Agency Costs, Net Worth, and Business Fluctuations

Bernanke & Gertler (1989)

Econ 210C Discussion
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Agency costs, net worth, and business fluctuations

- Incorporates financial frictions into a macro model (RBC)
- Firm balance sheets (net-worth) matter for economic fluctuations: both as a source and as a propagator
- Key mechanism: **agency costs** arising from asymmetric information between entrepreneurs and lenders

Model Overview

- Two generations each period (OLG): "y" and "o":
- A share η of each gen. are entrepeneurs, the rest are lenders
- Entrepeneurs are heterogeneous and indexed by ω, distributed $U[0,1]$
- Output can be consumed, invested in producing capital (available next period), or stored as inventories (return r)
- Inelastic labor supply by "y", normalized to 1
- Firms produce according to $Y_t = \theta_t F(K_t, L_t)$, where θ_t is iid with mean θ
- Savings are increasing in wage income

Entrepeneurs - 2 States

- Only get utility from consumption when old
- Endowed with a project that requires $x(\omega)$ units of output
- Project can generate κ_1 or κ_2 units of capital.
- $\kappa = \pi_1\kappa_1 + \pi_2\kappa_2$
- Indifference between investing and storing: $q_{t+1}\kappa = rx(\underline{\omega})$

Financial Frictions - Agency Costs

- An entrepeneur with $\omega < \underline{\omega}$ and $x(\omega) < S^e$ needs to borrow to invest
- Agency costs arise from asymmetric information between entrepeneurs and lenders
- Lenders cannot observe the true state of the project (high or low output)
- Entrepreneurs can misreport the state to avoid repayment
- Lenders can audit the project, but this incurs a cost γ per audit
- Aggregate capital will be $k_{t+1} = (\kappa - \gamma h_t) \underbrace{i_t}_{\text{No. projects}}$
- where h_t is the number of audits in period t.

Model: Entrepeneur problem

- **Entrepreneur's objective:**

$$U = \pi_1 [\, p\, c_a + (1-p)\, c_1 \,] + \pi_2\, c_2 \ ,$$

  where
  - c_1 = consumption if outcome reported low and *not* audited,
  - c_a = consumption if low outcome reported but audited,
  - c_2 = consumption in the high state,
  - p = probability of audit when a low outcome is reported.
- Lender gets the difference between project output and the entrepreneur's consumption

Model: Entrepreneur's problem

- **Optimal contract:** designed to maximize U subject to:
  1. *Participation Constraint:*

$$\pi_1\big[q\,\kappa_1 - (1-p)c_1 - p(c_a + \gamma q)\big] + \pi_2\big[q\,\kappa_2 - c_2\big] \;\geq\; \underbrace{r\left(x - S_e\right).}_{\text{Return on storing inventories}} \tag{C1}$$

  2. *Incentive compatibility (truth-telling):*

$$c_2 \;\geq\; \underbrace{(1-p)\left[c_1 + \overbrace{(\kappa_2 - \kappa_1)\,q}^{\text{Resale value of } \kappa}\;\right]}_{\text{Expected consumption if misreport high as low}} \quad . \tag{C2}$$

  3. *Limited liability and Feasibility:*

$$c_1 \geq 0, \qquad c_a \geq 0, \qquad 0 \;\leq\; p \;\leq\; 1\,. \tag{C3–C5}$$

Optimal Contract with CSV

The optimal contract has the structure defined by (C1)–(C5). We consider two cases:

Case 1: Full Collateralization (No Agency Problem)

If the entrepreneur's net worth is high enough that

$$q\,\kappa_1 \;\geq\; r\left(x - S_e\right),$$

then even in the low state, the entrepreneur can repay the lender's full required amount.

- *No incentive to misreport:* When $q\kappa_1$ covers the debt, the entrepreneur gains nothing by lying about the outcome. The optimal audit probability is $p^* = 0$.
- *Outcome:* The entrepreneur pays the lender $r(x - S_e)$ each period. His consumption is the residual output. For example, in the high state $c_2 = q\,\kappa_2 - r(x - S_e)$.

Case 2: Incomplete Collateralization

If

$$q\,\kappa_1 \;<\; r\left(x - S_e\right),$$

then the entrepreneur's net worth is insufficient to fully guarantee the loan. All contract constraints will bind at optimum:

- *No payoff in low state:* $c_1^* = c_a^* = 0$.
- *Incentive constraint binds:* Given $c_1^* = 0$, we have

$$c_2^* = (1-p)\big(\kappa_2 - \kappa_1\big)\,q\,.$$

- *Participation binds*

Solving for the Optimal Audit Probability p^* (I)

With $c_1^* = c_a^* = 0$ and $c_2^* = (1-p)(\kappa_2 - \kappa_1)q$, the participation constraint (C1) becomes:

$$\pi_1\Big[q\,\kappa_1 - p\,\gamma q\Big] \;+\; \pi_2\Big[q\,\kappa_2 - (1-p)(\kappa_2 - \kappa_1)q\Big] = r\,(x - S_e)\,.$$

Simplify LHS: $$\pi_1 q\,\kappa_1 + \pi_2 q\,\kappa_1 \;+\; p\,q\Big[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma\Big] = r\,(x - S_e)\,.$$

(We used $\pi_1 + \pi_2 = 1$ to combine terms $q\kappa_1$.)

Solving for the Optimal Audit Probability p^* (II)

Continue rearranging for p:

$$p\,q\Big[\pi_2(\kappa_2-\kappa_1)-\pi_1\gamma\Big] = r\,(x-S_e)-q\,\kappa_1\ ,$$

$$\Longrightarrow \qquad p^* = \frac{r(x-S_e)-q\,\kappa_1}{q\,\Big[\pi_2(\kappa_2-\kappa_1)-\pi_1\gamma\Big]}\,.$$

This is the optimal audit probability in the insufficient collateral case.

- p^* is lower the higher entrepreneur net worth S_e (more skin in the game $\rightarrow$ less monitoring needed).

Expected consumption is $\pi_2 \times (1-p^*) \times q(\kappa_2-\kappa_1)$

Investment Decision - With AI, who will undertake the project?

- Three types of entrepeneurs based on their efficiency ω:
  - Good $\rightarrow \omega \leq \underline{\omega}$

$$q\kappa - rx(\underline{\omega}) - q\pi_1\gamma = 0$$

  - their expected return is positive even if $p = 1$
  - Fair $\rightarrow \underline{\omega} < \omega \leq \overline{\omega}$
  - the expected return is profitable onlly if $p = 0$
  - Poor entrepeneurs are better off storing inventories
- Both good and fair cut-offs are increasing functions of q
- Let $S^*(\omega, q)$ be the savings above which entrepeneur can repay regardless of the state:

$$S^*(\omega, q) = x(\omega) - \frac{q}{r}\kappa_1$$

- This is decreasing in q and ω

Investment Decision - Who will undertake the project?

- Good entrepeneurs always want to invest
- Poor entrepeneurs never invest
- Fair entrepeneurs will enter a lottery

Fair entrepeneurs

There are three cases:

- Entrepeneurs with $S(\omega) < S^{'}$, prefers to store
- Entrepeneurs with $S^{'} \leq S(\omega)S^{*}$, invests with positive audit probability
- $S(\omega) > S^{*}$, invests under full collateralization

Expected consumption between $(0, S^{*})$ is convex:

- Entrepeneur with less than S^{*} would be better-of risking $S(\omega)$ for a lottery that pays S^{*} with $p = \frac{S^{e}}{S^{*}}$ and 0 otherwise
- For each ω, a fraction $g(\omega) = \frac{S^{e}}{S^{*}}$ wins the lottery and invests, with full collateralization
- At the end of the day, only a share of fair entrepneurs undertake their project

Fair Entrepeneurs - Lottery and Expected Consumption

Capital Supply Schedule

- The total supply of capital in the next period is the sum of capital invested by good and fair entrepeneurs (bad entrepeneurs contribute zero)
- In aggregate, the capital supply comes from
  1. Good entrepeneurs that are not audited
  2. Good entrepeneurs that are audited
  3. Fair entrepeneurs that win the lottery
- The auditing probability for good entrepeneurs satisfies:

$$p(\omega) = \max\left\{ \frac{rx(\omega) - \hat{q}\kappa_1 - rS^e}{\hat{q}[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma]}, 0 \right\}$$

Capital Supply Schedule

- Total capital supply in k_{t+1} is given by:

$$k_{t+1} = \eta \left[\kappa\overline{\omega}(q) - \int_0^{\underline{\omega}(q)} \pi_1 \gamma p(\omega, q) d\omega - \int_{\underline{\omega}(q)}^{\overline{\omega}(q)} \kappa(1 - g(\omega)) d\omega \right]$$

- With $\gamma = 0$, we get the frictionless model level of capital $\overline{\omega}\kappa\eta$. (All entrepeneurs $\omega \leq \overline{\omega}$ invest)
- With $\gamma > 0$, $k_{t+1} < \overline{\omega}\kappa\eta$ because some fair entrepeneurs do not invest, and some good entrepeneurs are audited.
- The capital supply schedule is upward-sloping $\rightarrow$ we will show this in the next slides.

Slope of the Capital Supply Schedule - Intuition

$$k_{t+1} = \eta \left[\kappa \overline{\omega}(q) - \int_0^{\underline{\omega}(q)} \pi_1 \gamma p(\omega, q) d\omega - \int_{\underline{\omega}(q)}^{\overline{\omega}(q)} \kappa (1 - g(\omega)) d\omega \right]$$

- **A**: the cut-off $\overline{\omega}$ is increasing in q. A higher price of capital tomorrow, allows less efficient projects to become profitable today.
- **B**: For a similar reason, the audit probability is decreasing in q. More profitable projects are less likely to be audited, because the expected return is higher. B gets smaller as q increases.
- **C**:$1 - g(\omega, q)$ is the share of fair entrepeneurs ω that do not invest. This is decreasing in q because as the expected return of the project is higher, more fair entrepeneurs invest.

Slope of the Capital Supply Schedule - Math

- We will compute $\frac{dk}{dq}$ step by step.
- First, compute dA/dq:

$$\frac{dA}{dq} = \kappa \frac{d\overline{\omega}}{dq}$$

- We know that $\overline{\omega}$ satisfies: $F(q, \overline{\omega}) = q\kappa - rx(\overline{\omega}) = 0$. Implicitly differentiating this with respect to q gives:

$$0 = \frac{\partial F}{\partial q} dq + \frac{\partial F}{\partial \overline{\omega}} d\overline{\omega}$$
$$\Rightarrow \quad \frac{d\overline{\omega}}{dq} = \frac{\kappa}{rx'(\omega)} > 0 \ .$$

- We know that $x'(\omega) > 0$ because $x(\omega)$ is increasing in ω.
- Then

$$\frac{dA}{dq} = \frac{\kappa^2}{rx'(\omega)} > 0$$

Slope of the Capital Supply Schedule - Math

- Next, compute $\frac{dB}{dq}$. We need to take into account that the limits of integration are a function of q.
- We will apply Leibnitz rule:

$$\frac{d}{dq}\int_{a(q)}^{b(q)} f(x,q)dx = f(b(q),q)\frac{db}{dq} - f(a(q),q)\frac{da}{dq} + \int_{a(q)}^{b(q)} \frac{\partial f}{\partial q} dx$$

- Step by step again
- Recall $\int_0^{\underline{\omega}(q)} \pi_1 \gamma p(\omega, q) d\omega$
- First term:

$$\pi_1 \gamma p(\underline{\omega}, q)\frac{d\underline{\omega}}{q} > 0$$

where $\frac{d\underline{\omega}}{dq}$ can be computed from the implicit function theorem:

$$\frac{d\underline{\omega}}{dq} = \frac{\kappa - \pi_1\gamma}{rx'(\omega)} > 0$$

Slope of the Capital Supply Schedule - Math

- Second term:

$$\pi_1 \gamma p(0, q) \frac{d0}{dq} = 0 \tag{1}$$

- Third term:
-

$$\int_0^{\underline{\omega}(q)} \underbrace{\frac{\partial p}{\partial q}}_{<0} d\omega < 0$$

- Combining:

$$\frac{dB}{dq} = \pi_1 \gamma p(\underline{\omega}, q) \frac{\kappa - \pi_1 \gamma}{rx'(\underline{\omega})} + \int_0^{\underline{\omega}(q)} \underbrace{\frac{\partial p}{\partial q}}_{<0} d\omega \tag{2}$$

Slope of the Capital Supply Schedule - Math - Recap

- So far we have:

$$\frac{dA}{dq} - \frac{dB}{dq} = \kappa \frac{d\overline{\omega}}{dq} - \pi_1 \gamma p(\underline{\omega}, q) \frac{d\underline{\omega}}{dq} - \int_0^{\underline{\omega}(q)} \underbrace{\frac{\partial p}{\partial q}}_{<0} d\omega \tag{3}$$

- Next, third term $C = \int_{\underline{\omega}(q)}^{\overline{\omega}(q)} \kappa(1 - g(\omega))d\omega$

$$\frac{dC}{dq} = \kappa \left[(1 - g(\overline{\omega})) \frac{d\overline{\omega}}{dq} - (1 - g(\underline{\omega})) \frac{d\underline{\omega}}{dq} - \int_{\underline{\omega}(q)}^{\overline{\omega}(q)} \underbrace{\frac{\partial g}{\partial q}}_{>0} d\omega \right] \tag{4}$$

- Combining all the parts, we have:

$$\frac{dA}{dq} - \frac{dB}{dq} - \frac{dC}{dq} = \kappa \frac{d\overline{\omega}}{dq} - \pi_1 \gamma p(\underline{\omega}, q) \frac{d\underline{\omega}}{dq} - \int_0^{\underline{\omega}(q)} \underbrace{\frac{\partial p}{\partial q}}_{<0} d\omega$$

$$- \kappa(1 - g(\overline{\omega})) \frac{d\overline{\omega}}{dq} + \kappa(1 - g(\underline{\omega})) \frac{d\underline{\omega}}{dq} + \kappa \int_{\underline{\omega}(q)}^{\overline{\omega}(q)} \underbrace{\frac{\partial g}{\partial q}}_{>0} d\omega$$

- Some rearranging gives:

$$\frac{\partial k_{t+1}}{\partial q_{t+1}} = \kappa g(\overline{\omega}) \frac{d\overline{\omega}}{dq} + (\kappa(1 - \underbrace{g(\underline{\omega})}_{0}) - \pi_1 \gamma p(\underline{\omega})) \frac{d\underline{\omega}}{dq} + \text{integral terms}(> 0)$$

$$\frac{\partial k_{t+1}}{\partial q_{t+1}} = \kappa g(\overline{\omega}) \frac{d\overline{\omega}}{dq} + \underbrace{(\kappa - \pi_1 \gamma p(\underline{\omega}))}_{>0} \frac{d\underline{\omega}}{dq} + \text{integral terms}(> 0)$$

Equilibrium and Dynamics

- In equilibrium, the capital stock is low relative to the full information case (price of capital is high)
- Higher net worth lowers auditing probabilities and increases the range of fair entrepeneurs that invest
- This shifts the capital supply schedule to the right, increasing the capital stock
- Shocks that increase S^e (eg. productivity shock) get amplified through the relaxation of financial constraints