Lecture 19: Lumpy Investment

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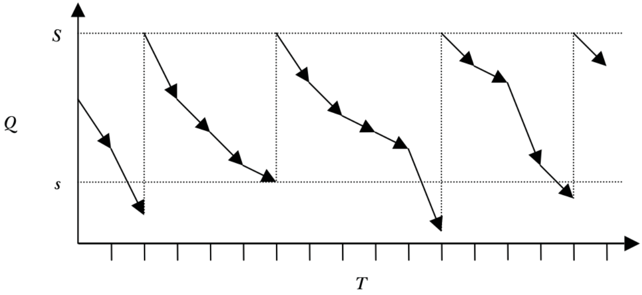
June 3, 2025

Fixing terms - (S,s) models

- The origin of the S,s terminology dates to models of inventory management
- A firm receives stochastic demand that decreases their inventories
- Every time the stock goes below s, firms would reorder inventories to stock back to a level S
- The problem is to find out the optimal s and S thresholds
- Today, we use the terms S, s models to where there is lumpy adjustment
  - Periods of inaction, followed by large adjustments
  - Usually, we rationalize economic decisions that are lumpy with the presence of fixed costs of adjustment
  - Applications: Investment theory, price setting, supply chain formation, exporting decisions, occupational choices, technology choices, energy transition, ...

(S,s) models - a picture

Figure 1
Operation of an (*S*, *s*) Policy with Upper and Lower Barriers



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- If too lumpy, models that imply smooth micro investment might be off
- May reflect increasing returns in the adjustment technology
- It may be better to invest a lot at once, rather than smooth it out

Doms and Dunne (1998)

Good example of a “facts” paper that reshaped a field

- Fundamental Question: How Lumpy is investment after all?
- Subsequent question: What does lumpiness depend on?

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Data

- U.S. data 1972-1988
- LRD from the Census Bureau
- Small sample: 13,702 establishments (of 350,000)
- Large establishments: account for 50% of manufacturing output

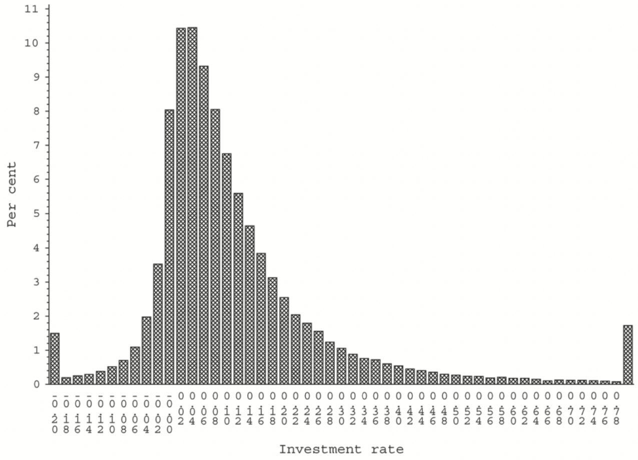
Doms and Dunne (1998)

Significant inaction

- In a given year, 80% of plants change their capital stock by < 10%
- ... 51.9% of plants ... by less than 2.5%

Distribution of Investment Rates

From Cooper and Haltwanger (2006). Same data. I/K. Skewed, large mass at 0, with fat tails



Distribution of Investment Rates

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TABLE 1

Summary statistics

Variable	LRD
Average investment rate	12·2% (0·10)
Inaction rate: investment	8·1% (0·08)
Fraction of observations with negative investment	10·4% (0·09)
Spike rate: positive investment	18·6% (0·12)
Spike rate: negative investment	1·8% (0·04)
Serial correlation of investment rates	0·058 (0·003)
Correlation of profit shocks and investment	0·143 (0·003)

LRD, Longitudinal Research Database.

Very low autocorrelation. Puzzling if you think that shocks (demand, productivity) are persistent

Distribution of Investment Rates

From Zwick and Mahon (2017). Stratified sample of tax returns. 1993 - 2010

(b) Summary Statistics

Variable	Unbalanced	Balanced
Average investment rate	11.9% (0.20, 3.23, 12.7)	10.4% (0.16, 3.60, 17.6)
Inaction rate	30.2%	23.7%
Spike rate	17.4%	14.4%
Serial correlation of investment rates	0.38	0.40
Aggregate investment rate	7.7%	6.9%
Spike share of aggregate investment	25.1%	24.4%

Higher autocorrelation (Does not include structures). Higher inaction (includes smaller firms)

Cross-sectional Patterns

There is some lumpiness, but compared to what? We need some benchmark

Doms and Dunne (1998)

Define two objects

- Growth rate of capital GK (why that formula?)

$$GK_{i,t} = \frac{I_{i,t} - \delta K_{i,t-1}}{0.5(K_{i,t} + K_{i,t-1})}$$

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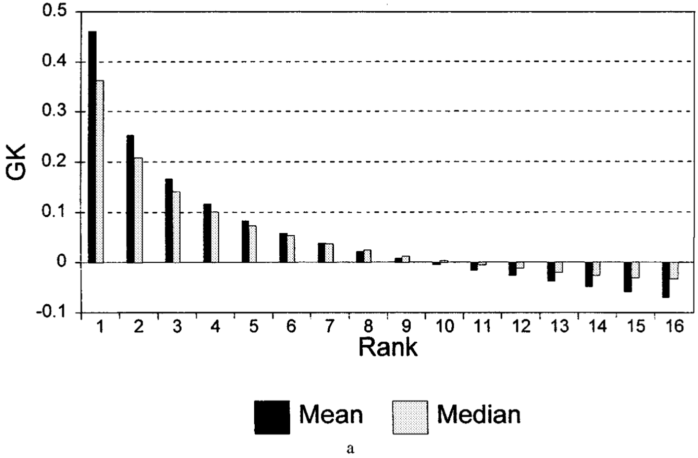
- And the (within-firm) share of investment in a given year

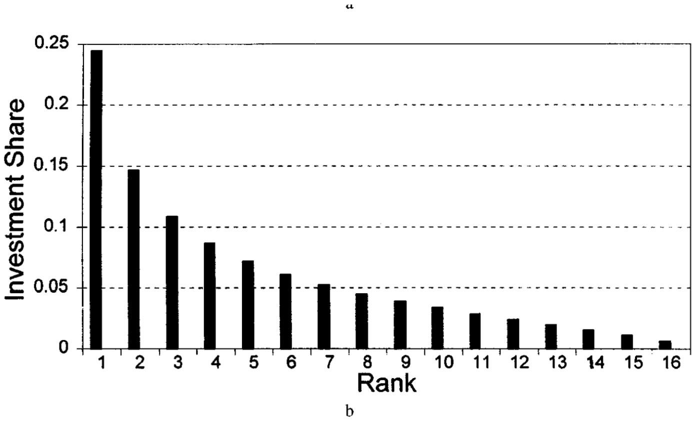
$$\text{Investment Share}_{i,t} = \frac{I_{i,t}}{\sum_\tau I_{i,\tau}}$$

Doms and Dunne (1998)



Rank is the year in which *GK* was the $n - th$ highest at the establishment level

Doms and Dunne (1998)



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Of course the previous two figures have decreasing patterns, duh

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- Define $k^*_{i,t}$ as the (log) desired level of capital at the firm

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- for ϵ distributed $N(\mu, \sigma)$

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- for ϵ distributed $N(\mu, \sigma)$
- And define $z_{i,t}$ as the gap between desired and actual capital stocks

$$z_{i,t} = k_{i,t} - k^*_{i,t}$$

Two views

Imagine there are no frictions to adjust your capital stock

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for $L < l < u < U$

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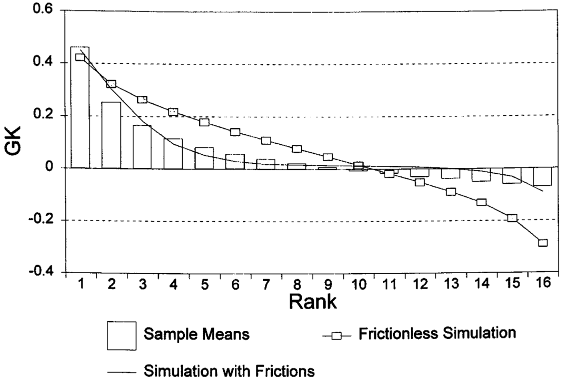

FIG. 3. Mean capital growth rates (GK) by rank, sample means, and simulated values.

Very simple example of “moment matching”

Further Evidence on Lumpiness

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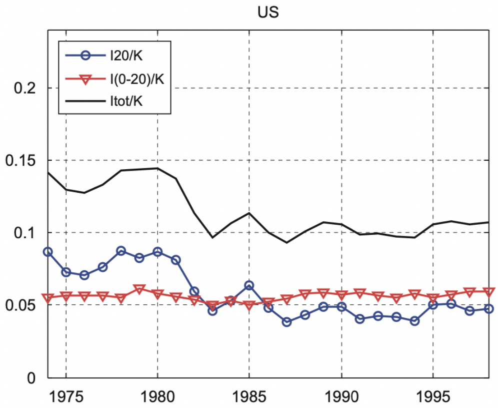
Arbitrary number of decompositions you can make, some are insightful!

Investment spikes appear to be important



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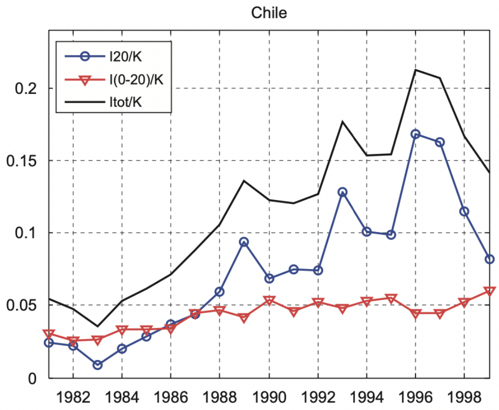

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Intensive vs. extensive margin

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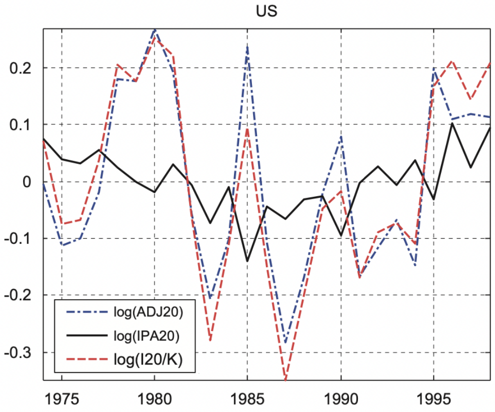
- $\frac{I_t^{20}}{K_t^{20}}$ is the avg. rate at firms with spikes. The intensive margin.
- $\frac{K_t^{20}}{K_t}$ is the (capital weighted) share of firms with spikes. The extensive margin.

$$\frac{\sum_i K_{i,t}\mathbb{1}_{spike}}{\sum K_{i,t}} = \frac{\sum_{i \in spike} K_{i,t}}{K_t} = \frac{K_t^{20}}{K_t}$$

Importance of the extensive margin



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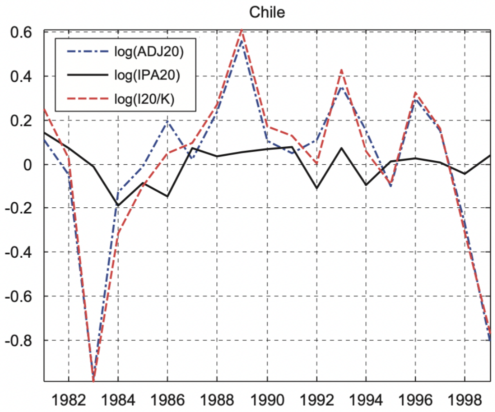

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- Equivalent to the real interest rate being constant

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- We covered some figures of that paper but we will take another look

Profits

- Firm profits are given by:

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- χ_{jt} are fixed costs of adjustment. Constant conditional on adjustment, and zero in the case of inaction
- Including χ and ψ is without loss. We could plausibly set them to zero
- Similar to the Golosov Lucas model but for capital

Firm's Problem

- Firms maximize firm value

$$V(k_j, a_j) = \max_{l_j} \left\{ a_{jt} k_j^{\theta} l_j^{\gamma} - w_t l_j \right\} + \max \left\{ V^n(k_j, a_j), V^a(k_j, a_j) - \chi_j w \right\}$$

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- We now need to specify what V^n and V^a look like

The value of non-adjusting

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- The value function in case of not-adjusting is

$$V^n(k_j, a_j) = \beta \mathbb{E} \left[V(k_j', a_j') | a_j \right]$$

- subject to

$$k_j' = (1 - \delta) k_j$$

The value of adjusting

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- The value function in case of adjusting is

$$V^a(k_j, a_j) = \max_{i_j} \left\{ -i - \frac{\psi}{2} \left(\frac{i_{jt}}{k_{jt}} \right)^2 k_j + \beta \mathbb{E} \left[V(k_j', a_j') | a_j \right] \right\}$$

- subject to

$$k_j' = (1 - \delta) k_j + i_j$$

The value of adjusting

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- subject to

$$k_j' = (1 - \delta) k_j + i_j$$

- We will assume that χ_j is iid across firms, and

$$\chi_j \sim U[\underline{\chi}, \bar{\chi}]$$

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- The value function in case of not-adjusting is

$$V^n(k_j, a_j) = \beta \mathbb{E} \left[V(k_j', a_j') | a_j \right]$$
$$k_j' = (1 - \delta) k_j$$

- The value function in case of adjusting is

$$V^a(k_j, a_j) = \max_{i_j} \left\{ -i - \frac{\psi}{2} \left(\frac{i_{jt}}{k_{jt}} \right)^2 k_j + \beta \mathbb{E} \left[V(k_j', a_j') | a_j \right] \right\}$$
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$$l(k_j, a_j) = \left(\frac{\nu a_j k_j^{\theta}}{w} \right)^{\frac{1}{1-\nu}}$$

The decision of adjusting

- Simple: you adjust if it is worth it

$$\text{adjust iff } V^a(k_j, a_j) - \chi w > V^n(k_j, a_j)$$

- Which implies an upper threshold for adjustment

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  - Stuff we do not understand that drives the decision to adjust at one time or another

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- Notice that conditional on adjustment investment has a q flavor
- Not a coincidence, we extended the framework to:
  - Heterogeneous firms
  - Idiosyncratic shocks
  - non-convex costs

Intuition of the Problem

- Let's turn to a simplified problem of a firm with profit function

$$\Pi(k, a) = ak^{\beta} - (\delta + r)k$$

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$$\Pi(k, a) = ak^{\beta} - (\delta + r)k$$

- And only random fixed costs of adjustment
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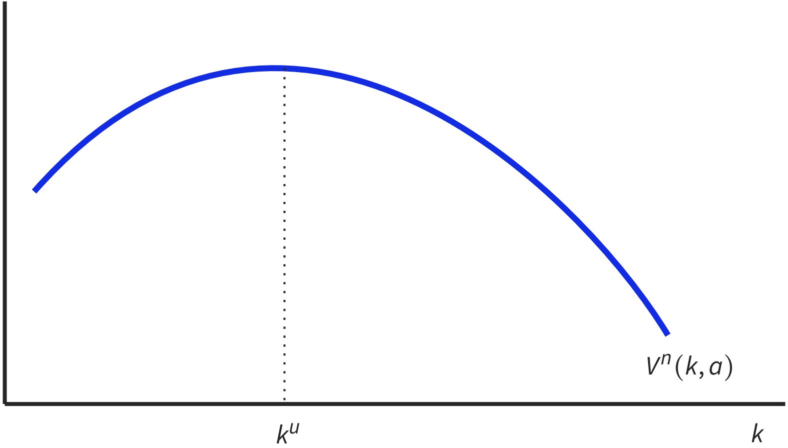
- This is the framework of Caballero and Engel (1999) in your required readings

Intuition of the Problem



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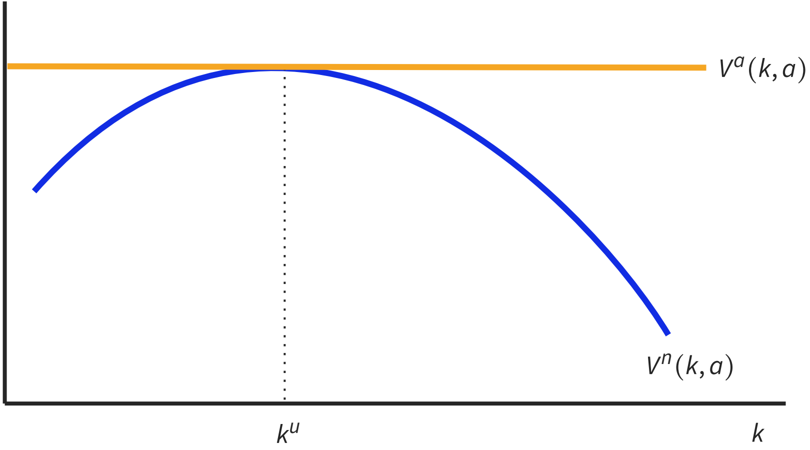
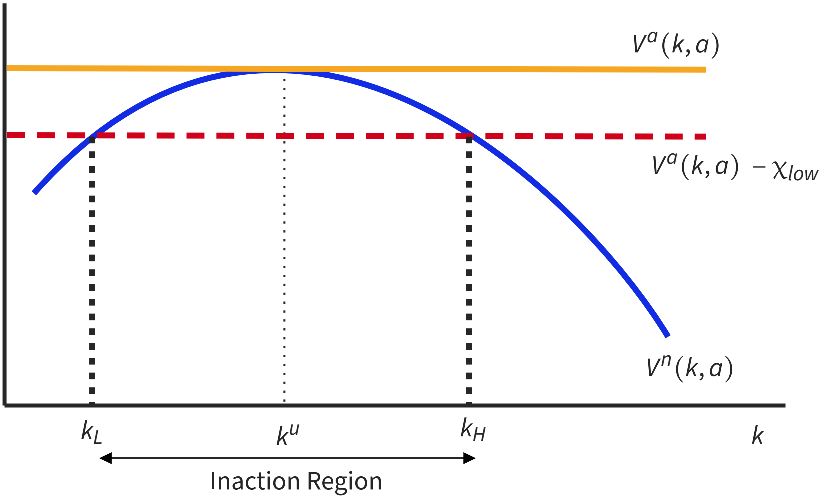
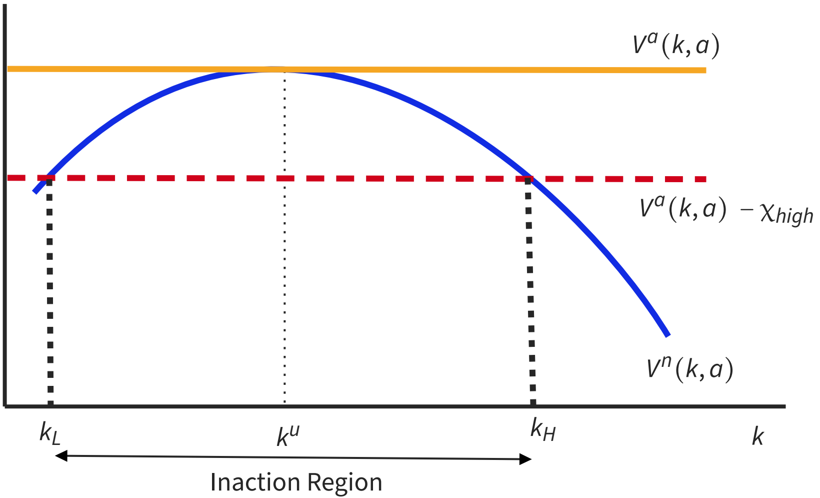




$V^a(k,a)$
$V^a(k,a) - \chi_{low}$
$V^n(k,a)$
k_L
k^u
k_H
k
Inaction Region



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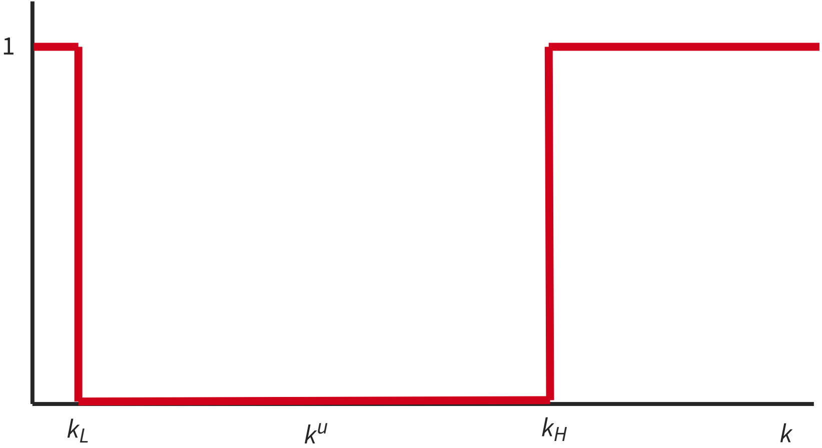
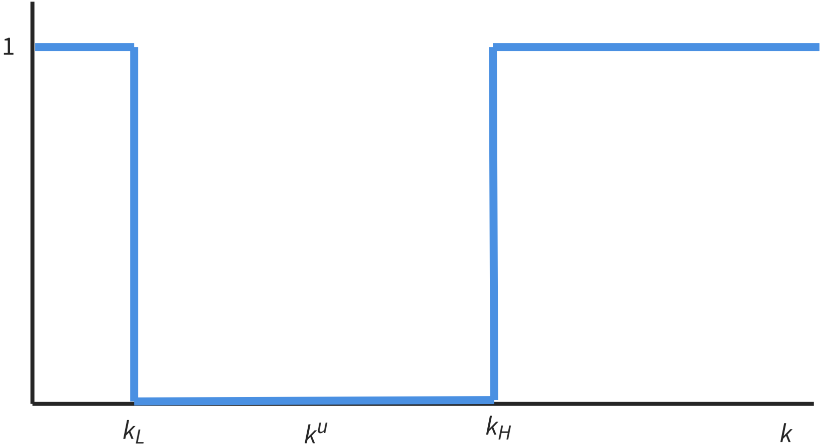
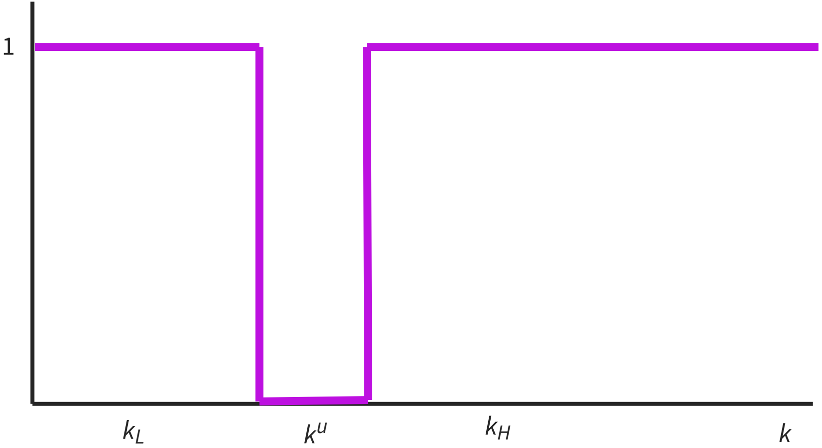
Adjustment for high χ



Adjustment for mid χ



Adjustment for low χ

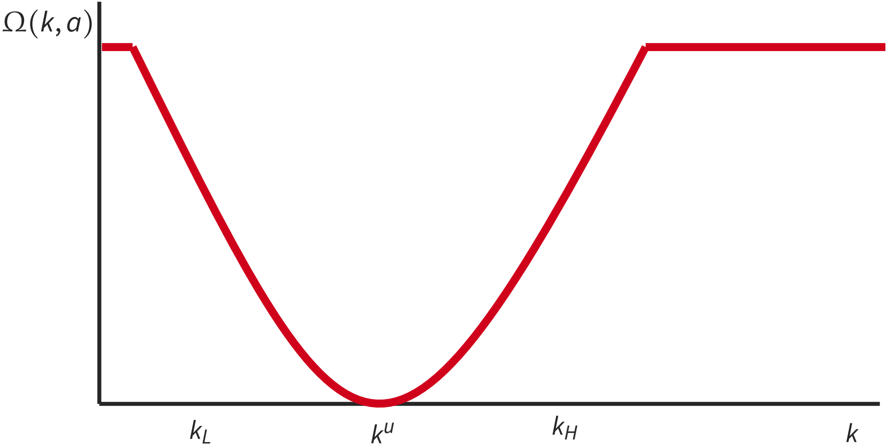


Variation in χ

Once we account for variation in χ, the adjustment region is probabilistic.

$$\Omega(k,a) = \text{Prob}(V^a(k,a) - \chi w > V^n(k,a))$$

Adjustment Hazard Function



Back to the more general problem

- I simulated for you the solution to the more general problem we considered before

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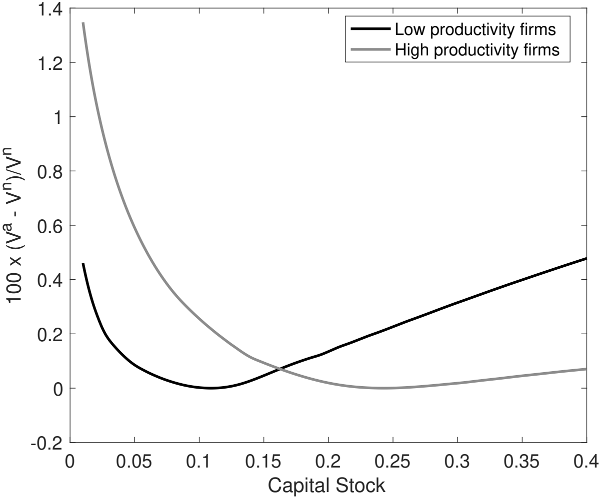
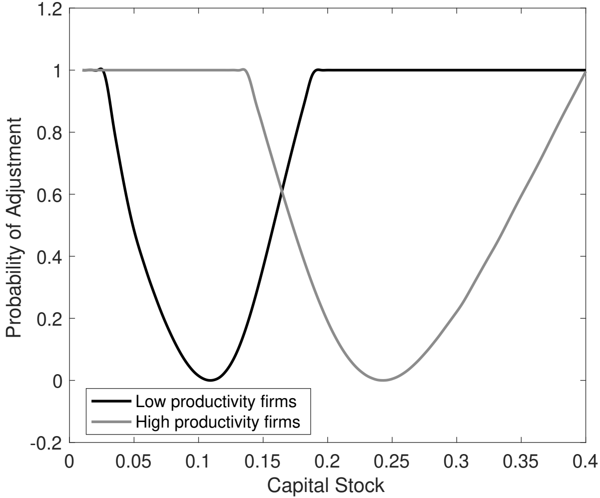
- I simulated for you the solution to the more general problem we considered before
- I chose standard values for the parameters

Value of Adjustment



Adjustment Probability



Cooper and Haltiwanger (2006)

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  - Indirect evidence on the value of a parameter, given the predictions of a model about a moment, rather than direct evidence on an elasticity driven by a natural experiment
- Use census data in order to estimate these cost of adjustment parameters

Cooper and Haltiwanger 2006

TABLE 3

Moments from illustrative models

Moment	LRD	No AC	CON	NC-F	NC-λ	TRAN
Fraction of inaction	0·081	0·0	0·038	0·616	0·588	0·69
Fraction with positive investment bursts	0·18	0·298	0·075	0·212	0·213	0·120
Fraction with negative investment bursts	0·018	0·203	0·0	0·172	0·198	0·024
Corr (i_{it}, i_{it-1})	0·058	−0·053	0·732	−0·057	−0·06	0·110
Corr (i_{it}, a_{it})	0·143	0·202	0·692	0·184	0·196	0·346

LRD, Longitudinal Research Database.

LRD: Data. No AC: no adjustment costs. CON: convex costs. NC F: Fixed costs. NC λ: fixed drops in productivity during adjustment. *Tran* Model of irreversibility.

Cooper and Haltiwanger 2006

TABLE 4

Parameter estimates: $\lambda = 1$

	Structural parameter estimates (S.E.)			Moments				
Spec.	γ	F	p_s	Corr (i, i_{-1})	Corr (i, a)	Spike$^+$	Spike$^-$	£$(\hat{\Theta})$
LRD				0·058	0·143	0·186	0·018	
all	0·049 (0·002)	0·039 (0·001)	0·975 (0·004)	0·086	0·31	0·127	0·030	6399·9
γ only	0·455 (0·002)	0	1	0·605	0·540	0·23	0·028	53,182·6
p_s only	0	0	0·795 (0·002)	0·113	0·338	0·132	0·033	7673·68
F only	0	0·0695 (0·00046)	1	−0·004	0·213	0·105	0·0325	7390·84

LRD, Longitudinal Research Database.

γ: convex cost. F: fixed cost. p_s transaction cost

Cooper and Haltiwanger 2006

- Structural estimation
- The model prefers a combination of costs

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- Firms have adjustment probability $\Lambda(x)$
- And conditional on adjustment they invest: $k^* - k = \left(\frac{k^* - k}{k}\right) k = \left(\frac{k^*}{k} - 1\right) k = (e^{-x} - 1)k$

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$$-x \approx \log(1 - x)$$

$$e^{-x} - 1 \approx -x$$

- and our expression becomes

$$\frac{I_t^A}{K_t^A} \approx \int -xf(x,t)dx = -X^A$$

- It **is** sufficient to know the *average disequilibrium* before adjustment.

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- You can understand the results we will have as very short-run responses. Before prices adjust.

Aggregate Shocks

- With aggregate shocks

$$\frac{I_t^A}{K_t^A} \approx \int (e^{-x} - 1)\Lambda(x) f(x + \delta + v_t, t-1) dx$$

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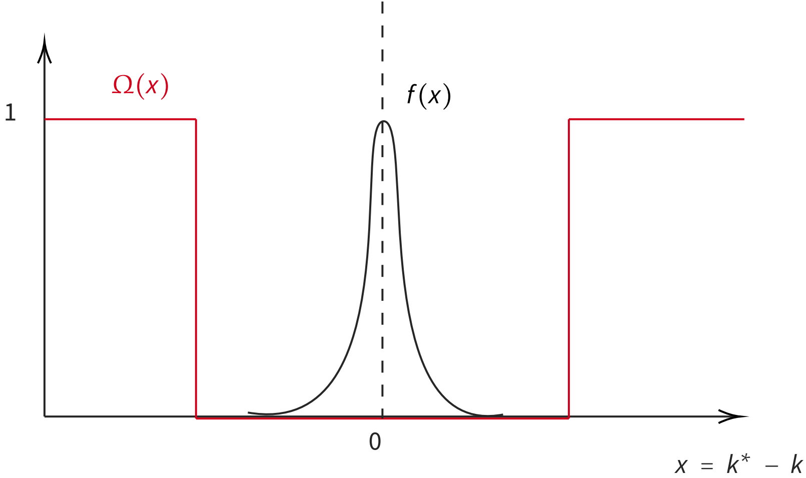
- Depreciation increases imbalances, as do positive aggregate shocks
- Ignoring changes in w and r imply that $\Lambda(.)$ is an invariant function

State dependence



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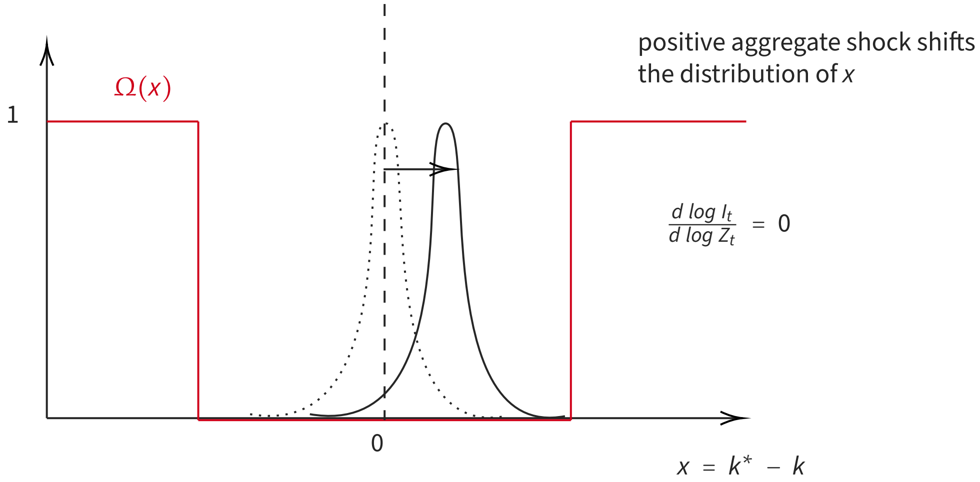

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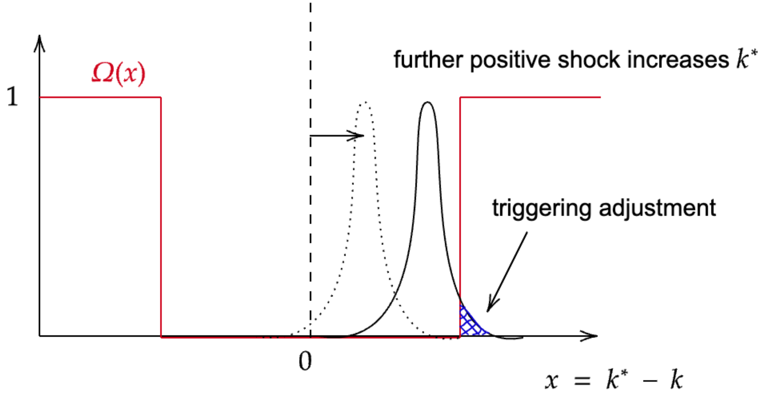

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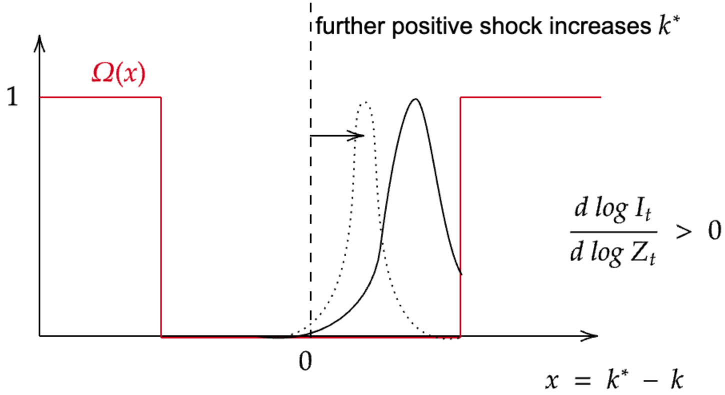

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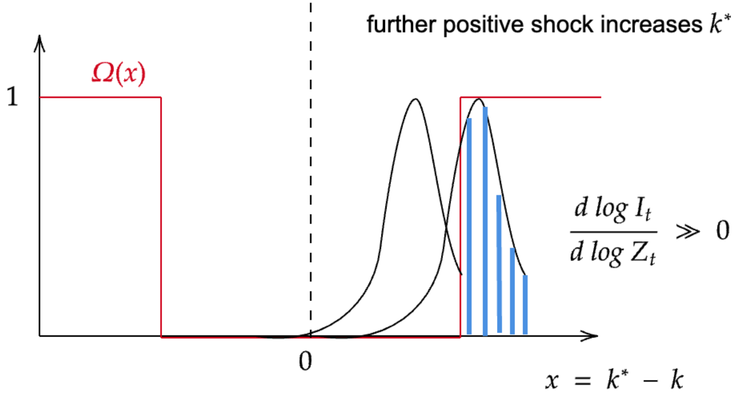

Pent-Up Demand

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Pent-Up Demand

- Aggregate investment depends on the mass of firms that are *close-enough* to adjustment
- Therefore shocks of the same size may have differential effects on investment
- As a function of the distribution of capital imbalances
- Which means that the *elasticity* of investment to an aggregate shock is state dependent
- as in: it depends on the distribution of the state variables of the model
- Remember your RBC model from 210A. Model is linear in logs. The elasticity of investment to a shock is invariant. Not guaranteed with fixed costs.

Applications to Recessions and Recoveries

- During a recession firms have “excess capital”
- Imagine a tax subsidy that makes positive investment cheaper
- If no firm is near the threshold, then the reform is ineffective
- Imagine a tax subsidy during a recovery
- If the mass of firms that would have adjusted in the absence of the subsidy is large...
- The reform subsidizes inframarginal firms, making tax subsidies more expensive per unit of investment

Caballero Engel (1999)

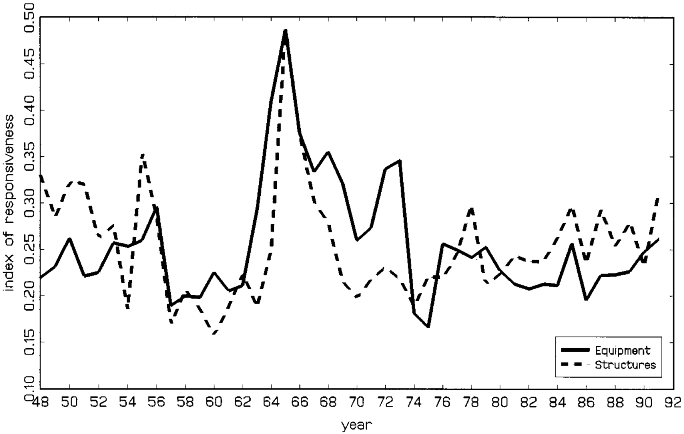

FIGURE 5

The sensitivity of investment to additional shocks is time-varying