NKM - SHORT REVIEW AND PRACTICE QUESTIONS

Discussion ECON 210C - Paula Donaldson

PLAN FOR TODAY

- 1. Short review of the New Keynesian Model
- 2. Practice question I
- 3. Practice question II

THREE-EQUATION NK MODEL

The log-linearized NK model boils down to three equations:

D-IS:
$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
NKPC:
$$\hat{\pi}_t = \kappa \underbrace{(\hat{y}_t - \hat{y}_t^{flex})}_{\text{Output Gap}} + \beta E_t\{\hat{\pi}_{t+1}\} + u_t$$
MR:
$$\hat{i}_t = \varphi_\pi \hat{\pi}_t + v_t$$

with

- three unknowns: \hat{i}_t , \hat{y}_t , and $\hat{\pi}_t$,
- productivity shocks drive the output gap $\hat{y}_t^{flex} = \frac{1+\phi}{\gamma+\phi}\hat{a}_t$,
- the monetary policy shock v_t.
- the cost push shock u_t

THREE-EQUATION NK MODEL - DYNAMIC IS

Dynamic IS: Relates output to future expectations of output and the real interest rate

1. Solve HH block for (non-linear) Euler:

$$1 = \beta E_t \left\{ (1+i_t) \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t \{ \Lambda_{t,t+1} R_{t+1} \}$$

2. Log-linearize around zero-inflation steady state:

$$\hat{c}_t = -\frac{1}{\gamma} \left(\hat{i}_t - E_t \{ \hat{\pi}_{t+1} \} \right) + E_t \{ \hat{c}_{t+1} \}$$

3. Substitute with market clearing Y = C and EIS $\sigma = 1/\gamma$ (with iterated version):

$$\begin{split} \hat{y}_t &= -\sigma \left(\hat{i}_t - E_t \{ \hat{\pi}_{t+1} \} \right) + E_t \{ \hat{y}_{t+1} \} \\ &\Rightarrow \hat{y}_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} \left(\hat{r}_{t+s+1} \right) \right\} \end{split}$$

THREE-EQUATION NK MODEL - NKPC

NKPC: inflation is expectations-augmented PDV of future marginal cost / markup deviations expressed in terms of output gap

1. Log-linearized price index (Dixit-Stiglitz + Calvo pricing)

$$P_t = \left\lceil \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon} \right\rceil^{\frac{1}{1-\epsilon}} \implies \hat{\pi}_t = (1-\theta) (\hat{p}_t^* - \hat{p}_{t-1})$$

2. Log-linearized reset price from firm problem written recursively

$$\begin{split} P_t^* &= (1+\mu)E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} \gamma_{t+s} P_{t+s}^{\epsilon-1}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \gamma_{t+k} P_{t+k}^{\epsilon-1}} \frac{W_{t+s}}{A_{t+s}} \right\} \\ &\Rightarrow \hat{p}_t^* = (1-\beta\theta)(\hat{p}_t + \hat{m}c_t) + \beta\theta E_t \{\hat{p}_{t+1}^*\} \end{split}$$

3. Combine and iterate for inflation in terms of marginal cost deviation:

$$\begin{split} \hat{\pi}_t &= \lambda \hat{m} c_t + \beta E_t \{ \hat{\pi}_{t+1} \}, \text{ where } \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \\ &\Rightarrow \hat{\pi}_t = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{m} c_{t+s} \right\} \end{split}$$

RTHREE-EQUATION NK MODEL - NKPC CONT.

4. Define marginal cost deviation in terms of output gap: output less natural (i.e., flexible price) level of output

$$\frac{\hat{m}c_{t}}{\Rightarrow} = \hat{w}_{t} - \hat{p}_{t} - \hat{a}_{t} = (\gamma + \varphi)\hat{y}_{t} - \varphi\hat{a}_{t}$$

$$\Rightarrow (\gamma + \varphi)\hat{y}_{t}^{flex} = \varphi \hat{a}_{t} \text{ for } \hat{m}c_{t}^{flex} = 0$$

$$\Rightarrow \hat{m}c_{t} = (\gamma + \varphi)(\hat{y}_{t} - \hat{y}_{t}^{flex})$$

5. Arrive at NKPC (with iterated version):

$$\hat{\pi}_{t} = \kappa(\hat{y}_{t} - \hat{y}_{t}^{flex}) + \beta E_{t}\{\hat{\pi}_{t+1}\} \text{ where } \kappa = \lambda(\gamma + \varphi)$$

$$\hat{\pi}_{t} = \kappa(\hat{y}_{t} - \hat{y}_{t}^{flex}) + \beta E_{t}\{\hat{\pi}_{t+1}\} \text{ where } \kappa = \lambda(\gamma + \varphi)$$

$$= (\gamma + \varphi)\hat{\gamma}_{t} - (\gamma + \varphi)\hat{\gamma}_{t}^{flex}$$

$$= (\gamma + \varphi)\hat{\gamma}_{t}^{flex}$$

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$$= (\gamma + \varphi)\hat{\gamma}_{t}^{flex}$$

THREE-EQUATION NK MODEL - MONETARY POLICY RULE

Central banks sets the nominal interest rate according to an interest rate (Taylor) rule

1. Log-linearized monetary rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

PRACTICE QUESTIONS

Consider the standard NK model with $\hat{y}_t^{\textit{flex}}$ normalized to zero:

$$\begin{split} \hat{y}_t &= -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} \\ \hat{\pi}_t &= \kappa \hat{y}_t + \beta E_t\{\hat{\pi}_{t+1}\} \\ \hat{i}_t &= \varphi_\pi \hat{\pi}_t + v_t \end{split}$$

- (20pts) Assume $v_t = \rho_V v_{t-1} + \epsilon_t^V$ with $\epsilon_t^V \sim N(0, \sigma_V^2)$. Solve for the equilibrium levels of $\hat{y}_t, \hat{\pi}_t, \hat{i}_t$, and $\hat{r}_t = \hat{i}_t \mathbb{E}_t \hat{\pi}_{t+1}$ as a function of v_t .
- (20pts) Explain intuitively how a monetary policy shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 5 sentences)
- (15pts) Briefly explain the identification problem in estimating the effect of monetary policy shocks on real output. (max 5 sentences)
- **d (30pts)** Briefly explain two approaches to solving the identification problem. (max 5 sentences each)

Consider the standard NK model with $\hat{y}_t^{\textit{flex}}$ normalized to zero:

$$\begin{split} \hat{y}_t &= -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} & \quad \textbf{D} - \textbf{T} \textbf{S} \\ \hat{\pi}_t &= \kappa \hat{y}_t + \beta E_t\{\hat{\pi}_{t+1}\} & \quad \textbf{N} \textbf{k} \textbf{PC} \\ \hat{i}_t &= \varphi_\pi \hat{\pi}_t + v_t & \quad \textbf{\pi} \textbf{2} \end{split}$$

(20pts) Assume $v_t = \rho_V v_{t-1} + \epsilon_t^V$ with $\epsilon_t^V \sim N(0, \sigma_V^2)$. Solve for the equilibrium levels of $\hat{y}_t, \hat{\pi}_t, \hat{i}_t$, and $\hat{r}_t = \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}$ as a function of v_t .

Replace NR in D-Is:

$$\hat{Y}_t = -\sigma \left[\phi_H \hat{\pi}_t + v_t - E_t \hat{\pi}_{t+1} \right] + E_t \hat{Y}_{t+1}$$

$$\hat{Y}_t = -\sigma \phi_H \hat{\pi}_t - \sigma v_t + \sigma E_t \hat{\pi}_{t+1} + E_t \hat{Y}_{t+1}$$

$$+ NKPC$$

Guess:

$$\hat{y}_{t} = \underline{Y}_{Y} V_{t}$$

$$\hat{\pi}_{t} = \underline{Y}_{\pi} V_{t}$$

$$V_{t} = \rho V_{t-1} + \varepsilon^{V}_{t}$$

$$V_{$$

$$\begin{aligned}
\pi_t &= \kappa \, \mathring{y}_t + \beta E_t [\tilde{\pi}_{t\pi}] \\
\Psi_{\pi} v_t &= \kappa \, \Psi_y \, V_t + \beta \rho \, \Psi_{\pi} \, V_t \\
\left[\Psi_{\pi} - \kappa \, \Psi_y - \beta \rho \, \Psi_{\pi} \right] v_t &= 0
\end{aligned}$$

$$\begin{aligned}
(1 - \beta \rho) \, \Psi_{\pi} &= \kappa \, \Psi_y \\
\Psi_{\eta} &= (1 - \beta \rho) \, \Psi_{\pi}
\end{aligned}$$

$$\frac{1}{2} \int_{\Gamma} \frac{1-\rho}{\rho} \int_{\Gamma$$

$$\frac{2}{2} + \epsilon \phi_{\pi} \Psi_{\pi} + \sigma - \sigma \rho \Psi_{\pi} - \rho \Psi_{y} \int_{1}^{2} v_{t} = 0$$

$$\Rightarrow \langle 1 - \rho \rangle \Psi_{y} = \sigma (\rho - \phi_{\pi}) \Psi_{\pi} - \sigma$$

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$$\frac{C(\rho - \phi_{\pi}) \Psi_{\pi} - \frac{\sigma}{(1-\rho)}}{(1-\rho)} = \frac{(1-\beta \rho) \Psi_{\pi}}{k}$$

$$\int_{0}^{\infty} G(\rho - \phi_{\pi}) \qquad (1-\beta \rho) \psi_{\pi} - \sigma$$

$$\left[\frac{G(\rho-\phi_{\pi})}{(1-\rho)} - \frac{(1-\beta\rho)}{\kappa}\right]\Psi_{\pi} = \frac{G}{1-\rho}$$

$$G\kappa(\rho-\phi_{\pi}) - (1-\rho)(1-\beta\rho) \quad W = G$$

$$\frac{(1-\beta)\kappa}{(1-\beta)(1-\beta)} \frac{1}{\sqrt{\pi}} = \frac{(1-\beta)}{\pi}$$

$$\Psi_{\pi} = \frac{\kappa \sigma}{\sigma \kappa (\rho - \phi_{\pi}) - (1 - \rho)(1 - \beta \rho)}$$

$$\Psi_{\gamma} = \frac{(1-\beta \rho)}{\kappa} \frac{\kappa \sigma}{\kappa (\rho - \phi_{\pi}) - (1-\rho)(1-\beta \rho)}$$
Eq. Denels:

 $= (1 + \phi_{\pi} \psi_{\pi}) \vee_{+} - \rho \psi_{\pi} \vee_{+}$

 $= (1 + (\phi_{\pi} - \rho) \Psi_{\pi}) V_{t}$

$$\hat{y}_{t} = \Psi_{y} \vee_{t} : \hat{\pi}_{t} = \Psi_{\pi} \vee_{t}$$

$$\hat{C}_{t} = \emptyset_{\pi} \Psi_{\pi} \vee_{t} + \vee_{t} = (1 + \emptyset_{\pi} \Psi_{\pi}) \vee_{t}$$

$$\hat{C}_{t} = \emptyset_{\pi} \underbrace{\Psi_{\pi} \vee_{t}}_{\pi_{t}} + \vee_{t} = \underbrace{\hat{C}_{t} - \hat{C}_{t} \hat{C}_{t+1}}_{\pi_{t}}$$

Consider the standard NK model with \hat{y}_t^{flex} normalized to zero:

$$\begin{split} \hat{y}_t &= -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\} \\ \hat{\pi}_t &= \kappa \hat{y}_t + \beta E_t\{\hat{\pi}_{t+1}\} \\ \hat{i}_t &= \phi_\pi \hat{\pi}_t + v_t \end{split}$$

(20pts) Explain intuitively how a monetary policy shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 5 sentences)

(15pts) Briefly explain the identification problem in estimating the effect of monetary policy shocks on real output. (max 5 sentences)

(30pts) Briefly explain two approaches to solving the identification problem. (max 5 sentences each)

NEW KEYNESIAN MODEL - PRACTICE QUESTION

1. Cost-push shocks

Consider the standard new Keynesian model

$$\hat{x}_t = E_t \hat{x}_{t+1} - E_t (\hat{i}_t - \hat{\pi}_{t+1} - \hat{r}_{t+1}^n)$$
(1)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t \tag{2}$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t, \qquad \phi_\pi > 1 \tag{3}$$

- (a) Interpret each of the equations (1)-(3) (max 2 sentence each).
- (b) Assume $\hat{r}_t^n = 0$ and $u_t = \rho_u u_{t-1} + \epsilon_t^u$ with $\epsilon_t^u \sim N(0, \sigma_{\epsilon^u}^2)$. Solve for the equilibrium levels of \hat{x}_t , \hat{i}_t , and $\hat{r}_t = \hat{i}_t E_t \hat{\pi}_{t+1}$ as a function of u_t .
- (c) Explain intuitively how a supply shock affects the output gap, inflation, the nominal interest rate, and the real interest rate. (4 sentences should suffice.)
- (d) Use your solution to express the loss function $L = \vartheta var(\hat{x}_t) + var(\hat{\pi}_t)$ as a function of the model parameters, where $var(\hat{x}_t)$ is the variance of the output gap and $var(\hat{\pi}_t)$ is the variance of inflation.
- (e) Show that the optimal interest rate rule satisfies $\phi_{\pi} = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$.
- (f) Using the optimal ϕ_{π} , show that $\hat{x}_t = \frac{\kappa}{\vartheta(1-\beta\rho_{\infty})} \hat{\pi}_t$.
- (g) The optimal monetary policy under discretion is $\hat{x}_t = -\frac{\kappa}{\theta}\hat{\pi}_t$. Does the optimal ϕ_{π} deliver a better, a worse, or the same loss? Explain intuitively. (No derivation should be necessary.)

(a) Interpret each of the equations (1)-(3)

Guess:
$$\hat{X^f} = \hat{A^x} N^f$$

(1-BPu) (1-Pu) - K(Pu- pr)

(1-BPu) (1- /u) - K (Pu- + T)

(b) Assume $\hat{r}_t^n = 0$ and $u_t = \rho_u u_{t-1} + e_t^u$. Solver for the equilibrium levels of \hat{x}_t , $\hat{\pi}_t$, \hat{i}_t and \hat{r}_t as a function of u_t .

$$A_{\mu} = \frac{(1 - \beta b^{\mu})(1 - b^{\mu}) - \kappa (b^{\mu} - \phi^{\mu})}{(1 - \beta b^{\mu})(1 - b^{\mu}) - \kappa (b^{\mu} - \phi^{\mu})}$$

$$A_{\mu} = \frac{1 - b^{\mu}}{(1 - \beta b^{\mu})(1 - b^{\mu}) - \kappa (b^{\mu} - \phi^{\mu})}$$

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(c) Explain intuitively how a cost-push shock affects the output, inflation, the nominal interest rate, and the real interest rate. (max 4 sentences)



(d) Use your solution to express the loss function $L = \vartheta var(\hat{x}_t) + var(\hat{\pi}_t)$ as a function of the model parameters, where $var(\hat{x}_t)$ and $var(\hat{\pi}_t)$ are the variances of the output gap and inflation, respectively.

(e) Show that the optimal interest rate rule satisfies $\phi_{\pi} = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$

$$= \frac{\theta (\rho_{u} - \phi_{\pi})^{2} + (1 - \rho_{u})^{2}}{[(1 - \beta_{u})(1 - \rho_{u}) - k (\rho_{u} - \phi_{\pi})]^{2}} A$$

$$L = \frac{\theta (\rho_{u} - \phi_{\pi})^{2} + (1 - \rho_{u})^{2}}{[(1 - \beta_{u})(1 - \rho_{u}) - k(\rho_{u} - \phi_{\pi})]^{2}} A$$

$$f(\phi_{\pi}) = \theta (\rho_{u} - \phi_{\pi})^{2} + (1 - \rho_{u})^{2}$$

$$g(\phi_{\pi}) = \left[(1 - \beta P_{u})(1 - P_{u}) - k(P_{u} - \phi_{\pi}) \right]^{2}$$

$$L = f(\phi_{\pi}) g(\phi_{\pi}) \cdot A$$

$$\frac{\partial L}{\partial \phi_{\pi}} = \left[f'(\phi_{\pi}) g(\phi_{\pi}) + f(\phi_{\pi}) g'(\phi_{\pi}) \right] A$$

$$\begin{aligned}
\xi(\phi_{\pi}) &= \theta \left(\left(\left(\left(\phi_{\pi} \right)^{2} + \left(\left(\left(- \rho_{\pi} \right)^{2} \right)^{2} + \left(\left(\left(\rho_{\pi} \right)^{2} \right)^{2} \right) \right) \right) \\
\xi(\phi_{\pi}) &= \left[\left(\left(\left(\left(\rho_{\pi} \right) \right) \left(\left(\left(\rho_{\pi} \right) \right) + \left(\rho_{\pi} \right) \right) \right] - 2 \right] \\
&= \left[\xi(\phi_{\pi}) \cdot g(\phi_{\pi}) \cdot A \right] \\
\frac{2L}{2\phi_{\pi}} &= \left[\xi^{1}(\phi_{\pi}) \cdot g(\phi_{\pi}) + \left(\xi(\phi_{\pi}) \cdot g^{1}(\phi_{\pi}) \right) \right] A
\end{aligned}$$

$$S_{1}(\phi_{\pi}) = -2\theta \left(\rho_{N} - \phi_{\pi} \right)$$

$$S_{1}(\phi_{\pi}) = -2\kappa \left[(1 - \beta \rho_{N}) (1 - \rho_{N}) - \kappa (\rho_{N} - \phi_{\pi}) \right]^{3}$$

$$\frac{\partial L}{\partial \rho_{\pi}} = \frac{-20(\rho_{u} - \phi_{\pi})}{[(1 - \rho_{u})^{(1-\rho_{u})} - k(\rho_{u} - \phi_{\pi})]^{2}} = 0$$

$$= -0(\rho_{u} - \phi_{\pi})[(1 - \rho_{u})^{(1-\rho_{u})} - k(\rho_{u} - \phi_{\pi})] - k[0(\rho_{u} - \phi_{\pi})^{2} + (1-\rho_{u})^{2}]$$

$$= -0(\rho_{u} - \phi_{\pi})[(1 - \rho_{u})^{(1-\rho_{u})} - k(\rho_{u} - \phi_{\pi})^{2} + (1-\rho_{u})^{2}]$$

$$-\theta (\rho_{N} - \phi_{\pi}) [(i - \beta \rho_{N})(1 - \rho_{N}) - \kappa(\rho_{N} - \phi_{\pi})] - \kappa [\theta (\rho_{N} - \phi_{\pi})^{2} + (1 - \rho_{N})^{2}] = 0$$

$$-\theta (\rho_{N} - \phi_{\pi}) (1 - \beta \rho_{N})(1 - \rho_{N}) + \kappa \theta (\rho_{N} - \phi_{\pi})^{2} - \kappa \theta (\rho_{N} - \rho_{\pi})^{2} - \kappa (1 - \rho_{N})^{2} = 0$$

$$\rho_{\mu} - \phi_{\pi} = -\frac{\kappa (1 - \rho_{\mu})}{\theta (1 - \beta \rho_{\mu})(1 - \rho_{\mu})} + \rho_{\mu}$$

$$\rho_{\pi}^{*} = \frac{\kappa (1 - \rho_{\mu})}{\theta (1 - \beta \rho_{\mu})(1 - \rho_{\mu})} + \rho_{\mu}$$

(f) Using the optimal
$$\phi_{\pi}$$
, show that $\hat{x}_t = \frac{\kappa}{\vartheta(1-\beta\rho_u)}\hat{\pi}_t$

$$\hat{X}_{t} = \frac{\pi_{t}}{1-\rho_{u}} \left(\rho_{u} \rho_{u} - \frac{\kappa (s-\rho_{u})}{2} \right)$$

$$\hat{\chi}_{e} = -\hat{\pi}_{e} \frac{\kappa}{2(1.32)}$$

(g) The optimal monetary policy under discretion is $\hat{x}_t = \hat{\eta}_t \hat{x}_t$. Does the optimal ϕ_π deliver a better, a worse or the same loss? Explain intuitively. (NO derivatiosn needed)

A smaller coefficient, 121, on $X_t = 2 \pi_t$ implies a better trade-off for the CB.

note: some trade-off if cost-push shock has zero persistence. (Ph=0)

(g) The optimal monetary policy under discretion is $\hat{x}_t = \frac{\kappa}{\vartheta} \hat{\pi}_t$.. Does the optimal φ_{π} deliver a better, a worse or the same loss? Explain intuitively. (NO derivatiosn needed)

(g) The optimal monetary policy under discretion is $\hat{x}_t = \frac{\kappa}{\vartheta} \hat{\pi}_t$.. Does the optimal φ_{π} deliver a better, a worse or the same loss? Explain intuitively. (NO derivatiosn needed)