Lecture 13: Costly State Verification

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Financial Markets

- Core role of financial markets:
  - Match savers with borrowers
  - Allow funds to flow from those that have more funds than projects to those that have more projects than funds
- Core friction in financial markets:
  - Agency problem with asymmetric information
  - Savers know less about projects than borrowers
  - Borrowers can take too much risk / put in too little effort / lie about outcome (moral hazard)
  - Saver can’t observe borrower’s / project’s “type” (adverse selection)

Costly State Verification Model

- Entrepreneur has a project that requires 1 unit of resources
- Entrepreneur has wealth $W < 1$
- Must obtain $1 - W$ of outside financing
- Expected output of project: γ
  - Heterogeneous across projects
  - Publicly observable (no adverse selection)
- Actual output is distributed $Y \sim U[0, 2\gamma]$

Costly State Verification Model

- Outside investor must bear some risk
  - Output realization can be lower than $1 - W$
- For simplicity: Entrepreneur and investor are risk neutral
- Outside option: Technology that yields a net return of r
- Perfect competition among outside investors
  - Implies expected net return to outside investors is r

Costly State Verification Model

- Socially efficient to undertake project if $\gamma > 1 + r$
- Entrepreneur undertakes project if

$$E[Y - P] > (1 + r)W$$

where P denotes payments to outside investor

Costly State Verification Model

- Entrepreneur knows actual output but investor does not
- Outside investor must pay c to verify actual output
- Costly state verification model of asymmetric information (Townsend, 1979)

Equilibrium with Symmetric Information

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$$E\left[\frac{(1+r)(1-W)}{\gamma}\gamma\right] = (1+r)(1-W)$$

- Entrepreneur's expected income is

$$\gamma - (1+r)(1-W) = (1+r)W + \gamma - (1+r)$$

which exceeds $(1 + r)W$ if $\gamma \geq 1 + r$, so entrepreneur is better off

Optimal Contract with Asymmetric Info

- With asymmetric information, outside investor may need to verify output
- Expected payments to outside investor:

$$(1 + r)(1 - W) + \text{expected cost of verifying output}$$

- Efficiency dictates minimizing the cost of verifying output
- Entrepreneur gets output (exogenous) minus payments to investor
  - Wants to minimize payments to outside investor
  - Which means minimizing fraction of time investor needs to verifying output

Optimal Contract with Asymmetric Info

- Optimal contract: Simple debt contract
  - If $Y \geq D$, investor gets D and does not verify output
  - If $Y < D$, investor verifies output and gets Y
- Intuitively:
  - Entrepreneur "borrows" $1 - W$
  - Pays back D when she can
  - Defaults when $Y < D$
    (in which case investor verifies output and gets all of it)

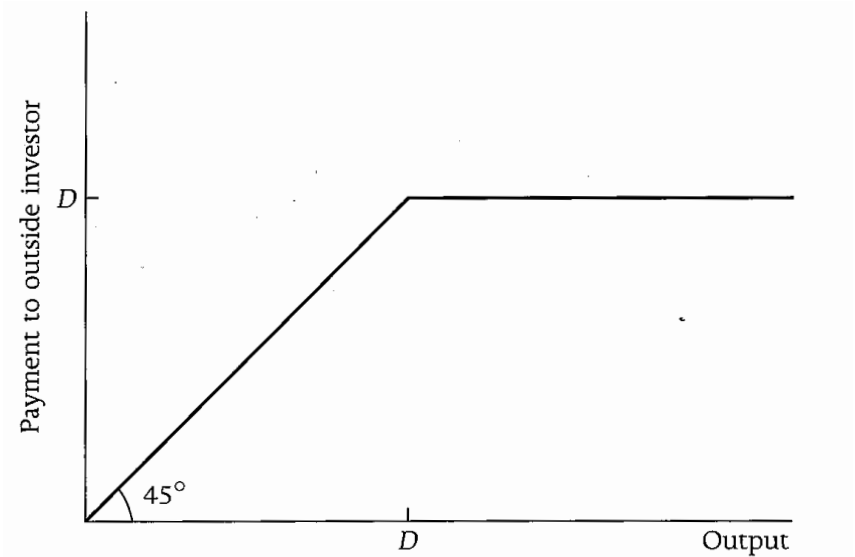


FIGURE 10.1 The form of the optimal payment function

Source: Romer (2019)

Why Debt?

- When investor does not verify output, payment cannot depend on actual output
  - Entrepreneur would have an incentive to misreport
  - Always report lowest non-verified output level
- Payment with verification cannot exceed payment without verification
  - Entrepreneur would have an incentive to misreport
  - Always report non-verified output levels when output is at higher levels
- Payment with verification cannot equal D
  - Might as well not verify and save verification cost

Why Debt?

- Payment is *D* whenever output exceeds *D*
  - If lower, possible to increase investor's expected returns and lower expected verification costs by raising payment to *D*
- Investor must verify if output is less than *D*
  - Entrepreneur cannot pay *D* in this case
  - So, investor must verify
- Payment equals output if output is less than *D*
  - If lower, possible to increase investor's expected returns and lower expected verification costs by raising payment to *Y*

Optimal *pure strategy* contract. Subgame perfect? Bankcruptcy as commitment device?

Investor Expected Net Revenue

$$R(D) = \begin{cases} \frac{2\gamma - D}{2\gamma} D + \frac{D}{2\gamma}\left(\frac{D}{2} - c\right) & \text{if} \quad D \leq 2\gamma \\ \gamma - c & \text{if} \quad D > 2\gamma \end{cases}$$

- Recall that $Y \sim U[0, 2\gamma]$
- If $D \leq 2\gamma$ probability output is above D is $(2\gamma - D)/(2\gamma)$
- Expected output if $Y < D$ is $D/2$

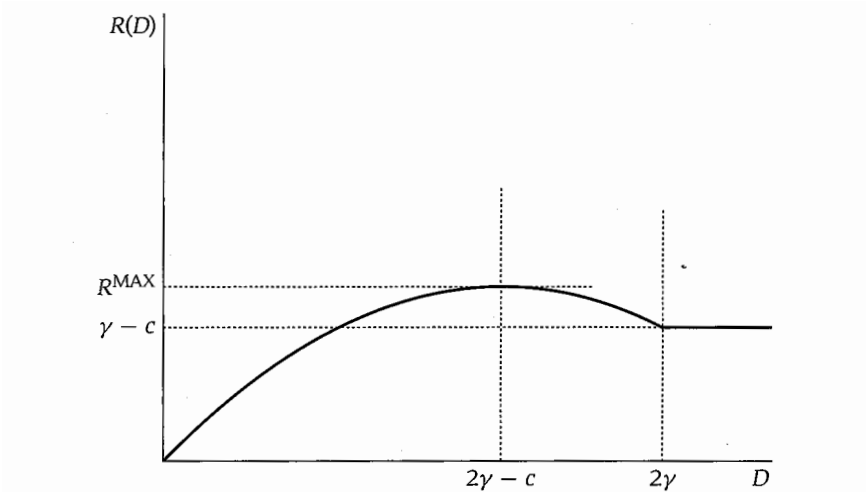
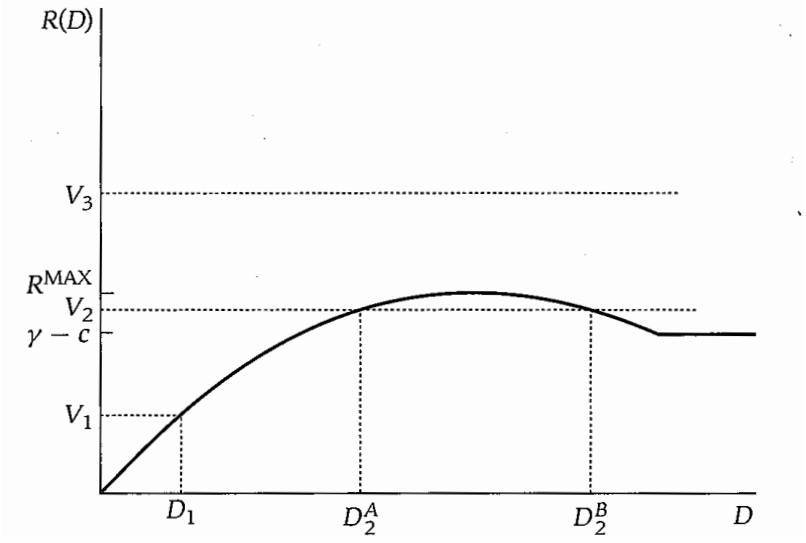


FIGURE 10.2 The investor's expected revenues net of verification costs

Source: Romer (2019). Raising D above $2\gamma - c$ is not worth the extra verification cost.



Source: Romer (2019). V_1, V_2, V_3 are three levels of required revenue $(1+r)(1-W)$.

Determination of D

- If required net revenue exceed R^{MAX}, project not funded
  - Example of **credit rationing**
- If required net revenue below R^{MAX}, project funded and equilibrium value of D given by smaller solution to

$$R(D) = (1 + r)(1 - W)$$

which is

$$D^* = 2\gamma - c - \sqrt{(2\gamma - c)^2 - 4\gamma(1 + r)(1 - W)}$$

(Competition among investors drives D down to lower value)

Expected Verification Costs

- Expected verification costs are

$$\begin{aligned} A(c,r,W,\gamma) &= \frac{D^*}{2\gamma}c \\ &= \left[\frac{2\gamma-c}{2\gamma} - \sqrt{\left(\frac{2\gamma-c}{2\gamma}\right)^2 - \frac{(1+r)(1-W)}{\gamma}}\right]c \end{aligned}$$

- Derivatives of $A(c,r,W,\gamma)$:

$$A_c > 0, \quad A_r > 0, \quad A_W < 0, \quad A_\gamma < 0$$

Will Project Be Undertaken?

Project will be undertaken if:

- Investor willing to lend:

$$R^{MAX} \geq (1+r)(1-W)$$

- Entrepreneur's return above outside option:

$$\gamma - (1+r)(1-W) - A(c,r,W,\gamma) \geq (1+r)W$$

Will Project Be Undertaken?

- With symmetric information project undertaken if $\gamma \geq (1 + r)$
- With asymmetric information, other considerations come into play
- Entrepreneur's wealth matters
  - Easier to finance when entrepreneur has more wealth (more skin in the game)
  - Less debt means lower D^* and therefore less default/verification

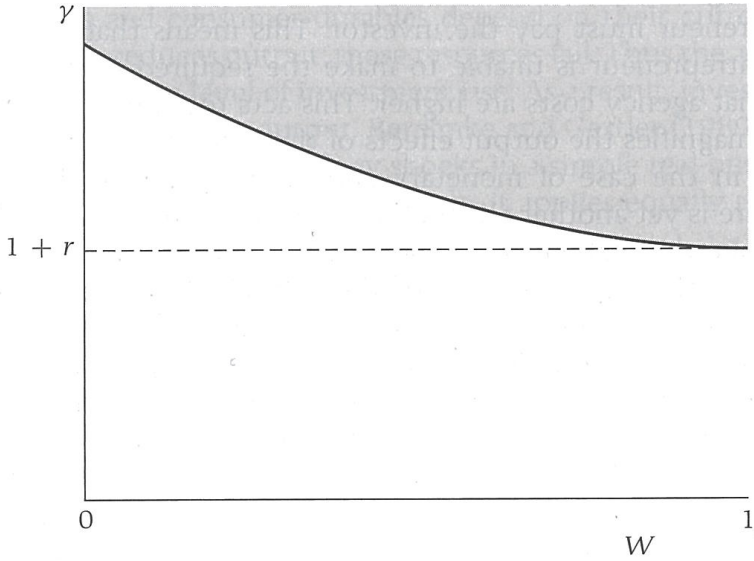
Cost of Outside Finance

- Cost of outside finance higher than of inside finance
  - Cost of inside finance:

$$1 + r$$

  - Cost of outside finance

$$1 + r + \frac{A(c, r, W, \gamma)}{1 - W}$$



Source: Romer (2019).

Will Project Be Undertaken?

- Financial system itself matters for whether projects are funded
- An increase in verification cost *c* reduces investment
- Two interpretations:
  - Efficiency of financial system in processing information and monitoring borrowers is important for investment
  - Financial system itself needs funding
    - Shocks to the financial system increase its cost of outside finance and reduce its ability to fund investment (e.g., Gertler-Kiyotaki 11)
    - Large empirical literature (e.g., Peek-Rosengren 00, Khwaja-Mian 08, Chodorow-Reich 14, Huber 18)

Financial Accelerator

- Financial frictions amplify macro shocks
  - Bad shock lowers profitability and entrepreneurial wealth
  - Increases agency costs
  - Resulting fall in investment amplifies initial shock
- Feedback loop can cause large amplification
  - Fall in investment can further weaken profitability and entrepreneurial wealth (especially in a Keynesian model)
  - Further increases agency costs and weakens investment.
- Strong enough feedback can result in multiple equilibria
- Key theoretical references: Bernanke-Gertler 89, Kiyotaki-Moore 97, Bernanke-Gertler-Gilchrist 99