

# The Aggregate Effects of Bank Lending Cuts

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## Abstract

A robust cross-sectional literature establishes that granular cuts in bank lending supply negatively affect firm-level outcomes due to imperfect substitutability of bank-firm relations and a degree of bank dependence. In this paper I make clear what these cross-sectional effects imply for the aggregate effects of overall lending cuts. I estimate the aggregate effect using a new model that captures the main cross-sectional patterns and allows for the central general equilibrium effects cross-sectional regressions difference out. I show that cross-sectional employment and credit elasticities are informative, but are not enough on their own, to obtain the aggregate effects of an overall credit supply shock. After imposing structure on the extent of reallocation of inputs, demand, and funds, I find large aggregate effects of credit supply shocks on aggregate output, so cross-sectional effects survive aggregation in general equilibrium.

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# Introduction

Motivated to understand the effects of bank funding disruptions on *aggregate* allocations, a large literature exploiting cross-sectional variation in bank exposure across firms, show that granular shocks to the supply of bank lending have causal effects on real and financial outcomes at the firm and regional levels (Khwaja and Mian, 2008; Chodorow-Reich, 2014; Huber, 2018). The main reason to rationalize these effects exist is the imperfect substitutability of financing options for firms. These cross-sectional elasticities identify relative rather than aggregate effects by design, so the mapping from them to the aggregate effects of aggregate credit supply shocks is not clear.

What can cross-sectional estimates of granular shocks then tell us, if anything, about the aggregate effects of overall bank funding shocks? This question requires an economist to translate the cross-sectional effects of granular shocks into the aggregate effects of symmetric shocks that affect every lender.

In order to answer this question, I use a model that allows for the main empirical patterns highlighted by the cross-sectional literature. As in cross-sectional studies, firms may potentially borrow from multiple banks and use different forms of external finance. As highlighted by the empirical literature, substitutability across banks and between forms of external finance may be imperfect. I allow for flexibility in the patterns of substitution between banks and between different forms of external finance (see Paravisini (2008)).

The model is designed to make clear the ways in which these ingredients affect the aggregate economy and that firm and regional regressions difference away. These channels are the reaction of aggregate input prices, and of national deposit real interest rates to credit supply shocks. In cross-sectional regressions, the variation of interest rates and input prices that is common to every firm, or every region regardless of their bank exposure will be soaked in a time intercept, even when this variation is crucial to understand the macroeconomic implications of such shocks.

The model clarifies that there are two sets of mechanisms causing firm-level elasticities

ties to not aggregate up exactly, and that in general is not possible to sign the deviation between cross-sectional and aggregate effects. First, inputs and demand may flow from firms with unhealthy lenders into firms with healthy lenders, and the elasticity to which that happens depends on the structure of these markets.<sup>1</sup> This channel implies that cross-sectional elasticities will overestimate aggregate effects. The second one is that due to competition for deposits across banks, a shock to an individual bank will spillover to the balance sheet of lenders not directly exposed to the shock. The elasticity at which deposits move across banks, and the elasticity at which firms substitute bank credit will govern how borrowers of ex-ante healthy lenders will be affected. This channel implies that cross-sectional elasticities will underestimate aggregate effects. Furthermore, even if labor markets were completely local and goods completely non-tradeable, regional-level elasticities would not aggregate up, since localized bank shocks will trigger movements in national deposit and lending rates.

I discipline the elasticities of substitution of credit between banks and between sources of finance using two cross-sectional regressions that are the best practice in the empirical assessment of the firm-level effects of bank shocks. These regressions estimate the differential impact of a bank shock on credit and employment firms with stronger ties to the affected bank.

I show that cross-sectional regressions of firm-level employment growth and credit growth on exposure to granular bank-lending shocks are informative about the aggregate effects of symmetric shocks. These regressions contain information on the extent of bank credit substitutability, and the strength of this margin of substitution is relevant for aggregate output after symmetric shocks that affect all the banks, and after granular shocks that affect only a subset of lenders.

I show that bank-credit growth regressions with firm fixed effects are interesting in their own right, not just as a robustness exercise. Although the elasticity of substitution

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<sup>1</sup>see the online appendix of Chodorow-Reich (2014), who makes a similar point in a model where substitution between banks and forms of finance that are central to this paper are muted

across banks does not affect aggregate output after symmetric shocks that affect all the banks, it is relevant for the cross-sectional and aggregate consequences of the granular shocks affecting a subset of banks for which we have good causal effects estimates. So the firm fixed-effect estimator provides useful information on its own when moving from the cross-sectional effects of granular shocks to the aggregate effects of symmetric shocks.

Firm-level credit and employment regressions suffer from an observational equivalence problem. It is not possible to distinguish economies with different degrees of financial frictions by using these firm-level regressions exclusively. The reason is that although cross-sectional elasticities of output are decreasing in the ability of firms to substitute between financial sources, they are increasing in the elasticity with which inputs and demand reallocate across firms. Only after pinning down the strength of reallocation of demand and inputs it is possible to use the regressions to uncover the substitutability of finance sources.

I extend the model to allow for spillovers of funding shocks across banks driven by deposit competition, and estimate the two key elasticities of substitution numerically. The idea behind the identification is the following. After a bank funding shock, firms that can replace funding from the affected bank with financing from other banks will experience little change in their credit or output due to the shock. However, if firms cannot substitute across banks but can avoid bank credit altogether, they will take on much less bank credit, but their output losses will be small. This tension implies that elasticities of firm credit and employment identify the two key parameters in the model. I target the regression coefficients estimated by Huber (2018), but the methodology can be adapted to target estimates in other settings.

I estimate an elasticity of output to lending-supply of 0.2.<sup>2</sup> This number means that an aggregate bank funding shock that triggers a 1 percent drop in aggregate lending causes a decline of aggregate output of 0.2 percent. Under an alternative parametrization of the

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<sup>2</sup>The aggregate effects I estimate correspond to the average treatment effects identified in the cross-sectional analysis.

model that assumes an infinite elasticity of reallocating inputs across firms and targets the same observed cross-sectional estimates, the estimated elasticity is three times smaller, highlighting the quantitative relevance of the observational equivalence problem I presented before. The reason is that more flexible labor markets imply larger cross-sectional elasticities after the same shock. To target the same microeconomic patterns, frictions in the banking sector must be smaller when input markets are more elastic.

To discipline the easiness of reallocation of inputs across firms, I use two sources of evidence. First, I use direct evidence from Webber (2015) who documents an inelastic firm-specific labor supply curve. To hire additional labor, a firm must pay a higher than average wage. Second, I use the *indirect effects of bank lending cuts*. After extending the model to contain two regions that produce non-tradable goods, I show that in economies with flexible input markets, unexposed firms operating in local markets with high average exposure outperform non-exposed firms in local markets with low average exposure. Evidence on the indirect effects reported by Huber (2018) documents the opposite. Both pieces of evidence favor models where the aggregate consequences of bank shocks are large.

Finally, I compare the magnitude of the elasticity I obtain in general equilibrium with the (partial equilibrium) back-of-the-envelope calculation. Back-of-the-envelope calculations add up the differences of every firm's outcome with respect to that of a control firm with zero direct exposure to the shock. Under my benchmark parameterization, the general equilibrium output drop is 70% the partial equilibrium one. Under an alternative parametrization with perfectly elastic labor markets, the general equilibrium effects are five times smaller than in partial equilibrium. Overall, the interpretation of the evidence through the lens of the model implies that general equilibrium forces do not cause the micro patterns to vanish.

**Literature Review** This paper addresses the long-standing question of the relevance of bank health disruptions on aggregate economic performance. Bernanke (1983) stated that cuts in the supply of bank lending make credit more expensive, potentially affecting the aggregate economy. I analyze the relevance of cuts in the supply of bank lending to firms in determining drops in aggregate production.<sup>3</sup>

To measure the effects of an aggregate lending cut on aggregate output, I rely on a large and robust empirical literature that inquires about the effects of bank health in a cross-section of firms and banks. This cross-sectional literature exploits variation in bank exposure to funding shocks and variation in the exposure of firms and regions to different banks. This body of evidence concludes that bank disruptions affect the allocation of firm credit, as in Khwaja and Mian (2008);<sup>4</sup> firm outcomes like employment and sales, presumably because of the existence of sticky firm-bank relationships, as in Chodorow-Reich (2014);<sup>5</sup> regional outcomes, as in the seminal work by Rosengren and Peek (2000).<sup>6</sup>, and across countries, as in Biermann and Huber (2019), and Xu (2020).

This paper contributes to the broad literature that uses cross-sectional estimates to investigate the macroeconomic effects of macro and micro shocks. The approach I follow in this study uses causal effects measured in cross-sectional settings as inputs to measure an aggregate elasticity, in this case the elasticity of aggregate output to aggregate lending shocks. Nakamura and Steinsson (2017) survey the literature and discuss its challenges. Particularly relevant is the work of Sraer and Thesmar (2018) studying the aggregate effects of collateral constraints.

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<sup>3</sup>Benmelech, Frydman, and Papanikolaou (2019) document that large firms with maturing bonds tried to access bank loans after debt markets froze up during the Great Depression. Firms located in regions with more affected banks suffered larger employment losses during the Depression.

<sup>4</sup>Other examples of this literature are Gan (2007); Schnabl (2012); Iyer, Peydro, da Rocha Lopes, and Schoar (2013); Benetton and Fantino (2018); Jiminez, Mian, Peydro, and Saurina Salas (2014); and Cingano, Manaresi, and Sette (2016).

<sup>5</sup>The strength of the relationship between firms and banks depends on the closeness of the entities. The literature has explore geography as a key driver of closeness, like in Degryse and Ongena (2005), Agarwal and Hauswald (2010), Brevoort, Wolken, and Holmes (2010), Nguyen (2019), or Huber (2018). The consequences of bank-firm relationships in the cross-section has been widely studied. An example is Darmouni (2017).

<sup>6</sup>Other examples are Ashcraft (2005); Greenwood, Mas, and Nguyen (2014); and Huber (2018).

In the modeling front, I embed a discrete choice problem in a macroeconomic model with heterogeneous firms. This approach is similar to the Ricardian models in the Eaton and Kortum (2002) spirit, used to characterize trade flows between countries. Instead, however, I use it to characterize the flow of credit from banks to firms and firms' decisions about how much to borrow.

## 1 A Model of Credit Dependence of Multi-Bank Firms

In this section I present a model that is flexible to incorporate the credit and output effects observed in the cross-section after a granular credit supply shock, and use it for analyzing the effect of bank health on aggregate output. The model features a continuum of firms, a discrete number of bank types, and a representative household. Firms borrow from multiple banks simultaneously. Banking relationships are imperfectly substitutable in the sense that the relative demand for funding from a particular bank is downward sloping, not horizontal. Self-finance is an imperfect substitute for bank credit. This model is static and makes a number of simplifications that I will relax later in the paper.

### 1.1 Firms

There is a continuum of monopolistic competitive firms producing differentiated varieties. Firms are indexed by  $j$  in the unit interval. The demand schedule for each variety is given by:

$$Y_j = Y P_j^{-\eta}, \quad (1)$$

where  $P_j$  is the relative price of variety  $j$ ,  $Y_j$  is the quantity demanded of each variety and  $Y$  is aggregate demand. The aggregate price level is set to be the numeraire.

Each variety is produced by mixing a continuum of intermediates indexed by  $\omega$ . The firm aggregates the intermediates via a CES function with elasticity of substitution  $\sigma$ .

$$Y_j = \left( \int_0^1 (y_j(\omega))^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \quad (2)$$

Each intermediate good  $\omega$  is produced with labor with a linear production function, and a firm-wide productivity shifter  $z$

$$y_j(\omega) = z_j l_j(\omega). \quad (3)$$

## 1.2 Financing

For a given intermediate, firms decide whether to self-finance or look for funding from a bank. Firms that choose bank financing must select an individual bank to finance each intermediate. Different financing options offer different terms, and firms face shifters that reflect the comparative advantage of financing an intermediates with a given financing option.

Because firms need to finance a continuum of tasks, the cost of funds for the firm, which determines its marginal cost, does not depend on the realization of the financing cost of any particular intermediate, but on structural parameters that capture how substitutable bank credit is for self-finance, and the substitutability of credit from a particular bank. In the two next subsections I introduce these discrete choice problems.

### 1.2.1 Shopping for a lower rate

The total cost of financing intermediate  $\omega$  is given by  $TC_j(\omega)$ , which consists of the wage bill and the financing costs of financing the wage bill, which after replacing the production function is given by

$$TC_j(\omega) = \frac{w_j}{z_j} R_j(\omega) y_j(\omega), \quad (4)$$

where  $R_j(\omega)$  is the effective interest rate firm  $j$  must pay to finance the expenditures of intermediate  $\omega$ . As part of its cost minimization problem, firm  $j$  looks for the cheapest



financing option.

In particular, at the intermediate level, the firm chooses

$$R_j(\omega) = \min \left\{ \min_{b \in \mathcal{B}} \left\{ \frac{R_b}{\epsilon_{jb}(\omega)} \right\}, \frac{R_{\mathcal{S}}}{\epsilon_{j\mathcal{S}}(\omega)} \right\}. \quad (5)$$

Here  $\mathcal{B}$  and  $\mathcal{S}$  denote a set of banks and an self-finance options ( $N_{\mathcal{S}} = 1$  without loss),  $b$  indexes one out of the  $N_{\mathcal{B}}$  banks in the economy. The effective cost the firm perceives if it were to choose a financing option is equal to the cost of funds of that option  $R$ , over a shifter, that captures all the idiosyncratic reasons why one option may be better for some intermediates than others. For example, some projects of the firm entail high verification costs, as in ?, so the firm may prefer to self-finance them. Other projects benefit from the know-how of a specific expert bank or benefit from financing in a particular location.

I assume the vector  $\epsilon$ , which collects the shifters for the banks and the self-financing option is drawn from a nested Fréchet Distribution

$$F_j(\epsilon) = \exp \left\{ - \sum_{s \in (\mathcal{B}, \mathcal{S})} \bar{\varphi}_s \left( \sum_{b=1}^{N_s} T_{jb} \epsilon_{sb}^{-\theta} \right)^{\frac{s}{\theta}} \right\},$$

This distribution has been used by Dingel, Meng, and Hsiang (2019), and by Lashkaripour and Lugovskyy (2018), and it extends the Fréchet distribution common in the Ricardian model of international trade of Eaton and Kortum (2002). An analogy comes to mind from the literature in international trade. When deciding where to import from, a destination country shops around many locations and chooses the origin with the lowest effective price.. The Fréchet shifters capture the fact that some countries have a comparative advantage in the production of some goods, and that within each country, some cities have comparative advantage. In this application, the shifters capture the firm's preference to finance a given task with a given financing option. It is akin to a productivity shock that depends on the financing source and the intermediate. The nested Fréchet distribution captures the variation in the advantage of financing a given task both across banks (some

banks are better than others) and across financing options (some intermediates are better for bank financing).

The  $T_{jb}$  parameters capture the strength of the long-term banking relationship between firm  $j$  and bank  $b$ , or the absolute advantage of bank  $b$  in providing funding for firm  $j$ .

Under the assumptions stated before, we can characterize the share of expenditures financed with each bank conditional on choosing bank financing  $\nu_{jb}$  and the cost of bank credit for the firm  $R_{jB}$ :

$$R_{jB} = \left( \sum_{b \in \mathcal{B}} T_{jb} R_b^{-\theta} \right)^{-1/\theta}. \quad (6)$$

The share of borrowing needs that firm  $j$  gets from bank  $b$  is given by:

$$\nu_{jb} = \frac{T_{jb} R_b^{-\theta}}{\sum_k T_{jk} R_k^{-\theta}}. \quad (7)$$

The borrowing shares depend on  $\theta$ , which is the elasticity of substitution of funding from a specific bank, and on  $T_{jb}$ , which is the relative strength of the banking relationship between firm  $i$  and bank  $b$ . The share of expenditures financed with the banking sector  $s_j$ , is given by

$$s_j = \frac{\bar{\varphi} R_{jB}^{-\varphi}}{\bar{\varphi} R_{jB}^{-\varphi} + (1 - \bar{\varphi}) R_{jS}^{-\varphi}}, \quad (8)$$

and the effective cost of funds for the firm is given by the cost of funds index  $R_{jt}$

$$R_j = \left( \bar{\varphi} R_{jB}^{-\varphi} + (1 - \bar{\varphi}) R_{jF}^{-\varphi} \right)^{-1/\varphi}. \quad (9)$$

This discrete choice block is a microfoundation of the desired financial structure of the firm. When bank credit becomes more expensive ( $R_{jtB} \uparrow$ ), the firm moves away from bank lending ( $s_{jt} \downarrow$ ). The elasticity at which the substitution occurs is given by  $\varphi$ .

Figure (1) plots the share of financing from the banking sector as a function of the cost of funds for different values of  $\varphi$ . The figure shows that as  $\varphi$  increases, the relative

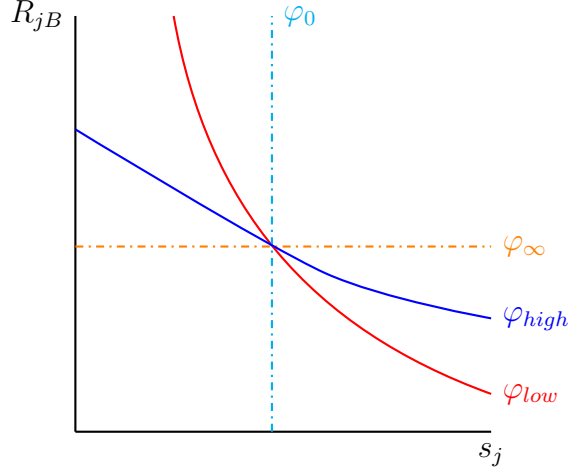


Figure 1: Relative demand schedule for bank credit

*Note:* The figure shows the ratio of bank loans to financing needs of the firm  $s_j$  as a function of the effective cost of bank loans  $R_{jB}$  for several values of the elasticity of substitution between bank loans and self-finance  $\varphi$ .

demand schedule for bank funds becomes more elastic. In the limit, when  $\varphi \rightarrow \infty$  the demand curve becomes horizontal, and firms are perfectly elastic in switching between bank funding and self-finance. On the other side, when  $\varphi$  becomes smaller, the share of bank financing is less sensitive to its cost.

When  $\theta$  is higher, the demand curves for funding for a particular bank become flatter, which I show in Figure (2). In the limit, when  $\theta \rightarrow \infty$  the demand curve becomes horizontal, and firms are perfectly elastic in switching between banks. On the other side, when  $\theta$  tends to zero, the share of bank financing from bank  $b$  is less sensitive to bank  $b$ 's lending rate.

### 1.3 Workers

There is a representative household. It consumes and supplies labor. The household maximizes the utility function, which for simplicity I assume takes a GHH form.

$$U(C_t, L_t) = \frac{1}{1-\gamma} \left( C_t - \frac{L_t^{\phi+1}}{1+\phi} \right)^{1-\gamma} \quad (10)$$

Where  $L_t$  is an aggregator of the labor supply to different firms in the economy:

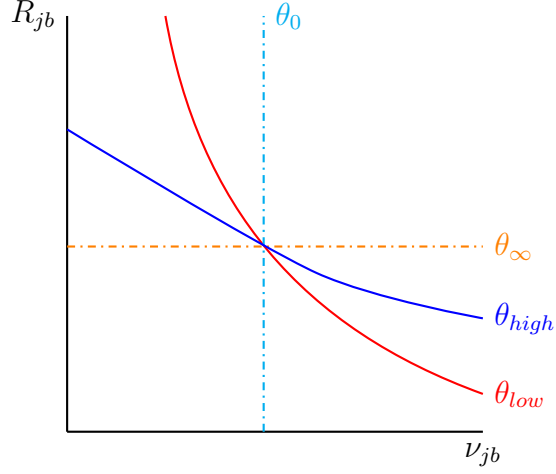


Figure 2: Relative demand schedule for bank-specific credit

*Note:* The figure shows the ratio of bank  $b$  credit to total bank credit chosen by firm  $j$ ,  $\nu_{jb}$ , as a function of the effective cost of bank loans from bank  $b$   $R_{jb}$  for several values of the elasticity of substitution between banks  $\theta$ .

$$L_t = \left( \int L_{jt}^{\frac{1+\alpha}{\alpha}} dj \right)^{\frac{\alpha}{1+\alpha}}. \quad (11)$$

Workers maximize utility subject to a budget constraint  $\int w_{jt} L_{jt} = C_t$ . Households supply labor according to the following relationship:

$$L_t = w_t^{1/\phi}, \quad (12)$$

$$L_{jt} = L_t \left( \frac{w_{jt}}{w_t} \right)^\alpha, \quad (13)$$

where  $w_t$  is defined as  $L_t^\phi$ . Workers demand higher pay in order to work more hours at the same firm. When  $\alpha \rightarrow \infty$ , the labor market operates under a single wage rate  $w_{jt} = w_t \forall j$ .

## 1.4 Other Aspects of the Model

In this model, it is assumed that lending rates are exogenous. Later in the full model, I will specify the bank problem that gives rise to the lending rates in equilibrium as a

function of the market structure and the ease of securing funding. I also assumed that the profits belong to the workers. The problem of the firm owners is included in the full model as well.

## **2 Characterization**

The focus of this section is to characterize the elasticity of aggregate output to an exogenous hike in the cost of funds of a particular bank, and to a similar shock that affect the whole banking sector symmetrically. I focus on the aggregation problem for the case in which the no-sorting identifying assumption of the cross-sectional regression holds exactly in the economy.

In this section I present two main results. First, aggregate and cross-sectional effects on output of a rate hike of an individual bank are different, and it is a priori unclear which of them is larger. The difference in magnitude is dominated by the difference in the Frisch elasticity of the labor supply and the easiness to reallocate demand and inputs across firms. When it is easy to reallocate labor and demand across firms, then up to a second order the cross-sectional effects of output are larger.

Second, although greater frictions in the banking sector, in the form of low elasticities of substitution of funds between banks and between funding alternatives increase the output losses caused by lending rate hikes, it is not possible to back out from a single cross-sectional elasticity the structural parameters that determine the response of aggregate output.

### **2.1 The Aggregate Effects of Loan Term Changes in One Bank**

All the results in this section exploit the following assumption, which I impose in order to abstract from identification concerns of the cross-sectional studies

**Assumption 1.** *There is no sorting in financial relations. That is, firm-level productivity  $z_j$  and the strength of bank lending relationships  $T_{jb}$  are independent. I rule out the possibility that banks more prone to lending rate hikes are linked to firms more prone to lower productivity draws.*

As I show in the appendix, under assumption 1, aggregate output is given by equation 14, where the expectation operator is taken across the continuum of firms:

$$Y = \left( \frac{\eta}{\eta - 1} \right)^{-1/\phi} \mathbb{E} \left( z_j^{\frac{(\eta-1)(\alpha+1)}{\alpha+\eta}} \right)^{\frac{(1+\phi)(\alpha+\eta)}{\phi(\eta-1)(\alpha+1)}} \mathbb{E} \left( R_j^{\frac{-(\eta-1)\alpha}{\alpha+\eta}} \right)^{\frac{(1+\phi\eta)}{\phi(\eta-1)}} \mathbb{E} \left( R_j^{\frac{-\eta(\alpha+1)}{\alpha+\eta}} \right)^{\frac{(1-\phi\alpha)}{\phi(\alpha+1)}}. \quad (14)$$

The first results of this section hold under the following assumption:

**Assumption 2.** *Assume the lending terms of all banks except one are kept constant at an arbitrary level  $R$ , as is the self-financing rate. At these rates, the level of output coming from equation 14 is defined as  $\bar{Y}$ . For an arbitrary bank  $b$ , the lending terms are disrupted to  $Re^u$ , for a positive and sufficiently small  $u$ .*

Note that after making an assumption about the distribution of  $T_{jb} \forall j, b$ , and setting an arbitrary level of lending terms, we can compute the behavior of output numerically according to equation 14. Assumption (2) helps in finding closed-form results on the aggregate output effects of lending term disruptions, presented in Proposition (1).

**Proposition 1.** *The aggregate effects of granular shocks. Under Assumption (2), up to the second order, the log change of output caused by an increased in the lending cost of bank  $b$  is given by:*

$$\Delta \log Y = -\frac{1}{\phi} \left( \bar{s} \mu_b u + \Omega s^2 (\sigma_b^2 + \mu_b) \frac{u^2}{2} - \theta \bar{s} (\mu_b - \sigma_b^2 - \mu_b^2) \frac{u^2}{2} - \varphi \bar{s} (1 - \bar{s}) (\sigma_b^2 + \mu_b^2) \frac{u^2}{2} \right) \quad (15)$$

$$\Delta \log Y \approx -\frac{1}{\phi} \bar{s} u \left( \bar{\nu}_b - \theta \frac{u}{2} \Upsilon_1 - \varphi (1 - \bar{s}) \frac{u}{2} \Upsilon_2 \right) + \Omega s \Upsilon_2 \frac{u^2}{2}, \quad (16)$$

where  $\bar{\nu}_b = \int_0^1 T_{jb} dj$  is the average market share of bank  $b$  in the symmetric equilibrium,  $\Upsilon_1 = (\mathbb{E}(T_{jb}(1 - T_{jb}) - \sigma_b^2))$ ,  $\Upsilon_2 = (\sigma_b^2 + \bar{\nu}_b^2)$ , and  $\Omega = \frac{\eta - \alpha + \eta\alpha(1-\phi)}{\phi(\alpha+\eta)}$  are constants.

*Proof: See Appendix*

Proposition (1) shows that for a sufficiently small shock  $u$  to the lending terms of one bank, the response of output depends on three terms. The first term measures the di-

rect effect of the shock up to second order, abstracting from any substitution in financing sources. The drop in output will be proportional to the relevance of the affected bank  $\bar{s}\bar{\nu}_b$ , weighted by the Frisch elasticity of labor supply  $1/\phi$ . When the labor supply is inelastic, the increase in the cost of funds in the aggregate will be compensated for by a fall in the aggregate wage, limiting the fall in output. The second term captures a counteracting force from the ability of the firms in the economy to substitute the affected bank. Importantly,  $\theta$ , the cross-bank elasticity of substitution, helps determine this second term. In a similar way, the third term captures the ability of the economy to avoid using bank credit altogether, which is determined by  $\varphi$ .

Proposition (2), shows that the response of output depends on observables, like the average bank-dependence of the real sector,  $\bar{s}$ , the average market share of the disrupted bank,  $\bar{\nu}_b$ , or the dispersion of the market shares,  $\sigma_b^2$ . It also depends on well-studied parameters like the Frisch elasticity of labor supply (see Chetty et al. (2011)), the elasticity of substitution across goods (see Broda and Weinstein (2006)), or the firm-specific elasticity of labor supply (see Webber (2015)) . The output response also depends on two less-studied parameters: the elasticity of substitution of funding from a given bank  $\theta$ , and the elasticity of substitution of bank-credit  $\varphi$ . In later sections of the paper I discuss the strategy I use to recover these parameters from the cross-sectional evidence and use them to estimate the effects of an aggregate bank disruption.

## 2.2 The Aggregate Effects of Overall Loan Term Disruptions

Now I extend the results in Proposition (1) for a generalized disruption in the loan terms of all the banks. Proposition (2) presents the main result of this section, using Assumption (3).

**Assumption 3.** *Assume the lending terms of all banks are disrupted from  $R$  to  $Re^u$ , for a positive and sufficiently small  $u$ . Keep the self-finance rate equal to  $R$ .*

**Proposition 2.** *The Aggregate Effects of Symmetric Shocks: Under Assumption (3), up to a second order, the fall of output triggered by a symmetric increase in the lending terms of all the banks is given by:*

$$\Delta \log Y \approx -\frac{1}{\phi} \left( \bar{s}u - \varphi \bar{s}(1 - \bar{s}) \frac{u^2}{2} + \Omega s \Upsilon_2 \frac{u^2}{2} \right). \quad (17)$$

*Proof: See Appendix*

Proposition (2) shows that the elasticity of substitution between banks  $\theta$  is irrelevant at the aggregate level after a symmetric shock. However, the elasticity of substitution away from bank lending  $\varphi$  is still important through its second-order effect on aggregate output. Up to a first order approximation, the response of aggregate output is determined by observable and usual parameters.

### 2.3 The Cross-Sectional Elasticity of Bank-Funding Shocks

Under the conditions stated in Assumption (2), Proposition (3) characterizes the determinants of the cross-sectional differences in output with respect to a notional control firm that has zero exposure to the disrupted bank  $b$  ( $T_{cb} = 0$ ).

**Proposition 3.** *Under Assumption (2), up to a second order approximation, the average cross-sectional effect on output of a lending rate hike of bank  $b$  from  $R$  to  $Re^u$ , with respect to the production of a firm with zero direct exposure to bank  $b$   $\bar{Y}_{ct}$ , is given by:*

$$\mathbb{E}(\Delta \log Y_{jt} - \Delta \log Y_{ct}) \approx -\frac{\eta\alpha}{\alpha + \eta} \left( \bar{s}\bar{\nu}_b u + \mathbb{E}(T_{jb}^2) \bar{s}^2 \frac{u}{2} - \theta \bar{s}^2 \frac{u^2}{2} \mathbb{E}(T_{jb}(1 - T_{jb})) - \varphi \bar{s}(1 - \bar{s}) \frac{u^2}{2} \mathbb{E}(T_{jb}^2) \right), \quad (18)$$

where  $\bar{\nu}_b = \int_0^1 T_{jb} dj$  is the average market share of bank  $b$  in the steady state,  $\sigma_b^2 = \text{var}(T_{jb})$  is the variance of market shares of bank  $b$  across firms,  $\bar{s}$  is the credit dependence in the steady state, and  $\log \bar{Y}$ , is the steady state level of output.

*Proof: See Appendix*



From Proposition (3) the effect is larger when the shocked bank is more important ( $\nu_b$  is large), when firms are credit dependent ( $\bar{s}$  is high), when the elasticities of substitution between banks ( $\theta$ ) and away from bank-credit ( $\varphi$ ) are low, and when it is easy to reallocate demand from one firm to another ( $\eta$  and  $\alpha$ ) are high.

## 2.4 The Identification Challenge

Even if we observe the cross-sectional effects on output, we would be missing equations to back out  $\varphi$ , which is the relevant variable for understanding the aggregate effects of a symmetric shock. In particular, many combinations of  $\theta$  and  $\varphi$  can produce the same cross-sectional patterns, a point I will expand on later.

## 3 Identification

In this section I use the static model to illustrate how the patterns in the data identify  $\theta$  and  $\varphi$ , the key parameters of the model. I use the insight in this section to estimate the full model I introduce in the following section. I start by introducing two cross-sectional estimates used in the literature, the elasticity of firm-level output and the elasticity of credit after a bank shock.

### 3.1 The Elasticity of Firm Production

The elasticity of firm production to a disruption in the terms of loans of bank  $b$  can be estimated through the following regression:

$$\Delta \log Y_f = \beta_0 + \beta_{output} T_{jb} + \epsilon_f, \quad (19)$$

where  $\Delta$  is the difference operator between a pre-period, that I will assume to be equal to the symmetric equilibrium of the model, and pos-period, when a shock of size  $u$  that

increases the interest rate of bank  $b$  from  $R$  to  $Re^u$  occurs. The right hand side variable is the pre-existing exposure of firm  $j$  to bank  $b$ , measured by  $T_{jb}$ , which I assume to be exogenous.

$T_{jb}$ , the pre-existing borrowing share of firm  $j$  with bank  $b$  is given by historical reasons as in Huber (2018), or assumed to be endogenous but instrumented as in Chodorow-Reich (2014). The main empirical concern is that banks that are more prone to receiving funding shocks are also more likely to pick bad firms, which would induce a correlation between lending and firm outcomes even in absence of a causal link. The empirical literature has addressed that problem by using an instrumental variables (IV) approach.. The idea is to find variation in the ability of some banks to give out loans that is not correlated with the quality of the firms they lend to. For a discussion of sorting between firms and banks see Chang, Gomez, and Hong (2020)

The elasticity of production with respect to pre-existing exposure is characterized in Proposition (4)

**Proposition 4.** *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on pre-existing exposures, accurate up to a second-order, is given by the following expression*

$$\beta_{output} = -\frac{\eta\alpha}{\alpha + \eta} \bar{s}u \left( 1 + \bar{s}\mathcal{M}\frac{u}{2} - \theta(1 - \mathcal{M})\frac{u}{2} - \varphi(1 - \bar{s})\mathcal{M}\frac{u}{2} \right). \quad (20)$$

For a constant  $\mathcal{M} = \left( \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \right)$

*Proof: See Appendix*

Equation (20) makes clear that up to a second order, as the elasticity of substitution across banks ( $\theta$ ) and the elasticity of substitution away from bank credit ( $\varphi$ ) increase, the firm-level effects on output of a bank disruption become smaller. On top of the frictions in the banking sector, the structure of the goods market ( $\eta$ ), and the structure of labor markets  $\alpha$  determine the cross-sectional effects of the bank disruption. When  $\alpha$  tends to infinity, the cross-sectional effects tend to  $\eta$ . When both  $\alpha$  and  $\eta$  tend to infinity, the

cross-sectional effects diverge, since in this situation all production would take place in the firms with the lowest marginal costs.

### 3.2 The Elasticity of Firm Borrowing

We turn to the effects of bank disruptions on firm bank-credit. We will use the following specification:

$$\Delta \log \text{Loans}_j = \beta_0 + \beta_{\text{credit}} T_{jb} + \epsilon_j, \quad (21)$$

where  $\Delta$  is the difference operator between a pre-period, which I assume to be equal to the symmetric equilibrium of the model, and pos-period, when a shock of size  $u$  that increases the interest rate of bank  $b$  from  $R$  to  $Re^u$  occurs. The independent variable is the pre-existing exposure of firm  $j$  to bank  $b$ , measured by  $T_{jb}$ . Gan (2007), Khwaja and Mian (2008), Schnabl (2012), and Iyer, Peydro, da Rocha Lopes, and Schoar (2013), among others are examples of this approach.

**Proposition 5.** *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on the pre-existing exposure, accurate up to a second order, is given by the following expression*

$$\beta_{\text{credit}} = \frac{1 + \alpha}{\alpha} \beta_{\text{output}} - \varphi(1 - \bar{s})u \left( 1 + \frac{u}{2} \frac{1 - s^2}{1 - s} \mathcal{M} + \varphi \bar{s} \frac{u}{2} \mathcal{M} - \theta \frac{u}{2} (1 - s) (1 - \mathcal{M}) \right) \quad (22)$$

$$\text{For a constant } \mathcal{M} = \left( \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})} \right)$$

*Proof: See Appendix*

Proposition 5 shows that on top of the effect on output times a multiplier (first term), there is a first order effect of the elasticity of substitution of bank credit  $\varphi$  on firm credit. When firms are more elastic in substituting away from bank credit, credit falls by more. The substitutability across banks limits the fall in credit since it limits the size of both terms. The intuition is clear, when firms can more easily move across banks, they find

less necessary to reduce bank credit and suffer smaller output losses, with direct effects on credit demand.

### 3.3 Identification Argument

The elasticity of credit becomes larger (more negative) when  $\varphi$  is larger and when  $\theta$  is smaller. The elasticity of output becomes larger when both  $\varphi$  and  $\theta$  are smaller. Therefore it is possible to back out the values of these two coefficients once we take a stance on the other coefficients that determine the cross-sectional elasticities.

The identification argument is represented in Figure 3. The figure presents two locus of points in the space  $\varphi - \theta$ , which produce a given estimate for the elasticity of credit and production, after taking a stance on the other parameters of the economy.

Start by placing yourself on point  $b_1$ , in the locus of  $\beta_{\text{credit}}$ . Now arbitrarily increase the value of  $\varphi$ . Since a larger  $\varphi$  causes the elasticity to be larger in absolute value, in order to keep the elasticity constant we must move  $\theta$  in a direction that compensates for the change in  $\varphi$ . That is, we need to make  $\theta$  larger, making firms more elastic with respect to a given bank. This argument implies that the locus of points ( $\varphi$  and  $\theta$ ) that keeps the regression coefficient  $\beta_{\text{credit}}$  constant is upward sloping.

Now place yourself on top of point  $a_1$  on the locus of  $\beta_{\text{output}}$ . Once again move to a larger value of  $\varphi$ . When firms are more elastic to substitute bank credit, the elasticity of production becomes smaller in absolute value. In order to keep its value constant, we need firms to be less able to switch from the affected lender, making  $\theta$  smaller. Therefore, the locus of points is downward sloping.

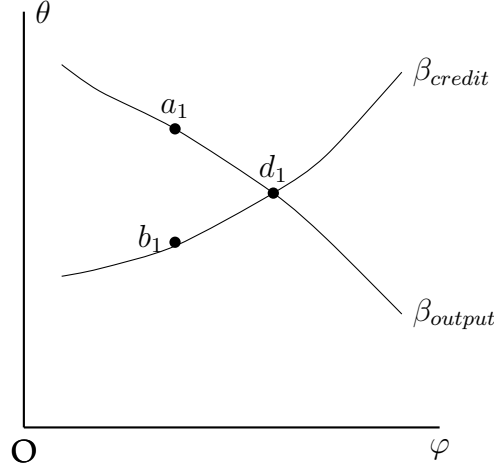


Figure 3: Identification argument for  $\varphi$  and  $\theta$ .

*Note:* The figure plots the loci of points ( $\varphi$  and  $\theta$ ) that achieve a given value for  $\beta_{output}$  and  $\beta_{credit}$  after taking a stance on the other parameters. The intersection of the two loci gives the value of  $\theta$  and  $\varphi$ .

### 3.4 Firm Fixed Effects Estimator

Although I will not use a firm fixed-effect regression to calibrate the model<sup>7</sup>, in this section I will discuss what economic mechanisms are identified by fixed-effect regressions.

The result of this section states that firm fixed-effect regressions provide information about  $\theta$ , the elasticity of substitution of funds across banks, and no information about the elasticity of substitution of bank credit ( $\varphi$ ), the substitutability of goods in the goods market ( $\eta$ ), or the ability to reallocate labor across firms ( $\alpha$ ). The result carries significant economic content. Firm fixed-effects regressions compare firm reallocations of credit demand across banks, abstracting from any economic mechanism that operates across firms ( $\eta$  and  $\alpha$ ), or that does not depend on a firm-bank pair ( $\varphi$ ).

In the previous section I showed that  $\theta$  is irrelevant up to the second order to determine drops in aggregate output after an aggregate disruption of the banking sector, then the fixed-effect regression estimation is, on its own, uninformative about such aggregate experiment. However, firm fixed-effect regressions are still important. By identifying  $\theta$ ,

<sup>7</sup>The reason is that I will use the reported elasticities by Huber (2018), which does not present firm fixed effect regressions.

they can be combined with other cross-sectional regressions to recover  $\phi$ , or studying  $\theta$  is interesting to determine aggregate output fluctuations after an idiosyncratic bank shock. The log of loan sizes is given by

$$\log \text{Loans}_{jb} = \log \mathcal{C} + \frac{(\eta - 1)(\alpha + 1)}{\alpha + \eta} \log z_j - \left( \frac{\eta(\alpha + 1)}{\alpha + \eta} - \varphi \right) \log R_j - \varphi \log R_{jB} + \log \nu_{jb}, \quad (23)$$

for a constant  $\mathcal{C}$  that captures aggregate and firm-level outcomes that are soaked into the constant term. Demeaning this object to compute the within-firm loan variation across banks, and computing a before-after difference, yields an expression for  $\Delta \text{Loans}_{jb} = \Delta \log \nu_{jb} - \Delta \log \bar{\nu}_{jb}$ . The LHS variable in a firm fixed-effect regression is the relative change in the borrowing share of a given bank with respect to the change in this objects in the average bank that a firm uses. Up to a second-order approximation, the firm fixed-effect estimator yields

$$\beta_{\text{fixed effect}} = -\theta u + \theta^2 \frac{u^2}{2} \left( 1 - \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})} \right). \quad (24)$$

Equation (24) makes clear that up to a second order, the fixed-effect estimator only identifies  $\theta$ , the elasticity of substitution of funds across banks.

Just as important, the previous sections of the paper showed that after a shock that affects all banks symmetrically,  $\theta$ , the elasticity of substitution of funding across different banks, is irrelevant in determining aggregate fluctuations up to a second order. Therefore, a firm fixed-effect regression on its own does not provide any information about such experiment. It does provide information with regards to the aggregate effects of a disruption to a subset of banks, although other parameters values, including  $\varphi$ , are needed in order to reach a conclusion on that experiment as well.

### 3.5 Observational Equivalence

Cross-sectional estimates comparing firm effects after an idiosyncratic bank shock suffer from an observational equivalence problem. Equation (20) makes clear that the firm-level output effects of a lending cut depend on two distinct terms. The first one  $\frac{\eta\alpha}{\eta+\alpha}$  captures the structure of labor and goods markets, while the second term  $(1 - \theta\frac{u}{2}\mathcal{M}_1 - \varphi(1 - \bar{s})\frac{u}{2}\mathcal{M}_2)$  captures the structure of financial markets.

The cross-sectional elasticity of output would be indistinguishable for an econometrician in worlds with distinct insubstitutabilities of financial sources. The elasticity could be similar in an economy where goods and labor can move easily across firms ( $\eta$  and  $\alpha$  large) and financial sources are very substitutable ( $\theta$  and  $\varphi$  large), and another world where the reverse happens. A priori, it is not possible to assert insubstitutabilities in finance from a large cross-sectional estimate, nor reject them from small elasticities.

It is only possible to use cross-sectional elasticities to learn about the extent of financial frictions once we pin down the elasticities that govern the ability of the economy to reshuffle demand and input after a shock that affects a subset of the firms. The mechanisms that induce observational equivalence in the model are well-studied objects in economics, so using the data plus the model the observational equivalence problem can be solved, this is one of the main contributions of this paper.

## 4 Full Model

In this section I embed the simple model in a consumption/savings general equilibrium model in order to make the total amount of deposits endogenous, and let banks set lending rates as a response to balance sheet disruptions. The basics of the model are the same as in the simple model, and here I only present the new blocks of the model.

Time is continuous. Space is contained in a  $[0, 1]$  interval.  $N_B$  banks are uniformly spaced in this interval. Firms are distributed uniformly over space. I take as primitive of

the model the closeness of firm  $j$  to bank  $b$ , and denote it by  $T_{jb}$  as in the simple model.  $T_j$ , a vector of size  $N_B \times 1$  specifies the closeness of firm  $j$  with each bank, and given by  $T_{j,b} = \max\{1 - \bar{d} \times d_{j,b}, 0\}$  where  $d_{j,b}$  is the distance between firm  $j$  and bank  $b$ , and  $\bar{d}$  is a constant that determines how the distance between a firm-bank pair affects the ease of creating banking relationships. In the extreme where  $\bar{d} = 0$ , firms are equally likely to borrow from banks regardless of their distance. When  $\bar{d}$  increases, firms only use banks that are close to them.

Each firm is owned by an entrepreneur, with utility function  $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$ . Each entrepreneur solves the following problem:

$$\max_c \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt.$$

They maximize utility subject to the budget constraint:

$$\dot{a}_{it} = r_{it}^d a_{it} + \pi_{i,t}^* - c_{it}$$

That is, entrepreneurs earn interest income at rate  $r_{it}^d$  on their wealth  $a_{it}$ , earn profits  $\pi_{i,t}^*$  and consume  $c_{it}$ . The effective rate of deposits  $r_{it}^d$  is a weighted average of the deposit rates at different banks  $r_{dt} = \sum_k \omega_{kt} r_{dkt}$ . And weights given by  $\omega_{bt} = \frac{R_{bdt}^X}{\sum_k R_{kdt}^X}$ . This functional form for the deposit shares is chosen to be symmetric with the way that firms allocate their loan demand across banks. Profits are given by  $P_{jt}Y_{jt} - w_{jt}L_{jt}R_{jt}$ , prices are given by  $\frac{\eta}{\eta-1}MC_{jt}$ , and marginal costs are given by  $MC_{jt} = \frac{w_{jt}}{z_{jt}}R_{jt}$ .

## 4.1 Banks

Banks compete by setting rates.<sup>8</sup> Banks understand the structure of demand of each firm, but do not internalize the aggregate consequences of their actions. That is, banks take the aggregate wage and aggregate output as given, but they understand that firms can sub-

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<sup>8</sup>In this model I assume that the loan market is cleared using lending rates. In the data, banks offer multi-dimensional contracts that differ in terms, covenants, size of credit lines, on top of variation on prices. Some of this variation has been covered by Payne (2018) and Chodorow-Reich and Falato (2017).



stitute towards other banks, or substitute away from bank credit, and that firm optimal scale is decreasing in its cost of funds. I allow for banks to price-discriminate across firms.

The profits that bank  $b$  gets from its relationship with firm  $j$  are given by

$$\Pi_{jb} = w_j L_j s_j \nu_{jb} (R_b - R_{bd}).$$

I am saving on the notation by eliminating the time subscript. The condition

$$R_{bj} = R_{bd} \frac{\tilde{\theta}_{jb}}{\tilde{\theta}_{jb} - 1}$$

For  $\tilde{\theta}_{jb} = \theta + \nu_{jb}(\varphi - \theta) + \left(\eta^{\frac{1+\alpha}{\alpha+\eta}} - \varphi\right) \nu_{jb} s_j$ , characterizes the optimal pricing of the loans for each bank.

A bank with zero mass ( $\nu \rightarrow 0$ ) faces an elasticity of substitution  $\theta$ , the elasticity at which firms switch banks. A monopolist bank ( $\nu \rightarrow 1$ ) that lends to firms that are fully dependent on bank credit ( $s \rightarrow 1$ ), faces an elasticity of substitution  $\eta^{\frac{1+\alpha}{\alpha+\eta}}$ , the elasticity at which higher costs translate into lower firm scale and correspondingly to lower loan demand. The elasticity is positive since  $\varphi, \eta$ , and  $\theta$  are positive, and  $\nu_{bj}$  and  $s_j$  are between zero and one. Banks charge variable markups. This is an important departure from models with constant elasticities of substitution.

The balance sheet of the bank is given by:<sup>9</sup>

$$\text{Loans}_{bt} = \text{Deposits}_{bt} + \text{Equity}_{bt}, \quad (25)$$

where the loans granted by a bank are the integral of the loans given to each firm in the economy, given by

$$\text{Loans}_{bt} = \int_0^1 \text{Loans}_{jbt} dj = \int_0^1 s_{jt} w_{jt} L_{jt} \nu_{bjt} dj. \quad (26)$$

Similarly, deposits are equal to the integral of the deposits that the bank gets from all

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<sup>9</sup>In the model, I interpret  $\text{Equity}_{bt}$  as another source of funding for the bank that is different than deposits. It could well be thought of as a generic source of funding.

entrepreneurs in the economy

$$\text{Deposits}_{bt} = \int_0^1 \text{Deposits}_{jbt} dj = \int_0^1 \omega_{bt} a_{jt} dj. \quad (27)$$

I assume that  $\text{Equity}_{bt}$  is exogenous, and that banks are owned by agents outside the economy. It is simple to change that assumption on the ownership of the banking sector.

The supply of deposits at a given bank depends positively on its deposit rate, while the demand for loans depends negatively on it, through its negative relationship with the lending rate and the positive relationship between lending and deposit rates. Therefore, after a decrease in the right-hand side of the balance sheet, the bank will respond by increasing the deposit and lending rates accordingly, balancing out its balance sheet again.

The aggregate state vector is  $\mathbb{S} = (\mathbf{Equity}, X)$ , where  $\mathbf{Equity}$  is a  $K \times 1$  vector of the equity of each of the  $K$  banks in the economy, and  $X$  is the distribution of entrepreneurs over their individual state-space  $(z, a, \mathbf{T})$ .

1. Entrepreneur's optimization. Taking  $w(\mathbb{S})$ ,  $R_k(\mathbb{S})$ ,  $R_k^d(\mathbb{S})$  as given, entrepreneurs maximize utility and their firms maximize profits.
2. Household problem. Taking  $w(\mathbb{S})$  as given, households maximize utility
3. Banks problem. Taking  $R_k^d(\mathbb{S})$ , banks set  $R_k(\mathbb{S})$  to maximize profits.
4. Market Clearing.  $w(\mathbb{S})$ ,  $R_k^d(\mathbb{S})$ , are such that labor market clears  $L^s(\mathbb{S}) = \int l(z, a, \mathbf{T}, \mathbb{S}) X(dz, da, d\mathbf{T})$ , and banks' balance sheet holds  $\text{Deposits}_k(\mathbb{S}) + \text{Equity}_k = \text{Loans}_k(\mathbb{S})$

## 4.2 Solution Method

Since there are only a handful of banks, a shock to the financial conditions of a bank will create aggregate disturbances. Therefore, when agents are formulating their policy functions, they need to forecast the behavior of the input prices in the economy—namely,

the wage rate and the deposit rate at each bank. In order to do so, agents need to forecast the behavior of the cross-sectional distribution of entrepreneurs and banks, which is an infinite-dimensional object. I take advantage of methods developed by Ahn, Kaplan, Moll, Winberry, and Wolf (2018). In particular, the solution will be globally accurate with respect to the individual state space, and will be a linear approximation with respect to the aggregate shocks.

## 5 Discussion of Assumptions

I assume that the substitutability across banks, and across forms of finance are different economic objects, so they can take different values. This assumption is taken in line with the corporate finance literature that points that local bank finance is difficult to substitute (Paravisini (2008)). The alternative would be to assume that substitution between banks and between forms of external finance is governed by the same parameter, which would simplify the identification argument.

Second, I assume that the interaction of between firms are limited to competition for demand and inputs. Importantly, I exclude the possibility of input output networks, or agglomeration externalities as in Huber (2018), which would increase the aggregate effects for a given size of a cross-sectional elasticity. I also leave aside the effects of bank shocks on firm entry and exit. However the model captures in a stylized way that the interaction of firms for demand and inputs, and the interaction of banks competing for deposits and allocating loans are the first order factors in aggregating up the cross-sectional elasticities in this literature.

Third, I make the problem of the banks simple. The main exercise of this manuscript is to come close to the quasi-experimental evidence produced in corporate finance and empirical macroeconomics, which tries to find conditions where bank shocks are as good as random. I label a funding shock as an “equity” shock, but that labeling is not conse-

quential for the exercise. What is important is that access to additional external finance for the bank, which in the model takes the form of deposits, comes with a cost, which creates variation in the lending decisions of the banks. To focus on the aggregation exercise, I leave aside many interesting issues in the banking literature like bank default, deposit runs, and bank regulation . See for instance Egan et al. (2017) and Corbae and D’erasmo (2020).

Finally, throughout the model I assume that workers are hand to mouth. Results for the case where workers face uninsurable idiosyncratic labor income risk and hold deposits are similar to those of the main exercise.

## 6 Estimation

The parametrization of the model takes two steps. The majority of the parameters are calibrated. Most of these parameters are well studied and I fix them at standard values. I use microdata to calibrate a subset of parameters that are not widely used in macroeconomic models but for which we have good evidence. Then, the key parameters of the model,  $\theta$  and  $\varphi$ , are estimated to target the patterns observed in cross-sectional studies of the bank lending channel that were introduced in the previous sections.

I offer a preview of the results of this section. In the benchmark calibration, the values of  $\theta$  and  $\varphi$  I estimate are low, implying low ability to adjust to bank shocks. As an illustration, Under an alternative specification of the labor market, (high  $\alpha$ ), the values of  $\theta$  and  $\varphi$  that are consistent with the cross-sectional elasticities are large.

On top of evidence from labor economics that advocates for an economy with a low  $\alpha$ , I use an additional cross-sectional moment from the banking literature as a sanity check. I extend the model to have two symmetric regions. In models with flexible labor markets within the region ( $\alpha$  is high), the indirect effects of bank shocks are positive. This means that a firm without exposure to a shocked bank in a region where the average exposure

to the troubled bank is high will outperform an unexposed firm in a region where the average exposure to the troubled bank is low. This prediction is at odds with the evidence, as Huber (2018) has documented. Only when there are substantial rigidities in local labor markets, the model is consistent with the sign of the indirect effect. Therefore, the model rejects the limit of high  $\alpha$ , consistent with the micro evidence from labor economics.

## 6.1 Calibration of Standard Parameters

Table 1 lists the parameters that I fix throughout the estimation. The intertemporal elasticity of substitution is set to a standard value of  $1/2$ . The Frisch elasticity of labor supply is 0.75, as suggested by Chetty et al. (2011). This value is significantly lower than what is used in most macro models. A highly elastic labor supply will increase the aggregate effects of a bank shock, by making it more difficult for wages to go down after a negative shock, increasing the elasticity of output to bank funding shocks.

I set  $\eta$ , the elasticity of substitution across goods equal to 4, within the range of estimates in Broda and Weinstein (2006). I set the discount rate  $\rho$  equal to 0.03 per year as in Itskhoki and Moll (2019). I set the persistence of the shock  $\rho_E$  at 0.95, consistent with the persistence used by Gertler and Kiyotaki (2015). I set the parameters of the productivity Poisson process to target the volatility of 0.056 and a persistence of 0.9 as chosen by Winberry (2018).

I set the number of banks in the economy  $N_B$  equal to 10 equal-sized banks. This number replicates the across-Metropolitan Statistical Area median Herfindahl-Hirschmann Index (HHI) of 0.11 coming from data from the Community Reinvestment Act (CRA) data that report business loans for 2006 in the U.S. As an alternative, using call reports data at the national level, the HHI of commercial and industrial loans (C&I) for 2006 is 0.05, implying 20 equal-sized banks. However, this number underestimates the degree of concentration in C&L loans, since firms prefer banks that are closer to them (see Nguyen (2019)). The parameter  $d$ , controls how many banking relationships each firm will have.

Parameter	Description	Value
$1/\gamma$	Intertemporal Elasticity of Substitution	1/2
$\rho$	Discount Rate	0.03
$\eta$	Elasticity of Substitution - Goods Market	4
$1/\phi$	Frisch Elasticity of Labor Supply	0.75
$z$	Two-State Markov Process	0.9 - 1.1
$\lambda$	Intensity of Poisson productivity shock	1/3
$\mathcal{B}$	Number of Banks in the Economy	10
$\rho_E$	Persistence of Equity Shock	0.95
$d$	Distance Coefficient	3 bank relationships
$\chi$	Elasticity of deposits to deposit rates	5

Table 1: Externally calibrated parameters

*Note:* The table presents the parameters of the model that I calibrate externally.

I fix  $d$  so that firms have three banking relationships on average, as reported by Huber (2018). I set  $\chi$ , the parameter that governs how much deposits flow out of a bank with lower deposit rates to 5, matching the semi-elasticity reported by Drechsler et al. (2017).

## 6.2 Estimation of Key Parameters

Using the relative effects in the data as target moments to estimate the full model, I structurally estimate the parameters values for  $\theta$ , the elasticity of substitution of firms across banks, and  $\varphi$ , the elasticity at which firms switch away from bank credit. The idea behind the identification is the same as exposed in the identification section, with the difference that the full model gives dynamics to simulate a simulated panel dataset, and that the model is globally accurate with respect to individual policy functions, which are more accurate than the second-order Taylor expansions we introduced before. Specifically, I simulate a panel of firms over time after a bank funding shock. With the simulated data, I run a regression analysis that replicates the cross-sectional analysis, after collapsing a set of periods before and after the shock into two bins, the pre-period and the post-period. Table (2) specifies the microeconomic targets of the calibration. For a detailed discussion of the regressions, please refer to the identification section.

Moment	Source	Value
Bank Credit Elasticity	Huber (2018)	-0.166
Output Elasticity	Huber (2018)	-0.044

Table 2: Microeconomic Targets

*Note:* This table shows the two statistics I will target with  $\theta$  and  $\varphi$ . These are the values of cross-sectional elasticities on firm employment and credit introduced in the identification section. The targets come from Huber (2018).

### 6.3 Estimation of the Equity Shock

Complete this XXX.

### 6.4 Sensitivity of Cross-Sectional Elasticities to Structural Parameters

Before showing the estimation of the model, I illustrate the effect of  $\theta$  and  $\varphi$  in determining the cross-sectional moments and the effect of different values of  $\alpha$  in shifting the effect of these two parameters.

Figures 4 and 5 show the effect of changing  $\theta$  for two values of  $\alpha$ , on the cross-sectional moments of credit and production, respectively, while keeping the rest of the parameters in the model fixed. As is intuitive from previous sections, a higher value of  $\theta$ , by increasing the flexibility of firms on switching across banks, decreases the cross-sectional elasticities of both output and credit. In the limit, where  $\theta \rightarrow \infty$ , the elasticities tend to zero. Figures 4 and 5 make an additional point. Because the elasticity is larger in absolute value when labor markets do not have any frictions, the value of  $\theta$  that is consistent with a given elasticity is significantly larger when  $\alpha \rightarrow \infty$  than when  $\alpha$  is low. Therefore, in order to match the same cross-sectional elasticities,  $\theta$  will be lower in an economy with labor and demand insubstitutabilities.

Figures 6 and 7 perform the same exercise for the elasticity at which firms move away from bank credit ( $\varphi$ ). These figures show that the identification argument holds beyond the second order approximation we did in the simple model. When  $\varphi$  increases the output effects of the shock are smaller, but the credit effects of the same shock are larger.

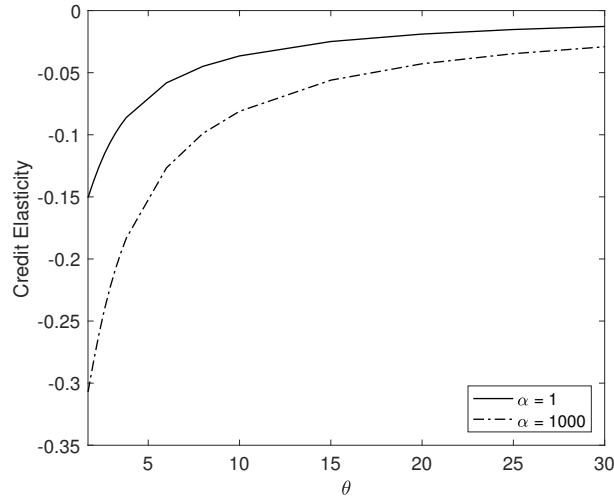


Figure 4: Sensitivity of the cross-sectional elasticity of credit on  $\theta$ .

*Note:* This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of  $\theta$ , the elasticity of substitution of funding across banks. I conduct this exercise for two different values of  $\alpha$ : first for a market with  $\alpha \rightarrow \infty$ , and second, for a low level of  $\alpha$  when there are substantial difficulties in moving labor across firms.

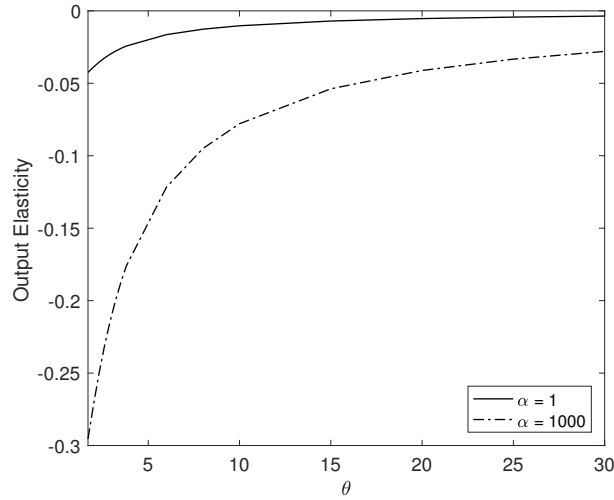


Figure 5: Sensitivity of the cross-sectional elasticity of output on  $\theta$ .

*Note:* This figure shows the cross-sectional elasticity of output in response to a bank shock for different values of  $\theta$ , the elasticity of substitution of funding across banks. I conduct this exercise for two different values of  $\alpha$ : first for a market with  $\alpha \rightarrow \infty$ , and second, for a low level of  $\alpha$  when there are substantial difficulties in moving labor across firms.



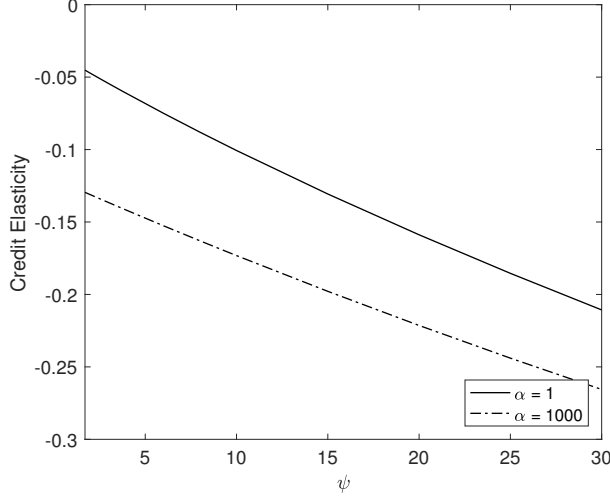


Figure 6: Sensitivity of the cross-sectional elasticity of credit on  $\varphi$

*Note:* This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of  $\varphi$ , the elasticity of substitution of bank credit. I conduct this exercise for two different values of  $\alpha$ : first for a market with  $\alpha \rightarrow \infty$ , and second, for a low level of  $\alpha$  when there are substantial difficulties in moving labor across firms.

With respect to  $\alpha$ , Figure (7) shows that for frictionless labor markets, the value of  $\varphi$  that is consistent with a given elasticity is higher than for markets with frictions. The intuition for this result is the same as for the results that involved  $\theta$ . Under a frictionless labor market, the cross-sectional effects are larger since it is easier to move labor across firms. In the case of Figure (6), when  $\alpha$  is larger, which increases the losses of a given shock, firms move away from credit by more, explaining why the schedule of  $\alpha = 1000$  is below from the schedule for  $\alpha = 1$ .

## 7 Estimated Parameters

In this section I report the combination of  $\theta$  and  $\varphi$  that match the values of the observed moments as reported in Table 2. I report the values that fit the cross-sectional moments in models where  $\alpha = 1$  and  $\alpha \rightarrow \infty$ , with the purpose of showing that the estimated structural parameters are vastly different depending on the assumed structure of the labor market.

The estimated parameters in Table (3) led me to reject that firms and banks operate in

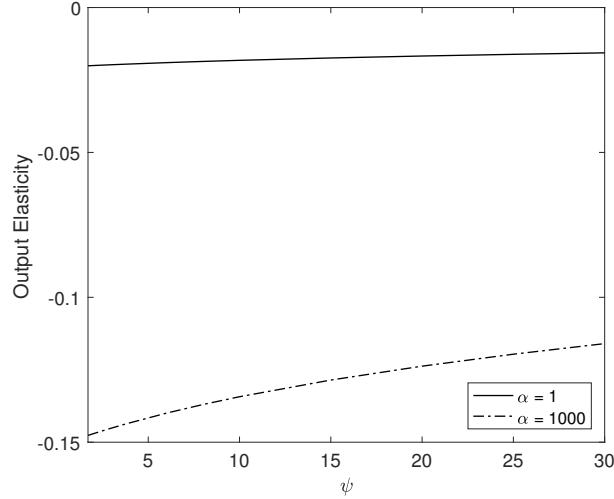


Figure 7: Sensitivity of the cross-sectional elasticity of output on  $\varphi$

*Note:* This figure shows the cross-sectional elasticity of output in response to a bank shock for different values of  $\varphi$ , the elasticity of substitution of bank credit. I conduct this exercise for two different values of  $\alpha$ : first for a market with  $\alpha \rightarrow \infty$ , and second, for a low level of  $\alpha$  when there are substantial difficulties in moving labor across firms.

Parameter	Description	Value ( $\alpha = 1$ )	Value ( $\alpha = 1000$ )
$\theta$	Substituability Across Banks	1.5	6.5
$\varphi$	Inverse credit Dependence	4.5	20

Table 3: Estimated Elasticities of Substitution

*Note:* This Table shows the values of  $\theta$  and  $\varphi$  that target the cross-sectional elasticities shown in Table 2. I estimate  $\theta$  and  $\varphi$  for two values of  $\alpha$ . A benchmark case when  $\alpha = 1$  and another when  $\alpha \rightarrow \infty$ , which reflects an economy where there is perfect mobility of labor across firms.

markets of perfect substitutability, which is the limit of  $\theta \rightarrow \infty$  and  $\varphi \rightarrow \infty$ . The numbers in the table alone do not tell us quantitatively, how important are deviations from perfect substitutability, an answer that I provide in the next section.

Table (3) makes clear the importance of the structure of the labor market. Under elastic labor markets, the parameters are larger, implying that firms are more flexible in reacting to a bank shock. Therefore the effects of bank shocks will be lower.

We have shown how  $\alpha$ , the parameter that governs the extent of frictions in the labor market, is important in this model. The reason is that the extent of real rigidities in the model change the extent to which demand and inputs can be reallocated across firms. When there are substantial frictions in reallocating labor across firms, the model requires substantial frictions in banking as well, in order to match the cross-sectional moments. On the other side, with frictionless labor markets, the banking sector must be relatively flexible, or the model would predict cross-sectional elasticities that are larger than the ones observed in the data. The question becomes how to distinguish across values of  $\alpha$ . I use two sources of evidence: direct evidence on the value of  $\alpha$ , and indirect evidence showing that additional cross-sectional patterns in the banking sector reject the case of labor markets with low frictions.

In particular, Webber (2015) document an inelastic firm-specific labor supply. This evidence has already been used in the literature by Chodorow-Reich (2014), and I show that in a more flexible model with flexible patterns of substitution of firm funding, the extent of these frictions is still important. I also use an additional cross-sectional moment, the *indirect effects* of bank lending cuts, to distinguish across models. The indirect effects measure how a firm without direct exposure to the shocked bank that operates in a region where other firms are highly exposed behaves with respect to another firm without direct exposure to the troubled bank that operates in a region where firms are not highly exposed to the troubled bank. Huber (2018) reports that the indirect effects of bank-lending cuts are negative. This means that unexposed firms in exposed regions underperform

unexposed firms in unexposed regions.

I extend the model to illustrate the behavior of the indirect effects. Specifically, I extend the model to have 2 symmetric regions. The regions are segmented in the markets for goods and labor. That is, each firm produces non-tradeable goods, and people cannot move across regions. However, there is partial financial integration. Lending relationships are determined by distance, regardless of geographical barriers. Therefore, firms may borrow from banks in their home or a foreign region, but must sell their products and hire their workers in the local region. As before, the extent to which workers can move across firms *within the same region* is given by the parameter  $\alpha$ :

$$\Delta \log Y_{jr} = \beta_0 + \beta_1 \nu_{jr,pre} + \beta_2 \nu_{jr,pre}^- + \epsilon_{jr}. \quad (28)$$

Equation (28) presents the regression we will run to get the reduced-form indirect effects. The dependent variable is the log change of an outcome of interest (in this case output) of firm  $j$  located in region  $r$ , and the right-hand-side variables are the pre-existing lending relationship of the same firm and the average exposure of the firms in region  $r$ .  $\beta_2$  is the coefficient of interest; it captures the change in outcomes of a firm with  $\nu_{jr,pre} = 0$  in a region where the average exposure is complete  $\nu_{jr,pre}^- = 1$ , with respect to a firm with zero direct exposure  $\nu_{j-r,pre} = 0$  in a region  $-r$  where the average exposure is also zero  $\nu_{jr,pre}^- = 0$ .

To give a clear sense of the effect of  $\alpha$  in the model, I show the effect of different values of this parameter on the three cross-sectional patterns I have documented so far: the elasticity of credit, the elasticity of output, and the indirect effects. In order to provide a clean intuition, I fix all the other values of the parameters at arbitrary values, including  $\theta$  and  $\varphi$ . This approach is in contrast to the previous results where I estimated  $\varphi$  and  $\theta$  for different values of  $\alpha$ .

Figures (8) and (9) illustrate an argument that is familiar by now. When labor markets exhibit less frictions, the direct cross-sectional effects increase in absolute value. This

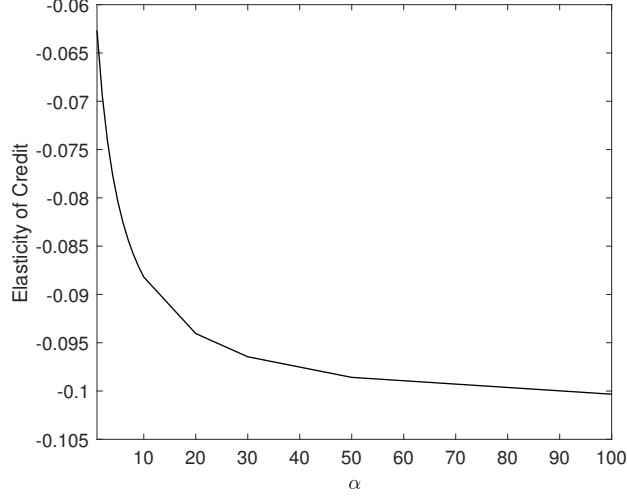


Figure 8: Sensitivity of the cross-sectional effects on credit of an idiosyncratic bank shock to  $\alpha$

*Note:* This Figure shows the cross-sectional effect on credit to a bank shock for different values of  $\alpha$ , the extent of frictions in the labor market. All the other parameters are fixed in their calibrated values, except  $\theta$  and  $\varphi$  which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

happens because the wedge between marginal costs between firms with and without exposure to the shock increases. As a consequence, the wedge between prices, production, and credit demand increases as well.

Figure (10) plots the indirect effects of the lending shock for different values of  $\alpha$ . The figure makes clear that as labor markets become more efficient, the indirect effects of a lending shock become more positive. That is, an unexposed firm in an exposed region experiences a outperforms an unexposed firm in an exposed region. On the contrary, Huber (2018) reports that firms in exposed regions underperform unexposed firms in exposed regions. Although the confidence intervals on the indirect effects reported by Huber (2018) are wide, they reject positive values of the indirect effects, which means that the model rejects values of  $\alpha$  greater than 1.

The insight that the model rejects perfectly competitive labor markets by using the indirect effects is key in the estimation of the aggregate effects of bank shocks. As Figure (10) shows, only values of  $\alpha < 1$  can rationalize negative indirect effects. Therefore, we can reject the limit of frictionless labor markets, and with it, the small elasticities of output

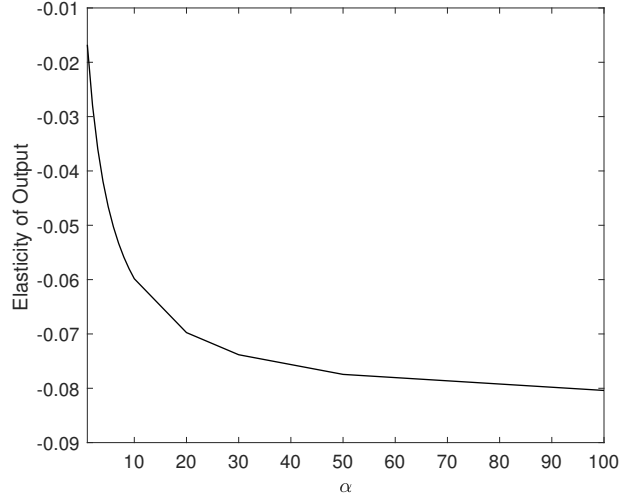


Figure 9: Sensitivity of the cross-sectional effects on output of an idiosyncratic bank shock to  $\alpha$

*Note:* This Figure shows the cross-sectional effect on output to a bank shock for different values of  $\alpha$ , the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except  $\theta$  and  $\varphi$  which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

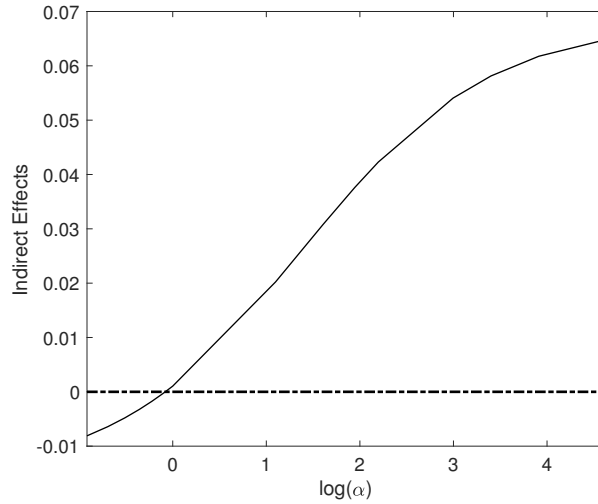


Figure 10: Sensitivity of the indirect effects on credit of an idiosyncratic bank shock to  $\alpha$ .

*Note:* This figure shows the indirect effects of a bank shock for different values of  $\alpha$ , the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except  $\theta$  and  $\varphi$ , which are fixed at an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

to lending they entail.

## 8 Discussion

### 8.1 The Aggregate Effects of Bank Supply Shocks

In this section I analyze the aggregate effects of a cut in the supply of bank lending. In particular I compute the ratio between the integral of the discounted value of aggregate output drops over the integral of the discounted value of the funding shock. Formally, I compute an elasticity  $\varepsilon^M$  as follows:

$$\varepsilon^M = \frac{\int_0^T e^{-\rho t} (\log(Y_t) - \log(\bar{Y})) dt}{\int_0^T e^{-\rho t} \log(\text{Lending}_t) - \log(\bar{\text{Lending}}) dt}. \quad (29)$$

The reason to compute the elasticity of output to lending in this way is that output may exhibit different persistence than total lending, and that the shock that is feeding the economy is persistent, inducing additional responses in output and lending beyond the response on impact. Note as well that the elasticity is computed with respect to lending, not with respect to the shock. There are two reasons for this. First, the policy-relevant variable is the reduced ability of banks to make loans—or to put it another way, the drop in the right-hand-side of the balance sheet of the banking sector. Second, this definition admits comparisons with back-of-the-envelope aggregations that cross-sectional studies make by abstracting from general equilibrium effects.

$\varepsilon^M$  should be interpreted as the elasticity of output to lending caused by a shock in the supply of bank lending. It is the macroeconomic equivalent of an instrumental variables (IV) specification. In an IV, we compute regressions between two endogenous variables, and find an instrument that affects the right-hand-side variable (lending in this case), and that only affects the dependent variable (aggregate output), through its effect on lending.<sup>10</sup>

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<sup>10</sup>Computing an elasticity between two endogenous variables in macroeconomics is commonplace. The

Calibration	$\alpha = 1$	$\alpha = 1000$
Benchmark (%)	19.63	6.73

Table 4: Elasticity of Aggregate Output to Aggregate Bank Lending

*Note:* This table shows the elasticity of output to lending to bank lending. Each column shows the elasticity of output to bank lending for two assumptions of the labor market. One where there are meaningful frictions in the labor market ( $\alpha = 1$ ), and for a case where labor markets are frictionless.

The result of this section is an estimation of this elasticity, and I will show the sensitivity of the elasticity for both experiments with respect to the key parameters of the model. As before, we will consider results for two extreme values of  $\alpha$ , the extent of rigidities in the labor market.

## 8.2 The Aggregate Effects of a Symmetric Bank Shock

We start by performing an experiment in which every bank in the economy is shocked at the same time. This experiment is interesting for several reasons. One, this type of shock captures the attention of macroeconomists and policy experts. Second, it speaks to situations without meaningful heterogeneous exposure to the shock, where using the cross-section to estimate effects is implausible. However, we will inquire how the knowledge of the structural parameters we gained from the cross-sectional estimates extrapolates to an aggregate shock.

Figures (11) and (12) show the effects of the key parameters,  $\varphi$  and  $\theta$ , in determining the output effects of an idiosyncratic shock. The x-axis of these figures is the value of one parameter, and the y-axis is the elasticity of aggregate output to aggregate lending after an aggregate bank shock. The solid line shows the preferred case when  $\alpha = 1$ , and the dashed line shows the case of frictionless labor markets, when  $\alpha \rightarrow \infty$ . The marker in each line shows the estimated value of the parameter for each case.

Figure (11) shows that higher values of  $\varphi$ , which decrease the extent of financial frictions, diminishes the elasticity of output to lending. Under frictionless labor markets, the

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Phillips Curve slope for instance is the elasticity of inflation to unemployment caused by a demand shock. Interest rate parities relates exchange rates to interest rate differentials.



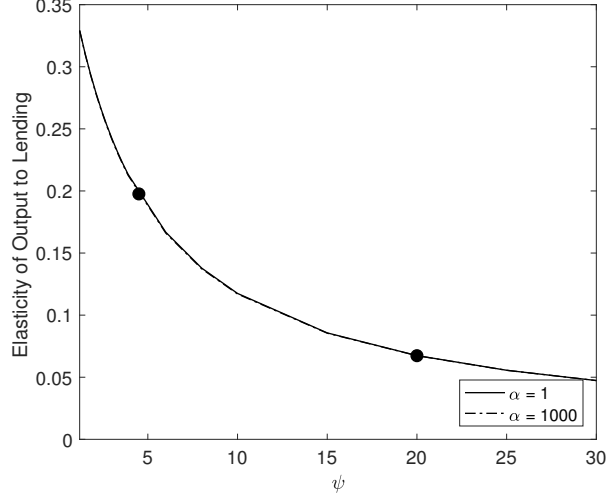


Figure 11: Sensitivity of the aggregate effects of an aggregate bank shock to  $\varphi$

*Note:* This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of  $\varphi$ , the elasticity of credit dependence. We perform this exercise for two different values of  $\alpha$ . First for a frictionless labor market, where  $\alpha \rightarrow \infty$ . And second, for a low level of  $\alpha$  when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for  $\varphi$ . The dot on each line represents the estimated value for  $\varphi$  and the correspondent output drop.

estimated parameter of 20, implies that the elasticity of output to lending is one third the elasticity estimated when there are meaningful frictions in the labor market. The solid and dashed line are over the other for two indicating that other than  $\varphi$ , no other parameters that differ across the two parametrizations of the model  $\alpha$  or  $\theta$  change the size of the elasticity.

On the other side, Figure (12) shows that  $\theta$  is not quantitatively relevant for determining the aggregate elasticity since the lines are flat around the estimated values. This is true even when  $\theta$  is relevant at determining the cross-sectional responses, as shown in previous sections. This result indicates that irrespective of the value of  $\theta$ , the response of output to lending is the same. It does not mean that  $\theta$  is irrelevant in the aggregate. To think about this issue it is useful to remember that the elasticity of output to lending is equal to the elasticity of output to the shock, divided by the elasticity of lending to the shock. The flatness of the elasticity of output to lending indicates that the behavior of lending follows the same pattern.

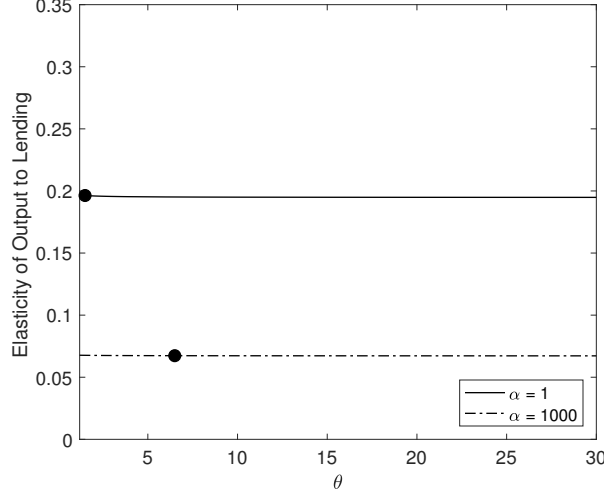


Figure 12: Sensitivity of the aggregate effects of an aggregate bank shock to  $\theta$

*Note:* This Figure shows the aggregate output drop to a bank shock for different values of  $\theta$ , the substitutability of funds across banks. We perform this exercise for two different values of  $\alpha$ . First for a frictionless labor market, where  $\alpha \rightarrow \infty$ . And second, for a low level of  $\alpha$  when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for  $\theta$ . The dot on each line represents the estimated value for  $\varphi$  and the correspondent output drop.

### 8.3 The Aggregate Effects of a Granular Bank Shock

So far, I presented results about the effects on aggregate output of a cut in the supply of bank lending of the whole banking sector, a truly aggregate shocks. However, granular bank lending cuts have aggregate consequences in the model. The reason is that banks in the model are large entities. In this section I illustrate the macroeconomic effects of an idiosyncratic bank shock. I measure the elasticity of aggregate output to the cut in the supply of bank lending of one entity with the following elasticity:

$$\varepsilon^{M,b} = \frac{\int_0^T e^{-\rho t} (\log(Y_t) - \log(\bar{Y})) dt}{\int_0^T e^{-\rho t} \log(\text{Lending}_{bt}) - \log(\bar{\text{Lending}}_b) dt}. \quad (30)$$

Where  $\varepsilon^{M,b}$  is the macro elasticity of output after a cut in lending of bank  $b$ . The interpretation of the elasticity is the same as before. It is the macroeconomic equivalent of an instrumental variable regression, where after taking a stance in a source of variation, we compare the effect of that shock on two exogenous variables.

The main result of this section is that opposed to the case of a truly aggregate shock,

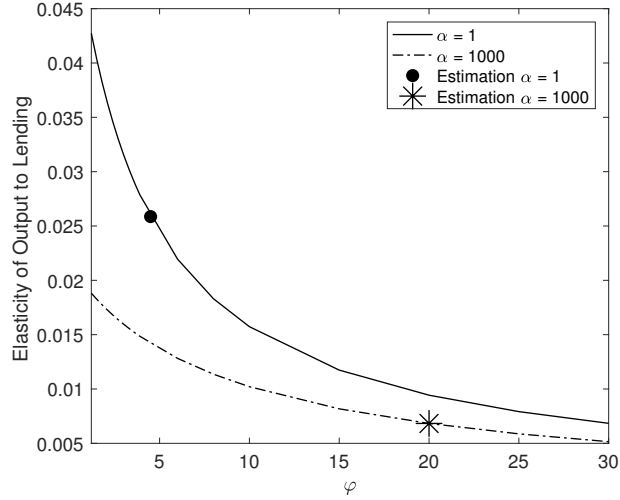


Figure 13: Sensitivity of the aggregate effects of an idiosyncratic bank shock to  $\varphi$

*Note:* This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of  $\varphi$ , the elasticity of credit dependence. We perform this exercise for two different values of  $\alpha$ . First for a frictionless labor market, where  $\alpha \rightarrow \infty$ . And second, for a low level of  $\alpha$  when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for  $\varphi$ . The dot on each line represents the estimated value for  $\varphi$  and the correspondent output drop.

in this case,  $\theta$  the elasticity of substitution of funds across different banks is important in determining aggregate outcomes. The economic intuition behind this result is clear. When one bank suffers a given shock that induces the bank to offer less attractive loan terms to its customers, the elasticity at which firms switch away from the affected bank dictates their change in marginal costs and their output as a consequence. This result is the numerical equivalent of the qualitative argument presented in the theoretical sections of the paper, that shows that when one bank is disrupted, both  $\theta$  and  $\varphi$  are important in determining the aggregate response of output.

Figure (13) shows on the x axis the elasticity of substitution away from bank credit, and on the y axis, the elasticity of aggregate output to idiosyncratic bank lending. Here, I estimate an elasticity of 0.025, which means that if the shocked bank (that had a bank share of 10 percent) cuts its lending by 1 percent, then aggregate output will fall by 0.025 percent. The figure also shows that when  $\alpha \rightarrow \infty$ , the case of perfect labor mobility, this elasticity would be roughly 0.007.

Figure (14) shows on the x axis the elasticity of substitution across banks, and on the

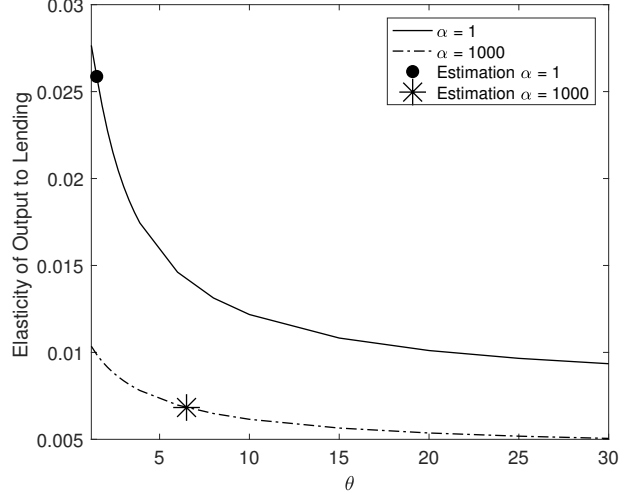


Figure 14: Sensitivity of the aggregate effects of an idiosyncratic bank shock to  $\theta$

*Note:* This Figure shows the aggregate output drop to a bank shock for different values of  $\theta$ , the substitutability of funds across banks. We perform this exercise for two different values of  $\alpha$ . First for a frictionless labor market, where  $\alpha \rightarrow \infty$ . And second, for a low level of  $\alpha$  when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for  $\theta$ . The dot on each line represents the estimated value for  $\varphi$  and the correspondent output drop.

y axis, the elasticity of aggregate output to idiosyncratic bank lending. This figure makes clear that  $\theta$ , the elasticity of substitution across banks, is important in determining the aggregate response of aggregate output to an idiosyncratic bank shock.

The fact that the elasticity is lower is no surprise, as illustrated in the theoretical section of the paper, the effect of a disruption of one bank is weighted by its market share in the pre-period. What is worth emphasizing is that the elasticity of substitution of funding across banks is now relevant to determine aggregate fluctuations. The estimation of the model suggests that a 10 percent drop in lending of a bank with 10 percent market share would generate a drop in aggregate activity of 0.25 percent.

## 8.4 Comparing General to Partial Equilibrium

An important use of the parametrized model is to compare the estimated aggregate bank-lending channel to the alternative measure when general equilibrium effects are ignored. These aggregations are important because after estimating a result in the cross-section us-

ing micro data and regression analysis, empirical researchers want to assess the potential of their findings to have aggregate implications.

Back-of-the-envelope (partial equilibrium) aggregations measure the difference in any given firm outcome between each firm in the economy with respect to the least exposed firm to the shock, a control firm which we denote with  $c$ . In the model we can present an intertemporal version of the partial equilibrium aggregation in present value given by the following expression

$$\varepsilon^{cs} = \frac{\int_0^T e^{-\rho t} \int_0^1 (\log(Y_{jt}) - \log(Y_{ct})) dj dt}{\int_0^T e^{-\rho t} \int_0^1 \log(\text{Borrowing}_{jt}) - \log(\text{Borrowing}_{ct}) dj dt}. \quad (31)$$

To compare the general and partial equilibrium aggregations, I simulate an experiment in which I shock only one bank. The parametrization of the model indicates that the partial equilibrium aggregation ( $\varepsilon^{cs}$ ) is 10 percent higher than the general equilibrium response ( $\varepsilon^M$ ). This message is important. The preferred estimation of the model, that is consistent with many patterns documented over the years in the corporate finance literature, indicates that general equilibrium forces of the model do *not* cause the micro patterns to vanish in the aggregate.

However this result does not need to hold, and it depends on the parameters we have estimated. For instance, under an alternative model with frictionless labor market frictions, the partial equilibrium aggregation is only one fifth of the general equilibrium effect. Meaning that extrapolating from cross-sectional estimates in such a world would lead researchers to overestimate the relevance of the firm credit channel by a factor of five. However, such a world with frictionless labor markets is at odds with the evidence.

Figure (15) shows how the extent of financial frictions in the model, the substitution from bank credit ( $\varphi$ ), and the  $\theta$ , change the ratio between the general equilibrium and the partial equilibrium elasticities for two parametrizations of the labor market. In particular, it shows that the General Equilibrium aggregation can be higher or lower than the partial equilibrium one as  $\theta$  and  $\varphi$  change. It also shows that general equilibrium effects

are stronger when labor markets work better, as illustrated in the theoretical sections of the paper. It shows that the ratio between general equilibrium and partial equilibrium elasticities is more or less stable, and higher for a model with input market frictions.

However, although Figure (15) presents important information with respect to the output effects of a given lending drop, it does not answer the question of whether back-of-the-envelope aggregations over or underestimate drops in output. The reason is that for the same shock, the aggregate and the cross-sectional drop in *lending* are different. Specifically, Figure (15) shows that for each 1 percent of a lending drop caused by the shock, output reacts by with a given elasticity. However the two aggregations differ in the percent change in lending they exploit. The general equilibrium aggregation exploits the drop in aggregate lending, while the partial equilibrium one exploits the differential change in lending across banks.

Figure (16) shows the ratio of the output aggregations, which means the ratio of the numerators of  $\varepsilon^M$  and  $\varepsilon^{cs}$ . The figure makes several points. First, it shows that across the parameter space, in principle the general equilibrium effects on output can be larger, similar, or smaller than is implied by partial equilibrium estimates. However, the estimation of the model imposes restrictions on the size of the difference. In my benchmark estimation the output drops in GE are around 70 percent those implied in PE.. In an alternative calibration with perfect mobility in labor the ratio would be around 1/6. That is, labor market immobility elevates the ratio of the output effects in GE to PE aggregations by a factor of 4.

## 9 Conclusion

The aggregate effects of cuts in the supply of bank lending are difficult to measure using aggregate time-series because bank funding disruptions coincide with other shocks that affect loan demand, and because banks are sensitive to drops in economic conditions

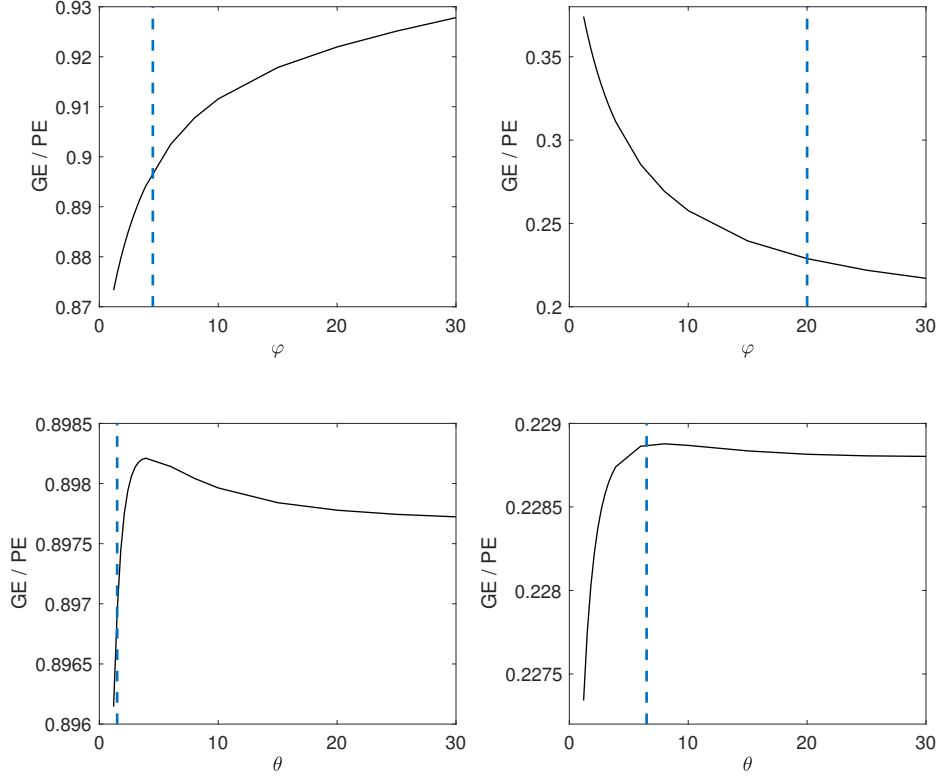


Figure 15: Ratio of the aggregate elasticity to back-of-the-envelope aggregations

*Note:* This figure shows four panels. The left column shows figures when there are significant frictions in the labor market  $\alpha = 1$ . The right column shows the case when  $\alpha \rightarrow \infty$ . The top row shows results for the elasticity of substitution away from bank credit  $\varphi$ , while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank,  $\theta$ . Each panel shows the ratio between the elasticity of aggregate output to aggregate bank lending ( $\epsilon^M$ ), to the back-of-the-envelope aggregation  $\epsilon^{cs}$ . The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.

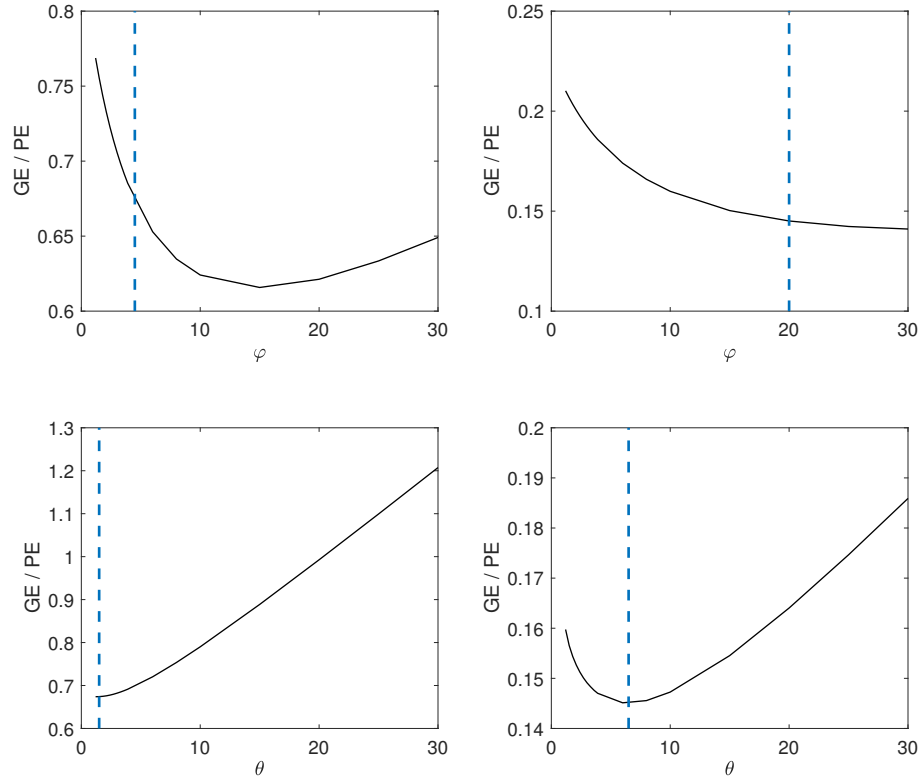


Figure 16: Ratio of the aggregate output drop with respect to back-of-the-envelope aggregations

*Note:* This figure shows four panels. The left column shows figures when there are significant frictions in the labor market  $\alpha = 1$ . The right column shows the case when  $\alpha \rightarrow \infty$ . The top row shows results for the elasticity of substitution away from bank credit  $\varphi$ , while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank,  $\theta$ . Each panel shows the drop of aggregate output to the drop in output inferred from a back-of-the-envelope-aggregation. The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.



creating reverse causality concerns.

Using direct and indirect evidence on the cost of reallocating inputs across firms, and on the relative effects of bank shocks on firm outcomes and credit, I conclude that the aggregate consequences of bank lending cuts are large. When lending drops by 1 percent due to a disruption in bank funding, aggregate output is reduced by 0.2 percent.

This elasticity depends on the extent of bank dependence, and this paper uses cross-sectional evidence to recover this elasticity. Although the ease with which firms can borrow from different banks is relevant in the cross-section, it is not quantitatively relevant in determining the aggregate effect of an aggregate bank shock. Taking a stance on the frictions needed to reallocate inputs and demand across firms is important, even under the experiment of an aggregate shock where all firms are shocked symmetrically. This happens because, in order to target the same cross-sectional moments, frictionless input and demand markets require banking frictions to be milder than in an economy with substantive frictions in reallocating inputs and demand.

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# Appendix for Online Publication

## A Proofs Section 2

### A.1 Derivation of Aggregate Output in the simple model

We start with the expression of firm-level labor demand. I save on the time subscript for brevity.

$$L_j = \left( \frac{\eta}{\eta - 1} \right)^{-\eta} Y z_j^{\eta-1} w_j^{-\eta} R_j^{-\eta}$$

The firm takes as given the labor supply curve  $w_j = w \left( \frac{L_j}{L} \right)^{\frac{1}{\alpha}}$ . Plugging this relationship into the labor demand equation, we get the following:

$$L_j = \left( \frac{\eta}{\eta - 1} \right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}$$

By elevating to the power  $\frac{\alpha+1}{\alpha}$ , integrating over firms, and elevating to the power  $\frac{\alpha}{\alpha+1}$ , we get an expression for aggregate labor:

$$L = \left( \frac{\eta}{\eta - 1} \right)^{-\eta} Y w^{-\eta} \mathbb{E} \left( z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} \right)^{\frac{\alpha+\eta}{\alpha+1}} \mathbb{E} \left( R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}} \right)^{\frac{\alpha+\eta}{\alpha+1}}$$

This expression is useful because it let us to plug in the aggregate labor supply equation, replacing away  $L$

$$w = \left( \frac{\eta}{\eta - 1} \right)^{-\frac{\eta\phi}{1+\eta\phi}} Y^{\frac{\phi}{1+\eta\phi}} \mathbb{E} \left( z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} \right)^{\frac{\alpha+\eta}{\alpha+1} \frac{\phi}{1+\eta\phi}} \mathbb{E} \left( R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}} \right)^{\frac{\alpha+\eta}{\alpha+1} \frac{\phi}{1+\eta\phi}}$$

Since  $Y_j = z_j L_j$ , then

$$Y_j = \left( \frac{\eta}{\eta - 1} \right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{\eta\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}$$

Taking the  $\frac{\eta-1}{\eta}$  power, integrating over all the firms, and taking the power  $\frac{\eta}{\eta-1}$ , we get an expression for  $Y$ . By replacing the expressions we derived for  $L$  and  $w$ , we get the result

$$Y = \left( \frac{\eta}{\eta-1} \right)^{-1/\phi} \mathbb{E} \left( z_j^{\frac{(\eta-1)(\alpha+1)}{\alpha+\eta}} \right)^{\frac{(1+\phi)(\alpha+\eta)}{\phi(\eta-1)(\alpha+1)}} \mathbb{E} \left( R_j^{\frac{-(\eta-1)\alpha}{\alpha+\eta}} \right)^{\frac{(1+\phi\eta)}{\phi(\eta-1)}} \mathbb{E} \left( R_j^{\frac{-\eta(\alpha+1)}{\alpha+\eta}} \right)^{\frac{(1-\phi\alpha)}{\phi(\alpha+1)}}$$

## A.2 Proof of Proposition 1

Start from equation (14). We express log output as

$$\begin{aligned} \log Y = & -\frac{1}{\phi} \log \left( \frac{\eta}{\eta-1} \right) + \left( \frac{1+\eta\phi}{(\eta-1)\phi} + \frac{1-\alpha\phi}{1+\eta\phi} \right) \log \left( \int_0^1 z_j^{(\eta-1)\frac{\alpha+1}{\alpha+\eta}} dj \right) \\ & + \frac{1+\eta\phi}{\phi(\eta-1)} \log \int_0^1 R_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} dj + \frac{1-\alpha\phi}{\phi(\alpha+1)} \log \int_0^1 R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}} dj \end{aligned} \quad (32)$$

Then we will do a second-order Taylor expansion around a point where all the lending rates take value of  $R$ . According to assumption 2, all the lending rates will stay at that level, except for the lending rate of an arbitrary bank  $b$  that will suffer a disruption of its lending terms to  $Re^u$  for a positive and sufficiently small  $u$ . Define  $\bar{Y}$  as the value of output when the  $z_j$  is the same for each firm and all the rates are kept at  $R$ .

For  $x > 0$  we can approximate  $R_j^{-x}$  according to

$$R_j^{-x} \approx \bar{R}_j^{-x} \left( 1 - x\bar{s}_j\bar{\nu}_{bj}u + x^2\bar{\nu}_{bj}^2\bar{s}_j^2\frac{u^2}{2} + x\bar{\nu}_{bj}(1-\nu_{bj})\bar{s}_j\theta\frac{u^2}{2} + \varphi\bar{\nu}_{bj}^2x\bar{s}_j(1-\bar{s}_j)\frac{u^2}{2} \right). \quad (33)$$

Which uses that  $\frac{d\nu_{jb}}{d\log R_b} = -\theta\nu_{jb}(1-\nu_{jb})$  and that  $\frac{ds_j}{d\log R_b} = -\varphi\nu_{jb}s_j(1-s_j)$ . In the point around we are taking the second-order Taylor expansion,  $\bar{\nu}_{ujb} = T_{jb}$  and  $s_j = \bar{s}$ , where  $\bar{s}$  is the share of bank credit when all the lending rates are set at  $R$ .

Applying expectations across firms, we can find an expression for  $\mathbb{E}R_j^{-x}$

$$\mathbb{E}R_j^{-x} \approx \bar{R}_j^{-x} \left( 1 - x\bar{s}\mathbb{E}\bar{\nu}_{bj}u + x^2\mathbb{E}(\bar{\nu}_{bj}^2)\bar{s}^2\frac{u^2}{2} + x\mathbb{E}(\bar{\nu}_{bj}(1-\nu_{bj}))\bar{s}\theta\frac{u^2}{2} + \varphi\mathbb{E}(\bar{\nu}_{bj}^2)x\bar{s}(1-\bar{s})\frac{u^2}{2} \right). \quad (34)$$

Applying logs and taking differences with respect to the initial point

$$\Delta \mathbb{E}R_j^{-x} \approx -x\bar{s}\mathbb{E}\bar{\nu}_{bj}u + x^2\mathbb{E}(\bar{\nu}_{bj}^2)\bar{s}^2\frac{u^2}{2} + x\mathbb{E}(\bar{\nu}_{bj}(1-\nu_{bj}))\bar{s}\theta\frac{u^2}{2} + \varphi\mathbb{E}(\bar{\nu}_{bj}^2)x\bar{s}(1-\bar{s})\frac{u^2}{2} \quad (35)$$

by replacing  $x$  for the exponents in each term of equation (33) we get the result.

$$\Delta \log Y = -\frac{1}{\phi} \left( \bar{s} \mu_b u + \Omega s^2 (\sigma_b^2 + \mu_b) \frac{u^2}{2} - \theta \bar{s} (\mu_b - \sigma_b^2 - \mu_b^2) \frac{u^2}{2} - \varphi \bar{s} (1 - \bar{s}) (\sigma_b^2 + \mu_b^2) \frac{u^2}{2} \right) \quad (36)$$

$$\text{Where } \Omega = \frac{\eta - \alpha + \eta \alpha (1 - \phi)}{\phi(\alpha + \eta)}$$

### A.3 Proof of Proposition 2

Every bank increases its cost from  $\bar{R}_b$  to  $Re^u$ .

In a similar way to the proof in Proposition 1, up to a second-order approximation

$$\Delta \mathbb{E} R_j^{-x} \approx -x \bar{s} u + x^2 \bar{s}^2 \frac{u^2}{2} + x \bar{s} \theta \frac{u^2}{2} + \varphi x \bar{s} (1 - \bar{s}) \frac{u^2}{2} \quad (37)$$

Replacing  $x$  for the exponents in equation (33) we get the result

$$\Delta \log Y = -\frac{1}{\phi} \left( \bar{s} u + \Omega s^2 \frac{u^2}{2} - \varphi \bar{s} (1 - \bar{s}) \frac{u^2}{2} \right) \quad (38)$$

### A.4 Proof of Proposition 3

Firm-level output can be expressed as a function of aggregate variables and firm-level shifters as in:

$$Y_j = \left( \frac{\eta}{\eta - 1} \right)^{-\eta \frac{\alpha}{\alpha + \eta}} Y^{\frac{\alpha}{\alpha + \eta}} z_j^{\frac{\eta(\alpha + 1)}{\alpha + \eta}} L^{\frac{\eta}{\eta + \alpha}} w^{-\eta \frac{\alpha}{\alpha + \eta}} R_j^{-\eta \frac{\alpha}{\alpha + \eta}}$$

Define as  $Y_j^c$  the level of output of firms that do not have any relationship with shocked bank  $b$ . The difference between any particular firm with a relationship with bank  $b$  and a control firm is given by:

$$\log Y_j - \log Y_j^c = -\eta \frac{\alpha}{\alpha + \eta} (\log R_j - \log R_j^c) + \eta \frac{\alpha + 1}{\alpha + \eta} (z_j - z_j^c)$$

Now we take expectations across firms, and using the assumption of no sorting, we cancel out the productivity term. That is, firm-level productivity is independent of the existence of bank relationships.

$$\mathbb{E}(\log Y_j - \log Y_j^c) = -\eta \frac{\alpha}{\alpha + \eta} \mathbb{E}(\log R_j - \log R_j^c)$$

A second-order Taylor expansion of  $R_j$  with respect to a disruption of the lending



terms of bank  $b$  as stated in Assumption 2, around a symmetric point where all the lending rates are equal to  $R$  yields:

$$R_j \approx \bar{R}_j \left( 1 + \bar{s}\bar{\nu}_{jb}u + \bar{s}^2\bar{\nu}_{jb}^2\frac{u^2}{2} - \theta\bar{s}\bar{\nu}_{jb}(1 - \bar{\nu}_{jb})\frac{u^2}{2} - \varphi\bar{s}(1 - \bar{s})\bar{\nu}_{jb}^2\frac{u^2}{2} \right)$$

And  $\bar{R}_j^c = \bar{R}_j$ . By plugging combining these expressions we get the result:

$$\mathbb{E}(\log Y_j - \log Y_j^c) = -\frac{\eta\alpha}{\alpha + \eta} \left( \bar{s}\mathbb{E}(T_{jb})u(1 + \mathbb{E}(T_{jb})s\frac{u}{2}) - \theta\bar{s}\mathbb{E}(T_{jb}(1 - T_{jb}))\frac{u^2}{2} - \varphi\bar{s}(1 - \bar{s})\mathbb{E}(T_{jb}^2)\frac{u^2}{2} \right)$$

## B Proofs Identification Section

### B.1 Output regression

Firm-level output can be written as:

$$Y_j = \left( \frac{\eta}{\eta - 1} \right)^{-\frac{\eta\alpha}{\alpha + \eta}} Y^{\frac{\alpha}{\alpha + \eta}} z_j^{\eta\frac{\alpha}{\alpha + \eta}} w^{-\eta\frac{\alpha}{\alpha + \eta}} R_j^{-\eta\frac{\alpha}{\alpha + \eta}} \quad (39)$$

Taking logs we get:

$$\log Y_j = -\frac{\eta\alpha}{\alpha + \eta} \log \left( \frac{\eta}{\eta - 1} \right) + \frac{\alpha}{\alpha + \eta} \log Y + \eta\frac{\alpha}{\alpha + \eta} \log z_j - \eta\frac{\alpha}{\alpha + \eta} \log w - \eta\frac{\alpha}{\alpha + \eta} \log R_j \quad (40)$$

I will collapse the first, second, and fourth term into a single term called  $\log \Theta_t$ , which is common to all the firms, and will therefore become irrelevant in computing the result.

$$\log Y_j = \log \Theta_t + \eta\frac{\alpha}{\alpha + \eta} \log z_j - \eta\frac{\alpha}{\alpha + \eta} \log R_j \quad (41)$$

Taking temporal differences we get:

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta\frac{\alpha}{\alpha + \eta} \Delta \log z_j - \eta\frac{\alpha}{\alpha + \eta} \Delta \log R_j \quad (42)$$

A second-order Taylor expansion of  $\log R_j$  that coincides with assumption 2 yields:

$$\log R_j \approx \log \bar{R}_j + \bar{s}T_{jb}u + \bar{s}^2T_{jb}^2\frac{u^2}{2} - \theta\bar{s}T_{jb}(1 - T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1 - \bar{s})T_{jb}^2\frac{u^2}{2} \quad (43)$$

Taking temporal differences with respect to a pre-period where  $\log R_j = \log \bar{R}_j$ , yields:

$$\Delta \log R_j \approx \bar{s}T_{jb}u + \bar{s}^2T_{jb}^2\frac{u^2}{2} - \theta\bar{s}T_{jb}(1 - T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1 - \bar{s})T_{jb}^2\frac{u^2}{2} \quad (44)$$

Plugging this expression into equation 42 yields a second order approximation of firm-level output after one shock suffers an increase in its lending terms.

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta \frac{\alpha}{\alpha + \eta} \Delta \log z_j - \eta \frac{\alpha}{\alpha + \eta} \left( \bar{s}T_{jb}u + \bar{s}^2T_{jb}^2\frac{u^2}{2} - \theta\bar{s}T_{jb}(1 - T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1 - \bar{s})T_{jb}^2\frac{u^2}{2} \right) \quad (45)$$

The cross-sectional regression of log output changes on pre-existing exposure can be computed by a simple regression estimated by OLS:

$$\Delta \log Y_j = \beta_0 + \beta_{output}T_{jb} + \epsilon_f \quad (46)$$

In this setting the exposure in the preperiod to the affected bank is just  $T_{jb}$ .

The regression coefficient is given by the covariance between  $\Delta \log Y_j$  and  $T_{jb}$ . Since all the firms have the same  $\Delta \log \Theta_t$  regardless of their specific  $T_{jb}$ , then the effect on aggregates of the shock is absorbed by the intercept. The regression coefficient in the population is given by:

$$\begin{aligned} \beta_{output} &= \frac{cov(\Delta \log Y_j, T_{jb})}{var(T_{jb})} \\ \beta_{output} &= \eta \frac{\alpha + 1}{\alpha + \eta} \frac{cov(\Delta z_j, T_{jb})}{var(T_{jb})} - \frac{\eta\alpha}{\alpha + \eta} \bar{s}u \left( 1 + \bar{s} \frac{u}{2} \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} - \theta \frac{u}{2} \left( 1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \right) \right. \\ &\quad \left. - \varphi(1 - \bar{s}) \frac{u}{2} \left( \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \right) \right) \end{aligned}$$

Further, imposing the no-sorting condition implies that the first term is zero.

$$\begin{aligned} \beta_{output} &= -\frac{\eta\alpha}{\alpha + \eta} \bar{s}u \left( 1 + \bar{s} \frac{u}{2} \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} - \theta \frac{u}{2} \left( 1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \right) \right. \\ &\quad \left. - \varphi(1 - \bar{s}) \frac{u}{2} \left( \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \right) \right) \end{aligned}$$

This is the main result. The regression coefficient in the population is larger when

consumers are more elastic in reallocating demand across varieties ( $\eta$  higher), when labor markets work without frictions  $\frac{\alpha}{\alpha+\eta}$ . Both substitution across banks and substitution away from bank credit make the elasticity less negative. Note the term  $\left(1 - \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})}\right)$  in the second term that accompanies the  $\theta$ . Since the shifter  $T$  are between 0 and 1, the covariance can be equal to the variance if the  $T$  terms only take either 0 or 1. When that is the case, firms are completely dependent of one bank. Therefore  $\theta$  becomes irrelevant.

## B.2 Bank-credit regression

Firm-level credit in logs can be written as a

$$\log Q_j = \log \Sigma + \log s_j + \frac{1+\alpha}{\alpha}(\log Y_j - \log z_j)$$

Therefore, an OLS regression

$$\Delta \log Q_j = \beta_0 + \beta_{credit} T_{jb} + \xi_j, \quad (47)$$

can be expressed simply as:  $\beta_{credit} = \beta_{share} + \frac{1+\alpha}{\alpha} \beta_{output}$ , where  $\beta_{share} = \frac{\text{cov}(\Delta s_j, T_{jb})}{\text{var}(T_{jb})}$ .

Up to a second order  $\Delta \log s_j$  is given by:

$$\Delta \log s_j = -\varphi \left( T_{jb}(1-s_j)u + T_{jb}^2(1-s^2)\frac{u^2}{2} - \theta T_{jb}(1-T_{jb})(1-s)\frac{u^2}{2} + \varphi s_j(1-s_j)T_{jb}^2\frac{u^2}{2} \right)$$

Therefore,

$$\begin{aligned} \beta_{credit} = \frac{1+\alpha}{\alpha} \beta_{output} - \varphi(1-\bar{s})u \left( 1 + \frac{u}{2} \frac{1-s^2}{1-s} \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})} + \varphi \bar{s} \frac{u}{2} \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})} \right. \\ \left. - \theta \frac{u}{2}(1-s) \left( 1 - \frac{\text{cov}(T_{jb}^2, T_{jb})}{\text{var}(T_{jb})} \right) \right) \end{aligned}$$