Lecture 3: Investment Theory with Heterogeneity

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Outline

- Short refresher on Asset Pricing
- Kekre Lenel (2022)
- Bierdel, Drenik, Herreño, Ottonello (2025)
- Ottonello Winberry (2022)
- Catherine, Chaney, Huang, Sraer, Thesmar (2021)
- WInberry (2021)

Fundamental Asset Pricing Equations

- When thinking about returns of any asset:

$$\mathbb{E}_t(M_{t,t+1}R_{t+1}) = 1$$

- When thinking about a risk-free return:

$$\frac{1}{\mathbb{E}_t M_{t,t+1}} = R^f_{t+1}$$

- When thinking about excess returns:

$$\mathbb{E}_t(M_{t,t+1}(R^a_{t+1} - R^b_{t+1})) = 0$$

Fundamental Asset Pricing Equations

M is the stochastic discount factor

- The right discount rate to value a claim to future payments
- Different asset pricing theories take a stance on what the right M is
- Consumption-based asset pricing is one of such stances: M is given by the contribution of a payment to the marginal utility of consumption
- Asset Pricing by John Cochrane is in my view the best reference

Fundamental Asset Pricing Equations

- Consumer's Euler equation.
- In the margin, indifference between paying 1 unit of goods today and receive R_{t+1} units tomorrow

$$u'(c_t) = \beta \mathbb{E}_t[u'(c_{t+1})R_{t+1}]$$

- Or

$$\mathbb{E}_t(M_{t,t+1}R_{t+1}) = 1$$

- for

$$M_{t,t+1} = \beta \frac{u'(c_{t+1})}{c_t}$$

- making obvious that M is a random variable

Fundamental Asset Pricing Equations

$$\mathbb{E}_t(M_{t,t+1}R_{t+1}) = 1$$

- Remembering that $cov(x, y) = \mathbb{E}(xy) - \mathbb{E}(x)\mathbb{E}(y)$

$$cov_t(M_{t,t+1}R_{t+1}) + \mathbb{E}_t M_{t,t+1}\mathbb{E}_t R_{t,t+1} = 1$$

- And remembering that the risk-free rate is risk-free

$$\mathbb{E}_t R_{t,t+1} = R^f_{t+1}(1 - cov_t(M_{t,t+1}R_{t+1}))$$

- An asset has a positive excess return iff:

$$cov_t(M_{t,t+1}R_{t+1}) < 0$$

Fundamental Asset Pricing Equations

$$cov_t(M_{t,t+1}R_{t+1}) < 0$$

- $M_{t,t+1}$ is increasing in $u'(c_{t+1})$
- And as a consequence decreasing in c_{t+1}
- a negative covariance between M and R implies a positive covariance between R and c_{t+1}
- Positive excess returns for assets that pay badly in bad states of the world (= pay well in good states)
- Benchmark examples:
  - Insurance pays in terrible states of the world. Negative excess returns
  - The stock market pays well in good times. Positive excess returns.

The Campbell Shiller Decomposition

- In what comes it will be useful to know a decomposition of excess returns
- Intuitively if excess returns are increased, then two things could have happened
  - There were news about future dividend growth
  - Future excess returns are expected to decrease

The Campbell Shiller Decomposition

- This derivation is from Bernanke and Kuttner (2005)
- There is a risky asset with return R_{t+1}.

$$R_{t+1} = \frac{P_{t+1} + D_t}{P_t}$$

- P prices, D dividends. In logs

$$r_{t+1} = \log(P_{t+1} + D_t) - \log P_t$$

- sums inside a log are the worst. We will need an approximation.
- linearize $\log(P_{t+1} + D_t)$

$$\Delta \log(P_{t+1} + D_t) \approx k + \frac{P}{P+D} p_{t+1} + \frac{D}{P+D} d_t - \frac{P}{P+D} p_t - \frac{D}{P+D} d_{t+1} = k + \rho \Delta p_{t+1} + (1-\rho) \Delta d_t$$

- Express in levels

$$\log(P_{t+1} + D_t) \approx k + \rho p_{t+1} + (1-\rho) d_t$$

The Campbell Shiller Decomposition

$$\log(P_{t+1} + D_t) \approx k + \rho p_{t+1} + (1 - \rho)d_t$$

- Define δ_{t-1} as the inverse log price dividend ratio. Substitute into r_{t+1}

$$r_{t+1} = k - \rho\delta_{t+1} + \delta_t + \Delta d_t$$

- Express with lag operators

$$r_{t+1} = k + (1 - \rho L^{-1})^{-1}\delta_t + \Delta d_t$$

- Solve for δ_t

$$\delta_t = \sum_{i=0}^{\infty} \rho^i (r_{t+i+1} - \Delta d_{t+i}) - k/(1 - \rho)$$

- Substitute into the equation for r

$$r_{t+1} - \mathbb{E}_t r_{t+1} = -\sum_{i=1}^{\infty} \rho^i (\mathbb{E}_{t+1} - \mathbb{E}_t) r_{t+i+1} + \sum_{i=0}^{\infty} \rho^i (\mathbb{E}_{t+1} - \mathbb{E}_t) \Delta d_{t+i+1}$$

- If returns were higher than expected is either because there were positive news about future dividends, or the future returns are now expected to be lower.

The Campbell Shiller Decomposition

- Any return $r_{t+1} = r^f_{t+1} + r^e_{t+1}$. r^f_{t+1} known in t, so $r^f_{t+1} - \mathbb{E}_t r^f_{t+1} = 0$
- Use it to express the equation in terms of excess returns

$$r^e_{t+1} - \mathbb{E}_t r^e_{t+1} = -\sum_{i=1}^{\infty} \rho^i (\mathbb{E}_{t+1} - \mathbb{E}_t)(r^e_{t+i+1} + r^f_{t+i+1}) + \sum_{i=0}^{\infty} \rho^i (\mathbb{E}_{t+1} - \mathbb{E}_t)\Delta d_{t+i+1}$$

Kekre Lenel (2022)

Risk Premia Effects of Monetary Policy

- *Expansionary monetary policy lowers risk premia*
- Why?
- Let's think about the stock market
- Expansionary monetary policy must change the covariance between the relevant SDF and stock returns
- When the CB lowers rates inflation increases
- Dilutes debt holdings of more risk averse households
- Transfer to more risk averse households. Wealth tilts towards people with more risk tolerance
- Need less compensation for taking risk

Setting

- Heterogeneous investors that can invest in capital or bonds
- The return on capital is risky. It is a function of a future productivity shock
- The return on bonds is not. Assumption: The central bank commits to zero inflation
- The portfolio weight on capital ω^i

$$\omega^i \approx \frac{1}{\gamma^i} \frac{\mathbb{E}_0 \log R_1^k - \log R_1 + \frac{1}{2}\sigma^2}{\sigma^2}$$

- high ω if γ (risk aversion) is low
- Can you verbalize how to get this expression?
- Why is it not an explicit function of consumption c, or savings a?

Risk Premia

$$\frac{d\left[\mathbb{E}_0 \log R_1^k - \log R_1\right]}{d\epsilon_0^m} = \gamma\sigma^2 \int_0^1 \frac{d\left[a_0 / \int_0^1 a_0^{i'} di'\right]}{d\epsilon_m^0}(1-\omega_0^i)di$$

- If wealth is flowing towards people with high capital shares
- then the risk premium goes down
- The return on capital is decreasing in investment
- An expansionary monetary policy shock has an additional effect. It reallocates wealth to people who are prone to invest
- Or High Marginal Propensity to take Risk (MPR)

Redistribution of wealth

Given (17) and defining $n_0 \equiv \int_0^1 n_0^i di$, the change in its wealth share is in turn

$$\frac{d\left[n_0^i / \int_0^1 n_0^{i'} di'\right]}{d\epsilon_0^m} = \frac{1}{n_0}\left[-\frac{1+i_{-1}}{P_0} B_{-1}^i \frac{d \log P_0}{d\epsilon_0^m} + \left(k_{-1}^i - \frac{n_0^i}{n_0} k_{-1}\right)\left(\frac{d\pi_0}{d\epsilon_0^m} + (1-\delta)\frac{dq_0}{d\epsilon_0^m}\right)\right]. \quad (21)$$

Effects on quantities

Proposition 2. *The change in investment in response to a monetary shock is*

$$\frac{dk_0}{d\epsilon_0^m} = -\frac{k_0}{1-\alpha+\chi^x}\left[\frac{d\left[\mathbb{E}_0\log(1+r_1^k)-\log(1+r_1)\right]}{d\epsilon_0^m} + \frac{d\log(1+r_1)}{d\epsilon_0^m}\right]. \tag{18}$$

The change in consumption $c_0 \equiv \int_0^1 c_0^i di$ *in response to a monetary shock is*

$$\frac{dc_0}{d\epsilon_0^m} = \frac{1-\beta}{\beta} q_0(1+\chi^x)\frac{dk_0}{d\epsilon_0^m}.$$

The change in output $y_0 \equiv \ell_0^{1-\alpha}k_{-1}^{\alpha}$ *in response to a monetary shock is*

$$\frac{dy_0}{d\epsilon_0^m} = \frac{dc_0}{d\epsilon_0^m} + q_0\left(1+\chi^x\frac{x_0}{k_0}\right)\frac{dk_0}{d\epsilon_0^m}. \tag{19}$$

		$\frac{A^i}{W\ell^i}$	
		$\geq p60$	$< p60$
$\frac{Qk^i}{A^i}$	$\geq p90$	**Group** a Share households: 4% $\sum_{i \in a} W\ell^i / \sum_i W\ell^i$: 3% $\sum_{i \in a} A^i / \sum_i A^i$: 18% $\sum_{i \in a} Qk^i / \sum_{i \in a} A^i$: 2.0	**Group** c Share households: 60% $\sum_{i \in c} W\ell^i / \sum_i W\ell^i$: 83% $\sum_{i \in c} A^i / \sum_i A^i$: 23% $\sum_{i \in c} Qk^i / \sum_{i \in c} A^i$: 1.1
	$< p90$	**Group** b Share households: 36% $\sum_{i \in b} W\ell^i / \sum_i W\ell^i$: 14% $\sum_{i \in b} A^i / \sum_i A^i$: 58% $\sum_{i \in b} Qk^i / \sum_{i \in b} A^i$: 0.5	

Table II: heterogeneity in wealth to labor income and the capital portfolio share

Notes: observations are weighted by SCF sample weights.

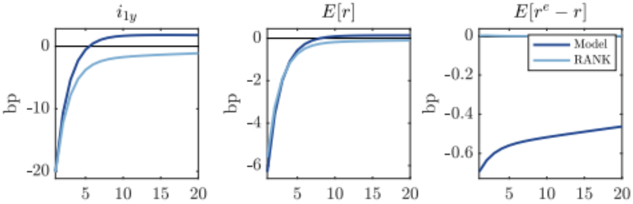

Figure 2: expected returns after negative monetary policy shock

Notes: series are quarterly (non-annualized) measures, except for the 1-year nominal bond yield Δi_{1y}. Impulse responses are the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

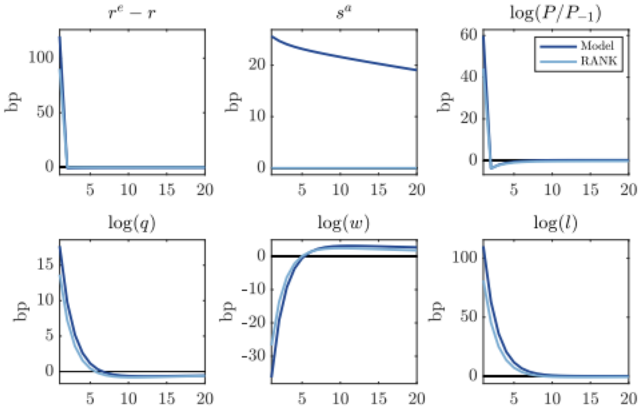

Figure 3: redistribution after negative monetary policy shock

Notes: see notes accompanying Figure 2 on construction of impulse responses.

% Real stock return	Data [90% CI]	Model	RANK
Dividend growth news	33% [-13%,71%]	52%	65%
−Future real rate news	8% [-6%,21%]	16%	35%
−Future excess return news	59% [19%,108%]	32%	0%

Table VII: Campbell-Shiller decomposition after monetary shock

Notes: estimates from data correspond to Table I. Comparable estimates obtained in the model assuming a debt/equity ratio of 0.5 on a stock market claim.

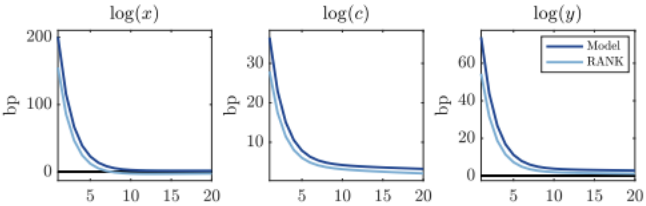

Figure 4: quantities after negative monetary policy shock

Notes: see notes accompanying Figure 2 on construction of impulse responses.

Bierdel, Drenik, Herreño, Ottonello (2025)

Motivation

- Information asymmetries are a salient phenomenon of real asset markets
  - Capital heterogeneous in quality, sellers have more info than buyers (Akerlof 1970)
- Changes in degree of asymmetric info play key role in classic narrative of crises
  - E.g., subprime crisis, Euro crisis, historical events in Reinhart Rogoff 2011
- How capital heterogeneity and asymmetric information affect the macroeconomy?
- This paper: Micro-to-macro approach, motivated by two ideas:
  1. Capital markets are illiquid, involving delayed trade (Ramey Shapiro 2001)
  2. In illiquid mkts, asymmetric info distorts terms-of-trade (Guerrieri Shimer Wright 2010)

$\Longrightarrow$ Study AI by measuring the liquidity of different capital units listed for trade

What We Do

1. **GE capital-accumulation model with illiquid mkts and asymmetric info**
    - AI distorts listed prices and selling prob of sellers of high-quality capital

      $\Longrightarrow$ Degree of AI can be identified from relationship between prices and duration
2. **Measurement of distortions from asymmetric information**
    - Dataset on capital units listed for trade, with listed price, duration, characteristics
    - Empirical evidence consistent with AI distorting listed prices and liquidity
        - Distortions increase during Euro crisis
3. **Quantifying the macroeconomic implications of asymmetric information**
    - Changes in degree of asymmetric info have large macro effects
        - Measured $\uparrow$ in degree of AI during Euro crisis leads to 2% output slump, slow recovery

Related Literature

- **Asymmetric information in asset markets**
  - Akerlof 1970, Stiglitz Weiss 1981, Guerrieri Shimer Wright 2010, Delacroix and Shi, 2013
  - Eisfeldt, 2004; Kurlat, 2013; Guerrieri and Shimer, 2014; Bigio, 2015; Lester et al., 2018,...

  Here: Use microlevel data on assets listed for trade to measure degree of AI

- **Misallocation and capital reallocation**
  - Eisfeldt Rampini 2006, Restuccia Rogerson 2008, Hsieh Klenow 2009, Hopenhayn, 2014, Cui 2017, Lanteri 2018, David Venkateswaran 2019, Kehrig and Vincent 2019, ...

  Here: Quantify misallocation due to AI in the aggregate economy

- **Illiquid asset markets**
  - Ramey Shapiro 2001, Gavazza 2011, Kermani Ma, 2020
  - Kurmann Petrosky-Nadeau 2007, Cao Shi 2017, Ottonello 2017, Wright et al. 2018,...
  - Lagos Rocheteau Wright 2017, Kaplan Violante 2014, ...

  Here: Leverage search-and-matching frictions to measure degree of AI

Outline

1. **Model**
2. The Micro Effects of Asymmetric Information
3. Measurement
4. The Macro Effects of Asymmetric Information

Model Overview

- Neoclassical capital-accumulation model
  - Discrete infinite time, final and capital goods
- Additional ingredients:
  1. Capital-quality heterogeneity
  2. Decentralized capital markets
  3. Asymmetric information

Neoclassical Block

- **Households**
  - Preferences $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)\gamma_n^t$
  - Access to a linear technology to produce new capital goods using final goods
- **Firms** (owned by households)
  - Technology $y_{jt} = f_t(\mathcal{K}_{jt}, l_{jt}) \equiv \mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$
  - Every period: prob. φ a firm exits the economy, new mass of firms φ enters

Neoclassical Block

- **Households** $\Rightarrow$ hold unemployed capital, capital sellers
  - Preferences $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)\gamma_n^t$
  - Access to a linear technology to produce new capital goods using final goods
- **Firms** $\Rightarrow$ hold employed capital, capital buyers
  - Technology $y_{jt} = f_t(\mathcal{K}_{jt}, l_{jt}) \equiv \mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$
  - Every period: prob. φ a firm exits the economy, new mass of firms φ enters

Capital-quality heterogeneity

- Capital stock composed of infinitesimal indivisible units
- Capital units are heterogeneous in two dimensions
  1. observed quality $\omega \in \Omega \equiv [\omega_1, \ldots, \omega_{N_\omega}]$
     - e.g., location, type, size, number of rooms,...
  2. unobserved quality $a \in \mathcal{A} \equiv [a_1, \ldots, a_{N_a}]$
     - private information of owner
     - e.g., physical conditions, atmosphere for customers, neighbors,...
- Capital services of unit i: $\omega_i a_i$
  - Firm j's capital input: $\mathcal{K}_{jt} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k_{jt+1}(\omega, a)$
- Maintenance costs: $\delta \omega_i a_i$ (depreciation)

Decentralized Capital Market

- Capital goods traded in a decentralized mkt with **search-and-matching frictions**
- Search is directed (Shimer 1996, Moen 1997, Menzio Shi 2011)
- Organized in a continuum of submarkets, indexed by $\boldsymbol{(\omega, \hat{a}, q)} \in \Omega \times \mathcal{A} \times \mathbb{R}_+$
  - Sellers list price $q(\omega, a)$ and announced quality $\hat{a}(\omega, a)$
  - Buyers dedicate hours of work to search and match $v_t(\omega, \hat{a}, q)$
- Cobb-Douglas matching technology in each submarket (w/ matching elasticity η)
- Tightness $\theta_t(\omega, \hat{a}, q)$: ratio of buyers' hours of search to capital posted by sellers
- Sellers' matching probability $p(\theta_t(\cdot))$, buyers' matching yield/hour $\mu_t(\theta_t(\cdot))$

Asymmetric Information

- Buyers have access to **information-revealing technology** (Menzio Shi 2011)
  - prob ψ: buyer learns the true type (ω, a) of the capital good
  - prob $1 - \psi$: inspection is uninformative

ψ parameterizes the degree of asymmetric information in the economy

Asymmetric Information

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  ψ parameterizes the degree of asymmetric information in the economy

- **Trading protocol upon inspection**
  - If no new info is revealed: trade at listed price q
  - If quality a is revealed (and there are gains from trade): trade at adjusted price

    $$q_t^P(\omega, a, \hat{a}, q) = \begin{cases} q \text{ if } a \geq \hat{a} \\ \min\{\text{bargaining price}, q\} \text{ if } a < \hat{a} \end{cases}$$

  - In the paper we provide sufficient conditons for generic $q_t^P(\omega, a, \hat{a}, q)$

Bargaining | Sufficient conditions post-inspection price | Timing

Households' Capital Accumulation

- Law of motion households' unemployed capital of type (ω, a)

$$k_{Ht+1}(\omega,a) = \underbrace{(1 - p(\theta_t(\omega, \hat{a}_t(\omega,a), q_t(\omega,a))))k_{Ht}(\omega,a)}_{\text{unsold capital}} + \underbrace{g(\omega,a)i_t}_{\text{investment}} + \underbrace{\varphi K_{Ft}(\omega,a)}_{\text{separations}}$$

where $g : \Omega \times \mathcal{A} \to [0,1]$ governs the production of different capital qualities

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where $g : \Omega \times \mathcal{A} \to [0,1]$ governs the production of different capital qualities

- Euler equation for investment

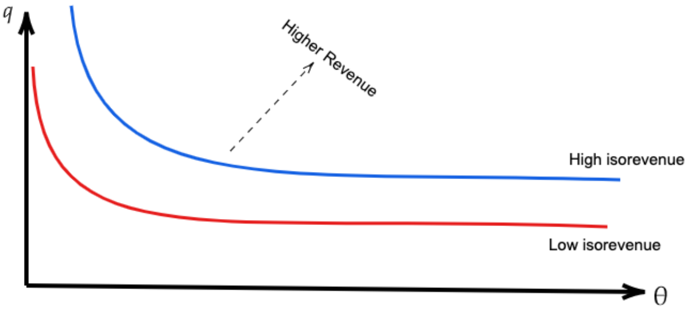
$$1 = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} g(\omega,a)\lambda_t(\mathbf{k})\nu^{\mathrm{s}}_{t+1}(\omega,a,\mathbf{k}),$$

Households' recursive problem

Households as Capital Sellers

- Marginal value of capital

$$\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = \max_{\hat{a},q} \; p(\theta_t(\omega,\hat{a},q))((1-\psi)q + \psi q_t^P(\omega,a,\hat{a},q))$$
$$+ (1 - p(\theta_t(\omega,\hat{a},q)))\left(\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, k_{Ht+1}(\mathbf{k})) - \delta\omega a\right)$$



Firms' Capital Accumulation

- Law of motion firms' employed capital of type (ω, a)

$$k_{jt+1}(\omega, a) = \underbrace{\sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega, \hat{a}, q)\mu_t\left(\theta_t\left(\omega, \hat{a}, q\right)\right) v_{jt}(\omega, \hat{a}, q)\,\mathrm{d}q}_{\text{purchases of capital type } (\omega, a)} + \underbrace{k_{jt}(\omega, a)}_{\text{initial capital}}$$

where $\iota_t(a|\omega, \hat{a}, q)$: share of capital quality a in submarket $(\omega, \hat{a}, q)$

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where $\iota_t(a|\omega, \hat{a}, q)$: share of capital quality a in submarket $(\omega, \hat{a}, q)$

- Marginal value of capital

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right],$$

where $Z_t \equiv \alpha\left(\frac{\gamma^t(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$

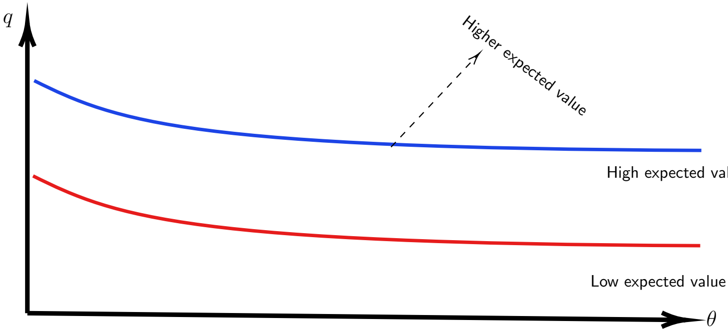
Firms' recursive problem

Firms as Capital Buyers

- Optimal search activity across submarkets

$$\underbrace{((1-\psi)q+\psi\mathbb{E}_a(q_t^P(\omega,a,\hat{a},q))|\omega,\hat{a},q))}_{\text{Expected price}} \geq \underbrace{\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega,a)|\omega,\hat{a},q\right)}_{\text{Expected value}} - \underbrace{\frac{w_t}{\mu_t(\theta(\omega,\hat{a},q))}}_{\text{Search cost}},$$

with equality if $v_t(\omega,\hat{a},q)>0$

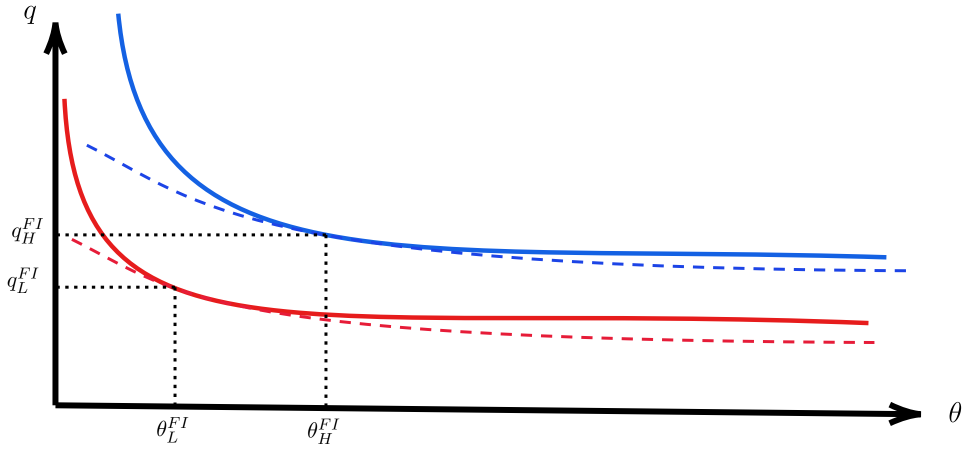


Outline

1. Model
2. **The Micro Effects of Asymmetric Information**
3. Measurement
4. The Macro Effects of Asymmetric Information

Equilibrium Under Full Information

- Suppose $\Omega = \{\omega_L, \omega_H\}$, $\mathcal{A} = \{\overline{a}\}$ + focus on BGP ▸ BGP



Prices and Duration for Observed Capital Qualities

Prediction I: Under **FI** there is a **negative relationship between prices and duration**

$$q^{\mathrm{FI}}(\omega_H, a) > q^{\mathrm{FI}}(\omega_L, a) \text{ and } p\left(\theta^{\mathrm{FI}}(\omega_H, a)\right) > p\left(\theta^{\mathrm{FI}}(\omega_L, a)\right)$$

Proposition

Intuition: Submarkets with higher quality attract more buyers resulting on higher prices and higher matching probability for sellers

$$\underbrace{(1-\eta)\left(\nu^{\mathrm{b}}(\omega, a) - \Lambda \nu^{\mathrm{s}}(\omega, a)\right)}_{\text{benefit purchasing quality } (\omega, a)} = \underbrace{\frac{w}{\mu\left(\theta(\omega, a)\right)}}_{\text{search cost}}$$

Equilibrium Under Asymmetric Information

- We illustrate for $\mathcal{A} = \{a_L, a_H\}$ and $\Omega = \overline{\omega}$ (Paper: characterization with multiple types)

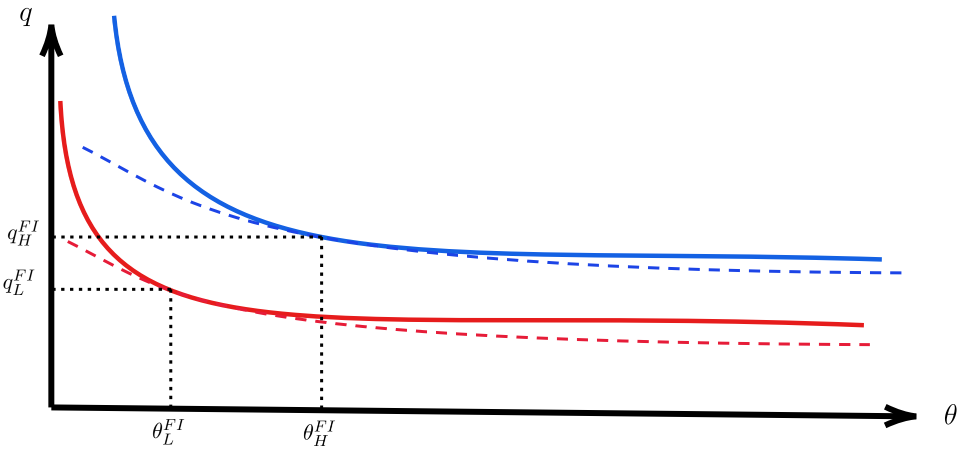
Proposition

There exists a unique fully-revealing separating equilibrium that satisfies the intuitive criterion; there are no pooling equilibria.

- Equilibrium resembles that of the Spence (1973) model
  - "Effort" corresponds to selling with a lower probability
  - High-quality sellers have a lower marginal cost of not trading than low-quality sellers

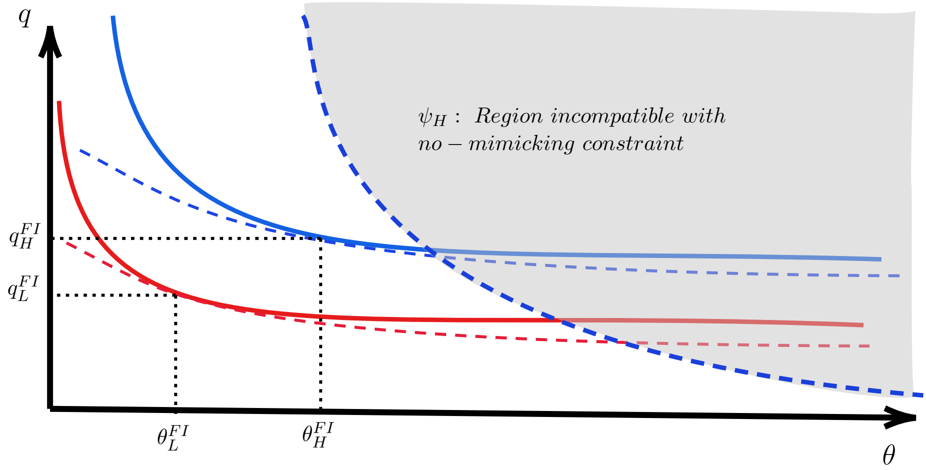
Definition of equilibrium | Definition of types of equilibrium | Conditions for separating | Sketch proof

Equilibrium Under Asymmetric Information



Constructing Eq.

Equilibrium Under Asymmetric Information



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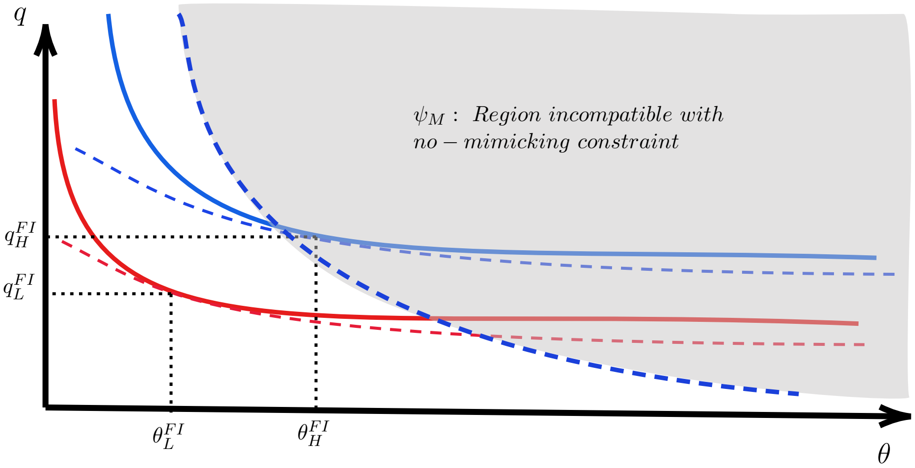
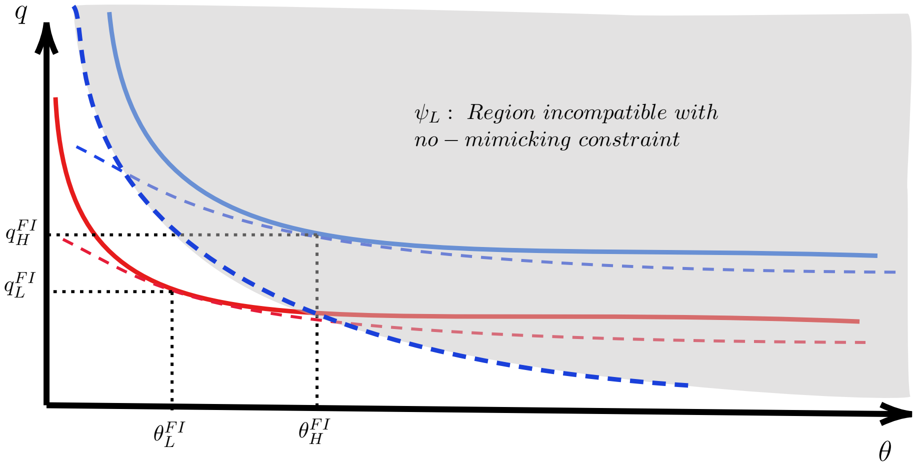


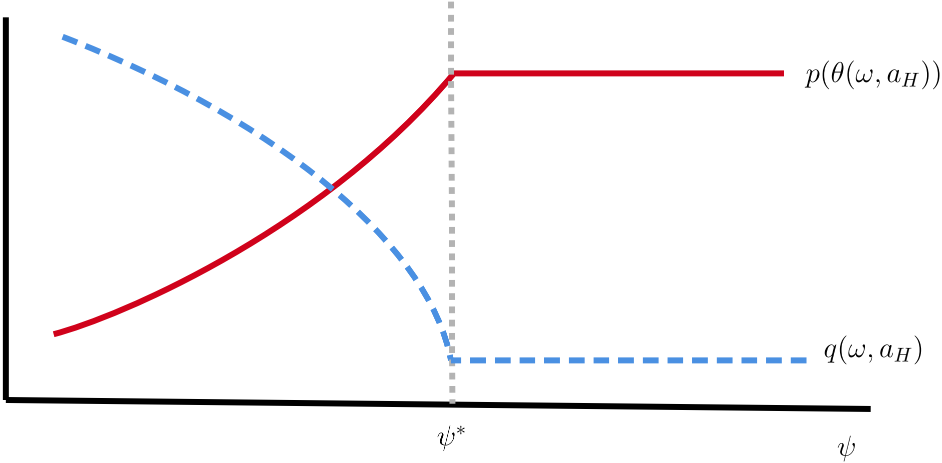
▸ Constructing Eq.

Equilibrium Under Asymmetric Information



▸ Constructing Eq.

Equilibrium Under Asymmetric Information



Prices and Duration for Unobserved Capital Quality

Prediction II: **AI** (i.e., $\psi < \psi^*$) affects **terms of trade of high-quality capital**

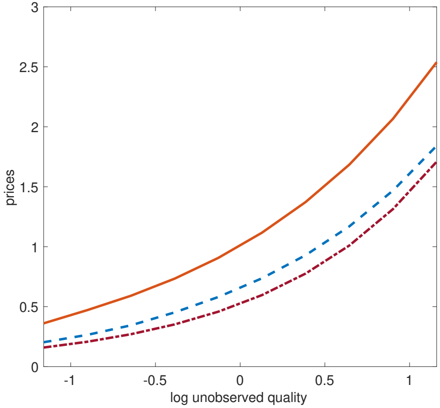
$$q(\omega, a_H) > q^{FI}(\omega, a_H) \text{ and } p\left(\theta(\omega, a_H)\right) < p\left(\theta^{FI}(\omega, a_H)\right)$$

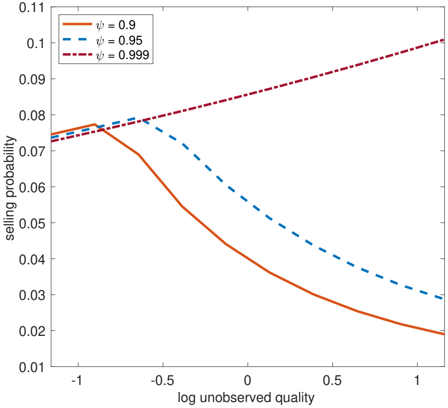
▸ Proposition

- Intuition: a_H chooses higher price to signal its quality, willing to accept lower trading probability
- Distortions governed by ψ: $\left.\frac{\partial \frac{p(\theta(\omega, a_L))}{p(\theta(\omega, a_H))}}{\partial \psi}\right|_{\psi < \psi^*} < 0$

$\Longrightarrow$ **Relationship between prices and duration is informative about ψ**

Prices and Duration for Multiple Types: A Quantitative Illustration





▸ Identification (Paper provides formal identification results)

Outline

1. Model
2. The Micro Effects of Asymmetric Information
3. **Measurement**
4. The Macro Effects of Asymmetric Information

The Data

- Panel of capital structures posted for sale and rent
  - Retail, office space, and warehouses
  - Monthly listed price
  - Information on listed characteristics: location, age, size, number of rooms, etc.
  - Duration and monthly search intensity (clicks and emails received)
- Source: *Idealista*, leading online platform in the real estate market in Europe
- Coverage: 8.5 million observations from Spain
  - $>$ 1.1 million capital units
  - Period: 2005–2018

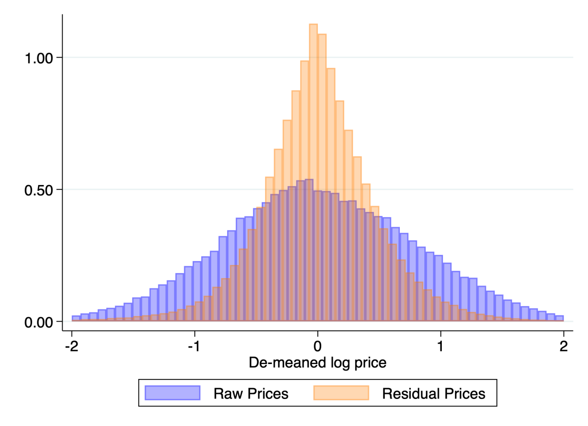
Online Platform | Benchmarking | Descriptive statistics | Dist. Duration

Price Variation Explained by Listed Characteristics

(Log) price per sq. ft. of property i in location l in month t:

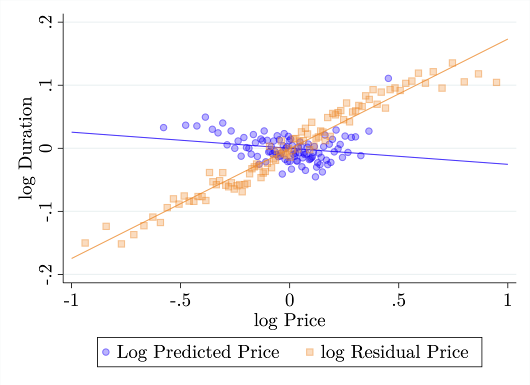
$$log(q_{it}) = \nu_{l(i)t} + \gamma X_i + \varepsilon_{it}$$

	St. Dev.	R^2
Raw data	0.83	0.00
Year	0.79	0.09
Location	0.59	0.49
Year $\times$ Location $\times$ Type	0.56	0.54
. . . $+$ Area	0.53	0.59
. . . $+$ Age	0.53	0.59
Benchmark	0.51	0.62



▸ Location ▸ Time

Relationship Between Duration and Prices



	(1) log Duration	(2) log Duration
log Price	0.013***	
	(0.004)	
log Predicted Price		-0.025**
		(0.011)
log Residual Price		0.148***
		(0.004)
Observations	456351	439680
R^2	0.226	0.228

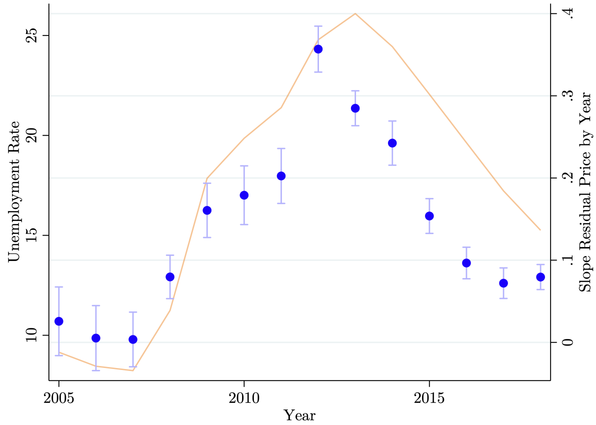
$$\log Duration_{it} = \beta_0 + \beta_1 \hat{\varepsilon}_{it} + \beta_2 \widehat{log(q_{it})} + u_{it}$$

Consistent with model predictions for observed and unobserved capital quality

Robust to other measures

Clicks Rental market

The Euro Crisis and the Slope between Duration and Residual Prices



Alternative Interpretations and Additional Results

1. Homogeneous capital qualities and sellers' indifference between prices & duration
   - Expected net present revenues are increasing in residual prices ▸ Details
2. Homogeneous capital qualities and sellers' heterogeneity in holding costs
   - Holding costs that equalize net present revenues to that of higher residual prices appear unplausibly large ▸ Details
3. Buyers' wealth heterogeneity
   - Similar relationship between duration and residual prices observed at different absolute price ranges ▸ Details

Outline

1. Model
2. The Micro Effects of Asymmetric Information
3. Measurement
4. **The Macro Effects of Asymmetric Information**

Parameterization

Two-step procedure

1. **Fix a subset of parameters to standard values**
2. Calibrate by targeting moments on model simulated data

Parameter	Description	Value
β	Discount factor	0.9966
α	Share of capital	0.35
δ	Depreciation rate	0.0074
γ	Technology growth	1.004
γ_n	Population growth	1.0027
φ	Firms' exit rate	0.0027
η	Curvature matching technology	0.8
ϕ	Bargaining power of seller	0.5

Parameterization

Two-step procedure:

1. Fix a subset of parameters to standard values
2. **Calibrate by targeting moments on model simulated data**

Parameter	Description	Value	Target	Model	Data
ψ	Accuracy information tech.	0.9795	Regression coefficient	0.148	0.148
σ_ω	SD observed quality	0.72	SD log predicted prices	0.65	0.65
σ_a	SD unobserved quality	0.58	SD log residual prices	0.51	0.51
$\bar{m}$	Matching efficiency	0.267	Mean duration	11.46	11.44

- Benchmark: targeting the slope during the Euro crisis, get $\psi = 0.96$
- Model matches untargeted slope between duration and predicted prices Regression

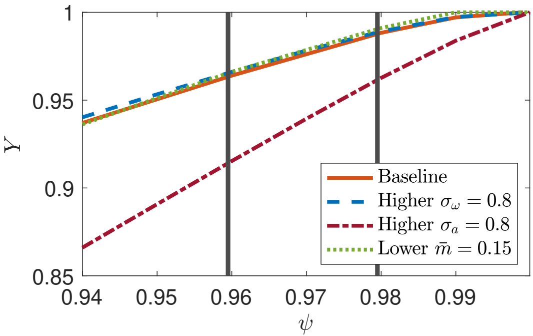
Contour plots

Aggregation and Macro Channels

$$
\begin{aligned}
Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t{}^{\alpha} \\
&= \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega \in \omega} \sum_{a \in \mathcal{A}} K_t(\omega, a) \right] \left[\mathbb{E}\left(\omega a\right) \left(1 - \mathbb{E}\left(u_t(\omega, a)\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega, a)\right) \right] \right)^{\alpha}
\end{aligned}
$$

- L_t: labor used in production
- $\mathcal{K}_t \equiv \int \mathcal{K}_{jt} \, \mathrm{d}j$: aggregate capital input
  - $\sum_{\omega \in \omega} \sum_{a \in \mathcal{A}} K_t(\omega, a)$: aggregate capital stock
  - $\mathbb{E}\left(u_t(\omega, a)\right)$: average capital unemployment rate
  - $\mathbb{C}ov\left(\omega a, u_t(\omega, a)\right)$: selection into unemployment

Asymmetric Information and Economic Activity



	Change	Contribution
$Y/Y^{FI} - 1$	1.22%	100%
$\mathcal{K}/\mathcal{K}^{FI} - 1$	2.55%	74%
$K/K^{FI} - 1$	1.12%	32%
$u - u^{FI}$	0.92%	25%
$cov - cov^{FI}$	0.01	16%
$L/L^{FI} - 1$	0.5%	26%

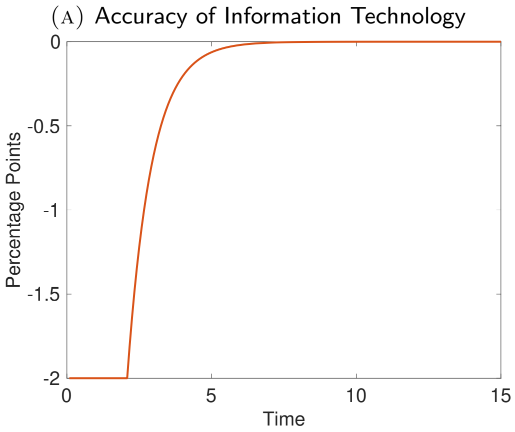
Alternative parameterizations
Investment
Capital-unemployment
Quality

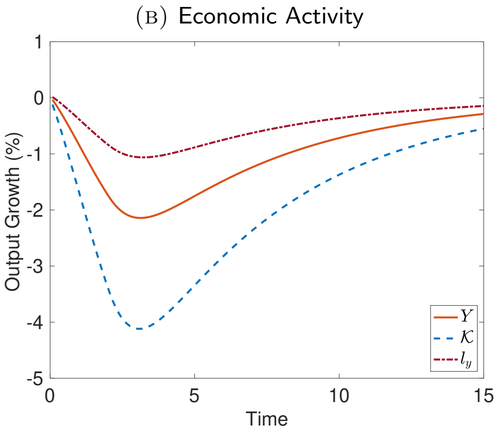
Crisis Experiment

- Study macro dynamics from changes in the degree of asymmetry information
- Assume at $t = 0$ the economy experiences an unexpected change in the accuracy of information technology
- Captures classic narrative during economic crises:
  - Sellers innovate to make their assets more opaque
  - Buyers realize the lower quality of information-revealing technologies
- Discipline change and persistence of ψ_t with dynamics of the slope between duration and residual prices observed during the Euro crisis

Contour plots

Macroeconomic Reponses to Changes in Information Technologies

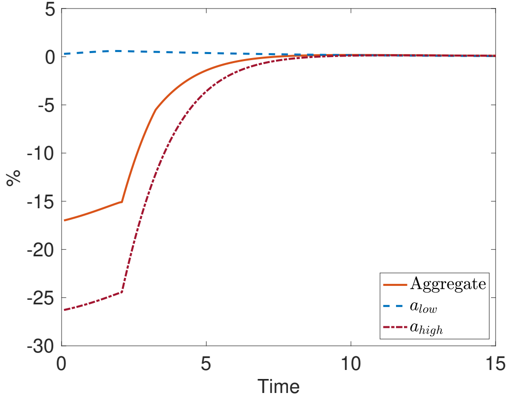




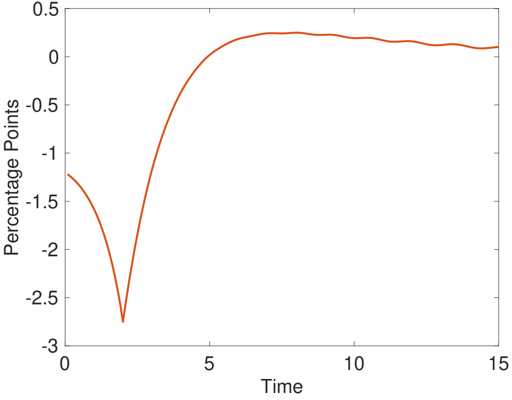
▸ Additional variables

Macroeconomic Reponses to Changes in Information Technologies

(A) Selling Probabilities



(B) Expected Marginal Value of Capital



Conclusions

- Capital heterogeneity and trading frictions in capital markets have important macro implications
- Large elasticity of economic activity to changes in degree of asymmetric information
- Transmission mechanism: liquidity of capital and its effects on investment and capital allocation
- Role for studying policies aimed at preventing signaling
  - Measurement developed in the paper can help inform policies

Appendix

Firms' Problem

$$V_{Ft}(\mathbf{k}) = \max_{\{l,\{v(\omega,\hat{a},q)\geq 0\},\{k'(\omega,a)\}\}} \mathbb{E}_a[div + \Lambda_{t,t+1}((1-\varphi)V_{Ft+1}(\mathbf{k}') + \varphi V^{\text{exit}}_{t+1}(\mathbf{k}'))],$$

subject to:

$$div = \left(\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a k'(\omega,a)\right)^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l - \delta \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a k'(\omega,a)$$

$$- \sum_{\omega\in\Omega}\sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} [(\psi \sum_{a\in\mathcal{A}} \iota_t(a|\omega,\hat{a},q) q^P_t(\omega,a',\hat{a},q) + (1-\psi)q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t] v(\omega,\hat{a},q)\,\mathrm{d}q$$

$$k'(\omega,a) = \sum_{\hat{a}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) v(\omega,\hat{a},q)\,\mathrm{d}q + k(\omega,a)$$

where $\mathbb{E}_a[\cdot]$: expectation under the belief function $\pi_t(a|\omega,\hat{a},q)$

Back

Households

- Idiosyncratic state: $\mathbf{k} \equiv \begin{bmatrix} k(\omega_1, a_1) & \dots & k(\omega_{N_\omega}, a_1) \\ \dots & \dots & \dots \\ k(\omega_1, a_{N_a}) & \dots & k(\omega_{N_\omega}, a_{N_a}) \end{bmatrix}$

- Households' problem

$$V_{Ht}(\mathbf{k}) = \max_{\{c,h,\{k'(\omega,a),\hat{a}(\omega,a),q(\omega,a)\},i\geq 0\}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

subject to:

$$c\gamma_n^t + i + \delta_t(\mathbf{k}) = w_t h \gamma_n^t + Div_{Ft} + \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \left[(1-\psi)q(\omega,a) + \psi q_t^P(\omega,a,q)\right] p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))k(\omega,a)$$

$$k'(\omega,a) = (1 - p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))k(\omega,a) + ig(\omega,a) + \varphi K_{Ft-1}(\omega,a),$$

Back

Post-inspection Bargaining

- Surplus from the match

$$S(\omega, a) = \nu^{\mathrm{b}}(\omega, a) - [\Lambda\nu^{\mathrm{s}}(\omega, a) - \delta\omega a]$$

- Post-inspection price

$$q^{P}(\omega, a, \hat{a}, q) = \min\{\phi\nu^{\mathrm{b}}(\omega, a) + (1-\phi)[\Lambda\nu^{\mathrm{s}}(\omega, a) - \delta\omega a], q\} \quad \text{if } a < \hat{a} \text{ with } \phi < \eta$$

▸ Back

Sufficient Conditions for Post-inspection Price

The inspection-adjusted price function $q_t^P(\omega, a, \hat{a}, q) : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to \mathbb{R}_+$ has the following properties:

1. it is non-decreasing in the true quality:

$$\forall (a, a') \in \mathcal{A}^2 \text{ such that } a' > a, \ \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ \ : \ q_t^P(\omega, a', \hat{a}, q) \geq q_t^P(\omega, a, \hat{a}, q),$$

2. it is non-increasing in the announced quality:

$$\forall (\hat{a}, \hat{\hat{a}}) \in \mathcal{A}^2 \text{ such that } \hat{a} > \hat{\hat{a}}, \ \forall (\omega, a, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ \ : \ q_t^P(\omega, a, \hat{a}, q) \leq q_t^P(\omega, a, \hat{\hat{a}}, q),$$

3. it is weakly lower (resp. higher) than the buyer's (resp. seller's) value for the unit:

$$q_t^P(\omega, a, \hat{a}, q) \in [\min(q, \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a), \min(q, \nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}))]$$

   $\forall \omega \in \Omega, \hat{a}, a \in \mathcal{A}, q \in \mathbb{R}+, \mathbf{K}_{Ht} \in \mathbb{R}+,$

4. it is such that buyers obtain at least a fraction $1 - \eta$ of the surplus:

$$q_t^P(\omega, a, \hat{a}, q) \leq \eta\nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}) + (1 - \eta)\left(\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right),$$

5. it does not decrease "too fast" as the announced quality increases, i.e.:

$$\frac{\eta(\nu_t^{\mathrm{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i)}{q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i} \geq \frac{q^B(\omega, a_j, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}{q^B(\omega, a_j, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}$$

   $\forall a_j < a_i < a_k \in \mathcal{A}, \omega \in \Omega.$

Back

Timing

i. Exit shocks are realized, and mass φ of new firms enter the economy.

ii. Households choose the capital units they list for sale, their prices, and their announced qualities, which are perfectly observed by all agents. Incumbent non-exiting firms and new firms search and match with potential capital units to buy.

iii. Firms conduct inspections of matched capital units and decide whether to buy them or not.

iv. Incumbent non-exiting firms and new firms hire workers, produce final goods, and pay wages. Firms that exit the economy transfer their capital to households. All agents holding capital pay a maintenance cost δ per unit of effective capital in terms of final goods. Households invest in new capital units and consume.

Back

Equilibrium

Definition (Competitive Equilibrium)

Given initial conditions $\mathbf{K}_{H0}$ and $(\mathbf{k}_{j0})_{j\in[0,1]}$, a PBE consists of a sequence of household value and policy functions; firm value and policy functions; market tightness functions; belief functions; wages; and aggregate variables for all $t \geq 0$ such that

1. Given wages and market tightness, $V_{Ht}(\mathbf{k})$, $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k})$, $c_t(\mathbf{k})$, $i_t(\mathbf{k})$, $\mathbf{k}_{Ht}(\mathbf{k})$, $\hat{a}_t(\omega, a, \mathbf{k})$, and $q_t(\omega, a, \mathbf{k})$ solve households' problem.
2. Given wages, market tightness, and discount factors, $V_{Ft}(\mathbf{k})$, $\nu_t^{\mathrm{b}}(\omega, a)$, $l_t(\mathbf{k})$, $\mathbf{k}_{Ft+1}(\mathbf{k})$, and $\{v_t(\omega, \hat{a}, q)\}$ solve firms' problem.
3. Market tightness functions satisfy buyer's optimality condition.
4. Beliefs $\pi_t(a|\omega, \hat{a}, q)$ are consistent with sellers' strategies using Bayes' rule when possible.
5. The labor market clears.
6. Aggregate variables are consistent with individual policies.

Definition (Balanced growth path)

A balanced growth path is defined as a competitive equilibrium in which the sequence $\{c_t, k_{Ht}(\omega, a), k_{Ft}(\omega, a), \hat{a}_t(\omega, a), q_t(\omega, a), \theta_t(\omega, \hat{a}_t, q_t), w_t, \Lambda_{t,t+1}, Z_t\}_{t\geq 0}$ satisfies:

1. Per-capita consumption c_t, wages w_t and productivity Z_t grow at rate γ.
2. For all (ω, a), the stock of capital held by firms and households ($k_{Ft}(\omega, a)$ and $k_{Ht}(\omega, a)$, respectively) grows at rate $\gamma\gamma_n$.
3. For all (ω, a), submarket choices $a_t(\omega, a)$ and $q_t(\omega, a)$, and market tightness $\theta_t(\omega, \hat{a}_t, q_t)$, are constant.
4. The discount factor satisfies $\Lambda_{t,t+1} = \beta\gamma_n/\gamma$.

Back

Proposition

Under full information, the price and market tightness for capital of quality (ω, a) are given by

$$q^{FI}(\omega, a) = \eta\nu^{\mathrm{b}}(\omega, a) + (1-\eta)\Lambda\nu^{\mathrm{s}}(\omega, a)$$

and

$$\theta^{FI}(\omega, a) = \left(\frac{\bar{m}(1-\eta)}{\chi}\left(\nu^{\mathrm{b}}(\omega, a) - \Lambda\nu^{\mathrm{s}}(\omega, a)\right)\right)^{1/\eta},$$

where $\chi \equiv w_t/\gamma^t$.

Back

Types of Equilibria

Definition (Types of equilibria)

- A **pooling eq.** is a competitive eq. in which $q(\omega, a_j) = q(\omega, a_{j'})$ and $\hat{a}(\omega, a_j) = \hat{a}(\omega, a_{j'})$ for $a_j \neq a_{j'}$ with positive probability.
- A **fully revealing separating** eq. is a competitive eq. in which $q(\omega, a_j) = q(\omega, a_{j'})$ and $\hat{a}(\omega, a_j) = a_j$ for $a_j \neq a_{j'}$.

▸ Back

Separating equilibrium ▸ Back

A competitive equilibrium is separating if and only if for each quality (ω, a), sellers choose a strategy $(\hat{a}(\omega, a), q(\omega, a))$ to maximize their objective

$$\nu_t^{\mathrm{s}}(\omega, a) = \max_{\{q(\omega,a),\hat{a}(\omega,a)\}} p\left(\theta\left(\omega, \hat{a}(\omega, a), q(\omega, a)\right)\right)\left[(1-\psi)q(\omega, a) + \psi q^P(\omega, a, \hat{a}(\omega, a), q)\right]$$
$$+ \left(1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\right)\left[\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right]$$

subject to no-mimicking constraints

$$\nu_t^{\mathrm{s}}(\omega, a') \geq p\left(\theta\left(\omega, \hat{a}(\omega, a), q(\omega, a)\right)\right)\left[(1-\psi)q(\omega, a) + \psi q^P(\omega, a', \hat{a}(\omega, a), q)\right]$$
$$+ \left(1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\right)\left[\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a') - \delta\omega a'\right] \quad \text{for all } a' \neq a$$

and a market tightness given by buyers' optimal search decisions

$$\mathbb{E}_a\left((1-\psi)q(\omega, a) + \psi q^P(\omega, a, \hat{a}(\omega, a), q) \,|\, \omega, \hat{a}(\omega, a), q(\omega, a)\right)$$
$$= \mathbb{E}_a\left(\nu^b(\omega, a)\right) - \frac{w_t}{\mu_t\left(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))\right)}$$

Sketch of the Proof ▸ Back

1. Show that capital seller's optimization is
   - linear in capital holdings
   - independent across capital qualities
2. Solve the unconstrained full-information problem
3. Induction on the size of the set of unobserved qualities, ordered by value
4. Show that on a BGP, $a \to \Lambda_{t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a) - \delta\omega a$ is increasing in a

Sketch of the Proof (Cont.) ▸ Back

Recursive proof:

- Assume strategies of k lower unobserved qualities are determined according to iterative procedure and this is unique fully revealing eq. on $\{a_1, ..., a_k\}$:
  - If incentive-compatibility constraints of lower qualities do not rule out the full-information strategy for k+1, show that it is the only possible one
  - Else, show that any strategy that satisfies the incentive-compatibility constraints of lower unobserved qualities can be an equilibrium
    - Use the D1 criterion to reduce the set of possible strategies to single one
- Show this strategy makes type $k+1$ unwilling to mimick lower types
- Show the constructed strategy makes all types in $\{a_1, ..., a_{k+1}\}$ unwilling to deviate to an off-equilibrium sub-market using the D1 criterion

Recursive characterization

- Fix ω, order unobs. qualities $\mathcal{A} = \{a_1, ..., a_{N_a}\}$ and impose $\hat{a}(\omega, a) = a$
- Construct the equilibrium recursively:

1. **Initialize:** The lowest type a_1 always chooses the full information strategy

$$q_t^{FI}(\omega, a_1) = \nu_t^b(\omega, a_1) - \frac{w_t}{\mu_t(\theta^{FI}(\omega, a_1))}$$

and

$$p'(\theta^{FI}(\omega, a_1)) \left(\nu_t^b(\omega, a_1) - \left(\frac{\Lambda\gamma_n}{\gamma}\nu_{t+1}^{\mathrm{s}}(\omega, a_1) - \delta\omega a_1\right)\right) = \chi,$$

  - Full information is the unconstrained optimal strategy
  - No one wants the lowest type's payoff, hence its sellers can always pick it

Recursive characterization (Cont.)

- Fix ω, order unobs. qualities $\mathcal{A} = \{a_1, ..., a_{N_a}\}$ and impose $\hat{a}(\omega, a) = a$
- Construct the equilibrium recursively:

2. **Recursion:** Suppose $(q(\omega, a_1), ...q(\omega, a_{k-1}))$ already constructed and construct $q(\omega, a_k)$

Recursive characterization (Cont.)

Case 1: Type a_k always chooses the full information strategy if no lower type wants to mimick it. Formally, if for all $i < k$:

$$\nu_t^{s}(\omega, a_i) \geq p\left(\theta^{FI}(\omega, a_k)\right)\left[(1-\psi)q^{FI}(\omega, a_k) + \psi q^{P}(\omega, a_i, a_k, q^{FI}(\omega, a_k))\right]$$
$$+ \left(1 - p(\theta^{FI}(\omega, a_k))\right)\left[\Lambda_{t+1}\nu_{t+1}^{s}(\omega, a_i) - \delta\omega a_i\right]$$

Then type a_k chooses the full information strategy characterized by:

$$q_t^{FI}(\omega, a_k) = \nu_t^{b}(\omega, a_k) - \frac{w_t}{\mu_t(\theta^{FI}(\omega, a_k))}$$

and

$$p'(\theta^{FI}(\omega, a_k))\left(\nu_t^{b}(\omega, a_k) - \left(\frac{\Lambda\gamma_n}{\gamma}\nu_{t+1}^{s}(\omega, a_k) - \delta\omega a_k\right)\right) = \chi.$$

$\Rightarrow$ Full information is the unconstrained optimal strategy and is always chosen if available

Recursive characterization (Cont.)

Case 2: Otherwise, type a_k chooses the lowest price above its full information price that satisfies all no-mimicking constraints of lower types. Formally, for all $i < k$, let $\underline{\theta_i}$ be the lowest solution θ to:

$$\begin{aligned}\nu_t^{\mathrm{s}}(\omega, a_i) = & p\left(\theta\right)\left[(1-\psi)q(\omega, a_k, \theta) + \psi q^P\left(\omega, a_i, a_k, q(\omega, a_k, \theta)\right)\right] \\ & + (1-p(\theta))\left[\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) - \delta\omega a_i\right]\end{aligned}$$

where

$$q(\omega, a_k, \theta) = \nu_t^b(\omega, a_k) - \frac{w_t}{\mu_t(\theta)}.$$

Then, $\theta(\omega, a_k) = \text{Min}_{\{i<k\}}\underline{\theta_i}$ and $q(\omega, a_k) = q(\omega, a_k, \theta(\omega, a_k))$.

$\Rightarrow$ Sellers choose the price closest to their full information strategy that allows them to separate from lower qualities

Proposition

Let $\psi^* \in [0,1]$ be defined as

$$p(\theta^{FI}(\omega,a_L))\left[q^{FI}(\omega,a_L) - \Lambda\nu^{\mathrm{S},FI}(\omega,a_L) + \delta\omega a_L\right]$$
$$= p(\theta^{FI}(\omega,a_H))\left[(1-\psi^*)q^{FI}(\omega,a_H) + \psi^* q_t^P(\omega,a_H,q) - \Lambda\nu^{\mathrm{S},FI}(\omega,a_L) + \delta\omega a_L\right]$$

Then, for a given ω, the seller of quality a_L chooses the same terms of trade as under full information. For sellers of quality a_H, there are two cases:

1. $\psi \geq \psi^*$: the incentive-compatibility constraint is not binding and $\theta(\omega,a_H)$ solves

$$p'(\theta(\omega,a_H))\left(\nu^{\mathrm{b}}(\omega,a_H) - \Lambda\nu^{\mathrm{s}}(\omega,a_H)\right) = \chi$$

2. $\psi < \psi^*$: the incentive compatibility constraint is binding and $\theta(\omega,a_H)$ solves

$$p(\theta^{FI}(\omega,a_L))\left(q^{FI}(\omega,a_L) - \Lambda\nu^{\mathrm{s}}(\omega,a_L)\right)$$
$$= p(\theta(\omega,a_H))\left((1-\psi)q(\omega,a_H) + \psi q_t^P(\omega,a_H,q) + \Lambda\nu^{\mathrm{s}}(\omega,a_L)\right)$$

Back

Constructing the Equilibrium

- For **low type**:

$$\nu^{\mathrm{s}}(\omega, a_L) = \max_{\{q(\omega,a_L)\}} p\left(\theta\left(\omega, a_L, q(\omega, a_L)\right)\right) q(\omega, a_L)$$

$$+ \left(1 - p(\theta(\omega, a_L, q(\omega, a_L)))\right)\left(\Lambda\nu^{\mathrm{s}}(\omega, a_L) - \delta\omega a_L\right)$$

subject to buyer's indifference

$$\mu\left(\theta(\omega, a_L, q(\omega, a_L))\right)\left[\nu^{\mathrm{b}}(\omega, a_L) - q(\omega, a_L)\right] = w$$

Constructing the Equilibrium

- For **high type**:

$$\nu^{\mathrm{s}}(\omega, a_H) = \max_{\{q(\omega,a_H)\}} p\left(\theta\left(\omega, a_H, q(\omega, a_H)\right)\right) q(\omega, a_H)$$

$$+ \left(1 - p(\theta(\omega, a_H, q(\omega, a_H)))\right)\left(\Lambda \nu^{\mathrm{s}}(\omega, a_H) - \delta \omega a_H\right)$$

subject to buyer's indifference

$$\mu\left(\theta(\omega, a_H, q(\omega, a_H))\right)\left[\nu^{\mathrm{b}}(\omega, a_H) - q(\omega, a_H)\right] = w$$

and no-mimicking constraint

$$\nu^{\mathrm{s}}(\omega, a_L) \geq p\left(\theta\left(\omega, a_H, q(\omega, a_H)\right)\right)\left[(1-\psi) q(\omega, a_H) + \psi q_t^P(\omega, a_L, \hat{a}(\omega, a_H), q(\omega, a_H))\right]$$

$$+ \left(1 - p(\theta(\omega, a_H, q(\omega, a_H)))\right)\left[\Lambda \nu^{\mathrm{s}}(\omega, a_L) - \delta \omega a_L\right]$$

Back

Mapping Model to Data

- Researcher observes micro data on capital units listed for sale with:
  - Prices, duration, vector of obseved characteristics X_i
- Assume independent log-normal distributions for capital qualities with variances $\{\sigma_\omega^2, \sigma_a^2\}$ and $\log \omega_i = \tau X_i$
- Consider estimating the following regressions:

$$\log(q_i) = \iota_\omega X_i + \varepsilon_i^q$$

$$\log(Duration_i) = \upsilon_\omega \hat{q}_i + \upsilon_q \hat{\varepsilon}_i^q + \varepsilon_i^d$$

- Let $\hat{q}_i = \hat{\iota}_\omega X_i$: predicted prices, $\hat{\varepsilon}_i^q$: residual prices
- Proposition: If $\left(\frac{w_t}{\mu_t(\theta(\omega,\hat{a},q))}\right)/\nu_t^{\mathrm{b}}(\omega,a) \to 0$ and $\varphi \to 0$, then up to first-order, $(\psi, \sigma_\omega^2, \sigma_a^2)$ are identified by the estimated moments $\hat{\upsilon}_q$, $\hat{\sigma}_\omega^2 \equiv Var(\hat{q}_i)$ and $\hat{\sigma}_a^2 \equiv Var(\hat{\varepsilon}_i^q)$.

Sketch proof Quantitative Back

Identification

Step 1: Assume $\frac{\frac{w_t}{\mu_t(\theta(\omega,a))}}{\nu_t^{\mathrm{b}}(\omega,a)} \to 0$ (small relative search costs) and $\varphi \to 0$ (small exit rate)

$$\implies \log q_t(\omega, a) \approx \log \omega + \iota_t + \log a$$

Step 2: Assume $\log \omega_i = \tau X_i$ and estimate $\log q_i = \iota_\omega X_i + \iota_{t(i)} + \varepsilon_i^q$

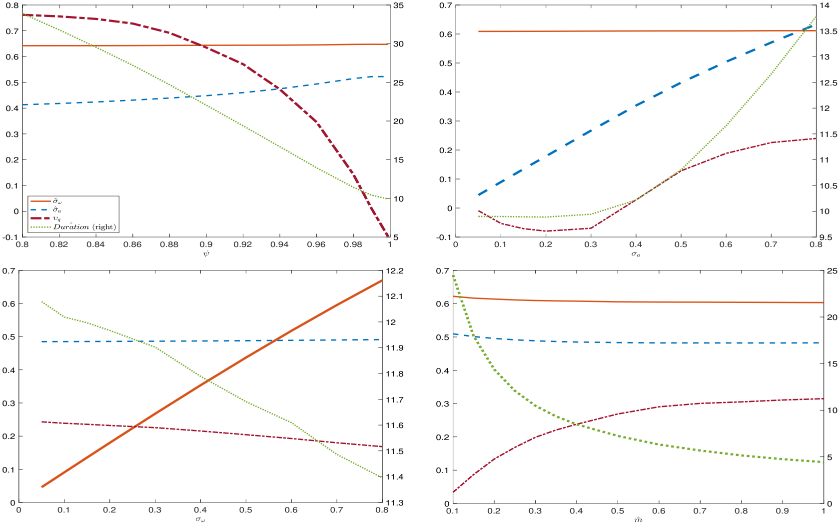
$$\implies (\iota_\omega X_i, \varepsilon_i^q) \to (\log \omega_i, \log a_i)$$

Step 3: Estimate $\log(Duration_i) = \upsilon_\omega \log(\omega_i) + \upsilon_q \log(q_i) + \iota_{t(i)} + \varepsilon_i^d$

$$\implies \text{up to first order } \upsilon_q = \Upsilon_q(\psi) \implies \psi = \Upsilon_q^{-1}(\upsilon_q)$$

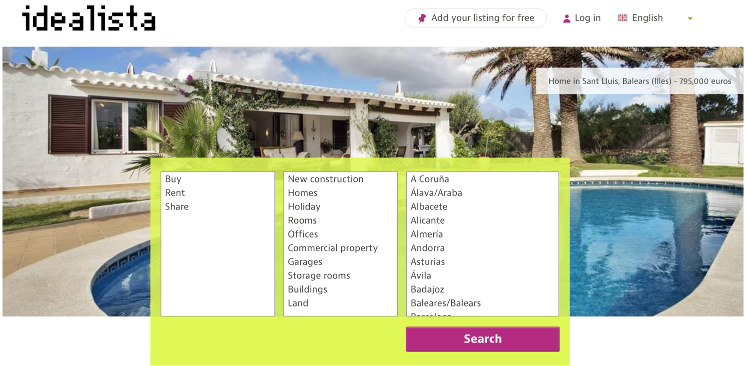
Back

Identification Illustration



Back

The Online Platform



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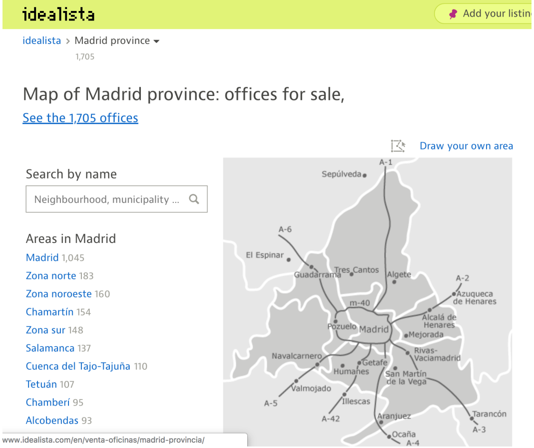

Back

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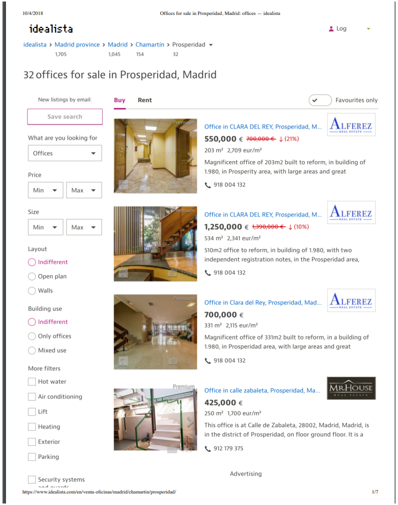

10/4/2018 Offices for sale in Prosperidad, Madrid: offices — idealista

idealista Log

idealista > Madrid province > Madrid > Chamartín > Prosperidad
1,705 1,045 154 32

32 offices for sale in Prosperidad, Madrid

New listings by email:
Save search

What are you looking for
Offices

Price
Min Max

Size
Min Max

Layout
- Indifferent
- Open plan
- Walls

Building use
- Indifferent
- Only offices
- Mixed use

More filters
- Hot water
- Air conditioning
- Lift
- Heating
- Exterior
- Parking
- Security systems and guards

Buy Rent Favourites only

Office in CLARA DEL REY, Prosperidad, M...
550,000 € ~~700,000 €~~ ↓ (21%)
203 m² 2,709 eur/m²
Magnificent office of 203m2 built to reform, in building of 1.980, in Prosperity area, with large areas and great
918 004 132

Office in CLARA DEL REY, Prosperidad, M...
1,250,000 € ~~1,390,000 €~~ ↓ (10%)
534 m² 2,341 eur/m²
510m2 office to reform, in building of 1.980, with two independent registration notes, in the Prosperidad area,
918 004 132

Office in Clara del Rey, Prosperidad, Mad...
700,000 €
331 m² 2,115 eur/m²
Magnificent office of 331m2 built to reform, in a building of 1.980, in Prosperidad area, with large areas and great
918 004 132

Office in calle zabaleta, Prosperidad, Ma...
425,000 €
250 m² 1,700 eur/m²
This office is at Calle de Zabaleta, 28002, Madrid, Madrid, is in the district of Prosperidad, on floor ground floor. It is a
912 179 375

Advertising

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Back

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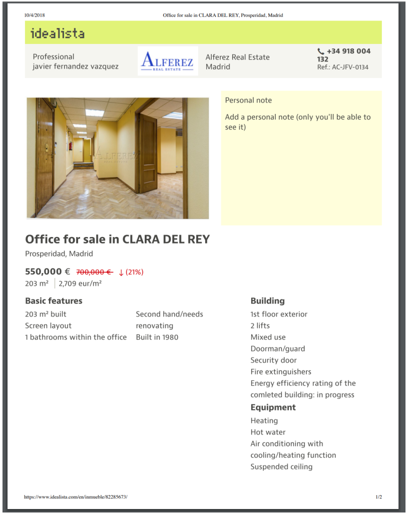

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Personal note

Add a personal note (only you'll be able to see it)

Office for sale in CLARA DEL REY

Prosperidad, Madrid

550,000 € ~~700,000 €~~ ↓ (21%)

203 m² | 2,709 eur/m²

Basic features

203 m² built
Screen layout
1 bathrooms within the office
Second hand/needs renovating
Built in 1980

Building

1st floor exterior
2 lifts
Mixed use
Doorman/guard
Security door
Fire extinguishers
Energy efficiency rating of the comleted building: in progress

Equipment

Heating
Hot water
Air conditioning with cooling/heating function
Suspended ceiling

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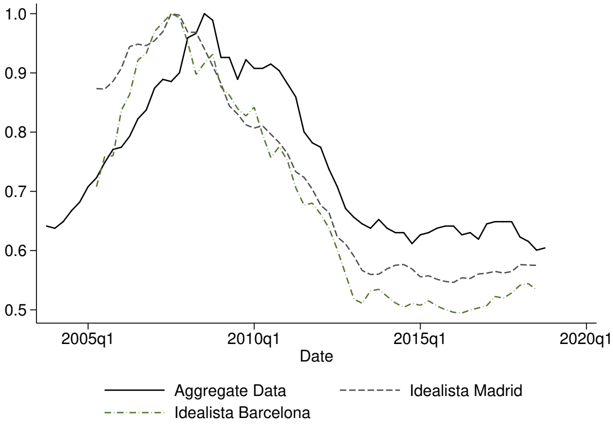
Back

Benchmarking the Data: Price Indices



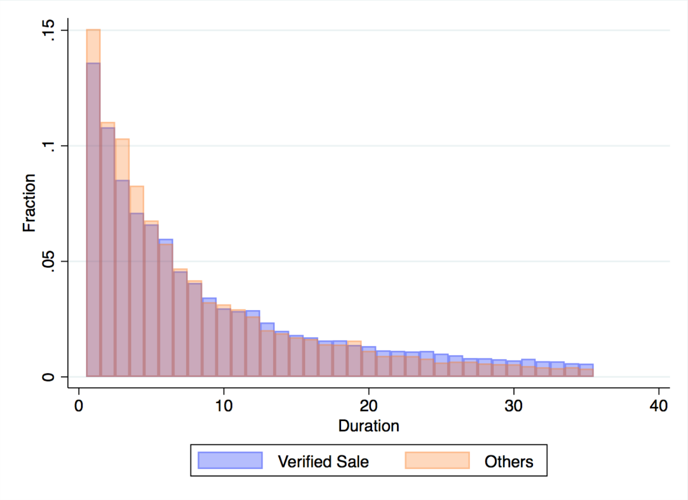
Back

Descriptive Statistics

	Mean	Std. dev.
Price	162.27	131.71
Duration	10.47	11.21
Construction Date	1987.63	19.50
Area	3008.89	4619.22
New	0.05	0.22
Needs Restoration	0.14	0.35
Good Condition	0.81	0.40
Rooms	2.31	2.99
Restrooms	1.21	1.54
Heating	0.27	0.45
AC	0.64	0.48
E-Mails	2.75	2.12
Views	799.91	1273.95
Clicks	44.28	59.08
Number of Obs.	4.4e+05	4.4e+05

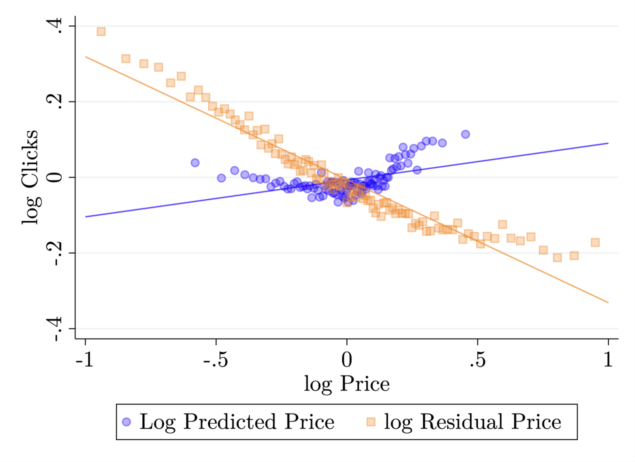
Back

Distribution of Duration



Back

Prices and Clicks



Back

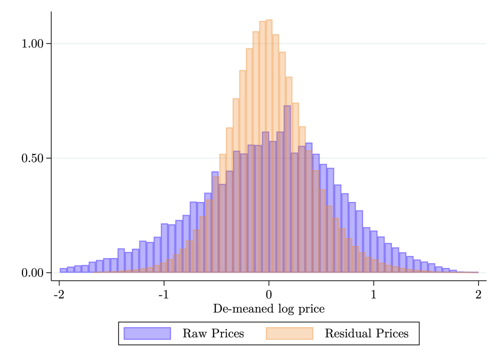
Prices and Clicks

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.027***		-0.226***	
	(0.008)		(0.006)	
log Predicted Price		0.248***		0.116***
		(0.012)		(0.011)
log Residual Price		-0.270***		-0.272***
		(0.007)		(0.007)
Constant	3.299***	2.265***	4.509***	2.889***
	(0.035)	(0.056)	(0.029)	(0.051)
Observations	398260	387213	386163	386163
R^2	0.000	0.035	0.421	0.425
Fixed Effects	No	No	Yes	Yes

Back

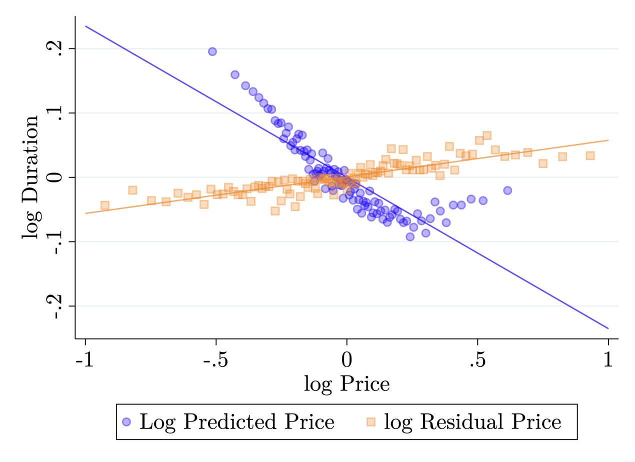
Price Variation Explained by Listed Characteristics: Rental Market

	St. Dev.	R^2
Raw data	0.77	0.00
Year	0.75	0.04
Year $\times$ Location	0.55	0.48
. . . $\times$ Type	0.55	0.48
. . . $+$ Area	0.51	0.55
. . . $+$ Age	0.51	0.56
Benchmark	0.48	0.60



Back

Prices and Duration: Rental Market



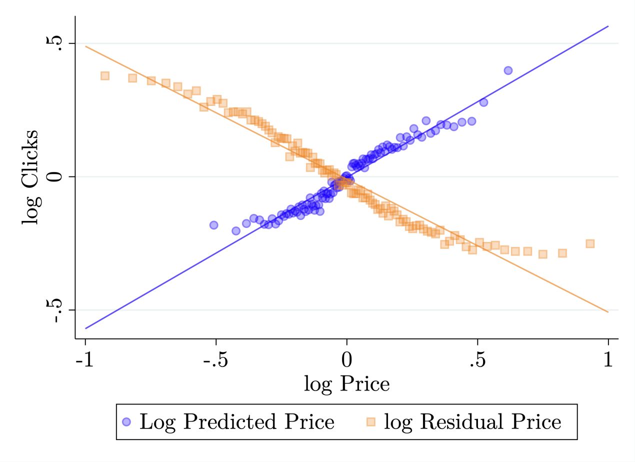
Back

Prices and Duration: Rental Market

	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	-0.092***		-0.012***	
	(0.004)		(0.003)	
log Predicted Price		-0.175***		-0.228***
		(0.006)		(0.007)
log Residual Price		0.032***		0.032***
		(0.004)		(0.004)
Constant	1.848***	1.838***	1.857***	1.832***
	(0.004)	(0.004)	(0.000)	(0.001)
Observations	696874	680553	680553	680553
R^2	0.007	0.014	0.182	0.186
Fixed Effects	No	No	Yes	Yes

Back

Prices and Clicks: Rental Market



Back

Prices and Clicks: Rental Market

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.176***		-0.195***	
	(0.011)		(0.007)	
log Predicted Price		0.528***		0.565***
		(0.017)		(0.010)
log Residual Price		-0.351***		-0.353***
		(0.009)		(0.009)
Constant	3.883***	3.950***	3.829***	3.957***
	(0.013)	(0.013)	(0.001)	(0.002)
Observations	578653	567847	566704	566704
R^2	0.013	0.088	0.437	0.460
Fixed Effects	No	No	Yes	Yes

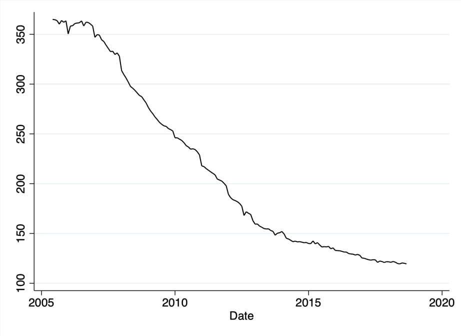
Back

Frequency and Size of Price Changes

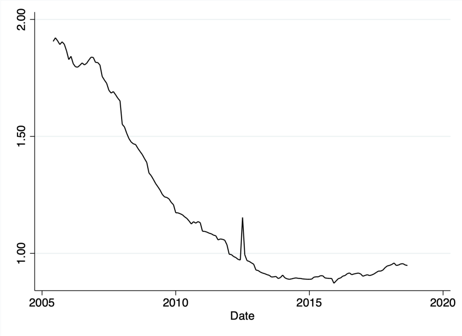
Statistic	Value
Frequency of Price Changes	0.09
Frequency of Price Increases	0.03
Frequency of Price Decreases	0.06
Absolute Size of Price Changes	0.13
Absolute Size of Price Increases	0.15
Absolute Size of Price Decreases	0.12

Back

Evolution of Prices over Time



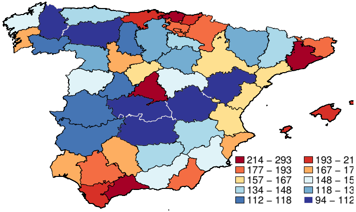
(A) Avg. Price for Sale



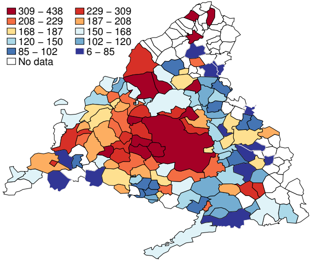
(B) Avg. Price for Rent

▸ Back

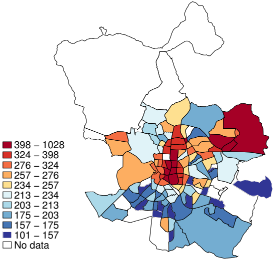
Geographical Dispersion of Prices



(A) Spain



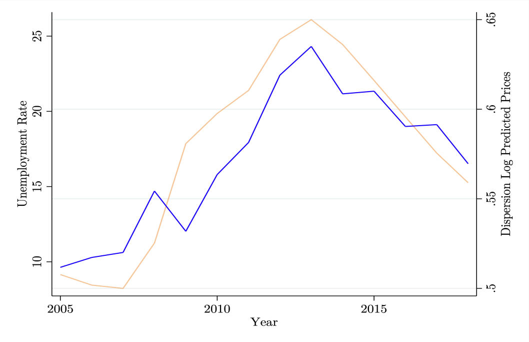
(B) Province of Madrid

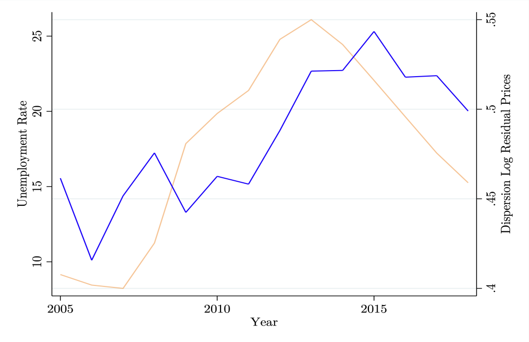


(C) City of Madrid

Back

Price Dispersion Over Time





Back

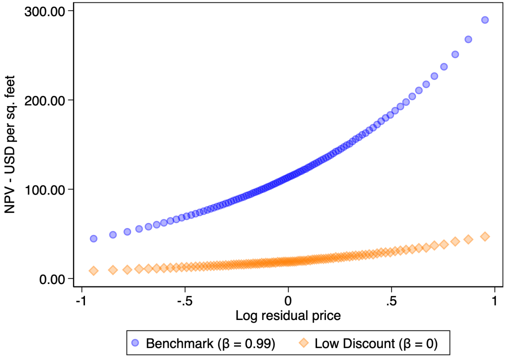
Residual Prices and Expected Net Present Revenue

Back

Expected net present revenue from choosing residual price ε_{it}:

$$\mathcal{R}(\varepsilon_{it}, \beta) \equiv \sum_{t=0}^{\infty} \beta^t (1 - p(\varepsilon_{it}))^t p(\varepsilon_{it}) e^{\varepsilon_{it}} = \frac{p(\varepsilon_{it}) e^{\varepsilon_{it}}}{(1 - \beta(1 - p(\varepsilon_{it})))},$$

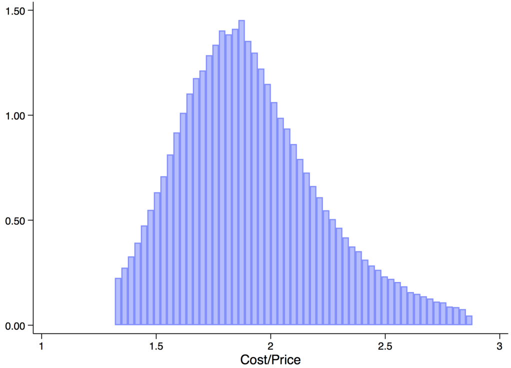
$p(\varepsilon_{it})$: selling probability implied by the empirical relationship



Residual Prices and Holding Costs ▸ Back

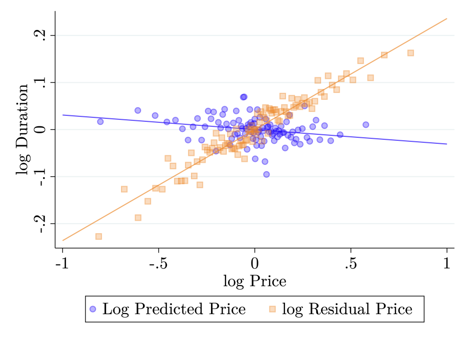
Cost that would make sellers indifferent between choosing that residual price and the highest observed residual price:

$$\frac{p(\varepsilon_{it})e^{\varepsilon_{it}} - \xi(\varepsilon_{it},\beta)(1-p(\varepsilon_{it}))}{1-\beta(1-p(\varepsilon_{it}))} = \frac{p(\bar{\varepsilon}_{it})e^{\bar{\varepsilon}_{it}} - \xi(\varepsilon_{it},\beta)(1-p(\bar{\varepsilon}_{it}))}{1-\beta(1-p(\bar{\varepsilon}_{it}))}$$

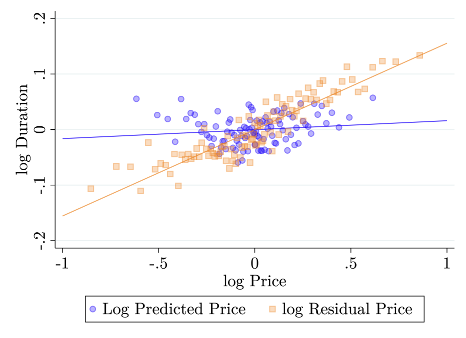


Relationship between Duration and Prices by Total Price

Back



(A) Below Median Total Price



(B) Above Median Total Price

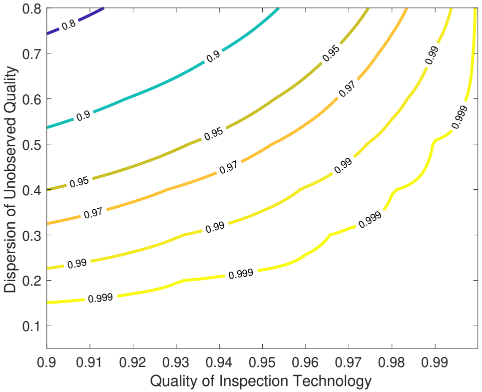
Regression Moments

	Data $\log$ Duration	Model $\log$ Duration
$\log$ Predicted Price	-0.025	-0.086
$\log$ Residual Price	0.148	0.148
Constant	2.14	1.99

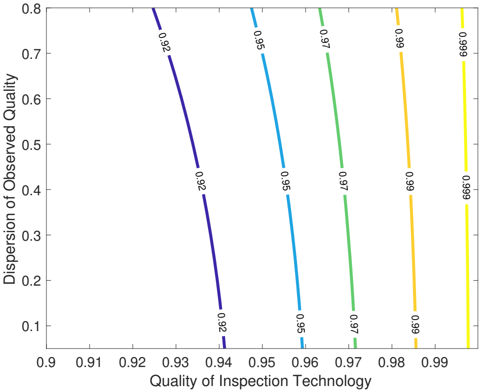
- The model is consistent with
  1. Average duration
  2. Elasticity of duration w.r.t. residual prices
  3. **Elasticity of duration w.r.t. predicted prices**

Back

Output and $\{\psi, \sigma_a, \sigma_\omega\}$



(A) $\{\psi, \sigma_a\}$



(B) $\{\psi, \sigma_\omega\}$

▸ Back parameterization ▸ Back crisis

Robustness to Alternative Parameterizations

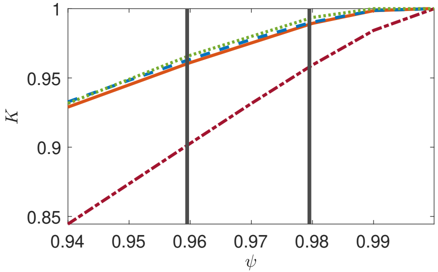
Calibration	$Y/Y^{FI}-1$	$\mathcal{K}/\mathcal{K}^{FI}-1$	$K/K^{FI}-1$	$u-u^{FI}$
Baseline	-1.22%	-2.55%	-1.12%	0.83%
TIOLI (ϕ)	-1.44%	-2.93%	-1.46%	0.72%
Higher φ	-1.5%	-3.15%	-1.24%	1.09%
Inelastic Labor Supply (ξ)	-1.22%	-2.55%	-1.12%	0.83%
Incomplete Observed Charact.	-1.17%	-2.46%	-1.06%	0.78%

- ϕ, ξ do not significantly alter the aggregate effects
- Higher φ increases separations, amplifying the macro effects
- φ calibrated to exit rate $+$ K reallocation among public firms (Eisfeldt and Shi, 2018)

Back

Macro Channels: Investment

$$
\begin{aligned}
Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha} \\
&= \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega \in \boldsymbol{\omega}} \sum_{a \in \mathcal{A}} \boldsymbol{K_t(\omega, a)} \right] \left[\mathbb{E}\left(\omega a\right) \left(1 - \mathbb{E}\left(u_t(\omega, a)\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega, a)\right) \right] \right)^{\alpha}
\end{aligned}
$$



Investment channel:

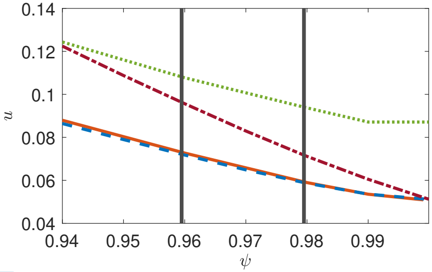
Higher information asymmetries

$\Rightarrow$ lower returns to producing capital goods

Back

Macro Channels: Capital Unemployment

$$
\begin{aligned}
Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha} \\
&= \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega \in \omega} \sum_{a \in \mathcal{A}} K_t(\omega, a) \right] \left[\mathbb{E}\left(\omega a\right) \left(1 - \mathbb{E}\left(\boldsymbol{u_t(\omega, a)}\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega, a)\right) \right] \right)^{\alpha}
\end{aligned}
$$

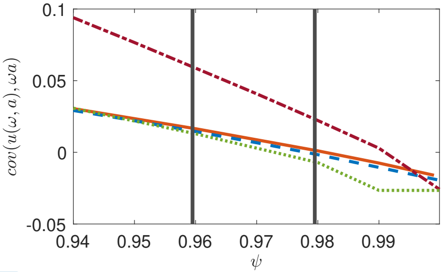


Capital-unemployment channel:
Higher information asymmetries
$\Rightarrow$ lower trading probabilities,
longer unemployment durations

Back

Macro Channels: Quality of Employed Capital

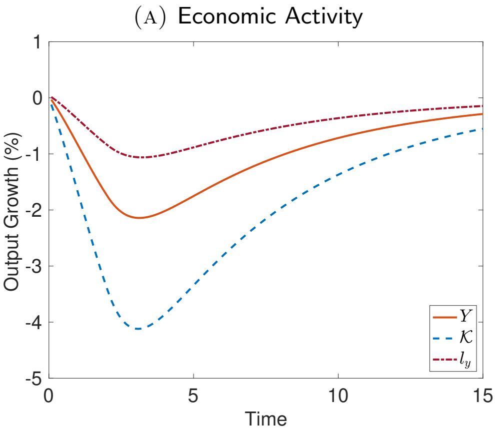
$$\begin{aligned} Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha} \\ &= \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega \in \omega} \sum_{a \in \mathcal{A}} K_t(\omega, a) \right] \left[\mathbb{E}\left(\omega a\right) \left(1 - \mathbb{E}\left(u_t(\omega, a)\right)\right) - \boldsymbol{\mathbb{C}ov\left(\omega a, u_t(\omega, a)\right)} \right] \right)^{\alpha} \end{aligned}$$

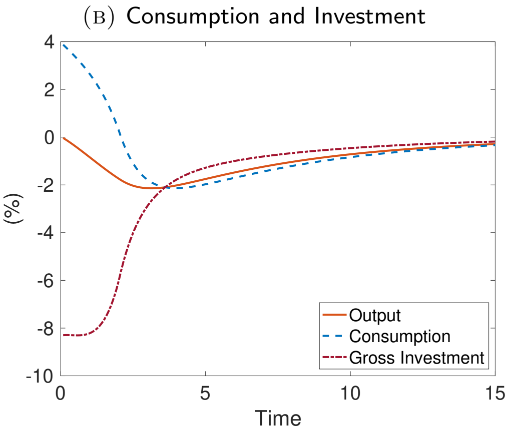


Employed-capital quality channel:
Higher information asymmetries
$\Rightarrow$ relative decrease in trading probabilities of high-quality capital

▸ Back

Macroeconomic Reponses to Changes in Information Technologies





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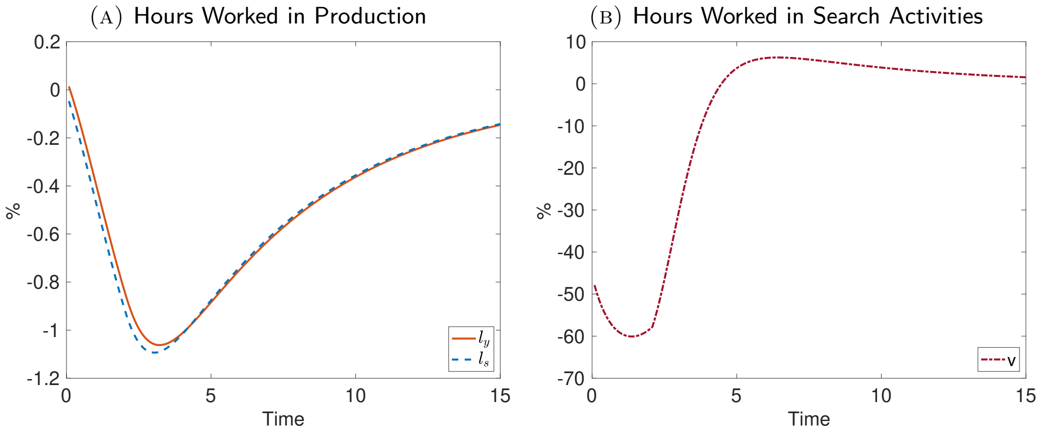
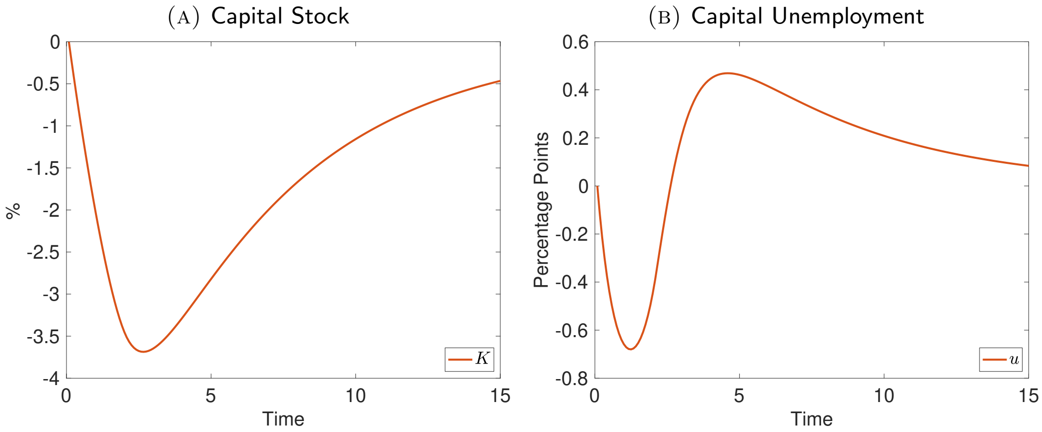
Macroeconomic Reponses to Changes in Information Technologies



Back

Macroeconomic Reponses to Changes in Information Technologies



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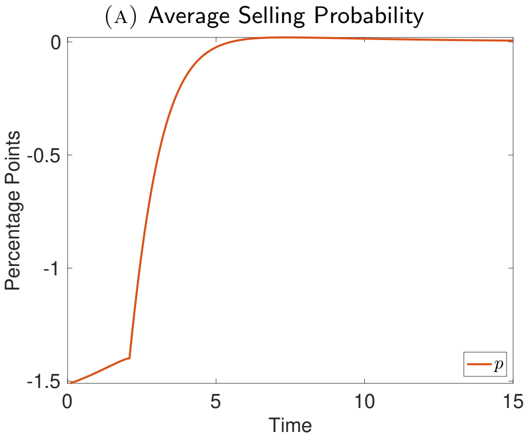

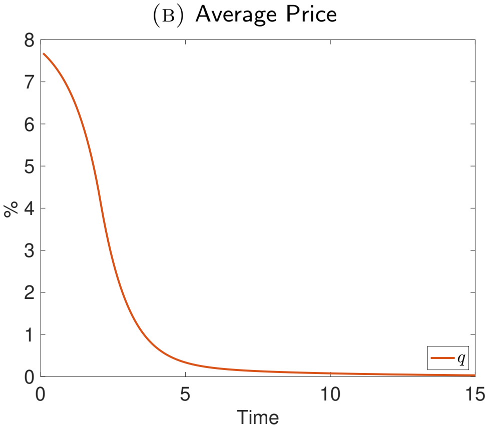

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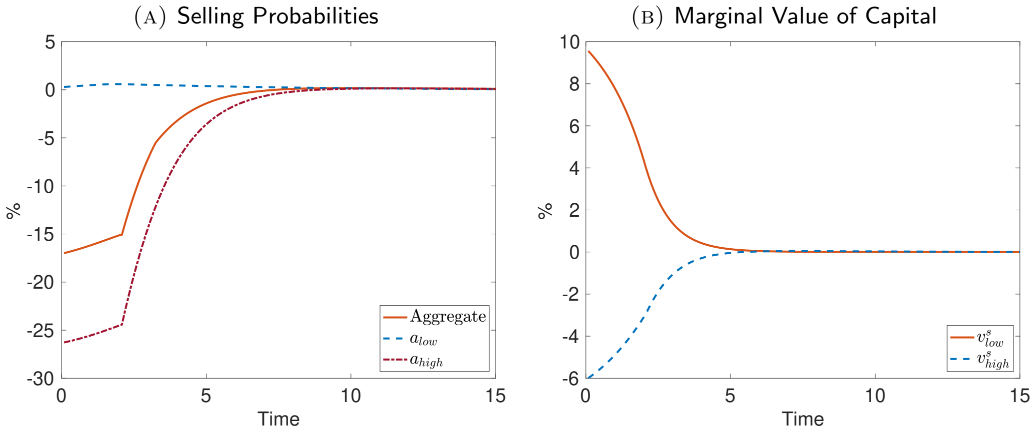


Back

Liquidity and AI Effects on $\nu^{\mathrm{s}}(\omega, a, \mathbf{k})$ ▸ Back

- Frictionless benchmark: $\nu^{\mathrm{s}}(\omega, a, \mathbf{k}) = \frac{(Z-\delta)\omega a}{1-\Lambda}$
- Search frictions + AI:

$$\nu^{\mathrm{s}}(\omega, a, \mathbf{k}) = \frac{Z\omega a}{(1-\Lambda)} \left[1 - \underbrace{\frac{(1-\Lambda(1-\varphi))\,(1-p(\omega,a))}{(1-(1-p(\omega,a))\,\Lambda(1-\varphi))}}_{\text{Illiquidity Discount}} \right]$$

$$- \underbrace{\frac{\chi\theta(\omega,a)}{(1-\Lambda)} \frac{[1-\Lambda(1-\varphi)]}{(1-(1-p(\omega,a))\,\Lambda(1-\varphi))}}_{\text{Search costs}}$$

$$- \frac{\delta\omega a}{(1-\Lambda)}$$

Ottonello Winberry (2022)

Motivation

- Investment is the most cyclical component of aggregate demand
- Investment Channel of Monetary Policy
- What determines the strength of this effect?
- Underlying notion of state-dependence

Two Possibilities

- Two possibilities on which firms respond more:
  - More constrained firms: Monetary policy expansions ease financial frictions. More constrained firms respond by more. Financial accelerator story
  - Less constrained firms: More constrained firms have steeper marginal cost curves, so they react by less to the same aggregate demand shock
- Ultimately an empirical question

Specification

- Basic specification

$$\Delta \log k_{j,t+1} = \alpha_j = \alpha_j + \alpha_{st} + \beta(x_{jt-1} - \mathbb{E}_j(x_{jt}))\epsilon_t^m + \Gamma' Z_{jt-1} + e_{jt}$$

- Where ϵ^m is determined using HFI

$$\epsilon_t^m = \tau(t) \times (ffr_{t+\Delta_+} - ffr_{t-\Delta_-})$$

- Size of the window: -15 to +45 minutes
- Thoughts? Identifying assumption?
- Stronger or weaker assumption than time series variation using HFI shocks?

Basic Result

TABLE III

HETEROGENEOUS RESPONSES OF INVESTMENT TO MONETARY POLICY[a]

	(1)	(2)	(3)	(4)	(5)
leverage × ffr shock	−0.69	−0.57		−0.26	−0.14
	(0.29)	(0.27)		(0.35)	(0.58)
dd × ffr shock			1.14	1.01	1.16
			(0.41)	(0.40)	(0.47)
ffr shock					2.14
					(0.61)
Observations	219,402	219,402	151,027	151,027	119,750
R^2	0.113	0.124	0.141	0.142	0.151
Firm controls	no	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	no
Time clustering	yes	yes	yes	yes	yes

[a]Results from estimating $\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta(x_{jt-1} - \mathbb{E}_j[x_{jt}])\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + e_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, $x_{jt} \in \{\ell_{jt}, \mathrm{dd}_{jt}\}$ is leverage or distance to default, $\mathbb{E}_j[x_{jt}]$ is the average of x_{jt} for firm j in the sample, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing x_{jt-1}, sales growth, size, current assets as a share of total assets, an indicator for fiscal quarter, and the interaction of demeaned financial position with lagged GDP growth. Standard errors are two-way clustered by firms and quarter. We have normalized the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive shock corresponds to a decrease in interest rates. We have standardized $(\ell_{jt} - \mathbb{E}[\ell_{jt}])$ and $(\mathrm{dd}_{jt} - \mathbb{E}[\mathrm{dd}_{jt}])$ over the entire sample. Column (5) removes the sector-quarter fixed effect α_{st} and estimates $\Delta \log k_{jt+1} = \alpha_j + \alpha_{sq} + \gamma \varepsilon_t^{\mathrm{m}} + \beta(x_{jt-1} - \mathbb{E}_j[x_{jt}])\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}_1' Z_{jt-1} + \mathbf{\Gamma}_2' Y_{t-1} + e_{jt}$, where Y_t is a vector with four lags of GDP growth, the inflation rate, and the unemployment rate.

Dynamic Response

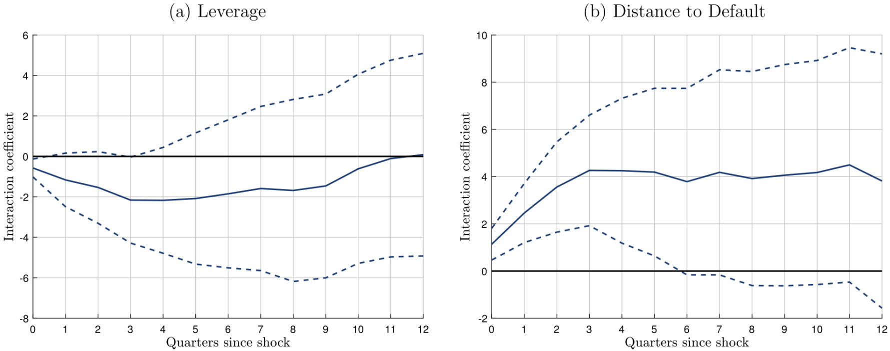


FIGURE 1.—Dynamics of differential response to monetary shocks. Notes: dynamics of the interaction coefficient between financial positions and monetary shocks over time. Reports the coefficient β_h over quarters h from $\log k_{jt+h} - \log k_{jt} = \alpha_{jh} + \alpha_{sth} + \beta_h (x_{jt-1} - \mathbb{E}_j[x_{jt}])\varepsilon_t^{\mathrm{m}} + \Gamma'_h Z_{jt-1} + e_{jth}$, where all variables are defined in the notes for Table III. Dashed lines report 90% error bands.

Model - Financial structure

- No aggregate uncertainty
- MIT shock later on
- Firms can borrow in defaultable debt
  - This is the optimal contract of Costly State Verification models (Townsend 1979)
  - Backbone of financial accelerator models (BGG 1999)
- Retain earnings, not issue equity (think of infinite costs of equity issuance)
- Without financial frictions need to keep track of only net worth, not k and b separately. Not possible with financial frictions
- Economics: External finance premium/One unit of external finance is more costly

Capital Producers

- Capital producer sector
- Relative price of investment q
- q-theory FOC

$$q_t = \frac{1}{\Psi'(I_t/K_t)}$$

Retailers - NK firms

- Set prices subject to Rotemberg (1982) frictions
- Relative price of retail goods p
- Gives rise to a standard NK Phillips Curve

Lenders

- Intermediary, gets funds from the household, lends to firms
- CSV block. Upon default (or verification) the lender gets α fraction of the market value of the firm stock
- Price contracts at $\mathcal{Q}(z, k', b')$ to get zero profits (free entry in the background)

Production Firms

- DRS
- exit shocks
- Fixed costs of operation
- Need a source of variation that suddenly brings firms closer to default
- Capital quality shock *We view capital quality shocks as capturing unmodeled forces which reduce the value of the firm's capital, such as frictions in the resale market, breakdown of machinery, or obsolescence.*
- Effective units of capital ωk
- Firms decide whether to default or not

When to default

- A firm receives a capital-quality shock ω
- The firm has some debt b and the value of its capital goes down ωk
- Its net worth $n = \max_l p_t z(\omega k)^{\theta} l^{\nu} - w_t l + q_t(1-\delta)\omega k - b\frac{1}{\Pi_t} - \xi$ goes down
- $\exists\, \underline{n}$ such that the firm cannot respect the non-negativity on equity issuance

$$n - q_t k' + \mathcal{Q}(z, k', b')b' \geq 0$$

Main Mechanism

Impact on Decision Rules. The optimal choice of investment k' and borrowing b' satisfy the following two conditions:

$$q_t k' = n + \frac{1}{R_t(z, k', b')} b', \tag{9}$$

$$\left(q_t - \varepsilon_{Q,k'}(z, k', b') \frac{Q_t(z, k', b') b'}{k'}\right) \frac{R_t^{\text{sp}}(z, k', b')}{1 - \varepsilon_{R,b'}(z, k', b')}$$
$$= \frac{1}{R_t} \mathbb{E}_t[\text{MRPK}_{t+1}(z', k')]$$

$$+ \frac{1}{R_t} \frac{\mathbb{C}ov_t(\text{MRPK}_{t+1}(z', \omega' k'), 1 + \lambda_{t+1}(z', \hat{n}_{t+1}(z', \omega', k', b')))}{\mathbb{E}_t[1 + \lambda_{t+1}(z', \hat{n}_{t+1}(z', \omega', k', b'))]}$$
$$- \frac{1}{R_t} \mathbb{E}_{\omega'}[v_{t+1}^0(\omega', k', b') g_z(\underline{z}(\omega', k', b') | z) \hat{\underline{z}}_{t+1}(\omega', k', b')], \tag{10}$$

Intuition Main Mechanism

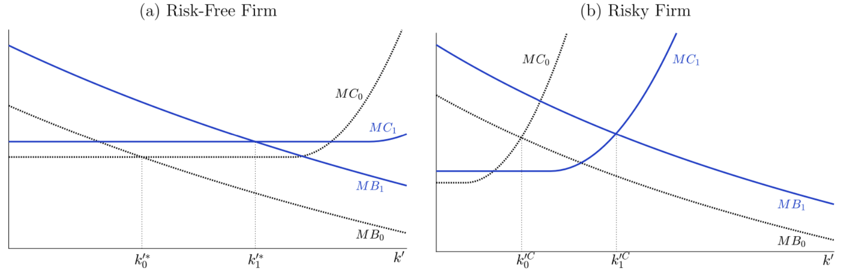


FIGURE 2.—Response to monetary policy for risk-free and risky firms. Notes: Marginal benefit and marginal cost curves as a function of capital investment k' for firms with same productivity. Left panel is for a firm with high initial net worth and right panel is for a firm with low initial net worth. Marginal cost curve is the left-hand side of (10) and marginal benefit the right-hand side of (10). Dashed black lines plot the curves before an expansionary monetary policy shock, and solid blue lines plot the curves after the shock.

Intuition Main Mechanism

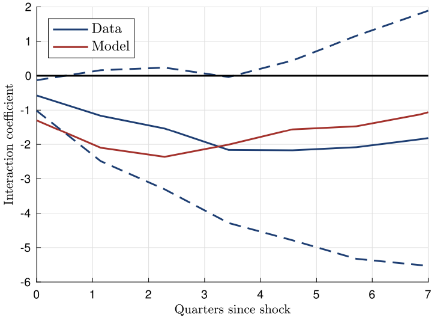


FIGURE 5.—Dynamics of differential responses, model vs. data. Notes: dynamics of the interaction coefficient between leverage and monetary shocks. Reports the coefficient β_h over quarters h from $\log k_{jt+h} - \log k_{jt} = \alpha_{jh} + \alpha_{sth} + \beta_h(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t^{\mathrm{m}} + \Gamma_h' Z_{jt-1} + \Gamma_{2h}'(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])Y_{t-1} + e_{jt}$, where all table notes from Columns (1) and (2) of Table VII apply. Dashed lines report 90% error bands.

Intuition Main Mechanism

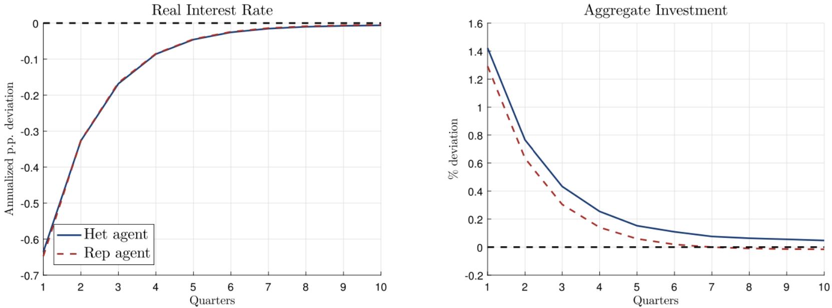


FIGURE 8.—Aggregate impulse responses in full model vs. rep firm model. Notes: "Het agent" refers to calibrated heterogeneous firm model from the main text. "Rep agent" refers to a version of the model in which the heterogeneous production sector is replaced by a representative firm with the same production function and no financial frictions.

Intuition Main Mechanism

TABLE VIII

AGGREGATE RESPONSE DEPENDS ON INITIAL DISTRIBUTION[a]

(everything rel. to steady state)	Bad distribution	Medium distribution
Avg. capital response	0.67	0.84
Avg. net worth	0.48	0.75
Frac. risky constrained	1.37	1.17

[a] Dependence of aggregate response on initial distribution. We compute the change in aggregate capital for different initial distributions as described in the main text. "Bad distribution" corresponds to $\widehat{\omega} = 1$ and "Medium distribution" corresponds to $\widehat{\omega} = 0.5$.

Catherine, Chaney, Huang, Sraer, Thesmar (2021)

Motivation

- Cross-sectional effects of having more collateral on firm-investment
- Broad literature of firm excess sensitivity
- What are the TFP and output effects of collateral constraints?

Cross-Sectional Elasticity

$$\frac{i_{it}}{k_{it}} = a + \beta \frac{REValue_{it}}{k_{i,t-1}} + Offprice_{it} + \Gamma' X_{it} + \nu_{it}$$

- Chaney, Sraer, Thesmar (2012) AER paper all about this
- Exogenous shock to real estate value, increases the value of collateral, which increases debt capacity and investment for financially-constraint firms

Production

$$q_{it} = e^{z_{it}} \left(k_{it}^{\alpha} l_{it}^{1-\alpha} \right)$$

- Firm-level productivity AR(1)
- Downward-sloping demand curves

$$q_{it} = Q p_{it}^{-\phi}$$

- Curvature in the revenues minus wage bill

$$\pi(z_{it}, k_{it}) = bQ^{1-\theta} w^{-(1-\alpha)\theta/\alpha} e^{z_{it}\theta/\alpha} k_{it}^{\theta},$$

- For $\theta = \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)}$
- Why is it important?

Capital adjustment frictions

- Law of motion of capital stock

$$k_{it+1} = k_{it} + i_{it} - \delta k_{it}$$

- Convex costs of adjustment

$$\frac{c}{2}\left(\frac{i}{k}\right)^2 k$$

Financial Frictions

- interest rate spread on debt m
- Cost of issuing equity. If cash-flows are x, post-issuance

$$G(x) = x(1 + e1_{x<0})$$

- Collateral constraint

$$(1 + r)d_{it+1} \leq s((1 - \delta)k_{it+1} + \mathbb{E}(p_{t+1}|p_t) \times h)$$

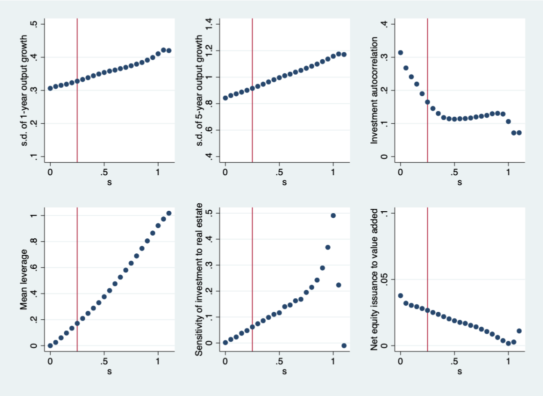
- s parameterize loose or tight the constraint is
- h is the amount of real estate (common across firms)
- Friction comes from limited enforcement
- h is a parameter

Estimation

- Autocorrelation of investment rates to infer the adjustment cost c
- This is usual in investment models (see Cooper and Haltiwanger, 2006)
- Use the cross-sectional elasticity β in an SMM to estimate s
- Use data on equity issuances to estimate e

Capital or Financial frictions?

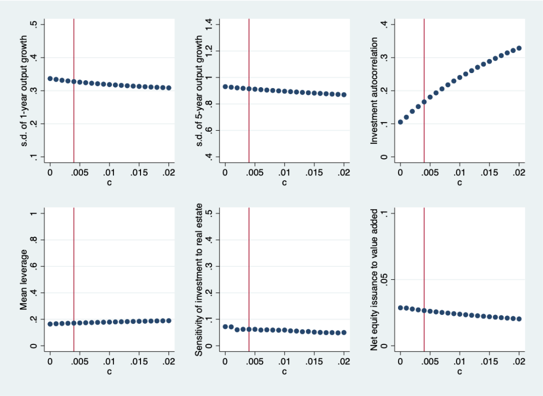
Figure E.1: Sensitivity of moments to pledgeability s



Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary s from 0 to 1. For each value of s that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of s.

Capital or Financial frictions?

Figure E.2: Sensitivity of moments to adjustment costs c



Note: In this figure, we set all estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column 3, Panel A in Table 2. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary c from 0 to 0.02. For each value of c that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel

GE Block

- Aggregate production Q is CES
- Resource constraint

$$Q_t = C_t + I_t + AC_t$$

- Quasi linear utility

$$L_t^s = \bar{L} w_t^{\epsilon}$$

Results

Table 3: **Aggregate Effects of Collateral Constraints**

Specification:	(1) Model 1 $c = 0, e = +\infty$	(2) Model 2 $c > 0, e = +\infty$	(3) Model 3 $c > 0, e > 0$
Panel A: *General equilibrium results*			
Δ log(TFP)	0.031	0.027	0.014
Δ log(Output)	0.151	0.120	0.071
Δ log(wage)	0.101	0.080	0.048
Δ log(L)	0.051	0.040	0.024
Δ log(K)	0.282	0.215	0.137
Panel B: *Partial equilibrium results, holding Q fixed only*			
Δ log(TFP)	0.012	0.012	0.005
Δ log(Output)	0.110	0.088	0.052
Δ log(wage)	0.073	0.059	0.035
Δ log(L)	0.037	0.029	0.017
Δ log(K)	0.240	0.185	0.117
Panel C: *Partial equilibrium results, holding (Q, w) fixed*			
Δ log(TFP)	-0.040	-0.029	-0.020
Δ log(Output)	0.400	0.320	0.189
Δ log(wage)	-	-	-
Δ log(L)	0.400	0.320	0.189
Δ log(K)	0.531	0.417	0.254

Note: This table reports the results of the counterfactual analysis for different SMM parameter estimates. The general equilibrium analysis is described in Section 4 and reported in Panel A. Columns (1)-(3) correspond to the three different models described in Columns (1)-(3) of Table 2: Column (1) assumes no adjustment cost ($c = 0$) and infinite cost of equity issuance ($e = +\infty$). Column (2) allows for adjustment cost but still assumes infinite cost of equity issuance. Column (3) also allows for finite cost of equity issues. Panel B implements the same methodology, except that it holds the aggregate demand shifter Q constant, but the wage w clears the labor market. Panel C holds both the aggregate demand shifter Q and wage w constant. Results in both panels are shown as log deviations from the constrained estimated model to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – reported in the same column, Table 2, panel A – but do not face a constraint on equity issuance ($e = 0$). In this unconstrained benchmark, investment reaches first best, but firms still benefit from the debt tax shield. *Reading*: In column 1 (no adjustment cost, no equity issuance), the aggregate TFP loss compared to a benchmark without financing constraints is 3.1%.

Results

- The results depend a lot on the persistence of productivity ρ
- Why?

Mispecification

- Two alternatives to estimate the model
  - Estimate the structural parameters Θ to target (among others) β
  - Estimate the structural parameters Θ to target (among others) debt to capital ratios
- Which is better?
- Offer one metric: Effects of model mispecification
- Also: Effect of measurement error

Mispecification

- Idea: Complicate the model
  - Intangible capital
  - Mismeasured capital
  - Economic depreciation $\neq$ accounting depreciation
  - Secured debt
- Estimate the extended and restricted (benchmark) model with data generated by the extended model
- What is the effect on the counterfactuals of TFP and output of model mispecification

Mispecification

Table 6: Estimation Error and Distance from Correct Specification

Relative error in estimation of:	log TFP loss		log Output loss	
Misspecified SMM targets:	β	Leverage	β	Leverage
	(1)	(2)	(3)	(4)
Misspecification parameters:				
Intangible capital share ($\boldsymbol{I}$)	-.0056	-.41	-.0021	-.39
Unobserved physical capital share (U)	-.19	-.34	-.18	-.33
Price measurement error ($\boldsymbol{\sigma_u}$)	.12	-.0033	.11	-.0058
Unobserved debt capacity - need ($\boldsymbol{d_0}$)	.028	1.2	.041	1.2
Fixed unsecured debt ($\boldsymbol{\kappa}$)	.098	-.43	.075	-.42
Actual tax rate - 33% ($\boldsymbol{\tau} - 0.33$)	-.73	-.54	-.68	-.49
Constant	.063	.14	.065	.13
Observations	4,000	4,000	4,000	4,000
R^2	0.32	0.74	0.29	0.73

Note: We simulate datasets from 4,000 alternative models. Each alternative model correspond to the baseline model augmented in six different dimensions described in Section 5.3.3. Six "misspecification" parameters control the degree of departure from the baseline model along these dimensions: $\boldsymbol{\Theta} = (\boldsymbol{\mathcal{I}}, \boldsymbol{U}, \boldsymbol{\sigma_u}, \boldsymbol{d_0}, \boldsymbol{\kappa}, \boldsymbol{\tau})$. We estimate the baseline (misspecified) model on these 4,000 datasets using two separate approaches: one estimation targets leverage; another targets the reduced-form moment β. We then regress:

$$\frac{\hat{X}_i - X_i}{\frac{1}{N}\sum_j X_j} = a + b\frac{\boldsymbol{\mathcal{I}}_i}{\max_j \boldsymbol{\mathcal{I}}_j} + c\frac{\boldsymbol{U}_i}{\max_j \boldsymbol{U}_j} + d\frac{\boldsymbol{\sigma}_{\boldsymbol{u},i}}{\max_j \boldsymbol{\sigma}_{\boldsymbol{u},j}} + e\frac{\boldsymbol{d}_{0,i}}{\max_j \boldsymbol{d}_{0,j}} + f\frac{\boldsymbol{\kappa}_i}{\max_j \boldsymbol{\kappa}_j} + g\frac{\boldsymbol{\tau}_i - 0.33}{\max_j(\boldsymbol{\tau}_j - 0.33)} + \epsilon_i$$

where X stands for the estimated TFP/output losses and i index alternative models. Standard errors are omitted because they are irrelevant in this cross-section of simulations, but the number is large enough to ensure smooth, linear, relationships as shown in Appendix Figures E.7 and E.8. *Reading*: When the fraction of intangible capital increases from 0 to .5 (maximum misspecification), the misspecification bias on TFP losses estimated by targeting leverage increases from zero (correctly specified) to 41% of the average TFP loss in the cross-section.

Alternative measures

Table V

Average Leverage Ratios and β Using Alternative Definition

Source: Compustat. The sample corresponds to the sample of firms in Chaney, Sraer, and Thesmar (2012). We calculate the average leverage ratio and estimate β under specific sources of misspecification. We use the following Compustat items: at is total assets; dltt is total long-term debt; dlc is debt in current liability; che is cash and short-term investment; ppent is property, plant, and equipment; capx is capital expenditures; xrd is R&D expense; xsga is selling, general and administrative expenses; act is total current assets; and ap is account payables. k_{int} is intangible capital, and $k_{\text{int}}^{\text{off-bs}}$ is its off-balance-sheet counterpart, from Peters and Taylor (2017); lease corresponds to leased operating capital and is calculated following an approach similar to Rampini and Eisfeldt (2009). For each firm-year, we compute l_{it}, the ratio of lagged one-year rental commitments (mrc1) to the rental cost of assets, which we measure as depreciation (dp) plus 10% of total assets (at). We trim observations for which this ratio is above one or below zero, and set it to zero when mrc1 is missing. We then multiply this ratio by total assets (at) to estimate the value of operating capital and implicit debt, assuming leverage being one for operating capital. To calculate PV(lease), we start from the next five years of commitments (mtr1-5), spread expected commitment (mrtca) equally over these five years, and calculate the present value of these commitments at a 10% discount rate. K corresponds to the capital stock calculated using a perpetual inventory method. For each firm, we take PPE (ppent) in the first fiscal year post-1981, depreciate it every year at 6% as in Midrigan and Xu (2014), and increase it with capital expenditures (capx) and decrease it with sales of property (sppe). Firm-clustered s.e. are between parentheses.

	Definition	D	Assets	K	I	Leverage = D/Assets	β
1	Standard	dltt+dlc-che	at	ppent	capx	0.093 (0.007)	0.060 (0.007)
2	Intangible	dltt+dlc-che	at+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int}	capx + xrd+.3×xsga	0.072 (0.004)	0.083 (0.011)
3	Leasing 1	dltt+dlc-che+lease	at+lease	ppent+lease+lease-lease(t-1)	capx	0.202 (0.006)	0.065 (0.010)
4	Leasing 2	dltt+dlc-che+PV(lease)	at	ppent	capx	0.130 (0.008)	0.060 (0.007)
5	Account payables	dltt+dlc-che+ap	at	ppent+act	capx+act-act(t-1)	0.201 (0.008)	0.028 (0.007)
6	Real depreciation	dltt+dlc-che	at+K-ppent	K	capx	0.074 (0.006)	0.070 (0.012)
7	All adjustments	dltt+dlc-che +lease+ap	at+K-ppent +lease+$k_{\text{int}}^{\text{off-bs}}$	ppent+k_{int} +lease+act	capx+act-act(t-1) +lease-lease(t-1) +(1-τ)(xrd+.3×xsga)	0.184 (0.005)	0.037 (0.022)

Winberry (2021)

Neoclassical Firms very sensitive to changes in the Real Interest Rate

- Time is discrete time, each period is a year.
- Simplest determination of capital $\delta = 0$

$$AF_k = r$$

- Assume that $F(K, L) = K^{\alpha} L^{1-\alpha}$. Therefore:

$$1 + r_t = 1 + A_t \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

- Make a log-linear approximation. Hatted variables are log changes:

$$\hat{r}_t = \frac{r}{1+r} \left(\hat{a}_t - (1-\alpha)\hat{k}_t + (1-\alpha)\hat{l}_t \right)$$

- where $\hat{r}_t = \log \frac{1+r_t}{1+r}$

Neoclassical Firms very sensitive to changes in the Real Interest Rate

$$\hat{k}_t = -\hat{r}_t \left(\frac{1+r}{(1-\alpha)r} \right) + \frac{\hat{a}_t}{1-\alpha} + \hat{l}_t$$

- Assume an exogenous decrease of 1% in interest rates.
- Capital would have to increase 31.5%
- Including reasonable depreciation would change this number to 14%.
- Letting labor increase would further increase this number
- Assume 100% of GDP could be transformed to capital.
- Capital-output ratios are between 2 and 4 (depending on land and housing)
- To increase capital by 31%, it would take 61%-124% of GDP
- With δ: to increase capital by 14%, it would take 28-56% of GDP

Another way of seeing the same

- Another way of illustrating the same issue is to compute the semi-elasticity of investment to interest rates
- Imagine firms with DRS

$$y_j = z\epsilon_j k_j^{\alpha}$$

- Then

$$\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -\frac{1}{\delta}\frac{1}{1-\alpha}\left(\frac{1+r_t}{r_t+\delta}\right)$$

- As $\alpha \to 1$, the semi-elasticity becomes infinite
- Under $\alpha = 0.7$, $\delta = 0.025$ $r_t = 0.01$
- The semi-elasticity is equal to -3,847

Capital Adjustment Frictions

- Very large literature
- 70s: Abel (1979)
- 80s: Hayashi (1982)
- 90s: Doms and Dunne (1998), Caballero (1999), Caballero and Engel (1999)
- 2000s: Thomas (2000), Cooper and Haltiwanger (2006), Khan and Thomas (2003, 2008), Gourio Kashyap (2007)
- Just to name a few

Some Context

- Capital accumulation models tend to have adjustment costs
- One reason is what we saw before
- in CT: Without any costs, in a standard model investment functions are not well-defined
- Convex Adjustment costs. Two main results:
  - Investment is a function of q: The marginal value of one extra unit of capital

$$\frac{i_{jt}}{k_{jt}} = h(q_t)$$

  - Marginal (q) and average (Q) values of capital are equal, when some conditions apply

$$q_t = Q_t$$

- Very tractable problem. Block in medium-scale DSGE models

Some Context

Issue:

- Evidence of lumpiness of investment at the individual level
- Lumpy investment: Periods of inaction followed by spikes in investment
- Obviously convex adjustment costs do not get that
- Documented originally by Doms and Dunne (1998)
- The literature proposed fixed costs of adjustment as a possible answer
- Cooper and Haltiwanger (2006) interpret the microdata as exhibiting both convex and non-convex costs
- For the purpose of our class: Does micro-level frictions of capital adjustment matter in the aggregate?

Metric

- What does it mean that micro frictions “matter” for the aggregate
- Is the response to shocks the same in models with and without fixed costs
- One particular dimension receives interest: Pent-up demand
- Or in more technical jargon, state-dependence of the elasticity of investment to aggregate shocks
- Is the response of investment to a TFP shock higher or lower in a recession?
- RBC model: It’s the same
- Alternative: Pent-up demand, the elasticity depends on the distribution of capital imbalances
- At the start of the recovery firms have “excess capital”, so an additional shock may not trigger large adjustments

Early findings

- Response by Thomas (2002): No
- Micro level lumpiness is irrelevant
- Meaning: Models with and without lumpiness as observed in the data have the same aggregate dynamics

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$$\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -\frac{1}{1-\alpha}\frac{1}{\delta}\frac{1+r_t}{r_t+\delta}$$

- Under a reasonable calibration:
- $\alpha = 0.7$, $\delta = 0.025$, $r_t = 0.01$: $\frac{\partial i_{jt}/i_{jt}}{\partial r_t} = -3,847$
- r_t is an equilibrium outcome, so much depends on how r_t behaves.
- The standard model has very strong strategic substitutability
- That others do not adjust induces higher incentives to adjust
- Mediated by the response of the real interest rate to aggregate shocks

Generic Setting

Firms have a DRS production function

$$y = e^z e^a k^\alpha n^\gamma$$

a captures idiosyncratic productivity (iid across firms)

$$a_{it} = \rho_a a_{t-1} + \epsilon_{it}\sigma_a.$$

z captures aggregate productivity

$$z_t = \rho_z z_{t-1} + \xi_t \sigma_z.$$

Firms discount period τ future profits with the household stochastic discount factor $\Lambda_{t,t+\tau}$

Setting

$$V(k,a,\chi,\mathcal{S}) = \max_n \left[e^z e^a k^\alpha n^\gamma - w(\mathcal{S})n\right] + \max\left[V^n(k,a,\chi,\mathcal{S}), V^a(k,a,\chi,\mathcal{S}) - \chi w(\mathcal{S})\right]$$

The value function conditional on non-adjustment is given by:

$$V^n(k,a,\chi,\mathcal{S}) = \mathbb{E}(\Lambda(\mathcal{S},\mathcal{S}')V(k',a',\chi',\mathcal{S}')|a,\mathcal{S}),$$

subject to

$$k' = k(1-\delta)$$

The value function conditional on adjustment is given by:

$$V^a(k,a,\chi',\mathcal{S}) = \max_i -i - \phi\left(\frac{i}{k}\right)^2 k + \mathbb{E}((\Lambda(\mathcal{S},\mathcal{S}')V(k',a',\chi',\mathcal{S}')|a,\mathcal{S}),$$

subject to

$$k' = k(1-\delta) + i$$

.

Setting

In the background there is a representative household that supplies labor, and consumes.

- There is a labor supply function in the background
- The Stochastic Discount Factor will capture household preferences for consumption smoothing

Habits in Consumption

- Fix the dynamics of r by changing optimal consumption decisions

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \log\left(C_t - \chi \frac{N_t^{1+\xi}}{1+\xi} - X_t \right)$$

$$X_t = \lambda \hat{C}_t$$

$$\hat{C}_t = C_t - \chi \frac{N_t^{1+\xi}}{1+\xi}$$

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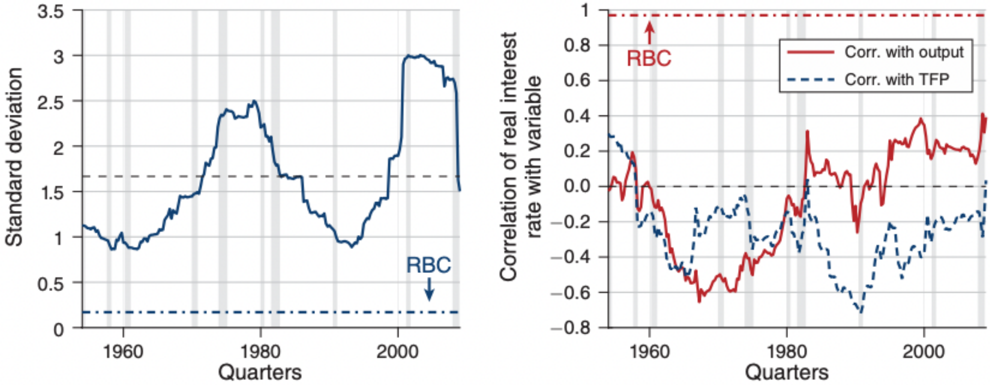

Figure 1. Stability of Cyclical Dynamics of Risk-Free Rate

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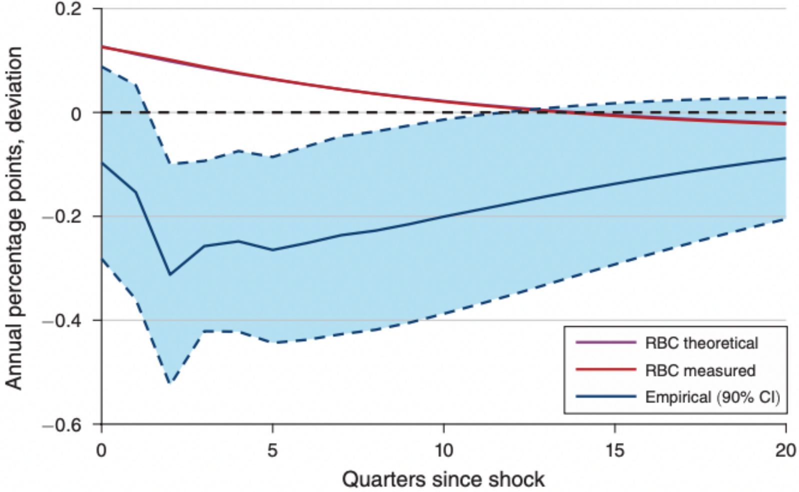

Figure 2. Impulse Response of the Real Interest Rate to TFP Shock

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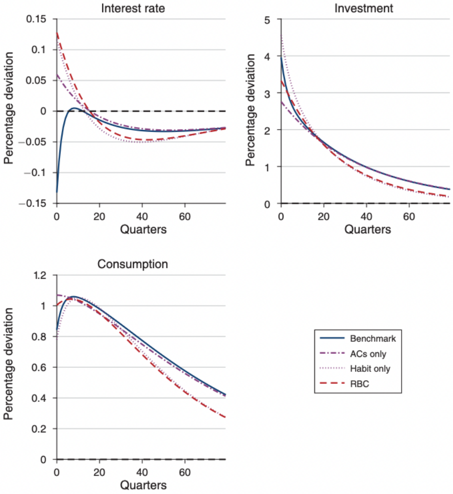

Figure 3. Identification of Habit Formation and Adjustment Costs

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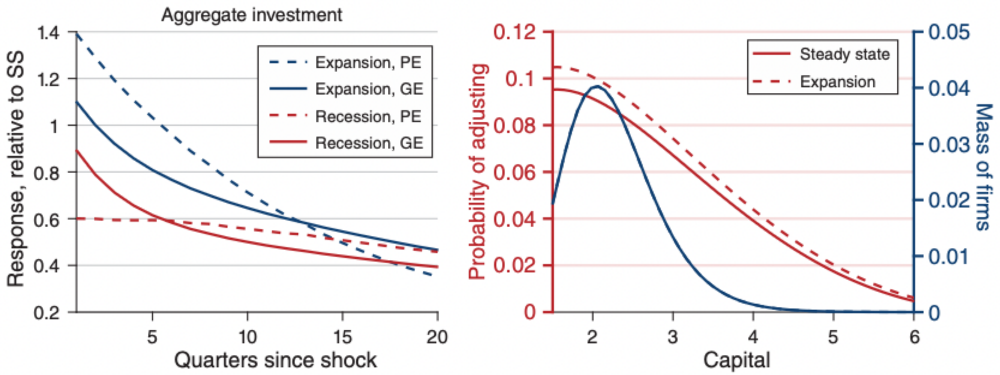

Figure 5. Procyclical Impulse Responses of Aggregate Investment

How to tell models apart?

- Koby and Wolf (2021) proposal: Use Zwick and Mahon (2017)
- Semi-elasticity of investment to bonus depreciation reforms
- Preview: Semi-Elasticity of investment in the data is consistent with Winberry (2021), not with Khan and Thomas (2008)