Problem Set 1. 210C
Due Friday 18th.

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1. Find the problem set in the online appendix of Nakamura and Steinsson (2018) Identification in Macroeconomics paper. You can find it in their websites. Solve it.

2. A representative household wants to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t), \tag{1}$$

where C_t is a CES bundle between goods and services with elasticity of substitution η and weight parameters φ_g and φ_s

$$C_t = \left(\varphi_g^{1/\eta} C_{g,t}^{\frac{\eta-1}{\eta}} + \varphi_s^{1/\eta} C_{s,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{2}$$

C_{jt} is also a CES bundle that aggregates a continuum of varieties i in sector $j \in g, s$. Formally

$$C_{jt} = \left(\int_0^1 C_{ijt}^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}, \tag{3}$$

for $j \in g, s$.

The household maximizes utility subject to a series of budget constraints

$$\int_0^1 P_{igt} C_{igt} di + \int_0^1 P_{ist} C_{ist} di + B_{t+1} \leq B_t(1 + i_{t-1}) + W_t N_t + T_t$$

taking output prices P_{ijt}, input prices W_t, nominal interest rates i_{t-1}, and lump-sum transfers T_t as given, as well as an initial condition for B in period zero.

   (a) Solve the cost minimization problem for each sector j. That is, taking the value of C_{jt} as given, find the allocation across varieties C_{ijt} $\forall i$, as a function of prices P_{ijt}, sectoral demand C_{jt}, the Lagrange multiplier with respect to the shape of the sectoral CES aggregator, and parameters.

   (b) Find an expression for the ideal sectoral price index, that is, a price index P_{jt} such that $P_{jt} C_{jt} = \int_0^1 P_{ijt} C_{ijt} di$.

(c) Solve the cost minimization problem across sectors. That is, taking the value of C_t as given, find the allocation across sectors C_{jt}, as a function of sectoral price indices P_{jt}, aggregate demand C_t, the Lagrange multiplier with respect to the shape of the CES aggregator across sectors, and parameters.

(d) Find an expression for the ideal aggregate price index, that is, a price index P_t such that $P_t C_t = P_{st} C_{st} + P_{gt} C_{gt}$.

(e) Find the optimality conditions of the household for bond holdings, labor supply, and consumption.

(f) Firms in each sector j produce with a technology linear in labor that depends on sector-specific productivity. $Y_{ijt} = A_{jt} L_{ijt}$. Firms are monopolistic competitors, and take wages as given. Firms understand the structure of demand, and must satisfy demand $Y_{ijt} = C_{ijt}$. Solve the profit maximization problem of the firms and find an expression for the optimal price firms set. Profits are given by $(P_{ijt} Y_{ijt} - W_t L_{ijt})$, so there is a fully integrated labor market in the economy.

(g) Are firms charging the same prices within sectors? Are firms charging the same price across sectors? Are firms charging the same markup within sectors? Are firms charging the same markup across sectors?

(h) Do markups depend on η, ρ, or both η, and ρ? What is the intuition?

(i) Will firms across sectors sell different quantities? What economic objects determine the differences (if any)? What is the intuitive explanation?