Problem Set 1

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April 10, 2023

This problem set is due on April 19th on canvas. You should submit only one .zip file. The answer to the questions that require writing will be on a single .pdf file.

1. Discuss each of the required assumptions to move from q to Q. Why do you need each assumption? Feel free to refer to the proof in Hayashi (1982) or the one we covered in class.

2. Give an example of an economic setup (a model with a friction, or feature) in which each of the assumptions needed to move from q to Q is violated. Discuss whether the violation would imply that q became smaller or larger than Q in equilibrium, or whether it is not possible to tell.

3. In the neoclassical model with convex adjustment costs, under certain conditions, q is a sufficient statistic for investment rates. Consider a model where the adjustment technology is non-convex, but all the other assumptions of the $q - theory$ are maintained. Read Caballero and Leahy (1993) *The demise of marginal q* (it is alright to read selectively), and provide answer to the following two questions. You may use math in providing your answer, and you must be as precise as possible.

   (a) What do the authors mean when they say that in the case of non-convex costs, the investment function is not well defined? Is it the same as we said in lecture for the case without any type of adjustment costs?

   (b) Why is it that average Q may do a better job in investment equations than marginal q? Is average Q a sufficient statistic then?

4. Imagine that the Q theory is a sensible representation of the economy. That is, investment rates at the firm level are given by:

$$\frac{i_j}{k_j} = -\frac{1}{\varphi} + \frac{1}{\varphi} q_j + \epsilon_j$$

, where ϵ captures mistakes that managers do when investing, and we assume ϵ is distributed normal with mean zero and independent across firms. Assume that in the economy the conditions behind $Q = q$ hold.

However, as an econometrician you do not observe Q, but you observe a noisy measurement $\tilde{Q}_j = Q_j + \nu_j$. ν_j is independent of q_j, and is iid across firms with variance σ^2.

(a) Show that if the $\sigma^2 = 0$, then you can recover the parameter $1/\varphi$ by ols if you ran the regression

$$\frac{i_j}{k_j} = \beta_0 + \beta_1 \tilde{Q}_j + u_j$$

(b) What value of $\hat{\beta}$ will you recover by running OLS in the presence of measurement error?

(c) Imagine you want to test if the Q theory holds, so you include a variable, called CF that captures the cash flow of the company. The cash flow of the company comoves with Q, that is $cov(CF_j, Q_j) > 0$. What can you say about the value of $\hat{\beta}_2$ you would recover if you ran the regression

$$\frac{i_j}{k_j} = \beta_0 + \beta_1 \tilde{Q}_j + \beta_2 \tilde{CF}_j + \xi_j$$

5. The purpose of this question is to guide you as you code a model in partial equilibrium of a heterogeneous firms model in which firms invest in physical capital, hire labor, and produce a homogeneous good. The firms will face convex adjustment costs. It is important that the answer to this question is given by a zip file, that includes a file called "main", in which after unpacking the folder, and running the file, we can get all the figures, tables and reports we will ask you. If we cannot run your code with minimal effort, we will conclude that it does not run properly.

Setting

The problem we will solve has the following structure.

$$V(k,a) = \max_n \left[e^a k^\alpha n^\gamma - wn\right] + -i - \phi\left(\frac{i}{k}\right)^2 k + \beta\mathbb{E}(V(k',a')|a),$$

subject to

$$k' = k(1-\delta) + i.$$

The exogenous process a, captures the behavior of log TFP. And is given by:

$$a_{it} = \rho_a a_{t-1} + \epsilon_{it}\sigma_a.$$

(a) Create a .m file in which you input the following parameters.
  - $\beta = 0.99$. Discount factor.
  - $\phi = 0.1$. Convex cost of adjustment parameter
  - $w = 3$. The wage rate
  - $\alpha = 0.21$. Capital in the production function
  - $\gamma = 0.64$. Labor in the production function
  - $\delta = 0.025$. The depreciation rate.
  - $\sigma_a = 0.007$. The volatility of firm-level TFP shocks
  - $\rho_a = 0.8$. The autocorrelation of firm-level TFP shocks.

(b) Write an .m file that creates grids for log firm-level productivity a, and physical capital k. The grid for k should have $k_n = 100$ points with equal distancing between 0.01 and 0.4. The grid for a should have $a_n = 4$ points and take the values of $-0.245, -0.08, 0.08, 0.245$. You may want to use the function linspace in matlab.

(c) Write a function called Markov_AR, that take as inputs a_n the number of points of the grid for a, ρ_a, and σ_a, and produce a probability matrix $a_n \times a_n$ with the transition probability of the process

$$a_t = \rho_a a_{t-1} + \epsilon\sigma_a,$$

where ϵ is a standard normal shock (zero mean, unity variance). You may want to google the Tauchen method.

(d) Create a function called state_space, that creates 2 matrices, $k_n \times a_n$, called kk, and aa. Each column of the matrix kk should contain the grid for k you created, and each row of aa should have the grid you created for a.

(e) Create a .m file called *vf_iteration*

(f) Start by solving for the optimal level of labor in the problem. The outcome of this process should be a matrix $k_n \times a_n$.

(g) In *vf_iteration*, Define profits as $[e^a k^\alpha n^\gamma - wn]$. The outcome of this process should be a matrix $k_n \times a_n$.

(h) In *vf_iteration*, generate a variable called v_0 that is equal to the profits you defined before.

(i) Generate a function *v_adj_a* that takes a value matrix $V(k, a)$ and a transition probability for a $P(a, a')$, and creates the expected future value $\mathbb{E}(V(k, a')|a)$.

(j) Use your previous work to write in your program *vf_iteration* a code that iterates over the value function to solve the problem. In doing this, you may want to consider the following hints. One of the key challenges is to tell the computer that you want to evaluate the value function at a point $k' = k(1-\delta) + i$. The problem is that k' may be or may not be one of the values of the grid, and that you have to choose i optimally. What I recommend you doing, but it's completely up to you how you solve the model, is that you use the functions *spline*, and the function *ppval*, which create interpolation of a vector and evaluates the interpolation in arbitrary points. That you create a very thin grid for $investment$, in which you evaluate the value function $\mathbb{E}(V(k, a')|a)$ that you created in an array of values k', where $k' = k(1-\delta) + i$, and that you choose the value of investment that maximizes the value function. The value function is judged to converge when starting from a guess $\tilde{V}(k, a)$, it produces a value function sufficiently close to $\tilde{V}(k, a)$. That is, the core of your code will take the form of a loop in which you iterate over values of V until you are sufficiently close to a fixed point.

(k) Using your solution in the code *vf_iteration*, plot the value function, and the investment rate i/k as a function of k in the x-axis, including different lines for each value of a.