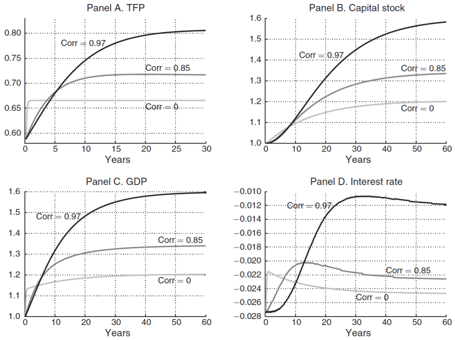

FIGURE 4. TRANSITION DYNAMICS FROM DISTORTED INITIAL WEALTH DISTRIBUTION

Notes: Parameter values are $\alpha = 1/3$, $\rho = \delta = 0.05$, and $\lambda = 1.2$, consistent with the external-finance-to-GDP ratio for India (see Table E1). For the benchmark exercise, I use corr $= \exp(-(1/\theta)) = 0.85$ and $\sigma\sqrt{1/\theta} = 0.56$. The lines for corr $= 0$ and corr $= 0.97$ vary θ while holding constant $\mathrm{var}(\log z) = \sigma^2/2$. Initial wealth shares are given by (29) with $m = -0.5$.

Several polar cases:

- Static model: Losses from financial frictions arbitrarily large
- Steady State differences in Dynamic model
  - i.i.d. firm level productivity: arbitrarily large TFP losses, instantaneous transitions.
  - Permanent firm level productivity: TFP losses go to zero, very long transitions
- For realistic persistence
  - meaningful steady state losses, transition dynamics take time

The right persistence parameter

Moll (2014) advocates for persistence between 0.75-0.97 using evidence from Gourio (2008), Asker, Collard-Wexler and De Loecker (2014)

- Go back to figure 2 and discuss

Moving Forward

- Appeal of the model is its tractability
- Perhaps unrealistic features
  - Financial frictions matter not only for scale of production but entry/sectoral allocation
  - Decreasing returns to scale could be important. We are assuming the best firm can undertake the whole production of the economy
  - ...