Illiquid Lemon Markets and the Macroeconomy*

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Abstract

We study the macroeconomic implications of asymmetric information in capital markets. We build a quantitative capital-accumulation model in which capital is traded in illiquid markets, with sellers having more information about capital quality than buyers. Asymmetric information distorts the terms of trade for sellers of high-quality capital, who list higher prices and are willing to accept lower trading probabilities to signal their type. Led by the model's predictions, we measure the distortions from asymmetric information by studying the relationship between listed prices and trading probabilities in a unique dataset of individual capital units listed for trade. By combining the empirical measurement with the model, we show that information asymmetries can play a quantitatively large role during economic crises when the degree of asymmetric information deteriorates.

Keywords: Asymmetric information, investment, misallocation, trading frictions.

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1 Introduction

Information asymmetries are a prominent feature of real asset markets. As considered in Akerlof (1970)'s seminal work, capital units are heterogeneous in their qualities, and sellers tend to have an informational advantage over buyers. Given that information asymmetries can have important implications for allocations in the capital market, a key question is how these microlevel distortions affect investment and economic activity at the macro level. This question is particularly relevant during economic crises, which are frequently characterized by a decline in agents' ability to assess the quality of assets (see, for example, Gorton, 2008, and references therein).

In this paper, we study the aggregate effects of asymmetric information by combining a quantitative capital-accumulation model with microlevel data on capital markets. Our approach is motivated by two ideas. First, empirical evidence shows that capital markets are illiquid, with capital units remaining listed for significant periods before being traded (e.g., Ramey and Shapiro, 2001). Second, theory indicates that in illiquid markets, asymmetric information distorts the behavior of high-quality capital sellers, who signal their type by listing higher prices and accepting lower trading probabilities (Guerrieri, Shimer, and Wright, 2010; Guerrieri and Shimer, 2014; Chang, 2018). Together, these findings suggest that the distortions arising from asymmetric information can be measured by studying the liquidity of different capital units listed for trade, and that the role of information asymmetries during economic downturns can be linked to how these measured distortions vary during such periods.

To implement this approach, we begin by developing a capital-accumulation model with asymmetric information and illiquid capital markets. We then measure the degree of asymmetric information using microlevel data on capital units listed for trade and combine it with the model to quantify the aggregate effects of asymmetric information. Our analysis indicates that even in economies that are close to full information, information asymmetries can play a quantitatively large macroeconomic role during economic crises when the degree of asymmetric information increases. These effects stem from the impact of asymmetric-information distortions on aggregate investment, the share of idle capital, and the quality of employed capital.

Our model embeds three key ingredients in the neoclassical capital-accumulation framework. First, capital units are heterogeneous in quality (i.e., their output in production). Second, information about capital quality is privately held by the owner of a capital unit. Buyers have access to an information-revealing technology that, with a certain probability, reveals the true quality of the unit. The accuracy of this information technology governs the degree of asymmetric information in the economy. Third, the trade of capital takes place in decentralized markets, in which sellers announce a capital quality and choose at what price to list their capital units, and buyers choose at what price and announced quality to search. We provide conditions whereby the model features a unique fully revealing separating equilibrium, in which sellers announce the true quality of their capital. This separating equilibrium resembles that of the classical model of Spence (1973), in which low types have a high marginal cost of effort and choose not to mimic the education levels of high types. In the context of asset markets, the equivalent marginal effort exerted by high-quality types corresponds to selling with a lower probability: Insofar as there is a probability of buyers detecting the true capital quality, high-quality sellers have a lower marginal cost of not trading than low-quality sellers.

Using the model, we show how the distortions that arise from asymmetric information can be linked to cross-sectional patterns of capital units listed for trade. When capital quality is observed by buyers, high-quality capital attracts more buyers and has a higher selling probability than low-quality capital. However, when capital quality is unobserved by buyers, high-quality capital sellers choose to signal their quality and separate from sellers of low-quality assets. They do so by listing high-quality capital at such high prices that sellers of low-quality capital would not want to mimic their pricing behavior; in turn, higher prices attract fewer buyers and result in lower trading probabilities. The less accurate buyers' information technology, the larger the price sellers of high-quality capital choose in order to separate from low-quality capital, and the larger the covariance between capital units' listed prices and their expected duration on the market. Therefore, by studying the empirical relationship between listed prices and duration, a researcher can measure the degree of information asymmetry in capital markets.

We then apply our proposed measurement approach to a novel dataset of capital units listed for trade. Our dataset contains the history of nonresidential structures (i.e., retail and office space) listed for sale and rent in Spain from one of Europe's biggest online real estate platforms, `Idealista`. The data contain rich information on each unit, including the listed price, exact location, size, age, and other characteristics. Given the dataset's panel structure, we can compute each unit's duration on the platform and the search intensity it attracted, measured by the number of clicks and emails received in a given month.

We document a set of empirical facts consistent with the presence of distortions that arise from asymmetric information in capital markets. First, we show that the component of capital units' listed prices that reflects publicly observed characteristics in the listing (i.e., the predicted price from a hedonic regression of prices on the set of characteristics included in each listing in a narrowly defined market) is negatively correlated with the unit's duration on the market. This empirical fact is consistent with the model's prediction for observed capital quality: Since predicted prices are obtained from observable characteristics, properties with better characteristics (which are reflected by a higher predicted price) have a shorter average duration on the market. Second, we show that the component of a capital unit's price that is orthogonal to the characteristics publicly observed in the listing (i.e., the residual from the hedonic regression described above) is positively related to the unit's duration on the market. This fact is consistent with the presence of asymmetric information about capital characteristics not observed in the listing and the fact that owners of higher capital quality choose higher prices to signal their type, which are associated with lower trading probabilities. Furthermore, our measurement indicates that the degree of asymmetric information features cyclical properties. In particular, the correlation between duration and residual exhibits a strong comovement with economic activity, displaying a sharp increase during the Euro crisis and revealing more significant information asymmetries during economic downturns.

Finally, we integrate our empirical measurements with the model to quantify the aggregate effects of asymmetric information. Our model features three main channels through which information asymmetries in capital markets affect aggregate output. First, higher information asymmetries lead to a lower capital stock. This is because higher information asymmetries are associated with lower revenue for sellers of high-quality capital and, consequently, lower returns to producing capital goods. Second, higher information asymmetries lead to a higher unemployment rate of capital. As information asymmetries increase, so do the listed prices of high-quality capital, which decreases the selling probability and increases the unemployment duration of listed units. Third, a higher degree of asymmetric information is associated with a lower average quality of employed capital. This is because information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities.

By disciplining the degree of asymmetric information in the model with the cross-sectional patterns of capital units listed for trade, we find that changes in the degree of asymmetric information have a large macroeconomic effect. Our measurement indicates that in the steady state, the economy features moderate levels of asymmetric information, with the probability of a lemon's going unnoticed being close to 2%. However, the economy displays large aggregate responses to changes in information technologies. For instance, an unanticipated decline in the accuracy of information technologies akin to that measured during the Euro crisis (with a 2 p.p. increase in the probability of a lemon's going unnoticed) leads to a more than 2% decline in economic activity, followed by a slow recovery. This suggests that policies designed to alleviate information asymmetries in asset markets (e.g., Guerrieri and Shimer, 2014) can play an important role in stabilizing economic downturns.

Related Literature First, our paper is related to the literature on asymmetric information in asset markets, pioneered by Akerlof (1970); Stiglitz and Weiss (1981); and Myers and Majluf (1984), among others. Our framework particularly builds on theories that study these frictions in decentralized markets (e.g., Guerrieri et al., 2010; Delacroix and Shi, 2013; Chang, 2018). Our paper also contributes to the literature on the macroeconomic effects of asymmetric information (e.g., Eisfeldt, 2004; Kurlat, 2013; Bigio, 2015). We contribute to this literature by using a "micro-to-macro" approach, which uses microlevel data on assets listed for trade to measure the degree of asymmetric information, and study their macroeconomic implications.

Second, the paper is related to the literature on misallocation (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008); capital reallocation (e.g., Ramey and Shapiro, 2001; Eisfeldt and Rampini, 2006; Lanteri, 2018; Eisfeldt and Shi, 2018); and asset specificity (e.g., Caballero and Hammour, 1998; Kermani and Ma, 2022). We contribute to this literature by showing that asymmetric information can constitute a sizable source of capital illiquidity and affect the allocation of capital in the economy.[1]

Third, our paper is related to the literature that studies the role of search-and-matching frictions in asset markets. This includes a large body of work on financial markets (see Lagos, Rocheteau, and Wright, 2017, and references therein); housing markets (e.g., Wheaton, 1990; Krainer, 2001; Caplin and Leahy, 2011; Piazzesi, Schneider, and Stroebel, 2020); and physical capital markets (e.g., Kurmann and Petrosky-Nadeau, 2007; Gavazza, 2011; Cao and Shi, 2017; Ottonello, 2017; Wright, Xiao, and Zhu, 2018, 2020; Cui, Wright, and Zhu, 2021). We

[1]The form of misallocation we study builds on that of Gavazza (2016), who uses business-aircraft data to study the welfare effects of trading frictions in the allocation of assets.

contribute to this literature by incorporating asymmetric information in the study of markets characterized by search-and-matching frictions. This interaction is central to our measurement of the degree of asymmetric information and understanding its role in driving economic crises.

Layout The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the effects of asymmetric information in capital markets and discusses how the degree of asymmetric information can be measured from micro-data moments. Section 4 applies this measurement to our dataset and presents a set of empirical facts linked to model predictions. Section 5 combines the model and empirical measurement and quantifies the aggregate effects of asymmetric information, and Section 6 concludes.

2 Model

2.1 Environment

Time is discrete and infinite, and there is no aggregate uncertainty. Final goods are perishable and can be used for consumption or investment. Capital goods are storable and can be used, together with labor services, to produce final goods.

Agents, preferences, and technology The economy is populated by a unit mass of identical households and a unit mass of firms owned by the representative household. Households have preferences over consumption described by the lifetime utility function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)\gamma_n^t$, where c_t and h_t denote per capita consumption and hours worked in period t, respectively; $\gamma_n \geq 1$ denotes the gross population growth within the representative household;[2] $u(\cdot, \cdot)$ is a continuous and differentiable function, increasing in the first argument and decreasing in the second; $\beta \in (0, 1)$ is the subjective discount factor; and $\mathbb{E}_t$ denotes the expectation conditional on the information set available in period t. Households have access to a linear technology to produce new capital goods using final goods.

A continuum of identical firms with measure one have access to a constant-returns-to-scale technology to produce final goods using capital and labor as inputs, $y_{jt} = f_t(\mathcal{K}_{jt}, l_{jt}) \equiv \mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$, where y_{jt}, $\mathcal{K}_{jt}$, and l_{jt} denote the output, capital input, and labor input of firm

[2] We include population and technology growth in the model to better match the investment rates observed in the data, which are sizable flows for capital markets.

j in period t, respectively; $\gamma \geq 1$ denotes the exogenous growth rate of labor-augmenting technology in the economy; and $\alpha \in (0, 1)$. Each period, with i.i.d. probability φ, a firm receives an exit shock and must exit the economy; exiting firms cannot produce, and transfer their capital holdings to households. After exit shocks are realized, a new mass φ of firms enter the economy. In this setup, operating firms will be capital buyers and households capital sellers (selling new capital or capital from exiting firms).

The model features three main departures from the neoclassical capital-accumulation model: heterogeneity in capital quality, a decentralized market for capital, and information frictions. We describe each of these elements next.

Capital-quality heterogeneity Studying information asymmetries in capital markets requires introducing heterogeneity in these goods. To do so, we consider an environment in which the capital stock is composed of infinitesimal indivisible units (i.e., capital goods are available to trade in integer quantities only, and agents hold a mass of these units). Capital units are heterogeneous in two dimensions: an "observed quality" $\omega \in \Omega \equiv [\omega_1, \ldots, \omega_{N_\omega}]$, with $\omega_r < \omega_s$ for $r < s$, and an "unobserved quality" $a \in \mathcal{A} \equiv [a_1, \ldots, a_{N_a}]$, with $a_r < a_s$ for $r < s$. While the observed quality ω of a unit is assumed to be perfectly observable by all market participants, unobserved quality a is the private information of the owner of the capital unit and the source of asymmetric information in the model, which is further discussed below. The capital services a capital unit provides are determined by these qualities, with the capital services of a capital unit i being given by $\omega_i a_i$. Capital services employed as input in production by firm j are then given by $\mathcal{K}_{jt} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k_{jt+1}(\omega, a)$, where $k_{jt+1}(\omega, a)$ is the mass of capital of quality (ω, a) employed in production by firm j in period t. In our application, we interpret capital quality broadly as representing any characteristic that increases the marginal product of capital. For example, in the case of retail space, capital quality can capture the physical conditions of the property, the ability of the store to attract customers, or the atmosphere of the space for customers.

Terms of trade in the decentralized market for capital In models of asymmetric information, search-and-matching frictions can play a key role in shaping equilibrium outcomes, because they provide a signaling mechanism for sellers to differentiate themselves through price-setting behavior (see, for example, Guerrieri et al., 2010; Chang, 2018). Based on the empirical evidence of studies that characterize the trading process in capital markets (e.g., Gavazza, 2011; Ottonello, 2017), we assume that capital goods are traded in a decentralized market subject to search-and-matching frictions.

The decentralized capital market is organized in a continuum of submarkets, indexed by $(\omega, \hat{a}, q)$, where ω is the observed quality, $\hat{a}$ is the unobserved quality announced by the seller, and q is the listed price. Search is directed: Sellers can choose at what announced unobserved quality and price to list their capital units; buyers can choose at what observed quality, announced unobserved quality, and price to search, and dedicate labor to search and match.[3] In each submarket $(\omega, \hat{a}, q)$ the market tightness, denoted by $\theta_t(\omega, \hat{a}, q)$, is defined as the ratio between buyers' hours of search and the mass of capital listed by sellers.[4] In visiting submarket $(\omega, \hat{a}, q)$ in period t, sellers face a probability $p(\theta_t(\omega, \hat{a}, q))$ of finding a potential buyer for their unit and buyers match at a rate $\mu_t(\theta_t(\omega, \hat{a}, q))$ per hour of search potential units to buy, where $p(\theta) = \min\{\overline{m}\theta^{1-\eta}, 1\}$ with $\eta \in (0, 1)$.[5]

When sellers list a capital unit in submarket $(\omega, \hat{a}, q)$, they commit to allowing potential buyers to inspect the unit using the technology described below. If no new information about the capital quality is revealed during the inspection or if the inspection indicates that capital quality is not below that announced (i.e., $a' \geq \hat{a}$), sellers and buyers commit to trade the capital unit at the listed price q. If the inspection reveals that the true quality of the capital a' is lower than the announced quality (i.e., $a' < \hat{a}$) and there are gains from trade between the buyer and seller, then trade occurs at the inspection-adjusted price $q_t^P(\omega, a', \hat{a}, q) \leq q$. Here, we assume that the transacted price $q_t^P(.)$ results from a Nash bargaining problem.[6] In Appendix A, we relax this assumption and show that the equilibrium characterization

[3]The assumed directed-search framework is similar to that used by Shimer (1996), Moen (1997), and Menzio and Shi (2011) in the labor market and Ottonello (2017) in the capital market. For a recent survey of the literature on directed search in labor, housing, and monetary economics, see Wright, Kircher, Julien, and Guerrieri (2019).

[4]Following the directed search literature (see, e.g., Moen, 1997; Menzio and Shi, 2011), in submarkets that are not visited by any seller, $\theta_t(\omega, \hat{a}, q)$ is an out-of-equilibrium conjecture that helps determine the equilibrium.

[5]The functional form of the matching probability can be obtained from a Cobb-Douglas matching technology $\mathcal{M}_t(k^s(\omega, \hat{a}, q), \gamma^t v^s(\omega, \hat{a}, q)) = \min\{\overline{m}(k^s(\omega, \hat{a}, q))^\eta (\gamma^t v^s(\omega, \hat{a}, q))^{1-\eta}, k^s(\omega, \hat{a}, q)\}$, where $k^s(\omega, \hat{a}, q)$ and $v^s(\omega, \hat{a}, q)$ denote the mass of capital listed by sellers and hours dedicated by buyers to search in submarket $(\omega, \hat{a}, q)$, respectively, and $\overline{m} > 0$; given the labor-augmenting technology in the production of final goods, the labor-augmenting technology in the matching sector is necessary for a balanced-growth path.

[6]More specifically, we assume that the transacted price is the minimum between the listed price and the bargained price, which reflects the commitment to sell at the listed price if favorable to the buyer. The only assumption we impose on the bargaining problem is that the seller's bargaining power ϕ satisfies $\phi \leq \eta$, so that the equilibrium bargained price is weakly lower than the price sellers would obtain when announcing the quality truthfully.

remains the same under general post-inspection trading protocols. Finally, if the inspection reveals that the quality of the capital a' is such that $a' < \hat{a}$ and there are no gains from trade, the match is dissolved without trade.

Finally, since our main focus is on capital markets, we assume that final goods and labor services are traded in Walrasian markets.

Information structure An information asymmetry arises because capital quality has a component that is private information to its owner, a_i. We are interested in studying how the degree of asymmetric information in the economy affects capital accumulation. For this, we assume that after having searched and matched with a capital unit and before purchasing it, buyers have access to a technology to inspect the unit. Similar to Menzio and Shi (2011), this information-revealing technology is such that in any submarket $(\omega, \hat{a}, q)$ there is a probability ψ that the buyer learns the true type (ω, a) of the capital good and a probability $1 - \psi$ that the inspection is uninformative. Hence, ψ parameterizes the degree of asymmetry of information in the economy, nesting the cases with full-information when $\psi = 1$ and with complete asymmetric information when $\psi = 0$ (since there cannot be any discovery of the unobserved quality). As will be seen later, the inspection technology effectively acts as a punishment for not truthfully revealing the unobserved capital quality.

It is worth noting that the information asymmetry is present both when households sell newly produced units and when they sell used capital from exiting firms. The underlying assumption is that households know the unobserved quality of the new capital they produce, as well as that of the capital transferred to them from the firms they own. Appendix C.2.1 shows that this formulation is isomorphic to one with a representative intermediary operating in a competitive market. In this alternative environment, the intermediary purchases new capital from households and used capital from exiting firms, with full knowledge of its quality, and then resells the capital to other uninformed firms. In Appendix C.2.1, we formally establish an equivalence result between these two interpretations.

Information asymmetry requires that we specify agents' beliefs about the type of capital available for sale, given a listed price and observable characteristics. We assume that all potential buyers have the same beliefs. We describe beliefs by the mapping $\pi_t(a|\omega, \hat{a}, q)$: $\Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to [0, 1]$, which denotes the probability that a unit of capital is of unobserved type a, given observed type ω, announced quality $\hat{a}$, and price q. After purchasing a unit of capital, buyers obtain full information about its quality. Sellers are assumed not to have recall on the capital quality of their units sold.

Timing The timing of events within each period is as follows:

(i) Exit shocks are realized, and a mass φ of new firms enter the economy. Firms that exit the economy transfer their capital to households.

(ii) Households choose the capital units they list for sale, their prices, and their announced qualities, which are perfectly observed by all agents. Incumbent non-exiting firms and new firms search and match with potential capital units to buy.

(iii) Firms inspect the matched capital units and decide whether to buy.

(iv) Incumbent non-exiting firms and new firms hire workers, produce final goods, and pay wages. All agents holding capital units pay a maintenance cost δ per unit of effective capital in terms of final goods. Households invest in new capital units and consume.

2.2 Optimization

Households Each period, households produce new capital goods. They do so by choosing their total investment in terms of final goods i_t and the resulting quality of new capital is exogenous and random, governed by the distribution function $g : \Omega \times \mathcal{A} \rightarrow [0, 1]$, which describes the measure of new capital of each quality. Since households do not have access to a production technology, their capital revenue comes from selling these newly produced units of capital together with unemployed capital transferred by exiting firms to operating firms. The evolution of capital holdings by households is then given by

$$k_{Ht+1}(\omega, a) = (1 - p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a))))(k_{Ht}(\omega, a) + \varphi K_{Ft}(\omega, a)) + g(\omega, a)i_t, \quad (1)$$

where $k_{Ht+1}(\omega, a)$ denotes capital of quality (ω, a) held by the household at the end of period t; $\hat{a}_{Ht}(\omega, a)$ and $q_{Ht}(\omega, a)$ denote the household's choice of announced capital quality and price to list units of quality (ω, a); $p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a)))(k_{Ht}(\omega, a) + \varphi K_{Ft}(\omega, a))$ denotes the mass of capital of type (ω, a) matched with buyers given the household's choice of submarket; and $K_{Ft}(\omega, a)$ denotes the aggregate capital of quality (ω, a) held by firms at the beginning of period t—a fraction φ of which is transferred to households by firms that exit. For expositional simplicity, equation (1) abstracts from households that list a unit of capital in multiple submarkets and from capital not being sold following an inspection that reveals a different quality from that announced (Appendix A shows that this does not happen in equilibrium).

We write the household's optimization problem recursively. At the beginning of a period, the individual state for the household is a matrix of its capital holdings, given by $\mathbf{k} \equiv \begin{bmatrix} k(\omega_1, a_1) & ... & k(\omega_{N_\omega}, a_1) \\ ... & ... & ... \\ k(\omega_1, a_{N_a}) & ... & k(\omega_{N_\omega}, a_{N_a}) \end{bmatrix}$. The recursive problem of the representative household is then given by

$$V_{Ht}(\mathbf{k}) = \max_{\{c,h,\{k'(\omega,a),\hat{a}(\omega,a),q(\omega,a)\},i\geq 0\}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'), \tag{2}$$

subject to the budget constraint

$$\begin{aligned} & c\gamma_n^t + i + \delta \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a \left[(1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))))(k(\omega, a) + \varphi K_{Ft}(\omega, a))\right] \\ & = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \left[(1-\psi)q(\omega, a) + \psi q^P(\omega, a, \hat{a}(\omega, a), q)\right] p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))(k(\omega, a) + \varphi K_{Ft}(\omega, a)) \\ & + w_t h \gamma_n^t + Div_{Ft} \end{aligned}$$

and the law of motion for capital

$$k'(\omega, a) = (1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))))(k(\omega, a) + \varphi K_{Ft}(\omega, a)) + ig(\omega, a),$$

where Div_{Ft} denotes the dividends transferred by firms in period t. The optimal level of investment (provided that $i > 0$) is characterized by the Euler equation

$$1 = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} g(\omega, a)\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta\gamma_n \frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$ is the household's discount factor, $\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy functions for capital accumulation associated with problem (2), and $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega,a)} \frac{1}{u_{ct}(\mathbf{k})\gamma_n^t}$ is the household's marginal value of capital of type (ω, a) measured in final goods, which satisfies the following recursive problem:

$$\nu_t^{\mathrm{s}}(\omega,a,\mathbf{k}) = \max_{\{\hat{a},q\}} \; p(\theta_t(\omega,\hat{a},q))((1-\psi)q + \psi q^P(\omega,a,\hat{a},q)) \\ + (1-p(\theta_t(\omega,\hat{a},q)))\left(\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega,a,k_{Ht+1}(\mathbf{k})) - \delta\omega a\right). \tag{3}$$

Finally, the optimal labor supply is given by the first-order condition $u_{ht}(\mathbf{k}) = u_{ct}(\mathbf{k})w_t$.

Firms Firms accumulate capital by buying it from sellers in the decentralized market, which requires paying for hours of labor to search for potential units that are a good match for the firm. Abstracting from the possibility that firms might want to sell capital (which, as shown in Appendix A, does not occur in equilibrium), their capital holdings evolve according to

$$k_{jt+1}(\omega,a) = \sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) v_{jt}(\omega,\hat{a},q)\,\mathrm{d}q + k_{jt}(\omega,a), \tag{4}$$

where $v_{jt}(\omega,\hat{a},q)$ denotes the hours of work hired by firms to search and match with sellers in submarket $(\omega,\hat{a},q)$; $\mu_t\left(\theta_t\left(\omega,\hat{a},q\right)\right)v_{jt}(\omega,\hat{a},q)$ the mass of capital matched by these workers; and $\iota_t(a|\omega,\hat{a},q)$ the share of capital units of quality a found in submarket $(\omega,\hat{a},q)$.

Conditional on not exiting, the recursive problem of the firm is given by

$$V_{Ft}(\mathbf{k}) = \max_{\{l,\{v(\omega,\hat{a},q)\geq 0\},\{k'(\omega,a)\}\}} \; \mathbb{E}_a[div + \Lambda_{t,t+1}((1-\varphi)V_{Ft+1}(\mathbf{k}') + \varphi V_{t+1}^{\mathrm{exit}}(\mathbf{k}'))], \tag{5}$$

subject to the definition of dividends (in terms of final goods transferred to households)

$$div = \left(\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a k'(\omega,a)\right)^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l - \delta \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a k'(\omega,a) \\ - \sum_{\omega\in\Omega}\sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} [(\psi \sum_{a\in\mathcal{A}} \iota_t(a|\omega,\hat{a},q) q_t^P(\omega,a,\hat{a},q) + (1-\psi)q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t] v(\omega,\hat{a},q)\,\mathrm{d}q$$

and the law of motion for capital

$$k'(\omega,a) = \sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) v(\omega,\hat{a},q)\,\mathrm{d}q + k(\omega,a), \tag{6}$$

where $\mathbb{E}_a[\cdot]$ denotes the expectation under the belief function $\pi_t(a|\omega,\hat{a},q)$; $\Lambda_{t,t+1}$ denotes households' discount factor; $V_t^{\mathrm{exit}}(\mathbf{k}) \equiv \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} k(\omega,a)\nu_t^{\mathrm{s}}(\omega,a,\mathbf{K}_{Ht})$ denotes the household's value of exiting firms with capital holdings $\mathbf{k}$; w_t denotes the wage rate in period t; and $\mathbf{K}_{Ht}$ denotes the matrix of capital holdings by households in period t, which is taken as given by individual firms. Problem (5) abstracts from the scenario in which, after the inspection, trade does not occur, and there are no gains from trade for quality $a' < \hat{a}$ (Appendix A provides general conditions for $q_t^P(\omega, a, \hat{a}, q)$ for which this does not happen in equilibrium).

The following result characterizes firms' optimal choices of capital and labor.

***Proposition* 1.** *The firm's value function $V_{Ft}(\mathbf{k})$ is linear in capital stocks—i.e., it can be expressed as $V_{Ft}(\mathbf{k}) = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \nu_t^{\mathrm{b}}(\omega, a)k_t(\omega, a)$. This marginal value of capital holdings satisfies the recursive problem*

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right], \tag{7}$$

where $Z_t \equiv \alpha\left(\frac{\gamma^t(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$. Labor demand in the production of final goods is linear in $\mathcal{K}_t$ and given by $l_t(\mathcal{K}_t) = \mathcal{K}_t \times \left(\frac{(1-\alpha)\gamma^{t(1-\alpha)}}{w_t}\right)^{\frac{1}{\alpha}}$.

Proof. All proofs are relegated to Appendix A.2. ■

Proposition 1 implies that the buyer's value of a capital unit of a given quality does not depend on other capital holdings. In particular, the value of capital with quality (ω, a) is given by the utility flow generated by its production plus its continuation value, which takes into account the probability of exiting production and becoming a seller of capital.

Finally, a firm's optimal search activity across different submarkets is characterized by

$$v_t(\omega, \hat{a}, q)\left(\underbrace{((1-\psi)q + \psi\mathbb{E}_a(q^P(\omega, a, \hat{a}, q)|\omega, \hat{a}, q))}_{\text{Expected price}} + \underbrace{\frac{w_t}{\mu_t(\theta(\omega, \hat{a}, q))}}_{\text{Search cost}} - \underbrace{\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right)}_{\text{Expected value}}\right)^+ = 0, \tag{8}$$

for all $(\omega, \hat{a}, q)$ (with $(x)^+ \equiv \max(x, 0)$), which shows that firms are willing to search for capital in a given submarket if the expected marginal cost of purchasing capital in that market, including its expected price and search cost, does not exceed its expected value. Given that submarkets differ in their price q, firms are indifferent between buying capital with the same expected value in different submarkets, insofar as units with a higher price have an associated higher matching rate $\mu_t(\omega, \hat{a}, q)$.

2.3 Equilibrium

Appendix A.1 provides the formal definition of the competitive equilibrium of the overall economy. Here, we focus on the equilibrium characterization of the signaling game. In the rest of this section, and for the analytical characterization of the equilibrium, we restrict our attention to the equilibrium in the balanced-growth-path and the fully-revealing separating solution of the signaling game (Appendix A.1 shows this is the unique type of equilibria).

Equilibrium characterization Since the strategy space contains both the announcement of the unobserved quality and the posted price, sellers can signal their unobserved quality and separate from each other by differing along either of these two dimensions. As in Guerrieri et al. (2010), we characterize the equilibrium as an allocation that solves the following sequence of constrained optimization problems $\mathcal{P}_j(\omega)$.

***Definition* 1.** *For a given observed quality ω and aggregate variables, the solution to problem $\mathcal{P}_j(\omega)$ is a vector $(q(\omega, a_j), \hat{a}(\omega, a_j))$ that solves*

$$\begin{aligned}\nu^{\mathrm{s}}(\omega, a_j) = \max_{\{q(\omega,a_j),\hat{a}(\omega,a_j)\}} & p\left(\theta\left(\omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right)\right)\left[(1-\psi)q(\omega, a_j) + \psi q^P(\omega, a_j, \hat{a}(\omega, a_j), q)\right] \\ & + (1 - p(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j))))\left[\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_j) - \delta\omega a_j\right] \end{aligned} \tag{9}$$

subject to

$$\begin{aligned}&\mathbb{E}_a\left((1-\psi)q(\omega, a_j) + \psi q^P(\omega, a, \hat{a}(\omega, a_j), q) \,|\, \omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right) \\ &\qquad = \mathbb{E}_a\left(\nu^{\mathrm{b}}(\omega, a)\right) - \frac{w_t}{\mu\left(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j))\right)},\end{aligned} \tag{10}$$

and

$$\begin{aligned}\nu^{\mathrm{s}}(\omega, a_{j'}) \geq\; & p\left(\theta\left(\omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right)\right)\left[(1-\psi)q(\omega, a_j) + \psi q^P(\omega, a_{j'}, \hat{a}(\omega, a_j), q)\right] \\ & + (1 - p(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j))))\left[\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_{j'}) - \delta\omega a_{j'}\right] \quad \textit{for all } j' < j.\end{aligned} \tag{11}$$

In problem $\mathcal{P}_j(\omega)$, the seller of capital with quality (ω, a_j) announces an unobserved quality $\hat{a}(\omega, a_j)$ and lists a price $q(\omega, a_j)$ to maximize expected revenues subject to two constraints. The first constraint is the buyer's search optimality condition, which pins down the market tightness for a given set of beliefs and seller's choices. In addition, the seller is constrained by a set of no-mimicking conditions, which require that sellers of lower quality weakly prefer their own terms of trade rather than mimicking the terms of trade chosen by the seller of unobserved quality a_j. Hence, an allocation that solves the above sequence of optimization problems effectively describes a separating equilibrium. We will focus on separating equilibria in which sellers of different unobserved qualities truthfully reveal their unobserved quality, which we defined above as a fully revealing separating equilibria.[7]

As is well known in the signaling games literature, the sequence of problems $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ may admit multiple solutions, each with the corresponding equilibrium supported by appropriate out-of-equilibrium beliefs. Indeed, the definition of a fully revealing separating equilibrium does not impose any constraint on off-equilibrium beliefs, which can potentially lead to multiplicity. Therefore, we impose more structure on these beliefs by considering equilibria that satisfy the *D1 criterion* of Cho and Kreps (1987)—an equilibrium refinement commonly used in signaling games. This criterion first identifies the set of sellers who are more likely to deviate from equilibrium choices and then requires that buyers have beliefs consistent with this set when observing a deviation. Last, it eliminates equilibria in which a seller's payoff from the deviation under the worst buyer's consistent belief is not equilibrium dominated. This refinement is enough to establish the existence and uniqueness of equilibrium, which we analyze in the following section.

3 The Micro Effects of Asymmetric Information

This section uses the model to understand how asymmetric information distorts capital markets. Section 3.1 examines how the accuracy of information technologies affects the prices and duration of capital units listed for trade. For pedagogical purposes, the analysis is based on two quality types. Appendix A.1 characterizes the equilibrium in the general case. Based on these model predictions, Section 3.2 discusses how the degree of asymmetric information can be identified from micro-data moments.

[7]In our analysis, we omit unreasonable separating equilibria in which sellers of different unobserved qualities choose to set different prices (so buyers do indeed identify their different unobserved qualities) and announce different, but untrue, qualities. In Appendix A, we argue that these alternative equilibria are dominated by the equilibrium analyzed here.

3.1 Asymmetric Information, Prices, and Duration

We begin by focusing on a simple case that can be characterized analytically, with $\Omega = \{\omega_L, \omega_H\}$ and $\mathcal{A} = \{a_L, a_H\}$ (where L and H denote low and high qualities, respectively). Assuming that depreciation costs are small relative to the values of sellers (i.e., $\delta \to 0$), we first show how the terms of trade change in the cross-section of observed characteristics. The analysis then describes how asymmetric information affects the trade of units with different unobserved characteristics.

Observed capital quality We begin by focusing on the cross-sectional predictions of the model for units with different observed capital qualities. To isolate these differences, we set $a_L = a_H = \overline{a}$. In this case, the solution to the seller's problem is characterized by the first-order condition

$$p'(\theta(\omega, \overline{a})) \left(\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, \overline{a}) \right) = \chi$$

for all $\omega \in \Omega$, where we have substituted the expression for the price $q(\omega, a)$ derived from the buyer's optimal search strategy. The optimal choice of market tightness balances the marginal benefit of a higher trading probability (left-hand side) with the reduction in price required by potential buyers in order to visit the chosen submarket (right-hand side). The following proposition formalizes this result by deriving the optimal price of capital and market tightness for each type of capital under full information.

***Proposition* 2.** *If $a_L = a_H = \overline{a}$, the price and market tightness for capital of quality ω are given by*

$$q(\omega, \overline{a}) = \eta \nu^{\mathrm{b}}(\omega, \overline{a}) + (1 - \eta) \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, \overline{a})$$

and

$$\theta(\omega, \overline{a}) = \left(\frac{\bar{m}(1 - \eta)}{\chi} \left(\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, \overline{a}) \right) \right)^{1/\eta}.$$

Proposition 2 shows that the equilibrium price is a weighted average of the seller's and buyer's value of capital, and the selling probability is an increasing function of the surplus $\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, \overline{a})$. This optimal choice is graphically represented in Figure 1 for types $(\omega_L, \overline{a})$ and $(\omega_H, \overline{a})$ and a given search cost χ. Dashed lines represent the iso-cost curves of buyers, with the highest one corresponding to the high-quality ω_H. These curves denote the combination of prices and purchase probabilities that generate the same expected cost to buyers and are derived from equation (10). Curves are downward-sloping because buyers are indifferent between submarkets if higher prices are associated with higher matching rates with sellers. They are increasing in ω because buyers can obtain higher revenues by using capital of higher quality. Similarly, solid lines denote the iso-revenue curve of sellers—i.e., the combination of prices and market tightness that produce the same expected revenues—and are derived from Equation (9). These are downward-sloping because the seller is willing to accept a lower price if the sale's probability increases. Note that the iso-revenue curves have a lower slope for high-quality capital. This occurs because the seller's outside option (i.e., the continuation value) increases with the quality of its capital, which leads the seller to require lower "compensation" in terms of a higher sale probability for a given reduction in the price. In equilibrium, sellers choose the submarket that maximizes their utility subject to buyers' indifference curves.

[8]

Proposition 2 and Figure 1 show that under full information, the price of a unit of capital and its matching rate are increasing in its quality, which implies the following result.

***Corollary* 1.** *If $a_L = a_H = \overline{a}$, capital units with higher prices match at a higher rate: $q^{\mathrm{FI}}(\omega_H, \overline{a}) > q^{\mathrm{FI}}(\omega_L, \overline{a})$ and $p\left(\theta^{\mathrm{FI}}(\omega_H, \overline{a})\right) > p\left(\theta^{\mathrm{FI}}(\omega_L, \overline{a})\right)$.*

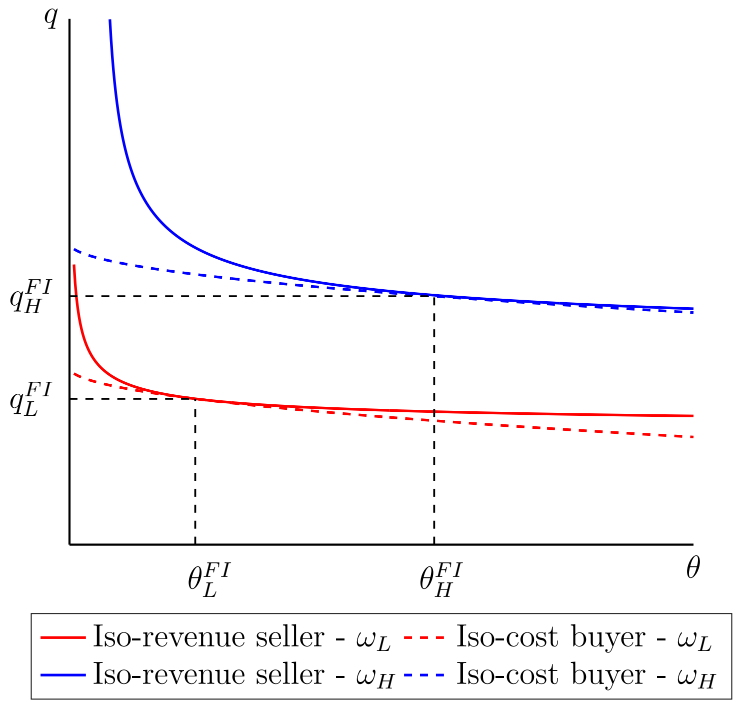
To understand the intuition behind this corollary, replace the equilibrium price of capital in the optimal search strategy of the buyer to obtain

$$(1-\eta)\left(\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, \overline{a})\right) = \frac{\theta(\omega, \overline{a})\chi}{p\left(\theta(\omega, \overline{a})\right)}. \tag{12}$$

Equation (12) requires that in equilibrium, the seller's net benefit from buying a unit of capital must be equal to its expected search cost. As in standard models of directed search, the surplus (given by $\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, \overline{a})$) is "split" according to the elasticity of the matching function. Thus, since the price of capital scales with the buyer's value less than proportionally ($\eta < 1$), the net gain of buying capital is increasing in this value. By non-arbitrage, the

[8] To see that the solution is unique, notice that the dotted line is less convex than the solid line (formally, the second-order derivative is lower for the buyer's indifference condition). This implies that it is possible to construct a strictly monotonous transformation of θ, such that the dotted line depicts a linear relationship while the solid line remains strictly convex. Replacing θ with this transformation, we obtain the standard problem of finding the utility-maximizing intersection of strictly convex preference curves and a convex budget set, which has a unique solution. Alternatively, note that the second-order condition of the seller's problem is satisfied because $p''(\theta) < 0$ for $p(\theta) < 1$, which is the empirically relevant case.

Figure 1: Competitive Equilibrium under Full Information



expected search cost must be higher for capital units with higher quality—and thus higher value—which implies that buyers (sellers) of these units match at a lower (higher) rate.

Unobserved capital quality Next, we consider the solution to the seller's problem under asymmetric information. To isolate the differences, we set $\omega_L = \omega_H = \overline{\omega}$. As expected, capital of the lowest unobserved quality a_L is sold under the full-information terms of trade. However, the choice of the seller of quality a_H might be affected by information frictions. In this case, the solution to the seller's problem is characterized by the first-order condition

$$p'(\theta(\overline{\omega}, a_H)) \left(\nu^{\mathrm{b}}(\overline{\omega}, a_H) - \frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{s}}(\overline{\omega}, a_H) \right) = \chi + \zeta(\overline{\omega}, a_H) \tag{13}$$

and the complementary slackness condition

$$\begin{aligned} &\zeta(\overline{\omega}, a_H) \Big[p(\theta^{FI}(\overline{\omega}, a_L)) \left(q^{FI}(\overline{\omega}, a_L) - \frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{s}}(\overline{\omega}, a_L) \right) \\ &- p(\theta(\overline{\omega}, a_H)) \left((1-\psi) q(\overline{\omega}, a_H) + \psi q^P(\overline{\omega}, a_L, a_H, q) - \frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{s}}(\overline{\omega}, a_L) \right) \Big] = 0, \end{aligned} \tag{14}$$

where $\zeta(\overline{\omega}, a_H)$ denotes the Lagrange multiplier of the no-mimicking constraint, which requires that the lower type a_L does not want to mimic the choices made by the higher type a_H. Notice that the constraint incorporates the fact that sellers who mimic the choices of sellers with other qualities sell at the posted price only when the inspection is uninformative. The presence of the inspection stage introduces a small deviation from the standard signaling model à la Spence (1973). For high values of ψ—i.e., when the extent of asymmetric information is not severe—sellers might not need to signal their quality, since the probability of detection is high. This intuition is formalized in the following proposition, which is a special case of Proposition 5 for the two-type example considered here.

***Proposition* 3.** *Let $\psi^* \in [0,1]$ be defined by*

$$p(\theta^{FI}(\overline{\omega}, a_L)) \left[q^{FI}(\overline{\omega}, a_L) - \frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{S}}(\overline{\omega}, a_L) \right] \\ = p(\theta^{FI}(\overline{\omega}, a_H)) \left[(1-\psi^*) q^{FI}(\overline{\omega}, a_H) + \psi^* q^P(\overline{\omega}, a_L, a_H, q) - \frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{S}}(\overline{\omega}, a_L) \right].$$

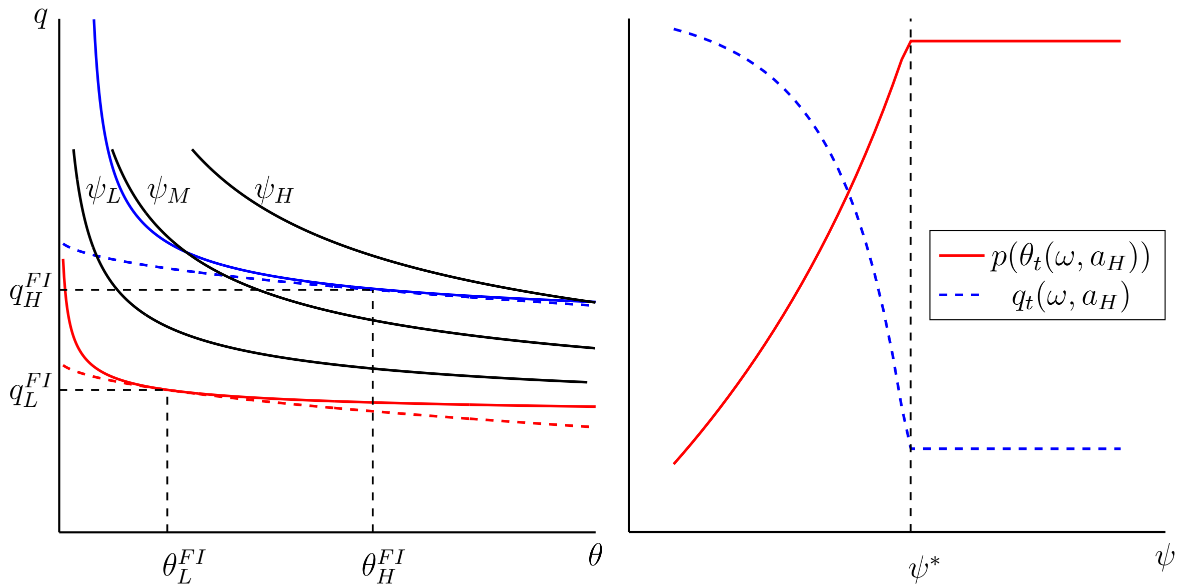
The seller of quality a_L chooses the same terms of trade as under full information. For sellers of quality a_H, there are two cases:

(i) $\psi \geq \psi^*$: the incentive-compatibility constraint is not binding and $\theta(\overline{\omega}, a_H)$ solves the optimality condition (13) with $\zeta(\overline{\omega}, a_H) = 0$.

(ii) $\psi < \psi^*$: the incentive compatibility constraint is binding (i.e., $\zeta(\overline{\omega}, a_H) > 0$) and $\theta(\overline{\omega}, a_H)$ solves (14). The optimal terms of trade satisfy $q(\overline{\omega}, a_H) > q^{FI}(\overline{\omega}, a_H)$ and $p(\theta(\overline{\omega}, a_H)) < p(\theta^{FI}(\overline{\omega}, a_H))$. Therefore, the difference in the expected time to sell across qualities increases as ψ decreases—i.e., $d\left[p(\theta^{FI}(\overline{\omega}, a_L))/p(\theta(\overline{\omega}, a_H))\right]/d\psi < 0$. Thus, if information asymmetries are strong enough (i.e., ψ is low enough), then $p(\theta(\overline{\omega}, a_H)) < p^{FI}(\theta(\overline{\omega}, a_L))$.

We illustrate the equilibrium under asymmetric information in Figure 2. In a fully revealing separating equilibrium with signaling, the outcome in the submarket for the lowest quality capital is the same as the one obtained under full information (see Figure 1). However, the outcome in the submarket for high-quality capital could be distorted by the fact that sellers maximize the expected value subject to the no-mimicking constraint, whereby low-quality sellers do not have a strict preference for participating in the same submarket. The different possibilities are illustrated in Panel (A) of Figure 2. In addition to the iso-revenue and iso-cost curves shown in Figure 1, the figure includes the no-mimicking constraint behind the

Figure 2: Competitive Equilibrium under Asymmetric Information



(A) Contraints Arising from AI (B) Optimal Allocations under AI

complementary slackness condition (14) for three values of ψ: $\psi_L < \psi_M < \psi^* < \psi_H$. For a given price, any market tightness to the right of the solid black lines violates the constraint.

When the information technology is good enough (e.g., ψ_H in Figure 2), the seller of high-quality capital can choose the full-information market tightness. Sellers of low-quality capital do not want to mimic this choice because, with a high probability, the inspection reveals their lower quality and they end up selling at a lower price.

As ψ decreases below ψ^* (e.g., ψ_M in Figure 2), sellers of low-quality capital are more likely to be able to sell without being detected by the inspection. Then the full-information tightness violates the constraint, and the seller chooses a higher price and a lower tightness to signal the higher quality of capital. Optimal tightness is determined by the intersection between the no-mimicking constraint and the buyer's isocost curve evaluated at a_H. This lower sale probability is more costly for low-quality sellers given the information-revealing technology, which could reveal their true type and lead to a low sale price. Because of these additional delays, low-quality sellers weakly prefer their own submarket. If the informativeness of the inspection is very low (e.g., ψ_L in Figure 2), then the required signaling in the form of delays is such that capital of higher quality ends up selling with a lower probability than low-quality capital. Therefore, ψ governs the potential distortions to terms of trade from information asymmetries (see Panel (B) of Figure 2).

Thus, when $\psi < \psi^*$, the optimal terms of trade become distorted relative to the full-information case. Note that, even if the underlying equilibrium is fully separating (i.e., no mimicking occurs on the equilibrium path), it is the off-equilibrium *threat* of being detected that introduces the distortion.[9]

Multiple capital qualities So far, we have focused on the simple case with two types of capital qualities, which can be characterized analytically. We now provide a quantitative illustration to show how the effects of asymmetric information on prices and duration extend to a setting with multiple types of observed and unobserved capital quality. For this, we assume that the observed and unobserved qualities are distributed according to two independent log-normal distributions with variance σ_j^2 for $j \in \{\omega, a\}$.[10] For the quantitative exercises presented in this section, we use the calibrated version of the model discussed in detail in Section 5.

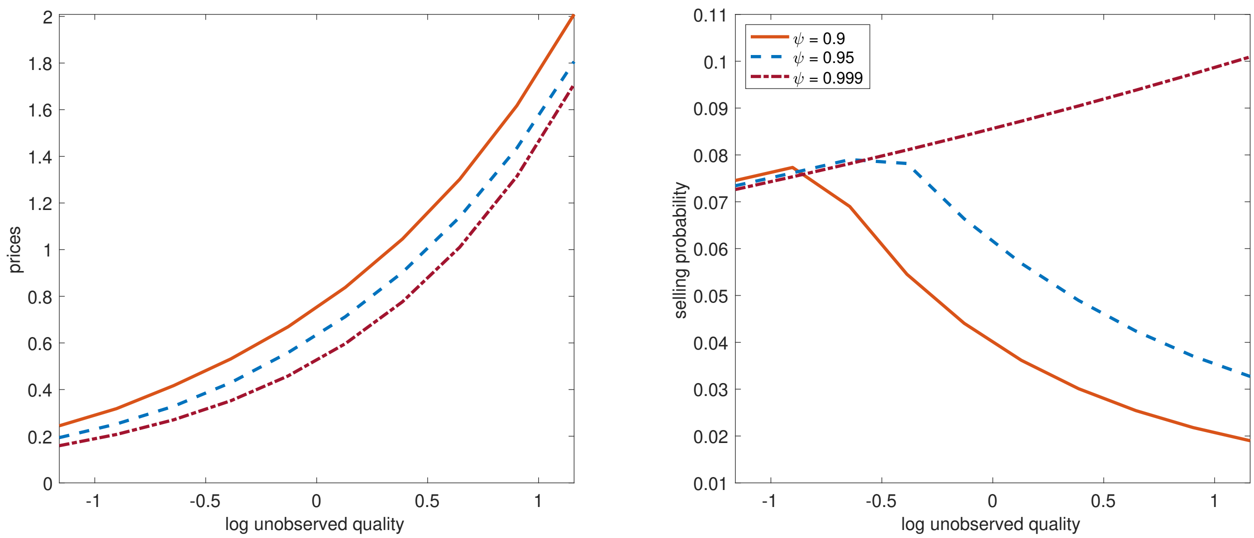
Figure 3 depicts listed prices and associated selling probabilities for different levels of unobserved capital quality. The dotted red line shows that in the limiting case of $\psi \to 1$, in which the information technology approaches full information, sellers of higher quality units list them at a higher price and sell them with a higher probability. As discussed in the analytical example above, this is because, under full information, capital units of high quality are relatively more attractive to buyers, which leads to a higher trading probability for sellers.

The blue and orange lines show that as the accuracy of the information technology declines (i.e., a lower value of ψ), the relationship between listed prices and unobserved capital quality becomes negative and steeper. This is because a more imprecise information technology creates stronger incentives for sellers of low-quality capital to mimic higher-quality sellers. In turn, high-quality capital sellers respond to the inferior information technology by increasing the listed price of their units, which separates them from low-capital-quality sellers, who are not willing to bear the cost of the associated lower trading probabilities. For

[9] In this off-equilibrium threat, the inspection technology acts as a punishment for mimicking. As can be seen from the no-mimicking condition (14), a higher quality of the inspection technology has a similar effect on the punishment as a lower post-inspection price. The fundamental feature that allows high-quality sellers to signal their type is the fact that $(1-\psi)q(\overline{\omega}, a_H) + \psi q^P(\overline{\omega}, a_L, a_H, q) < q(\overline{\omega}, a_H)$—i.e., for the low-type seller, the expected price in a given submarket is lower than the price the high-type seller would receive.

[10] The distributional assumption is without loss of generality, but we adopt it to operationalize our quantitative analysis. We normalize the mean of both types of capital qualities to one. The assumption that qualities ω and a are independent is also without loss of generality, since one could interpret the observable quality as the conditional expected quality $\omega + \mathbb{E}(a|\omega)$ and the unobserved quality as the residual $a - \mathbb{E}(a|\omega)$. When solving the model numerically, we truncate these log-normal distributions to the support $[-2\sigma_j, 2\sigma_j]$ for $j \in \{\omega, a\}$.

FIGURE 3: Capital market outcomes for different accuracies of information technologies



Note: The left panel shows the equilibrium price for units of different unobserved quality on the x-axis. The right panel shows the selling probabilities for units of different unobserved quality. The three lines correspond to three values of the accuracy of the information-revealing technology, ψ: a low value $\psi = 0.9$, an intermediate value $\psi = 0.95$, and a value high enough to take the economy to the full information limit, $\psi = 0.999$. Other model parameters are set to their calibrated values from Section 5.

this reason, an increase in the degree of asymmetric information captured by an inferior information technology is associated with higher average prices and higher duration on the market, particularly at the top of the distribution of capital qualities.

3.2 Identification

Based on the model predictions, we now discuss how the parameters linked to the degree of asymmetric information and capital heterogeneity can be identified from microlevel data. Our distributional assumption regarding (ω, a) implies that the model features three parameters linked to the degree of asymmetric information and capital heterogeneity, which are the most novel part of the model: $\{\psi, \sigma_\omega, \sigma_a\}$. We first illustrate our strategy under some specific assumptions that allow us to derive analytical results; below, we use a parameterized version of the model to show our strategy using model-simulated data.

Analytical illustration To provide an empirical measurement of the model predictions, we assume that a researcher observes micro-data on capital units listed for sale with the following information: the price of each unit listed in every period t, $\{q_{it}\}$; the duration of each unit while listed for trade $\{Duration_{it}\}$; and a vector of observable characteristics $\{X_i\}$ (e.g., location, size, number of rooms, etc.).[11] We further assume that the observable characteristics map onto observable efficiency units of capital according to $\log \omega_i = \tau X_i$, where τ is an unknown vector. Consider estimating the following regressions using these data:

$$\log(q_{it}) = \iota_\omega X_i + \varepsilon^q_{it}, \tag{15}$$

$$\log(Duration_{it}) = \upsilon_\omega X_i + \upsilon_q \log(q_{it}) + \varepsilon^d_{it}, \tag{16}$$

where ε^q_{it} and ε^d_{it} are random error terms. Regression (15) is a "hedonic regression," which projects listed prices on the observed capital quality of each unit. Henceforth, we refer to $\hat{q}_{it} = \hat{\iota}_\omega X_i$ as "predicted prices" and $\hat{\varepsilon}^q_{it}$ as "residual prices." Intuitively, by estimating this regression, we can approximate the variance of observed and unobserved capital qualities with the variance of predicted and residual prices, $\hat{\sigma}_\omega \equiv \mathrm{Var}(\hat{q}_{it})$ and $\hat{\sigma}_a \equiv \mathrm{Var}(\hat{\varepsilon}^q_{it})$. Regression (16) projects the duration of each listed unit on their observed characteristics and listed price. The estimated coefficient $\hat{\upsilon}_q$ measures the slope between log duration and the component of the price that is orthogonal to its observed quality (i.e., the residual $\hat{\varepsilon}^q_{it}$), which, following the discussion in Section 3.1, is informative of the degree of asymmetric information. The following proposition formalizes this mapping between model parameters and data moments:

***Proposition* 4.** *Assume $\left(w_t/\mu_t(\theta(\omega, \hat{a}, q))\right)/\nu^{\mathrm{b}}_t(\omega, a) \to 0$ and $\varphi \to 0$. Then, up to a first-order approximation, $(\sigma_\omega, \sigma_a, \psi)$ are identified by the estimated moments $\hat{\sigma}_\omega$, $\hat{\sigma}_a$, and $\hat{\upsilon}_q$.*

Proposition 4 imposes two assumptions (which are relaxed in the quantitative analysis of the identification below). First, expected search costs are small relative to the buyer's value of a unit of capital. Second, the exit rate of firms is approximately zero. The first assumption ensures that the price of a unit of capital mainly reflects its expected value to the buyer. The second assumption ensures that this value is mainly determined by the net present value of the stream of dividends generated by the unit of capital (and not by its future resale value in case of exit).

Given these assumptions, the residual in equation (15) fully captures the unobserved quality of the unit of capital (i.e., $\varepsilon^q_{it} = \log a_i$). Thus, we can directly measure the volatility of the distribution of the observed and unobserved quality with $\hat{\sigma}_\omega$ and $\hat{\sigma}_a$, respectively. In addition, up to a first-order approximation, the assumptions imply that the estimated

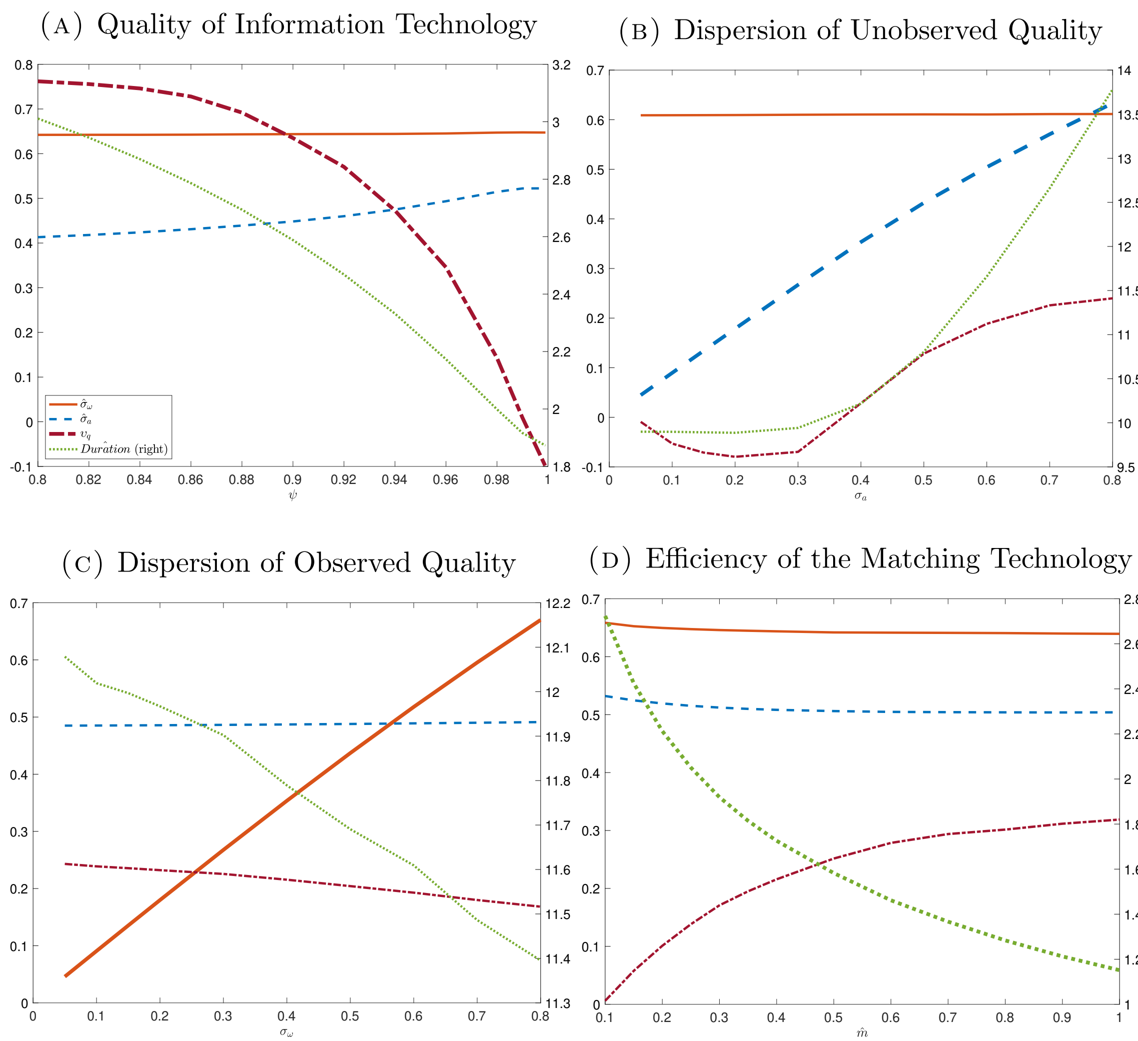
[11] Section 5.2 shows how the quantitative results of the paper are affected if the researcher only observes a subset of the capital characteristics observed by market participants.

regression coefficient $\hat{\upsilon}_q$ represents the elasticity of the (log) selling probability $p(\theta(\omega, a))$ to the unobserved quality a evaluated at the average qualities. In the previous section, we showed how this elasticity is a monotonic function of ψ, which governs the degree of information asymmetries. As the asymmetry of information increases, sellers of high-quality capital choose a lower selling probability to signal their higher quality. Thus, the regression coefficient υ_q is informative of the degree of information asymmetries captured by ψ.

Quantitative illustration Our analytical identification results above were obtained under a set of simplifying assumptions. To show that the identification strategy holds more generally, Figure 4 illustrates the behavior of the key moments $(\hat{\sigma}_\omega, \hat{\sigma}_a, \hat{\upsilon}_q)$ as we change the value of the parameters $(\sigma_\omega, \sigma_a, \psi)$. Given our calibration strategy in Section 5, it is useful to expand our discussion here to incorporate one additional parameter, the match efficiency $\bar{m}$, and one additional identifying moment, the unconditional average duration on the market. Panel (A) shows that changes in the quality of information technology ψ have a monotonic effect on regression coefficient $\hat{\upsilon}_q$ and unconditional average duration. Panel (B) shows that changes in the standard deviation of unobserved quality σ_a have a positive and almost linear effect on the variance of the regression residuals $\hat{\sigma}_a$, as expected. In addition, more dispersed unobserved qualities naturally increase the incentives to mimic and induce sellers of higher qualities to signal their quality more strongly, which also increases regression coefficient $\hat{\upsilon}_q$ and average duration. Panel (C) shows that changes in the standard deviation of observed quality σ_ω also have a positive and almost linear effect on the variance of predicted prices $\hat{\sigma}_\omega$. As we further discuss below, the dispersion of observed qualities has a small interaction with the moments associated with asymmetric information, since information frictions distort the terms of trade of units of capital for a given observed quality ω. Finally, Panel (D) shows that increases in the efficiency of matching technology $\bar{m}$ decrease the average duration of capital units. However, since a higher trading probability makes signaling harder (i.e., higher matching efficiency reduces overall delay in the market), regression coefficient $\hat{\upsilon}_q$ increases.

To summarize, $\hat{\sigma}_\omega$ and $\hat{\sigma}_a$ are directly informed by the dispersion of observed and unobserved qualities. The degree of asymmetric information ψ and matching efficiency $\bar{m}$ are separately identified by the fact that the regression coefficient $\hat{\upsilon}_q$ and average duration positively comove with ψ, but move in opposite directions in response to changes in $\bar{m}$.

FIGURE 4: Illustration of Identification Strategy



Note: The figures report moments $\{\hat{\sigma}_\omega, \hat{\sigma}_a, \hat{\upsilon}_q, \text{Avg. Duration}\}$ computed from model-simulated data as we change the values of the parameters $\{\sigma_\omega, \sigma_a, \psi, \bar{m}\}$. Remaining model parameters are set to their calibrated values from Section 5.

4 Measurement

This section applies our proposed measurement to a novel dataset of capital units listed for trade. Section 4.1 describes the data. Section 4.2 presents a set of cross-sectional facts linked to the model predictions. Section 4.3 discusses additional evidence linked to alternative interpretations of these facts.

4.1 Data

Our data consist of a rich panel of nonresidential structures (retail, office, and industrial space) listed for sale and rent. The source of these data is Idealista, one of Europe's leading online real estate intermediaries.[12] The frequency of the panel is monthly and includes the universe of capital units that were listed on the platform between 2005 and 2018. The data include information during the period each listing was active online. The dataset includes approximately 12 million observations for Spain, where an observation corresponds to a property–month pair. Overall, these observations come from over 1.2 million different capital units. Appendix B provides more details on the data. In particular, Appendix B.1 describes how the online platform works. Appendix B.2 discusses the representativeness of the dataset and shows that the data from the online platform are consistent with the aggregate evolution of prices observed in Spain during the period of analysis.

For each property, we observe a wide range of characteristics detailed in the listing, including the address of the property, its construction year, its area, the number of rooms, and whether the property has heat or air conditioning, among others. We also observe the main variables discussed in our model measurement—namely, the capital unit's listed price, which we observe for each property at monthly frequency, and its duration on the market, which we compute as the number of months the unit is listed on the platform.[13] Our dataset also features information about the search volume in each month, which we measure by the number of views and clicks each listing receives and the number of emails the seller receives from potential buyers through the platform.

Table 1 presents descriptive statistics on prices, duration, search intensity, and some of the characteristics included in the listing. Although we focus the analysis in this section on properties listed for sale, which have a more direct mapping to our model assumptions, Appendix B.4 shows that we observe similar empirical patterns for properties listed for rent. The average sale price per square foot is $160 (expressed in constant 2017 dollars), and the average duration on the market is 11.5 months. Properties are relatively old, with an average age of around 26 years. Each listing is, on average, viewed 850 times per month and receives 46 clicks and 3 emails per month.

[12] `Idealista` is the leading online platform in the real estate market in Spain (see `Comparison of users` and `Comparison of platform`). For other papers using data from online platforms in the real estate market, see Piazzesi et al. (2020).

[13] The platform asks sellers why they decided to close the listing. Figure B6 in Appendix B.3 compares the histograms of duration for two groups of listings: those that closed the listing because the property was rented or sold and those that do not provide an explanation. Those histograms are virtually identical. It is worth noting that Idealista is a paid service, so it is costly for the seller to keep a dormant listing after the property has been sold or rented.

Table 1: Descriptive Statistics

	Mean	St. Dev.
Price	160.10	125.94
Duration	11.53	12.11
Construction Date	1987.51	19.61
Area	3,008.56	4,675.15
New	0.05	0.21
Rooms	2.33	3.01
Restrooms	1.21	1.54
Heating	0.27	0.44
AC	0.63	0.48
E-Mails	2.95	2.03
Views	856.22	1,299.68
Clicks	46.49	57.50
Number of Obs.	4.1e+05	4.1e+05

Note: "Price" is the price per square foot in constant 2017 dollars. "Duration" is the number of months a property was listed in the database. "Construction date" is the year the property was built. Property area is measured in square feet. "New" is a categorical variable that takes the value 1 if the property is new. "Rooms" is the number of separate rooms the property has, and the same for "Restrooms." "Heating" and "AC" are categorical variables that take the value 1 when the property has heating and air conditioning technologies. "Emails" is the number of times a property receives an email from a potential customer per month. "Views" is the number of times a property appeared on the screen of a potential customer per month. "Clicks" is the number of times a potential customer clicked on the property listing to see its details per month. For those variables that change over time, we first take the average of the variable within each listing and report the average of that variable across listings.

4.2 Cross-sectional Empirical Facts

We now use our data to provide a set of facts about the cross-section of listed capital units associated with the model's microlevel predictions.

Measuring predicted and residual prices Following the model's identification strategy described in Section 3.2, we begin by measuring the component of a listed price that can be predicted based on the property's characteristics included in the listing. We do so by estimating the following hedonic pricing regression:

$$\log(q_{it}) = \nu_{l(i)t} + \gamma X_i + \varepsilon_{it}, \tag{17}$$

where q_{it} is the real price per square foot of capital unit i in location $l(i)$, listed in month t; $\nu_{l(i)t}$ are location-by-time fixed effects; X_i is a set of observable characteristics included in the listing; and ε_{it} is a random error term.[14] Similar to Section 3.2, using the estimated coefficients $\{\hat{\nu}_{lt}, \hat{\gamma}\}$, we refer to $\hat{q}_{it} \equiv \hat{\nu}_{lt} + \hat{\gamma} X_i$ as "(log) predicted prices" and $\hat{\varepsilon}_{it} \equiv \log(q_{it}) - \hat{q}_{it}$ as "(log) residual prices."

Using the estimated model (17), Table 2 shows that more than 60% of the variation in listed prices can be accounted for by characteristics included in the listing. The geographic dimension plays a salient role and explains almost 50% of the differences in listed prices. To illustrate this, Appendix Figure B7 shows large differences in sale prices across regions at different levels of aggregation. These maps demonstrate that locations vary significantly in their capital prices. Table 2 also shows that the time dimension explains 12% of the variation in listed prices—which is substantially smaller than the geographic dimension, despite the large fluctuations in capital prices Spain experienced during the Euro crisis (illustrated in Appendix Figure B8). Finally, Table 2 shows that the standard deviation of residual prices, which we obtain after including all available controls, is 0.45, which is approximately 60% of the variation observed in the raw data. Figure 5 shows the distribution of price residuals, which illustrates the relevance of the dispersion in prices not accounted for by characteristics in the listings.

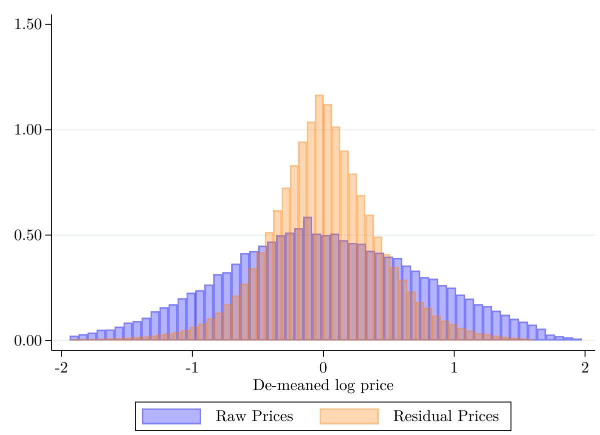
Table 2: Price Variation Accounted for by Listed Characteristics

	St. Dev.	R^2
Raw data	0.75	0.00
Year	0.70	0.12
Location	0.54	0.48
Year $\times$ Location $\times$ Type	0.49	0.58
... + Area	0.46	0.62
... + Age	0.46	0.62
Benchmark	0.45	0.64

Note: This table reports the R^2 and standard deviation of residuals from estimating equation (17). The row labeled Raw data presents statistics for demeaned raw log prices. The following rows include fixed effects in the regression. Year and location denote fixed effects. Type (office and retail space or warehouse), area, and age are sets of fixed effects for each of these characteristics. The last row includes additional controls for the variables listed in Table 1.

[14]Location fixed effects are defined, for each unit, at the finest geographic level possible in the platform: neighborhood level in the case of big cities like Madrid or Barcelona and city level in smaller cities. Results are similar if we focus only on cities that have available neighborhood information. In model (17), we focus on the average listed price during the lifetime of the listing. Table B1 in Appendix B.3 shows that when we estimate a version of model (17) using the entire panel dataset and include a listing fixed effect, less than 2% of the variation in prices can be accounted for by properties that change their price during the lifetime of the listing (i.e., $R^2 = 0.98$). To understand this result, Table B2 in Appendix B.3 shows that only 9% of listings change price in a given month.

Figure 5: Distribution of Price Residuals



Note: This figure shows the distribution of log prices per square foot relative to its mean for the raw data and price residuals after including the fixed effects in Table (2).

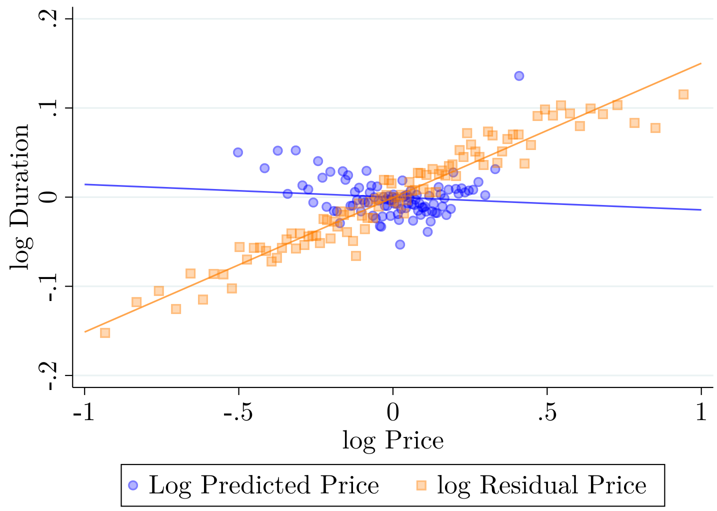
Relationship between prices and duration Guided by our model predictions, we now analyze the relationship between units' predicted and residual prices and their duration on the market. Figure 6 shows that units with higher predicted prices tend to have a shorter duration on the market, while units with higher residual prices tend to have a longer duration. Table 3 presents the same results in a regression framework. In column (1), we regress (log) duration on (log) prices and obtain a mildly negative and statistically significant relation. In the second column, we split the (log) price into two components—predicted and residual prices—and run the same regression. While we obtain a negative and statistically significant relationship between duration and predicted prices, we obtain a positive and statistically significant relationship between duration and residual prices.[15] In the last two columns, we estimate similar regressions but include location-time-property-type fixed effects and obtain similar results.[16]

Table B3 and Appendix Figure B10 reproduce the same analysis by replacing duration

[15] A priori, the degree of asymmetric information could vary across different market segments, such as the primary market for new units versus the secondary market for used units. In Appendix Table B4, we estimate market-specific price-duration slopes based on self-reported unit conservation status ("new construction," "pre-owned/good condition," "pre-owned/requires renovation"). While the relationship is attenuated for new units—consistent with the idea that information asymmetries may be smaller in the market for new capital—the differences are not statistically significant. This suggests that prices reflect revenue-generating potential broadly defined, rather than solely physical condition. Moreover, the differences are quantitatively small and are overshadowed by the cyclical variation we present below.

[16] The reason for including time-location fixed effects in the regression is to allow for the process of duration on the market to differ over time and location (e.g., the match efficiency could be market-specific). However, the theory predicts that if a better observable location contributes positively to the quality of the property, it should also positively affect the trading probability. Therefore, including fixed effects also absorbs part of this effect.

Figure 6: Relationship between Duration and Prices



Note: This figure shows the relationship between log prices and duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (17)). Figures show a binned scatter plot of each relationship after controlling for location-time-type (office, retail space, or warehouse) fixed effects.

Table 3: Prices and Duration

	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	-0.011***		0.124***	
	(0.004)		(0.005)	
log Predicted Price		-0.100***		-0.023*
		(0.006)		(0.013)
log Residual Price		0.145***		0.145***
		(0.005)		(0.005)
Constant	2.159***	2.583***	1.509***	2.216***
	(0.021)	(0.029)	(0.022)	(0.061)
Observations	398761	398761	398761	398761
R^2	0.000	0.009	0.257	0.258
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The left-hand-side variable is the log duration of a listing, and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of duration on prices. Column 2 regresses duration on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at location-time level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

with the average monthly clicks received by a listing (as a proxy for search intensity). Results are consistent with those found for duration. Properties with high predicted prices receive more clicks on average, which is consistent with a shorter duration, and properties with high residual prices receive fewer clicks on average, which is consistent with a longer duration. This last set of results is consistent with listed prices' important role in attracting or repelling potential buyers by affecting their search behavior.

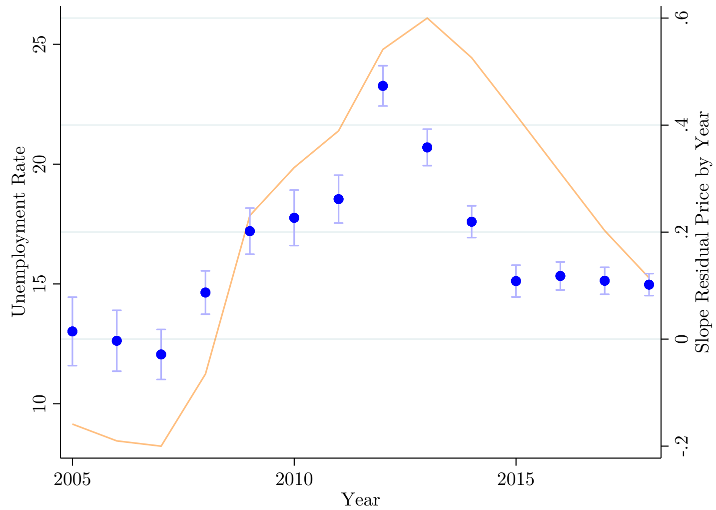
Through the lens of the model, the different relations residual and predicted prices have with duration suggest an important role of information asymmetries. When higher prices stem from listed characteristics, such as the location of the unit—which can be perfectly observed by buyers—they tend to be associated with shorter time to sell. When high prices cannot be easily linked to observable characteristics, they are associated with a longer time to sell. Under the null hypothesis of full information, according to our model, residual prices reflect characteristics of properties not observed by the econometrician but observed by market participants. Thus, we should expect a negative relation with duration, as is the case with predicted prices. The fact that we estimate a positive relation provides evidence that the extent of asymmetric information is not zero. This conclusion is more formally supported in the estimation exercise of the model, which allows us to provide a quantitative magnitude of the deviation from full information.

Cyclical properties Economic downturns are often characterized as times when information about the quality of assets deteriorates. To study this in our data, we estimate the relationship between a unit's duration on the market and its residual price separately for each year in our sample. Figure 7 shows that the slope between duration and residual prices exhibit substantial cyclical variation, showing a strong comovement with the fluctuations of labor unemployment and more than doubling during the Euro crisis that started in 2008. In the context of our model, these patterns are consistent with the degree of asymmetric information increasing during downturns. Motivated by this evidence, in the next section we examine the macroeconomic effects of changes in the accuracy of information technologies ψ.

4.3 Discussion of Alternative Interpretations

So far, we have interpreted the cross-sectional facts through the lens of our model with decentralized capital markets and asymmetric information. In Appendix B.5, we provide

FIGURE 7: Cyclical Fluctuations in the Slope between Duration and Residual Prices



Note: This figure shows the relationship between log duration and residual prices over the business cycle. Blue points represent the regression coefficient of log residual prices when estimating specification (4) in Table 3 for each year in the sample separately (vertical bars denote 95% confidence intervals). The solid line depicts the national unemployment rate in Spain during the sample period (source: Statistical Agency of Spain INE).

additional evidence to indicate that these empirical patterns would be hard to account for by alternative explanations that do not involve asymmetric information.

To briefly summarize: We first explore the possibility that the positive relationship between residual prices and duration on the market results from sellers' indifference across these variables (an explanation akin to that of Burdett and Mortensen, 1998, for labor and product markets). To study whether the trade-off between residual prices and duration can account for their positive relationship in the data, we compute the expected net present discounted revenue for properties with different residual prices under alternative preferences. The results show that the expected net present discounted revenue monotonically increases in the listed price, which indicates that sellers' indifference cannot explain the observed relationship between residual prices and duration. A related explanation of the patterns in the data could be agents' departures from rational behavior—e.g., residual prices reflect outright mistakes. Although it would be difficult to rule out the presence of such behavior fully, our net present value calculations would suggest that all sellers, except those posting the *highest* residual prices, would have to be making mistakes. Furthermore, given the magnitudes of residual prices, note that the mistakes would need to be arguably large. We then analyze the possibility that sellers have varying expenses associated with holding onto a property (e.g., maintenance costs, taxes, and debt service costs) that they must pay every period until the property is sold. If some sellers have higher costs than others, they might have to sell quickly and at a lower price. We use our data to determine the minimum cost that would make it reasonable for a seller to choose a lower residual price and find that this is implausibly large (i.e., on average, the cost of holding 1 square foot of a property for 1 additional month would have to be larger than the price at which the owner can sell that unit). We also consider the possibility that differences in buyers' liquidity for different units could explain our facts. However, the positive relationship between residual prices and duration holds for both high- and low-priced units (in terms of total price), which indicates that buyer liquidity is not the main factor driving the results.

5 The Macro Effects of Asymmetric Information

This section combines the model and empirical measurement to study the macroeconomic effects of asymmetric information. Section 5.1 discusses model parameterization. Section 5.2 explores the impact of asymmetric information on steady-state macroeconomic variables, and Section 5.3 conducts crisis experiments to study the role of asymmetric information in economic downturns.

5.1 Calibration

We calibrate the model in two steps. First, we fix a subset of parameters. Second, we calibrate the remaining parameters—which govern the degree of trading frictions—to match the key data moments discussed in Section 4.

Fixed parameters The parameters we fix in the calibration are detailed in Table 4. The model is calibrated at monthly frequency. A subset of these parameters is shared with the neoclassical stochastic-growth model and set to standard values from the literature. For preferences, we assume the period utility $u(c,h) = \log\left(c - \varpi\gamma^t \frac{1}{1+\xi} h^{1+\xi}\right)$, as in Greenwood, Hercowitz, and Huffman (1988), and set the Frisch elasticity of labor supply, $1/\xi$, to one.[17]

[17]These preferences, proposed by Greenwood et al. (1988), eliminate the wealth effect on labor supply and are commonly used in environments with shocks that primarily affect investment. In our framework, they help ensure labor dynamics that are more consistent with standard business cycle empirical patterns. In particular, with separable preferences, the model produces similar dynamics for output and capital input as in our baseline model, but in response to a negative shock, it generates an initial contraction of labor followed by a subsequent boom not typically observed during crises. By contrast, as discussed in Section 5.3, these

Table 4: Fixed Parameters

Parameter	Description	Value
β	Discount factor	0.9966
ξ	Frisch elasticity of labor supply	1
α	Share of capital	0.35
δ	Depreciation rate	0.0074
γ	Technology growth	1.004
γ_n	Population growth	1.0027
φ	Firms' exit rate	0.0027
η	Curvature matching technology	0.8
ϕ	Bargaining power of seller	0.5

Note: This table reports the parameters we fix in the calibration. The frequency of the model is monthly.

We set the disutility of labor, ϖ, to target steady-state hours worked, $\overline{h} = 1/3$, and the discount factor to $\beta = 0.996$, which is associated with a 4% annual rate of time preference. Regarding the firm's technology, we set the share of capital to $\alpha = 0.35$ (consistent with Fernald, 2014). We set the depreciation rate to $\delta = 0.0074$, which corresponds to an annual rate for nonresidential capital of 8.5% (source: BEA, Fixed Asset tables); the growth rate of technical progress to $\gamma = 1.004$, which is associated with an annual technology growth rate of 1.6%—the growth rate per worker the U.S. economy experienced from 1980 to 2015 (data source: BEA)—and the population growth rate to $\gamma_n = 1.0027$, which is associated with an annual growth rate of the working-age population in the period of analysis of 1% (population aged 15–64, data source: Federal Reserve Bank of St. Louis and OECD). For the exit rate of firms, which governs separation flows, we set $\varphi = 0.0027$, which corresponds to the 3.2% average exit rate of U.S. establishments, obtained as a weighted average of exit rates for establishments of different sizes reported by the U.S. Census Bureau. For search-and-matching frictions, we set the curvature of the matching technology to $\eta = 0.8$ (as estimated by Ottonello, 2017) and the bargaining parameter to $\phi = 0.5$ as a benchmark (used in the context of labor markets, for example, by Shimer, 2010), and analyze how the results vary with alternative parameter values.

Fitted parameters We calibrate the remaining parameters, $\{\psi, \sigma_\omega, \sigma_a, \overline{m}\}$, following the identification strategy proposed in Section 3.2 and targeting four key data moments measured in Section 4 and reported in Table 5. These moments are the slope in the regression of

preferences lead to employment dynamics that more closely track those of output and are more aligned with standard business cycle patterns.

duration on residual prices, which is informative of the quality of information technology ψ; the standard deviations of predicted and residual prices, which are mostly governed by the standard deviation of capital qualities, σ_ω and σ_a, respectively; and the average selling probability, which is mostly governed by the matching efficiency $\bar{m}$. The calibration strategy for these parameters proceeds as follows. For a given set of parameters, we compute the equilibrium choices of prices and transaction probabilities for each type of capital. Then, we simulate the evolution of multiple units of capital, generate a sample of listed units (similar to that of listed properties in our dataset), and perform the same measurement analysis to obtain those moments in the model-simulated data as we performed on the data in Section 4. Finally, we use a minimum-distance estimator to choose parameter values that match the moments in the data. Table 5 shows that our parameterized model matches fairly well the moments targeted in our calibration. Table 6 also reports the results from running, in model-simulated data, the regressions between duration and predicted and residual prices considered in the empirical analysis in Section 4, which indicates that the model is aligned with the (untargeted) relationship between duration and predicted prices.

Table 5 also reports the parameters obtained from the calibration. The calibrated parameter for the accuracy of information technology is $\psi = 0.98$, which indicates that the probability a lemon goes unnoticed is 2%. Therefore, the economy features moderate levels of information asymmetry in the steady state. For use as a benchmark in our quantitative exercises, we also estimate the accuracy of the information technology that would correspond to an economy with the larger degree of asymmetric information measured during the Euro crisis. As shown in Figure 7, in this episode the slope between duration and residual prices reached a level of 0.38, which would correspond to a level of $\psi = 0.96$ (keeping the rest of the model parameters constant).

TABLE 5: Fitted Parameters and Targeted Moments

Parameter	**Description**	**Value**	**Target**	**Model**	**Data**
ψ	Accuracy information tech.	0.9795	Regression coefficient	0.148	0.148
σ_ω	SD observed quality	0.72	SD log predicted prices	0.65	0.65
σ_a	SD unobserved quality	0.58	SD log residual prices	0.51	0.51
$\bar{m}$	Matching efficiency	0.267	Mean duration	11.46	11.44

Note: This table reports the parameters we calibrate by minimizing the distance between four moments in the data and their counterparts in the simulated model.

Table 6: Relationship between Duration and Prices: Data and Model

	Data log Duration	Model log Duration
log Predicted Price	-0.023	-0.086
log Residual Price	0.145	0.148
Constant	2.216	1.99

Note: This table reproduces the regression coefficients in the data and the model. The dependent variable is log duration on the market, which we regress on a constant and our measures of predicted and residual prices. Refer to the empirical section for further details.

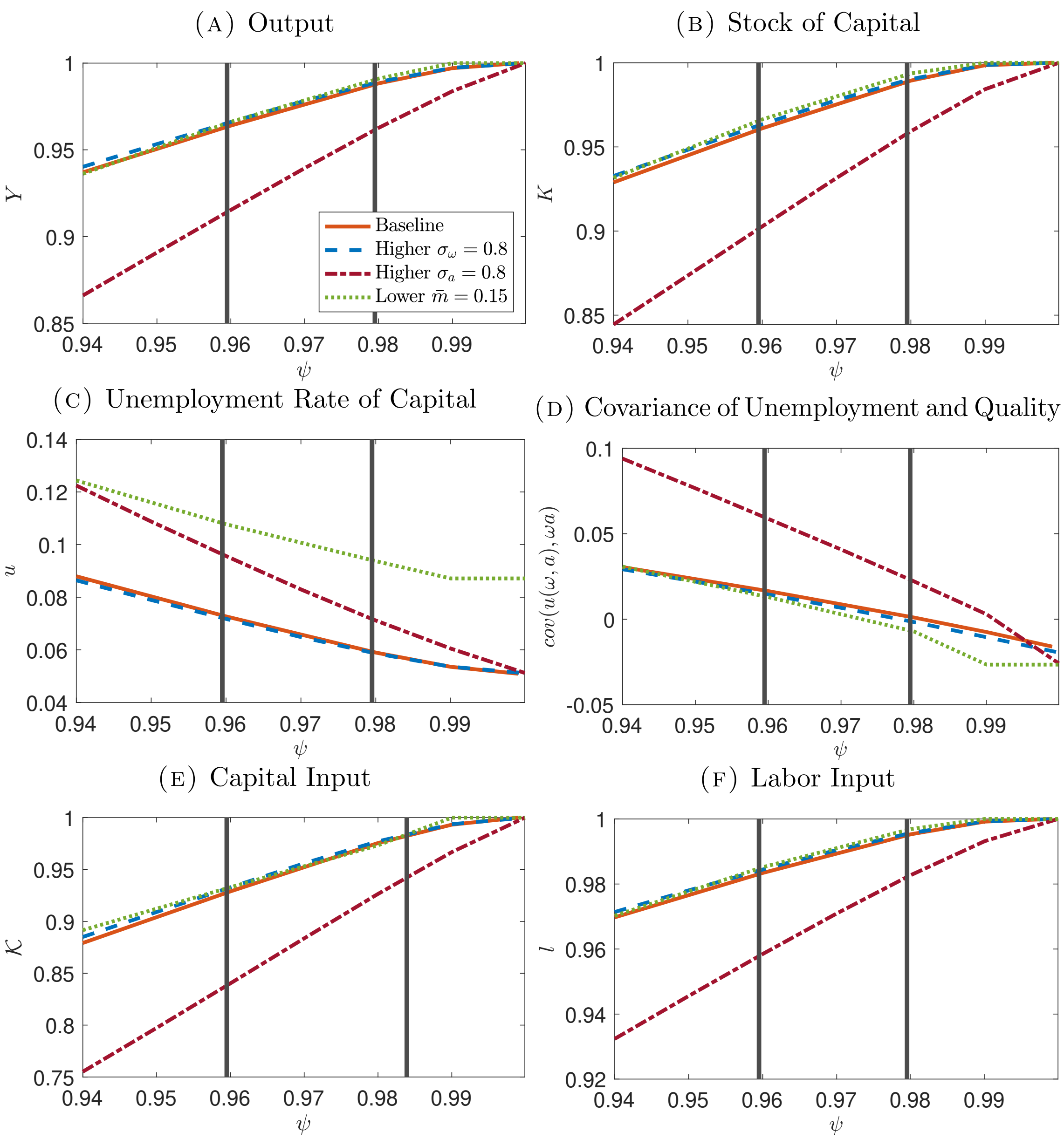
5.2 Steady-state Analysis

Output effects of asymmetric information. Figure 8 shows the equilibrium level of aggregate variables as a function of ψ (each variable is normalized by its corresponding value under full information). Panel (A) shows how steady-state aggregate output varies with the degree of asymmetric information in the economy. Given that the baseline economy features a moderate degree of asymmetric information ($\psi = 0.98$), moving to an economy with full information also involves modest output gains (i.e., a level of output 1.5% larger relative to the baseline economy). However, the economy features a large elasticity of output to changes in the degree of asymmetric information. For instance, a 2 p.p. permanent increase in the degree of asymmetric information to $\psi = 0.96$ (i.e., the level observed during the Euro crisis) is associated with a 2% decline in steady-state output. This high sensitivity of economic activity to changes in the degree of asymmetric information suggests that information asymmetries can play an important role in economic fluctuations, as we further analyze in Section 5.3.

Panel (A) of Figure 8 also shows how the output effects of asymmetric information vary for different degrees of capital heterogeneity and matching efficiency. Each line corresponds to a different parameterization, which reports the effects of varying ψ relative to the value under full information for that parameterization. On the one hand, these results indicate that varying the degree of heterogeneity in observed capital quality σ_ω or the matching efficiency $\bar{m}$ does not substantially affect the output effects of asymmetric information.[18] On the other hand, the model naturally predicts a larger elasticity of output losses with respect to ψ when the dispersion of unobserved qualities the information technology is expected to uncover is

[18] Note that, because in each line we are normalizing output relative to their respective level under full information, these results do not indicate that changing σ_ω or $\bar{m}$ does not have an effect on output. Instead, it shows that the model predicts a small interaction effect between ψ and model parameters associated with capital heterogeneity and search frictions.

Figure 8: Degree of Asymmetric Information and Macroeconomic Variables



Note: This figure shows the decomposition of the macroeconomic effects of changes in the quality of the information technology for four parameterizations of the model. The solid orange line shows our baseline calibration. We also report a calibration with a higher variance of observed quality in dashed blue; a calibration with a higher variance of unobserved quality in dash-dot red; and a calibration with a lower efficiency of the matching technology in dotted green. All numbers are reported as a percentage of the value under full information except for the capital unemployment rate, which is reported in levels.

higher. For instance, in an economy that features a standard deviation of observed capital quality, σ_a, 38% larger than in our baseline economy, the effects of eliminating asymmetric information are 2.25 times larger than in our baseline economy. Motivated by this finding, in the robustness analysis below we study how imperfectly measuring the degree of asymmetric information in the economy affects our results.

Aggregate channels. To decompose the channels through which asymmetric information affects economic activity, we can express aggregate output, $Y_t \equiv \int y_{jt}\,\mathrm{d}j$, as

$$
\begin{aligned}
Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha} \\
&= \left(\gamma^t L_t\right)^{1-\alpha} \left(K_t \left[\mathbb{E}\left(\omega a\right)\left(1-\mathbb{E}\left(u_t(\omega,a)\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega,a)\right)\right]\right)^{\alpha},
\end{aligned}
\tag{18}
$$

where $L_t \equiv h_t(\mathbf{k})\gamma_n^t - \int\int \sum_{\omega}\sum_{\hat{a}} v_{jt}(\omega,\hat{a},q)\,\mathrm{d}q\,\mathrm{d}j$ denotes labor used in production; $\mathcal{K}_t \equiv \int \mathcal{K}_{jt}\,\mathrm{d}j$ aggregate capital input used in production; $K_t \equiv \sum_{\omega\in\omega}\sum_{a\in\mathcal{A}} K_t(\omega,a)$ measured capital stock; and $u_t(\omega,a)$ and $K_t(\omega,a)$ denote the aggregate unemployment rate and the aggregate stock of capital of type (ω,a), respectively. Equation (18) indicates that there are four channels through which asymmetric information can affect aggregate output: the capital stock, the unemployment rate of capital, the allocation of employed capital (measured by the covariance between capital productivity and the unemployment rate of capital), and the labor input. Figure 8 shows how each of these channels are affected by the degree of asymmetric information and Table 7 reports their contribution to the effect of asymmetric information in output, relative to an economy with full information. We next describe each of these channels.

First, Panel (B) of Figure 8 shows that a lower accuracy of the information technology, ψ, is associated with a lower capital stock. This is because higher information asymmetries are associated with lower revenues for sellers of high-quality capital, which decreases the returns to producing capital goods. Relative to the economy under full information, the capital stock is 1.12% lower and contributes 32% of the output effects of asymmetric information. Second, Panel (C) of Figure 8 shows that higher information asymmetries lead to a higher unemployment rate of capital. As information asymmetries increase, so do the listed prices of high-quality capital sellers, which decreases selling probabilities and increases the duration of unemployment of listed units up to 1 p.p. relative to the full-information rate (5%), and contributes 25% of the output effects of asymmetric information. Third, this unemployment effect is compounded by the fact that information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities (see Panel (D)), although this channel has the smallest independent effect on output (with a contribution of 16% to the output effects of asymmetric information). By combining these three effects, Panel (E) shows that a higher degree of asymmetric information is associated with a lower effective capital input of

Table 7: Decomposition of Output Effects

	Change	Contribution
$Y/Y^{FI}-1$	-1.22%	100%
$\mathcal{K}/\mathcal{K}^{FI}-1$	-2.55%	74%
$K/K^{FI}-1$	-1.12%	32%
$\mathbb{E}\left(u(\omega,a)\right)-\mathbb{E}\left(u^{FI}(\omega,a)\right)$	0.92%	25%
$\mathbb{C}ov\left(\omega a,u(\omega,a)\right)-\mathbb{C}ov\left(\omega a,u^{FI}(\omega,a)\right)$	0.01	16%
$L/L^{FI}-1$	-0.5%	26%

Note: This table decomposes the output effects of the calibrated degree of asymmetric information into the channels shown in equation (18). The second column reports the percentage change of each variable in the baseline calibration relative to the full-information equilibrium. The third column reports the contribution of each channel to the output effect by computing the counterfactual level of output that would arise if only one channel is present at a time, while keeping the remaining aggregate variables at their full-information level. Effects are reported as a share of the total decline in output due to asymmetric information.

approximately 2.55% relative to the full-information benchmark, which accounts for most of the output effects of asymmetric information. Panel (f) shows that the lower equilibrium level of capital input reduces the demand for labor and results in 0.5% lower labor input.

Finally, to benchmark the effects of asymmetric information in our model, we conduct an exercise in the spirit of Hsieh and Klenow (2009) and compute its implications for measured total factor productivity (TFP). We define measured TFP in an economy with inspection technology ψ as $\hat{\text{TFP}}_t(\psi) \equiv \frac{Y_t(\psi)}{(h_t(\psi))^{1-\alpha} K_t(\psi)^\alpha}$, where $Y_t(\psi)$, $h_t(\psi)$, and $K_t(\psi)$ denote aggregate output, hours worked, and measured capital, respectively, in an economy with inspection level ψ, and all other parameters set as in our baseline calibration. We find that moving ψ from its full-information level ($\psi = 1$) to the baseline calibration value ($\psi = 0.9795$) leads to a 0.54% decline in measured TFP, which represents 44% of the output decline due to asymmetric information reported in Table 7. These effects appear plausible, as they are not larger than, for example, the output losses typically attributed to the allocation of inputs across establishments or firms in modern economies (see, for example, Midrigan and Xu, 2014; Ottonello and Winberry, 2024).[19]

Robustness. Table 8 shows that our quantitative results for the aggregate effects of asymmetric information are robust to several alternative model parameterizations. First, we study the role of the inspection-adjusted price $q^P(\omega, a, \hat{a}, q)$, which in our baseline calibration

[19] A key difference between the effects on measured total factor productivity in this exercise and those in the misallocation literature is that our effects stem from heterogeneous capital quality, whereas the misallocation literature typically studies the consequences of homogeneous capital not being efficiently allocated across heterogeneous firms. An interesting area for future research would be to combine the sources of misallocation emphasized in our paper with those in the heterogeneous firms literature.

is determined through a Nash bargaining problem. We now consider a version of the model in which the buyer has greater bargaining power: if an inspection reveals that the seller's quality is below what was announced, the buyer makes a take-it-or-leave-it (TIOLI) offer to the seller. Table 8 shows that, once we recalibrate the model under this assumption, we obtain aggregate effects of asymmetric information that are similar to those in our baseline parameterization.[20]

Second, we consider an economy in which the exit rate of firms (and thus the associated reallocation of capital) is larger, and as a result, the steady-state level of capital unemployment is also larger. Third, we consider an economy with an inelastic labor supply. All of these variants feature aggregate effects that are quantitatively similar to our baseline economy, ranging from 100% of the baseline results in an economy with inelastic labor supply to 1.22 times our baseline results in an economy with a higher exit rate of firms.

In addition, we study how our quantitative results vary when we change the set of characteristics we observe for listed capital units in the dataset in Section 4. This exercise is motivated by the concern that our dataset may feature an incomplete set of characteristics relative to those observed by market participants, which translates into an estimation of a larger dispersion of unobserved capital qualities and potentially larger output effects of asymmetric information. To address this concern, we recalibrate the model assuming that our dataset only includes information on the price per square foot, the duration on the market, and the time and location of listed capital units; that is, it does not include any additional information about listed capital units, such as the age, type, number of rooms, etc.[21] Appendix Table C1 reports how the moments targeted in the calibration vary in the dataset with fewer observable characteristics. As expected, we estimate a lower dispersion for predicted prices and a larger dispersion for residual prices. Importantly, we estimate a smaller slope between residual prices and duration. Through the lens of the model, this can be explained by the differences in slopes between duration and predicted prices and duration and residual prices: As we reduce the set of characteristics observed by the researcher, the slope between duration

[20] When we recalibrate the model under the take-it-or-leave-it offer assumption, the calibrated value of the inspection technology that matches the same moments in Table 5 is 0.96, which is lower than the baseline value of 0.98. This is because, as seen from the no-mimicking constraint (11), a lower post-inspection price has a qualitatively similar effect on the incentives to mimic as a more accurate inspection technology, and thereby reduces the value of ψ required to match the data.

[21] Our exercise is in the spirit of Romer (1986): While we cannot assess how our results would vary if the data were to include more observable characteristics than they currently have, we can still study how results would vary if they were to include fewer.

and residual prices becomes more influenced by characteristics that are actually observed by market participants, which moves the slope between duration and residual prices more in the direction of that between duration and predicted prices. Therefore, the calibration to the dataset with fewer observable characteristics features a higher σ_a but a lower ψ, which have opposing effects on the aggregate effects of asymmetric information (see Appendix Figure C1). Table 8 shows that these two parameter changes roughly offset each other, leading to conclusions about the aggregate effects of asymmetric information that are similar to those in our baseline calibration.

TABLE 8: Robustness of Macro Effects to Different Calibrations

Calibration	$Y/Y^{FI}-1$	$\mathcal{K}/\mathcal{K}^{FI}-1$	$K/K^{FI}-1$	$u-u^{FI}$
Baseline	-1.22%	-2.55%	-1.12%	0.83%
TIOLI	-1.44%	-2.93%	-1.46%	0.72%
Higher φ	-1.5%	-3.15%	-1.24%	1.09%
Inelastic Labor Supply	-1.22%	-2.55%	-1.12%	0.83%
Incomplete Observed Characteristics	-1.17%	-2.46%	-1.06%	0.78%

Note: This table shows the macroeconomic effects of asymmetric information on output, the capital input, the capital stock, and the unemployment rate of capital for the benchmark calibration and alternative parameterizations in which we calibrate the model to match the same moments. The *TIOLI* ("take-it-or-leave-it") calibration assigns a bargaining power of zero to the seller if an inspection reveals that the capital quality is below that announced (i.e., $\phi = 0$). The *Higher* φ calibration increases the value of firm exit to match the number of exits plus the share of capital that is reallocated among public firms in the United States as reported by Eisfeldt and Shi (2018). The *Inelastic Labor Supply* version of the model keeps the labor supply fixed at its baseline level of 1/3. The *Incomplete Observed Characteristics* calibration is one in which we purposely ignore a set of observable characteristics in order to analyze the effect of perturbing the availability of information about observed characteristics on our results. In all these exercises, we recalibrate the parameters $\{\psi, \sigma_\omega, \sigma_a, \overline{m}\}$ to target the four key data moments reported in Table 5.

Finally, one limitation of the quantitative analysis is that the micro-level data on individual capital units' listed prices and durations are available only for capital structures. In light of this, our baseline analysis adopted a parsimonious strategy in which we set up a model with a single type of capital good and used these data to inform the degree of asymmetric information across all capital units. In Appendix C.2.2, we adopt an alternative strategy in which we consider a model with separate capital types for structures and equipment, using micro-level data only to inform the degree of asymmetric information in capital structures. To provide a lower bound on the effect of asymmetric information on the macroeconomy, we assume that the trade of equipment is frictionless. As reported in Appendix C.2.2, we find results that are qualitatively similar to our baseline analysis, with the quantitative aggregate effects of asymmetric information being almost a rescaling by the share of structures in the aggregate stock of capital. Given that capital structures represent a large share of the capital stock (close to 50% in the U.S. economy, based on Fernald, 2014), we conclude from this exercise that asymmetric information in capital markets has large macroeconomic effects even when only capital structures are subject to these frictions.[22]

5.3 Crisis Experiments

Degree of asymmetric information. We now study how shocks that primarily affect the degree of asymmetric information can lead to macroeconomic crises. We begin by considering those induced by transitory declines in the accuracy of information-revealing technologies. This experiment is motivated by the dynamics of capital markets observed during the Euro crisis (documented in Figure 7), in which the slope between duration and residual prices sharply increased during the economic downturn and then gradually recovered. Through the lens of our model, such dynamics can be accounted for by a transitory decline in the accuracy of information-revealing technologies, ψ, which increases the degree of asymmetric information in the economy (see Figure 4). Appendix C.3 shows that, in our model, the magnitude of the changes in the slope between duration and residual prices observed during the Euro crisis cannot be accounted for by other shocks likely to have occurred during this period, such as changes in productivity, the dispersion of unobserved capital qualities, changes in discount factors, or exit rates.

The decline in the accuracy of information-revealing technologies that we consider aligns with classic narratives of economic crises, which emphasize the deterioration of agents' ability to assess asset quality. For instance, Gorton (2008) describes financial panics, including the Great Recession, as episodes in which agents lose information about the location and magnitude of default risks. In the context of real assets, this loss of information may arise from two key phenomena observed during crises. First, an increase in real estate foreclosures, which reduces the value of neighboring units (see, for example, Campbell, Giglio, and Pathak, 2011; Towe and Lawley, 2013) and makes it harder for buyers to assess their quality. Second, shifts in demand across products and locations, which affect the profitability of capital units and introduce uncertainty about their future value—e.g., a restaurant in an area experiencing a

[22] It is worth noting that, while this exercise provides a useful lower bound on the macroeconomic effects of asymmetric information, we do not consider the presence of asymmetric information in capital equipment implausible. A classic example is the used car market, as considered by Akerlof (1970).

demand shock (see, for example, Rembert, 2017; Stroebel and Vavra, 2019, for evidence on the geographical reallocation of employment and demand during recessions). In our framework, a decline in the accuracy of information technology captures the idea that the deterioration in agents' ability to assess asset quality is greater for buyers than for sellers, who own the capital and may have more information about neighboring foreclosures or demand shifts.[23]

To implement this crisis experiment, we assume that at $t = 0$ the economy experiences an unexpected and transitory decline in the accuracy of information technologies, ψ_t. We parameterize the magnitude and persistence of this shock to induce a change in the slope between duration and residual prices akin to that observed during the Euro crisis. In particular, as shown in Panel (A) of Figure 9, we assume that at $t = 0$ the economy experiences a 2 p.p. decline in the accuracy of information technologies to $\psi_0 = 0.96$, which lasts for 3 years and reverts to its steady-state value following a first-order autoregressive process.[24] The resulting increase in the slope of 20 p.p. replicates the behavior of the slope in the data.

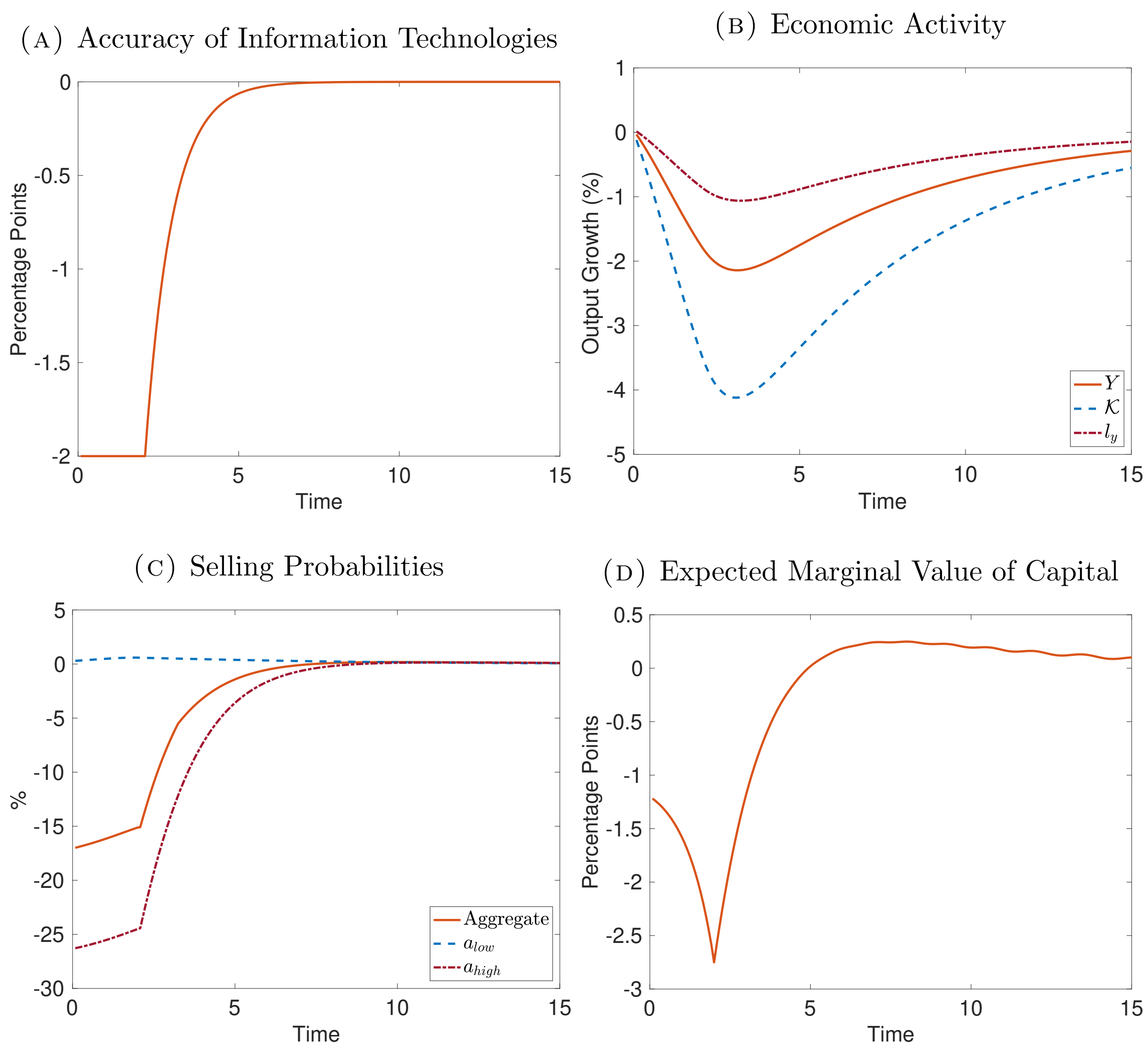
Panel (B) of Figure 9 shows that increases in the degree of asymmetric information induced by declines in the accuracy of information technologies lead to large and persistent contractions in economic activity. In particular, following the shock, the economy experiences a 2% output contraction, and it takes more than 5 years to recover half of its decline. Panel (B) also shows that this contraction in economic activity is primarily driven by declines in the capital input. This occurs because, as shown in Panel (C), with a less accurate information technology, sellers of high-quality capital are willing to accept lower selling probabilities to signal their unobserved quality. For a given capital stock, this implies an increase in the unemployment rate of capital, particularly among capital units of high quality. In addition, as shown in Panel (D), the lower selling probabilities of high-quality capital lead to a decline in the expected returns of producing new capital goods, and thus in aggregate investment. In sum, a decline in the accuracy of information technology acts like a tax on high-quality capital, by lowering the incentives to invest and worsening the allocation of capital.

In addition to the increase in the slope between duration and residual prices, Appendix Figure C3 shows that the Euro crisis was also characterized by a sizable increase in the

[23] It is worth noting that the decline in the accuracy of information technologies we describe may be triggered by other aggregate shocks, such as a decline in TFP, which could, in turn, lead to increased foreclosures or demand reallocation. Our baseline exercise isolates the effect of deteriorating information accuracy by considering it in the absence of additional shocks.

[24] More specifically, we assume that during the recovery, the accuracy of information technologies follows the process $\psi_t = \rho_\psi \psi_{t-1}$. We parameterize this process with $\rho_\psi = 0.94$ to match the half-life of the slope between duration and residual prices observed in Figure 7.

Figure 9: Macroeconomic Responses to Changes in Information Technologies



Note: This figure shows the impulse responses of output, capital input, and labor input to an unexpected decline in the accuracy of information technologies ψ_t. Panel (A) depicts the assumed path for ψ_t considered in the exercise. Panel (B) shows the response of aggregate output Y_t, capital input $\mathcal{K}_t$, and labor input L_t. The horizontal axis displays years after the shock. Impulse responses are expressed in percentage deviations from the detrended steady state. Panel (C) shows the selling probabilities ($p(\theta)$) for the lowest type in dashed blue, for the highest type in dash-dot crimson, and the weighted average selling probability in solid red. Panel (D) shows the behavior of the expected marginal value of capital, computed as $\sum_a \sum_\omega \nu^s_{t+1}(\omega, a) g(\omega, a)$.

dispersion of price residuals. Through the lens of our model, such dynamics can be accounted for by an increase in the dispersion of unobserved capital qualities, σ_a (see Figure 4), which, as discussed in Section 5.2, can influence the effects of changes in information technologies. Motivated by this fact, Appendix Figure C3 reports the impact of a deterioration in the quality of information technologies under different levels of unobserved capital quality. These results show that when the degree of unobserved capital quality is 24% larger than in our baseline framework—akin to the increase observed during the Euro crisis—the output effect of a deterioration in the accuracy of information technologies is 18% larger than that reported in

Figure 9. This result is consistent with the literature showing that increases in the dispersion of unobserved capital qualities play a central role in accounting for the macroeconomic effects of asymmetric information (e.g., Eisfeldt, 2004; Kurlat, 2013; Bigio, 2015).

Other crisis experiments. Our analysis so far has concentrated on the effects of shocks that primarily affect the degree of asymmetric information. To complement this analysis, Appendix Figure C2 shows how the presence of asymmetric information shapes the response to other shocks (i.e., changes in total factor productivity, the discount factor, and the exit rate of firms). Results indicate that the extent to which information asymmetries amplify the response to these shocks critically depends on the steady-state degree of asymmetric information. For the baseline economy, which features a moderate degree of asymmetric information, the aggregate responses to these shocks are similar to those in an economy with full information. However, in economies with large steady-state levels of asymmetric information, the responses differ more substantially from those in an economy with full information. For instance, in an economy in which the probability of a lemon's going unnoticed is 4%, the cumulative output effect of a shock that increases the exit rate of firms is 26% higher than in an economy with full information.[25] The intuition behind this result is that asymmetric information affects the amount of capital being reallocated at each point in time from capital sellers to capital buyers. Therefore, an increase in the exit rate of firms raises the effective amount of capital subject to information frictions at any given time.

6 Conclusion

In this paper, we show that information asymmetries in capital markets can have important macroeconomic implications and affect an economy's investment, capital allocation, and economic activity. This conclusion emerges from adopting a micro-to-macro approach, which combines microlevel data on capital units listed for trade with a quantitative capital-accumulation model that features illiquid capital markets and asymmetric information. The results of our paper suggest the importance of studying capital-market policies designed to address potential inefficiencies that arise from information asymmetries. For example, our quantitative framework is useful for investigating the effects of policies that aim to

[25] As a reference, the exit of establishments during the Covid-19 recession in the U.S. increases by roughly one-third (see Decker and Haltiwanger, 2022).

reduce information asymmetries (e.g., prevent signaling). In addition, our results suggest the relevance of further studying agents' incentives for developing information technologies that mitigate information frictions (e.g., in a version of the model in which the accuracy of information technologies is endogenous). We leave these analyses for future research.

References

Akerlof, G. (1970): "The market for 'Lemons': Quality uncertainty and the market mechanism," *Quarterly Journal of Economics*, 84, 488–500.

Bigio, S. (2015): "Endogenous liquidity and the business cycle," *American Economic Review*, 105, 1883–1927.

Burdett, K. and K. L. Judd (1983): "Equilibrium price dispersion," *Econometrica*, 955–969.

Burdett, K. and D. T. Mortensen (1998): "Wage differentials, employer size, and unemployment," *International Economic Review*, 257–273.

Caballero, R. J. and M. L. Hammour (1998): "The macroeconomics of specificity," *Journal of Political Economy*, 106, 724–767.

Campbell, J. Y., S. Giglio, and P. Pathak (2011): "Forced sales and house prices," *American Economic Review*, 101, 2108–2131.

Cao, M. and S. Shi (2017): "Endogenously procyclical liquidity, capital reallocation, and q," Working paper.

Caplin, A. and J. Leahy (2011): "Trading frictions and house price dynamics," *Journal of Money, Credit and Banking*, 43, 283–303.

Chang, B. (2018): "Adverse selection and liquidity distortion," *The Review of Economic Studies*, 85, 275–306.

Cho, I.-K. and D. M. Kreps (1987): "Signaling games and stable equilibria," *Quarterly Journal of Economics*, 102, 179–221.

Cui, W., R. Wright, and Y. Zhu (2021): "Endogenous liquidity and capital reallocation," *Available at SSRN 3881116*.

Decker, R. A. and J. Haltiwanger (2022): "Business entry and exit in the COVID-19 pandemic: A preliminary look at official data," FEDS Notes. Washington: Board of Governors of the Federal Reserve System, May 06, 2022, https://doi.org/10.17016/2380-7172.3129.

Delacroix, A. and S. Shi (2013): "Pricing and signaling with frictions," *Journal of Economic Theory*, 148, 1301–1332.

Eisfeldt, A. L. (2004): "Endogenous liquidity in asset markets," *Journal of Finance*, 59, 1–30.

Eisfeldt, A. L. and A. A. Rampini (2006): "Capital reallocation and liquidity," *Journal of Monetary Economics*, 53, 369–399.

Eisfeldt, A. L. and Y. Shi (2018): "Capital reallocation," *Annual Review of Financial Economics*, 10, 361–386.

Fernald, J. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," Tech. rep., Federal Reserve Bank of San Francisco, working paper 2012-19.

Gavazza, A. (2011): "The role of trading frictions in real asset markets," *American Economic Review*, 101, 1106–1143.

——— (2016): "An empirical equilibrium model of a decentralized asset market," *Econometrica*, 84, 1755–1798.

Gorton, G. B. (2008): "The panic of 2007," Tech. rep., National Bureau of Economic Research.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 402–417.

Guerrieri, V. and R. Shimer (2014): "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *American Economic Review*, 104, 1875–1908.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse selection in competitive search equilibrium," *Econometrica*, 78, 1823–1862.

Guren, A. M. (2018): "House price momentum and strategic complementarity," *Journal of Political Economy*, 126, 1172–1218.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

Kermani, A. and Y. Ma (2022): "Asset specificity of nonfinancial firms*," *The Quarterly Journal of Economics*, 138, 205–264.

Krainer, J. (2001): "A theory of liquidity in residential real estate markets," *Journal of Urban Economics*, 49, 32–53.

Kurlat, P. (2013): "Lemons markets and the transmission of aggregate shocks," *American Economic Review*, 103, 1463–89.

Kurmann, A. and N. Petrosky-Nadeau (2007): "Search frictions in physical capital markets as a propagation mechanism," Working Paper 07-12, Centre Interuniversitaire sure le Risque, les Politiques Économiques and l'Emploi.

Lagos, R., G. Rocheteau, and R. Wright (2017): "Liquidity: A new monetarist perspective," *Journal of Economic Literature*, 55, 371–440.

Lanteri, A. (2018): "The market for used capital: Endogenous irreversibility and reallocation over the business cycle," *American Economic Review*, 108.

Menzio, G. and S. Shi (2011): "Efficient search on the job and the business cycle," *Journal of Political Economy*, 119, 468–510.

Midrigan, V. and D. Y. Xu (2014): "Finance and misallocation: Evidence from plant-level data," *American economic review*, 104, 422–458.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

Myers, S. C. and N. S. Majluf (1984): "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, 13, 187–221.

Ottonello, P. (2017): "Capital unemployment," Working paper.

Ottonello, P. and T. Winberry (2024): "Capital, ideas, and the costs of financial frictions," Tech. rep., National Bureau of Economic Research.

Piazzesi, M., M. Schneider, and J. Stroebel (2020): "Segmented housing search," *American Economic Review*, 110, 720–59.

Ramey, V. A. and M. D. Shapiro (2001): "Displaced capital: A study of aerospace plant closings," *Journal of Political Economy*, 109, 958–992.

Rembert, M. (2017): "Creative Destruction & Inter-Regional Job Reallocation during the Great Recession," *Journal of Regional Analysis and Policy*, 48, 77–91.

Restuccia, D. and R. Rogerson (2008): "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 11, 707–720.

Romer, C. (1986): "Spurious volatility in historical unemployment data," *Journal of Political Economy*, 94, 1–37.

Shimer, R. (1996): "Contracts in a frictional labor market," Tech. rep., Massachusetts Institute of Technology, mimeo.

——— (2010): *Labor Markets and Business Cycles*, Princeton University Press.

Spence, M. (1973): "Job market signaling," *Quarterly Journal of Economics*, 87, 355–374.

Stiglitz, J. E. and A. Weiss (1981): "Credit rationing in markets with imperfect information," *American Economic Review*, 71, 393–410.

Stroebel, J. and J. Vavra (2019): "House prices, local demand, and retail prices," *Journal of Political Economy*, 127, 1391–1436.

Towe, C. and C. Lawley (2013): "The contagion effect of neighboring foreclosures," *American Economic Journal: Economic Policy*, 5, 313–335.

Wheaton, W. C. (1990): "Vacancy, search, and prices in a housing market matching model," *Journal of Political Economy*, 98, 1270–1292.

Wright, R., P. Kircher, B. Julien, and V. Guerrieri (2019): "Directed search and competitive search: A guided tour," *Journal of Economic Literature*.

Wright, R., S. X. Xiao, and Y. Zhu (2018): "Frictional capital reallocation I: Ex ante heterogeneity," *Journal of Economic Dynamics and Control*, 89, 100–116.

——— (2020): "Frictional capital reallocation with ex post heterogeneity," *Review of Economic Dynamics*, 37, S227–S253.

A Theory Appendix

In this section, we provide all proofs of our theory. Instead of focusing on a specific post-inspection trading protocol, as we did in the main text, here we provide a set of general sufficient conditions that the inspection-adjusted price function must satisfy and generalize the proof to any protocol that satisfies the following assumption.

***Assumption* 1.** *The inspection-adjusted price function* $q_t^P(\omega, a, \hat{a}, q) : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to \mathbb{R}_+$ *has the following properties:*

(i) it is non-decreasing in the true quality:

$$\forall (a, a') \in \mathcal{A}^2 \text{ such that } a' > a,\ \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ : q_t^P(\omega, a', \hat{a}, q) \geq q_t^P(\omega, a, \hat{a}, q),$$

(ii) it is non-increasing in the announced quality:

$$\forall (\hat{a}, \hat{\hat{a}}) \in \mathcal{A}^2 \text{ such that } \hat{a} > \hat{\hat{a}},\ \forall (\omega, a, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ : q_t^P(\omega, a, \hat{a}, q) \leq q_t^P(\omega, a, \hat{\hat{a}}, q),$$

(iii) it is weakly lower (resp. higher) than the buyer's (resp. seller's) value for the unit:

$$q_t^P(\omega, a, \hat{a}, q) \in [\min(q, \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a), \min(q, \nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}))]$$

$\forall \omega \in \Omega, \hat{a}, a \in \mathcal{A}, q \in \mathbb{R}+, \mathbf{K}_{Ht} \in \mathbb{R}+,$

(iv) it is such that buyers obtain at least a fraction $1 - \eta$ *of the surplus:*

$$q_t^P(\omega, a, \hat{a}, q) \leq \eta\nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}) + (1 - \eta)\left(\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right),$$

(v) it does not decrease "too fast" as the announced quality increases, i.e.:

$$\frac{\eta(\nu_t^{\mathrm{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i)}{q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i} \geq \frac{q^B(\omega, a_j, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}{q^B(\omega, a_j, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}$$

$\forall a_j < a_i < a_k \in \mathcal{A}, \omega \in \Omega.$

The first assumption requires that for a given announced quality, sellers obtain a weakly higher post-inspection price the higher their true revealed quality is. Relatedly, the second assumption states that after the inspection reveals the true quality, sellers of higher announced quality are weakly worse off. This protocol captures a variety of contractual arrangements that punish sellers for lying about the true quality of their units. For example, these assumptions allow for a post-inspection bargaining price that sanctions sellers more severely when the difference between true and announced quality increases. They also allow for sanction-less bargaining, which we consider in our quantitative analysis. The third assumption states that the inspection-adjusted price is bounded by the buyer's valuation of the unit and the seller's outside option, which corresponds to its continuation value if the transaction does not happen. This implies that a transaction occurs as long as the gains from trade are positive. Notice that this assumption also incorporates the seller's commitment to sell at the initially posted price q. The fourth assumption requires the inspection-adjusted price to be weakly lower than the price the seller would obtain under full information, which we derive below. Intuitively, after units are inspected and their true qualities are revealed, sellers should not be able to transact at a higher price than they would have received if all information were publicly available. The post-inspection price we propose can capture situations in which sellers who lie about the quality of their units are "punished" with a lower transacted price (e.g., by increasing the buyer's bargaining power). The final assumption limits how large this "punishment" can be. This is a sufficient condition that ensures that the separating equilibrium derived below has sellers truthfully reporting their quality and rules out pathological equilibria.

In the quantitative analysis, we use a standard Nash bargaining protocol to determine post-inspection price $q^P(\omega, a, \hat{a}, q)$. At the end of this section, Lemma 2 shows that such Nash solution satisfies the above assumptions if the seller's bargaining power satisfies $\phi \leq \eta$.

A.1 Equilibrium Definition and Equilibrium Characterization with Multiple Types

Definition of Competitive Equilibrium

We now define the economy's competitive equilibrium and two types of equilibria: pooling and separating. We restrict attention to pure strategy equilibria, which characterize the unique solution under the D1 equilibrium refinement.

Definition 2. Competitive Equilibrium

Given initial conditions $\mathbf{K}_{H0}$ and $(\mathbf{k}_{j0})_{j\in[0,1]}$, a perfect Bayesian equilibrium under asym-

metric information consists of a sequence of household value functions $\{V_{Ht}(\mathbf{k}), \nu^{\mathrm{s}}_t(\omega, a, \mathbf{k})\}$ *and policy functions* $\{c_t(\mathbf{k}), h_t(\mathbf{k}), i_t(\mathbf{k}),\ \mathbf{k}_{Ht+1}(\mathbf{k}), \hat{a}_t(\omega, a, \mathbf{k}), q_t(\omega, a, \mathbf{k})\}$*; firm value functions* $\{V_{Ft}(\mathbf{k}), \nu^{\mathrm{b}}_t(\omega, a)\}$ *and policy functions* $\{l_t(\mathbf{k}), div_t(\mathbf{k}),\ \mathbf{k}_{Ft+1}(\mathbf{k}), \{v_t(\omega, \hat{a}, q)\}\}$*; market tightness functions* $\{\theta_t(\omega, \hat{a}, q)\}$*; belief functions* $\{\pi_t(a|\omega, \hat{a}, q)\}$*; wages* $\{w_t\}$*; discount factors* $\{\Lambda_{t,t+1}\}$*; and aggregate variables* $\{\mathbf{K}_{Ht+1}, \mathbf{K}_{Ft+1},\ Div_{Ft}, \iota_t(a|\omega, \hat{a}, q)\}$ *for all* $t \geq 0$ *such that*

(i) Given wages and market tightness, the household's value functions $V_{Ht}(\mathbf{k})$ *and* $\nu^{\mathrm{s}}_t(\omega, a, \mathbf{k})$ *solve* (2) *and* (3) *with associated policy functions* $c_t(\mathbf{k})$, $i_t(\mathbf{k})$, $\mathbf{k}_{Ht+1}(\mathbf{k})$, $\hat{a}_t(\omega, a, \mathbf{k})$*, and* $q_t(\omega, a, \mathbf{k})$ *for all* $(\omega, a) \in \Omega \times \mathcal{A}$.

(ii) Given wages, market tightness, and discount factors, a firm's value functions $V_{Ft}(\mathbf{k})$ *and* $\nu^{\mathrm{b}}_t(\omega, a)$ *solve* (5) *and* (7) *with associated policy functions* $l_t(\mathbf{k})$, $\mathbf{k}_{Ft+1}(\mathbf{k})$*, and* $\{v_t(\omega, \hat{a}, q)\}$ *for all* $(\omega, \hat{a}) \in \Omega \times \mathcal{A}$.

(iii) The market tightness function satisfies (8) *in all submarkets.*

(iv) The belief function $\pi_t(a|\omega, \hat{a}, q)$ *is consistent with sellers' strategies using Bayes' rule when possible.*

(v) The labor market clears: $\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\int_{q\in\mathbb{R}_+} v_t(\omega, \hat{a}, q)\,\mathrm{d}q + \int l_t(\mathbf{k}_{jt})\,\mathrm{d}j = h_t(\mathbf{k})\gamma_n^t$.

(vi) The discount factor satisfies $\Lambda_{t,t+1} = \lambda_t(\mathbf{K}_{Ht})$.

(vii) Aggregate variables are consistent with individual policies: $\mathbf{K}_{Ht+1} = \mathbf{k}_{Ht+1}(\mathbf{K}_{Ht})$, $\mathbf{K}_{Ft+1} = \int \mathbf{k}_{Ft+1}(\mathbf{k}_{jt})\,\mathrm{d}j$, $Div_{Ft} = \int div_t(\mathbf{k}_{jt})\,\mathrm{d}j$*, and*

$$\iota_t(a|\omega, \hat{a}, q) = \frac{\mathbb{I}_{\{\hat{a}\ =\ \hat{a}_t(\omega,a,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q\ =\ q_t(\omega,a,\mathbf{K}_{Ht})\}}K_{Ht}(\omega, a)}{\sum_{a_j\in\mathcal{A}}\mathbb{I}_{\{\hat{a}\ =\ \hat{a}_t(\omega,a_j,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q\ =\ q_t(\omega,a_j,\mathbf{K}_{Ht})\}}K_{Ht}(\omega, a_j)},$$

for all $(\omega, \hat{a}, q)$ *such that* $\hat{a}$ *and* q *are part of the set of policy functions associated with the household's problem.*

Definition of Balanced-Growth Path

***Definition* 3.** *A balanced-growth path is defined as a competitive equilibrium in which the sequence*

$\{c_t, k_{Ht}(\omega, a), k_{Ft}(\omega, a), \hat{a}_t(\omega, a), q_t(\omega, a), \theta_t(\omega, \hat{a}_t, q_t), w_t, \Lambda_{t,t+1}, Z_t\}_{t\geq 0}$ *satisfies:*

(i) Per capita consumption c_t*, wages* w_t*, and productivity* Z_t *grow at rate* γ.

(ii) *For all* (ω, a), *the stock of capital held by firms and households* ($k_{Ft}(\omega, a)$ *and* $k_{Ht}(\omega, a)$, *respectively) grows at rate* $\gamma\gamma_n$.

(iii) *For all* (ω, a), *submarket choices* $a_t(\omega, a)$ *and* $q_t(\omega, a)$, *market tightness* $\theta_t(\omega, \hat{a}_t, q_t)$ *are constant over time.*

(iv) *The discount factor satisfies* $\Lambda_{t,t+1} = \frac{\beta\gamma_n}{\gamma}$.

Definition of Equilibrium in the Signaling Game

We consider two types of equilibria, defined as follows:

***Definition* 4.** *A pooling equilibrium is a competitive equilibrium in which sellers of different unobserved qualities list the same price and announce the same quality with strictly positive probability—i.e.,* $q(\omega, a_j) = q(\omega, a_{j'})$ *and* $\hat{a}(\omega, a_j) = \hat{a}(\omega, a_{j'})$. *Similarly, a separating equilibrium is a competitive equilibrium in which sellers of different unobserved qualities list either different prices or different qualities—i.e.,* $q(\omega, a_j) \neq q(\omega, a_{j'})$ *or* $\hat{a}(\omega, a_j) \neq \hat{a}(\omega, a_{j'})$. *Among those, a fully revealing separating equilibrium is a separating equilibrium in which sellers of a given unobserved quality announce their true unobserved quality—i.e.,* $\hat{a}(\omega, a_j) = a_j$.

Equilibrium Characterization

The following proposition establishes the existence and uniqueness of equilibrium, and characterizes its properties.

***Proposition* 5.** *The balanced-growth-path fully revealing separating equilibrium is characterized by the following solution to the sequence of problems* $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ *for all* $\omega \in \Omega$, *which is constructed recursively:*

(i) *The seller of the lowest unobserved quality* a_1 *chooses the full-information strategy* $\hat{a}(\omega, a_1) = a_1$, $q(\omega, a_1) = q^{FI}(\omega, a_1)$, *and* $\theta(\omega, \hat{a}(\omega, a_1), q(\omega, a_1)) = \theta^{FI}(\omega, a_1)$, *which is characterized by*

$$q^{FI}(\omega, a_1) = \nu^{\mathrm{b}}(\omega, a_1) - \frac{\chi}{\mu(\theta^{FI}(\omega, a_1))} \tag{A.1}$$

and

$$p'(\theta^{FI}(\omega, a_1)) \left(\nu^{\mathrm{b}}(\omega, a_1) - \left(\frac{\beta\gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, a_1) - \delta\omega a_1 \right) \right) = \chi,$$

where $\chi \equiv w_t/\gamma^t$.

(ii) The seller of any unobserved quality $a_k > a_1$ announces his true quality—i.e., $\hat{a}(\omega, a_k) = a_k$. Regarding the terms of trade, there are two cases to consider:

(a) If none of the constraints (11) evaluated at all $l \leq k-1$ bind, then the seller of quality a_k chooses the full-information terms of trade—i.e., $q(\omega, a_k) = q^{FI}(\omega, a_k)$ and $\theta(\omega, \hat{a}(\omega, a_k), q(\omega, a_k)) = \theta^{FI}(\omega, a_k)$.

(b) If at least one of the constraints (11) binds for $l \leq k-1$, then let $\underline{\theta}_l^k$ denote the lowest θ that solves

$$\nu^{\mathrm{s}}(\omega, a_l) = p(\theta)\left((1-\psi)q(\omega, a_k) + \psi q^P(\omega, a_l, \hat{a}(\omega, a_k), q)\right) + (1-p(\theta))\left(\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_l) - \delta\omega a_l\right),$$

where $q(\omega, a_k) = \nu^{\mathrm{b}}(\omega, a_k) - \frac{\chi}{\mu(\theta)}$. The seller of quality a_k chooses $\theta(\omega, a_k) = \min\left\{\underline{\theta}_j^k,\ j \in [1, k-1]\right\}$ and the corresponding price, as long as $\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_k) - \delta\omega a_k \geq \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_l) - \delta\omega a_l$ for all $l < k$. In this case, the optimal market tightness is lower than under the full-information terms of trade—i.e., $\theta(\omega, a_k) < \theta^{FI}(\omega, a_k)$.

Finally, there are no pooling equilibria.

Excluding the seller of the lowest unobserved quality a_1 who is never affected by the information asymmetry, Proposition 5 describes two distinct situations. In the first case, sellers can choose the unconstrained optimum of their objective since no other seller wants to mimic them when they adopt this strategy. Formally, constraints (11) drop out, and the optimal terms of trade are characterized by the first-order condition and the buyer's indifference condition (10). We refer to this unconstrained solution as the *full-information* terms of trade. As shown above, this situation arises when the inspection is informative enough—i.e., when ψ is high enough. The second case emerges when at least one other seller wants to mimic the unconstrained solution, which is formally characterized by at least one of the constraints being violated at the full-information solution. Intuitively, a relatively uninformative inspection facilitates mimicking by sellers of lower unobserved qualities. Sellers of high unobserved quality must then adapt their strategy to disincentivize mimicking by lower types and thereby signal their true quality.

We show that if sellers' values are increasing in the unobserved quality (which is always true for realistically low depreciation rates), then the optimal signaling strategy consists of choosing a lower tightness and a higher price than under full information so that the tightest constraint is just binding. This forms the unique fully revealing separating equilibrium that satisfies the D1 criterion. The proposition also states that the signaling game does not feature any pooling equilibria, in which sellers of different unobserved qualities choose the same submarket with positive probability.

A.2 Proofs

A.2.1 Proof of Proposition 1

Here we prove a more general version of Proposition 1, in which we endogenize all selling and buying decisions of firms and households. Let us denote by $b^{FI}(\omega, \hat{a}, q, a) \in \{0, 1\}$ the decision of buyers to purchase the unit on submarket $(\omega, \hat{a}, q)$ conditional on learning from the inspection that it is of quality a. Similarly, let $b(\omega, \hat{a}, q) \in \{0, 1\}$ be the decision of buyers to purchase the unit conditional on visiting the submarket $(\omega, \hat{a}, q)$ and not learning the true quality from the inspection. Let $s(\omega, a)$ be the seller's decision to post the unit of quality (ω, a) for sale. In what follows, we drop the subscript j of an individual firm.

Household's problem. The recursive optimization of the household can be written as

$$V_{Ht}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a),c,h,i>0\}}} u(c, h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

subject to the per-period budget constraint

$$c\gamma_n^t + i + \delta \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}} \omega a \left(k'(\omega, a) - ig(\omega, a)\right) = w_t h\gamma_n^t + x^{\mathrm{s}} - x^{\mathrm{b}} + Div_{Ft},$$

the law of motion of capital of quality (ω, a)

$$k'(\omega, a) = k^{\mathrm{b}}(\omega, a) - k^{\mathrm{s}}(\omega, a) + k(\omega, a) + ig(\omega, a) + \varphi K_{Ft}(\omega, a),$$

and the nonnegativity constraints $v(\omega, \hat{a}, q) \geq 0\ \forall(\omega, \hat{a}, q) \in \Omega\times\mathcal{A}\times\mathbb{R}_+$, where total purchases of quality (ω, a) are given by

$$k^{\mathrm{b}}(\omega, a) = \sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega, \hat{a}, q)[\psi b^{FI}(\omega, \hat{a}, q, a) + (1-\psi)b(\omega, \hat{a}, q)]\mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v(\omega, \hat{a}, q)\, \mathrm{d}q,$$

total sales are given by

$$\begin{aligned}k^{\mathrm{s}}(\omega,a) = \Big(&\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) \\ &+ (1-\psi)b_t(\omega,\hat{a}(\omega,a),q(\omega,a))\Big)p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)\left(k(\omega,a)+\varphi K_{Ft}(\omega,a)\right),\end{aligned}$$

total costs of buying capital are given by

$$\begin{aligned}x^{\mathrm{b}} = \sum_{\hat{a}\in\mathcal{A}}\sum_{\omega\in\Omega}\int_{q\in\mathbb{R}_+}\Big[&(\psi\sum_{a\in\mathcal{A}}\iota_t(a|\omega,\hat{a},q)q_t^P(\omega,a,\hat{a},q)b^{FI}(\omega,\hat{a},q,a) \\ &+ (1-\psi)qb(\omega,\hat{a},q))\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)+w_t\Big]v(\omega,\hat{a},q)\,\mathrm{d}q,\end{aligned}$$

and total revenues from selling capital are given by

$$\begin{aligned}x^{\mathrm{s}} = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\Big[&\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)q^P(\omega,a,\hat{a}(\omega,a),q(\omega,a)) \\ &+ (1-\psi)b_t(\omega,\hat{a}(\omega,a),q(\omega,a))q(\omega,a)\Big]p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)\left(k(\omega,a)+\varphi K_{Ft}(\omega,a)\right).\end{aligned}$$

The optimal level of investment, provided that $i>0$, is given by the first-order condition

$$1 = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}g(\omega,a)\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega,a,\mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta\gamma_n\frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$, $\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy function for capital accumulation associated with problem (2), and $\nu_t^{\mathrm{s}}(\omega,a,\mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega,a)}\frac{1}{u_{ct}(\mathbf{k})\gamma^t}$ is the marginal value of capital of type (ω,a) measured in final goods, which satisfies the recursive problem (its notation anticipates the result whereby households only sell capital, which is derived below):

$$\begin{aligned}\nu_t^{\mathrm{s}}(\omega,a,\mathbf{k}) = &\max_{\{\hat{a}(\omega,a),q(\omega,a)\}} \\ &s(\omega,a)p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))\Big[\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)q_t^P(\omega,a,\hat{a}(\omega,a),q(\omega,a))+(1-\psi)b_t(\omega,\hat{a}(\omega,a),q(\omega,a))q(\omega,a)\Big] \\ &+\Big(1-(\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)+(1-\psi)b_t(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))\Big)\left[\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega,a,\mathbf{k}_{Ht+1}(\mathbf{k}))-\delta\omega a\right].\end{aligned}$$

Firm's problem. The recursive optimization problem faced by firms can be written as

$$V_F(\mathbf{k}) = \max_{\substack{\{l,v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a)\}}}\mathbb{E}_a[div+\Lambda'\left((1-\varphi)V_F'(\mathbf{k}')+\varphi V_{t+1}^{\mathrm{exit}}(\mathbf{k}')\right)],$$

subject to the nonnegativity constraints $v(\omega,\hat{a},q)\geq 0\ \forall(\omega,\hat{a},q)\in\Omega\times\mathcal{A}\times\mathbb{R}_+$, the definition of per-period dividends

$$div=\left(\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega ak'(\omega,a)\right)^{\alpha}(\gamma^t l)^{1-\alpha}-wl-\delta\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega ak'(\omega,a)-x^b+x^s,$$

and the law of motion of capital of quality (ω,a)

$$k'(\omega,a)=k^{\mathrm{b}}(\omega,a)-k^{\mathrm{s}}(\omega,a)+k(\omega,a), \tag{A.2}$$

where total purchases of quality (ω,a) are given by

$$k^{\mathrm{b}}(\omega,a)=\sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}\iota_t(a|\omega,\hat{a},q)[\psi b^{FI}(\omega,\hat{a},q,a)+(1-\psi)b(\omega,\hat{a},q)]\mu_t(\theta(\omega,\hat{a},q))v(\omega,\hat{a},q)\,\mathrm{d}q, \tag{A.3}$$

total sales are given by

$$\begin{aligned}k^{\mathrm{s}}(\omega,a)=&\big(\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)+(1-\psi)b(\omega,\hat{a}(\omega,a),q(\omega,a))\big)\\&\times p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)k(\omega,a),\end{aligned} \tag{A.4}$$

total costs of buying capital are given by

$$\begin{aligned}x^{\mathrm{b}}=&\sum_{\hat{a}\in\mathcal{A}}\sum_{\omega\in\Omega}\int_{q\in\mathbb{R}_+}\Big[(\psi\sum_{a\in\mathcal{A}}\iota_t(a|\omega,\hat{a},q)q^P(\omega,a,\hat{a},q)b^{FI}(\omega,\hat{a},q,a)\\&+(1-\psi)qb(\omega,\hat{a},q))\mu\left(\theta\left(\omega,\hat{a},q\right)\right)+w_t\Big]v(\omega,\hat{a},q)\,\mathrm{d}q,\end{aligned} \tag{A.5}$$

and total revenues from selling capital are given by

$$\begin{aligned}x^{\mathrm{s}}=&\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\Big[\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)q^P(\omega,a,\hat{a}(\omega,a),q(\omega,a))\\&+(1-\psi)b(\omega,\hat{a}(\omega,a),q(\omega,a))q(\omega,a)\Big]p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)k(\omega,a).\end{aligned} \tag{A.6}$$

The recursive problem of the firm features a static choice of labor demand and only depends on the number of efficiency units of capital $\mathcal{K}'=\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega ak'(\omega,a)$. The first-order condition with respect to l is given by

$$\mathcal{K}'^{\alpha}\gamma^{t(1-\alpha)}(1-\alpha)l^{-\alpha}=w_t,$$

which can be rewritten as

$$l = \mathcal{K}' \left(\frac{(1-\alpha)\gamma^{t(1-\alpha)}}{w_t} \right)^{\frac{1}{\alpha}}.$$

Hence, labor demand is linear in $\mathcal{K}'$, which proves the last part of Proposition 1. We can express the revenue from production as

$$\Phi_t(\mathbf{k}') = \mathcal{K}'^{\alpha}(\gamma^t l)^{1-\alpha} - w_t l.$$

Replacing our expression for the optimal labor demand, we obtain that $\Phi_t(\mathbf{k}') = Z_t \mathcal{K}'$, where

$$Z_t \equiv \alpha \left(\frac{\gamma^t (1-\alpha)}{w_t} \right)^{\frac{1-\alpha}{\alpha}}.$$

Given this result, we can now re-express the problem of the firm as

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q), q(\omega,a), \\ b(\omega,\hat{a},q), b^{FI}(\omega,\hat{a},q,a), \\ s(\omega,a), \hat{a}(\omega,a), k'(\omega,a)\}}} \mathbb{E}_a \left[(Z_t - \delta) \mathcal{K}' - x^b + x^s \right] + \Lambda_{t,t+1} \left((1-\varphi) V_{Ft+1}(\mathbf{k}') + \varphi V^{\text{exit}}_{t+1}(\mathbf{k}') \right),$$

subject to (A.2), (A.3), (A.4), (A.5), (A.6), and the nonnegativity constraint $v(\omega, \hat{a}, q) \geq 0 \; \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+$.

Next, we conjecture that $V_{Ft}(\mathbf{k}) = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \nu^{\text{b}}_t(\omega, a) k(\omega, a)$. Let us denote by $\xi_t(\omega, \hat{a}, q)$ the Lagrange multiplier associated with the nonnegativity constraint for vacancies in all submarkets $(\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$. The first-order condition with respect to $v(\omega, \hat{a}, q)$ is

$$\mathbb{E}_a[\psi b^{FI}(\omega, \hat{a}, q, a) + (1-\psi) b(\omega, \hat{a}, q)] \mu_t \left(\theta\left(\omega, \hat{a}, q\right)\right) \left((Z_t - \delta)\omega a + \Lambda_{t,t+1}((1-\varphi)\nu^{\text{b}}_{t+1}(\omega, a) + \varphi \nu^{\text{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1})) \right) =$$
$$[(\psi \mathbb{E}_a q^P_t(\omega, a, \hat{a}, q) b^{FI}(\omega, \hat{a}, q, a) + (1-\psi) q b(\omega, \hat{a}, q)) \mu_t \left(\theta\left(\omega, \hat{a}, q\right)\right) + w_t] + \xi_t(\omega, \hat{a}, q),$$

together with the complementary slackness condition $\xi_t(\omega, \hat{a}, q) v(\omega, \hat{a}, q) = 0$. These conditions do not depend on the firm's individual capital holdings and state that the purchased units of capital are bought at a cost equal to their marginal value. We multiply the first-order condition above by $v(\omega, \hat{a}, q)$ and replace it in the objective of the firm, which then becomes

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a)\}}} \mathbb{E}_a \Bigg\{ \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}}$$

$$\Bigg[(Z_t - \delta)\omega a \left(1 - [\psi b^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b(\omega, \hat{a}(\omega, a), q(\omega, a))] p\left(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))\right) s(\omega, a)\right) k(\omega, a)$$

$$+ [\psi b^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q_t^P(\omega, a, \hat{a}(\omega, a), q) + (1-\psi) b(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a)] p\left(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))\right) s(\omega, a) k(\omega, a)$$

$$+ \Lambda_{t,t+1}((1-\varphi)\nu^{\mathrm{b}}_{t+1}(\omega, a) + \varphi \nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}))$$

$$\times \left(1 - [\psi b^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b(\omega, \hat{a}(\omega, a), q(\omega, a))] p\left(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))\right) s(\omega, a)\right) k(\omega, a)\Bigg]\Bigg\},$$

where we have used the fact that the total cost of the newly purchased units equals their marginal value, so the terms x^b and k^b cancel each other. This shows the linearity of the firm's value function with respect to $\mathbf{k}$. In what follows, we show that households are the sellers and firms are the buyers in the capital market. We also show that under certain assumptions about $q_t^P(\omega, a, \hat{a}(\omega, a), q)$, buyers always choose to buy the capital unit after matching with a seller.

Firm's selling decision and household's buying decision. Here, we show that firms buy capital but do not sell it, and that households sell capital but do not buy it (although they invest to produce capital). For this, recall that the value of a capital unit is symmetric among all firms and households because it does not depend on individual capital holdings.

Notice that the problem of households is a particular case of firms' problem with productivity Z_t set to zero. Hence, the marginal value of a capital unit for firms is larger than the marginal value of a capital good for the household as long as $Z_t > 0$. As a consequence, if firms want to sell (the value from operating the unit of capital is lower than the value from selling it), then households also prefer to sell, as they cannot obtain a higher value from this unit than an operating firm would. Hence, there is no market for the unit considered. Similarly, if households do not want to sell (they obtain a higher value by keeping the unit), firms also will not want to sell. Again, there will be no market for the unit considered, since no one wants to sell. This implies that we can simplify the problem: Households never buy capital (otherwise there are no sellers), and firms never sell capital as long as $Z_t > 0$ (otherwise there are no buyers). Thus, the optimal firm's policy is $s(\omega, a) = 0$, which simplifies the firm's marginal value of capital of quality $k(\omega, a)$ to

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right],$$

which proves the second result of Proposition 1. This result also simplifies the household's marginal value of a capital unit to

$$\begin{aligned}
\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = & \max_{\{q_t(\omega,a),\hat{a}_t(\omega,a)\}} p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big[\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q_t^P(\omega, a, \hat{a}(\omega, a), q) \\
& + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a)\Big] \\
& + \Big(1 - (\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big) \\
& \times \left[\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}_{Ht+1}(\mathbf{k})) - \delta\omega a\right].
\end{aligned}$$

Optimal purchase decision. Here, we characterize the buyer's optimal purchase decision. There are two cases to consider: Either the inspection is unsuccessful and only $(\omega, \hat{a})$ is known, or the inspection is successful and the true type (ω, a) is revealed. We handle the cases successively.

In the first case, the firm's first-order condition with respect to $b(\omega, \hat{a}, q)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b(\omega, \hat{a}, q)} = (1-\psi)\mu_t(\theta(\omega, \hat{a}, q))[\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right) - q].$$

Hence, the optimal purchase policy when the inspection is not informative is given by

$$b(\omega, \hat{a}, q) = \begin{cases} 1 \text{ if } \mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right) \geq q \\ 0 \text{ otherwise} \end{cases}.$$

In the second case, the firm's first-order condition with respect to $b^{FI}(\omega, \hat{a}, q, a)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b^{FI}(\omega, \hat{a}, q, a)} = \psi\mu_t(\theta(\omega, \hat{a}, q))[\nu_t^{\mathrm{b}}(\omega, a) - q_t^P(\omega, a, \hat{a}, q)].$$

Hence, the optimal purchase policy when the inspection is informative is given by

$$b^{FI}(\omega, \hat{a}, q, a) = \begin{cases} 1 \text{ if } \nu_t^{\mathrm{b}}(\omega, a) \geq q_t^P(\omega, a, \hat{a}, q) \\ 0 \text{ otherwise} \end{cases}.$$

Given the partial derivative with respect to $b^{FI}(\omega, \hat{a}, q, a)$ derived above, Assumption 1 ensures that a transaction always takes place after an informative inspection.

A.2.2 Proof of Proposition 5

The proof is split into five steps we list below. It makes use of the particular case of the full-information case from Proposition 2, for which a separate proof is presented below. The proof proceeds as follows:

Step 1: We describe the link between prices and market tightness under a fully revealing separating equilibrium.

Step 2: We construct the unique, fully revealing separating equilibrium during the transition path recursively. We proceed in two substeps:

(a) First, we show that under certain conditions, the full-information allocation can be sustained when $\psi < 1$.

(b) Second, we prove the existence and uniqueness of a separating equilibrium when the full-information optimum is not part of the possible strategies under asymmetric information.

Step 3: We apply the result from the previous step to a balanced-growth path.

Step 4: We show that there cannot be a pooling equilibrium if $a \to \phi q^P(\omega, a, \hat{a}, q) - \nu^{\mathrm{b}}(\omega, a)$ is monotonous in a during the transition path.

Step 5: We show that there is no pooling equilibrium on the balanced-growth path by applying the previous step.

Step 1: Prices and market tightness in a fully revealing separating equilibrium. The following Lemma characterizes the equilibrium prices and market tightness in a fully revealing separating equilibrium.

Lemma* 1. *Equilibrium market tightness in a fully revealing separating equilibrium

In a fully revealing separating allocation, sellers never misreport their true unobserved *quality—i.e.,* $\hat{a}(\omega, a) = a \ \forall(\omega, a) \in \Omega \times \mathcal{A}$. *Then, market tightness is given by*

$$\theta(\omega, \hat{a}, q) = \mu_t^{-1}\left(\frac{w_t}{\nu^{\mathrm{b}}(\omega, \hat{a}) - (1-\psi)q - \psi q^P(\omega, \hat{a}, \hat{a}, q)}\right). \tag{A.7}$$

Proof. In a fully revealing separating allocation, the vector $(\omega, \hat{a}, q)$ reveals unobserved quality a by definition. Hence, using the indifference condition of buyers, we obtain

$$\theta(\omega, \hat{a}, q) = \mu_t^{-1}\left(\frac{w_t}{\mathbb{E}_a(\nu^{\mathrm{b}}(\omega, \hat{a}) - (1-\psi)q - \psi q^P(\omega, a, \hat{a}, q)|\omega, \hat{a}, q)}\right).$$

Since the allocation is fully revealing, $\hat{a}(\omega, a) = a$. Therefore, we can drop the expectation term. We then obtain the result of Lemma 1. ■

Step 2: Recursive construction of the unique fully revealing separating equilibrium. Let us consider the case with $\psi < 1$. The case in which $\psi = 1$, corresponding to full information, is solved in the proof of Proposition 2 below. We first define the notation used in this step of the proof. In what follows, we fix $\omega \in \Omega$ and a time t and omit all references to ω and t (there will be no ambiguity: All variables depend on t except for the continuation values taken at $t+1$). We note $\bar{v}(a) = \Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a) - \delta\omega a$ the continuation value of a seller of unobserved quality a and observed quality ω. We also note $V(a) = p(\theta(\omega, q(\omega, a)))\left[\nu^{\mathrm{b}}(\omega, a) - \bar{v}(a)\right] - \chi\theta(\omega, q(\omega, a))$, where $\chi \equiv \frac{w_t}{\gamma^t}$. The set of qualities $\mathcal{A}$ is ordered, and we note $\mathcal{A}_k = \{a \in \mathcal{A} | a \leq a_k\}$ the subset of its k lowest elements. We denote $q^P(\omega, a_i, a_j, q) = min(q, q^B(\omega, a_i, a_j))$ so that the post-inspection price is equal to some $q^B(\omega, a_i, a_j)$ unless the seller committed to a lower price pre-inspection. Finally, we use Lemma 1 to define $q(a, \theta) = \nu^{\mathrm{b}}(\omega, a) - \frac{w_t}{\mu_t(\theta)}$ as the price corresponding to quality a and tightness θ. We will show the following assertion by induction on $k \in [1, N_a]$.

Assertion 1. Assertion at rank $\mathbf{k} \in [\mathbf{1}, \mathbf{N_a}]$

The unique separating fully revealing equilibrium allocation $\Theta_k = \{\theta(a_1), \ldots, \theta(a_k)\}$ *on* $\mathcal{A}_k$ *that satisfies the D1 criterion is constructed recursively:*

(i) The seller of the lowest unobserved quality a_1 *chooses the full-information strategy* $\hat{a}(\omega, a_1) = a_1$, $q(\omega, a_1) = q^{FI}(\omega, a_1)$ *and* $\theta(\omega, \hat{a}(\omega, a_1), q(\omega, a_1)) = \theta^{FI}(\omega, a_1)$, *which is characterized by*

$$q^{FI}(\omega, a_1) = \nu^{\mathrm{b}}(\omega, a_1) - \frac{w}{\mu(\theta^{FI}(\omega, a_1))}$$

and

$$p'(\theta^{FI}(\omega,a_1))\left(\nu^{\mathrm{b}}(\omega,a_1)-\bar{v}(\omega,a_1)\right)=\chi.$$

(ii) The seller of any unobserved quality $a_i > a_1$ signals his true quality—i.e., $\hat{a}(\omega,a_i)=a_i$. Regarding the terms of trade, there are two cases to consider:

(a) If for all $l < i$, the constraint (11) evaluated at $q^{FI}(\omega,a_i)$ and $\theta^{FI}(\omega,a_i)$ is slack, then the seller of quality a_i chooses the full-information terms of trade—i.e., $q(\omega,a_i)=q^{FI}(\omega,a_i)$ and $\theta(\omega,\hat{a}(\omega,a_i),q(\omega,a_i))=\theta^{FI}(\omega,a_i)$.

(b) If at least one of the constraints (11) binds for $l < i$, then let $\underline{\theta}_l^i$ denote the lowest solution θ to

$$\nu^{\mathrm{s}}(\omega,a_l)=p\left(\theta\right)\left((1-\psi)\nu^{\mathrm{b}}(\omega,a_i)+\psi q^P(\omega,a_l,\hat{a}(\omega,a_i),q)\right)+\left(1-p(\theta)\right)\bar{v}(\omega,a_l)-\chi\theta.$$

The seller of quality a_i chooses

$$\theta(\omega,a_i)=min\left\{\underline{\theta}_j^i,\ j\in\{1,\ldots,i-1\}\right\}$$

and the corresponding price, as long as $\bar{v}(\omega,a_i)\geq\bar{v}(\omega,a_l)$ for all $l < i$. In this case, the optimal market tightness is lower than under the full-information terms of trade—i.e., $\theta(\omega,a_i) < \theta^{FI}(\omega,a_i)$.

Initialization: $\mathcal{A}_1=\{a_1\}$. We begin the construction by noting that since in the set $\mathcal{A}_1$ there is no lower type that needs to be disincentivized from mimicking for type a_1 and type a_1 does not want to mimic any higher type because there is none, we have that in any separating equilibrium in $\mathcal{A}_1$:

$$\begin{cases}\hat{a}(a_1)=a_1\\ q(a_1)=q^{FI}(a_1)\end{cases},\tag{A.8}$$

which proves the assertion for $k=1$.

Recursion. Let us fix $k\in[2,N_a]$ and suppose that the assertion is true for $k-1$.

Step 2(a): The full-information optimum can be sustained under asymmetric information. We first study the case in which the full-information terms of trade can be sustained for quality a_k. Given the sequence Θ_{k-1}, the full-information strategy of the seller of quality a_k can be part of a fully revealing separating equilibrium if and only if no seller of a lower quality wants to deviate from its current strategy to mimic her. Formally, the incentive compatibility constraint must be just binding or slack for every quality $a_i \leq a_k$ when type a_k implements its full-information allocation:

$$\begin{aligned}&V(a_i) \equiv p(\theta(a_i)) \left[\nu^{\mathrm{b}}(a_i) - \bar{v}(a_i)\right] - \chi\theta(a_i) \geq \\ &p(\theta^{FI}(a_k)) \left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_i, a_k, q^{FI}(a_k)) - \bar{v}(a_i)\right] - (1-\psi)\chi\theta^{FI}(a_k).\end{aligned} \tag{A.9}$$

The seller of quality a_k is then allowed to implement its full-information strategy, which maximizes its unconstrained objective conditional on $\hat{a}(a_k) = a_k$. Since a_k is the highest quality on $\mathcal{A}_k$, the seller would not be able to obtain a higher value by mimicking a lower quality. Then, since no seller has an incentive to deviate to mimic quality a_k, the previous allocation Θ_{k-1} remains.

The last step is to discuss off-equilibrium beliefs conditional on being on other submarkets in which a_k is the announced quality. We can first rule out that quality a_k is expected by buyers in these other submarkets. Indeed, the seller of quality a_k is not better off deviating to any other tightness θ as it is achieving its unconstrained optimum. If the seller of any lower quality is better off deviating to a submarket $(\hat{a} = a_k, \theta)$ conditional on quality a_k being expected, this seller would be better off under a larger set of beliefs than the seller of quality a_k. The D1 criterion would then impose that its quality is the one expected instead of a_k, which rules out the deviation.

Finally, suppose that a seller of some quality $a_i < a_k$ has a profitable deviation by choosing $(\hat{a}(a_i) = a_k, \theta)$ and that the expected quality on this submarket is $a_j < a_k$. Then, since $q^B(a_i, a_k) \leq q^B(a_i, a_j)$ from Assumption 1, the seller of quality a_i would also have a profitable deviation absent quality a_k. Using the recursion at rank $k-1$, we know that this is not the case, since Θ_{k-1} is an equilibrium on $\mathcal{A}_{k-1}$. As a consequence, no seller of lower quality has a profitable deviation to either on-path or off-path submarkets in which a_k is announced, and quality a_k is never expected by buyers on any off-equilibrium submarket.

Thus, using the Assertion for $k-1$, we obtain a unique separating equilibrium in which the allocation is $\Theta_k = \Theta_{k-1} \cup \{\theta^{FI}(a_k)\}$ and all sellers announce their true quality.

Step 2(b): The full-information optimum cannot be sustained under asymmetric information. Let A_{k-1} denote the set of qualities that want to mimic sellers of quality a_k when they play their full-information market tightness. We now have that for all $a_j \in A_{k-1}$:

$$V(a_j) < p(\theta^{FI}(a_k)) \left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q^{FI}(a_k)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta^{FI}(a_k). \quad \text{(A.10)}$$

Let $R_j^k(\theta) = p(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(a_k,\theta)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta$. Then, $R_j^k(\theta) + \bar{v}(a_j)$ represents the revenue seller of quality a_j receives when mimicking the seller of quality a_k when the latter plays θ.

Single-peaked shape of $R_j^k(\theta)$. We next analyze the properties of the expected value from mimicking $R_j^k(\theta)$. We have two intervals to consider. Suppose first that $q(a_k,\theta) > q^B(a_j,a_k)$. Then, $q^P(a_j, a_k, q(a_k,\theta)) = q^B(a_j,a_k)$ does not depend on θ. The second-order derivative with respect to θ is then

$$p''(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^B(a_j,a_k) - \bar{v}(a_j)\right] < 0.$$

Hence, on this interval, the function is strictly concave.[26]

Now suppose that $q(a_k,\theta) \leq q^B(a_j,a_k)$. Then, $q^P(a_j,a_k,q(\theta,a_k)) = q(a_k,\theta)$ and the second-order derivative writes

$$p''(\theta)\left[\nu^{\mathrm{b}}(a_k) - \bar{v}(a_j)\right] < 0.$$

Hence, the revenue from mimicking is also strictly concave on that interval. The entire function is piecewise concave. Thus, it could have either one or two peaks.

We now show that it is indeed single-peaked. Let us define θ^B as the tightness at which the two concave parts connect: $q^B(a_j,a_k) = \nu^{\mathrm{b}}(a_k) - \frac{w}{\mu(\theta^B)}$. For $\theta > \theta^B$, the revenue from mimicking is equal to

$$R_j^k(\theta) = p(\theta)\left[\nu^{\mathrm{b}}(a_k) - \bar{v}(a_j)\right] - \chi\theta$$

[26] We have $\nu^{\mathrm{b}}(a_k) \geq \nu^{\mathrm{s}}(a_k) > \bar{v}(a_k) \geq \bar{v}(a_j)$, where the first inequality is due to discounting and the presence of search frictions, the second inequality is due to the definition of the value, and the last inequality comes from the assertion. In addition, $q^B(a_j,a_k) \geq \bar{v}(a_j)$ from Assumption 1.

and its maximum θ^* is characterized by the first-order condition:

$$p'(\theta^*)\left[\nu^{\mathrm{b}}(a_k) - \bar{v}(a_j)\right] - \chi = 0.$$

Substituting in the buyer's indifference condition, we obtain

$$q^* = \eta\nu^{\mathrm{b}}(a_k) + (1-\eta)\bar{v}(a_j).$$

Recall that the full information price of the seller of quality a_j satisfies

$$q^{FI}(a_j) = \eta\nu^{\mathrm{b}}(a_j) + (1-\eta)\bar{v}(a_j).$$

Since $\nu^{\mathrm{b}}(a_k) \geq \nu^{\mathrm{b}}(a_j)$ and by assumption $q^{FI}(a_j) \geq q^B(a_j, a_k)$, we obtain $q^* \geq q^{FI}(a_j) \geq q^B(a_j, a_k)$. Hence, the function $R_j^k(\theta)$ is strictly decreasing for $\theta > \theta^B$, which in turn implies that it is single-peaked.

Disincentivizing mimicking from lower types. Next, we derive the set of tightnesses (and corresponding prices) the seller of quality a_k can choose without having other types mimicking him.

Let $\tilde{\theta}_l$ denote the market tightness that causes the incentive-compatibility constraint between types a_k and $a_l < a_k$ to bind. Then, $\tilde{\theta}_l$ is characterized by the equation

$$p(\theta_l)(\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)) - \chi\theta_l = R_l^k(\tilde{\theta}_l). \tag{A.11}$$

Since $R_l^k(\cdot)$ is single-peaked, there are at most two values $\tilde{\theta}_l$ that satisfy the above equation. In addition, $\lim_{\theta\to+\infty} R_l^k(\theta) = -\infty$ and $\lim_{\theta\to 0} R_l^k(\theta) = 0$ and $p(\theta_l)(\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)) - \chi\theta_l > 0$. Hence, there are either two or no solutions to the equation above. Let us denote $\underline{\theta}_l^k$ and $\bar{\theta}_l^k$ the two solutions to (A.11), provided they exist. As $R_l^k(\cdot)$ is single-peaked, we have that for all $\theta \in [0, \underline{\theta}_l^k] \cup [\bar{\theta}_l^k, +\infty)$, the right-hand side of equation (A.11) is lower than its left-hand side. As a consequence, for any tightness on these two subintervals, the incentive-compatibility constraint is slack; i.e., type a_l does not want to mimic type a_k.

We now show that the sets $\left\{\underline{\theta}_l^k:\ l \in \{1, \dots, k-1\}\right\}$ and $\left\{\bar{\theta}_l^k:\ l \in \{1, \dots, k-1\}\right\}$ are non-empty. Because the seller of quality a_k cannot choose its full-information tightness, there exists at least one quality $a_j \in \mathcal{A}_{k-1}$ such that

$$R_j^k(0) = 0 < p(\theta_j)\left[\nu^{\mathrm{b}}(a_j) - \bar{v}(a_j)\right] - \chi\theta_j < R_j^k(\theta^{FI}(a_k)).$$

Using the inequality above and the fact that $R_j^k(\cdot)$ is continuous, we can find a $\tilde{\theta}_j < \theta^{FI}(a_k)$ such that equation (A.11) holds, which implies that $\underline{\theta}_j^k$ and $\bar{\theta}_j^k$ are well defined. As a consequence, the two sets are non-empty.

Let $\underline{\theta} = \min\left\{\underline{\theta}_l^k : \ l \in \{1, \ldots, k-1\}\right\}$ and $\bar{\theta} = \max\left\{\bar{\theta}_l^k : \ l \in \{1, \ldots, k-1\}\right\}$. It follows that if type a_k chooses any $\theta \geq \bar{\theta}$ or $\theta < \underline{\theta}$, then no seller of quality $a_j \in \mathcal{A}_{k-1}$ wants to mimic the seller of quality a_k. Finally, note that since $\underline{\theta}_j^k < \theta^{FI}(a_k)$ and $\bar{\theta}_j^k > \theta^{FI}(a_k)$, we necessarily have that $\underline{\theta} < \theta^{FI}(a_k)$ and $\bar{\theta} > \theta^{FI}(a_k)$.

Any tightness $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ is a possible value for a separating equilibrium. We just proved that for any $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ played by the seller of quality a_k, all types in $\mathcal{A}_{k-1}$ do not want to mimic, which ensures that a separating equilibrium can be constructed using any of these values. Note that without a further refinement of off-equilibrium beliefs, we can always find a set of buyers' beliefs that can sustain any θ in that set. We can, for example, assume that buyers believe that any off-path terms of trade are only picked by the lowest type and make any θ in the set part of a separating equilibrium. Also, note that any $\theta < \underline{\theta}$ or $\theta > \bar{\theta}$ imposes a cost to the seller without any further benefit, since the seller is already signaling its true type. Next, we follow the signaling literature and impose the D1 criterion, which is an equilibrium refinement that isolates the "most relevant" equilibrium.

The only values consistent with the D1 criterion are $\underline{\theta}$ or $\bar{\theta}$. Choose any $\theta_k \notin [\underline{\theta}, \bar{\theta}]$. The fact that $\theta_k > \bar{\theta}$ or $\theta_k < \underline{\theta}$ implies that the constraint "quality a_j does not want to mimic type a_k" is not binding; i.e.,

$$V(a_j) > p(\theta_k)\left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(a_k, \theta_k)) - \bar{v}(a_j)\right) - (1-\psi)\chi\theta_k.$$

Suppose first that $\theta_k < \underline{\theta}$. We now need to determine which seller is most likely to deviate and choose $(\hat{a} = a_k, \underline{\theta})$, so that we can set beliefs in accordance with the D1 criterion. We know that the seller of quality a_k is strictly better off if the price is $q(a_k, \underline{\theta})$, as the expected revenue of the seller of quality a_k is strictly increasing in market tightness for $\theta_k < \underline{\theta}$. At the same time, any seller of a quality lower than a_k would not, by construction, be better off by deviating to the submarket with tightness $\underline{\theta}$ and price $q(a_k, \underline{\theta})$. This implies that the seller of quality a_k is better off under a larger set of beliefs than other sellers, as he is better off under a larger set of prices (in the sense of inclusion). The D1 criterion then requires that type a_k is the one expected on any submarket with $\theta \in (0, \underline{\theta}]$. A symmetric reasoning implies that quality a_k is also expected on submarkets with $\theta \in [\bar{\theta}, +\infty)$.

We then invoke the fact that the unconstrained objective of sellers of quality a_k is strictly increasing on $(0, \underline{\theta}]$ because $\underline{\theta} < \theta^{FI}(a_k)$ and strictly decreasing on $[\bar{\theta}, +\infty)$ because $\bar{\theta} > \theta^{FI}(a_k)$. As a consequence, the seller of quality a_k has a profitable deviation for any $\theta_k < \underline{\theta}$ and $\theta_k > \bar{\theta}$, which leaves only $\underline{\theta}$ and $\bar{\theta}$ as possible equilibrium values after applying the D1 criterion.

The seller of quality a_k (weakly) prefers $\underline{\theta}$ to $\bar{\theta}$ if continuation values are (weakly) increasing in a. We proceed in two steps. We first show that the seller of quality a_k has a higher value at $\underline{\theta}$ if the incentive-compatibility constraint is binding with the same quality for $\underline{\theta}$ and $\bar{\theta}$. Then, we show that the same is true if the incentive-compatibility constraint binds with different qualities for $\underline{\theta}$ and $\bar{\theta}$.

Let $\nu^{\mathrm{s}}(a_k; \theta)$ denote the value of the seller of quality a_k when the market tightness is θ and the posted price is $q(a_k, \theta)$.

(i) Case 1: The same quality binds at $\underline{\theta}$ and $\bar{\theta}$. Let this quality be a_l.

We start with the subcase with $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) = q^B(a_l, a_k)$, which implies that $q(a_k, \bar{\theta}_l^k) \geq q^B(a_l, a_k)$. Since $\underline{\theta}_l^k < \theta^{FI}(a_k)$, we always have $q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) = q^B(a_l, a_k)$.

The binding incentive-compatibility constraint with quality a_l at tightness $\underline{\theta}_l^k$ can be written as

$$V(a_l) = p(\underline{\theta}_l^k) \left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - \bar{v}(a_l)\right) - (1-\gamma)\chi \underline{\theta}_l^k.$$

After adding and subtracting $(1-\psi)(1-p(\underline{\theta}_l^k))\bar{v}(a_k)$ on the right-hand side, we obtain

$$V(a_l) = (1-\psi)\nu^{\mathrm{s}}(a_k; \underline{\theta}_l^k) - (1-\psi)\bar{v}(a_k) + p(\underline{\theta}_l^k) \left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right].$$

A similar expression applies when evaluating the incentive-compatibility constraint at $\bar{\theta}_l^k$.

Subtracting the expression above from its counterpart at $\bar{\theta}_l^k$, we obtain

$$(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)\right]=\left(p(\bar{\theta}_l^k)-p(\underline{\theta}_l^k)\right)\left[\psi q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_l)\right]$$
$$+\psi p(\underline{\theta}_l^k)\left(q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))-q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))\right). \tag{A.12}$$

Given the subcase we started from, we have that $q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k)=q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))=q^B(a_l,a_k)$; therefore the second term on the left-hand side in (A.12) is zero. This yields

$$(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)\right]=\left(p(\bar{\theta}_l^k)-p(\underline{\theta}_l^k)\right)\left[\psi q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_l)\right].$$

Because $p(\bar{\theta}_l^k)>p(\underline{\theta}_l^k)$ and $q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))\geq\bar{v}(a_l)$ from Assumption 1, the right-hand side is (weakly) positive if continuation values are (weakly) increasing in unobserved quality. Hence, the left-hand side is (weakly) positive; i.e., $\underline{\theta}_l^k$ yields a (weakly) higher utility to the seller of quality a_k.

Let us now handle the remaining subcase with $q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))=q(a_k,\bar{\theta}_l^k)$. The binding incentive-compatibility constraint at $\bar{\theta}_l^k$ is given by

$$V(a_l)=p(\bar{\theta}_l^k)\left((1-\psi)\nu^{\mathrm{b}}(a_k)+\psi q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))-\bar{v}(a_l)\right)-(1-\gamma)\chi\bar{\theta}_l^k.$$

Substituting in $q^P(a_l,a_k,q(a_k,\bar{\theta}_l^k))=q(a_k,\bar{\theta}_l^k)=\nu^{\mathrm{b}}(a_k)-\frac{w}{\mu(\bar{\theta}_l^k)}$, we obtain

$$V(a_l)=p(\bar{\theta}_l^k)\left(\nu^{\mathrm{b}}(a_k)-\bar{v}(a_l)\right)-\chi\bar{\theta}_l^k$$

Next, we add and subtract $(1-p(\bar{\theta}_l^k))\bar{v}(a_k)$:

$$V(a_l)=\nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)-\bar{v}(a_k)+p(\bar{\theta}_l^k)\left[\bar{v}(a_k)-\bar{v}(a_l)\right].$$

As in the previous subcase, the binding incentive-compatibility constraint at $\underline{\theta}_l^k$ is given by

$$V(a_l)=\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\bar{v}(a_k)+p(\underline{\theta}_l^k)\left[\psi(q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))-q(a_k,\underline{\theta}_l^k))+\bar{v}(a_k)-\bar{v}(a_l)\right].$$

Subtracting these last two expressions, we finally obtain

$$\nu^{s}(a_k;\underline{\theta}_l^k)-\nu^{s}(a_k;\bar{\theta}_l^k)=\left(p(\bar{\theta}_l^k)-p(\underline{\theta}_l^k)\right)[\bar{v}(a_k)-\bar{v}(a_l)]+p(\underline{\theta}_l^k)\psi\left[q(a_k,\underline{\theta}_l^k)-q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))\right].$$

The first term on the right-hand side is (weakly) positive because continuation values are (weakly) non-decreasing in a by assumption. The second term is weakly positive by the commitment assumption on $q^P(\cdot)$. Hence, $\underline{\theta}_l^k$ is (weakly) preferred when the incentive-compatibility constraint binds with the same quality on both sides.

(ii) Case 2: Different qualities bind at $\underline{\theta}$ and $\bar{\theta}$. Let us now take $l,m<k$ with $m\neq l$ such that $\bar{\theta}=\bar{\theta}_m^k$ and $\underline{\theta}=\underline{\theta}_l^k$.

By definition of the bounds, we have $\bar{\theta}_l^k\leq\bar{\theta}_m^k$ and $\underline{\theta}_l^k\leq\underline{\theta}_m^k$. Since $\bar{\theta}_l^k$ is in the decreasing part of the revenue function $R_l^k(\theta)$ and $\bar{\theta}_l^k\leq\bar{\theta}_m^k$, we have that $R_l^k(\bar{\theta}_l^k)\geq R_l^k(\bar{\theta}_m^k)$. By definition of $\bar{\theta}_l^k$, the last inequality can be written as

$$V(a_l)>R_l^k(\bar{\theta}_m^k).$$

We subtract $V(a_m)$ from both sides of the last inequality and use the definition of $\bar{\theta}_m^k$ to obtain

$$V(a_l)-V(a_m)>p(\bar{\theta}_m^k)\left[\psi(q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))-q^P(a_m,a_k,q(a_k,\bar{\theta}_m^k))+\bar{v}(a_m)-\bar{v}(a_l))\right]. \tag{A.13}$$

Let us now compare the value obtained by the seller of quality a_k at the two market tightnesses $\bar{\theta}_m^k=\bar{\theta}$ and $\underline{\theta}_l^k=\underline{\theta}$. We start by analyzing the subcase $q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))<q(a_k,\bar{\theta}_m^k)$. As before, the binding incentive-compatibility constraint with respect to quality a_l can be written as

$$V(a_l)=(1-\psi)\nu^{s}(a_k;\underline{\theta}_l^k)-(1-\psi)\bar{v}(a_k)+p(\underline{\theta}_l^k)\left[\psi q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_l)\right].$$

We subtract the above expression from its counterpart evaluated at $\bar{\theta}_m^k$ and obtain

$$\begin{aligned}(1-\psi)\left[\nu^{s}(a_k;\underline{\theta}_l^k)-\nu^{s}(a_k;\bar{\theta}_m^k)\right]&=V(a_l)-V(a_m)\\&\quad+p(\bar{\theta}_m^k)\left[\psi q^P(a_m,a_k,q(a_k,\bar{\theta}_m^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_m)\right]\\&\quad-p(\underline{\theta}_l^k)\left[\psi q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_l)\right].\end{aligned}$$

Using inequality (A.13), we obtain

$$(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)\right] \geq (p(\bar{\theta}_m^k)-p(\underline{\theta}_l^k))\left[\psi q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))+(1-\psi)\bar{v}(a_k)-\bar{v}(a_l)\right]$$
$$+\psi p(\bar{\theta}_m^k)\left[q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))-q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))\right].$$

In this subcase, the second term on the right-hand side is zero. The first term on the right-hand side is positive, since $q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k)) \geq \bar{v}(a_l)$ and continuation values are increasing in a. Hence, $\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) \geq \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)$; therefore, $\underline{\theta}$ is preferred by the seller of quality a_k.

We now address the second subcase: $q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k)) = q(a_k,\bar{\theta}_m^k)$. In this case, the binding incentive-compatibility constraint at $\bar{\theta}_m^k$ can be written as

$$V(a_m)=\nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)-\bar{v}(a_k)+p(\bar{\theta}_m^k)\left[\bar{v}(a_k)-\bar{v}(a_m)\right]$$

and the binding incentive-compatibility constraint at $\underline{\theta}_l^k$ can be written as

$$V(a_l)=\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\bar{v}(a_k)+p(\underline{\theta}_l^k)\left[\psi(q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))-q(a_k,\underline{\theta}_l^k))+\bar{v}(a_k)-\bar{v}(a_l)\right].$$

Subtracting one from the other, we obtain

$$\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)=V(a_l)-V(a_m)+p(\bar{\theta}_m^k)\left[\bar{v}(a_k)-\bar{v}(a_m)\right]$$
$$-p(\underline{\theta}_l^k)\left[\psi(q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))-q(a_k,\underline{\theta}_l^k))+\bar{v}(a_k)-\bar{v}(a_l)\right].$$

Using inequality (A.13) to replace the difference $V(a_l)-V(a_m)$, substituting in the subcase expression $q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))=q(a_k,\bar{\theta}_m^k)$, and rearranging terms, we obtain the following inequality:

$$\nu^{s}(a_k;\underline{\theta}_l^k)-\nu^{s}(a_k;\bar{\theta}_m^k) \geq \left(p(\bar{\theta}_m^k)-p(\underline{\theta}_l^k)\right)\left[\bar{v}(a_k)-\bar{v}(a_l)\right]$$
$$+p(\bar{\theta}_m^k)\psi\left[q(a_k,\bar{\theta}_m^k)-q^P(a_m,a_k,q(a_k,\bar{\theta}_m^k))\right]$$
$$+p(\underline{\theta}_l^k)\psi\left[q(a_k,\underline{\theta}_l^k)-q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))\right].$$

All terms on the right-hand side are positive. Hence, the left-hand side is positive and $\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) \geq \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)$; therefore, $\underline{\theta}$ is preferred by the seller of quality a_k.

In conclusion, the only possible value for a separating equilibrium is $\underline{\theta}$ as long as continuation values are increasing in a. We also show that under beliefs set in accordance with the D1 criterion, there is no profitable deviation on $(0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$. This guarantees that no seller has an incentive to deviate to these submarkets.

Downward incentive-compatibility constraints. By construction, $\Theta_{k-1} \cup \{\underline{\theta}\}$ satisfies the "upward" incentive compatibility constraints, since no seller of a lower quality has an incentive to mimic a_k at $\underline{\theta}$. We now need to show that for this allocation, sellers have no incentives to mimic the strategy of qualities lower than themselves. The recursion at rank $k-1$ implies that all qualities lower than a_k satisfy the downward incentive-compatibility constraints, so that we only need to show the property for sellers of quality a_k. Let us fix $a_i < a_k$. We prove that type a_k does not want to mimic the strategy of type a_i. Let $\theta_k = \underline{\theta}$ and θ_i be the market tightness chosen by the seller of quality a_i in equilibrium.

(i) Case 1: $\theta_i < \theta_k$. By construction, $\theta_k \leq \theta^{FI}(a_k)$. We know that the unconstrained objective of sellers of quality a_k is strictly concave, with a maximum reached at $\theta^{FI}(a_k)$. This implies that $p(\theta)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta$ is strictly increasing in θ for $\theta \in [\theta_i, \theta_k]$, which yields

$$p(\theta_i)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_i < p(\theta_k)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_k.$$

We then use $\nu^{\mathrm{b}}(a_k) > \nu^{\mathrm{b}}(a_i)$, which implies

$$p(\theta_i)\left(\nu^{\mathrm{b}}(a_i) - \bar{v}(a_k)\right) - \chi\theta_i < p(\theta_k)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_k.$$

Intuitively, if $\theta_k > \theta_i$, then the seller of quality a_k benefits from both a higher selling probability and a higher price. As a consequence, the seller of quality a_k does not want to mimic the strategy of the seller of quality a_i.

(ii) Case 2: $\theta_i \geq \theta_k$. For this case, we need to introduce quality $a_j < a_k$ such that the "upward" incentive-compatibility constraint between a_k and a_j is binding. We proceed with three subcases:

(i) *Case 2.1: $a_i = a_j$.* The "upward" incentive-compatibility constraint between a_i and a_k is binding, i.e.:

$$p(\theta_k)\left[q(a_k)(1-\psi) + \psi q^P(a_i, a_k) - \bar{v}(a_i)\right] = p(\theta_i)\left[q(a_i) - \bar{v}(a_i)\right],$$

which can be rewritten as

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]=p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+(p(\theta_i)-p(\theta_k))(\bar{v}(a_k)-\bar{v}(a_i))$$
$$+\psi p(\theta_k)\left[q(a_k)-q^P(a_i,a_k)\right].$$

Since $\theta_i \geq \theta_k$, $q(a_k) \geq q^{FI}(a_k) > q^P(a_i,a_k)$ and continuation values are increasing, we have unambiguously:

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right].$$

Hence, the seller of quality a_k does not want to mimic type a_i's strategy.

(ii) *Case 2.2:* $a_j > a_i$. We start from the binding incentive-compatibility constraint between a_j and a_k:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right]=p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right].$$

Using the recursion, sellers of quality a_j satisfy the downward incentive-compatibility constraint with a_i:

$$p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_j)\right].$$

Injecting this inequality in the previous equation:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_j)\right]$$

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+\psi p(\theta_k)\left[q(a_k)-q^P(a_j,a_k)\right]$$
$$+(p(\theta_i)-p(\theta_k))(\bar{v}(a_k)-\bar{v}(a_j)).$$

Since $\theta_i \geq \theta_k$, $q(a_k) \geq q^P(a_j,a_k)$ and continuation values are increasing, we obtain

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right].$$

Hence, the seller of quality a_k does not want to mimic type a_i's strategy.

(iii) *Case 2.3:* $a_j < a_i$. We start again from the binding incentive-compatibility constraint between a_j and a_k:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right]=p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right].$$

This time we use the upward incentive-compatibility constraint between a_i and a_j:

$$p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right]\geq p(\theta_i)\left[q(a_i)(1-\psi)+\psi q^P(a_j,a_i)-\bar{v}(a_j)\right].$$

Injecting this inequality in the previous expression:

$$\begin{aligned}(1-\psi)p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq{}&(1-\psi)p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]\\&+(p(\theta_i)-p(\theta_k))((1-\psi)\bar{v}(a_k)-\bar{v}(a_j))\\&-\psi p(\theta_k)q^P(a_j,a_k)+\psi p(\theta_i)q^P(a_j,a_i).\end{aligned}$$

From Assumption 1 $q^P(a_j,a_i)\geq q^P(a_j,a_k)$, which yields

$$\begin{aligned}(1-\psi)p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq(1-\psi)p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+\\(p(\theta_i)-p(\theta_k))(\psi q^P(a_j,a_i)+(1-\psi)\bar{v}(a_k)-\bar{v}(a_j)).\end{aligned}$$

Since $\theta_i\geq\theta_k$, continuation values are increasing and $q^P(a_j,a_i)\geq\bar{v}(a_j)$ (from Assumption 1), the second term on the right-hand side is positive. As a consequence:

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]\geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right].$$

Thus, the downward incentive-compatibility constraint is satisfied.

As a result, the allocation $\Theta_{k-1}\cup\{\underline{\theta}\}$ satisfies all upward and downward incentive-compatibility constraints and forms a separating equilibrium. In order to conclude, we still need to analyze off-equilibrium beliefs and show the absence of strictly profitable deviations to off-equilibrium submarkets.

Off-equilibrium beliefs. Next, we show that no seller strictly improves its payoff by deviating to an off-equilibrium submarket. Beliefs on off-equilibrium submarkets are set in accordance with the D1 criterion, so that the expected quality on a given submarket is the one that is better off deviating under the largest set of beliefs, i.e., prices. For the equilibrium to survive the D1 criterion, it is sufficient to show that the seller of the expected quality on any off-equilibrium submarket does not have a strictly profitable deviation to that submarket. Since we focus on the seller who is better off under the largest set of beliefs, no other seller will obtain a strictly positive payoff from deviating to this submarket.

To prove this, fix an unobserved quality $a \in \mathcal{A}_k$. Conditional on being expected on a given off-equilibrium submarket $(\hat{a}, \theta')$, the seller of quality a faces the same objective function as in equilibrium, but evaluated at market tightness θ'. As a consequence, if in equilibrium the seller of unobserved quality a is implementing its full-information strategy, it cannot have a strictly profitable deviation to this submarket because its equilibrium strategy already maximizes its unconstrained objective. Hence, without loss, we can restrict our attention to the case in which the seller has a binding incentive-compatibility constraint with a lower unobserved quality on equilibrium.

Given a binding constraint, we know that the seller of quality a deviates from its full-information strategy in order to disincentivize mimicking from lower types by picking a lower tightness than it would have under full information. Using the concavity of the unconstrained objective of sellers with respect to market tightness, this implies that there is a range of tightnesses $[\underline{\theta}_a, \bar{\theta}_a]$, where the seller of quality a would be better off deviating conditional on being the only quality expected. Note that $\underline{\theta}_a$ is the market tightness picked by the seller of quality a on equilibrium. We therefore need to ensure that the seller of quality a is never expected on off-equilibrium submarkets $(\hat{a}, \theta')$ with $\theta' \in [\underline{\theta}_a, \bar{\theta}_a]$—i.e., that there is always a seller of lower quality who is better off on this submarket under a larger set of beliefs. We show this result in three steps corresponding to the three possible cases.

(i) Case 1: $\hat{a} = a$. Choose any $a \in \mathcal{A}_k$ and let $a_l < a$ be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. Let $q(a, \underline{\theta}_a)$ denote the price obtained by the seller of quality a in the separating equilibrium. Choose any $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$ and let q' be the corresponding price on the off-equilibrium submarket (a, θ'). The revenue of the seller of quality a at θ' would be $p(\theta')\,[q' - \bar{v}(a)] + \bar{v}(a)$. Hence, the net gain the seller of quality a would receive from deviating to θ' is

$$\Delta = p(\theta')[q' - \bar{v}(a)] - p(\underline{\theta}_a)\,[q(a, \underline{\theta}_a) - \bar{v}(a)]\,.$$

Similarly, the net gain the seller of quality a_l would receive from deviating to the same submarket, conditional on these beliefs, is

$$\Delta_l = p(\theta')[(1-\psi)q' + \psi q^P(a_l, a, q') - \bar{v}(a_l)] - p(\theta(a_l))\left[q(a_l, \theta(a_l)) - \bar{v}(a_l)\right]. \tag{A.14}$$

Let us now use the binding incentive-compatibility constraint between qualities a_l and a, which is given by

$$p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)] = p(\theta(a_l))\left[q(a_l, \theta(a_l)) - \bar{v}(a_l)\right].$$

Combining this equality with equation (A.14), we obtain

$$\Delta_l = p(\theta')[(1-\psi)q' + \psi q^P(a_l, a, q') - \bar{v}(a_l)] - p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)]. \tag{A.15}$$

The break-even price of the seller of quality a at (a, θ') that makes $\Delta = 0$, which we denote by $\tilde{q}$, is characterized by

$$p(\theta')\tilde{q} = (p(\theta') - p(\underline{\theta}_a))\bar{v}(a) + p(\underline{\theta}_a)q(a, \underline{\theta}_a). \tag{A.16}$$

Next, we show that $\Delta_l > 0$ when evaluated at this price. Suppose first that $\tilde{q} \leq q^B(a_l, a)$. Then, $q^P(a_l, a, \tilde{q}) = \tilde{q}$ and equation (A.15) becomes

$$\Delta_l = p(\theta')[\tilde{q} - \bar{v}(a_l)] - p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)].$$

Replacing in the expression for $\tilde{q}$ from (A.16), we obtain

$$\Delta_l = (p(\theta') - p(\underline{\theta}_a))[\bar{v}(a) - \bar{v}(a_l)] + p(\underline{\theta}_a)\psi\left(q(a, \underline{\theta}_a) - q^P(a_l, a, q(a, \underline{\theta}_a))\right).$$

Since $\theta' > \underline{\theta}_a$ and continuation values are increasing, the first term is positive. In addition, by Assumption 1, $q(a, \underline{\theta}_a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$ implies that the second term is also weakly positive. Hence, $\Delta_l > 0$; i.e., there exists a price $\tilde{q}$ for which there is a strictly profitable deviation for sellers of quality a_l but not for sellers of quality a.

Now suppose that $\tilde{q} > q^B(a_l, a)$. Then, $q^P(a_l, a, \tilde{q}) = q^B(a_l, a)$. Following similar steps, we evaluate (A.15) at price $\tilde{q}$ and replace the expression for $p(\theta')\tilde{q}$ using (A.16) to obtain

$$\Delta_l = (p(\theta') - p(\underline{\theta}_a))[\psi q^B(a_l, a) + (1-\psi)\bar{v}(a) - \bar{v}(a_l)] + p(\underline{\theta}_a)\psi \left(q^B(a_l, a) - q^P(a_l, a, q(a, \underline{\theta}_a))\right).$$

Since $\theta' > \underline{\theta}_a$, continuation values are increasing, $q^B(a_l, a) \geq \bar{v}(a_l)$, and $q^B(a_l, a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$ by Assumption 1, we can conclude again that $\Delta_l > 0$.

To summarize, for any submarket (a, θ') with $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$, sellers of quality a_l still find a strictly profitable deviation at the break-even price of sellers of quality a. This implies that sellers of quality a_l are better off under a larger set of beliefs (those that justify the price $\tilde{q}$) than sellers of quality a. The D1 criterion then requires that buyers do not expect to find units of quality a in these submarkets, which proves our claim in Case 1.

(ii) Case 2: $\hat{a} > a$. Let $a_j < a$ be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. Choose any $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$. Let q' denote the price on submarket $(\hat{a}, \theta')$. By deviating to this submarket, the seller of quality a obtains a change in expected revenues of

$$\Delta = p(\theta')\left[(1-\psi)q' + \psi q^P(a, \hat{a}, q') - \bar{v}(a)\right] - V(a). \tag{A.17}$$

Similarly, the change in revenue from this deviation for the seller of quality a_j is given by

$$\Delta_j = p(\theta')\left[(1-\psi)q' + \psi q^P(a_j, \hat{a}, q') - \bar{v}(a_j)\right] - V(a_j).$$

Hence, we have

$$\Delta_j - \Delta = V(a) - V(a_j) + p(\theta')\left[\psi(q^P(a_j, \hat{a}, q') - q^P(a, \hat{a}, q')) + \bar{v}(a) - \bar{v}(a_j)\right].$$

Let us use the binding incentive-compatibility constraint between a and a_j, which can be rewritten as

$$V(a_j) - V(a) = p(\underline{\theta}_a)\left[\psi(q^P(a_j, a, q(a)) - q(a)) + \bar{v}(a) - \bar{v}(a_j)\right].$$

Since $q(a) \geq q^{FI}(a) \geq q^B(a_j, a)$, $q^P(a_j, a, q(a)) = q^B(a_j, a)$. Substituting in the previous equation, we obtain

$$\Delta_j - \Delta \geq (p(\theta') - p(\underline{\theta}_a))(\bar{v}(a) - \bar{v}(a_j)) + \psi p(\theta')(q^P(a_j, \hat{a}, q') - q^P(a, \hat{a}, q')) + p(\underline{\theta}_a)\psi(q(a) - q^B(a_j, a)). \tag{A.18}$$

Let us now set q' such that $\Delta = 0$. First assume that $q' \leq q^B(a_j, \hat{a})$. Because $a_j < a$, $q^P(a_j, \hat{a}, q') \leq q^P(a, \hat{a}, q')$ and $q^P(a_j, \hat{a}, q') = q^P(a, \hat{a}, q') = q'$. Hence, the second term in (A.18) is zero. The other terms are positive as continuation values are increasing and $\theta' > \underline{\theta}_a$. As a consequence $\Delta_j \geq 0$: The seller of quality a_j is better off deviating for a larger set of prices than the seller of quality a.

We now consider the case in which $q^B(a, \hat{a}) \geq q' \geq q^B(a_j, \hat{a})$. The break-even price for the seller of quality a is such that

$$p(\theta')(1-\psi)q' = p(\underline{\theta}_a)q(a) + (p(\theta') - p(\underline{\theta}_a))\bar{v}(a) - \psi p(\theta')q^P(a, \hat{a}, q').$$

Provided that $q' \leq q^B(a, \hat{a})$, this equation becomes

$$p(\theta')q' = p(\underline{\theta}_a)q(a) + (p(\theta') - p(\underline{\theta}_a))\bar{v}(a). \tag{A.19}$$

Combining the expression for Δ_j, with the binding incentive-compatibility constraint between qualities a and a_j and the expression above that characterizes the break-even price q', we obtain

$$\Delta_j = p(\theta')\left(\psi q^B(a_j, \hat{a}) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)\right) - p(\underline{\theta}_a)\left(\psi q^B(a_j, a) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)\right).$$

We now combine equation (A.19) with the condition $q' \leq q^B(a, \hat{a})$ to obtain

$$p(\theta') \geq p(\underline{\theta}_a)\frac{q(a) - \bar{v}(a)}{q^B(a, \hat{a}) - \bar{v}(a)}.$$

Finally, we substitute this inequality into the previous equation to obtain

$$\Delta_j \geq p(\underline{\theta}_a)\left[\frac{q(a) - \bar{v}(a)}{q^B(a, \hat{a}) - \bar{v}(a)}\left(\psi q^B(a_j, \hat{a}) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)\right) - \left(\psi q^B(a_j, a) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)\right)\right].$$

The term between brackets is positive if

$$\frac{q(a) - \bar{v}(a)}{q^B(a, \hat{a}) - \bar{v}(a)} \geq \frac{\psi q^B(a_j, a) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)}{\psi q^B(a_j, \hat{a}) + (1-\psi)\bar{v}(a) - \bar{v}(a_j)}.$$

Since $q^B(a_j, a) \geq q^B(a_j, \hat{a})$, the right-hand side is an increasing function of ψ. We also have that $q(a) \geq q^{FI}(a)$. Therefore, a sufficient condition for the above inequality to hold is

$$\frac{q^{FI}(a) - \bar{v}(a)}{q^B(a, \hat{a}) - \bar{v}(a)} \geq \frac{q^B(a_j, a) - \bar{v}(a_j)}{q^B(a_j, \hat{a}) - \bar{v}(a_j)},$$

which is true by Assumption 1. As a consequence, the seller of quality a_j is better off deviating to submarket $(\hat{a}, \theta')$ for a larger set of beliefs than the seller of quality a, which therefore cannot be expected on that submarket.

Next let us discuss the remaining case in which $q' > q^B(a, \hat{a})$. Note that (A.17) implies that the q' that makes $\Delta = 0$ is a continuous, decreasing function of θ'. Hence, if $q' \geq q^B(a, \hat{a})$ for a given tightness θ, we have $q' \geq q^B(a, \hat{a})$ for all $\theta' \leq \theta$. This implies that the right-hand side of inequality (A.18) is a linear function of $p(\theta')$ on an interval of the form $(\underline{\theta}_a, \theta^B)$, where θ^B is defined as the market tightness in submarket $(\hat{a}, q')$ when $q' = q^B(a, \hat{a})$. The right-hand side of inequality (A.18) is positive at $\underline{\theta}_a$. We just showed that (A.18) is positive when $q' = q^B(a, \hat{a})$. Hence, it is positive in the entire range $(\underline{\theta}_a, \theta^B)$ or equivalently for any $q' \geq q^B(a, \hat{a})$.

Thus, for any submarket (a, θ') with $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$, sellers of quality a_j still find a strictly profitable deviation at the break-even price of sellers of quality a. So, the D1 criterion then requires that buyers do not expect to find units of quality a in these submarkets.

(iii) Case 3: $\hat{a} < a$. Let a_l be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. There are two cases to consider.

(i) *Case 3.1:* If $\hat{a} > a_l$, then the proof is identical to Case 1. Indeed, the only required change is to replace the announced quality in the expression of q^P. This change does not alter any argument made in that step of the proof.

(ii) *Case 3.2:* Suppose now that $\hat{a} \leq a_l$ and set q' to be the price on submarket $(\hat{a}, \theta')$. The net gain from deviating for sellers of quality a is

$$\Delta = p(\theta')[q' - \bar{v}(a)] - V(a),$$

where we have used the fact that quality a is higher than announced quality $\hat{a}$, so sellers of a still sell at price q' and not at the inspection-adjusted price $q^P(\cdot)$. Similarly, the net gain from deviating for sellers of quality a_l is

$$\Delta_l = p(\theta')(q' - \bar{v}(a_l)) - V(a_l).$$

The incentive-compatibility constraint between qualities a_l and a can be rewritten as

$$V(a_l) = V(a) + p(\underline{\theta}_a)\left[\psi(q^P(a_l, a, q(a, \underline{\theta}_a)) - q(a, \underline{\theta}_a)) + \bar{v}(a) - \bar{v}(a_l)\right].$$

Combining these three expressions, we obtain:

$$\Delta_l - \Delta = (p(\theta') - p(\underline{\theta}_a))(\bar{v}(a) - \bar{v}(a_l)) + p(\theta(a))\psi(q(a, \underline{\theta}_a) - q^P(a_l, a, q(a, \underline{\theta}_a))).$$

Continuation values are increasing, $\theta' > \underline{\theta}_a$ and $q(a, \underline{\theta}_a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$. Hence, $\Delta_l > \Delta$—i.e., sellers of quality a_l are better off under a larger set of prices (and the corresponding beliefs) than sellers of quality a. Hence, quality a cannot be expected on such off-equilibrium submarkets.

In conclusion, if a quality is expected on a given off-equilibrium submarket, the seller of this quality does not have a strictly profitable deviation to that submarket. As a consequence, the recursive construction of $\Theta_{k-1} \cup \{\theta_k\}$ as the only fully revealing separating equilibrium that survives the D1 criterion is justified.

Conclusion of the recursion. We proved that there exists a unique fully revealing equilibrium on $\mathcal{A}_k$ that satisfies the D1 criterion and that the construction is done using the procedure described in the assertion. Hence, the assertion is true at rank k, which concludes the induction.

A non-revealing separating equilibrium is always weakly dominated by a fully revealing separating equilibrium. For a given quality a_k, it is possible to construct bounds $\underline{\theta}$ and $\bar{\theta}$ on any submarket with announced quality $\hat{a} < a_k$ and similarly construct a separating, but not fully revealing, equilibrium. Because the inspection-adjusted price $q^P(\cdot)$ is weakly decreasing in the announced quality, the revenue from the mimicking of sellers of all lower qualities will be weakly *larger* when the announced quality is $\hat{a}$ than a_k. Thus, in order to disincentivize mimicking, the bound $\underline{\theta}$, conditional on an announced quality $\hat{a} < a_k$, will be lower than for an announced quality $\hat{a} = a_k$, and the bound $\bar{\theta}$ will be larger. At the same time, the transacted price obtained by the seller of quality a_k remains the same as the posted price (since there is no penalty for announcing a quality lower than its own). This implies that by announcing $\hat{a} < a_k$, the seller of quality a_k will have to choose a market tightness further from his full-information tightness than he would have had he signaled quality a_k, yielding a lower payoff. Therefore, non-revealing separating equilibria are at least weakly dominated by the fully revealing separating equilibrium. Since we assume $q^P(\cdot)$ to be weakly decreasing in the announced quality, we cannot rule out non-revealing separating equilibria as strictly dominated. If instead $q^P(\cdot)$ was strictly decreasing in the announced quality, then the fully revealing equilibrium would also be the unique separating equilibrium.

Step 3: Balanced-growth path under asymmetric information. The proof of Proposition 5 is then a simple application of the recursion above on $\mathcal{A} = \mathcal{A}_{N_a}$.

Step 4: There is no pooling equilibrium in transitional dynamics if $\psi q^P(\omega, a, \hat{a}, q) - \bar{v}(\omega, a)$ is monotonous in a. Assume that for all $(\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$, $\psi q^P(\omega, a, \hat{a}, q) - \bar{v}(\omega, a)$ is monotonous in a. Suppose there exists an equilibrium such that the subset of types $A \subset \mathcal{A}$ are pooled together in submarket $(\omega, \hat{a}, q)$ with some strictly positive probability. Let $\bar{a} = \max\{a : a \in A\}$. As in previous steps, we suppose they all have the same observed quality ω and omit it for convenience.

We next show that under the D1 criterion, there exists a strictly profitable deviation for sellers of quality $\bar{a}$, ruling out any equilibria in which pooling occurs with strictly positive probability. Let θ be the tightness on submarket $(\hat{a}, q)$ where pooling occurs. We can assume without loss of generality that buyers visit this submarket.[27] We then proceed by setting beliefs consistent with the D1 criterion on submarkets where $\hat{a}$ is the announced quality and θ' is the market tightness.

Consider a deviation of a seller of quality $a \in A$, with $a < \bar{a}$, to another submarket with the same announced quality $\hat{a}$ and market tightness θ'. The seller would be better off for any

[27] Otherwise, sellers of any quality could deviate to any submarket in which buyers would purchase for a strictly positive price, which would be a strictly profitable deviation.

price weakly greater than $\tilde{q}$, which is defined by the indifference condition:

$$p(\theta')((1-\psi)\tilde{q}+\psi q^P(a,\hat{a},\tilde{q}))+(1-p(\theta'))\bar{v}(a) = p(\theta)((1-\psi)q+\psi q^P(a,\hat{a},q))+(1-p(\theta))\bar{v}(a). \tag{A.20}$$

Let us now set beliefs. Let $\Delta_{\bar{a}}$ denote the net gain for the seller of quality $\bar{a}$ from deviating to a submarket with tightness θ' and price $\tilde{q}$:

$$\Delta_{\bar{a}} = p(\theta')((1-\psi)\tilde{q}+\psi q^P(\bar{a},\hat{a},\tilde{q}))+(1-p(\theta'))\bar{v}(\bar{a})-p(\theta)((1-\psi)q+\psi q^P(\bar{a},\hat{a},q))-(1-p(\theta))\bar{v}(\bar{a}).$$

Consider first the case with $\theta' < \theta$. Then, from equation (A.20) we have $\tilde{q} > q$. It can be verified that $q^P(\bar{a},\hat{a},q) - q^P(a,\hat{a},q)$ is weakly increasing in q. Therefore, $q^P(\bar{a},\hat{a},\tilde{q}) - q^P(a,\hat{a},\tilde{q}) \geq q^P(\bar{a},\hat{a},q) - q^P(a,\hat{a},q)$, which implies

$$\Delta_{\bar{a}} \geq (p(\theta)-p(\theta'))\left[\psi(q^P(a,\hat{a},q) - q^P(\bar{a},\hat{a},q)) + \bar{v}(\bar{a}) - \bar{v}(a)\right].$$

We now use the assumption that $\psi q^P(a,\hat{a},q) - \bar{v}(a)$ is monotonous in a. More specifically, assume the function is decreasing in a. This implies that the second term on the right-hand side is positive, as is the first term on the right-hand side. Hence, for all $a \in A$ and $\theta' < \theta$, the seller of quality $\bar{a}$ always has a larger benefit from deviating than the seller of inferior quality—by definition, at price $\tilde{q}$, the net benefit from deviating for sellers of quality a is zero. This ensures that the seller of quality $\bar{a}$ will be better off under the worst consistent beliefs. The D1 criterion then requires that quality $\bar{a}$ is expected in submarkets with tightness $\theta' < \theta$. We can now construct a profitable deviation for the seller of quality $\bar{a}$. Using the indifference condition of buyers, we have that $(1-\psi)q = \mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) - \frac{\chi}{\mu(\theta)}$. Given the monotonicity assumption, $\mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) = \nu^{\mathrm{b}}(\bar{a}) - \psi q^P(\bar{a},\hat{a},q) - \epsilon$ for some $\epsilon > 0$. The net gain from deviating to tightness θ' for the seller of quality $\bar{a}$ is then

$$\Delta_{\bar{a}} = p(\theta')[\nu^{\mathrm{b}}(\bar{a})-\bar{v}(\bar{a})]-p(\theta)\left[\mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) + \psi q^P(\bar{a},\hat{a},q) - \bar{v}(\bar{a})\right]+\chi(\theta-\theta'),$$

or

$$\Delta_{\bar{a}} = (p(\theta') - p(\theta))[\nu^{\mathrm{b}}(\bar{a}) - \bar{v}(\bar{a})] + \chi(\theta-\theta') + p(\theta)\epsilon.$$

Hence, we can find a θ' sufficiently close to θ such that the deviation yields a strictly positive $\Delta_{\bar{a}}$, which in turn implies a strictly profitable deviation for the seller of quality $\bar{a}$. Therefore, the pooling equilibrium cannot be sustained. If the function $\psi q^P(a,\hat{a},q) - \bar{v}(a)$ is instead increasing in a, we can make exactly the same symmetric reasoning with $\theta' > \theta$.

Step 5: There are no pooling equilibria on the balanced-growth path. Let us now apply the result from Step 4 to a balanced-growth-path equilibrium. Suppose that some qualities are pooled together at some announced quality $\hat{a}$ and market tightness θ. Let quality a be one of them and let q be the associated price.

The value of the seller of type a on the balanced-growth path is:

$$\nu^{\mathrm{s}}(a) = p(\theta)((1-\psi)q + \psi q^P(a,\hat{a},q)) + (1-p(\theta))(\Lambda\nu^{\mathrm{s}}(a) - \delta\omega a).$$

We multiply by Λ, subtract $\delta\omega a$, and reorganize the terms to obtain

$$\bar{v}(a)(1-\Lambda(1-p(\theta))) = -\delta\omega a + \Lambda p(\theta)((1-\psi)q + \psi q^P(a,\hat{a},q)).$$

Solving for $\bar{v}(a)$ and subtracting $\psi q^P(a,\hat{a},q)$ we obtain

$$\bar{v}(a) - \psi q^P(a,\hat{a},q) = \frac{1}{(1-\Lambda(1-p(\theta)))}\left[-\delta\omega a + \Lambda(1-\psi)p(\theta)q - \psi q^P(a,\hat{a},q)(1-\Lambda)\right].$$

Hence, $a \to \bar{v}(a) - \psi q^P(a,\hat{a},q)$ is monotonous in a for all unobserved qualities $a \in A$ that are pooled in submarket $(\hat{a},\theta)$. We can then apply the previous step, which rules out any pooling equilibrium on a balanced-growth path. ■

A.2.3 Proof of Proposition 2 and Corollary 1

We first characterize the equilibrium in transitional dynamics.

Transitional dynamics in the full-information case. We now characterize the equilibrium under full information—namely, the equilibrium when the signal is always informative of the unobserved quality ($\psi = 1$).

We fix the continuation value of sellers $\nu^{\mathrm{s}}_{t+1}(\omega,a)$ for all ω and a and describe the equilibrium in period t conditional on agents' continuation values. From Proposition 1, we have that the values of sellers and buyers are given by equations (3) and (7).

Under full information, the first-order condition with respect to vacancies posted is given by

$$\mu_t(\theta(\omega, a, q))(\nu_t^{\mathrm{b}}(\omega, a) - q_t^{FI}(\omega, a)) = w_t,$$

which relates the expected benefit from searching in a given submarket to the expected cost, and provides an indifference condition between sale prices and trading probabilities. Given this condition, the seller's maximization problem is then given by

$$\max_{\theta} p(\theta)\left(\nu_t^{\mathrm{b}}(\omega, a) - \frac{w_t\theta}{\gamma^t p(\theta)}\right) + (1 - p(\theta))(\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1}) - \delta\omega a),$$

which gives the following first-order condition with respect to θ:

$$p'(\theta)(\nu_t^{\mathrm{b}}(\omega, a) - \Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a) = \frac{w_t}{\gamma^t}.$$

We then replace the right-hand side using the indifference condition of buyers to obtain

$$(1 - \eta)(\nu_t^{\mathrm{b}}(\omega, a) - \Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a) = \nu_t^{\mathrm{b}}(\omega, a) - q_t^{FI}(\omega, a),$$

from which we can solve for the equilibrium full-information price

$$q_t^{FI}(\omega, a) = \eta\nu_t^{\mathrm{b}}(\omega, a) + (1 - \eta)(\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1}) - \delta\omega a).$$

To find the associated optimal market tightness, we replace this price in the seller's first-order condition to obtain

$$\theta_t^{FI}(\omega, a) = \left(\frac{\bar{m}\gamma^t}{w_t}(1 - \eta)(\nu_t^{\mathrm{b}}(\omega, a) - \Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a)\right)^{1/\eta}.$$

Balanced-growth path under full information. We next use these results to obtain closed-form solutions for values and terms of trade on the balanced-growth path equilibrium.

Let $\chi = \frac{w_t}{\gamma^t}$ denote the detrended wage on the balanced-growth path. From our expression above we have

$$q^{FI}(\omega, a) = \eta\nu^{\mathrm{b}}(\omega, a) + (1 - \eta)(\Lambda\nu^{\mathrm{s}}(\omega, a) - \delta\omega a)$$

and

$$\theta^{FI}(\omega, a) = \left(\frac{\bar{m}}{\chi}(1 - \eta)(\nu^{\mathrm{b}}(\omega, a) - \Lambda\nu^{\mathrm{s}}(\omega, a) + \delta\omega a)\right)^{1/\eta}.$$

From Proposition 1, the seller's and buyer's values on the balanced-growth path under full information are given by

$$\nu^{\mathrm{b}}(\omega, a) = (Z - \delta)\omega a + \Lambda \left[(1-\varphi)\nu^{\mathrm{b}}(\omega, a) + \varphi \nu^{\mathrm{s}}(\omega, a)\right],$$
$$\nu^{\mathrm{s}}(\omega, a) = q^{FI}(\omega, a) p(\theta^{FI}(\omega, a)) + \big(1 - p(\theta^{FI}(\omega, a))\big)\left(\Lambda \nu^{\mathrm{s}}(\omega, a) - \delta \omega a\right).$$

Replacing these values in the optimal market tightness $\theta^{FI}(\omega, a)$, we obtain

$$p(\theta^{FI}(\omega, a)) = \bar{m}\left(\frac{Z\omega a \bar{m}(1-\eta)}{\chi\left(1 - \Lambda(1-\varphi)(1 - \eta p(\theta^{FI}(\omega, a)))\right)}\right)^{\frac{1-\eta}{\eta}},$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dlogp(\theta^{FI}(\omega, a))}{dlog(\omega)} = \frac{1-\eta}{\eta}\frac{\left(1 - \Lambda\left(1-\varphi\right)\left(1 - \eta p(\theta^{FI}(\omega, a))\right)\right)}{\left(1 - \Lambda\left(1-\varphi\right)\left(1 - p(\theta^{FI}(\omega, a))\right)\right)} > 0.$$

Similarly, we replace the buyer's and seller's values in the optimal price $q^{FI}(\omega, a)$ to obtain

$$q^{FI}(\omega, a) = \frac{\omega a}{1-\Lambda}\left[\eta Z \frac{1 - \Lambda(1 - p(\theta^{FI}(\omega, a)))}{1 - \Lambda(1-\varphi)(1 - \eta p(\theta^{FI}(\omega, a)))} - \delta\right] \equiv \frac{\omega a}{1-\Lambda} F(p(\theta^{FI}(\omega, a))).$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dq^{FI}(\omega, a)}{d\omega} = \frac{a}{1-\Lambda}\left[F(p(\theta^{FI}(\omega, a))) + \omega F'(p(\theta^{FI}(\omega, a)))\frac{dp(\theta^{FI}(\omega, a))}{d\omega}\right],$$

where

$$F'(p(\theta^{FI}(\omega, a))) = \eta Z \Lambda \frac{1 - (1-\varphi)[\eta + \Lambda(1-\eta)]}{(1 - \Lambda(1-\varphi)(1 - \eta p(\theta^{FI}(\omega, a))))^2} > 0.$$

Thus, when $q^{FI}(\omega, a) \geq 0$, we obtain $\frac{dq^{FI}(\omega,a)}{d\omega} > 0$. Since qualities ω and a have similar effects on optimal terms of trade, the same comparative statics apply to changes in a under full information. ■

A.2.4 Proof of Proposition 3

In the case with $\mathcal{A} = \{a_L, a_H\}$ with $a_L < a_H$, the seller of quality a_L chooses the full-information price and market tightness. The strategy of the seller of quality a_H is then determined by the binding incentive-compatibility constraint between him and sellers of quality a_L, which is given by

$$p(\theta^{FI}(a_L))(q^{FI}(a_L)-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L) = \\ p(\theta(a_H))[(1-\psi)q(a_H)+\psi q^P(a_L,a_H,q(a_H))-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L].$$

There exists a threshold ψ^* that triggers the non-full-information solution. For a small degree of asymmetry of information, the constraint above might not be binding. Instead, the constraint becomes binding for a threshold value ψ^* defined by

$$p(\theta^{FI}(a_L))(q^{FI}(a_L)-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L) = \\ p(\theta^{FI}(a_H))[(1-\psi^*)q^{FI}(a_H)+\psi^* q^P(a_L,a_H,q^{FI}(a_H))-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L].$$

By Assumption 1, f$q^P(a_L,a_H,q^{FI}(a_H)) < q^{FI}(a_L)$; thus, we can rewrite the constraint and solve for ψ^* and obtain

$$\psi^* = \frac{\left(\nu^{S,FI}(a_H)-\nu^{S,FI}(a_L)\right)\left[1-\Lambda(1-p(\theta^{FI}(a_H)))\right]+\delta\omega\left(a_H-a_L\right)\left(1-p(\theta^{FI}(a_H))\right)}{p(\theta^{FI}(a_H))(q^{FI}(a_H)-q^P(a_L,a_H,q^{FI}(a_H)))}. \tag{A.21}$$

Thus, for ψ below ψ^*, the incentive-compatibility constraint evaluated at the full-information terms of trade is not satisfied. Then, the optimal market tightness for sellers of quality a_H is determined by

$$p(\theta^{FI}(a_L))(q^{FI}(a_L)-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L) \\ = p(\theta(a_H))\left((1-\psi)q(a_H)+\psi q^P(a_L,a_H,q(a_H))-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L\right).$$

Replacing the price $q(a_H)$ from the buyer's indifference condition, this condition can be rewritten as

$$p(\theta^{FI}(a_L))(q^{FI}(a_L)-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L) \\ = p(\theta(a_H))\left((1-\psi)\nu^b(a_H)+\psi q^P(a_L,a_H,q(a_H))-\Lambda\nu^{\mathrm{s}}(a_L)+\delta\omega a_L\right)-(1-\psi)\theta(a_H)\chi. \tag{A.22}$$

Comparative statics. Let us differentiate the constraint with respect to ψ in the region in which the incentive-compatibility constraint binds (i.e., $\psi \leq \psi^*$). Differentiating (A.22) with respect to ψ, we obtain

$$\frac{dlog(\theta(a_H))}{d\psi} = \frac{q(a_H) - q^P(a_L, a_H, q(a_H))}{(1-\eta)((1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) - \eta(1-\psi)\frac{\chi}{\mu(\theta(a_H))}}. \tag{A.23}$$

To sign this expression, we need to ensure that the denominator is positive, since $q(a_H) \geq q^{FI}(a_H) > q^{FI}(a_L) \geq q^P(a_L, a_H, q(a_H))$. Under full information, we know that

$$q^{FI}(a_H) = \nu^{\mathrm{b}}(a_H) - \frac{w}{\mu(\theta^{FI}(a_H))} = \eta\nu^{\mathrm{b}}(a_H) + (1-\eta)(\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H),$$

and hence

$$\frac{\chi}{\mu(\theta^{FI}(a_H))} = (1-\eta)(\nu^{\mathrm{b}}(a_H) - \Lambda\nu^{\mathrm{s}}(a_H) + \delta\omega a_H).$$

We also know that the binding incentive-compatibility constraint (A.22) has two solutions for $\theta(a_H)$ and that as long as $\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H \geq \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$, the lowest one will be chosen by the seller of quality a_H. In particular, this implies $\theta(a_H) \leq \theta^{FI}(a_H)$. Since $\mu(\theta)$ is a decreasing function of θ, we obtain

$$\frac{\chi}{\mu(\theta(a_H))} \leq (1-\eta)(\nu^{\mathrm{b}}(a_H) - \Lambda\nu^{\mathrm{s}}(a_H) + \delta\omega a_H).$$

The denominator in (A.23) can be rewritten as

$$\begin{aligned}(1-\eta)((1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) - \eta(1-\psi)\frac{\chi}{\mu(\theta(a_H))} = \\ (1-\eta)\left((1-\psi)\nu^{\mathrm{b}}(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\right) - (1-\psi)\frac{\chi}{\mu(\theta(a_H))}.\end{aligned}$$

Using the previous inequality and the fact that $q^P(a_L, a_H, q(a_H)) > \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$:

$$\begin{aligned}(1-\eta)\left((1-\psi)\nu^{\mathrm{b}}(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\right) - (1-\psi)\frac{\chi}{\mu(\theta(a_H))} > \\ (1-\eta)(1-\psi)\left[\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H - (\Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L)\right].\end{aligned}$$

Since continuation values are increasing, $\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H \geq \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$. This implies that the denominator in (A.23) is positive, which in turn implies

$$\frac{dlog(\theta(a_H))}{d\psi} > 0.$$

Thus, the optimal market tightness for sellers of quality a_H is increasing in the informativeness of the inspection. From the buyer's indifference condition, we can also conclude that their optimal posted price is decreasing in the informativeness of the inspection.

There exists a threshold $\underline{\psi}$ such that sellers' continuation values are increasing for $\psi \geq \underline{\psi}$. The last item we need to verify is that $\bar{v}(a_H) > \bar{v}(a_L)$, with $\bar{v}(a) \equiv \Lambda\nu^{\mathrm{s}}(a) - \delta\omega a$, in the case in which the incentive-compatibility constraint is binding so that we can guarantee the existence of the equilibrium.

Rewriting the incentive-compatibility constraint, we have

$$\nu^{\mathrm{s}}(a_L) - \bar{v}(a_L) = \nu^{\mathrm{s}}(a_H) - \bar{v}(a_H) + p(\theta(a_H))[\psi(q^P(a_L, a_H, q(a_H)) - q(a_H)) + \bar{v}(a_H) - \bar{v}(a_L)].$$

Using the fact that $q(a_H) > q^{FI}(a_H)$ and $q^P(a_L, a_H, q(a_H)) = q^B(a_L, a_H)$, this constraint can be written as

$$(1 - \Lambda(1 - p(\theta(a_H))))(\bar{v}(a_H) - \bar{v}(a_L)) = \Lambda\psi p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] - \delta\omega(a_H - a_L).$$

Hence, $\bar{v}(a_H) \geq \bar{v}(a_L) \geq 0 \iff \Lambda\psi p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$.

For δ low enough, there exists a ψ such that $\psi\Lambda p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$. As ψ decreases, $\theta(a_H)$ decreases—and since the function on the left-hand side is concave in $\theta(a_H)$, the left-hand side becomes increasing in ψ at optimal choices of $\theta(a_H)$. This implies that there exists a threshold $\underline{\psi}$ such that

$$\Lambda\underline{\psi} p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] = \delta\omega(a_H - a_L).$$

Thus, assuming that δ is low enough to have $\Lambda\psi^* p(\theta^{FI}(a_H))[q^{FI}(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$, then $\underline{\psi} \leq \psi^*$ and we obtain that the inequality is satisfied on the interval $[\underline{\psi}, 1]$, which in turn ensures that continuation values are increasing on that interval. Finally, notice that $\lim_{\delta \to 0} \underline{\psi} = 0$. That is, as the depreciation rate becomes negligible, it is always true that sellers' continuation values are increasing in the quality a.

There is no pooling equilibrium in the two-quality case. Using our earlier proof that rules out pooling equilibria, we need to verify the condition that for all $(\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$,

$\psi q^P(\omega, a, \hat{a}, q) - \nu^{\mathrm{s}}(\omega, a) + \delta\omega a$ is monotonous in a. Here, this condition is trivially satisfied. Indeed, the function is always monotonous over the set of unobserved qualities since there are only two (either one is greater than the other or the reverse). ■

A.2.5 Proof of Proposition 4

In a fully revealing separating equilibrium, the buyer's indifference condition is given by

$$q_t(\omega, a) = \nu_t^{\mathrm{b}}(\omega, a) - \frac{w_t}{\mu_t(\theta(\omega, a))},$$

where

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left((1-\varphi)\,\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right).$$

As $\varphi \to 0$, this value can be expressed as

$$\begin{aligned}\nu_t^{\mathrm{b}}(\omega, a) &\approx \omega a\left((Z_t - \delta) + \sum_{j=1}^{\infty}\left(\prod_{i=0}^{j}\Lambda_{t,t+i}\right)(Z_{t+j} - \delta)\right)\\ &\approx \omega a f(Z_t),\end{aligned}$$

where we have used the fact that Z_t follows a Markov process to summarize the effect of future productivity in terms of the function of current productivity $f(Z_t)$. Then, taking logs, this value can be expressed as

$$\log \nu_t^{\mathrm{b}}(\omega, a) \approx \log \omega + \log a + \iota_t,$$

where ι_t denote time fixed effects. Finally, making the assumption that search costs represent a small fraction of the buyer's value of a unit of capital—i.e., $\frac{\frac{w_t}{\mu_t(\theta(\omega,a))}}{\nu_t^{\mathrm{b}}(\omega,a)} \to 0$—we obtain the following expression for the price of a unit of capital:

$$\begin{aligned}\log q_t(\omega, a) &\approx \log \omega + \iota_t + \log a\\ &= \tau X + \iota_t + \log a,\end{aligned}$$

where the second line imposes the mapping between observed characteristics and observed efficiency units $\log \omega = \tau X$. Thus, using microdata we can regress

$$\log q_{it} = \iota_\omega X_i + \iota_t + \varepsilon^q_{it},$$

which results in a consistent estimator for τ (and thus ω) and the unobserved quality a from ι_ω and the residuals ε^q_{it}, respectively.

In the second step, we need to estimate the following regression:

$$\log(Duration_{it}) = \upsilon_\omega \log(\omega_{it}) + \upsilon_q \log(q_{it}) + \iota_t + \varepsilon^d_{it}.$$

Given our assumptions that yield $\log q_{it} \approx \log \omega_{it} + \iota_t + \log a_{it}$ and the independence between ω_{it} and a_{it}, we can recover $\upsilon_q = \frac{cov(\log Duration_{it}, \log a_{it})}{var(\log a_{it})}$.

To map this result to our model, note that $Duration_{it} \equiv Duration(\omega_{it}, a_{it}) \sim Geometric(1/p(\omega_{it}, a_{it}))$. Thus,

$$Duration(\omega_{it}, a_{it}) = \frac{1}{p(\omega_{it}, a_{it})} + \eta_{it},$$

with $\mathbb{E}(\eta_{it}|\omega_{it}, a_{it}) = 0$. Making a first-order approximation of $\log Duration_{it}$ around $\eta_{it} = 0$, we obtain

$$\log Duration(\omega_{it}, a_{it}) \approx \log \left(\frac{1}{p(\omega_{it}, a_{it})} \right) + p(\omega_{it}, a_{it})\eta_{it}.$$

Using the law of total variance and the approximation, the estimated covariance simplifies to

$$cov(\log Duration(\omega_{it}, a_{it}), \log a_{it}) \approx -cov\left(\log\left(p(\omega_{it}, a_{it})\right), \log a_{it}\right).$$

Taking a first-order approximation of the equilibrium price around the mean qualities $(\bar{\omega}, \bar{a})$, and of $\log a$ around $\bar{a}$,

$$\log\left(p(\omega_{it}, a_{it})\right) \approx \log\left(p(\bar{\omega}, \bar{a})\right) + \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial \omega_{it}}|_{\bar{\omega}, \bar{a}}}{p(\bar{\omega}, \bar{a})}\left(\omega_{it} - \bar{\omega}\right) + \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega}, \bar{a}}}{p(\bar{\omega}, \bar{a})}\left(a_{it} - \bar{a}\right)$$

and

$$\log a_{it} \approx \log \bar{a} + \frac{1}{\bar{a}}\left(a_{it} - \bar{a}\right).$$

Replacing this expression in the covariance,

$$
\begin{aligned}
cov\left(\log\left(p(\omega_{it},a_{it})\right),\log a_{it}\right) &\approx cov\left(\frac{\frac{\partial p(\omega_{it},a_{it})}{\partial \omega_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega},\bar{a})}\left(\omega_{it}-\bar{\omega}\right)+\frac{\frac{\partial p(\omega_{it},a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega},\bar{a})}\left(a_{it}-\bar{a}\right),\frac{1}{\bar{a}}\left(a_{it}-\bar{a}\right)\right)\\
&=\frac{\frac{\partial p(\omega_{it},a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega},\bar{a})}\bar{a}var\left(\frac{a_{it}}{\bar{a}}-1\right)\\
&\approx\frac{\frac{\partial p(\omega_{it},a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega},\bar{a})}\bar{a}var(\log a_{it}),
\end{aligned}
$$

where the second step follows from the independence of ω_{it} and a_{it}. Thus,

$$
\begin{aligned}
\upsilon_q &= \frac{cov(\log Duration_{it},\log a_{it})}{var(\log a_{it})}\\
&\approx -\frac{cov\left(\log\left(p(\omega_{it},a_{it})\right),\log a_{it}\right)}{var(\log a_{it})}\\
&\approx -\frac{\partial p(\omega_{it},a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}\frac{\bar{a}}{p(\bar{\omega},\bar{a})}\\
&\approx -\frac{\partial \log p(\omega_{it},a_{it})}{\partial \log a_{it}}|_{\bar{\omega},\bar{a}}.
\end{aligned}
$$

As shown in 3, our model predicts that $\frac{\partial \log p(\omega_{it},a_{it})}{\partial \log a_{it}}$ is a strictly monotonic function of ψ, which means ψ can be recovered by inverting the function $\upsilon_q(\psi)$.

A.3 Additional results

A.3.1 Bargaining as a special case of inspection-adjusted price function

***Lemma* 2.** *Suppose that $q^P(\omega,a,\hat{a},q)$ is determined by Nash bargaining. Let us denote ϕ the bargaining power of sellers so that*

$$q_t^P(\omega,a,\hat{a},q)=min(q,\phi\nu_t^{\mathrm{b}}(\omega,a)+(1-\phi)\left[\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)-\delta\omega a\right]). \tag{A.24}$$

Then, q_t^P satisfies Assumption 1 if and only if $\phi\leq\eta$.

Proof. Let us assume that the post-inspection price function $q^P(\cdot)$ is determined by a Nash bargaining protocol and the bargaining power of the seller is ϕ with $\phi<\eta$:

$$q_t^P(\omega,a,\hat{a},q)=\min\left(\phi\nu_t^{\mathrm{b}}(\omega,a)+(1-\phi)\left[\Lambda_t\nu_{t+1}^{\mathrm{s}}(\omega,a)-\delta\omega a\right],q\right). \tag{A.25}$$

Let us note $q_t^B(\omega, a, \hat{a}) = \phi\nu_t^{\text{b}}(\omega, a) + (1-\phi)\left[\Lambda_t\nu_{t+1}^{\text{s}}(\omega, a) - \delta\omega a\right]$ so that $q_t^P(\omega, a, \hat{a}, q) = min(q_t^B(\omega, \hat{a}, a), q)$. We now verify that this function satisfies all the conditions in Assumption 1. To do so, we make use of the result whereby both $\nu^{\text{b}}(\omega, a)$ and $\nu^{\text{s}}(\omega, a) - \delta\omega a$ are increasing in unobserved quality a, as we showed in the proof of Proposition 5.

(i) $q^P(\omega, a, \hat{a}, q)$ *is non-decreasing in the true quality:* For any two qualities $a' > a$,

$$q_t^P(\omega, a', \hat{a}, q) - q_t^P(\omega, a, \hat{a}, q)$$
$$= \phi(\nu_t^{\text{b}}(\omega, a') - \nu_t^{\text{b}}(\omega, a)) + (1-\phi)\left[\Lambda_t(\nu_{t+1}^{\text{s}}(\omega, a') - \nu_{t+1}^{\text{s}}(\omega, a)) - \delta\omega(a'-a)\right] \geq 0,$$

which proves the first condition.

(ii) $q^P(\omega, a, \hat{a}, q)$ *is non-increasing in the announced quality:* The second condition is trivially satisfied, since $q^P(\omega, a, \hat{a}, q)$ in (A.25) does not depend on the announced quality.

(iii) $q^P(\omega, a', \hat{a}, q)$ *is weakly lower (resp. higher) that the buyer's (resp. seller's) value for the unit:* This condition is also trivially satisfied, since $q^P(\omega, a', \hat{a}, q)$ in (A.25) is the minimum of q and a convex combination of $\nu^{\text{b}}(\omega, a)$ and $\Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a) - \delta\omega a$.

(iv) *Buyers obtain at least a fraction* $1-\eta$ *of the surplus:* The Nash bargaining solution implies that buyers get a fraction $1-\phi$ of the surplus. Hence, as long as $\phi \leq \eta$ this condition is satisfied.

(v) q_t^P *does not decrease too fast as the announced quality increases:* We need to show that

$$\frac{\eta(\nu_t^{\text{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_i) + \delta\omega a_i)}{q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_i) + \delta\omega a_i} \geq \frac{q_t^B(\omega, a_j, a_i) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_j) + \delta\omega a_j}{q_t^B(\omega, a_j, a_k) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_j) + \delta\omega a_j}.$$

Since $q^B(\cdot)$ does not depend on the announced quality (i.e., $q^B(\omega, a_j, a_k) = q^B(\omega, a_j)$), the right-hand side of the inequality is equal to one. Hence, the inequality simplifies to

$$\eta(\nu_t^{\text{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_i) + \delta\omega a_i) \geq q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_i) + \delta\omega a_i,$$

or

$$\eta\nu_t^{\text{b}}(\omega, a_i) + (1-\eta)(\Lambda_{t+1}\nu_{t+1}^{\text{s}}(\omega, a_i) - \delta\omega a_i) \geq q^P(\omega, a_i, a_k),$$

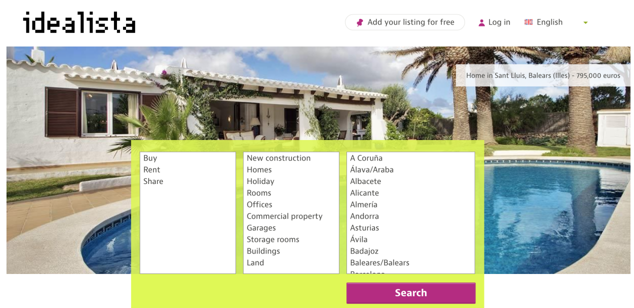
which is satisfied since it corresponds to the fourth condition of Assumption 1.

B Empirical Appendix

B.1 The online platform

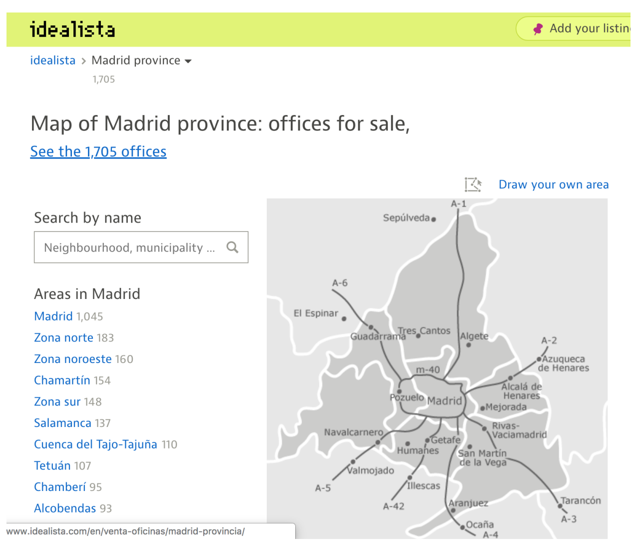
This subsection describes how the platform works. When entering the website, the buyer encounters the screen shown in Figure B1. The platform asks the client to choose a type of transaction (buy, rent, or find a shared space), the type of property (retail store, office, etc.), and the location.

FIGURE B1: Main Website



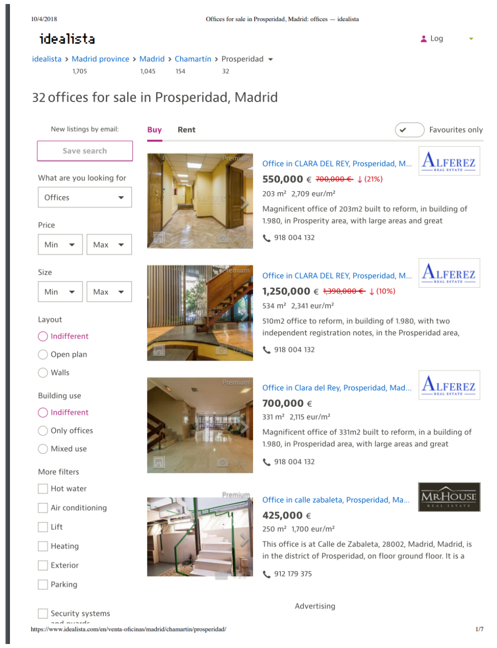
Once those options are selected (suppose the client wants to find a unit in Madrid—see Figure B2), then the website shows the number of properties available for sale by area in the city.

FIGURE B2: Options in Madrid



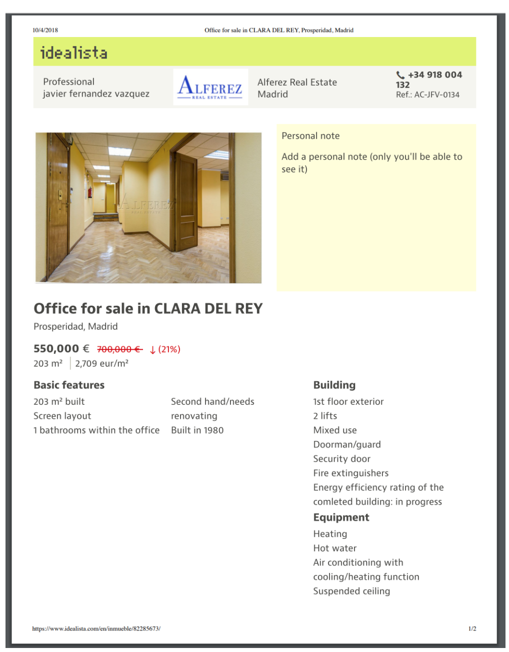
After choosing a narrower location within the city (not shown here), the client finds a scrolling list of available units that meet her requirements, as shown in Figure (B3). There, the user can include more filters depending on her requirements for layout and amenities.

FIGURE B3: Available Listings in a Narrow Location in Madrid



When the user finds a unit that may be to her taste and clicks on it, a window pops up with the details shown in Figure B4 and additional text details not shown here. The information the listing contains is the unit description with pictures, price, change in price, area, construction date, and other amenities and equipment.

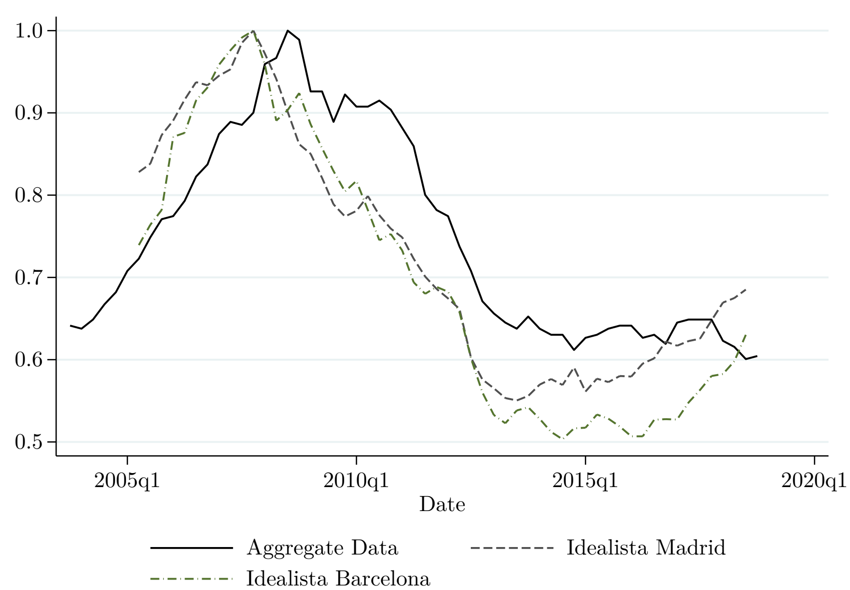
FIGURE B4: A Listing on the Website



B.2 Representativeness of the dataset

In this subsection, we analyze the representativeness of the dataset and show that our data are consistent with aggregate patterns observed in Spain over this period. More specifically, we show that the price index exhibits the patterns of aggregate data. Figure B5 shows the index of listed prices for properties for sale in our sample and the index of transacted prices of retail space in Spain (obtained from official transaction records). Both indexes are normalized to 1 at their respective peak. We highlight the fact that the fall in prices we observe is consistent and very similar in size to that observed for retail space in Spain during the recent financial crisis. Moreover, our index leads the aggregate index, which is expected since our index consists of listed prices, and it will take properties some months to exit the database, be registered as sales, and be recorded in national statistics. These patterns are consistent with the evidence presented in Guren (2018), who shows that the modal property sells at its listed price and that the average property sells within 1.6% of its listed price.

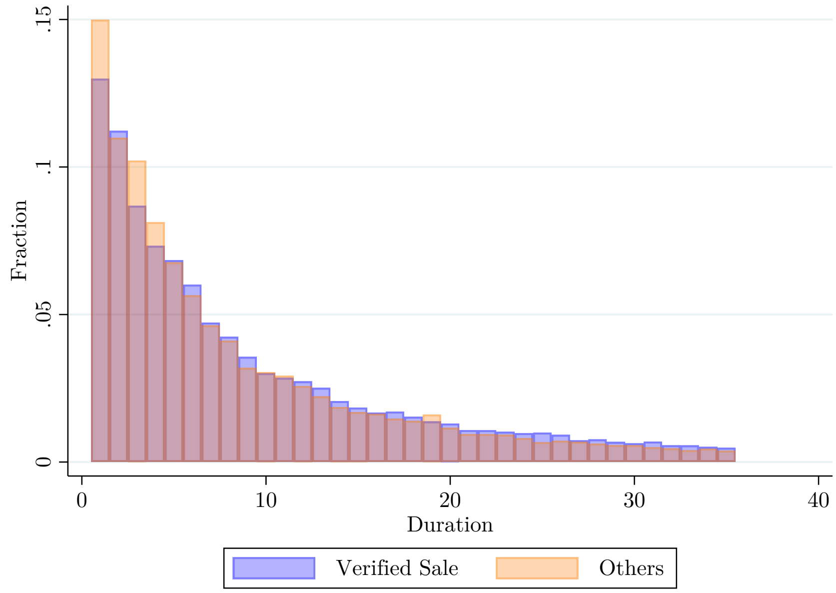
FIGURE B5: Price Index: Idealista Data versus Aggregate Data



Note: The solid line shows the price index for properties for sale in Barcelona and Madrid in our dataset. The dashed line shows the aggregate retail space price index gathered from the National Registry of Property (*Registradores de España*). All indices are normalized to their respective peak.

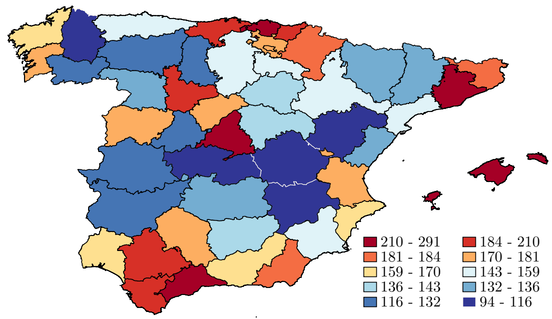
B.3 Additional Figures and Tables

Figure B6: Distribution of Duration: Confirmed Sales

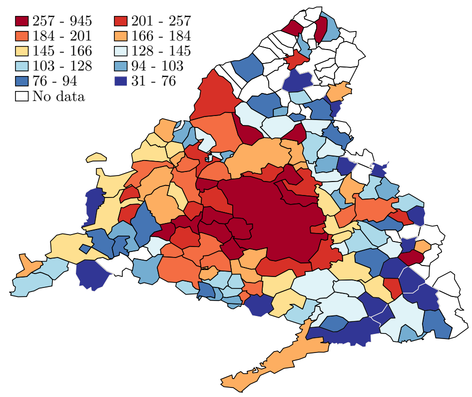


Note: This figure compares the histogram of duration for two subgroups of listings: those that, after removing the listing from the platform, explained that they did so because the property was rented or sold, and those that did not provide an explanation.

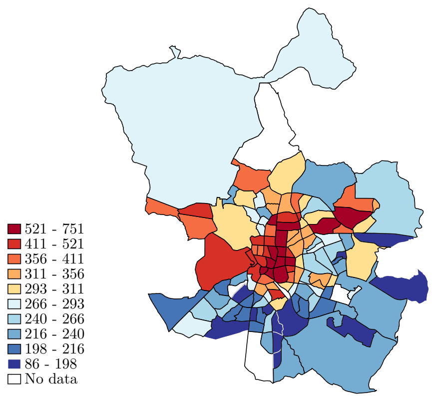
Figure B7: Capital Prices Across Locations



(A) Spain



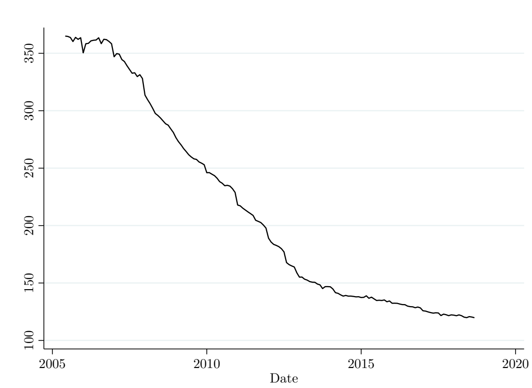
(B) Province of Madrid



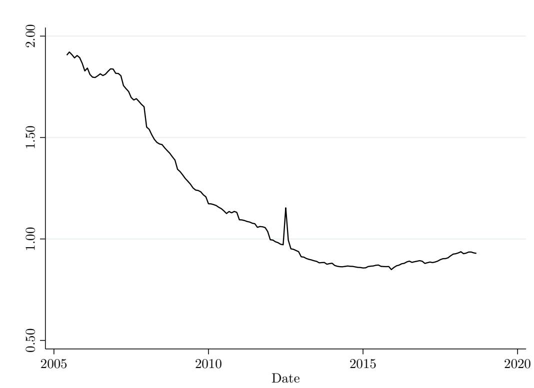
(C) City of Madrid

Note: Each map shows average prices by location expressed in constant 2017 dollars per square foot. Panel (A) shows average prices across provinces in Spain. Panel (B) zooms in on the province of Madrid to show substantial heterogeneity across municipalities within this province. Panel (C) shows that, after zooming in on the municipality of Madrid, there is still significant geographic dispersion of prices across neighborhoods.

Figure B8: Evolution of Prices of Capital Units



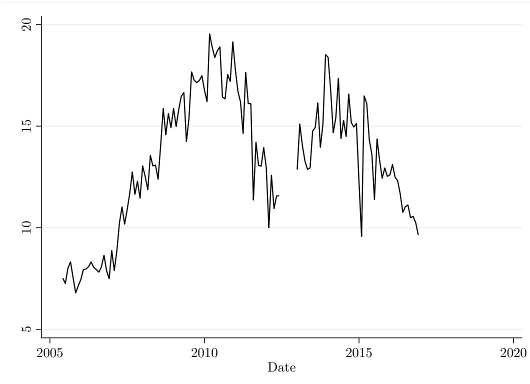
(A) Capital for Sale



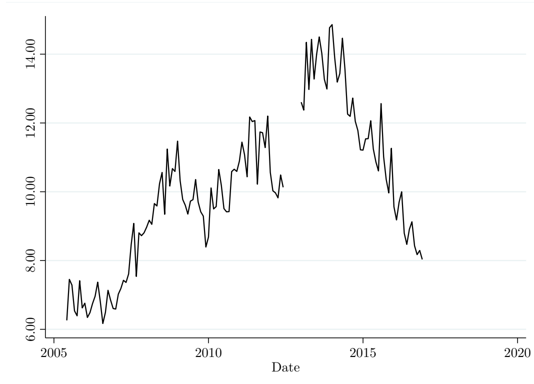
(B) Capital for Rent

Note: The left panel shows the evolution of mean prices at daily frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Prices are denominated in constant 2017 dollars per square foot. To compute these price indices, we averaged the prices of all active listings in a given day.

Figure B9: Evolution of Average Duration



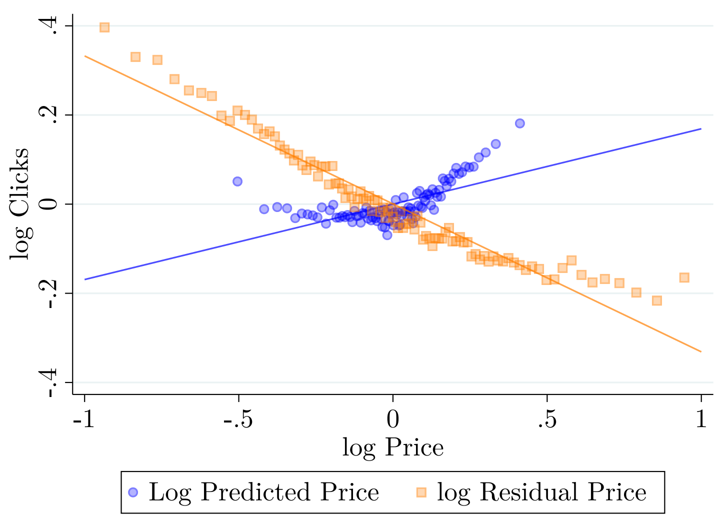
(A) Capital for Sale



(B) Capital for Rent

Note: The left panel shows the evolution of mean time to sell (in months) at monthly frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Time to sell is measured as the time difference between the entry and exit dates of each listing. Each observation contains the average time to sell for listings that entered the online platform in a given month.

Figure B10: Relationship between log Clicks and log Prices



Note: This figure shows the relationship between log prices and log average monthly clicks. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (17)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table B1: Price Variation Accounted by Property Fixed Effect

	St. Dev	R^2
Raw data	0.81	0.00
Year	0.76	0.10
Year + Location	0.57	0.50
Year × Location	0.55	0.53
... + Type	0.55	0.54
... + Area	0.52	0.59
... + Age	0.51	0.59
Benchmark	0.50	0.61
Property Fixed Effect	0.12	0.98

Note: This table extends Table 2 by including a property fixed effect, which gathers inference from properties that change their prices while they are active in the dataset. We find that after including property fixed effects, nonparametrically absorbing all the property's time-invariant price determinants, the standard deviation is 9% and the R^2 is roughly 0.98.

Table B2: Frequency of Price Changes for Capital

Statistic	Value
Frequency of Price Changes	0.09
Frequency of Price Increases	0.03
Frequency of Price Decreases	0.06
Absolute Size of Price Changes	0.14
Absolute Size of Price Increases	0.15
Absolute Size of Price Decreases	0.13

Note: This table presents price adjustment statistics for properties listed for sale. We first compute statistics on price changes within each property and then take averages across properties in a given time period. Finally, we compute the average over time. The first row shows the frequency of price changes, which is the average share of properties that exhibit a price change in a given month. The following two rows show the share of listings with price increases and decreases. The absolute size of price changes is computed as the absolute value of the log difference in prices over consecutive months (ignoring the zeros).

Table B3: Prices and Clicks

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.030***		-0.260***	
	(0.009)		(0.008)	
log Predicted Price		0.272***		0.195***
		(0.013)		(0.012)
log Residual Price		-0.323***		-0.324***
		(0.009)		(0.009)
Constant	3.381***	2.240***	4.755***	2.604***
	(0.038)	(0.058)	(0.036)	(0.058)
Observations	351128	351128	350259	350259
R^2	0.000	0.041	0.393	0.400
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log average monthly clicks on the two components of prices, residual and predicted prices. The left-hand-side variable is the log average monthly clicks of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of log clicks on prices. Column 2 regresses log clicks on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

TABLE B4: Prices and Duration by Conservation State

	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	-0.026***		0.113***	
	(0.008)		(0.007)	
log Price × Pre-owned-good condition	0.014**		0.012**	
	(0.007)		(0.005)	
log Price × Pre-owned-needs renovation	0.009		0.014***	
	(0.007)		(0.005)	
log Predicted Price		-0.126***		-0.054***
		(0.009)		(0.015)
× Pre-owned-good condition		0.022***		0.016***
		(0.007)		(0.005)
× Pre-owned-needs renovation		0.014**		0.013**
		(0.007)		(0.005)
log Residual Price		0.178***		0.116***
		(0.052)		(0.043)
× Pre-owned-good condition		-0.040		0.025
		(0.051)		(0.043)
× Pre-owned-needs renovation		-0.003		0.059
		(0.052)		(0.043)
Constant	2.171***	2.613***	1.509***	2.291***
	(0.021)	(0.029)	(0.022)	(0.064)
Observations	398761	398761	398761	398761
R^2	0.000	0.010	0.257	0.258
Fixed Effects	No	No	Yes	Yes

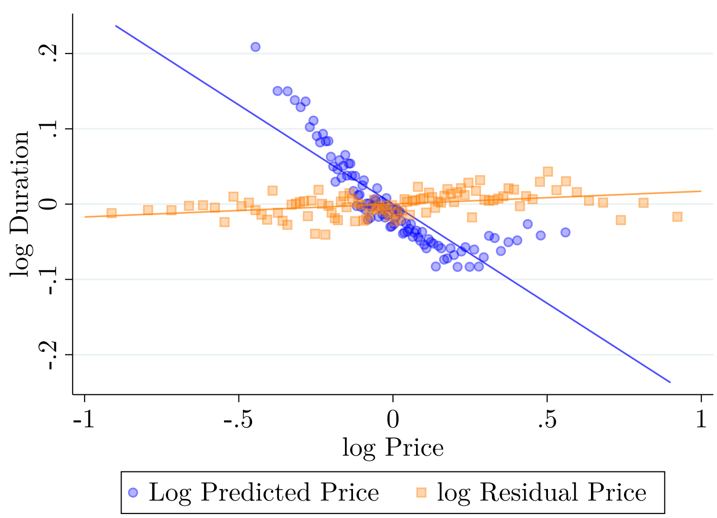
Note: This table extends the results in Table 3 by interacting each price variable with an indicator for the self-reported conservation status of the unit, which can be one of three options: new construction (the omitted category), pre-owned in good condition, and pre-owned and requires renovation. The left-hand-side variable is the log duration of a listing, and the right-hand-side variable is the mean price over the lifetime of the listing. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at location-time level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

B.4 Results for properties listed for rent

FIGURE B11: Distribution of Price Residuals for Rentals

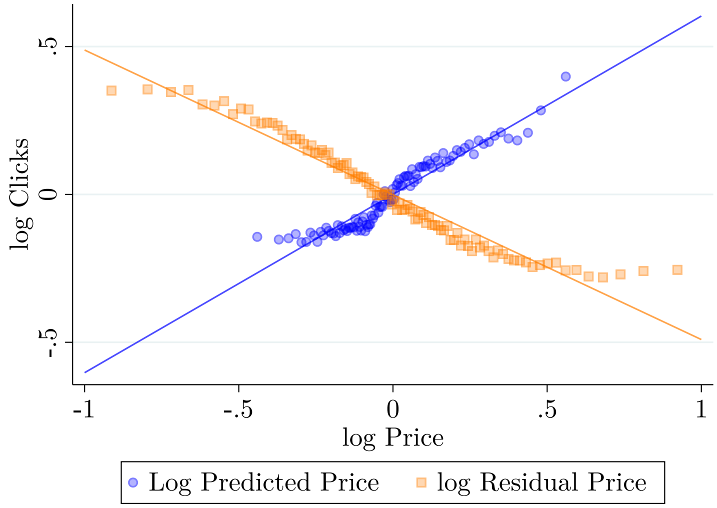
Note: This figure shows the distribution of log prices per square foot relative to its mean for the raw data and price residuals after including the fixed effects in Table (2).

FIGURE B12: Relationship between log Duration and log Prices for Rentals



Note: This figure shows the relationship between log prices and log duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (17)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Figure B13: Relationship between log Clicks and log Prices for Rentals



Note: This figure shows the relationship between log prices and log average monthly clicks. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (17)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table B5: Price Variation Accounted for by Listed Characteristics

	St. Dev.	R^2
Raw data	0.68	0.00
Year	0.66	0.05
Location	0.49	0.47
Year × Location × Type	0.45	0.56
... + Area	0.41	0.63
... + Age	0.41	0.63
Benchmark	0.40	0.64

Note: This table reports the R^2 and standard deviation of residuals from estimating equation (17). The row labeled Raw data presents statistics for the demeaned raw log prices. The following rows include the fixed effects in the regression. Year and location denote fixed effects. Type (office and retail space or warehouse), area, and age are sets of fixed effects for each of these characteristics. The last row includes additional controls for the variables listed in Table 1.

Table B6: Prices and Duration

	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	-0.138***		-0.046***	
	(0.005)		(0.005)	
log Predicted Price		-0.217***		-0.257***
		(0.007)		(0.008)
log Residual Price		0.003		0.003
		(0.006)		(0.006)
Constant	1.903***	1.893***	1.916***	1.887***
	(0.005)	(0.005)	(0.001)	(0.001)
Observations	632386	632386	632386	632386
R^2	0.011	0.017	0.234	0.236
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The left-hand-side variable is the log duration of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of duration on prices. Column 2 regresses duration on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

Table B7: Prices and Clicks

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.182***		-0.242***	
	(0.013)		(0.010)	
log Predicted Price		0.544***		0.613***
		(0.019)		(0.012)
log Residual Price		-0.442***		-0.443***
		(0.013)		(0.013)
Constant	3.962***	4.030***	3.883***	4.045***
	(0.013)	(0.013)	(0.002)	(0.002)
Observations	524750	524750	523702	523702
R^2	0.011	0.088	0.409	0.435
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log average monthly clicks on the two components of prices, residual and predicted prices. The left-hand-side variable is the log average monthly clicks of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of log clicks on prices. Column 2 regresses log clicks on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

B.5 Alternative Explanations for the Price-Duration Relationship

Sellers' indifference We begin by considering potential explanations for the positive relationship between residual price and duration on the market that rely on sellers' indifference across these variables: Although sellers prefer higher residual prices, this is also associated with longer duration on the market. The key for this interpretation is that the trade-off between residual prices and time to sell is such that they provide an equivalent expected revenue for sellers. This type of explanation is akin to that of labor- and product-market models such as those of Burdett and Judd (1983) and Burdett and Mortensen (1998).

To study whether this trade-off can explain the positive relationship observed between residual prices and duration in the data, we compute the expected net present discounted revenue for properties with different residual prices, given their observed trading probabilities implied by the relation in Figure 6. For this, we assume homogeneous risk-neutral sellers; in this case, the expected net present revenue from choosing residual price ε_{it} is given by

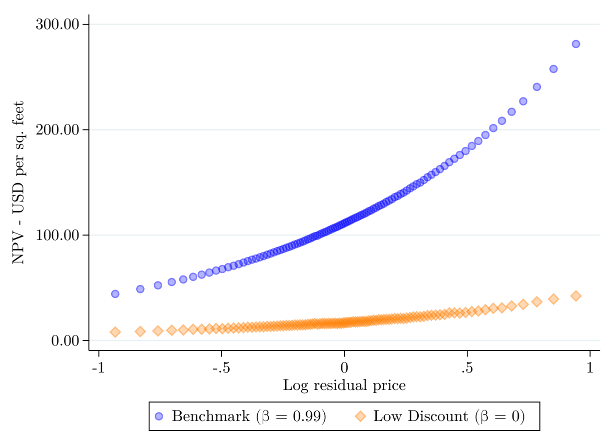
$$\mathcal{R}(\varepsilon_{it}, \beta) \equiv \sum_{t=0}^{\infty} \beta^t (1 - p(\varepsilon_{it}))^t p(\varepsilon_{it})\varepsilon_{it} = \frac{p(\varepsilon_{it})\varepsilon_{it}}{(1 - \beta(1 - p(\varepsilon_{it})))}, \tag{B.26}$$

where $p(\varepsilon_{it})$ is the associated selling probability implied by the empirical relationship between residual prices and duration, depicted in Figure 6, and $\beta \in [0, 1)$ is the discount factor used in the exercise.[28]

Figure B14 shows the results of this exercise for a wide range of discount factors ($\beta = 0.99$ and $\beta = 0$), which indicate that the expected net present discounted revenue is monotonically increasing in the listed price. Lower discount factors disproportionally affect properties that have lower trading probabilities and high prices, which flattens the net present value profile. However, even in the extreme scenario of $\beta = 0$, the relation between prices and duration in the data is such that we still find that the net present value is monotonically increasing in the listed price. These results indicate that sellers' indifference cannot explain the observed relationship between residual prices and duration: Any seller facing such a price-duration trade-off will maximize expected revenue by choosing the highest residual price we see in the data.

[28]Note that we abstract from price changes in this calculation and use the mean price instead, since the frequency of price changes is small. The trading probability in a given month is computed from the duration of each property.

Figure B14: Net Present Value of Price-Duration Trade-off



Note: This figure reports the net present value estimates from (B.26). The blue line ("Benchmark") shows the net present value for a discount factor of $\beta = 0.99$ given the empirical relationship between residual prices and average duration in the data. The orange line ("Low Discount") shows a similar net present value calculation for a discount factor of $\beta = 0$.

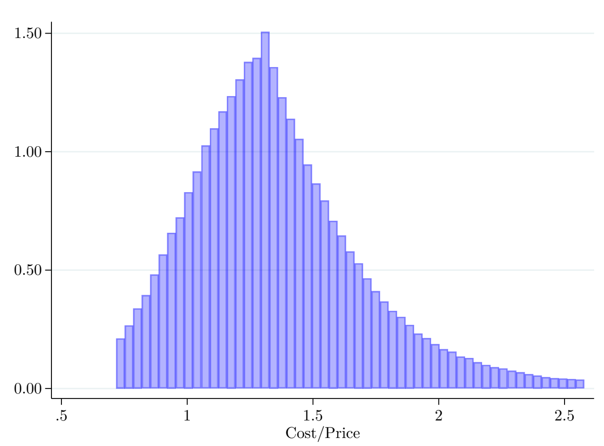
Sellers' heterogeneity We now consider whether the positive relationship between residual prices and duration can be explained by heterogeneity across sellers. First, the results presented in Figure B14 indicate that the positive relationship between residual prices and duration cannot be explained by heterogeneity in sellers' discount factors. To see this, note that, as shown in Figure B14, the expected net present revenue is increasing in both the computation with a high discount factor ($\beta = 0.99$) and with a low discount factor ($\beta = 0$). Therefore, if, under the preferences of the most impatient seller, a higher residual price with lower selling probability is preferred, the higher residual price would also be preferred under any other possible discount factor.

Second, we consider the possibility that sellers have heterogeneous holding costs. For this, assume that sellers must pay a fixed cost each period until the property is sold (e.g., maintenance costs, taxes, debt service costs, etc.). If sellers face different costs, then some sellers might be forced to list properties at low prices in order to sell their property faster, as would occur in a fire sale. Using our data, we ask how large must the cost be in order to rationalize a seller's choice of a lower residual price. Thus, for each residual price ε_{it}, we compute the (unobserved) cost, $\xi(\varepsilon_{it}, \beta)$, that would render risk-neutral sellers with discount factor β indifferent between choosing that residual price and the highest observed residual price ($\bar{\varepsilon}_{it}$) by solving the following condition:

$$\frac{p(\varepsilon_{it})\varepsilon_{it} - \xi(\varepsilon_{it}, \beta)(1 - \varepsilon_{it})}{1 - \beta(1 - \varepsilon_{it})} = \frac{p(\bar{\varepsilon}_{it})\bar{\varepsilon}_{it} - \xi(\varepsilon_{it}, \beta)(1 - p(\bar{\varepsilon}_{it}))}{1 - \beta(1 - p(\bar{\varepsilon}_{it}))}. \tag{B.27}$$

Figure B15 presents the results, which indicate that in order for differential holding costs to explain the differences in returns observed in the data, they must be extremely large. To illustrate, on average, the cost of holding 1 square foot of a property for one additional month would have to be larger than the price at which the owner can sell that unit. We conclude that it is unlikely that the bulk of the positive relation between residual prices and duration is explained by the presence of heterogeneous holding costs.

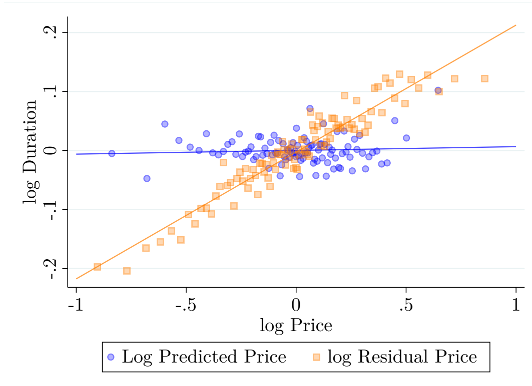
FIGURE B15: Required Holding Costs



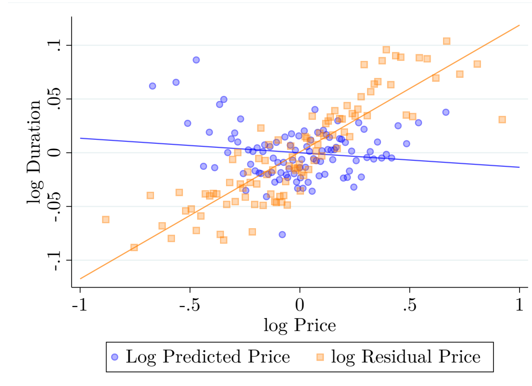
Note: This figure reports the distribution of holding costs obtained from (B.27) as a fraction of the property's price.

Buyers' heterogeneity Finally, we consider the possibility that our fact could be explained by the different liquidity of potential buyers of different units. For example, it could be the case that units with higher residual prices take longer to sell simply because fewer buyers can afford to buy expensive units. To explore this possibility, we reestimate the results in Figure 6 for two subsamples: units with total prices above and below the median. Figure B16 shows that the positive relationship between residual prices and duration is similarly present in each subsample, which indicates that heterogeneity in buyer's liquidity cannot be the main driver of our fact.

Figure B16: Relationship between Duration and Prices by Total Price



(A) Below Median Total Price



(B) Above Median Total Price

Note: This figure shows the relationship between log prices and duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (17)). Figures show a binned scatter plot of each relationship after controlling for location-time-type (offices, retail space, and warehouses) fixed effects. Panels (A) and (B) report results for the set of capital units listed for sale with total prices below and above the median, respectively.

C Appendix Quantitative Analysis

C.1 Additional Tables and Figures

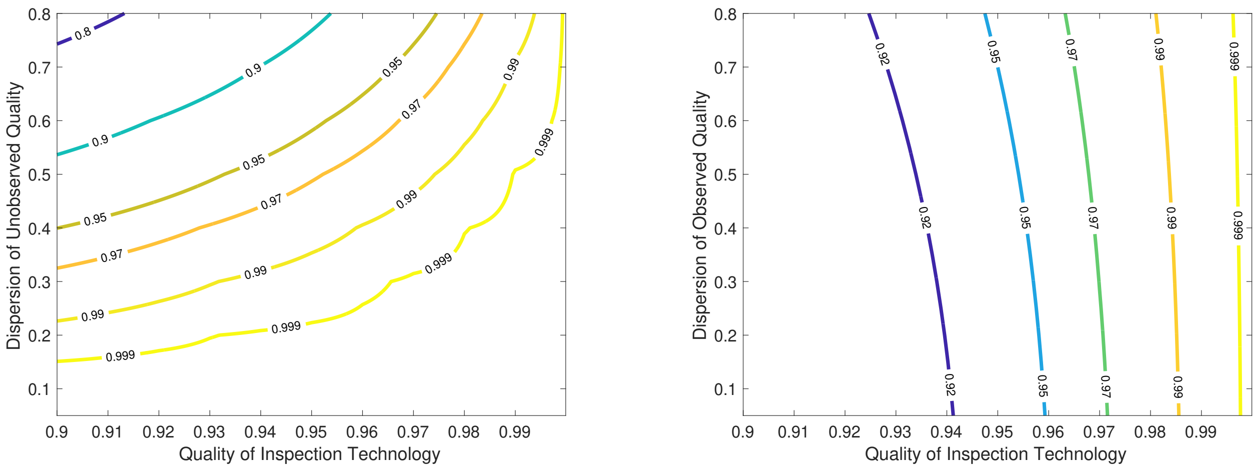
Table C1: Fitted Parameters and Targeted Moments — Robustness Exercise

Parameter	Description	Value	Target	Model	Data
ψ	Accuracy information technology	0.98	Regression coefficient	0.13	0.13
σ_ω	SD observed quality	0.69	SD log predicted prices	0.62	0.62
σ_a	SD unobserved quality	0.61	SD log residual prices	0.54	0.54
$\bar{m}$	Matching efficiency	0.27	Mean duration	11.4	11.4

Note: This table shows the parameters we calibrate by minimizing the distance between four moments in the data and in our simulated model for the case in which we exclude observables from the computation of predicted prices.

Figure C1: Output Response for Different Model Parameterizations

(A) Iso-output curves: ψ vs σ_a (B) Iso-output curves: ψ vs σ_ω



Note: This figure reports the effects of changing the precision of the inspection technology and the standard deviation of the observed and unobserved capital qualities on aggregate output. The left panel illustrates the loci of points (ψ, σ_a) that achieve a given level of output $Y(\psi, \sigma_a)$ as a fraction of the level of output with the same standard deviation of unobserved quality in the full information limit $Y(1, \sigma_a)$. The right panel shows the analog exercise but this time for the standard deviation of observed quality σ_ω. The figure shows that ψ and σ_a are substitutes. Either increases in σ_a or decreases in ψ decrease aggregate output. The effects of changes in ψ and σ_ω are almost orthogonal.

C.2 Extension to Baseline Model

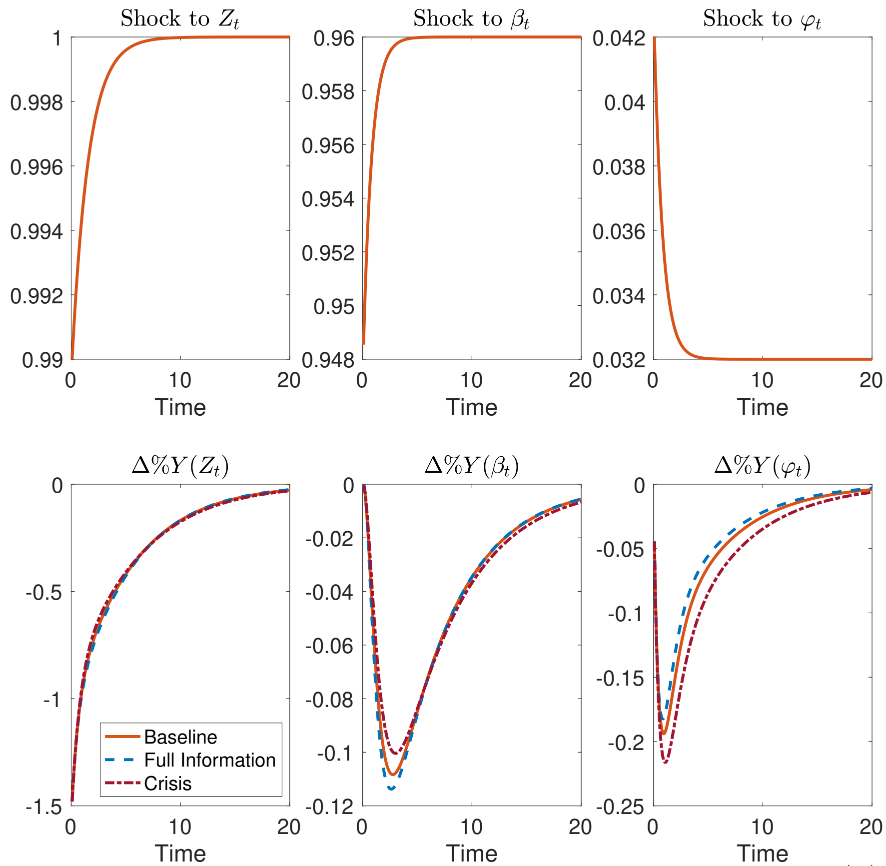
C.2.1 Model with Capital Intermediaries

In this section, we provide an alternative model setup in which the trade of capital features intermediaries, which we show is isomorphic to our baseline model. In this alternative formulation, we assume that the economy is populated by a representative intermediary who has access to a technology to perfectly observe the quality of capital units. The intermediary can purchase new capital units produced by households as well as used units from exiting firms and then sell them to active firms. In what follows, we focus on describing the parts of the model with intermediaries that differ from the baseline setup.

Households. Households produce capital units using a linear technology and sell capital units of quality (ω, a) to intermediaries at price $q_t^I(\omega, a)$. The recursive problem of the representative household is given by

$$V_{Ht}(\mathbf{k}) = \max_{\{c,h,i\geq 0\}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

Figure C2: Effect of macroeconomic shocks on aggregate output



Note: This figure shows the effect of shocks to the quality of the screening technology (ψ), TFP (Z), time preferences (β), and the exit rate of firms (φ). The top panel presents the shocks we feed into the model in levels. The bottom panel shows the effect on the percentage change in output as a consequence of the shock with respect to its long-run detrended value. Each figure in the bottom row has three lines: one for our benchmark calibration, one for a calibration of the full information limit ($\psi \to 1$), and one for a level of ψ that replicates the elasticity of duration to residual prices observed in the Euro Crisis.

subject to the budget constraint

$$c\gamma_n^t + i = w_t h \gamma_n^t + Div_{Ft} + Div_{It} + \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} k_{Ht}(\omega, a) q_t^I(\omega, a),$$

and the law of motion of capital holdings

$$k_{Ht+1}(\omega, a) = g(\omega, a) i_t,$$

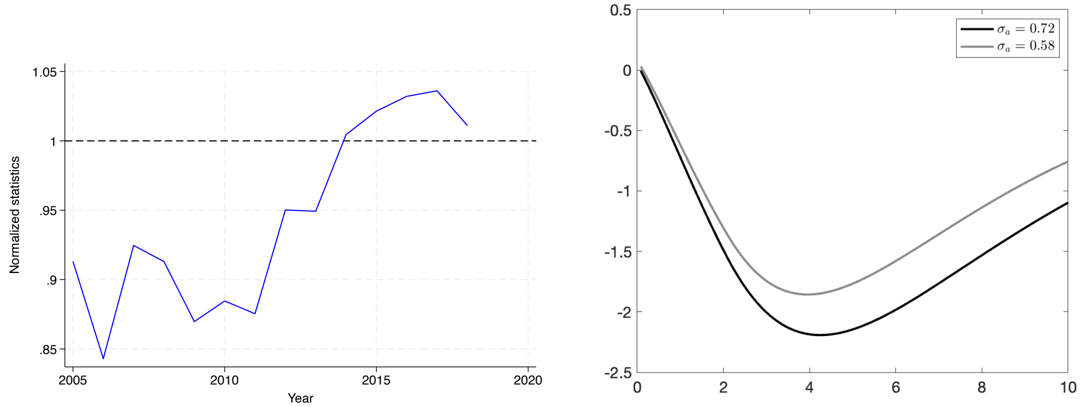
where Div_{It} denotes the dividends transferred by intermediaries in period t. The optimal level of investment (provided that $i > 0$) is characterized by

$$1 = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} g(\omega, a) \lambda_t(\mathbf{k}) q_{t+1}^I(\omega, a).$$

FIGURE C3: Complementarity Between ψ and σ_a

(A) Observed SD of Residual Prices

(B) Output Responses to Changes in ψ_t: The Role of σ_a



Note: Panel (A) reports the standard deviation of log residual prices observed in the data. Units are normalized to take a value of 1 at our calibration target. Panel (B) shows the impulse responses of output to an unexpected decline in the accuracy of information technologies, ψ_t, for different values of the dispersion of unobserved quality, σ_a. The horizontal axis displays years after the shock. Impulse responses are expressed as percentage deviations from the detrended steady state. See Section 5.3 for additional details about this exercise.

Firms. Operating firms accumulate capital by purchasing it from *intermediaries* in the decentralized market. Conditional on not exiting, the recursive problem of the firm remains identical to that in the baseline model. However, the value of an exiting firm that sells capital to intermediaries at the end of period t is now given by $V_t^{\text{exit}}(\mathbf{k}) \equiv \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} k(\omega,a)q_t^I(\omega,a)$, since unemployed capital is immediately sold to intermediaries in a frictionless manner.

Intermediaries. Intermediaries buy capital units from households and exiting firms and sell them to active firms. While the selling process is subject to the same frictions as in the baseline model, the buying process occurs in a frictionless and competitive market. Thus, the recursive problem of the intermediary is given by

$$V_{It}(\mathbf{k}) = \max_{\{k'(\omega,a),\hat{a}(\omega,a),q(\omega,a),x(\omega,a)\}} div_I + \Lambda_{t,t+1}V_{It+1}(\mathbf{k}'),$$

subject to the definition of dividends

$$\begin{aligned} div_I = &\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \left[(1-\psi)q(\omega,a) + \psi q^P(\omega,a,\hat{a}(\omega,a),q)\right] p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))\,(k(\omega,a)+x(\omega,a)) \\ &- \delta\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a\left[(1-p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))))\,(k(\omega,a)+x(\omega,a))\right] - \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} q^I(\omega,a)x(\omega,a). \end{aligned}$$

and the law of motion for capital holdings

$$k'(\omega, a) = (1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))) (k(\omega, a) + x(\omega, a)),$$

where $x(\omega, a)$ denotes the quantity of capital units bought from households and exiting firms. Following steps similar to those in Appendix A.2.1, we can show that the optimal level of purchased capital is characterized by the first-order condition

$$q_t^I(\omega, a) = \nu_t^{\mathrm{s}}(\omega, a).$$

This pricing condition, together with the market-clearing condition in the intermediated market,

$$\varphi K_{Ft}(\omega, a) + K_{Ht}(\omega, a) = x(\omega, a),$$

establishes the equivalence between the two models.

C.2.2 Model with Structures and Equipment

This section provides the details of the model with structures and equipment outlined in Section 5.2. As described in that section, to provide a lower bound on the effect of asymmetric information on the macroeconomy we assume that the trade of equipment is frictionless. In what follows, we focus on describing the parts of the model that differentiate structures and equipment, as well as the aspects of the quantitative analysis that differ from those in the baseline setup.

Technology. The production technology is now given by $y_{jt} = \mathcal{K}_{\mathrm{s}jt}^{\alpha_{\mathrm{s}}} k_{\mathrm{m}jt}^{\alpha_{\mathrm{m}}} (\gamma^t l_{jt})^{1-\alpha_{\mathrm{s}}-\alpha_{\mathrm{m}}}$, with $\alpha_{\mathrm{s}} + \alpha_{\mathrm{m}} \in (0, 1)$, where $\mathcal{K}_{\mathrm{s}jt}$ and $k_{\mathrm{m}jt}$ denote the quantities of structures and equipment used in production. We assume that equipment capital $k_{\mathrm{m}jt}$ is homogeneous in terms of efficiency units of capital and is traded in a centralized rental market with a rental rate $r_{\mathrm{m}t}$. Structures and equipment depreciate at rates δ_{s} and δ_{m}, respectively.

Households. Households have access to a linear technology to produce both types of capital. Let $\mathbf{k} \equiv (\mathbf{k}_{\mathrm{s}}, k_{\mathrm{m}})$. Then, the recursive problem of the representative household is given by

$$V_{Ht}(\mathbf{k}) = \max_{\{c,h,\{k'(\omega,a),\hat{a}(\omega,a),q(\omega,a)\},i_{\mathrm{s}}\geq 0,i_{\mathrm{m}}\}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'), \tag{C.28}$$

subject to the budget constraint

$$\begin{aligned}
&c\gamma_n^t + i_{\mathrm{m}} + i_{\mathrm{s}} + \delta_{\mathrm{s}} \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a \left[(1 - p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))))\left(k(\omega,a) + \varphi K_{Ft}(\omega,a)\right)\right] \\
&= \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \left[(1-\psi)q(\omega,a) + \psi q^P(\omega,a,\hat{a}(\omega,a),q)\right] p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))\left(k(\omega,a) + \varphi K_{Ft}(\omega,a)\right) \\
&+ r_{\mathrm{m}t}k_{\mathrm{m}} + w_t h\gamma_n^t + Div_{Ft},
\end{aligned}$$

and the law of motion for structures

$$k'(\omega,a) = (1 - p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))))\left(k(\omega,a) + \varphi K_{Ft}(\omega,a)\right) + ig(\omega,a)$$

and equipment

$$k'_{\mathrm{m}} = (1-\delta_{\mathrm{m}})k_{\mathrm{m}} + i_{\mathrm{m}}.$$

The optimal levels of investment (provided that $i_{\mathrm{m}} > 0$) are characterized by the following Euler equations:

$$1 = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} g(\omega,a)\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega,a,\mathbf{k})$$

and

$$1 = \lambda_t(\mathbf{k})(r_{\mathrm{m}t+1} + (1-\delta_{\mathrm{m}})).$$

Firms. Firms accumulate structures by purchasing them from sellers in the decentralized market and rent equipment in a centralized market. Conditional on not exiting, the recursive problem of the firm is given by

$$V_{Ft}(\mathbf{k}) = \max_{\{l,k_{\mathrm{m}},\{v(\omega,\hat{a},q)\geq 0\},\{k'(\omega,a)\}} \mathbb{E}_a[div + \Lambda_{t,t+1}((1-\varphi)V_{Ft+1}(\mathbf{k}') + \varphi V_{t+1}^{\mathrm{exit}}(\mathbf{k}'))], \tag{C.29}$$

subject to the flow of funds constraint

$$
\begin{aligned}
div = & \left(\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega a k'(\omega,a)\right)^{\alpha_{\mathrm{s}}}(k_{\mathrm{m}})^{\alpha_{\mathrm{m}}}(\gamma^t l)^{1-\alpha_{\mathrm{s}}-\alpha_{\mathrm{m}}} - w_t l - r_{\mathrm{m}t}k_{\mathrm{m}} - \delta_{\mathrm{s}}\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega a k'(\omega,a) \\
& - \sum_{\omega\in\Omega}\sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}[(\psi\sum_{a\in\mathcal{A}}\iota_t(a|\omega,\hat{a},q)q_t^P(\omega,a,\hat{a},q) + (1-\psi)q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t]v(\omega,\hat{a},q)\,\mathrm{d}q,
\end{aligned}
$$

and the law of motion for capital

$$
k'(\omega,a) = \sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}\iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)v(\omega,\hat{a},q)\,\mathrm{d}q + k(\omega,a). \tag{C.30}
$$

The rest of the environment closely follows that of the baseline specification after implementing minor changes to the definition of equilibrium.

Calibration. We set most of the model parameters to their corresponding values in the baseline calibration, except for those related to structures and equipment, the matching efficiency $\bar{m}$, and the accuracy of the information technology ψ. We set $\delta_{\mathrm{s}} = 0.0024$ and $\delta_{\mathrm{m}} = 0.0117$, which correspond to annual depreciation rates for nonresidential structures and equipment of 2.8% and 13.2%, respectively (Fernald, 2014). Next, we set $\alpha_{\mathrm{s}} = 0.47 \times \alpha$ to match the average weight of structures in U.S. capital income reported in Fernald (2014). The matching efficiency is set to $\bar{m} = 0.266$ and the accuracy of the information technology is set to $\psi = 0.9826$.

Note that the value of ψ that matches the same slope between residual prices and duration is higher than in the baseline calibration. The reason is that a lower maintenance cost of structures (i.e., a lower α_{s} relative to the baseline) reduces the cost sellers incur while waiting in the market to sell their properties, and thereby lowers the cost of mimicking for low-quality sellers. This induces high-quality sellers to signal their type by further increasing their price and lowering their trading probability relative to the baseline model. Therefore, to match the same slope between residual prices and duration, a higher ψ is required.

Quantitative results. Table C2 reports the percentage change in aggregate output and each production input in equilibrium relative to an economy with full information for the extended model with structures and equipment. Aggregate output falls by 0.57%. This decline is driven by a 1.92% decrease in the stock of structures used in production, which is partly determined by a 0.49% lower stock of structures, a 0.81 p.p. increase in the unemployment rate of structures, and the fact that information asymmetries disproportionately affect the trading probability of high-quality structures. In turn, the lower equilibrium level of structures reduces the demand for equipment by 0.57% and for labor by 0.23%.

Note that the effect of information asymmetries on aggregate output is almost a perfect rescaling of the output effect found in the baseline model, scaled by the 0.47 share of structures in the capital stock.

Table C2: Decomposition of Output Effects in a Model with Structures and Equipment

	Change
$Y/Y^{FI}-1$	-0.57%
$\mathcal{K}/\mathcal{K}^{FI}-1$	-1.92%
$\quad K/K^{FI}-1$	-0.49%
$\quad \mathbb{E}\left(u(\omega,a)\right)-\mathbb{E}\left(u^{FI}(\omega,a)\right)$	0.81
$\quad \mathbb{C}ov\left(\omega a, u(\omega,a)\right)-\mathbb{C}ov\left(\omega a, u^{FI}(\omega,a)\right)$	0.02
$k_m/k_m^{FI}-1$	-0.57%
$L/L^{FI}-1$	-0.23%

Note: This table reports the percentage change in each variable in the baseline calibration relative to the full-information equilibrium for the extended model with structures and equipment.

C.3 Accounting for Changes in the Slope Between Duration and Residual Prices

This section shows how the increase in the slope between duration and residual prices observed during the Euro crisis (documented in Figure 7) is difficult to account for by shocks other than the deterioration in the accuracy of information-revealing technologies, which we consider in our main crisis experiment. For this, Section C.3.1 studies changes in total factor productivity; Section C.3.2 changes in the dispersion of unobserved capital qualities; and Section C.3.3 changes in discount rates and the dispersion of trading needs.

C.3.1 Total Factor Productivity

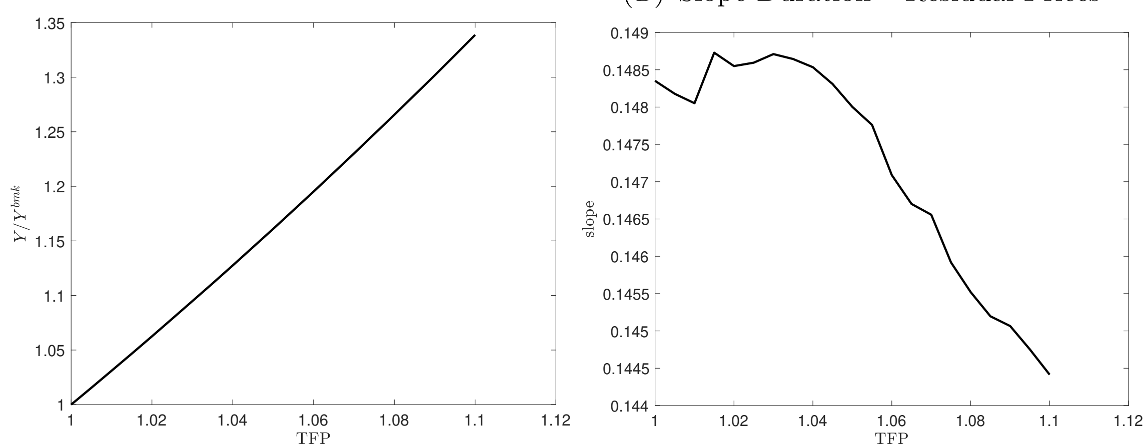
To study how changes in total factor productivity affect the slope between duration and residual prices, we extend firms' production technology to $y_{jt} = A_t\mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$, which includes a total factor productivity component, A_t, as the focus of our analysis. In terms of the equilibrium conditions of the model, this implies that Z_t (defined in the baseline model in Proposition 1) is now given by $Z_t \equiv \alpha A_t^{\frac{1}{\alpha}} \left(\frac{\gamma^t(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$. The rest of the model equilibrium conditions and results remain unchanged.

The top panels of Figure C4 consider the steady state of the model, with $A_t = \overline{A}$ for all t, and report the effects of varying $\overline{A}$ on equilibrium allocations. These results show that changes in $\overline{A}$ have a quantitatively negligible effect on the slope between duration and residual prices. Specifically, changes in $\overline{A}$ associated with a 30% change in aggregate output correspond to a change in the slope between duration and residual prices from 0.148 to 0.145.

Figure C4: The Effects of Changes in Total Factor Productivity

(A) Economic Activity

(B) Slope Duration – Residual Prices



Note: This figure reports the impact of changes in total factor productivity on aggregate output and the slope between log duration and residual prices. In Panel (A), output at each level of productivity is expressed relative to that of the benchmark calibration.

To further understand these results, we consider an analytical case with $\Omega = \{\bar{\omega}\}$, $\mathcal{A} = \{a_L, a_H\}$, and $\delta = 0$. The following proposition shows that if steady-state wages were an affine function of $\overline{Z} \equiv \alpha \overline{A}^{\frac{1}{\alpha}} \left(\frac{\gamma^t(1-\alpha)}{w_t(\overline{A})}\right)^{\frac{1-\alpha}{\alpha}}$, then changes in $\overline{A}$ would not affect trading probabilities.

***Proposition* 6.** *Consider the BGP of the model and assume that $\chi(\overline{Z}) = w_t(\overline{Z})/\gamma^t$ is approximately an affine function of $\overline{Z}$. Then, $(\nu^{\mathrm{s}}(\overline{\omega}, a_L), \nu^{\mathrm{s}}(\overline{\omega}, a_H), \nu^{\mathrm{b}}(\overline{\omega}, a_L), \nu^{\mathrm{b}}(\overline{\omega}, a_H), q(\overline{\omega}, a_H), q(\overline{\omega}, a_L))$ are affine functions of $\overline{Z}$ and $(\theta(\overline{\omega}, a_H), \theta(\overline{\omega}, a_L))$ are independent of $\overline{Z}$.*

Proof. We guess and verify the assertion. In the BGP, the values of the buyer, the seller, and the optimality condition for search activity on the equilibrium path can be written as

$$\nu^{\mathrm{s}}(\overline{\omega}, a) = \frac{p(\theta(\overline{\omega}, a))q(\overline{\omega}, a)}{(1 - p(\theta(\overline{\omega}, a)))\Lambda} \tag{C.31}$$

$$\nu^{\mathrm{b}}(\overline{\omega}, a) = \frac{\overline{Z}\bar{\omega}a}{1 - \Lambda(1-\varphi)} + \frac{\Lambda\varphi}{1 - \Lambda(1-\varphi)}\nu^{\mathrm{s}}(\overline{\omega}, a) \tag{C.32}$$

$$\nu^{\mathrm{b}}(\overline{\omega}, a) - q(\overline{\omega}, a) = \frac{\chi(\overline{Z})}{\mu(\theta(\overline{\omega}, a))} \tag{C.33}$$

We see from the first two equations that if $q(\overline{\omega}, a)$ is affine in $\overline{Z}$ and $\theta(\overline{\omega}, a)$ is independent of $\overline{Z}$, then $\nu^{\mathrm{s}}(\overline{\omega}, a)$ and $\nu^{\mathrm{b}}(\overline{\omega}, a)$ are also affine functions of $\overline{Z}$. From the last equation we see that if $q(\overline{\omega}, a)$ and $\nu^{\mathrm{b}}(\overline{\omega}, a)$ are affine in $\overline{Z}$, then $\theta(\overline{\omega}, a)$ is independent of $\overline{Z}$.

Finally, we need to verify the buyer's optimal choice of the terms of trade. If the optimal choice corresponds to the full-information allocation, then the optimal price is given by

$$q(\overline{\omega}, a) = \eta\nu^{\mathrm{b}}(\overline{\omega}, a) + (1-\eta)\Lambda\nu^{\mathrm{s}}(\overline{\omega}, a).$$

Hence, if values are affine in $\overline{Z}$, so is the price. If, instead, the incentive compatibility constraint of seller of quality a_H binds, then the optimal price is characterized by

$$\begin{aligned} &p(\theta^{FI}(\overline{\omega}, a_L))\left(q^{FI}(\overline{\omega}, a_L) - \Lambda\nu^{\mathrm{s}}(\overline{\omega}, a_L)\right) \\ &= p(\theta(\overline{\omega}, a_H))\left((1-\psi)q(\overline{\omega}, a_H) + \psi q^P(\overline{\omega}, a_L, a_H, q) - \Lambda\nu^{\mathrm{s}}(\overline{\omega}, a_L)\right), \end{aligned} \tag{C.34}$$

Recall that under Nash bargaining, $q^P(\overline{\omega}, a_L, a_H, q)$ is given by a linear combination of the seller's and buyer's values; thus, it is also affine in $\overline{Z}$. Since $\theta^{FI}(\overline{\omega}, a_L)$ is independent of $\overline{Z}$ and $q^{FI}(\overline{\omega}, a_L) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\overline{\omega}, a_L)$ are affine in $\overline{Z}$, so is the left-hand side of the equation. Similarly, since $\theta(\overline{\omega}, a_H)$ is independent of $\overline{Z}$, but $q(\overline{\omega}, a_H)$, $q^P(\overline{\omega}, a_L, a_H, q)$ and $\nu^{\mathrm{s}}(\overline{\omega}, a_L)$ are all affine in $\overline{Z}$, then the right-hand side of the equation is also affine in $\overline{Z}$. Thus, the proposed allocation satisfies the definition of the values, and the optimality conditions for search activity and prices. Using the uniqueness of the equilibrium, we conclude that if the wage is affine in $\overline{Z}$, then the market tightness—and the slope between duration and prices—must be independent of $\overline{Z}$.

Intuitively, since the buyer's net value of buying capital in a submarket and the expected cost of visiting that particular submarket (i.e., the expected labor costs) are both proportional to $\overline{Z}$, changes in steady-state productivity—and therefore in $\overline{Z}$—do not affect the trade-off that buyers face between low prices and high trading probabilities dictated by (C.33). In addition, since changes in productivity are aggregate and do not affect the relative marginal product of high- vs lower-quality capital, changes in aggregate productivity do not affect the incentive-compatibility constraint (C.34). Finally, because both the price of capital and its value to the seller are proportional to $\overline{Z}$, changes in aggregate productivity do not affect the seller's constraints and only proportionally scale its objective, without altering the optimal trading probability.

■

C.3.2 Dispersion in unobserved capital qualities

Our second experiment examines changes in the dispersion of unobserved capital qualities, σ_a, which is a central shock in the literature on the macroeconomic effects of asymmetric information (e.g., Eisfeldt, 2004; Kurlat, 2013; Bigio, 2015). In Section 5.3, we discussed how, through the lens of our model, the dispersion of unobserved capital qualities likely increased during the Euro crisis—as reflected in the rise in the dispersion of price residuals—and how this amplifies the macroeconomic effects of changes in the accuracy of information technologies.

We now study how changes in the dispersion of unobserved capital qualities affect the slope between duration and residual prices. To do so, we consider changes in σ_a that impact both the support of the distribution and the mass of capital units for each capital type, as illustrated in Panel (A) of Figure C6.[29]

Panel (B) shows how changes in σ_a affect the slope between duration and residual prices. The vertical lines indicate the steady-state level and the level that corresponds to an increase of 24%, which approximates the magnitude estimated to have occurred during the Euro crisis based on the maximum change in the dispersion of price residuals observed during this episode (see Figure C3 Panel (A)). The results show that this increase raises the slope between log duration and residual prices from 0.15 to 0.2, and accounts for 17% of the increase observed during the Euro crisis.

[29] As discussed in Section 3, in our calibrated model we truncate the log-normal distribution of unobserved capital qualities to the support $[-2\sigma_a, 2\sigma_a]$. Changing σ_a without altering the support of the distribution results in smaller aggregate effects (results available upon request). Therefore, to study the maximum effects of this shock, our baseline experiment considers changes in both the dispersion and the support of the distribution. Also, to capture the upper bound of these effects and for simplicity, our model experiment assumes permanent changes in the dispersion of unobserved capital qualities.

To further analyze this result, Panels (C) and (D) report the effect of changes in σ_a on sellers' choices in capital markets. These effects influence allocations along multiple dimensions. First, the distribution now includes units of lower capital quality, which, as discussed in Section 3.1, are optimally listed under the full-information terms of trade and are associated with lower prices and trading probabilities than units of slightly higher quality. Second, sellers of low-quality capital begin to deviate from their full-information allocation, since the presence of even lower quality lemons increases the threat of mimicking; this leads to higher listed prices and lower trading probabilities. Third, the distribution expands to include units of higher capital quality as well, which are optimally listed at lower trading probabilities to distinguish those sellers from lower-quality sellers. The relative strength of these effects leads to the nonmonotonic relationship between σ_a and the slope between duration and residual prices reported in Panel (B). Around the model's steady state, this figure shows that changes in σ_a have a relatively modest effect on the slope between duration and residual prices. Furthermore, Figure C5 shows that, in our model, no increase in the dispersion of unobserved capital qualities can account for the rise in the slope between duration and residual prices observed in the data.

From this analysis, we conclude that despite the relevance of changes in the dispersion of unobserved capital qualities for the macroeconomy, the changes observed during the Euro crisis are not likely to have generated shifts in the slope between duration and residual prices of the magnitude observed during this episode.

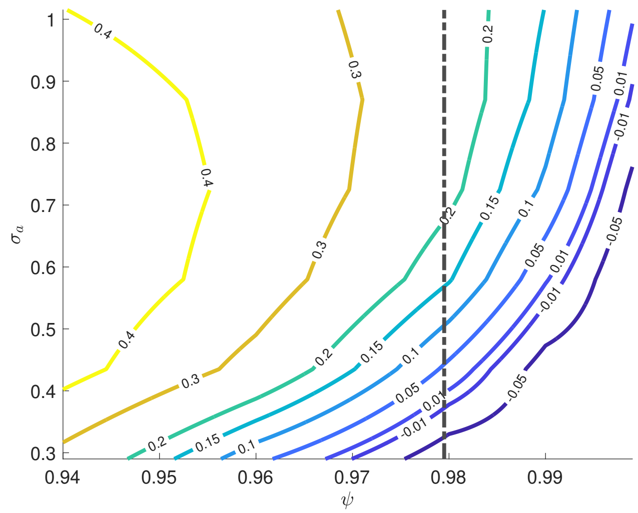
C.3.3 Sellers' discount rates and heterogeneity in trading needs

Our third exercise is motivated by the fact that the Euro crisis was characterized by financial distress that induced fire sales from capital sellers.[30] As discussed in Section B.5, this may have led some sellers to list their capital units in submarkets with a high trading probability, which affects the slope between duration and residual prices.

To quantify these forces in our model, we introduce heterogeneity in sellers' discount factors and examine the impact of changes in this heterogeneity (as would occur during a fire sale). To isolate the role of discount rate heterogeneity more clearly, we focus on the version of our model with capital intermediaries, described in detail in Section C.2.1—which, as shown

[30]See Ortega and Peñalosa (2012) and Jimeno and Santos (2014) for a detailed description of this crisis episode in Spain.

Figure C5: Slope between duration and residual prices as a function of ψ and σ_a



Note: This figure reports the slope between log duration and log residual prices as a function of two key structural parameters of the model, ψ and σ_a. The figure reports a set of contours of the slope in model-simulated data, where the value of each contour is made explicit on top of each line.

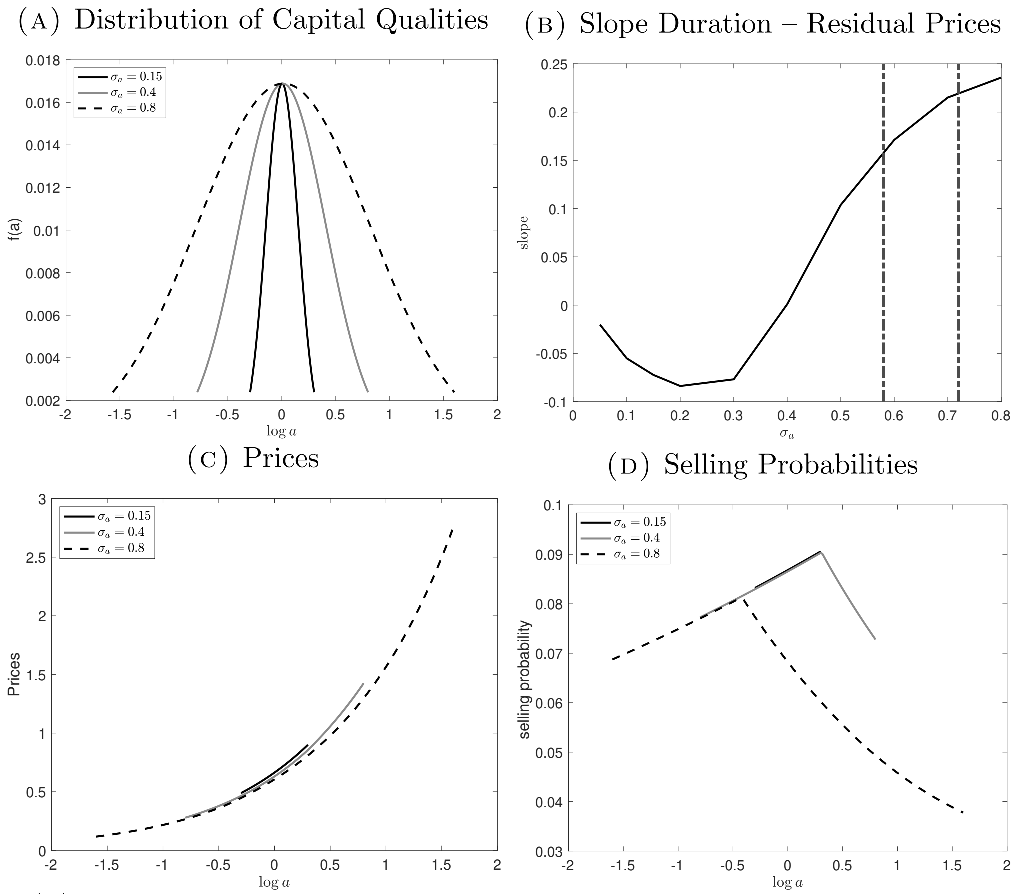
in that section, is isomorphic to our baseline model under homogeneous seller discount rates. For simplicity, we assume that there are two types of capital intermediaries, which differ in their discount factors, denoted β_{low} and $\beta_{high} \geq \beta_{low}$, each with equal mass. We define the difference in discount factors as $\epsilon_\beta \equiv \beta_{high} - \beta_{low}$ and set $\frac{1}{2}(\beta_{low} + \beta_{high}) = \beta$, which allows us to separately analyze the effects of dispersion in discount rates and their levels by varying ϵ_β and β.

To align this model as closely as possible with our baseline framework, we make two additional assumptions. First, we assume that at the beginning of each period, a random mass of households matches with a type of intermediary and trades the capital they produced at the end of the previous period (as well as the capital from exiting firms) in competitive markets, as described in Section C.2.1. This implies that households trade capital with intermediaries with discount rate β_j at the price $q^I_{jt}(\omega, a) = \nu^{\mathrm{s}}_{jt}(\omega, a)$, where, following the same steps as in Section 2, $\nu^{\mathrm{s}}_{jt}(\omega, a)$ is given by

$$\nu^{\mathrm{s}}_{jt}(\omega, a) = \max_{\{\hat{a}, q\}} \; p(\theta_t(\omega, \hat{a}, q))((1-\psi)q + \psi q^P(\omega, a, \hat{a}, q)) \\ + (1 - p(\theta_t(\omega, \hat{a}, q)))\,(\beta_j \frac{\gamma_n}{\gamma} \nu^{\mathrm{s}}_{jt+1}(\omega, a) - \delta\omega a),$$

for $\beta_j \in \{\beta_{low}, \beta_{high}\}$. Second, we assume that submarkets in the decentralized capital market are now indexed by $(\omega, \hat{a}, q, \beta_{\mathrm{s}})$, where β_{s} represents the sellers' discount rate, which is perfectly

Figure C6: The Effect of Changes in the Dispersion of Unobserved Capital Qualities



Note: Panel (A) depicts the distribution of unobserved capital qualities for different levels of σ_a. Panel (B) reports the impact of σ_a on the slope between log duration and residual prices. Panels (C) and (D) report listed prices and the corresponding effects on selling probabilities for different levels of unobserved capital quality.

observed by all market participants. This assumption allows us to restrict the asymmetric information problem to unobserved capital quality, as in our baseline framework.[31]

Figure C7 analyzes the effects of changes in the dispersion of discount factors, governed by ϵ_β. Panels (A) and (B) show how discount factors influence listed prices and associated selling probabilities across the distribution of unobserved capital quality (as in Figure 3

[31] Another possibility would be to assume that discount factors are heterogeneous and private information to each seller. Guerrieri and Shimer (2018) analyze this more complex scenario and find that when sellers have private information about both their asset quality and their preferences (discount factors), a multiplicity of equilibria arises that is immune to standard signaling game refinements, including those used in our paper (see Chang (2018) for an analysis of multivariate private information about sellers' holding costs). Furthermore, they show that different types of equilibria are generically not Pareto comparable, which makes equilibrium selection even more challenging. To incorporate this extension in our model, we could follow Guerrieri and Shimer (2018)'s main assumption that the average unobserved qualities are increasing in expected discounted continuation values (effectively assuming that discount factor heterogeneity is not the main driver of heterogeneity in discounted continuation values); see Chang (2018) for a similar monotonicity assumption. In their semi-separating equilibrium (which is unique under a stricter equilibrium definition), there is a positive relationship between expected asset quality and sale prices (and thus a negative relationship with trading probability), wchich resembles a "fuzzy" version of the relationship in our model.

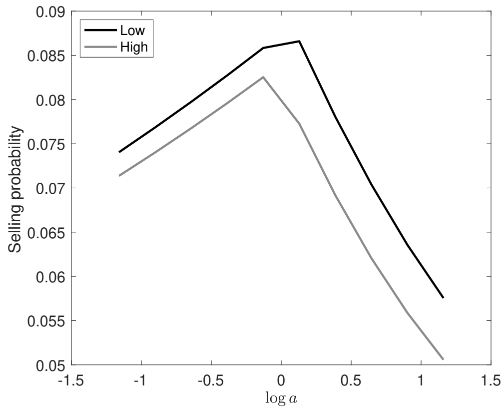
in the baseline model). For a given capital quality, sellers with low discount factors list their capital units at lower prices, which are associated with higher trading probabilities. In addition, a greater share of these sellers adopts their full-information allocation, as reflected in the rightward shift of the kink in the policy function for trading probabilities (Panel (A)) compared to sellers with high discount factors. This shift arises because low discount factors increase the cost of delaying sales, making it more costly for low-quality sellers to mimic high-quality types.

Panels (C) and (D) show that increases in the dispersion of discount factors lead to modest changes in average listed prices and trading probabilities. This is because, as implied by the results above, greater dispersion has offsetting effects across submarkets: it reduces prices and increases selling probabilities in submarkets with low discount factors (β_{low}), while having the opposite effect in those with high discount factors (β_{high}). These effects are roughly similar in magnitude, leaving the average listed price and trading probability nearly unchanged. As a result, Panel (D) shows that increases in the dispersion of discount factors lead to only modest increases in the slope between duration and residual prices. Panel (E) shows that these changes also have very small effects on economic activity.

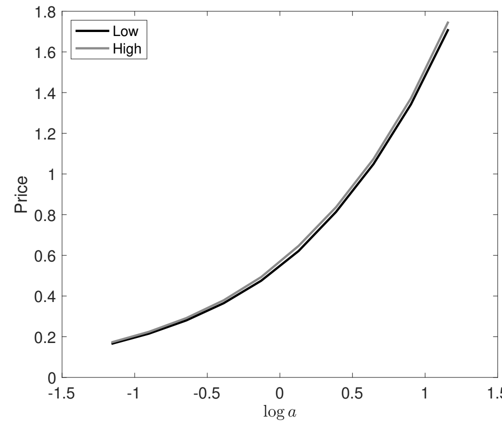
So far, our analysis has focused on increases in the dispersion of discount factors while holding their average level fixed. However, the firesale episodes motivating this exercise—such as those observed during the Euro crisis—likely involved a decline in the average seller discount factor. To analyze this case, Figure C8 examines how changes in sellers' average discount factors (i.e., changes in β) affect the slope between duration and residual prices and aggregate economic activity. Panel (A) shows that homogeneous declines in discount factors reduce the slope between duration and residual prices, resembling a decrease in the degree of asymmetric information. As discussed above, this occurs because lower discount factors lead more sellers to adopt their full-information allocation, bringing the slope closer to that observed under full information. Panel (B) shows that these declines also reduce economic activity through standard investment and capital stock channels. Therefore, while homogeneous declines in discount factors depress economic activity, they reduce the slope between duration and residual prices—opposite to what is observed during the Euro crisis.

Figure C7: The Effect of Discount Factor Heterogeneity

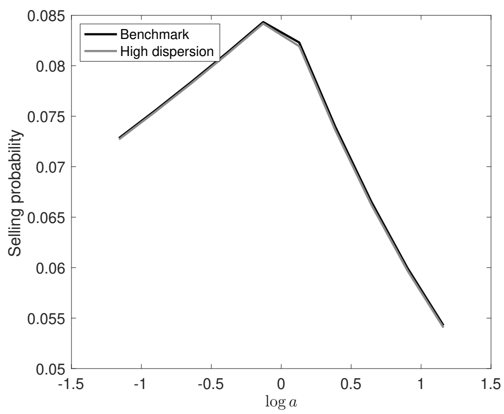
(A) Selling Probabilities



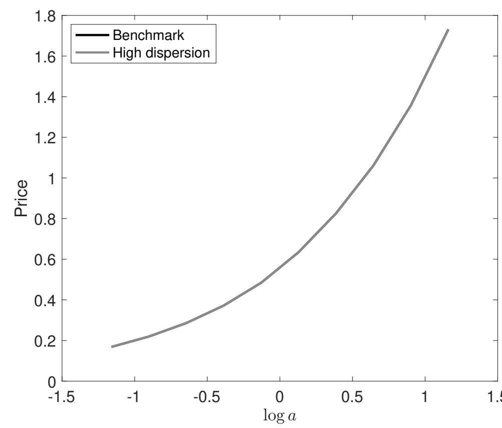
(B) Prices



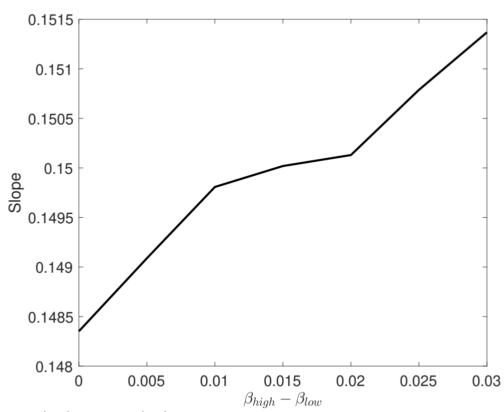
(C) Aggregate Selling Probabilities



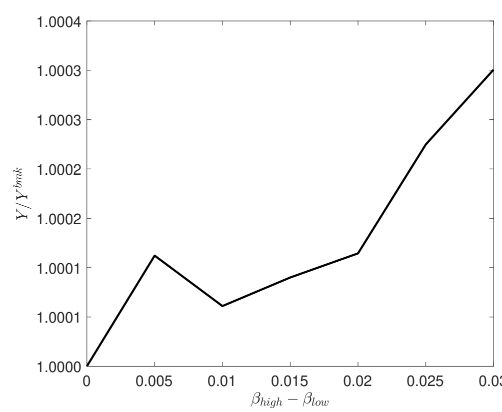
(D) Aggregate Prices



(E) Slope Duration – Residual Prices



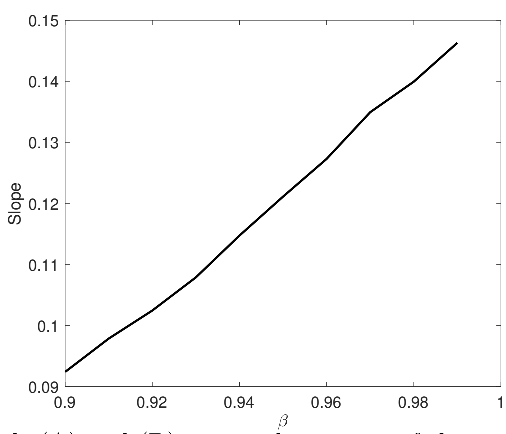
(F) Aggregate Output



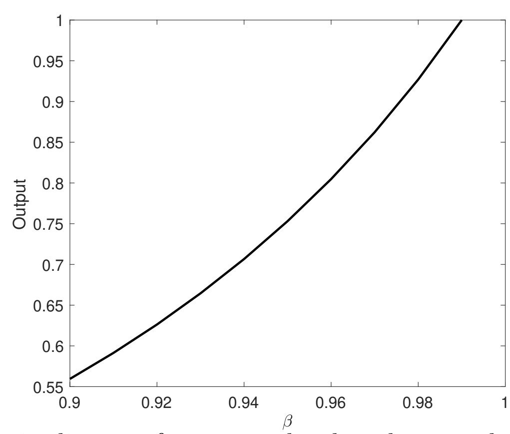
Note: Panels (A) and (B) report selling probabilities and listed prices for different levels of unobserved capital quality and different levels of discount factors (under the equilibrium with dispersed discount factors). Low and High discount factors are 3 percentage points above and below the calibrated value for β at an annual frequency (0.9966^{12}), respectively. Panels (C) and (D) plot the same variables shown in Panels (A) and (B) after aggregating across sellers with different discount factors and offers a comparison to our benchmark calibration. Panels (E) and (F) report the impact of changes in the dispersion of discount factors on the slope between log duration and residual prices and on aggregate output (expressed relative to that of the benchmark calibration).

Figure C8: The Effect of Homogeneous Changes in Discount Factors

(A) Slope Duration – Residual Prices



(B) Aggregate Output



Note: Panels (A) and (B) report the impact of changes in discount factors on the slope between log duration and residual prices and on aggregate output (expressed relative to that of the benchmark calibration).