Lecture 4: The New Keynesian Model

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  - Introduction of Rational Expectations
  - RBC theory ignoring monetary policy at first. Classical dichotomy holds
- Salt-water reply produced the "New Keynesian" model
  - Spend the 1980s on micro-foundations of price rigidities
  - Simple representative agent rational expectation models but with frictions
  - Focus on: Inefficient fluctuations, so role for policy. Deviations from the classical dichotomy, so role for monetary policy

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- We will discuss many of them. But we will start with the Calvo model. Why? It is simple and honestly beautiful. Theoretical starting point.

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  - You often hear about the "Calvo fairy", the exogenous occurrence of a chance to reset prices

Intuition

Imagine this problem

$$\max_{P_{it}, Y_{it}, L_{it}} P_{it} Y_{it} - W_t L_{it} \tag{1}$$

$$\text{subject to: } Y_{it} = C_t (P_{it}/P_t)^{-\theta}, \; Y_{it} = A_t L_{it} \tag{2}$$

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- This pricing block static. Why worry about tomorrow when I can change my price every period, my inputs are spot, and demand curves static.

Dynamic pricing

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$$\max_{P^*_{it}, Y_{i,t+k|t}, L_{i,t+k|t}} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \lambda^k \left(P^*_{it} Y_{i,t+k|t} - W_{t+k} L_{i,t+k|k} \right) \frac{1}{P_{t+k}}$$

subject to: $Y_{i,t+k|t} = C_{t+k} (P_{i,t+k|t} / P_{t+k})^{-\theta}$, $Y_{i,t+k|t} = A_{t+k} L_{i,t+k|t}$

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Optimality

- Plug the constraints in the objective and take FOC with respect to P_{it}^*. Will not do it here. Life is too short.

$$P_{it}^* = \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \lambda^k \Lambda_{t,t+k} \frac{W_{t+k}}{A_{t+k}} C_{t+k} P_{t+k}^{\theta-1}}{\mathbb{E}_t \sum_{k=0}^{\infty} \lambda^k \Lambda_{t,t+k} C_{t+k} P_{t+k}^{\theta-1}}$$

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- where $\mathcal{M} = \frac{\theta}{\theta-1}$ is the gross markup under flexible prices, and W/A is the nominal marginal cost
- In words: The firm chooses P^* to minimize the weighted distance of its profits to those of the flexible price eq., using as weights the probability that the price is active in period k, and the expected valuation of dividends by its owner in that period.

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- Notice: P_{it}^* is the same $\forall i$. Result of assumptions on the nature of shocks and competition.

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- Important economic assumption. No selection in price changes!
- Easy to show

$$P_t^{1-\theta} = (1-\lambda)(P_t^*)^{1-\theta} + \lambda P_{t-1}^{1-\theta}$$

Household

- Very easy. We pretty much did it in lecture 1 and 2
- Representative agent with preferences

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}-\chi\frac{N_t^{1+\varphi}}{1+\varphi}\right]$$

- Can save in nominal bonds:

$$P_tC_t + B_{t+1} \leq W_tN_t + B_t(1+i_{t-1})$$

- And a CES preference bundle and price index in the background

$$C_t = \left(\int_0^1 C_{it}^{\frac{\theta}{\theta-1}}di\right)^{\frac{\theta}{\theta-1}}, P_t = \left(\int_0^1 P_{it}^{1-\theta}di\right)^{\frac{1}{1-\theta}}$$

Optimality Conditions

- Labor supply

$$\frac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\gamma}$$

- Euler equation for nominal bonds

$$1 = \mathbb{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} (1 + i_t) \right)$$

- And CES demand curves in the background

$$C_{it} = C_t \left(\frac{P_{it}}{P_t} \right)^{-\theta}$$

Monetary Policy

- Very similar to the central bank we covered in lecture 1.
- Sets interest rates. Follows a Taylor Rule.
- May introduce random disturbances, like the shocks in Lecture 3.

$$(1 + i_t) = \beta^{-1} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} e^{v_t}$$

- More realistic versions include responses to output
- Not a statement on policy optimality. We will do that soon. Taylor rule more of an approximation to actual behavior

Market clearing

- Labor markets clear. $N_t = \int_0^1 L_{it} di$
- Good markets clear $C_{it} = Y_{it}$
- Bond markets clear $B_t = 0$
- We will **define** aggregate output as $Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta}{\theta-1}} di \right)^{\frac{\theta}{\theta-1}}$
- Why? It achieves $Y_t = C_t$.

Equilibrium Definition

An equilibrium is an allocation $\left\{C_{i,t+s} C_{t+s}, N_{t+s}, Y_{t+s}\right\}_{s=0}^{\infty}$, and a set of prices $\left\{i_{t+s}, W_{t+s}, P_{i,t+s} P_{t+s}\right\}_{s=0}^{\infty}$, along with exogenous processes $\{v_{t+s}, A_{t+s}\}_{s=0}^{\infty}$ such that

- Households optimize: Labor leisure, euler equation, and demand curves are satisfied taking prices as given.
- Firms optimize: Firm-prices are set optimally, price aggregation holds
- Central bank sets policy according to the Taylor rule
- Labor, bonds, and goods markets clear.

Log-linearize

- The model so far is difficult to analyze, too many non-linear equations
- We will log-linearize the model. Paula taught you how. Need to choose a linearization point. We will choose the equilibrium with flexible prices and zero inflation
- I will skip most of the log-linearization steps, as they are very standard. Check Gali's book if you are unsure how to do it.

Log Linearization: Inflation

- Price Index:

$$P_t = \left[\lambda P_{t-1}^{1-\theta} + (1-\lambda) P_t^{*\,1-\theta}\right]^{\frac{1}{1-\theta}}$$

- The price index can be log-linearized to get

$$\hat{p}_t = \lambda \hat{p}_{t-1} + (1-\lambda)\hat{p}_t^*$$

  – Equivalently written in terms of inflation:

$$\hat{\pi}_t = (1-\lambda)(\hat{p}_t^* - \hat{p}_{t-1})$$

  – Inflation is positive when new prices are higher than old prices.

Log Linearization: Reset Prices

- The reset price can be log-linearized as:

$$\hat{p}_t^* = (1 - \beta\lambda) E_t \left\{ \sum_{s=0}^{\infty} (\beta\lambda)^s \left(\hat{p}_{t+s} + \hat{mc}_{t+s} \right) \right\}$$

- We can write this recursively as:

$$\hat{p}_t^* = (1 - \beta\lambda)(\hat{p}_t + \hat{mc}_t) + \beta\lambda E_t\{\hat{p}_{t+1}^*\}$$

Log Linearization: Phillips Curve

- Subtract $\hat{p}_{t-1}$:

$$(\hat{p}_t^* - \hat{p}_{t-1}) = (1 - \beta\lambda)\hat{mc}_t + \hat{\pi}_t + \beta\lambda E_t\{\hat{p}_{t+1}^* - \hat{p}_t\}$$

- Plug into $\hat{\pi}_t = (1 - \lambda)(\hat{p}_t^* - \hat{p}_{t-1})$ to get an expectations-augmented Phillips curve:

$$\hat{\pi}_t = \alpha\hat{mc}_t + \beta E_t\{\hat{\pi}_{t+1}\}, \text{ where } \alpha = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$$

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
  - Expected inflation: Forward looking price setters choose higher prices now if inflation is expected to be high, as nominal marginal costs will rise.

Log Linearization: Phillips Curve

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
  - Two ways to think about marginal cost deviation:
  - Set higher prices to cover higher marginal cost.
  - When marginal costs are above desired level, markups are below desired level. Inflation as firms hike markup back to desired level. (In fact, $\hat{mc}_t = -\hat{\mu}_t$).
- Iterating forward,

$$\hat{\pi}_t = \alpha E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \right\}$$

  - Inflation is the PDV of future marginal cost / markup deviations from steady state.

Log Linearization: Real Marginal Costs

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$$

- Combine labor-leisure, production function, and $\hat{c}_t = \hat{y}_t$:

$$\hat{w}_t - \hat{p}_t = (\gamma + \varphi)\hat{y}_t - \varphi\hat{a}_t$$

- Consequently,

$$\hat{mc}_t = (\gamma + \varphi)\hat{y}_t - (1 + \varphi)\hat{a}_t$$

- Compare to flexible price case:

$$1 = \frac{P_t(i)}{P_t} = \mu\frac{W_t}{P_t}\frac{1}{A_t}$$

so

$$\hat{mc}_t^{flex} = 0, \qquad (\gamma + \varphi)\hat{y}_t^{flex} = (1 + \varphi)\hat{a}_t$$

where $\hat{y}_t^{flex}$ is called the *natural level of output*.

Real Marginal Costs in Terms of Output Gap

- Combine:

$$\hat{mc}_t = (\gamma + \varphi)\hat{y}_t - (1 + \varphi)\hat{a}_t$$
$$(\gamma + \varphi)\hat{y}_t^{flex} = (1 + \varphi)\hat{a}_t$$

  to write real marginal costs in terms of output gap $\tilde{y}_t$:

$$\hat{mc}_t = (\gamma + \varphi)(\hat{y}_t - \hat{y}_t^{flex})$$

- Real marginal costs go up (and markups go down) when the output gap is high.
  - To produce more than under flex prices, markup must be lower.
  - Marginal costs high because need to hire more workers, bidding up real wage.
  - Stronger when IES and labor supply elasticity are low.
  - In Gali textbook also stronger with DRS.

The New Keynesian Phillips Curve

- Plug back into the Phillips curve $\hat{\pi}_t = \alpha \hat{mc}_t + \beta E_t\{\hat{\pi}_{t+1}\}$

$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}, \text{ where } \kappa = \alpha(\gamma + \varphi)$$

  - This is the *New Keynesian Philips Curve*: an expectations augmented Phillips curve written in terms of the output gap.
  - It is the aggregate supply curve of the model
- Solving forward,

$$\hat{\pi}_t = \kappa E_t \left\{ \sum_{s=0}^{\infty} \beta^s (\hat{y}_{t+s} - \hat{y}_{t+s}^{flex}) \right\}$$

  - Inflation is an increasing function of future output gaps.
  - Output gap high $\Rightarrow$ marginal cost high and markups low $\Rightarrow$ raise markups.

Log Linearization: The Aggregate Demand Block

- Log-linearize Euler around zero-inflation:

$$\hat{c}_t = -\frac{1}{\gamma}\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) + E_t\{\hat{c}_{t+1}\}$$

  - Steady state nominal interest rate is $i_t = \rho = -\log\beta$.

- Combine with market clearing and use $\sigma = 1/\gamma$:

$$\hat{y}_t = -\sigma\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) + E_t\{\hat{y}_{t+1}\}$$

- This is the *dynamic IS curve*. It relates output to future expectations of output and the real interest rate.

Dynamic IS

- Iterating forward, the current output gap depends negatively on the gap between the real interest rate and the natural rate of interest (assuming return to steady state):

$$\hat{y}_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} \left(\hat{r}_{t+s+1} \right) \right\}$$

- If you want to figure out what happens to output in the NK model, you need to figure out what happens to the path of the real interest rate.
  - Output gap determined purely by intertemporal substitution. Not old Keynesian marginal propensities to consume / invest.
  - Intuition also works well for larger NK models.

The Three Equation Model

- In sum, the log-linearized NK model boils down to three equations:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

  with three unknowns: $\hat{i}_t$, $\hat{y}_t$, and $\hat{\pi}_t$ and an exogenous driving process for the output gap $\hat{y}_t^{flex}$ $(= \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t)$ and the monetary policy shock $\hat{v}_t$.
- Key new ingredient is NK Phillips curve:
  - $\beta E_t\{\hat{\pi}_{t+1}\}$: Price setters forward looking.
  - $\kappa \hat{y}_t$: Output $\uparrow \Rightarrow$ MC $\uparrow \Rightarrow$ markups $\downarrow \Rightarrow$ raise prices
- Determinacy: similar condition to lecture 1. See Gali.
- Note: Gali writes everything in terms of "gaps", $\tilde{y}_t = \hat{y}_t - \hat{y}_t^{flex}$.

Special Case: $\kappa \to \infty$

- Equivalent to flexible prices: $\lambda = 0$.
- The NK Phillips Curve becomes:

$$\hat{y}_t = \hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$$

- Output fluctuations arise only from productivity fluctuations.
- Monetary variables v_t have no real effect:
  - Drop in v_t lowers nominal rate and real interest rate.
  - All else equal increases output and marginal cost.
  - Prices today rise with marginal cost so that π_{t+1} is low and the real interest rate is constant.

$\Rightarrow$ With constant real interest rate output is unchanged.

Special Case: $\kappa = 0$

- Equivalent to perfectly rigid prices: $\theta = 1$. The NK Phillips Curve becomes $\hat{\pi}_t = 0$.
- Now output is demand determined:

$$\hat{y}_t = -\sigma \hat{i}_t + E_t\{\hat{y}_{t+1}\}$$
$$\hat{i}_t = v_t$$

- If $v_t = \rho_v v_{t-1} + \epsilon_t$, then

$$\hat{y}_t = -\frac{\sigma}{1-\rho_v} v_t$$

- Monetary variables $\hat{v}_t$ have a real effect:
  - Drop in v_t lowers nominal rate and real interest rate.
  - Inflation is constant so output expands with the lower real interest rate.
- With rigid prices, output is independent of productivity fluctuations.

Intermediate κ

- Assume

$$v_t = \rho_v v_{t-1} + \epsilon_t \text{ and } \hat{a}_t = 0$$

- Guess reduced form policy functions:

$$\hat{y}_t = \psi_{yv} v_t \text{ and } \hat{\pi}_t = \psi_{\pi v} v_t$$

- This gives:

$$\begin{aligned}
\psi_{\pi v} &= \kappa \psi_{yv} + \beta \rho_v \psi_{\pi v} \\
\psi_{yv} &= -\sigma(\phi_\pi \psi_{\pi v} + 1 - \rho_v \psi_{\pi v}) + \rho_v \psi_{yv}
\end{aligned}$$

- Solving by method of undetermined coeffs:

$$\psi_{yv} = -(1 - \beta \rho_v)\Psi_v \text{ and } \psi_{\pi v} = -\kappa \Psi_v$$

$$\text{where } \Psi_v = \frac{1}{(1 - \beta \rho_v)\gamma(1 - \rho_v) + \kappa(\phi_\pi - \rho_v)} > 0$$

Epistemiology

- NK model a response to the Lucas critique: “fully” optimizing agents but monetary policy has real effect.
- Extensive debate on how well NK model fits the data.
  - Centers on much more complex “medium-scale” models.
  - These are the simple NK model at its core with many additional “bells and whistles” (capital, habits, indexation, rigid wages, government, etc). See references in the syllabus.
  - Everyone agrees the simple three equation model does not match the data well.
- Should view simple model as an organizing framework.
  - Communicate results: everyone knows this model and how it works.
  - How should policy respond to shocks? Why?
  - Interpret policy actions through lens of NK model.