Lecture 6: Optimal Monetary Policy

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Goals

- How should monetary policy be conducted?
  - Broad qualitative consensus among policymakers
    - Central banks are in charge of maintaining low and stable inflation

      Explicit in the Federal Reserve Act of 1977
      Explicit in the Maastricht treaty as the goal of the ECB
      Written in the law and constitutions of countries around the world
    - Actual monetary policy assumes at least some degree of concern with stabilization of economic activity

      Explicit in the case of the US
      Not explicit in the case of the ECB
  - Suggests the central bank wants to minimize a “loss-function” that depends on “deviations” of output and inflation from “target”
  - Many questions
    - What is the relative weight of inflation vs. output?
    - What is the right measure of economic activity to stabilize?
    - What is the right inflation index to stabilize?
    - Should the price level or the inflation rate be stabilized?

Approach

- In this lecture we will see what the NK model has to say about optimal monetary policy
- The notion of optimality is to maximize the welfare of private agents
- This *utility-based* approach to policy design has a long tradition in public finance.
  - Examples are the optimal capital tax rate, optimal unemployment insurance, ...
- Notice our consumers do not care directly about prices. They care about leisure and consumption
- But as taxes can create deadweight losses, inflation can too. We saw two potential sources in Lecture 5
  - The aggregate markup can change, inducing too much or too little production
  - Price dispersion may change inducing allocative efficiency costs (misallocation). goods with the same marginal cost will have different prices

Loss Function

- The next few of slides provide a heuristic derivation of the Loss Function.
- I show the math only to make clear the connection between economic objects
- I will never ask you to know this math by heart in a final/qualifying exam
- Not because it’s too hard. Just because it’s pointless for you to memorize it
- I will just gloss over key steps in the lecture

- Notation:
  - U_t: $U(C_t, N_t)$.
  - U_t^n: U_t evaluated at the flexible price allocation
  - U: U_t in the steady state (no shocks whatsoever).
  - U_c and U_n: partial derivatives of U_t with respect to C, N evaluated in the steady state.
  - U_{cc}, U_{nn}: same thing for second derivatives.
  - Assume U is separable in C, N. Therefore $U_{cn} = U_{nc} = 0$.

Second-Order Approximation

- I will do a second order approximation of the utility function around U

$$U_t \approx U + U_c C \frac{C_t - C}{C} + U_n N \frac{N_t - N}{N} + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N} \right)^2$$

- For small deviations, log changes and percentage deviations are approximately equal
- use our functional form assumption that γ and φ are $-C(U_{cc}/U_c)$ and $N(U_{nn}/U_n)$, respectively and impose market clearing $C = Y$.

$$U_t \approx U + U_c C \hat{y}_t + U_n N \hat{n}_t + \frac{1 - \sigma}{2} U_c \hat{y}_t^2 + \frac{1 + \varphi}{2} \hat{n}_t^2$$

- collect terms and use the results on price dispersion of Lecture 5. (Full details Gali Appendix 4.A)

$$\frac{U_t - U}{U_c C} \approx -\frac{1}{2} \left(\theta var_i(\log P_{it}) + (\gamma + \varphi) \tilde{y}_t^2 \right)$$

Price Dispersion and Inflation

- Let me call $\Delta_t \equiv var_i \log P_{it}$ and let me call $\bar{P}_t = \mathbb{E}_i \log P_{it}$
- and note that due to Calvo.

$$\bar{P}_t - \bar{P}_{t-1} = \mathbb{E}_i(\log P_{it} - \bar{P}_{t-1}) = \lambda \cancelto{0}{\mathbb{E}_i(\log P_{it-1} - \bar{P}_{t-1})} + (1-\lambda)(\log P_t^* - \bar{P}_{t-1})$$

- Now $\Delta_t = var_i \log P_{it} = var_i(\log P_{it} - \bar{P}_{t-1})$ and use the definition of the variance

$$\Delta_t = \mathbb{E}_i\left[(\log P_{it} - \bar{P}_{t-1})^2\right] - (\cancelto{\bar{P}_t}{\mathbb{E}_i \log P_{it}} - \bar{P}_{t-1})^2$$

- And the Calvo property

$$\Delta_t = \lambda \mathbb{E}_i\left[(\log P_{it-1} - \bar{P}_{t-1})^2\right] + (1-\lambda)(\log P_t^* - \bar{P}_{t-1})^2 - (\bar{P}_t - \bar{P}_{t-1})^2$$

- use the first result in the side

$$\Delta_t \approx \lambda \Delta_{t-1} + \frac{\lambda}{1-\lambda}(\pi_t)^2$$

- Since $\bar{P} \approx \log P_t$. See Woodford (2003) Appendix E.2 for precise mathematical statement.

Welfare

- An affine transformation of the objective of the household

$$\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C}$$

- Use our second order approximation:

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\theta var_i(\log P_{it}) + (\gamma + \varphi)\tilde{y}_t^2\right)$$

- Iterate Δ_t forward and compute its present value (Details Woodford Chapter 6 2.2).

$$\sum_{t=0}^{\infty} \beta^t \Delta_t \propto \frac{\lambda}{(1-\lambda)(1-\lambda\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$$

- Finding

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2\right)$$

Result 1: In the Calvo model price dispersion is very costly

$$\mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

- Key Economics
  - Households care about consumption and leisure.
  - They don't like consuming too little, or working too much vs. the efficient allocation
  - Output gap captures the differences
  - Price dispersion is a symptom of misallocation and it reduces welfare
  - Due to Calvo, inflation and price dispersion are tightly linked. This depends on Calvo!
- Back of the envelope calculations
  - Imagine utility is log in C and linear in N. Then $\gamma + \varphi = 1$.
  - Typical values for $\theta \in [4, 7]$. Let's pick $\theta = 4$
  - If $\lambda = 0.9$ and $\beta = 0.995$, then $\alpha \approx 0.01$
  - So $\theta/\alpha \approx 330$
  - Inflation is waaaaaaaay more costly than output gaps

Optimal Policy

$$\max_{\pi_t, \tilde{y}_t} \mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

- Subject to

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n)$$

- We are looking for sequences for $\left\{\hat{i}, \tilde{y}, \hat{\pi}\right\}$ such that $\mathcal{W}$ is maximized

Optimal Policy

- It is useful to separate the problem in two
  - Pick a sequence for $\{\tilde{y}, \hat{\pi}\}$ that maximizes $\mathcal{W}$ subject to the Phillips curve

$$\max_{\pi_t, \tilde{y}_t} \mathcal{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\theta}{\alpha}\pi_t^2 + (\gamma + \varphi)\tilde{y}_t^2 \right)$$

  Subject to

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$$

  - Find a sequence for $\{\hat{i}\}$ that satisfies

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n)$$

- Solution
  - $\hat{\pi}_t = 0$, $\hat{y}_t = \hat{y}_t^n \; \forall t$ (so that $\tilde{y}_t = 0$) minimizes the objective and respects the Phillips curve
  - Pick $\hat{i}_t = \hat{r}_t^n \; \forall t$

Result 2: The Divine Coincidence

- The central bank does not want to stabilize output, it wants to stabilize the output gap!
- Divine Coincidence: The central bank can minimize output gap **and** inflation deviations simultaneously. No tradeoff.
- Notice: Central bank does not want to induce monetary policy shocks. $\hat{i}_t = \hat{r}_t^n \; \forall t$. Optimal policy 100% systematic.
- Notice: Optimal policy informationally very heavy. Central bank must now r_t^n perfectly.
- Shocks to the IS curve (demand shocks) are offset via movements of i

The Divine Coincidence: Intuition

- The flexible price allocation is optimal
- Nominal rigidities are the only constraint to reach this allocation
- If the constraint does not bind (happens with zero inflation), there is no distortion
- So neutralizing inflation implies neutralization of output gap

The Unrealistic Divine Coincidence

- If you were to ask central bankers, they would not agree that stabilizing one objective necessarily stabilizes the other.
- Rough response you would get. There are some shocks that increase inflation, and stabilizing inflation would push down output relative to the desired level
- Simple way in this model to capture that intuition: if the flexible price equilibrium is not efficient

Time-Varying Efficient Output Gap

- In our model so far, the outcome under flexible prices, $\hat{y}_t^n$, is also the efficient (first-best) outcome $\hat{y}_t^{eff}$.
- The central bank will face a trade-off and the divine coincidence will break once these are no longer the same.
- The welfare function will now penalize deviations of output from the efficient level of output $\hat{y}_t - \hat{y}_t^{eff}$. The Phillips Curve is:

$$
\begin{aligned}
\hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^n) + \beta E_t\{\hat{\pi}_{t+1}\} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} \underbrace{-\kappa(\hat{y}_t^n - \hat{y}_t^{eff})}_{\equiv u_t} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + u_t
\end{aligned}
$$

- We call u_t a "cost-push shock."
  - Exogenous increase in marginal costs.

Cost Push and the Labor Wedge

- What are cost push shocks?
  - Anything that moves the labor wedge beyond sticky prices.
- Let μ_t^W be the log of a time-varying exogenous wage markup:

$$\hat{w}_t - \hat{p}_t = \hat{\mu}_t^W + \varphi \hat{n}_t + \gamma \hat{c}_t$$

- Then the Phillips curve becomes:

$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + \alpha \hat{\mu}_t^W$$

- Intuition:
  - Higher mark-ups mean higher inflation and lower output.
  - Central bank wants to offset this inefficient shock, but can only move output and inflation in the same direction.

The Planning Problem With A Tradeoff

$$\frac{1}{2}E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\vartheta(\hat{y}_{t+s}-\hat{y}^{eff}_{t+s})^2+\hat{\pi}^2_{t+s}\right]\right\}$$

subject to

$$\hat{y}_t = -\sigma E_t\{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}^{eff}_t) + \beta E_t\{\hat{\pi}_{t+1}\} + u_t$$

- First stage is optimizing objective function subject to NKPC.
  - Cost push shock increases $\hat{\pi}_t$.
  - To offset it, can push down output relative to efficient output $\hat{y}_t - \hat{y}^{eff}_t$.
  - Thus there is now a tradeoff.
  - Intuitively, monetary policy shifts aggregate demand but u_t is a shock to aggregate supply, so there is a tradeoff.

The Planning Problem: Rules vs. Discretion

- Standard quadratic loss function with linear constraints.
  - But targets depend on expectations of future policy.
  - To see this, iterate forward to get:

$$\hat{\pi}_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\kappa \left(\hat{y}_{t+s} - \hat{y}_{t+s}^{eff} \right) + u_{t+s} \right] \right\}$$

$$\hat{y}_t = E_t \left\{ \sum_{s=0}^{\infty} \left[-\sigma \left(\hat{i}_{t+s} - \hat{\pi}_{t+s+1} \right) + g_{t+s} \right] \right\}$$

- This raises issues of credibility and time consistency of policy.
  - Central bank can influence outcomes today by “promising” outcomes tomorrow.
  - But are those promises credible?
  - See Gali 5.3 for the solution to the commitment case.

The Discretionary Problem

- For today, we assume that the central bank follows a *discretionary optimal policy*.
  - Cannot make credible commitments about future actions (hopefully have time to discuss why)
  - So optimize *taking expectations of future actions as given*.
- Solve

$$\min_{\hat{\pi}_t, \hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

where

$$\begin{aligned} F_t &= \frac{1}{2} E_t \left\{ \sum_{s=1}^{\infty} \beta^s \left[\vartheta(\hat{y}_{t+s} - \hat{y}_{t+s}^{eff})^2 + \hat{\pi}_{t+s}^2 \right] \right\} \\ f_t &= \beta \hat{\pi}_{t+1} + u_t \end{aligned}$$

are functions of expectations of future actions.

“Lean Against the Wind” Policy

$$\min_{\hat{\pi}_t, \hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

- The First order condition is

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta}\hat{\pi}_t$$

- “Lean Against The Wind” Policy.
  - In face of inflationary pressures from cost push shocks, *drive output below its efficient level to dampen rise in inflation*.
  - Extent to which it does so depends on:
    - κ, which determines reduced inflation per unit of output loss.
    - ϑ, the relative weight placed on output loss.
- Flip from “Old Keynesian” logic where stabilizing output at cost of inflation.

Inflation and Output Under Discretion

- Plug policy into Phillips curve:

$$\hat{\pi}_t = \frac{\vartheta\beta}{\vartheta + \kappa^2} E_t \hat{\pi}_{t+1} + \frac{\vartheta}{\vartheta + \kappa^2} u_t$$

- And iterate forward to get:

$$\hat{\pi}_t = \frac{\vartheta}{\vartheta(1 - \beta\rho_u) + \kappa^2} u_t$$

- Combine with optimality condition to get

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta(1 - \beta\rho_u) + \kappa^2} u_t$$

- So central bank lets output gap and inflation fluctuate in proportion to current value of cost push shock.
  - Intuition: Cost push increases inflation, central bank wants to smooth both so trades some inflation for output.

Interest Rate Under Discretion

- Plugging into dynamic IS:

$$\hat{y}_t = -\sigma E_t\{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t\{\hat{y}_{t+1}\} + g_t$$

  obtains

$$\hat{i}_t = \hat{r}^{eff}_{t+1} + \phi_\pi \hat{\pi}_t + \frac{g_t}{\sigma}$$

  where

$$\hat{r}^{e}_{t+1} = \frac{1}{\sigma} E_t\{\hat{y}^{eff}_{t+1} - \hat{y}^{eff}_t\}, \qquad \phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\sigma[\vartheta(1-\beta\rho_u) + \kappa^2]}$$

- Central bank implements the optimal outcome with what looks like an interest rate (Taylor) rule.