Problem Set 2. 210C
Due Monday May 11th.

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1. The model economy is the New Keynesian model, given by

$$\hat{\pi}_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t + u_t \tag{1}$$
$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}) \tag{2}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t \tag{3}$$

   u_t and v_t are shifters to the Phillips curve and the Taylor rule, respectively. These disturbances follow independent AR(1) processes, given by

$$u_t = \rho_u u_{t-1} + \epsilon_t \tag{4}$$
$$v_t = \rho_u v_{t-1} + \xi_t \tag{5}$$

   (a) Assume away the existence of u_t from the system. Make a guess that the solution is linear on the realization of v_t. Using the method of undetermined coefficients, find solutions for $\hat{\pi}_t$ and $\tilde{y}_t$.

   (b) Now assume away the existence of v_t from the system, but reintroduce u_t. Make a guess that the solution is linear on the realization of u_t. Using the method of undetermined coefficients, find solutions for $\hat{\pi}_t$ and $\tilde{y}_t$.

   (c) What did you learn about the covariance of inflation and the output gap as a function of the source of the shock? Discuss.

2. Under optimal monetary policy, in the simplest case we discussed in the slides (divine coincidence), the optimal interest rate was $\hat{i}_t = \hat{r}^n_t$. Imagine that a central bank then takes that literally to heart and you solve the new keynesian model given by

$$\hat{\pi}_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \tag{6}$$
$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}^n_t) \tag{7}$$
$$\hat{i}_t = \hat{r}^n_t \tag{8}$$

   (a) Check that $\hat{\pi}_t = \tilde{y}_t = 0$ is a possible solution to the model. Hint: A solution to the system does not violate any of the equations.

(b) Is this solution locally unique? In order to check uniqueness, use the state-space representation for the model and use the Blanchard-Khan method. Please state the conditions the eigenvalues should satisfy and check whether those conditions are indeed satisfied. Do you have a solution? Multiple solutions?

(c) Suppose that instead of following the rule $\hat{i}_t = \hat{r}_t^n$, the central bank follows the rule $\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t$ for $\phi_\pi > 1$. Repeat questions (a) and (b).

(d) Even though the two monetary policy rules produce the same sequence of interest rates in at least one equilibrium, the two rules are not identical as you hopefully discovered. Please state an intuitive explanation of why that is the case, and how an off-equilibrium threat by the central bank is a key aspect of the difference. Your answer should be **maximum one short paragraph**. Extreme wordiness will be penalized.

(e) Imagine an econometrician is interested in estimating the Taylor rule parameter ϕ_π. Please discuss the intuitive challenges in doing so. Can you think of a better alternative than running OLS regressions of interest rates on inflation rates? You are welcome to give your own ideas or gloss over papers you find online that discuss alternative approaches. Your answer should be **maximum one short paragraph**. Extreme wordiness will be penalized.

3. The economy has an Euler equation $\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \sigma \hat{r}_t$. Assume that the central bank has perfect control over the real interest rate, so the central bank is choosing r_t directly, as opposed to choosing only i_t, and r_t being given by the Fisher equation. Assume further that the economy is stationary so the economy is in steady state in the infinite future $\lim_{k\to\infty} \mathbb{E}_t \tilde{y}_{t+k} = 0$

   Imagine that at time t the central bank chooses the whole sequence of interest rates, and in particular it chooses $\hat{r}_{t+k} = 0$ for all k except for a single period $t + k^*$ for k^* finite, where the interest rate takes the value $-r$. Intuitively, the central bank is announcing in period t a one-period monetary policy change that will happen in the present (if $k^* = 0$) or in the future (if $k^* > 0$). In the latter case, this is precisely what Forward Guidance is: a present announcement about the future path of the interest rate.

   (a) Iterate forward the Euler equation using the terminal condition.

   (b) What is the value of the output gap in period t if the central bank chooses $k^* = 0$?

   (c) What is the value of the output gap in period t if the central bank chooses $k^* = 10$? $k^* = 100$?

   (d) Intuitively, do you think that the relative magnitude of the response of the output gap today to an interest rate cut that takes place today vs. one that takes place in the distant future is an appealing or unappealing property of the New Keynesian model?

   (e) Repeat your answers to questions (a) - (c) if the Euler equation had instead the following shape $\tilde{y}_t = \beta \mathbb{E}_t \tilde{y}_{t+1} - \sigma \hat{r}_t$, for a discount factor $\beta \in (0,1)$.

   (f) Relate your findings to the *Forward Guidance Puzzle*, and the *Discounted Euler Equation*. You will need to do a short google scholar search. You should not read full papers, but maybe read the abstract and parts of the introduction of one or two papers. Your answer should be **maximum one short paragraph** per concept. Extreme wordiness will be penalized.