Lecture 3: Consumption Theory with Heterogeneity

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Outline

- Short refresher on Consumption
- Forward Guidance using McKay Nakamura Steinsson (2016)
- Auclert Ronglie Straub (2023 - JPE) (IKC)
- Auclert Ronglie Straub (2020 - AER RR) (Jumps and Humps)
- McKay Wieland (2021)
- Beraja Zorzi (2024)
- Berger, Milbradt, Tourre, Vavra (2021) - Will not have time
- McKay Wieland (2022) - Will not have time

First submission

- Assessment of the average submission
  - Quality of idea not very ambitious
  - Quality of exposition not sophisticated
  - Execution not polished
  - Conceptual flaws
- You can clearly do better, this was below the bar by a significant margin
- Caveat 1: maybe I missed something in some submission. Caveat 2: maybe some of you had a negative personal shock.
- Mention mechanisms I want to ensure are not at work
- Presentations will be in week 11. You are not prepared to present the projects yet.

The Euler equation

- Market clearing

$$x_t = y_t - y_t^n = \hat{c}_t = c_t - c_t^n$$

- In log-linear terms

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1})$$

- or

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1})$$

- Notice that consumption tomorrow enters with a coefficient of 1
- Iterate the Euler equation forward

$$x_t = \mathbb{E}_t x_T - \frac{1}{\gamma}\sum_{\tau=t}^{T}(i_\tau - \mathbb{E}_t \pi_{\tau+1})$$

Partial Equilibrium - PIH

- Assume r_t is known and set at r
- Problem of one agent. Two ingredients.
  - Feasibility: PV of c = PV of y. Intertemporal Budget Constraint
  - Optimality: Sequence of Euler equations
- Feasibility:

$$\sum_{k=0}^{\infty} \frac{c_{t+k}}{(1+r)^k} = \sum_{k=0}^{\infty} \frac{y_{t+k}}{(1+r)^k}$$

- Optimality. For every pair of periods $t+k$, $t+k+1$ with CRRA

$$c_{t+k+1} = c_{t+k}\left((1+r)\beta\right)^{\gamma}$$

- Replace the euler equations into the ITBC

$$c_t \sum_{j=0}^{\infty} \beta^{\gamma j}(1+r)^{(\gamma-1)j} = \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$$

Partial Equilibrium - PIH

- What we call the consumption function
- Give me a sequence of y's and a sequence of r's, I give you a sequence for c's back

$$c_t \sum_{j=0}^{\infty} \beta^{\gamma j}(1+r)^{(\gamma-1)j} = \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$$

- $\frac{dc_t}{dy_t} = 1 - \beta^{\gamma}(1+r_t)^{\gamma-1}$
- if, $\beta(1+r) = 1$, then

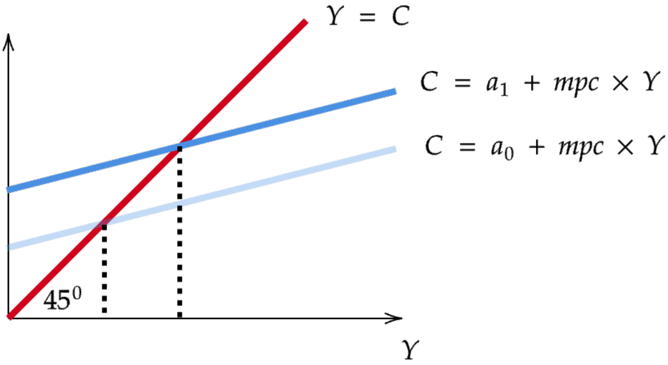
$$mpc_{t,t} = \frac{dc_t}{dy_t} = 1 - \beta$$

- A very small number. If $r = 2\%$ annual, then quarterly $mpc = 0.005$
- Intuition: Annuity of permanent income increased by way less than one dollar
- $mpc_{t,t+k} = \frac{dc_t}{dy_{t+k}} = (1-\beta)\left(\frac{1}{1+r}\right)^k$

General Equilibrium - PIH

- In general equilibrium, $C = Y$
- Channels when r_t changes:
  - From r to c. Dictated by $1/\gamma$. Intertemporal substitution.
  - From c to y. Market clearing.
  - From y to c. Very small. MPC close to zero. Keynesian cross is flat
  - From y to π. Slope of the phillips curve
  - from y and π to r: depends on MP rule and inflation expectations

Keynesian Cross



Forward Guidance

$$x_t = \mathbb{E}_t \hat{c}_T - \frac{1}{\gamma} \sum_{\tau=t}^{T} (i_\tau - \mathbb{E}_t \pi_{\tau+1})$$

- Let me assume the central bank controls the real rate

$$x_t = \mathbb{E}_t x_T - \frac{1}{\gamma} \sum_{\tau=t}^{T} r_\tau$$

- Imagine the interest rate has been $r_t = 0$
- And the economy is in steady state
- At time $t = 0$ the household learns about the following policy

$$r_t = \begin{cases} 0 & \text{if } t < t^* \\ -\bar{r} & \text{if } t = t^* \\ 0 & \text{if } t > t^* \end{cases}$$

The Euler equation

$$\hat{r}_t = \begin{cases} 0 & \text{if } t < t^* \\ -\bar{r} & \text{if } t = t^* \\ 0 & \text{if } t > t^* \end{cases}$$

- Between $t = 0$ and $t = t^*$ $\hat{r}_t = 0$
  - So $x_t = \bar{x}_1 \; \forall \; [0 \leq t \leq t^*]$
- The interest rate between t^* and $t^* + 1$ falls
- the relative price of consumption fell, so you consume more in t^* than in $t^* + 1$
  - So $x_{t^*+1} = \bar{x}_2 < \bar{x}_1$
- The interest rate never changes again
  - So $x_t = \bar{x}_2 \; \forall \; t > t*$

Forward Guidance

- The policy we considered

$$\hat{r}_t = \begin{cases} 0 & \text{if } t < t^* \\ -\bar{r} & \text{if } t = t^* \\ 0 & \text{if } t > t^* \end{cases}$$

- Under perfect foresight
- Creates a step-function for the output gap

$$x_t = \begin{cases} \bar{x}_1 & \text{if } t \le t^* \\ \bar{x}_2 & \text{if } t > t^* \end{cases}$$

- $\bar{x}_2 = 0$. Why? Monetary non-neutrality in the long-run
- $\bar{x}_1 = \frac{1}{\gamma}\bar{r}$

Forward Guidance

$$x_t = \begin{cases} \frac{1}{\gamma}\bar{r} & \text{if } t \leq t^* \\ 0 & \text{if } t > t^* \end{cases}$$

- So what?
- Bizarre!
  - Announcing a cut tomorrow or in large T has the same effect on output today
  - PV of CIRF of consumption is **increasing** on the horizon t^*
  - Forward guidance infinitely powerful on quantities

Forward Guidance

- How about inflation?
- Bizarre as well
- Remember the iterated-forward Phillips Curve

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t x_k$$

- Inflation response is front-loaded
- Current inflation is increasing on the discounted sums of expected output gaps
- Current inflation is increasing on t^* keeping $\bar{r}$ fixed

Mechanical intuition of the problem

- This is a problem with the Euler equation. Not a problem introduced by the NK block
- The source of the problem comes from the Euler equation being extremely forward looking
- There is no discounting in the log-linear Euler equation
- Mechanically, if you “discount” the Euler equation this problem will be diminished

Behavioral Explanation

- Angeletos and Huo (2021)
  - RA economy with no frictions:
$$a_t = \varphi \xi_t + \delta \mathbb{E}_t a_{t+1}$$
  - a could be consumption in the euler equation, or π in the PC.
  - Informational frictions (dispersed info, rational inattention)
  - The information friction outcome coincides with a representative agent economy with
$$a_t = \varphi \xi_t + \delta \omega_f \mathbb{E}_t a_{t+1} + \omega_b a_{t-1}$$
  - for some $\omega_f < 1$ (myopia), and a $\omega_b > 0$ (anchoring)

Incomplete Markets

- Households out of their Euler equations
- Borrowing constraints are a popular way of doing that
- Speaks closely to the Keynesian cross models
- Requires heterogeneity unless you want to assume that

The intuition for HANK models

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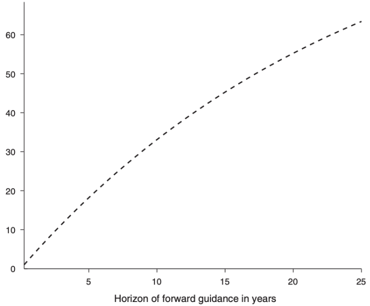


FIGURE 2. RESPONSE OF CURRENT INFLATION TO FORWARD GUIDANCE ABOUT INTEREST RATES AT DIFFERENT HORIZONS RELATIVE TO RESPONSE TO EQUALLY LARGE CHANGE IN CURRENT REAL INTEREST RATE

Source: McKay, Nakamura, Steinsson (2016)

The intuition for HANK models

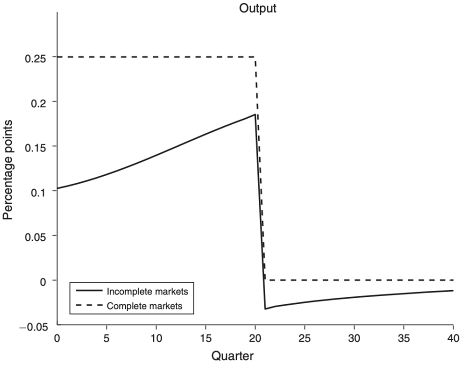


Figure 3. Response of Output to 50 Basis Point Forward Guidance about the Real Interest Rate in Quarter 20 (*With Real Interest Rates in All Other Quarters Unchanged*)

Source: McKay, Nakamura, Steinsson (2016)

The intuition for HANK models

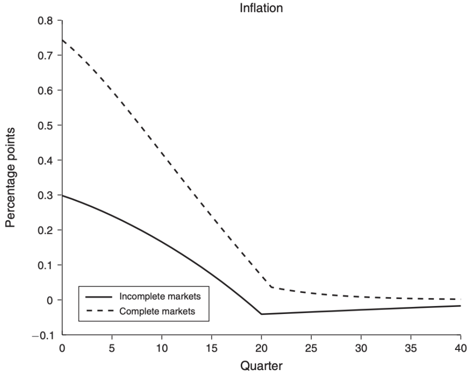


Figure 4. Response of Inflation to 50 Basis Point Forward Guidance about the Real Interest Rate in Quarter 20 (*With Real Interest Rates in all Other Quarters Unchanged*)

Source: McKay, Nakamura, Steinsson (2016)

The intuition for HANK models

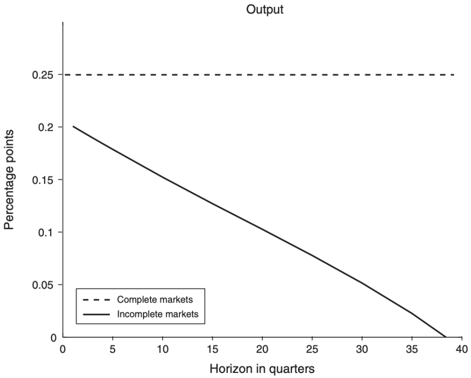


FIGURE 5. INITIAL RESPONSE OF OUTPUT TO 50 BASIS POINT FORWARD GUIDANCE ABOUT THE REAL INTEREST RATE FOR A SINGLE QUARTER AT DIFFERENT HORIZONS

Source: McKay, Nakamura, Steinsson (2016)

The intuition for HANK models

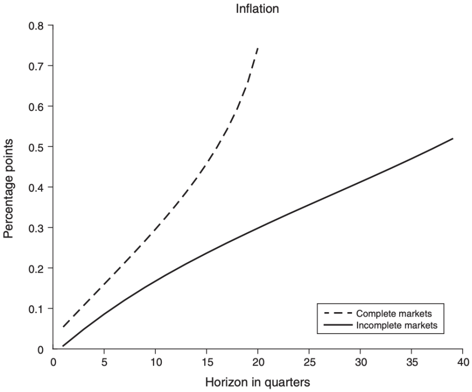


Figure 6. Initial Response of Inflation to 50 Basis Point Forward Guidance about the Real Interest Rate for a Single Quarter at Different Horizons

Source: McKay, Nakamura, Steinsson (2016)

Big Picture

- At the heart of the NK transmission mechanism is the Euler equation.
- Empirical evidence suggests that consumer behavior is much more nuanced than PIH / intertemporal substitution behavior.
- What are the lessons from these data for policy?
- Today: explore these questions in HANK style models.

Auclert, Rognlie, Straub (2024)

Simplified Consumer Problem

- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_{it})$$

- After-tax income:

$$z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it}\right)^{1-\lambda} = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}} Z_t$$

- Budget constraint:

$$c_{it} + \sum_j a_{it}^j = z_{it} + (1 + r_{t-1}) \sum_j a_{i,t-1}^j$$

- Nests: RA, TA, HA, HA-IL

Optimal Policy Rules

- Budget constraint with $Z_t = Y_t - T_t$:

$$c_{it} + \sum_j a_{it}^j = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}}(Y_t - T_t) + (1 + r_{t-1}) \sum_j a_{i,t-1}^j$$

- Idiosyncratic states: e_{it}.
- Assume aggregate variables follow a known sequence $\{Y_t - T_t, r_t\}_{t=0}^{\infty}$.
- Conditional on knowing these sequences, can solve for individual consumption and assets given any initial state $\{a_0^j\}, e_{i0}$:

$$c_t(\{a^j\}, e),\ \{a_t^j(\{a^j\}, e)\}, \qquad t \geq 0$$

- Why? Distribution only enters consumers problem through effect on $\{Y_t - T_t, r_t\}_{t=0}^{\infty}$. No longer have infinite state space.
- Then aggregate all consumption decisions to get:

$$C_t = \int c_t(\{a^j\}, e) d\Psi_t(\{a^j\}, e) = \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty})$$

New Keynesian Model in Sequence Space

- Consumption function:

$$\begin{aligned} C_t &= \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty}) \\ &\equiv \mathbb{C}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) \end{aligned}$$

- NKPC:

$$\pi_t = \mathbb{S}(\boldsymbol{Y} - \boldsymbol{Y}^*)$$

- Interest rate rule:

$$\boldsymbol{r} = \boldsymbol{\Phi}_Y(\boldsymbol{Y} - \boldsymbol{Y}^*) + \boldsymbol{\Phi}_\pi \boldsymbol{\pi} + \boldsymbol{\epsilon}^r$$

- Excess demand:

$$\boldsymbol{ED} = \mathbb{C}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) + \boldsymbol{G} - \boldsymbol{Y} = \boldsymbol{0}$$

Linearize to Solve with Linear Algebra

- Linearized Model:

$$
\begin{aligned}
\hat{\boldsymbol{C}} &= (\nabla_{\boldsymbol{Y}}\mathbb{C})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{T}}) + (\nabla_{\boldsymbol{r}}\mathbb{C})\hat{\boldsymbol{r}} \\
\hat{\boldsymbol{\pi}} &= (\nabla_{\boldsymbol{Y}}\mathbb{S})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^{*}) \\
\hat{\boldsymbol{r}} &= \boldsymbol{\Phi}_{Y}(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^{*}) + \boldsymbol{\Phi}_{\pi}\hat{\boldsymbol{\pi}} + \boldsymbol{\epsilon}^{r}
\end{aligned}
$$

  where the Jacobians of the model look like:

$$
\nabla_{\boldsymbol{Y}}\mathbb{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \cdots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \cdots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}
$$

- Equilibrium:

$$
\begin{aligned}
\hat{\boldsymbol{Y}} = {} & \left[\boldsymbol{\Phi}_{Y} + \boldsymbol{\Phi}_{\pi}(\nabla_{\boldsymbol{Y}}\mathbb{S})\right]^{-1}\left\{\left[\boldsymbol{\Phi}_{Y} + \boldsymbol{\Phi}_{\pi}(\nabla_{\boldsymbol{Y}}\mathbb{S})\right]^{-1} - \left[I - \nabla_{\boldsymbol{Y}}\mathbb{C}\right]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})\right\}^{-1} \times \\
& \times \left[I - \nabla_{\boldsymbol{Y}}\mathbb{C}\right]^{-1}\left[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C})\boldsymbol{\epsilon}^{r}\right]
\end{aligned}
$$

Demand and Supply Elasticities Determine GE

- Define:
  - Demand multiplier $\boldsymbol{\mu}^D \equiv [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}$
  - Demand elasticity $\boldsymbol{\eta}^{\boldsymbol{D}} \equiv -\nabla_{\boldsymbol{r}}\mathbb{C}$
  - Supply elasticity $\boldsymbol{\eta}^{\boldsymbol{S}} \equiv [\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1}$

$$\hat{\boldsymbol{Y}} = \underbrace{\boldsymbol{\eta}^{\boldsymbol{S}}[\boldsymbol{\eta}^{\boldsymbol{S}} + \boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}]^{-1}}_{\text{Demand Incidence}} \boldsymbol{\mu}^D \underbrace{[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + \boldsymbol{\eta}^{\boldsymbol{D}}\boldsymbol{\epsilon}^r]}_{\text{PE Excess Demand}}$$

$$+ \underbrace{\boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}[\boldsymbol{\eta}^{\boldsymbol{S}} + \boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}]^{-1}}_{\text{Supply Incidence}} \underbrace{\hat{\boldsymbol{Y}}^*}_{\text{PE Excess Supply}}$$

Implications:

- GE effects determined by a standard incidence formula.

$\Rightarrow$ Matrices of micro elasticities are sufficient statistics for GE effects.

Intertemporal Keynesian Cross

- Also assume no real rate change, so $\boldsymbol{\eta}^{\boldsymbol{S}}$ blows up in some meaningful sense.

$$\hat{\boldsymbol{Y}} = [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}}]$$

- This is the Intertemporal Keynesian Cross.
- Balanced budget: $\hat{\boldsymbol{G}} = \hat{\boldsymbol{T}}$

$$\begin{aligned}\hat{\boldsymbol{Y}} &= [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}[\boldsymbol{I} - (\nabla_{\boldsymbol{Y}}\mathbb{C})]\hat{\boldsymbol{G}} \\ &= \hat{\boldsymbol{G}}\end{aligned}$$

- The balanced budget multiplier is 1.

Fiscal Policies

- Proposition 4: In the RA model, the G-multiplier is 1.
  - Proof: The G-multiplier is 1 when budget is balanced, and we know for the RA model the timing of T does not matter.
- Proposition 5: in the TA model with μ constrained agents,

$$\hat{\boldsymbol{Y}} = \frac{1}{1-\mu}\left[\hat{\boldsymbol{G}} - \mu\hat{\boldsymbol{T}}\right]$$

  - Deficit-financed fiscal expansion ($\hat{T}_0 < \hat{G}_0$) has impact multiplier greater than 1.

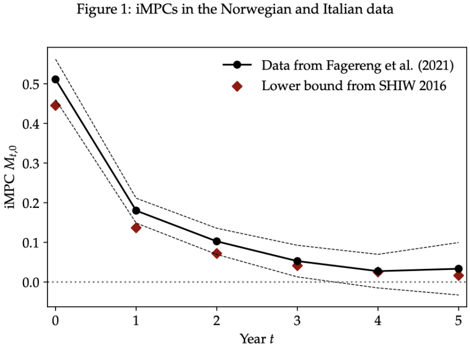
Measuring $\nabla_{\boldsymbol{Y}}\mathbb{C}$

- Demand multiplier: $\boldsymbol{\mu}^D \equiv \left[I - \nabla_{\boldsymbol{Y}}\mathbb{C}\right]^{-1}$ where

$$\nabla_{\boldsymbol{Y}}\mathbb{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \cdots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \cdots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

  Columns are PE IRFs of average consumption to income changes at different horizons.
- What evidence do we have?

Lottery Studies

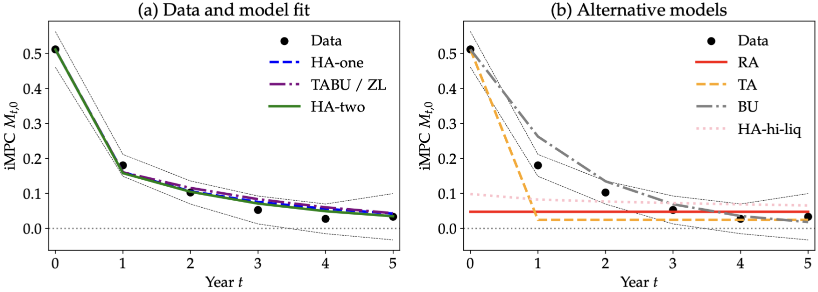


- Provides estimate of first column of $\nabla_{\boldsymbol{\gamma}}\mathbb{C}$.
- Need a model to extrapolate.

Source: Auclert, Rognlie, Straub (2024)

iMPCs across models

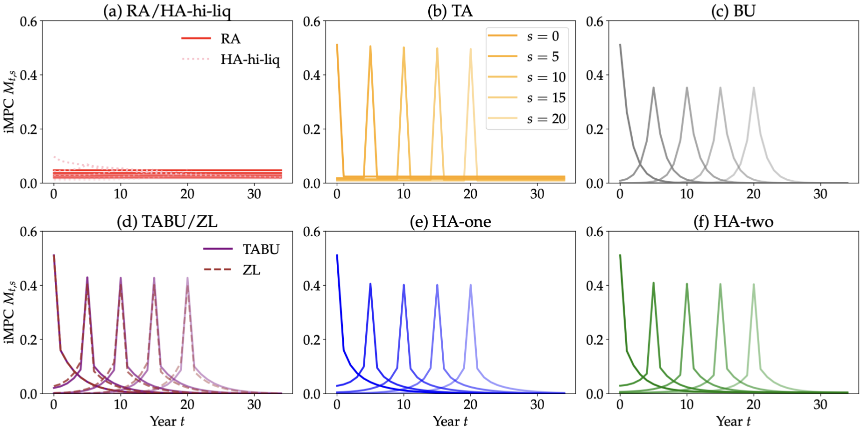
Figure 2: iMPCs in the Norwegian data and in several models



Notes: All models are calibrated to match $r = 0.05$. RA does not have any other free parameter. The single free parameter in BU (λ), TA (μ), HA-one (A/Z) and HA-two (ν) is calibrated to match $M_{00} = 0.51$. The additional free parameter in TABU and ZL (μ) is calibrated to match $M_{10} = 0.16$ (its value in the HA-one model). The HA-two and HA-hi-liq models are calibrated to an aggregate ratio of assets to post-tax income of $A/Z = 6.29$, its value in the model with capital in section 7.

iMPCs across models

Figure 3: iMPCs in eight standard models



Notes: The models are calibrated as in figure 1. The discount factors, which are relevant for anticipation, are reported in table 2.

Auclert, Rognlie, Straub (2020)

New Keynesian Model in Sequence Space

- Consumption function:

$$
\begin{aligned}
C_t &= \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty}) \\
&\equiv \mathbb{C}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r})
\end{aligned}
$$

- NKPC:

$$\pi_t = \mathbb{S}(\boldsymbol{Y} - \boldsymbol{Y}^*)$$

- Interest rate rule:

$$\boldsymbol{r} = \boldsymbol{\Phi}_Y(\boldsymbol{Y} - \boldsymbol{Y}^*) + \boldsymbol{\Phi}_\pi \boldsymbol{\pi} + \boldsymbol{\epsilon}^r$$

- Excess demand:

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Linearize to Solve with Linear Algebra

- Linearized Model:

$$
\begin{aligned}
\hat{\boldsymbol{C}} &= (\nabla_{\boldsymbol{Y}}\mathbb{C})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{T}}) + (\nabla_{\boldsymbol{r}}\mathbb{C})\hat{\boldsymbol{r}} \\
\hat{\boldsymbol{\pi}} &= (\nabla_{\boldsymbol{Y}}\mathbb{S})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*) \\
\hat{\boldsymbol{r}} &= \Phi_Y(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*) + \Phi_\pi\hat{\boldsymbol{\pi}} + \epsilon^r
\end{aligned}
$$

- Equilibrium:

$$
\begin{aligned}
\hat{\boldsymbol{Y}} = {} & [\Phi_Y + \Phi_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1}\{[\Phi_Y + \Phi_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1} - [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})\}^{-1}\times \\
& \times [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C})\epsilon^r]
\end{aligned}
$$

- Kaplan, Moll, and Violante (2018):
  - Small direct effect $\nabla_{\boldsymbol{r}}\mathbb{C}$
  - Large indirect effect $[I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}$
  - Product roughly constant $[I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})$

$\Rightarrow$ Should we care about the decomposition?

Add Investment

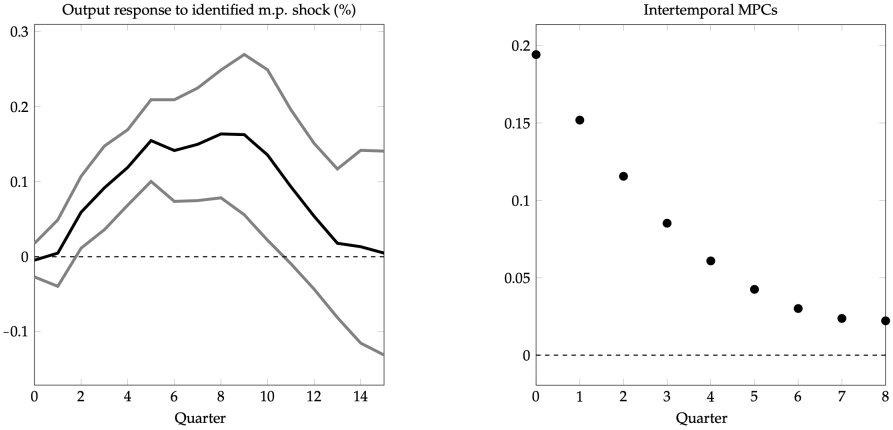
- Yes:

$$
\begin{aligned}
\hat{\boldsymbol{Y}} = {} & \left[\Phi_Y + \Phi_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})\right]^{-1}\times \\
& \times\left\{\left[\Phi_Y + \Phi_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})\right]^{-1} - \left[I - \nabla_{\boldsymbol{Y}}\mathbb{C} - \nabla_{\boldsymbol{Y}}\mathbb{I}\right]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C} + \nabla_{\boldsymbol{r}}\mathbb{I})\right\}^{-1}\times \\
& \times\left[I - \nabla_{\boldsymbol{Y}}\mathbb{C} - \nabla_{\boldsymbol{Y}}\mathbb{I}\right]^{-1}\left[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C} + \nabla_{\boldsymbol{r}}\mathbb{I})\epsilon^r\right]
\end{aligned}
$$

- Key challenge: How to set up model to match
  - Immediate consumption response to higher income (“jump”).
  - Delayed consumption, investment, output response to lower real interest rate (“hump”).

The Challenge

Figure 1: Macro Humps, Micro Jumps.



Model

- Household problem:

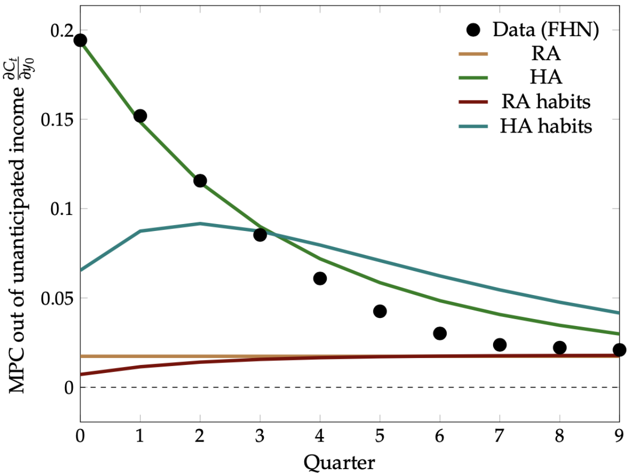
$$V_t(l,s) = \max_{c,l'} u(c) + \beta \mathbb{E}[V_{t+1}(l',s')|s]$$
$$c' + l' \leq (1+r_t)l + y_t e(s)$$
$$l' \geq 0$$

- Standard sequence space result:

$$C_t = \mathcal{C}(\{y_s, r_s\}_{s\geq 0}),\ t \geq 0$$

Matching Micro Data

Figure 2: Intertemporal MPCs $\partial C_t / \partial y_0$ in models and in the data



- Why does habit model have difficulty?

Inattention

- Households know current micro state $e(s)$.
- Households update info about aggregate shocks with iid probability $1 - \theta$ each period.
- Households know current r_t, Y_t but forecast r_{t+s}, Y_{t+s} using info from $t - k$.

$\Rightarrow$ Households always on budget constraint.

- Optimize subject to information from k periods ago:

$$V_t(l, s, k) = \max_{c, l'} u(c) + \beta \mathbb{E}_{t-k}[\theta V_{t+1}(l', s', k+1) + (1-\theta) V_{t+1}(l', s', 0) | s]$$

Estimation

- Two-step procedure.
- Calibrate household problem to micro data, so only need to compute household sequence space Jacobians once.
- Estimate information and macro parameters to match estimated IRF to monetary policy shocks.
- Sticky info micro Jacobian at t to shock at time s:

$$\mathcal{J}^{o,i}_{t,s} = \begin{cases} \theta \mathcal{J}^{o,i}_{t-1,s-1} + (1-\theta)\mathcal{J}^{o,i,FI}_{t,s} & t > 0, s > 0 \\ \mathcal{J}^{o,i,FI}_{t,s} & s = 0 \\ (1-\theta)\mathcal{J}^{o,i,FI}_{t,s} & t = 0, s > 0 \end{cases}$$

Calibration

Table 2: Calibrated and estimated parameters.

Panel A: Calibrated parameters

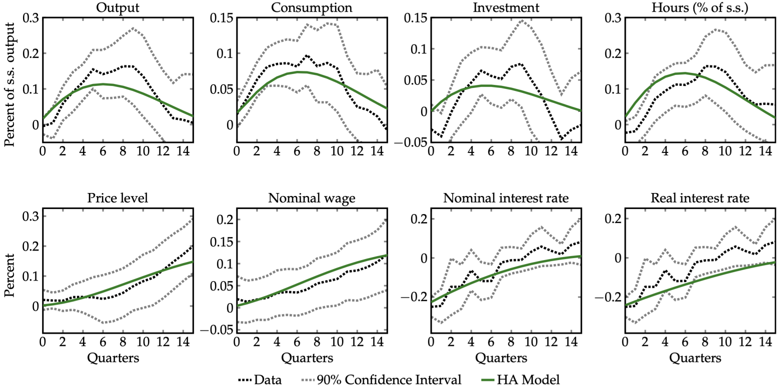
Parameter		Value
ν	EIS	1
ς	Frisch	0.5
β_g	Discount factors (p.a.)	Table 1
r	Real interest rate (p.a.)	0.050
α	Capital share	0.24
δ_K	Depreciation of capital (p.a.)	0.053
μ_p	Steady-state retail price markup	1.06
K/Y	Capital to GDP (p.a.)	2.23
L/Y	Liquid assets to GDP (p.a.)	0.23
ξ	Intermediation spread (p.a.)	0.065
G/Y	Spending-to-GDP	0.16
qB/Y	Government bonds to GDP (p.a.)	0.42
$\frac{1+r}{1+r-\delta}$	Maturity of government debt (a.)	5
ψ	Response of tax rate to debt (p.a.)	0.1
ϕ_π	Taylor rule coefficient	1.5

Panel B: Estimated parameters

Parameter		Value	std. dev.
θ	Household inattention	0.935	(0.01)
ϕ	Investment adj. cost parameter	9.639	(2.428)
ζ_p	Calvo price stickiness	0.926	(0.012)
ζ_w	Calvo wage stickiness	0.899	(0.016)
ρ^m	Taylor rule inertia	0.890	(0.01)
σ^m	Std. dev. of monetary shock	0.057	(0.005)

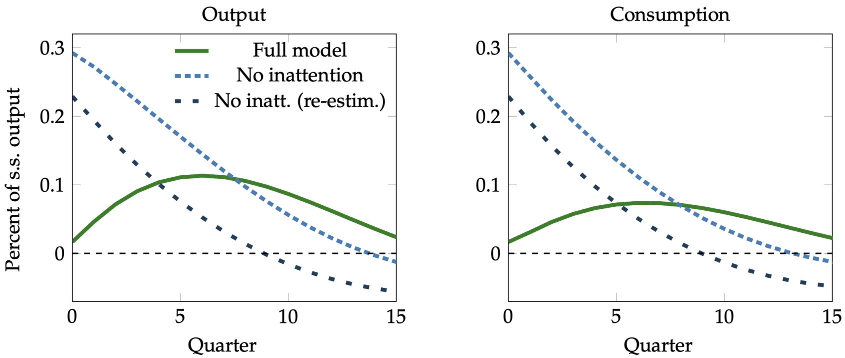
Matching the Humps

Figure 3: Impulse response to a monetary policy shock vs. model fit



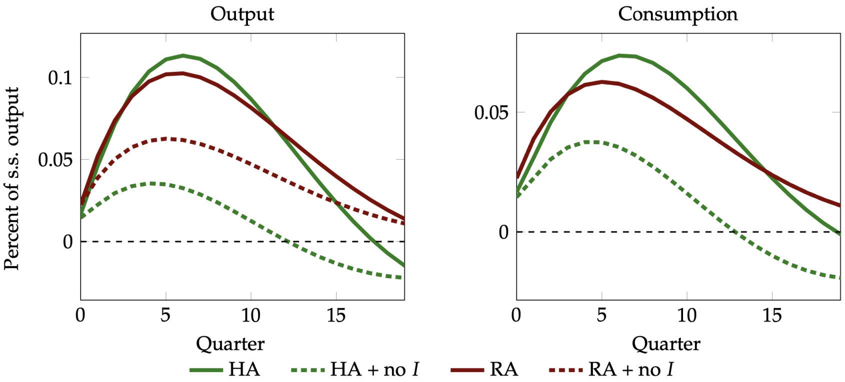
Role of Inattention

Figure 4: Impulse responses with and without inattention



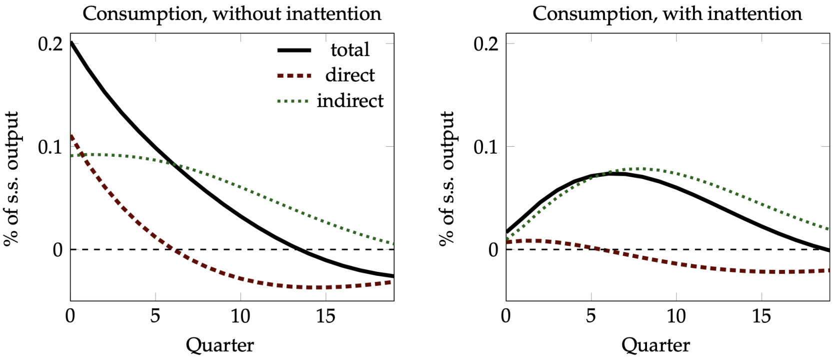
Importance of Investment

Figure 5: Role of investment in the transmission mechanism



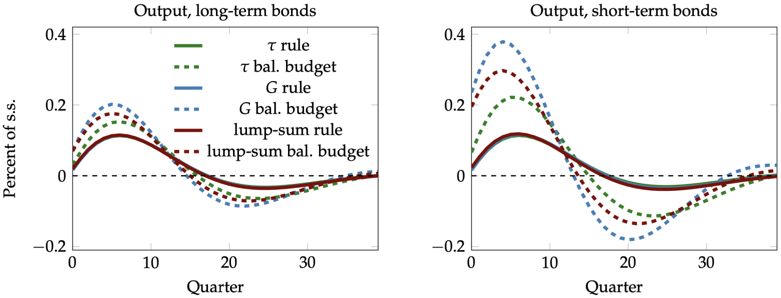
Direct vs Indirect Effects

Figure 6: Decomposition of consumption

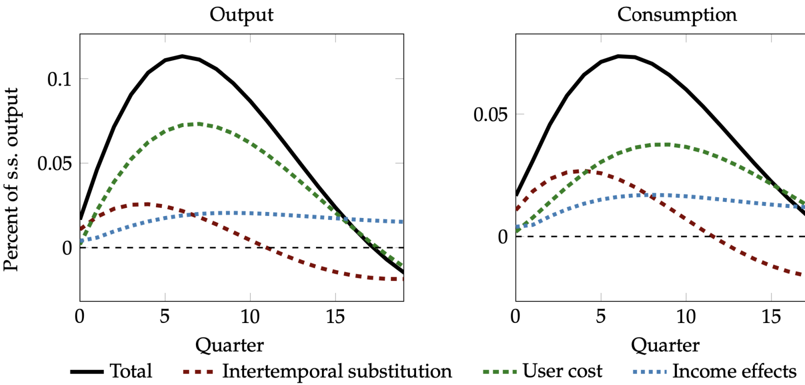


Aside: Role of Fiscal Policy

Figure 7: The role of fiscal policy for monetary transmission

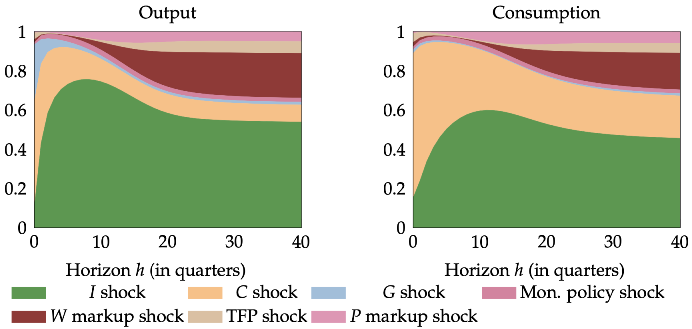


Starting the Transmission Mechanism



Reassessing the Importance of Investment

Figure 12: Forecast error variance decomposition for the HA model



McKay, Wieland (2021)

Does Central Bank Create or Borrow Demand?

- In standard NK model:

$$
\begin{aligned}
y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\
&= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T \to \infty} y_{t+1}
\end{aligned}
$$

- Central bank creates demand and output by changing r_t.
- Past r_t has no bearing on future AD.

$\Rightarrow$ Central bank can create AD subject to ZLB constraint.

- This paper: with durable goods, central bank is much more in the business of borrowing demand than creating demand.

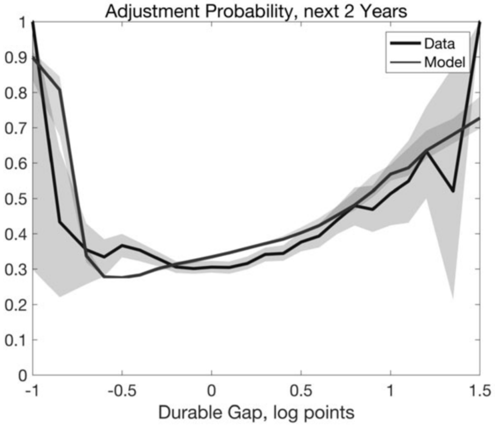
Model: Households

- Households consume non-durables and durables.

$$\max E_{i0} \int_{t=0}^{\infty} e^{-\rho t} u\left(c_{it}, q_{it} d_{it}\right) dt$$

- Durables subject to:
  - fixed adjustment cost (if adjust) $a'_{it} + p_t d'_{it} = a_{it} + (1-f) p_t d_{it}$
  - depreciation and maintenance cost $d_t d_{it} = -(1-\chi)\delta d_{it}$
  - operating cost (e.g. household utilities, gas, taxes) νd_{it}
  - match-quality shocks $q_{it} = 1$, drops to zero with intensity θ
- Idiosyncratic labor income risk
  $y_{it} = (1-\tau_t) Y_t z_{it}$ $\qquad$ $d \ln z_{it} = -\rho_z \ln z_{it} + \sigma_z d\mathcal{W}_{it}$
- Save in liquid assets $d_t a_{it} = r_t a_{it} + r_t^b a_{it} I_{\{a_{it}<0\}} - c_{it} + y_{it} - (\chi\delta p_t + \nu) d_{it}$
- Collateralized borrowing $a_{it} \geq -\lambda(1-f) p_t d_{it}$
- Sticky information (Carrol et al, 2018; Auclert et al, 2020).

Hazard Rate


Adjustment Probability, next 2 Years
Data
Model
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1
1.5
Durable Gap, log points

Intertemporal Shifting in the Data

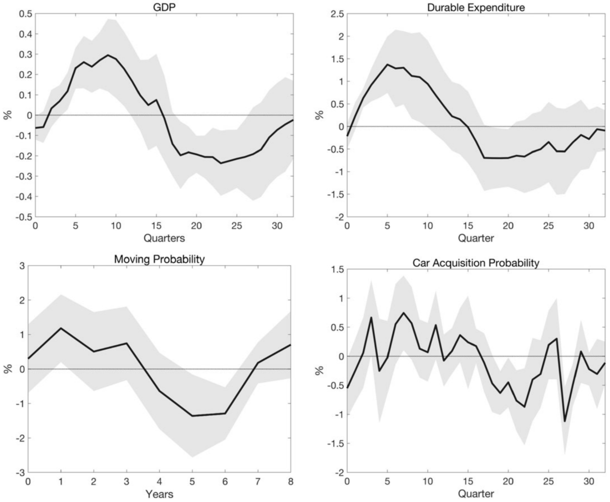


FIGURE 2.—Impulse response function of real GDP (top-left panel), real durable expenditure (top-right), probability of moving house (bottom-left), and probability of buying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence bands.

Intertemporal Shifting in the Data

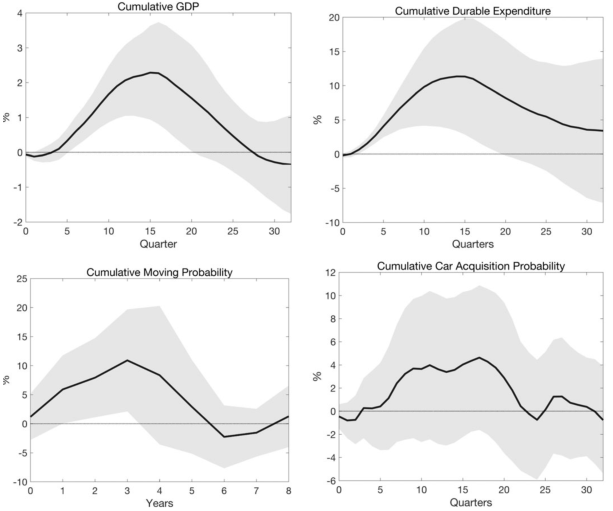


FIGURE 3.—Impulse response function of cumulative real GDP (top-left panel), cumulative real durable xpenditure (top-right), cumulative probability of moving house (bottom-left), and cumulative probability of uying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence ands.

Model vs Data

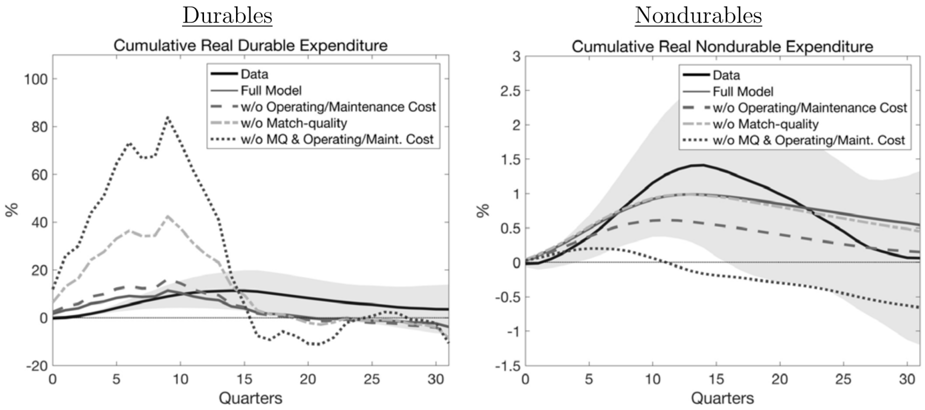
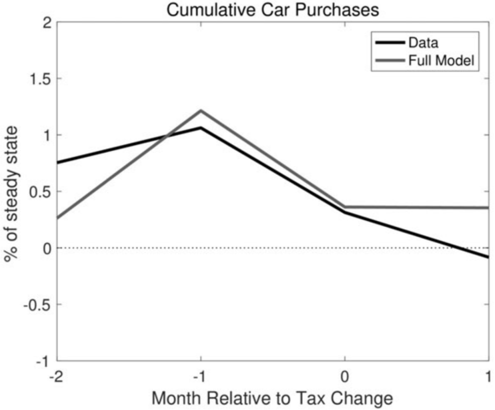


FIGURE 4.—Cumulative response of durable (left) and nondurable (right) expenditure to a simulated monetary policy shock. Model simulations feeding in the estimated impulse responses for (Y_t, r_t, p_t). Each panel shows the full model as well as the models that omit one or both of match-quality shocks and operating and maintenance costs.

Model vs Data



The Monetary Transmission Matrix

- Solve model with sequence space methods.
- The "monetary transmission matrix":

$$\mathcal{M} = \begin{pmatrix} \frac{d\hat{Y}_0}{dr_0} & \frac{d\hat{Y}_0}{dr_1} & \frac{d\hat{Y}_0}{dr_2} & \cdots \\ \frac{d\hat{Y}_1}{dr_0} & \frac{d\hat{Y}_1}{dr_1} & \frac{d\hat{Y}_1}{dr_2} & \cdots \\ \frac{d\hat{Y}_2}{dr_0} & \frac{d\hat{Y}_2}{dr_1} & \frac{d\hat{Y}_2}{dr_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- GE effect of monetary news on output gap.
  - Below-diagonal elements capture intertemporal-shifting effects.
  - Above-diagonal elements capture forward guidance effects.

IRF to r_0 and News

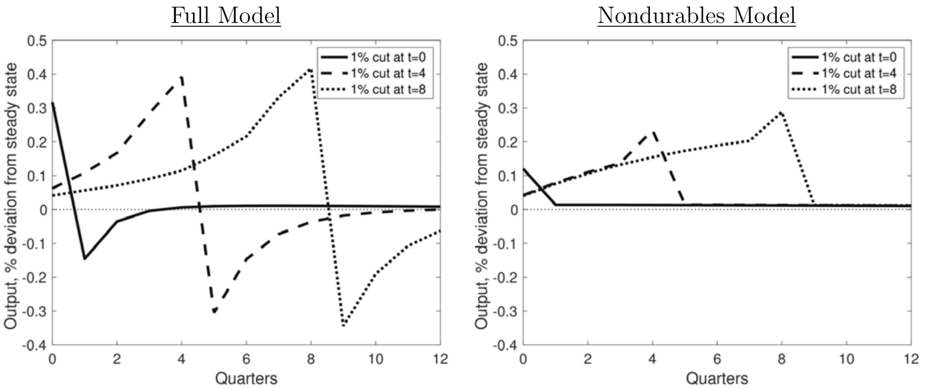


FIGURE 7.—Percentage change in output following a 1% reduction in the real interest rate at horizons $t = 0$ (solid line), $t = 4$ (dashed line), and $t = 8$ (dotted line). The left panel shows the effect in the full model, and the right panel shows the effect in the nondurables model.

Great Recession–Filtering the Data

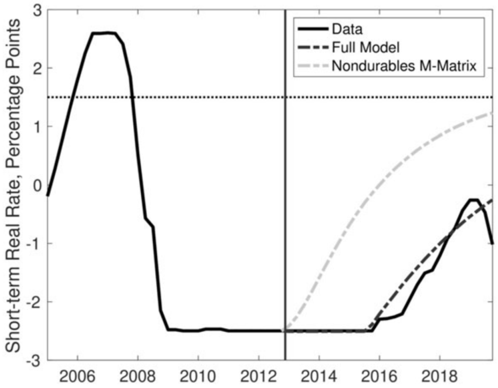
- Construct IRFs for shocks to Z_t, G_t, r_t^b, η_t.

- Match time series:
  - output gap,
  - durable spending,
  - real interest rate,
  - mortgage-treasury spread.

- Unique sequence of shocks that fit data exactly given initial condition. (Kalman filter with no observational error.)
  - No need for state transition matrix.

- Account for zero lower bound using monetary news shocks.

Model predicts slow normalization of real rate



Contribution of Intertemporal Shifting to r^*

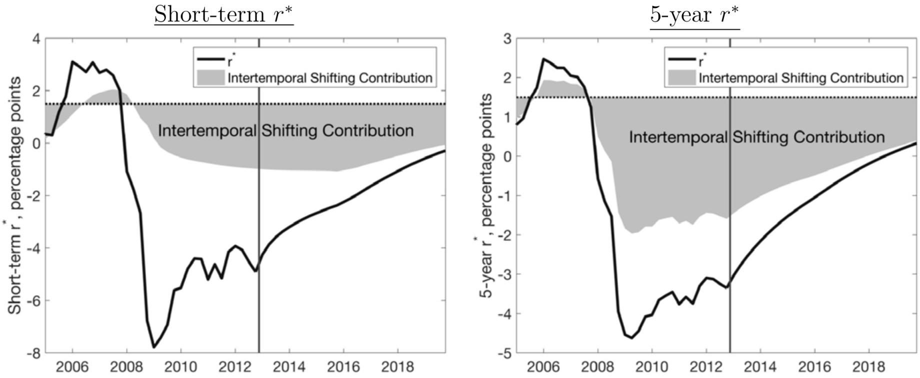
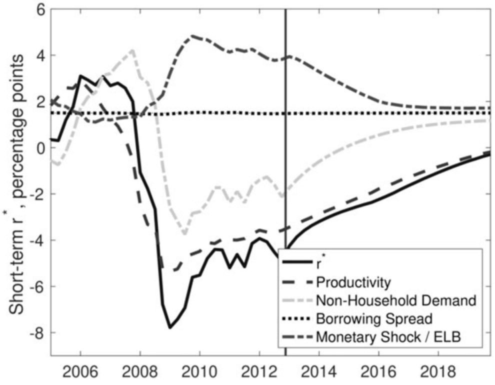


FIGURE 10.—Time series of the short-term natural rate of interest (left panel) and the 5-year natural rate of interest (right panel). The shaded area shows the contribution of intertemporal shifting effects of previous real interest rates to r^*. Forecast as of 2012Q4. The dotted horizontal line is the steady state real interest rate, equal to 1.5%.

Contribution Shocks to r^*



Questions

- Convincing?
- What role does the micro evidence play?

Beraja and Zorzi (2024)

Question

- How large a stimulus to close the output gap?
- Want to think about durables since important in determining MPCs as well as size and state-dependence.
- Why a model?

Model

- Value function:

$$V_t(x,\epsilon) = \max\{V_t^{adj}(x) - \epsilon, V_t^{noadj}(x)\}$$

  where $x = (d, m, y)$.
- Adjustment utility:

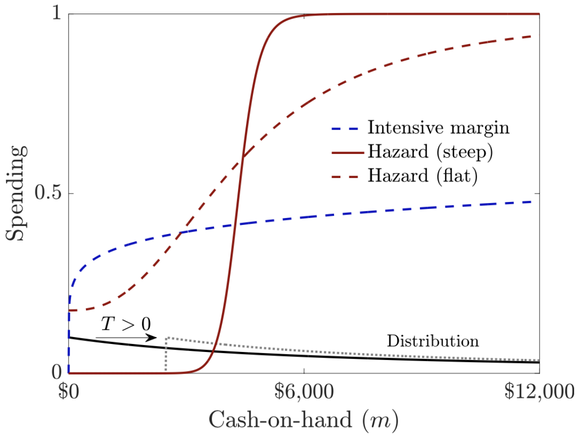
$$\begin{aligned} V_t^{adj}(x) = \max_{c,d',a'} & \; u(x,d') + \beta \int V_{t+1}(d',m',y',\epsilon') d\mathcal{E}(\epsilon)\Gamma(dy',y) \\ \text{s.t. } & \theta d' + c + m' = \mathcal{Y}(x,T_t) + ((1-\delta) - (1-\theta))d \\ & m' \geq 0 \end{aligned}$$

- No adjustment utility:

$$\begin{aligned} V_t^{noadj}(x) = \max_{c,a'} & \; u(x,d') + \beta \int V_{t+1}(d',m',y',\epsilon') d\mathcal{E}(\epsilon)\Gamma(dy',y) \\ \text{s.t. } & c + m' = \mathcal{Y}(x,T_t) - \iota\delta d - (1-\theta)(d-d') \\ & d' = (1-(1-\iota)\delta)d \\ & m' \geq 0 \end{aligned}$$

Importance of the Hazard

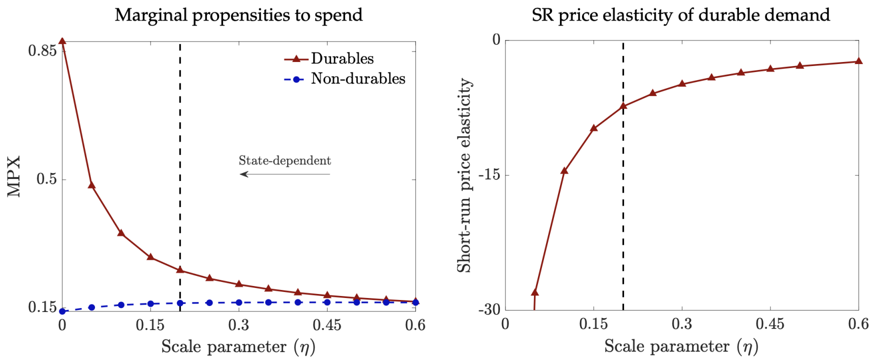
Figure 2.2: Hazard and intensive margin (fixing d and y)



Why is this a good model?

- How convincing did you find section 3?

Calibrating the Hazard (1)

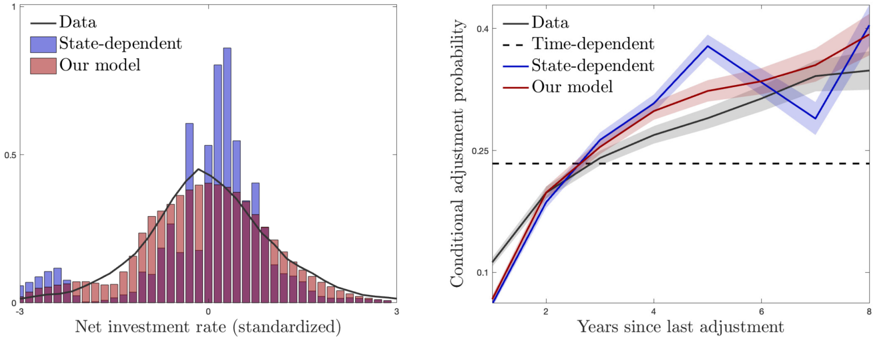


Calibrating the Hazard (2)

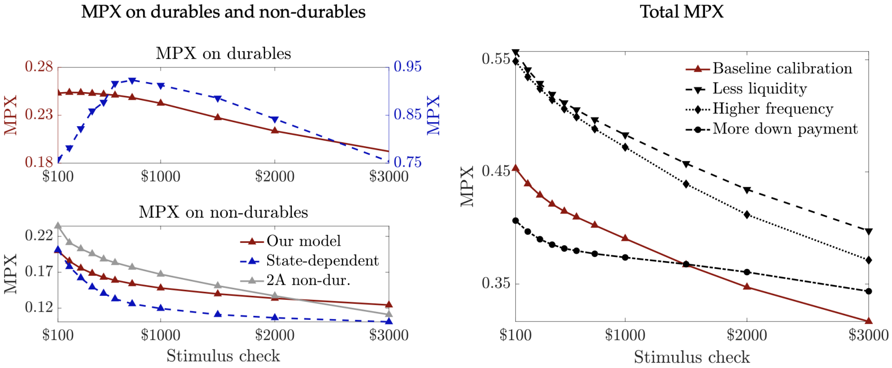
Figure 3.2: Distribution of adjustments and probability of adjustment

Distribution of net investment rates

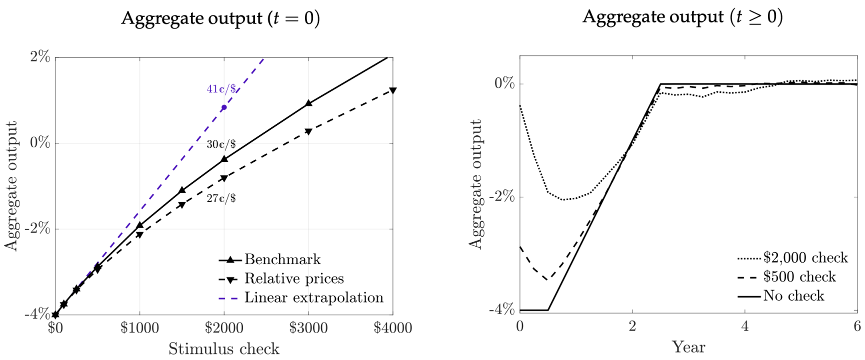
Conditional adjustment probability



MPX in the Model vs Alternatives



GE MPCs



Notes: The left panel plots aggregate output in the first quarter in deviations from steady state as a function of the size of stimulus checks. The solid curve is our benchmark model (Sections 5.1–5.2). The dashed black curve is a version with relative price movements between durables and non-durables ($\zeta \equiv 1/0.049$). The purple line extrapolates the response out of a $300 check. We also indicate the output increase (in cents per dollar sent) after a $2,000 stimulus check. The right panel reports the dynamic response of aggregate output in our benchmark model for stimulus checks of various sizes.

What did you think?

Berger, Milbradt, Tourre, Vavra (2021)

Refinancing (Pre-payment)

- Estimate effect of rate gap on refinancing probability.

$$prepay_{i,j,t} = \beta_{gapbin}\mathbf{1}(gapbin)_{j,t} + \beta_X X_{i,j,t} + \delta_i + \epsilon_{i,j,t}$$

- Identification assumption?

Refinancing (Pre-payment)

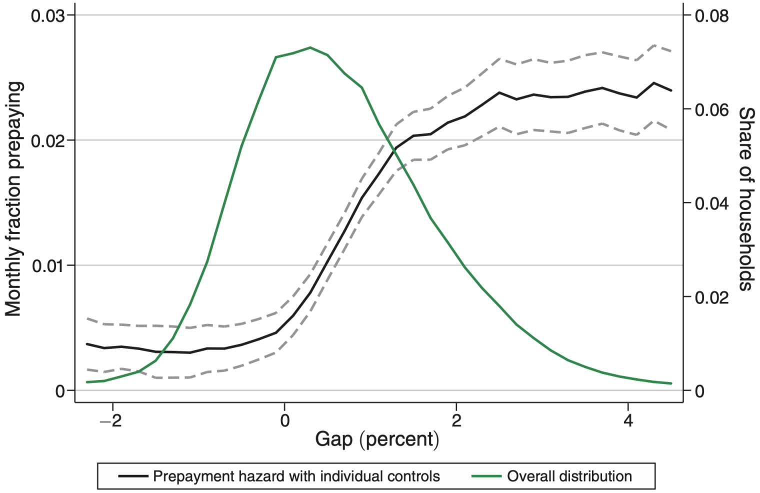


Figure 1. Prepayment Hazard with Individual Controls

Motives

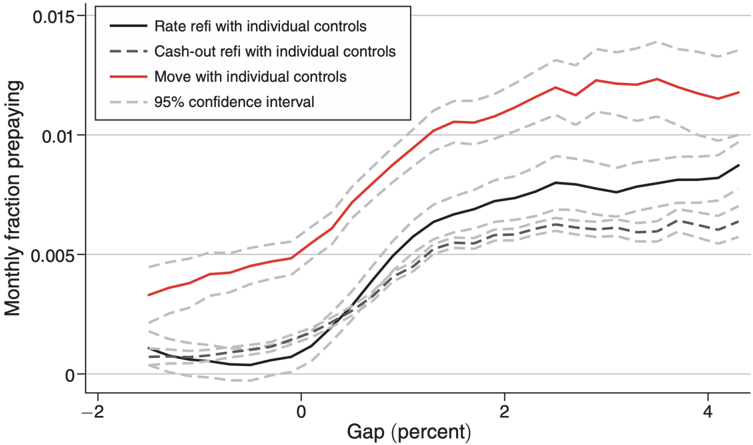


FIGURE 3. PREPAYMENT HAZARD DECOMPOSITION WITH INDIVIDUAL CONTROLS

Aggregate Correlation

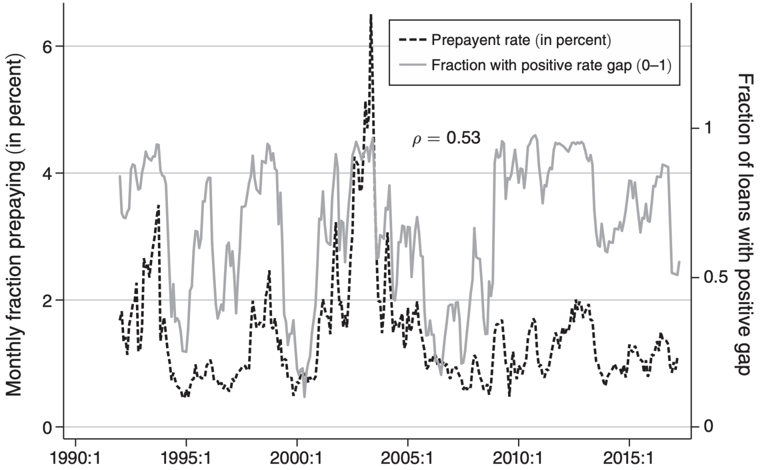


Figure 5. Prepayment versus Fraction with Positive Rate Gap Time Series

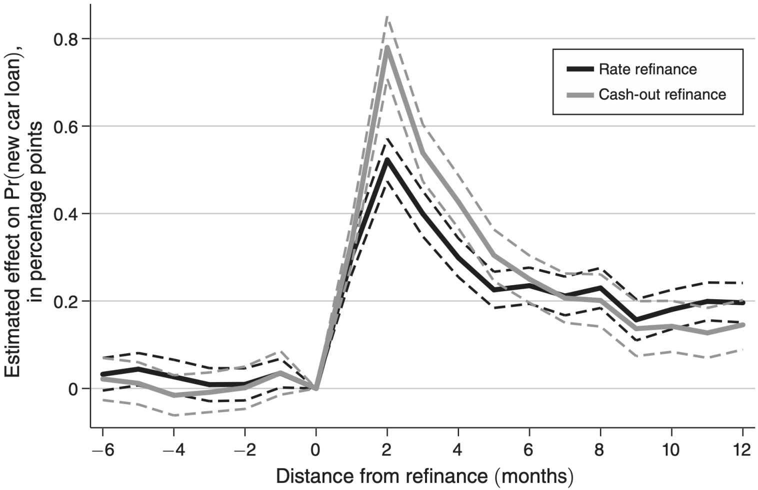
Spending

- Estimate effect of refinancing probability on spending.

$$\mathbf{1}(carloan)_{i,t} = \sum_k \mathbf{1}(refinanced)_{i,t-k} + \delta_i + \delta_t + \epsilon_{i,t}$$

- Identification assumption?

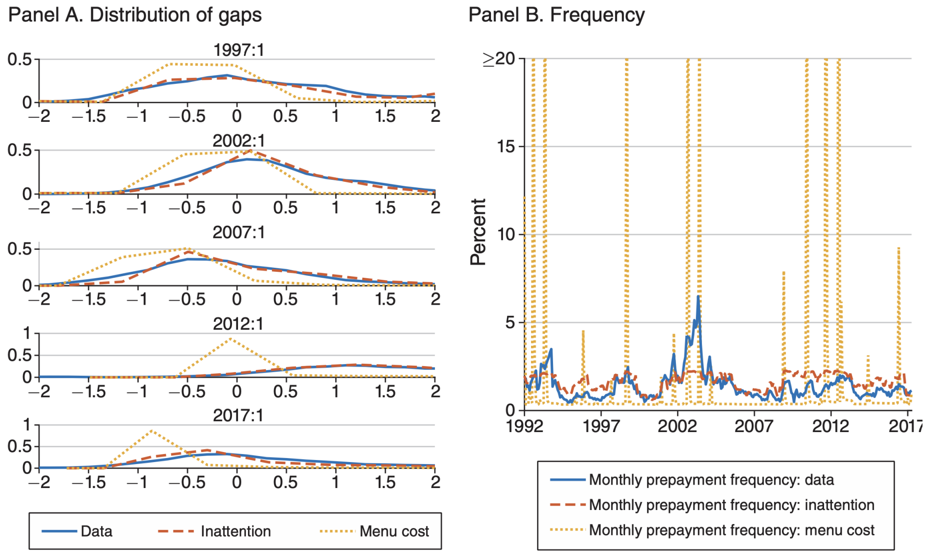
Spending



Rate refinance
Cash-out refinance
Estimated effect on Pr(new car loan), in percentage points
0.8
0.6
0.4
0.2
0
−6
−4
−2
0
2
4
6
8
10
12
Distance from refinance (months)

Why Do We Need a Model?

Gaps and Frequency



Hybrid Model

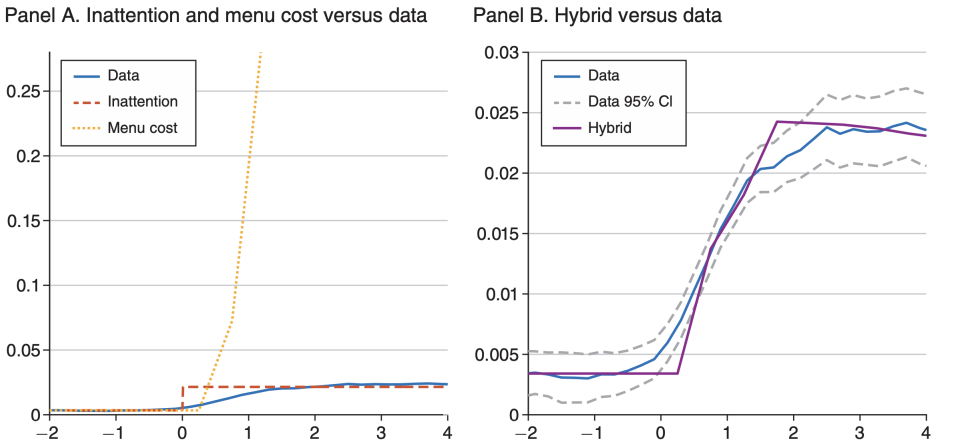


FIGURE 10. PREPAYMENT HAZARDS

State Dependence

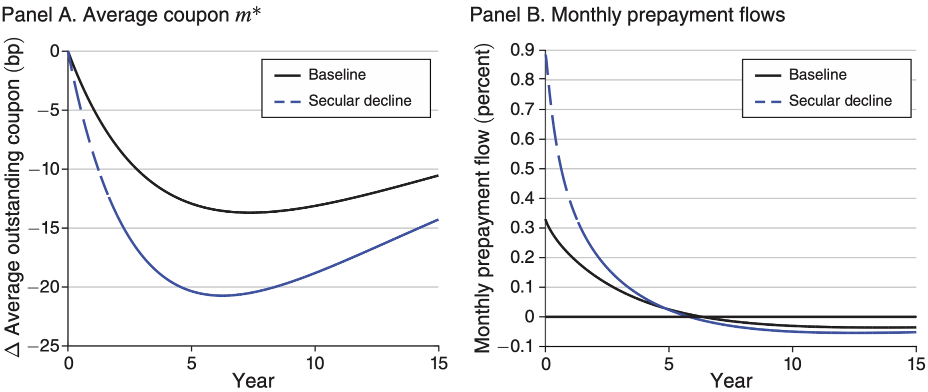


FIGURE 12. IMPULSE RESPONSE FUNCTIONS TO 100 BP DECLINE IN r

State Dependence

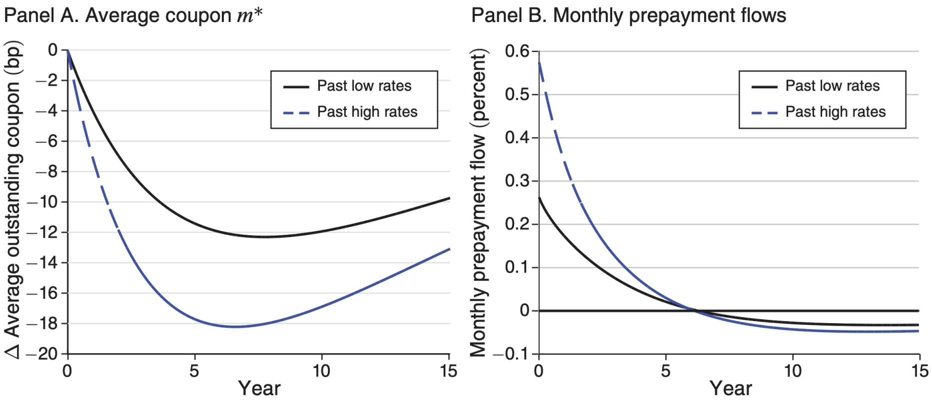
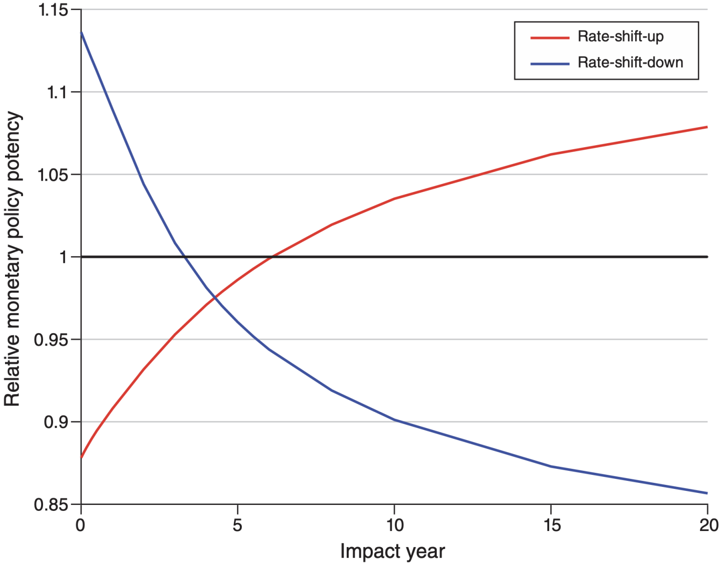


FIGURE 14. IRF OF AVERAGE COUPON m^* TO 100 BP DECLINE IN r

Shift in Policy Stance



Consumption Effects

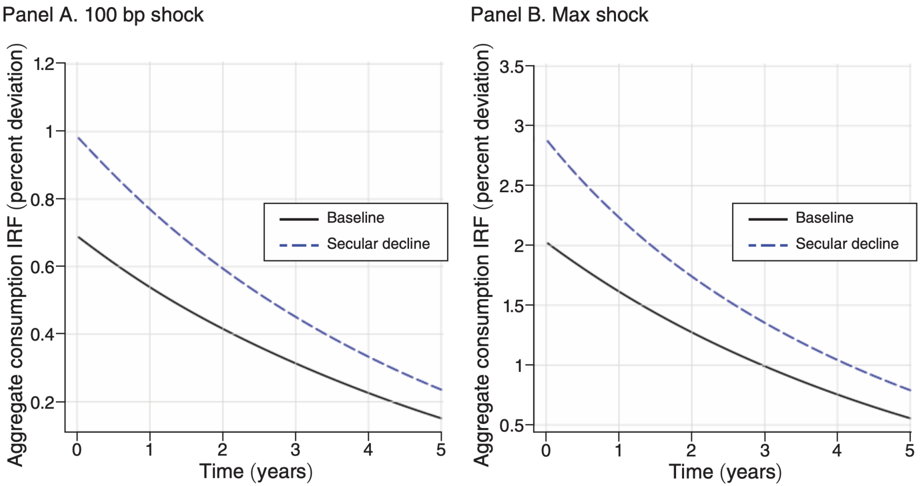


FIGURE 17. IRF OF CONSUMPTION—BASELINE VERSUS SECULAR DECLINE

Questions

- Convincing?

McKay, Wieland (2022)
Do not have time. Content for you.

Forward Guidance Puzzle

- In standard NK model:

$$
\begin{aligned}
y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\
&= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T \to \infty} y_{t+1}
\end{aligned}
$$

- Credible future real rate change has the same effect on current output regardless of how distant it is.

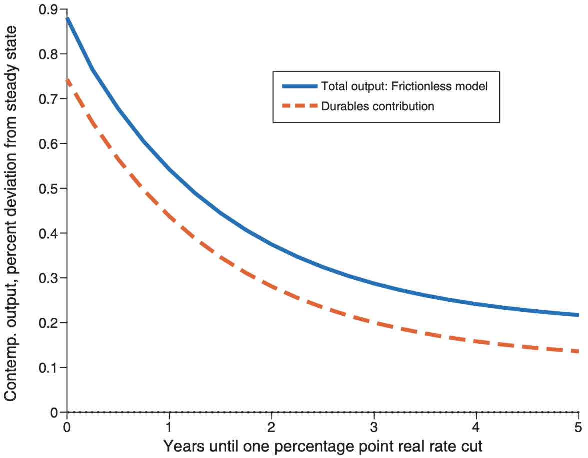
With Durable Goods

- MRS between durable and nondurable good is equal to user cost:

$$\left(\frac{\psi}{1-\psi}\frac{c_{it}}{d_{it}}\right)^{\frac{1}{\xi}} = p_t(r_t + \nu + \delta) - \dot{p}_t \equiv r_t^d$$

- Special role for contemporaneous real rate in durable goods demand.

Frictionless Durable Model



Lumpy durables

- Frictionless model inconsistent with lumpy nature of durable adjustment.

$\Rightarrow$ How credible are the results?

Fixed Cost Durable Model

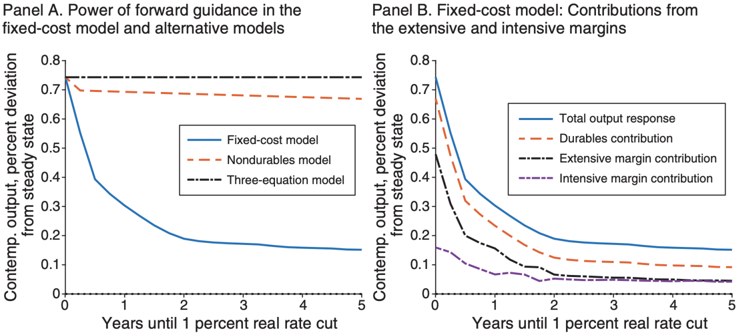


Figure 2. Contemporaneous Output Response to Forward Guidance in the Fixed-Cost Model

Extensive Margin

$$\frac{u(c_t^*, d_t^*) - u(c_t, d)}{V_{a,t}(a_t^*, d_t^*, z)} = r_t^d \left(d_t^* - d\right) + \left[r_t^d - (\nu + \delta\chi)p_t\right] fd + (c_t^* - c_t)$$
$$+ \frac{\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1}{1 - \lambda(1 - f)} \left\{ \frac{a}{p_t} \left[r_t^d - (\nu + \delta\chi)p_t\right] + z(1 - \tau_t)Y_t - c_t - (\nu + \delta\chi)p_t d \right\}$$

- Intuition: adjust now vs a little bit later.

Importance of Contemporaneous User Cost

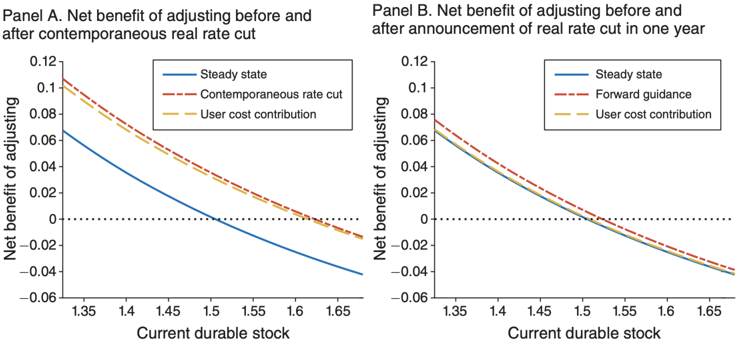


Figure 3. The Net Benefit of Making a Durable Adjustment

Intensive Margin

$$
\begin{aligned}
&\mathbb{E}_t \int_0^{\tau} e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d) ds \\
&\quad = \mathbb{E}_t e^{-\rho\tau} V^{adj}_{x,t+\tau} \left[r^d_{t,t+\tau} + e^{-\delta(1-\chi)\tau} p_{t+\tau} f \right] \\
&\qquad + \mathbb{E}_t \int_0^{\tau} e^{-\rho s} \Psi_{t+s} \left[r^d_{t,t+s} + (1-\lambda(1-f)) e^{-\delta(1-\chi)s} p_{t+s} \right] ds
\end{aligned}
$$

- Intuition: survival probability decreasing in time.

Long-term financing

- FOC for extensive margin the same when durable financed with long-term debt.
- Intuition: adjust now vs a little bit later, so expected change in financing cost matters for adjustment.

Questions

- Convincing?
- What role does the micro evidence play?