Illiquid Markets, Asymmetric Information, and Investment*

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Abstract

We study the macroeconomic implications of asymmetric information in capital markets. We build a quantitative capital-accumulation model in which capital is traded in illiquid markets, with sellers having more information about capital quality than buyers. Asymmetric information distorts the terms of trade for sellers of high-quality capital, who list higher prices and are willing to accept lower trading probabilities to signal their type. Guided by the model's predictions, we measure the distortions from asymmetric information by studying the relationship between listed prices and trading probabilities in a unique dataset of individual capital units listed for trade. Combining the empirical measurement with the model, we show that information asymmetries play a quantitatively large role during economic downturns, when we measure that the degree of asymmetric information deteriorates.

Keywords: Asymmetric information, investment, misallocation, trading frictions.

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1 Introduction

Information asymmetries are a salient phenomenon of real asset markets. As considered in the Akerlof (1970) seminal work, capital units are heterogeneous in their qualities, and sellers tend to have more information about them than buyers do. Given that capital accumulation plays a central role in the macroeconomy, a key question is how information asymmetries in physical capital markets affect aggregate investment.

In this paper, we study the aggregate effects of asymmetric information by combining a quantitative capital accumulation model with microlevel data on capital markets. Our approach is motivated by two ideas. First, empirical evidence using microlevel data shows that capital markets are illiquid, with capital units being listed for significant periods before being traded (e.g., Ramey and Shapiro, 2001). Second, theory indicates that in illiquid markers, asymmetric information distorts the behavior of high-quality capital sellers, who signal their type by listing higher prices and accepting lower trading probabilities (Guerrieri, Shimer, and Wright, 2010). Together, these findings suggest that the distortions from asymmetric information can be measured by studying the liquidity of different capital units listed for trade.

To implement this approach, we begin by developing a capital-accumulation model with asymmetric information and illiquid capital markets. At the macro level, the degree of asymmetric information in capital markets affects aggregate investment, the capital unemployment rate, and the average quality of employed capital. We then measure the degree of asymmetric information using microlevel data on capital units listed for trade and combine it with the model to quantify the aggregate effects of asymmetric information. Our analysis indicates that information asymmetries play a quantitatively large role during economic downturns, when the measured degree of asymmetric information increases.

Our model embeds three key ingredients in the neoclassical capital accumulation framework. First, capital units are heterogeneous in their quality (i.e., in terms of their output in production). Second, information about capital quality is private to the owner of a capital unit. Buyers have access to an information-revealing technology that with some probability reveals the true quality of the capital unit. The accuracy of this information technology characterizes the degree of asymmetric information in the economy. Third, the trading of capital occurs in decentralized markets, with sellers announcing a capital quality and choosing at what price to list their capital units and buyers choosing at what price and announced quality to search. We provide conditions whereby the model features a unique fully revealing separating equilibrium, in which sellers announce the true quality of their capital. This separating equilibrium resembles that of the classical model of Spence (1973), in which low types have a high marginal cost of effort and choose not to mimic the education levels of high types. In asset markets, the equivalent marginal effort exerted by high types corresponds to selling with a lower probability: Insofar as there is a probability of buyers detecting the true capital quality, high-quality sellers have a lower marginal cost of not trading than low-quality sellers.

Using the model, we show how distortions from asymmetric information can be linked to the cross-sectional patterns of capital units listed for trade. When capital quality is observed by buyers, high-quality capital attracts more buyers and has a higher selling probability than low-quality capital. However, when capital quality is unobserved by buyers, high-quality capital sellers choose to signal their type and separate from sellers of low-quality assets. They do so by choosing to list high-quality capital at such high prices that low-quality assets would not choose to mimic their pricing behavior; higher prices attract fewer buyers and are associated with lower trading probabilities. The less accurate buyers' information technology, the larger the price that sellers of high-quality capital choose to separate from low-quality capital, and the larger the covariance between capital units' listed prices and their expected duration on the market. Therefore, by studying the empirical relationship between listed prices and duration, a researcher can measure the degree of asymmetric information in capital markets.

We then apply our proposed measurement to a novel dataset of capital units listed for trade. Our dataset contains the history of nonresidential structures (retail and office space) listed for sale and rent in Spain by one of Europe's main online real estate platforms, `Idealista`; it contains rich information on each unit, including the listed price, exact location, size, age, and other characteristics. Given the data's panel structure, for each unit, we can compute the duration on the platform and the search intensity it attracted, measured by the number of clicks received in a given month.

We document a set of empirical facts consistent with the presence of distortions from asymmetric information in capital markets, which deteriorates during bad times. First, we show that the component of capital units' listed prices that reflect publicly observed characteristics in the listing (i.e., the predicted price from hedonic regression of prices on the set of characteristics included in each listing in a narrowly defined market) are negatively related to the unit's duration on the market. This empirical fact is consistent with the prediction of the model for observed capital quality: Since predicted prices are obtained from observable characteristics, properties with better characteristics (which are reflected by a higher predicted price) have a shorter average duration on the market. Second, we show that the component of a capital unit's price that is orthogonal to the characteristics that are publicly observed in the listing (i.e., the residual from the hedonic regression described above) is positively related to the unit's duration on the market. This fact is consistent with the presence of asymmetric information about capital characteristics not observed in the listing, and the fact that owners of higher capital quality choose higher prices to signal their type, which are associated with lower trading probabilities. Our measurement also indicates that the degree of asymmetric information deteriorates economic downturns. In particular, the slope between residual prices and duration exhibits a strong comovement with economic activity, with a sharp increase during the Euro crisis. This pattern is also observed at a geographic level, with regions that experience a larger decline in economic activity also experiencing a higher increase in the slope between residual prices and duration.

Finally, we combine our empirical measurements with the model to quantify the aggregate effects of asymmetric information. Our model features four channels through which capital-market information asymmetries affect aggregate output. First, higher information asymmetries lead to a lower capital stock. This is because higher information asymmetries are associated with a lower revenue for sellers of high-quality capital, which decreases the returns to producing capital goods. Second, higher information asymmetries lead to a larger unemployment rate of capital. As information asymmetries increase, so do the listed prices of high-quality capital sellers, which decreases the selling probability and increases the duration of unemployment of listed units. Third, a higher degree of asymmetric information is associated with a lower average quality of employed capital. This is because information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities. Finally, by distorting the terms of trade, asymmetric information affects search efforts and the demand for labor used in production.

By disciplining the degree of asymmetric information in the model with the cross-sectional patterns of capital units listed for trade, we find that small changes in the degree of asymmetric information have large macroeconomic effects. Our analysis indicates that, in the steady state, the economy features moderate levels of asymmetric information, with the probability of a lemon's going unnoticed being close to 2%. However, the economy features large aggregate responses to changes in information technologies. For instance, an unexpected decrease in the accuracy of information technologies akin to that measured during the Euro crisis (with a 2p.p. increase in the probability of a lemon going unnoticed) leads to a more than 2% decline in economic activity followed by a slow recovery. This suggests that policies aimed at preventing signaling in real asset markets (e.g., taxes intended to implement a pooling equilibrium) can play an important role in stabilizing economic downturns.

Related Literature First, our paper is related to the literature that studies asymmetric information in asset markets, pioneered by Akerlof (1970); Stiglitz and Weiss (1981); and Myers and Majluf (1984), among others. Our framework particularly builds on theories that study these frictions in decentralized markets (see, for example, Guerrieri et al., 2010; Delacroix and Shi, 2013). Our paper is also related to the body of work on the effects of asymmetric information in the macroeconomy (see, for example, Eisfeldt, 2004; Kurlat, 2013; Guerrieri and Shimer, 2014; Bigio, 2015; Lester, Shourideh, Venkateswaran, and Zetlin-Jones, 2019). We contribute to this literature by showing how microlevel data on assets listed for trade can be used to measure the degree of asymmetric information and by using this measurement to show that information asymmetries have quantitatively important aggregate effects.

Second, the paper is related to the literatures on misallocation (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008); capital reallocation (e.g., Ramey and Shapiro, 2001; Eisfeldt and Rampini, 2006; Lanteri, 2018); and asset specificity (e.g., Caballero and Hammour, 1998; Kermani and Ma, 2020). We contribute to this literature by showing that asymmetric information can constitute a sizable source of capital illiquidity and misallocation.[1]

Third, our paper is related to the literature that studies the role of search-and-matching frictions in asset markets. This includes a large body of work on financial markets (for a survey, see Lagos, Rocheteau, and Wright, 2017); housing markets (see, for example, Wheaton, 1990; Krainer, 2001; Caplin and Leahy, 2011; Piazzesi, Schneider, and Stroebel, 2020); and

[1]The form of misallocation we consider builds on that of Gavazza (2016), who uses business-aircraft data to study the welfare effects of trading frictions in the allocation of assets.

physical capital markets (see, for example, Kurmann and Petrosky-Nadeau, 2007; Gavazza, 2011; Cao and Shi, 2017; Ottonello, 2017; Wright, Xiao, and Zhu, 2018, 2020; Cui, Wright, and Zhu, 2021). Our paper contributes to this literature by demonstrating the relevance of the interaction between asymmetric information and search frictions.

Layout The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the effects of asymmetric information in capital markets and discusses how the degree of asymmetric information can be measured from micro-data moments. Section 4 applies this measurement to our dataset and presents a set of empirical facts linked to model predictions. Section 5 combines the model and empirical measurement and quantifies the aggregate effects of asymmetric information. Section 6 concludes.

2 Model

2.1 Environment

Time is discrete and infinite, and there is no aggregate uncertainty. Final goods are perishable and can be used for consumption or investment. Capital goods are storable and can be used in the production of final goods, together with labor services.

Agents, preferences, and technology The economy is populated by a unit mass of identical households and a unit mass of firms owned by the household. Households have preferences over consumption described by the lifetime utility function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)\gamma_n^t$, where c_t and h_t denote per capita consumption and hours worked in period t; $\gamma_n \geq 1$ denotes the gross population growth within the representative household;[2] $u(c,h) = \log(c) - \varpi \frac{1}{1+\xi} h^{1+\xi}$ with $\varpi > 0$ and $\xi > 0$; $\beta \in (0,1)$ is the subjective discount factor; and $\mathbb{E}_t$ denotes the expectation conditional on the information set available in period t. Households have access to a linear technology to produce new capital goods using final goods.

A continuum of identical firms with measure one have access to a constant-returns-to-scale technology to produce final goods using capital and labor as inputs, $y_{jt} = f_t(\mathcal{K}_{jt}, l_{jt}) \equiv \mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$, where y_{jt}, $\mathcal{K}_{jt}$, and l_{jt} denote the output, capital input, and labor input of firm

[2]We include population and technology growth in the model to better match the investment rates observed in the data, which are sizable flows for capital markets.

j in period t, respectively; $\gamma \geq 1$ denotes the exogenous growth rate of labor-augmenting technology in the economy; and $\alpha \in (0,1)$. Each period, with i.i.d. probability φ, a firm receives an exit shock and must exit the economy; firms that exit the economy cannot produce and transfer their capital holdings to households at the end of the period. After exit shocks are realized, a new mass φ of firms enter the economy. In this setup, operating firms will be capital buyers and households capital sellers (selling new capital or capital from exiting firms).

The model features three main departures from the neoclassical capital-accumulation model: heterogeneity in capital quality, a decentralized market for capital, and information frictions. We describe each of these elements next.

Capital-quality heterogeneity Studying information asymmetries in capital markets requires that we introduce heterogeneity in these goods. To do so, we consider an environment in which the capital stock is composed of infinitesimal indivisible units (i.e., capital goods are available to trade in integer quantities only, and agents hold a mass of these units). Capital units are heterogeneous in two dimensions: an "observed quality" $\omega \in \Omega \equiv [\omega_1, \ldots, \omega_{N_\omega}]$, with $\omega_r < \omega_s$ for $r < s$, and an "unobserved quality" $a \in \mathcal{A} \equiv [a_1, \ldots, a_{N_a}]$, with $a_r < a_s$ for $r < s$. While the observed quality ω of a unit is assumed to be perfectly observable by all market participants, unobserved quality a is the private information of the owner of the capital unit and is the source of asymmetric information in the model, which is further discussed below. The capital services a capital unit provides are determined by these qualities, with the capital services of a capital unit i being given by $\omega_i a_i$. Capital services employed as input in production by firm j are then given by $\mathcal{K}_{jt} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k_{jt+1}(\omega, a)$, where $k_{jt+1}(\omega, a)$ is the mass of capital of quality (ω, a) employed in production by firm j in period t. In our application we interpret capital quality broadly, as representing any characteristic that increases the marginal product of capital. For example, in the case of retail space, capital quality can capture the number of potential clients in a given geographic location or the quality of the store in attracting customers.

Terms of trade in the decentralized market for capital In models of asymmetric information, the presence of search-and-matching frictions can play an important role in determining the equilibrium, because it introduces a signaling device for sellers in choosing a price that reflects their different marginal benefits of trading (see, for example, Guerrieri et al., 2010). Based on the empirical evidence of studies that characterize trading capital markets (e.g., Gavazza, 2011; Ottonello, 2017), we assume that capital goods are traded in a decentralized market with search-and-matching frictions.

The decentralized capital market is organized in a continuum of submarkets, indexed by $(\omega, \hat{a}, q)$, where ω is the observed quality, $\hat{a}$ is the unobserved quality announced by the seller, and q is the listed price. Search is directed: Sellers can choose at what announced unobserved quality and price to list their capital units, and buyers can choose at what observed quality, announced unobserved quality, and price to search, and dedicate labor to search and match.[3] In each submarket $(\omega, \hat{a}, q)$, the market tightness, denoted by $\theta_t(\omega, \hat{a}, q)$, is defined as the ratio between buyers' hours of search and the mass of capital posted by sellers.[4] In visiting submarket $(\omega, \hat{a}, q)$ in period t, sellers face a probability $p(\theta_t(\omega, \hat{a}, q))$ of finding a potential buyer for their unit and buyers match with a mass $\mu_t(\theta_t(\omega, \hat{a}, q))$ of potential units to buy per hour of search, where $p(\theta) = \min\{\overline{m}\theta^{1-\eta}, 1\}$ with $\eta \in (0, 1)$.[5]

When sellers list a capital unit in submarket $(\omega, \hat{a}, q)$, they commit to allowing potential buyers to inspect the unit using the technology described below. If no new information about the capital quality is revealed during the inspection or if the inspection indicates that capital quality is not below that announced (i.e., $a' \geq \hat{a}$), sellers and buyers commit to trade the capital unit at the listed price q. If the inspection reveals that the true quality of the capital a' is lower than the announced quality (i.e., $a' < \hat{a}$) and there are gains from trade between the buyer and seller, then trade occurs at the inspection-adjusted price $q_t^P(\omega, a', \hat{a}, q) \leq q$. Here, we assume that the transacted price $q_t^P(.)$ results from a Nash bargaining problem.[6]

[3]The assumed directed search structure is similar to that used by Shimer (1996); Moen (1997) and Menzio and Shi (2011) in the labor market, and Ottonello (2017) in capital markets. For a recent survey of the literature on directed search in labor, housing, and monetary economics, see Wright, Kircher, Julien, and Guerrieri (2019).

[4]Following the directed search literature (see, for example Moen, 1997; Menzio and Shi, 2011), in submarkets that are not visited by any sellers, $\theta_t(\omega, \hat{a}, q)$ is an out-of-equilibrium conjecture that helps determine equilibrium.

[5]The functional form of the matching probability can be obtained from a Cobb-Douglas matching technology $\mathcal{M}_t(k^s(\omega, \hat{a}, q), \gamma^t v^s(\omega, \hat{a}, q)) = \min\{\overline{m}(k^s(\omega, \hat{a}, q))^{\eta}(\gamma^t v^s(\omega, \hat{a}, q))^{1-\eta}, k^s(\omega, \hat{a}, q)\}$, where $k^s(\omega, \hat{a}, q)$ and $v^s(\omega, \hat{a}, q)$ denote the mass of capital listed by sellers and hours dedicated by buyers to search in submarket $(\omega, \hat{a}, q)$, respectively, and $\overline{m} > 0$; given the labor-augmenting technology in the production of final goods, the labor-augmenting technology in the matching sector is necessary for a balanced-growth path.

[6]More specifically, we assume that the transacted price is the minimum between the listed price and the bargained price, which reflects the commitment to sell at the listed price if favorable to the buyer. The only assumption we impose on the bargaining problem is that the seller's bargaining power ϕ satisfies $\phi \leq \eta$, so that the equilibrium bargained price is weakly lower than the price sellers would obtain when announcing the quality truthfully.

In Appendix A, we relax this assumption and show that the equilibrium characterization remains the same under general post-inspection trading protocols. Finally, if the inspection reveals that the quality of the capital a' is such that $a' < \hat{a}$ and there are no gains from trade, then the match is dissolved without trade.

Finally, since our main focus is on capital markets, we assume that final goods and labor services are traded in Walrasian markets.

Information structure An information asymmetry arises because capital quality has a component that is private information to its owner, a_i. We are interested in studying how the degree of asymmetric information in the economy affects capital accumulation. For this, we assume that after having searched and matched with a capital unit and before purchasing it, buyers have access to a technology to inspect the unit. Similar to Menzio and Shi (2011), this information revealing technology is such that in any submarket $(\omega, \hat{a}, q)$, there is a probability ψ_t that the buyer learns the true type (ω, a) of the capital good and a probability $1 - \psi_t$ that the inspection is uninformative. Hence, ψ_t parameterizes the degree of asymmetry of information in the economy, nesting a full-information case when $\psi_t = 1$ and the case with complete asymmetric information when $\psi_t = 0$ (since there cannot be any discovery of the unobserved quality). To study the effects of changes in the degree of asymmetric information in the macroeconomy, we consider the possibility that the accuracy of the information technology ψ_t experiences exogenous changes, which can lead to economic fluctuations.

The information asymmetry requires that we specify agents' beliefs about the type of capital available for sale, given a listed price and observable characteristics. We assume that all potential buyers have the same beliefs. We describe beliefs by the mapping $\pi_t(a|\omega, \hat{a}, q) : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \rightarrow [0, 1]$, which denotes the probability that a unit of capital is of unobserved type a, given the observed type ω, the announced quality $\hat{a}$, and the price q. After purchasing a unit of capital, buyers obtain full information about its quality. Sellers are assumed not to have recall on the capital quality of their units sold.

Timing The timing of events within each period is as follows:

(i) Exit shocks are realized, and a mass φ of new firms enter the economy.

(ii) Households choose the capital units they list for sale, their prices, and their announced qualities, which are perfectly observed by all agents. Incumbent non-exiting firms and new firms search and match with potential capital units to buy.

(iii) Firms conduct inspections of matched capital units and decide whether to buy them or not.

(iv) Incumbent non-exiting firms and new firms hire workers, produce final goods, and pay wages. Firms that exit the economy transfer their capital to households. All agents holding capital pay a maintenance cost δ per unit of effective capital in terms of final goods. Households invest in new capital units and consume.

2.2 Optimization

Households Each period, households produce new capital goods. They do so by choosing their total investment in terms of final goods i_t, and the resulting quality of new capital is exogenous and random, governed by the distribution function $g : \Omega \times \mathcal{A} \to [0, 1]$, which describes the measure of new capital of each quality. Since households do not have access to a production technology, their capital revenue comes from selling these newly produced units of capital, together with unemployed capital transferred by exiting firms, to operating firms. The evolution of the capital holdings by households is then given by

$$k_{Ht+1}(\omega, a) = (1 - p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a)))k_{Ht}(\omega, a) + g(\omega, a)i_t + \varphi K_{Ft}(\omega, a), \qquad (1)$$

where $k_{Ht+1}(\omega, a)$ denotes capital of quality (ω, a) held by the household at the end of period t; $\hat{a}_{Ht}(\omega, a)$ and $q_{Ht}(\omega, a)$ denote the household's choice of announced capital quality and price to list units of quality (ω, a); $p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a)))k_{Ht}(\omega, a)$ denotes the mass of capital of type (ω, a) matched by buyers given the household's choice of submarket; and $K_{Ft}(\omega, a)$ denotes the aggregate capital of quality (ω, a) held by firms at the beginning of period t—a fraction φ of which is transferred to households by firms that exit. For expositional simplicity, equation (1) abstracts from households that post a unit of capital in multiple submarkets and from capital not being sold following an inspection that reveals a different quality from that announced (Appendix A shows that this does not happen in equilibrium).

We write households' optimization problem recursively. At the beginning of a period, the individual state for the household is a matrix of its capital holdings, given by $\mathbf{k} \equiv$

$\begin{bmatrix} k(\omega_1, a_1) & ... & k(\omega_{N_\omega}, a_1) \\ ... & ... & ... \\ k(\omega_1, a_{N_a}) & ... & k(\omega_{N_\omega}, a_{N_a}) \end{bmatrix}$. The recursive problem of the representative household is then given by

$$V_{Ht}(\mathbf{k}) = \max_{\{c,h,\{k'(\omega,a),\hat{a}(\omega,a),q(\omega,a)\},i\geq 0\}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

subject to the budget constraint

$$\begin{aligned} &c\gamma_n^t + i + \delta \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a\left[(1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))))k(\omega, a) + \varphi K_{Ft}(\omega, a)\right] \qquad (2) \\ &= \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \left[(1-\psi_t)q(\omega, a) + \psi_t q^P(\omega, a, \hat{a}(\omega, a), q)\right] p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))k(\omega, a) + w_t h\gamma_n^t + Div_{Ft} \end{aligned}$$

and the law of motion for capital

$$k'(\omega, a) = (1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))))k(\omega, a) + ig(\omega, a) + \varphi K_{Ft}(\omega, a),$$

where Div_{Ft} denote the dividends transferred by firms in period t. The optimal level of investment (provided that $i > 0$) is characterized by the Euler equation

$$1 = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} g(\omega, a)\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta\gamma_n \frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$ ($\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy functions for capital accumulation associated with problem (2.2)) and $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega,a)} \frac{1}{u_{ct}(\mathbf{k})\gamma_n^t}$ is the household's marginal value of capital of type (ω, a) measured in final goods, which satisfies the recursive problem:

$$\begin{aligned} \nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = \max_{\hat{a},q}\ & p(\theta_t(\omega, \hat{a}, q))((1-\psi_t)q + \psi_t q^P(\omega, a, \hat{a}, q)) \\ &+ (1 - p(\theta_t(\omega, \hat{a}, q)))\left(\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, k_{Ht+1}(\mathbf{k})) - \delta\omega a\right). \qquad (3) \end{aligned}$$

Finally, the optimal labor supply is given by the intratemporal first order condition $u_{ht}(\mathbf{k}) = w_t$.

Firms Firms accumulate capital by purchasing it from sellers in the decentralized market, which requires paying for hours of labor to search for potential units that are a good match for the firm. Abstracting from the possibility that firms might want to sell capital (which does not occur in equilibrium), the evolution of their capital holdings is given by

$$k_{jt+1}(\omega,a)=\sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}\iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)v_{jt}(\omega,\hat{a},q)\,\mathrm{d}q+k_{jt}(\omega,a), \tag{4}$$

where $v_{jt}(\omega,\hat{a},q)$ denotes the hours of work hired by firms to search and match with sellers in submarket $(\omega,\hat{a},q)$; $\mu_t\left(\theta_t\left(\omega,\hat{a},q\right)\right)v_{jt}(\omega,\hat{a},q)$ the mass of capital matched by these workers; and $\iota_t(a|\omega,\hat{a},q)$ the share of units of capital of quality a when searching in submarket $(\omega,\hat{a},q)$.

Conditional on not exiting, the recursive problem of the firm is given by

$$V_{Ft}(\mathbf{k})=\max_{\{l,\{v(\omega,\hat{a},q)\geq 0\},\{k'(\omega,a)\}\}}\ \mathbb{E}_a[div+\Lambda_{t,t+1}((1-\varphi)V_{Ft+1}(\mathbf{k}')+\varphi V^{\text{exit}}_{t+1}(\mathbf{k}'))], \tag{5}$$

subject to the definition of dividends

$$\begin{aligned}div=&\left(\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega a k'(\omega,a)\right)^{\alpha}(\gamma^t l)^{1-\alpha}-w_t l-\delta\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\omega a k'(\omega,a)\\ &-\sum_{\omega\in\Omega}\sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}[(\psi_t\sum_{a\in\mathcal{A}}\iota_t(a|\omega,\hat{a},q)q^P_t(\omega,a,\hat{a},q)+(1-\psi_t)q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)+w_t]v(\omega,\hat{a},q)\,\mathrm{d}q\end{aligned}$$

and the law of motion for capital

$$k'(\omega,a)=\sum_{\hat{a}\in\mathcal{A}}\int_{q\in\mathbb{R}_+}\iota_t(a|\omega,\hat{a},q)\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)v(\omega,\hat{a},q)\,\mathrm{d}q+k(\omega,a), \tag{6}$$

where $\mathbb{E}_a[\cdot]$ denotes the expectation under the belief function $\pi_t(a|\omega,\hat{a},q)$; div denotes dividends in terms of final goods transferred to households; $\Lambda_{t,t+1}$ denotes households' discount factor; w_t denotes the wage rate in period t; $V^{\text{exit}}_t(\mathbf{k})\equiv\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}k(\omega,a)\nu^{\text{s}}_t(\omega,a,\mathbf{K}_{Ht})$ denotes the household's value of exiting firms with capital holdings $\mathbf{k}$; and $\mathbf{K}_{Ht}$ denotes the matrix of capital holdings by households in period t, which is taken as given by individual firms. Problem (5) abstracts from the scenario in which, after the inspection, trade does not occur, and there are no gains from trade for quality $a'<\hat{a}$ (Appendix A provides conditions for $q^P_t(\omega,a,\hat{a},q)$ for which this does not happen in equilibrium).

The following result characterizes firms' optimal choices of capital and labor.

***Proposition* 1.** *The firm's value function $V_{Ft}(\mathbf{k})$ is linear in capital stocks—i.e., it can be expressed as $V_{Ft}(\mathbf{k})=\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\nu^{\text{b}}_t(\omega,a)k_t(\omega,a)$. This marginal value of capital holdings* *satisfies the recursive problem:*

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi \nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right], \tag{7}$$

where $Z_t \equiv \alpha \left(\frac{\gamma^t(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$*. The labor demand in the production of final goods is linear in* $\mathcal{K}_t$ *and given by* $l_t(\mathcal{K}_t) = \mathcal{K}_t \times \left(\frac{(1-\alpha)\gamma^{t(1-\alpha)}}{w_t}\right)^{\frac{1}{\alpha}}$.

Proof. All proofs are relegated to Appendix A.1. ■

Proposition 1 implies that the buyer's value of a capital unit of a given quality does not depend on other capital holdings. In particular, the value of capital with quality (ω, a) is given by the utility flow generated by its production plus its continuation value, which takes into account the probability of exiting production and becoming a seller of capital.

Firms' optimal search activity across different submarkets is characterized by

$$v_t(\omega, \hat{a}, q)\left(\underbrace{((1-\psi_t)q + \psi_t \mathbb{E}_a(q^P(\omega, a, \hat{a}, q)|\omega, \hat{a}, q))}_{\text{Expected price}} + \underbrace{\frac{w_t}{\mu_t(\theta(\omega, \hat{a}, q))}}_{\text{Search cost}} - \underbrace{\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right)}_{\text{Expected value}}\right)^{+} = 0, \tag{8}$$

for all $(\omega, \hat{a}, q)$ (with $(x)^+ \equiv \max(x, 0)$), which shows that firms are willing to search for capital in a submarket if the expected marginal cost of purchasing capital in that market, including its expected price and search cost, does not exceed its expected value. Given that submarkets differ in their price q, firms are indifferent between buying in different submarkets for the same expected value of capital only if those units with higher price have associated a higher rate $\mu_t(\omega, \hat{a}, q)$ at which the workers find a match.

2.3 Equilibrium

Definitions We now define the economy's competitive equlibrium, the balanced growth path, and types of equilibrium: pooling and separating. We restrict attention to pure strategy equilibria, which characterize the unique solution under the D1 equilibrium refinement.

Definition* 1. *Competitive Equilibrium

Given initial conditions $\mathbf{K}_{H0}$ *and* $(\mathbf{k}_{j0})_{j\in[0,1]}$*, a perfect Bayesian equilibrium under asymmetric information consists of a sequence of household value functions* $\{V_{Ht}(\mathbf{k}), \nu_t^{\mathrm{s}}(\omega, a, \mathbf{k})\}$

and policy functions $\{c_t(\mathbf{k}), h_t(\mathbf{k}), i_t(\mathbf{k}),\ \mathbf{k}_{Ht+1}(\mathbf{k}), \hat{a}_t(\omega, a, \mathbf{k}), q_t(\omega, a, \mathbf{k})\}$*; firm value functions* $\{V_{Ft}(\mathbf{k}), \nu_t^{\mathrm{b}}(\omega, a)\}$ *and policy functions* $\{l_t(\mathbf{k}), div_t(\mathbf{k}),\ \mathbf{k}_{Ft+1}(\mathbf{k}), \{v_t(\omega, \hat{a}, q)\}\}$*; market tightness functions* $\{\theta_t(\omega, \hat{a}, q)\}$*; belief functions* $\{\pi_t(a|\omega, \hat{a}, q)\}$*; wages* $\{w_t\}$*; discount factors* $\{\Lambda_{t,t+1}\}$*; and aggregate variables* $\{\mathbf{K}_{Ht+1}, \mathbf{K}_{Ft+1},\ Div_{Ft}, \iota_t(a|\omega, \hat{a}, q)\}$ *for all* $t \geq 0$ *such that*

(i) Given wages and market tightness, the household's value functions $V_{Ht}(\mathbf{k})$ *and* $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k})$ *solve* (2.2) *and* (3) *with associated policy functions* $c_t(\mathbf{k})$*,* $i_t(\mathbf{k})$*,* $\mathbf{k}_{Ht+1}(\mathbf{k})$*,* $\hat{a}_t(\omega, a, \mathbf{k})$*, and* $q_t(\omega, a, \mathbf{k})$ *for all* $(\omega, a) \in \Omega \times \mathcal{A}$*.*

(ii) Given wages, market tightness, and discount factors, a firm's value functions $V_{Ft}(\mathbf{k})$ *and* $\nu_t^{\mathrm{b}}(\omega, a)$ *solve* (5) *and* (7) *with associated policy functions* $l_t(\mathbf{k})$*,* $\mathbf{k}_{Ft+1}(\mathbf{k})$*, and* $\{v_t(\omega, \hat{a}, q)\}$ *for all* $(\omega, \hat{a}) \in \Omega \times \mathcal{A}$*.*

(iii) Market tightness functions satisfy (8) *in all submarkets.*

(iv) The belief function $\pi_t(a|\omega, \hat{a}, q)$ *is consistent with sellers' strategies using Bayes' rule when possible.*

(v) The labor market clears: $\sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \int_{q \in \mathbb{R}_+} v_t(\omega, \hat{a}, q)\, \mathrm{d}q + \int l_t(\mathbf{k}_{jt})\, \mathrm{d}j = h_t(\mathbf{k})\gamma_n^t$*.*

(vi) The discount factors satisfy $\Lambda_{t,t+1} = \lambda_t(\mathbf{K}_{Ht})$*.*

(vii) Aggregate variables are consistent with individual policies: $\mathbf{K}_{Ht+1} = \mathbf{k}_{Ht+1}(\mathbf{K}_{Ht})$*,* $\mathbf{K}_{Ft+1} = \int \mathbf{k}_{Ft+1}(\mathbf{k}_{jt})\, \mathrm{d}j$*,* $Div_{Ft} = \int div_t(\mathbf{k}_{jt})\, \mathrm{d}j$*;* $\iota_t(a|\omega, \hat{a}, q) = \frac{\mathbb{I}_{\{\hat{a}=\hat{a}_t(\omega,a,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q=q_t(\omega,a,\mathbf{K}_{Ht})\}}K_{Ht}(\omega,a)}{\sum_{a_j \in \mathcal{A}} \mathbb{I}_{\{\hat{a}=\hat{a}_t(\omega,a_j,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q=q_t(\omega,a_j,\mathbf{K}_{Ht})\}}K_{Ht}(\omega,}$ *for all* $(\omega, \hat{a}, q)$ *such that* $\hat{a}$ *and* q *are part of the set of policy functions associated with the household's problem.*

In the rest of this section, for analytical characterization of the equilibrium, we restrict our attention to the balanced-growth-path equilibrium, which is defined as:

Definition 2. Balanced-growth path

A balanced-growth path is defined as a competitive equilibrium in which the sequence $\{c_t, k_{Ht}(\omega, a), k_{Ft}(\omega, a), \hat{a}_t(\omega, a), q_t(\omega, a), \theta_t(\omega, \hat{a}_t, q_t), w_t, \Lambda_{t,t+1}, Z_t, \psi_t\}_{t \geq 0}$ *satisfies:*

(i) Per capita consumption c_t*, wages* w_t*, and productivity* Z_t *grow at rate* γ*.*

(ii) For all (ω, a)*, the stock of capital held by firms and households (*$k_{Ft}(\omega, a)$ *and* $k_{Ht}(\omega, a)$*, respectively) grows at rate* $\gamma\gamma_n$*.*

(iii) For all (ω, a), submarket choices $a_t(\omega, a)$ and $q_t(\omega, a)$, market tightness $\theta_t(\omega, \hat{a}_t, q_t)$, and the accuracy of information technology ψ_t are constant.

(iv) The discount factor satisfies $\Lambda_{t,t+1} = \frac{\beta\gamma_n}{\gamma}$.

Finally, for characterization, we consider two type of equilibrium—pooling and separating—defined as follows:

***Definition* 3.** *A pooling equilibrium is a competitive equilibrium in which sellers of different unobserved qualities post the same price and announce the same quality with strictly positive probability—i.e., $q(\omega, a_j) = q(\omega, a_{j'})$ and $\hat{a}(\omega, a_j) = \hat{a}(\omega, a_{j'})$. Similarly, a separating equilibrium is a competitive equilibrium in which sellers of different unobserved qualities post either different prices or different qualities—i.e., $q(\omega, a_j) \neq q(\omega, a_{j'})$ or $\hat{a}(\omega, a_j) \neq \hat{a}(\omega, a_{j'})$. Among those, a fully revealing separating equilibrium is a separating equilibrium in which sellers of a given unobserved quality announce their true unobserved quality—i.e., $\hat{a}(\omega, a_j) = a_j$.*

Equilibrium Characterization Since the strategy space contains both the announcement of the unobserved quality and the posted price, sellers can signal their unobserved quality and separate from each other by differing along any of these two dimensions. As in Guerrieri et al. (2010), we characterize the equilibrium as an allocation that solves the following sequence of constrained optimization problems $\mathcal{P}_j(\omega)$.

***Definition* 4.** *For a given observed quality ω and aggregate variables, the solution to problem $\mathcal{P}_j(\omega)$ is a vector $(q(\omega, a_j), \hat{a}(\omega, a_j))$ that solves*

$$\begin{aligned}\nu^{\mathrm{s}}(\omega, a_j) = &\max_{\{q(\omega,a_j),\hat{a}(\omega,a_j)\}} p\left(\theta\left(\omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right)\right) \left[(1-\psi)q(\omega, a_j) + \psi q^P(\omega, a_j, \hat{a}(\omega, a_j), q)\right] \\ &+ (1 - p(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j)))) \left[\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_j) - \delta\omega a_j\right] \end{aligned} \tag{9}$$

subject to

$$\begin{aligned}&\mathbb{E}_a\left((1-\psi)q(\omega, a_j) + \psi q^P(\omega, a, \hat{a}(\omega, a_j), q) \,|\, \omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right) \\ &\qquad = \mathbb{E}_a\left(\nu^{\mathrm{b}}(\omega, a)\right) - \frac{w_t}{\mu\left(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j))\right)},\end{aligned} \tag{10}$$

and

$$\nu^{\mathrm{s}}(\omega, a_{j'}) \geq p\left(\theta\left(\omega, \hat{a}(\omega, a_j), q(\omega, a_j)\right)\right)\left[(1-\psi)q(\omega, a_j) + \psi q^P(\omega, a_{j'}, \hat{a}(\omega, a_j), q)\right] \qquad (11)$$
$$+ (1 - p(\theta(\omega, \hat{a}(\omega, a_j), q(\omega, a_j))))\left[\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, a_{j'}) - \delta\omega a_{j'}\right] \quad \textit{for all } j' < j.$$

In problem $\mathcal{P}_j(\omega)$, the seller of capital with quality (ω, a_j) announces an unobserved quality $\hat{a}(\omega, a_j)$ and posts a price $q(\omega, a_j)$ to maximize expected revenues subject to two constraints. The first constraint is the buyer's search optimality condition, which pins down the market tightness for a given set of beliefs and seller's choices. In addition, the seller is constrained by a set of no-mimicking conditions, which require that sellers of lower quality weakly prefer their own terms of trade rather than mimicking the terms of trade chosen by the seller of unobserved quality a_j. Hence, an allocation that solves the above sequence of optimization problems effectively describes a separating equilibrium. We will focus on separating equilibria in which sellers of different unobserved qualities truthfully reveal their unobserved quality, which we defined above as fully revealing separating equilibria.[7]

The sequence of problems $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ may admit multiple solutions, each with the corresponding equilibria supported by appropriate out-of-equilibrium beliefs. Indeed, the definition of a fully revealing separating equilibrium does not impose any constraint on off-equilibrium beliefs, which can potentially lead to multiplicity. Therefore, we impose more structure on these beliefs by considering equilibria that satisfy the *D1 criterion* of Cho and Kreps (1987), which is an equilibrium refinement commonly used in signaling games. This criterion first identifies the set of sellers who are more likely to deviate from equilibrium choices. After requiring that buyers have beliefs consistent with this set when observing a deviation, the *D1 criterion* eliminates equilibria in which a seller's payoff from the deviation under the worst buyer's consistent belief is not equilibrium dominated. This refinement is enough to establish the existence and uniqueness of equilibrium:

Proposition 2. *The balanced-growth-path fully revealing separating equilibrium is characterized by the following solution to the sequence of problems $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ for all $\omega \in \Omega$, which is constructed recursively:*

[7] In our analysis, we omit unreasonable separating equilibria in which sellers of different unobserved qualities choose to set different prices (so buyers can identify their different qualities) and announce different, but untrue, qualities. In Appendix A, we argue that these alternative equilibria are dominated by the equilibrium analyzed here.

(i) The seller of the lowest unobserved quality a_1 chooses the full-information strategy $\hat{a}(\omega, a_1) = a_1$, $q(\omega, a_1) = q^{FI}(\omega, a_1)$, and $\theta(\omega, \hat{a}(\omega, a_1), q(\omega, a_1)) = \theta^{FI}(\omega, a_1)$, which are characterized by

$$q^{FI}(\omega, a_1) = \nu^{\mathrm{b}}(\omega, a_1) - \frac{\chi}{\mu(\theta^{FI}(\omega, a_1))} \tag{12}$$

and

$$p'(\theta^{FI}(\omega, a_1)) \left(\nu^{\mathrm{b}}(\omega, a_1) - \left(\frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, a_1) - \delta \omega a_1 \right) \right) = \chi,$$

where $\chi \equiv w_t / \gamma^t$.

(ii) The seller of any unobserved quality $a_k > a_1$ signals his true quality—i.e., $\hat{a}(\omega, a_k) = a_k$. Regarding the terms of trade, there are two cases to consider:

(a) If none of the constraints (11) evaluated at all $l \leq k-1$ bind, then the seller of quality a_k chooses the full-information terms of trade—i.e., $q(\omega, a_k) = q^{FI}(\omega, a_k)$ and $\theta(\omega, \hat{a}(\omega, a_k), q(\omega, a_k)) = \theta^{FI}(\omega, a_k)$.

(b) If at least one of the constraints (11) binds for $l \leq k-1$, then let $\underline{\theta}_l^k$ denote the lowest θ that solves

$$\begin{aligned} \nu^{\mathrm{s}}(\omega, a_l) &= p\left(\theta\right) \left((1-\psi) q(\omega, a_k) + \psi q^P(\omega, a_l, \hat{a}(\omega, a_k), q) \right) \\ &\quad + (1 - p(\theta)) \left(\frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, a_l) - \delta \omega a_l \right), \end{aligned}$$

where $q(\omega, a_k) = \nu^{\mathrm{b}}(\omega, a_k) - \frac{\chi}{\mu(\theta)}$. The seller of quality a_k chooses $\theta(\omega, a_k) = \min \left\{ \underline{\theta}_j^k,\ j \in [1, k-1] \right\}$ and the corresponding price, as long as $\frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, a_k) - \delta \omega a_k \geq \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{s}}(\omega, a_l) - \delta \omega a_l$ for all $l < k$. In this case, the optimal market tightness is lower than under the full-information terms of trade—i.e., $\theta(\omega, a_k) < \theta^{FI}(\omega, a_k)$.

Finally, there are no pooling equilibria.

Excluding the seller of the lowest unobserved quality who is never affected by the information asymmetry, Proposition 2 describes two distinct situations. In the first case, sellers can choose the unconstrained optimum of their objective, since no other seller wants to mimic them when they adopt this strategy. Formally, constraints (11) drop out and the optimal terms of trade are characterized by the first-order condition and the buyer's indifference condition (10). We denote this unconstrained solution as the full-information terms of trade.

As we will show below, this situation arises when the inspection is informative enough—i.e., when ψ is high enough. The second case arises when at least one other seller wants to mimic the unconstrained solution, which is formally characterized by at least one of the constraints being violated. Intuitively, a relatively uninformative inspection facilitates mimicking by sellers of lower unobserved qualities. Sellers of high unobserved quality must then adapt their strategy to disincentivize mimicking by lower types and thereby signal their true quality. Thus, the optimal terms of trade become distorted relative to the full-information case. We show that if the seller's values are increasing in the unobserved quality (which is true for realistically low depreciation rates), then the optimal signaling strategy consists of choosing a lower tightness and a higher price than under full information so that the tightest constraint is just binding. This forms the unique fully revealing separating equilibrium that satisfies the D1 criterion. The proposition also states that the signaling game does not feature any pooling equilibria, in which sellers of different unobserved qualities choose the same submarket with positive probability.

3 The Micro Effects of Asymmetric Information

This section uses the model to study how asymmetric information distorts capital markets. Section 3.1 studies how the accuracy of information technologies affects the prices and duration of capital units listed for trade. Based on these model predictions, Section 3.2 discusses how the degree of asymmetric information can be identified from micro-data moments.

3.1 Asymmetric Information, Prices, and Duration

We begin by focusing on a simple case that can be characterized analytically, with $\Omega = \{\omega_L, \omega_H\}$ and $\mathcal{A} = \{a_L, a_H\}$ (where L and H denote low and high qualities, respectively). We also assume that depreciation costs are small relative to the values of sellers (i.e., $\delta \to 0$). We first show how the terms of trade change in the cross-section of observed characteristics. We then describe how asymmetric information affects the trade of units with different unobserved characteristics. Throughout the section, we consider an information technology with a constant accuracy over time, i.e., $\psi_t = \psi$, and study the effects of changes in the parameter ψ.

Observed capital quality We begin by focusing on the cross-sectional predictions of the model for units with different observed capital qualities. To isolate these differences, we set $a_L = a_H = \overline{a}$. In this case, the solution to the seller's problem is characterized by the first-order condition

$$p'(\theta(\omega,\overline{a}))\left(\nu^{\mathrm{b}}(\omega,\overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega,\overline{a})\right) = \chi$$

for all $(\omega) \in \Omega$, where we have replaced the price $q(\omega, a)$ from the optimal search strategy of the buyer. The optimal choice of market tightness balances the marginal benefit of a higher trading probability (left-hand side) with the reduction in the price required by potential buyers in order to visit the chosen submarket (right-hand side). The following proposition formalizes this result by deriving the optimal price of capital and market tightness for each type of capital under full information.

***Proposition* 3.** *If $a_L = a_H = \overline{a}$, the price and market tightness for capital of quality ω are given by*

$$q(\omega,\overline{a}) = \eta\nu^{\mathrm{b}}(\omega,\overline{a}) + (1-\eta)\frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega,\overline{a})$$

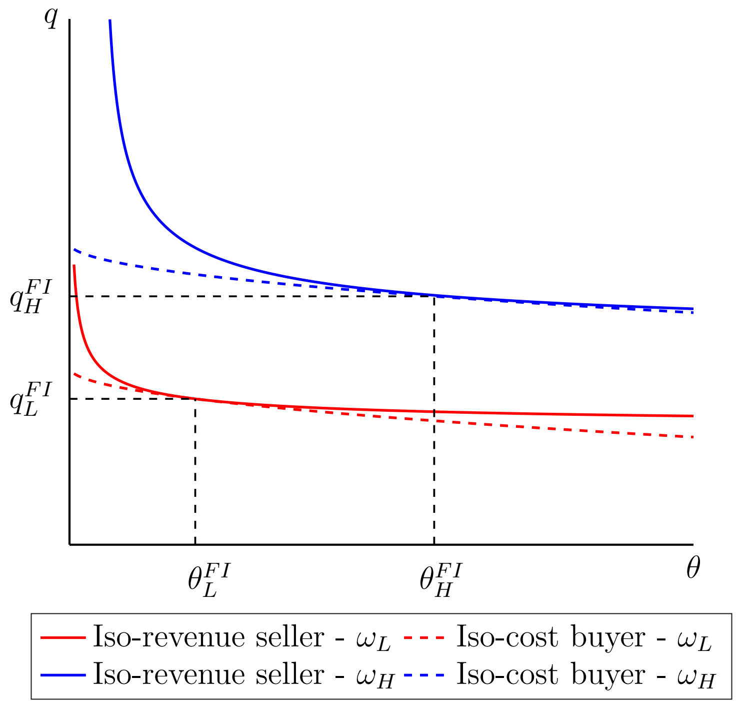
and

$$\theta(\omega,\overline{a}) = \left(\frac{\bar{m}(1-\eta)}{\chi}\left(\nu^{\mathrm{b}}(\omega,\overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega,\overline{a})\right)\right)^{1/\eta}.$$

Proposition 3 shows that the equilibrium price is a weighted average of the seller's and buyer's value of capital, and the selling probability is an increasing function of the surplus $\nu^{\mathrm{b}}(\omega,\overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega,\overline{a})$. This optimal choice is graphically represented in Figure 1 for types $(\omega_L,\overline{a})$ and $(\omega_H,\overline{a})$ and given search cost χ. Dashed lines represent the iso-cost curves of buyers, with the highest one corresponding to the high-quality ω_H. These curves denote the combination of prices and purchase probabilities that generate the same expected cost to buyers and are derived from equation (10). Curves are downward-sloping because buyers are indifferent between submarkets if higher prices are associated with higher matching rates with sellers. They are increasing in ω because buyers can obtain higher revenues by using capital of higher quality.

Similarly, solid lines denote the iso-revenue curve of sellers—i.e., the combination of prices and market tightness that produce the same expected revenues—and are derived from Equation (9). These are downward-sloping because the seller is willing to accept a lower price if the sale's probability increases. Note that the iso-revenue curves have a lower slope for high-quality capital. This results from the outside option (i.e., the continuation value)

FIGURE 1: Competitive Equilibrium under Full Information



of the seller increasing in the quality of its capital, which causes the seller to require lower "compensation" in terms of a higher sale probability for a given reduction in the price. In equilibrium, sellers choose the submarket that maximizes their utility subject to buyers' indifference curves.[8]

Proposition 3 and Figure 1 show that under full information, the price of a unit of capital and its matching rate are increasing in its quality, which implies the following result.

***Corollary* 1.** *If $a_L = a_H = \overline{a}$, capital units with higher prices match at a higher rate: $q^{\mathrm{FI}}(\omega_H, \overline{a}) > q^{\mathrm{FI}}(\omega_L, \overline{a})$ and $p\left(\theta^{\mathrm{FI}}(\omega_H, \overline{a})\right) > p\left(\theta^{\mathrm{FI}}(\omega_L, \overline{a})\right)$.*

To understand the intuition behind this corollary, replace the equilibrium price of capital in the optimal search strategy of the buyer to obtain

$$(1-\eta)\left(\nu^{\mathrm{b}}(\omega, \overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega, \overline{a})\right) = \frac{\theta(\omega, \overline{a})\chi}{p\left(\theta(\omega, \overline{a})\right)}. \tag{13}$$

Equation (13) requires that in equilibrium, the seller's net benefit from buying a unit of

[8]To see that the solution is unique, notice that the dotted line is less convex than the solid line (formally, the second-order derivative is lower for the buyer's indifference condition). This implies that it is possible to construct a strictly monotonous transformation of θ, such that the dotted line depicts a linear relationship while the solid line remains strictly convex. Replacing θ with this transformation, we obtain the standard problem of finding the utility-maximizing intersection of strictly convex preference curves and a convex budget set, which has a unique solution.

capital must be equal to its expected search cost. As in standard models of directed search, the surplus (given by $\nu^{\mathrm{b}}(\omega,\overline{a}) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\omega,\overline{a})$) is "split" according to the elasticity of the matching function. Thus, since the price of capital scales with the seller's value less than proportionally ($\eta < 1$), the net gain of buying capital is increasing in this value. By non-arbitrage, the expected search cost must be higher for capital units with higher quality—and thus higher value—which implies that buyers (sellers) of these units match at a lower (higher) rate.

Unobserved capital quality Next, we consider the solution to the seller's problem under asymmetric information. To isolate these differences, we set $\omega_L = \omega_H = \overline{\omega}$. As previously shown, capital of the lowest unobserved quality, a_L, is sold under the full-information terms of trade. However, the choice of the seller of quality a_H capital might be affected by information frictions. In this case, the solution to the seller's problem is characterized by the first-order condition

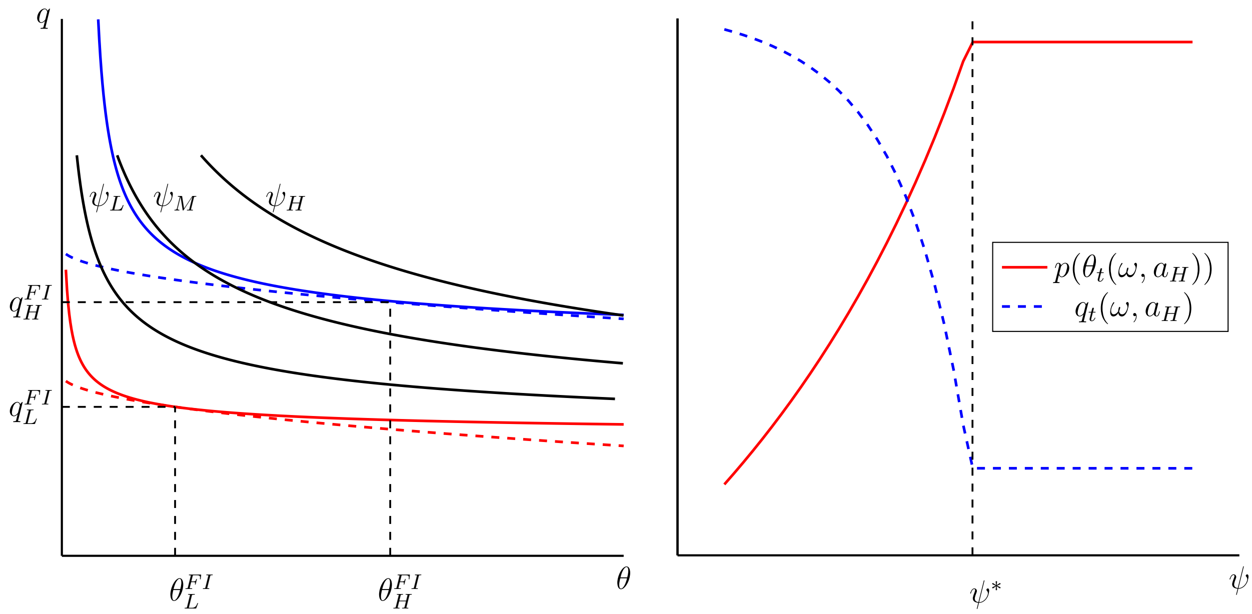
$$p'(\theta(\overline{\omega},a_H))\left(\nu^{\mathrm{b}}(\overline{\omega},a_H) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\overline{\omega},a_H)\right) = \chi + \zeta(\overline{\omega},a_H) \tag{14}$$

and the complementary slackness condition

$$\begin{aligned}\zeta(\overline{\omega},a_H)\Big[&p(\theta^{FI}(\overline{\omega},a_L))\left(q^{FI}(\overline{\omega},a_L) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\overline{\omega},a_L)\right) \\ &- p(\theta(\overline{\omega},a_H))\left((1-\psi)q(\overline{\omega},a_H) + \psi q^P(\overline{\omega},a_L,a_H,q) - \frac{\beta\gamma_n}{\gamma}\nu^{\mathrm{s}}(\overline{\omega},a_L)\right)\Big] = 0,\end{aligned} \tag{15}$$

where $\zeta(\overline{\omega},a_H)$ denotes the Lagrange multiplier of the no-mimicking constraint, which requires that the lower type a_L does not want to mimic the choices made by the higher type a_H. Notice that the constraint incorporates the fact that sellers who mimic the choices of sellers with other qualities sell at the posted price only when the inspection is uninformative. The presence of the inspection stage introduces a small deviation from the standard signaling model à la Spence (1973). For high values of ψ—i.e., when the extent of asymmetric information is not severe—sellers might not need to signal their quality, since the probability of detection is high. This intuition is formalized in the following proposition, which is a special case of Proposition 2 for the two-type example considered here.

FIGURE 2: Competitive Equilibrium under Asymmetric Information



(A) Contraints Arising from AI

(B) Optimal Allocations under AI

***Proposition* 4.** *Let $\psi^* \in [0,1]$ be defined as*

$$p(\theta^{FI}(\overline{\omega}, a_L)) \left[q^{FI}(\overline{\omega}, a_L) - \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{S}}(\overline{\omega}, a_L) \right]$$
$$= p(\theta^{FI}(\overline{\omega}, a_H)) \left[(1-\psi^*) q^{FI}(\overline{\omega}, a_H) + \psi^* q^P(\overline{\omega}, a_L, a_H, q) - \frac{\beta \gamma_n}{\gamma} \nu^{\mathrm{S}}(\overline{\omega}, a_L) \right].$$

The seller of quality a_L chooses the same terms of trade as under full information. For sellers of quality a_H, there are two cases:

(i) $\psi \geq \psi^*$: the incentive-compatibility constraint is not binding (i.e., $\zeta(\overline{\omega}, a_H) = 0$) and $\theta(\overline{\omega}, a_H)$ solves the optimality condition (14).

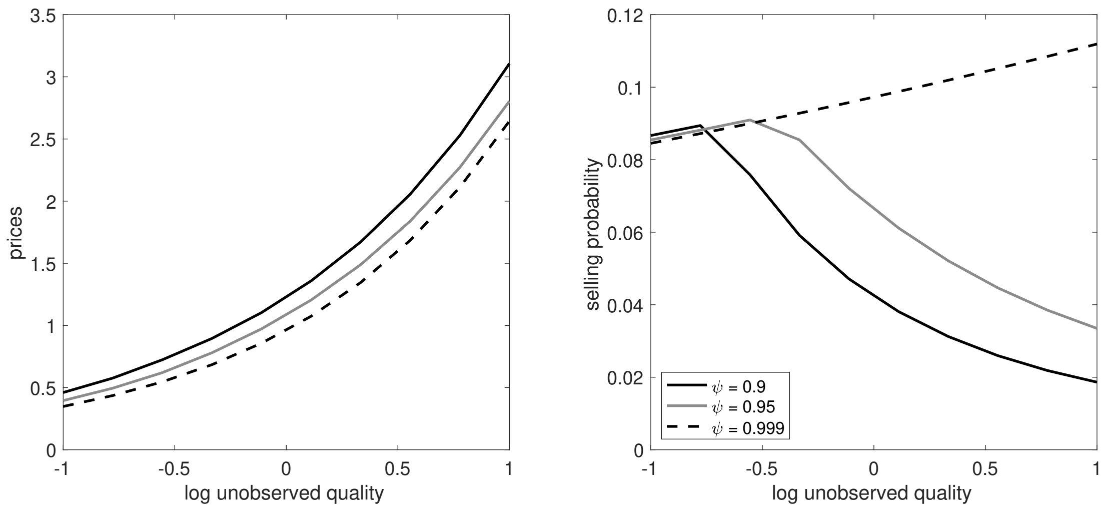
(ii) $\psi < \psi^*$: the incentive compatibility constraint is binding (i.e., $\zeta(\overline{\omega}, a_H) > 0$) and $\theta(\overline{\omega}, a_H)$ solves (15). Optimal terms of trade satisfy $q(\overline{\omega}, a_H) > q^{FI}(\overline{\omega}, a_H)$ and $p(\theta(\overline{\omega}, a_H)) < p(\theta^{FI}(\overline{\omega}, a_H))$. Therefore, the difference in the expected time to sell across qualities increases as ψ decreases—i.e., $d \left[\frac{p(\theta^{FI}(\overline{\omega}, a_L))}{p(\theta(\overline{\omega}, a_H))} \right] / d\psi < 0$. Thus, if information asymmetries are strong enough (i.e., ψ is low enough), then $p(\theta(\overline{\omega}, a_H)) < p^{FI}(\theta(\overline{\omega}, a_L))$.

We illustrate the equilibrium under asymmetric information in Figure 2. In a fully revealing separating equilibrium with signaling, the outcome in the submarket for the lowest quality capital is the same as the one obtained under full information (see Figure 1). However, the outcome in the submarket for high-quality capital could be distorted by the fact that sellers maximize the expected value subject to the constraint, whereby low-quality sellers do not have a strict preference for participating in the same submarket. The different possibilities are illustrated in Panel (a) of Figure 2. In addition to the iso-revenue and iso-cost curves shown in Figure 1, the figure includes the no-mimicking constraint behind the complementary slackness condition (15) for three values of ψ: $\psi_L < \psi_M < \psi^* < \psi_H$. For a given price, any market tightness to the right of the solid black lines violates the constraint.

When the information technology is good enough (e.g., ψ_H in Figure 2), the seller of high-quality capital can choose the full-information market tightness. Sellers of low-quality capital do not want to mimic this choice because with a high probability, the inspection reveals their lower quality and they end up selling at a lower price. As ψ decreases below ψ^* (e.g., ψ_M in Figure 2), sellers of low-quality capital are more likely to be able to sell without being detected by the inspection. Then, the full-information tightness violates the constraint and the seller chooses a higher price and a lower tightness to signal the higher quality of capital. The optimal tightness is determined by the intersection between the no-mimicking constraint and the buyer's isocost curve evaluated at a_H. This lower sale probability is more costly for low-quality sellers given the information revealing technology, which could reveal their true type and lead to a low sale price. Because of these additional delays, low-quality sellers weakly prefer their own submarket. If the informativeness of the inspection is very low (e.g., ψ_L in Figure 2), then the required signaling in the form of delays is such that capital of higher quality ends up selling with lower probability than low-quality capital. Therefore, ψ governs the potential distortions to terms of trade from information asymmetries (see Panel (b) of Figure 2).

Multiple capital qualities So far, we focused on a simple case with two types of capital qualities that can be characterized analytically. We now provide a quantitative illustration showing how the effects of asymmetric information on prices and duration extend to a setting with multiple types of observed and unobserved capital quality. For this, we assume that the observed and unobserved qualities are distributed according to two independent log-normal

FIGURE 3: Capital market outcomes for different accuracies of information technologies



Note: The left panel shows the equilibrium price for units of different unobserved quality on the x-axis. The right panel of the figure shows the selling probabilities for units of different levels of unobserved productivity. The three lines correspond to three values of the quality of information-revealing technology, ψ: a low value $\psi = 0.9$, a medium value $\psi = 0.95$, and a large value that takes the economy to the full information limit, $\psi = 0.999$. The rest of the model parameters are set to their calibrated values from Section 5.

distributions with variance σ_j^2 for $j \in \{\omega, a\}$.[9,10] For the quantitative exercsises presented in this section, we use the calibrated version of the model discussed in detail in Section 5.

Figure 3 depicts listed prices and associated selling probabilities for different levels of unobserved capital quality. The dotted black lines show that in the limiting case of $\psi \to 1$, in which the information technology approaches full information, sellers of higher quality list their units at a higher price and sell their units with a higher probability. As discussed in the analytical example above, this is because under full information, units of high capital quality are more attractive to buyers, which leads to a higher trading probability for sellers.

The solid gray lines show that as the quality of the information technology declines (i.e., lower values of ψ), the relationship between listed prices and unobserved capital quality becomes steeper. This is because a more imprecise information technology creates higher incentives for low-quality capital sellers to mimic higher quality sellers. In turn, high-quality capital sellers respond to the inferior information technology by increasing the listed price for their units, which separates them from low-capital-quality sellers, who are not willing to bear the cost of the associated low trading probabilities. For this reason, an increase in

[9]The distributional assumption is without loss of generality, but we make it to operationalize our quantitative analysis. We normalize the mean of both capital qualities to one. The assumption that qualities ω and a are independent is also without loss of generality, since one could interpret the observable quality as the conditional expected quality $\omega + \mathbb{E}(a|\omega)$ and the unobserved quality as the residual $a - \mathbb{E}(a|\omega)$.

[10]When solving the model numerically, we truncate these log-normal distributions to have the support $[-2\sigma_j, 2\sigma_j]$ for $j \in \{\omega, a\}$.

the degree of asymmetric information driven by a lower capital quality is associated with higher average prices and duration of listed units, particularly at the top of the distribution of capital qualities.

3.2 Identification

Led by the model predictions, we now discuss how the parameters linked to the degree of asymmetric information and capital heterogeneity can be identified from microlevel data. Our distributional assumption regarding (ω, a) implies that the model features three parameters linked to the degree of asymmetric information and capital heterogeneity, which are the most novel part of the model: $\{\psi, \sigma_\omega, \sigma_a\}$. We first illustrate our strategy under some specific assumptions that allow us to derive analytical results and below we use a parameterized version of the model to show our strategy using model-simulated data.

Analytical illustration To provide an empirical measurement of the model predictions, we assume that a researcher observes micro data on capital units listed for sale with the following information: the price of each unit listed in every period t, $\{q_{it}\}$; the duration of each unit while listed for trade $\{Duration_{it}\}$; and a vector of observable characteristics $\{X_i\}$ (e.g., location, size, number of rooms, etc.). We further assume that the observable characteristics map into observable efficiency units of capital according to $\log \omega_i = \tau X_i$, where τ is an unkown vector. Consider estimating the following regressions using these data:

$$\log(q_{it}) = \iota_\omega X_i + \varepsilon^q_{it}, \tag{16}$$

$$\log(Duration_{it}) = \upsilon_\omega X_i + \upsilon_q \log(q_{it}) + \varepsilon^d_{it}, \tag{17}$$

where ε^q_{it} and ε^d_{it} are random error terms. Regression (16) is a "hedonic regression," which projects listed prices on the observed capital quality of each unit. Henceforth, we refer to $\hat{q}_{it} = \hat{\iota}_\omega X_i$ as "predicted prices" and ε^q_{it} as "residual prices." Intuitively, by estimating this regression, we can approximate the variance of observed and unobserved capital qualities with the variance of predicted and residual prices, $\hat{\sigma}_\omega \equiv \mathrm{Var}(\hat{q}_{it})$ and $\hat{\sigma}_a \equiv \mathrm{Var}(\varepsilon^q_{it})$. Regression (17) projects the duration of each listed unit on their observed characteristics and listed price. The estimated coefficient $\hat{\upsilon}_q$ measures the slope between log duration and the component of the price that is orthogonal to its observed quality (i.e., the residual ε^q_{it}), which, following the discussion in Section 3.1, is informative regarding the degree of asymmetric information. The following proposition formalizes this mapping between model parameters and moments from the model-simulated data:

***Proposition* 5.** *Assume* $\left(\frac{w_t}{\mu_t(\theta(\omega,\hat{a},q))}\right)/\nu_t^{\mathrm{b}}(\omega,a) \to 0$ *and* $\varphi \to 0$*. Then, up to a first-order approximation,* $(\psi, \sigma_\omega^2, \sigma_a^2)$ *are identified by the estimated moments* $\hat{\sigma}_\omega$*,* $\hat{\sigma}_a$ *and* $\hat{\upsilon}_q$*.*

Proposition 5 imposes two assumptions (which are relaxed in the quantitative analysis of the identification below). First, expected search costs are small relative to the buyer's value of a unit of capital. Second, the exit rate of firms is approximately zero. The first assumption ensures that the price of a unit of capital is approximately equal to its expected value to the buyer. The second assumption ensures that this value is mainly determined by the net present value of the stream of dividends generated by the unit of capital (and not by its future resale value in case of exit).

Given these assumptions, the residual in equation (16) is equal to the unobserved quality of the unit of capital. Thus, we can directly measure the volatility of the distribution of the observed and unobserved quality with $\hat{\sigma}_\omega$ and $\hat{\sigma}_a$, respectively. In addition, up to a first-order approximation, the assumptions imply that the estimated regression coefficient $\hat{\upsilon}_q$ represents the elasticity of the (log) selling probability $p(\theta(\omega, a))$ to the unobserved quality a evaluated at the average qualities. In the previous section, we show how this elasticity is a monotonic function of the degree of information asymmetries. As the asymmetry of information increases, sellers of high-quality capital choose a lower selling probability to signal their higher quality. Thus, the regression coefficient υ_q is informative regarding the degree of information asymmetries captured by ψ.

Quantitative illustration Our analytical identification results above were obtained under a set of simplifying assumptions. To show that the identification strategy holds more generally, Figure 4 illustrates the behavior of the moments $(\hat{\sigma}_\omega, \hat{\sigma}_a, \hat{\upsilon}_q)$ as we change the value of the parameters $(\psi, \sigma_\omega, \sigma_a)$. Panel (a) shows that changes in the quality of information technology ψ have a monotonic effect on the regression coefficient υ_q and unconditional average duration. Panel (b) shows that changes in the standard deviation of the unobserved quality σ_a have a positive and almost linear effect on the variance of the regression residuals $\hat{\sigma}_a$, as expected. In addition, more dispersed unobserved qualities naturally increase the incentives to mimic and induce sellers of higher qualities to signal their quality more strongly, which also increases the regression coefficient $\hat{\upsilon}_q$ and average duration. Panel (c) shows that changes in the standard deviation of the observed quality σ_ω also have a positive and almost linear effect on the variance of predicted prices $\hat{\sigma}_\omega$. As we further discuss below, the dispersion of observed qualities has a small interaction with the moments associated with asymmetric information, since information frictions distort the terms of trade of units of capital for a given observed quality ω. Finally, Panel (d) shows that increases in the efficiency of the matching technology $\bar{m}$ decrease the average duration of capital units. However, since a higher trading probability makes signaling harder (i.e., a higher matching efficiency reduces overall delay in the market), the regression coefficient $\hat{\upsilon}_q$ increases. To summarize, $\hat{\sigma}_\omega$ and $\hat{\sigma}_a$ are directly informed by the dispersion of observed and unobserved qualities. The regression coefficient $\hat{\upsilon}_q$ and the matching efficiency $\hat{\bar{m}}$ are identified by the fact that the regression coefficient and duration positively comove with $\hat{\upsilon}_q$, but move in opposite directions in response to changes in $\hat{\bar{m}}$.

4 Measurement

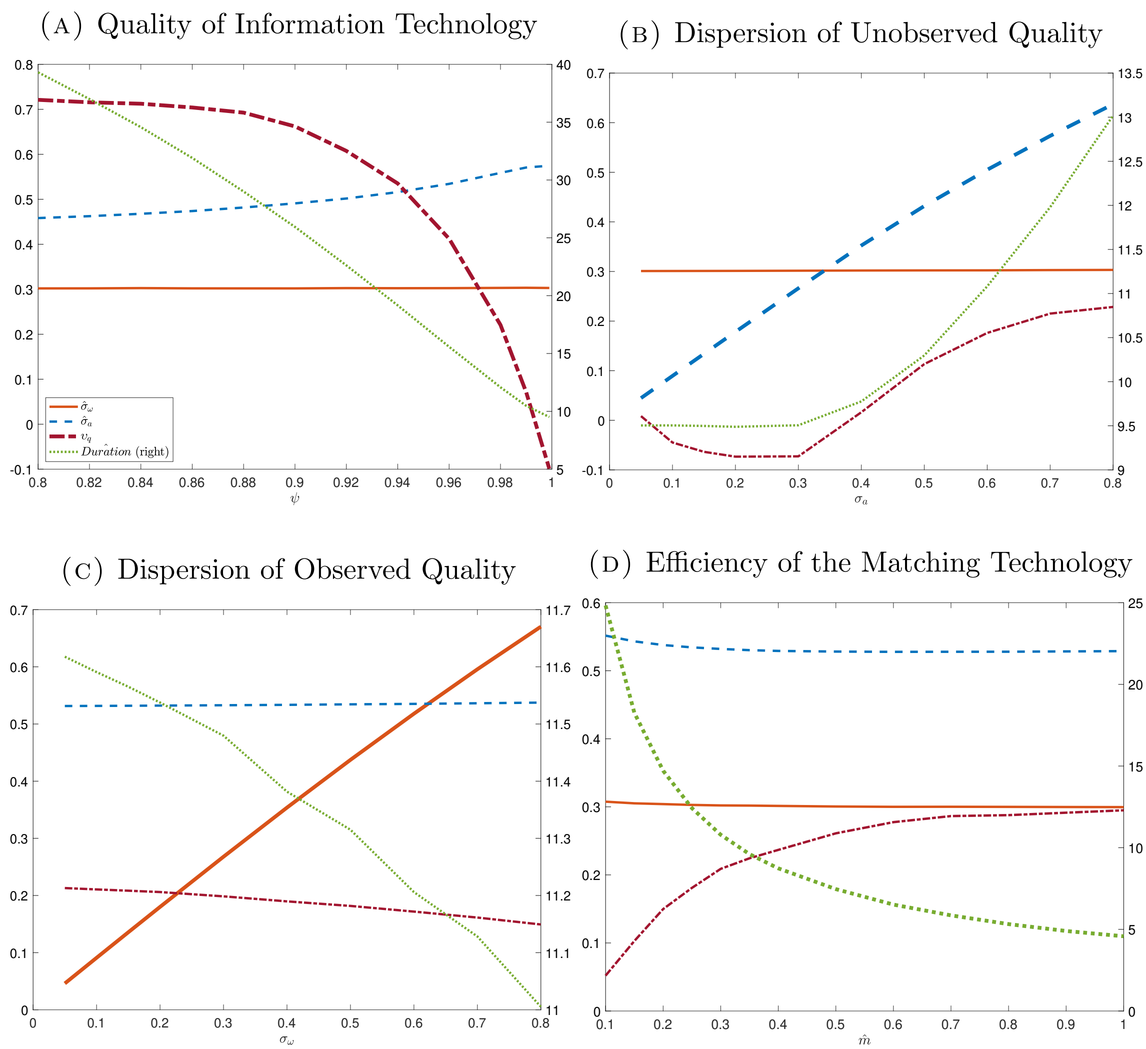
This section applies our proposed measurement to a novel dataset of capital units listed for trade. Section 4.1 describes the data. Section 4.2 presents a set of cross-sectional facts linked to the model predictions. Section 4.3 presents additional evidence linked to alternative interpretations of these facts.

4.1 Data

Our data consist of a rich panel of nonresidential structures (warehouses and retail and office space) listed for sale and rent. The source of these data is `Idealista`, one of Europe's leading online real estate intermediaries.[11] The frequency of the panel is monthly and includes the universe of capital units that were listed on the platform between 2005 and 2018. The data contain information during the period each listing was active online. The dataset includes approximately 8.9 million observations for Spain, where an observation corresponds to a property–month pair. Overall, these observations come from over 1.15 million different capital units. Appendix B provides more details on the data. In particular, Appendix B.1 describes how the online platform works. Appendix B.2 discusses the representativeness of the dataset

[11] `Idealista` is the leading online platform in the real estate market in Spain (see `Comparison of users` and `Comparison of platform`). For other papers using data from online platforms in the real estate market see Piazzesi et al. (2020).

FIGURE 4: Identification Illustration



(A) Quality of Information Technology

(B) Dispersion of Unobserved Quality

(C) Dispersion of Observed Quality

(D) Efficiency of the Matching Technology

Note: This figure reports the moments $\{\hat{\sigma}_\omega, \hat{\sigma}_a, \upsilon_q, \hat{\text{Duration}}\}$ computed model-simulated data, as we change the values of the parameters $\{\psi, \sigma_\omega, \sigma_a, \bar{m}\}$. The rest of the model parameters are set to their calibrated values from Section 5.

and shows that data from the online platform are consistent with aggregate patterns observed in Spain during the period of analysis in terms of the aggregate evolution of prices and the timing of sales.

For each property, we observe a wide range of characteristics detailed in the listing, including the address of the property, its construction year, its area, the number of rooms, and whether the property has heat or air conditioning, among others. We also observe the main variables discussed in our model measurement—namely, the capital unit's listed price, which we observe for each property at a monthly frequency, and its duration on the market, which we compute as the number of months the unit is listed on the platform.[12] Our dataset

[12]The platform asks sellers why they decided to close the listing. Figure B7 in Appendix B.3 compares

also features information about the search volume in each month, which we measure by the number of views and clicks each listing receives and the number of emails the seller receives from potential buyers through the platform.

Table 1 presents descriptive statistics on the listed characteristics, prices, duration, and search. Although we focus the analysis in this section on properties listed for sale, which have a more direct mapping to our model assumption, Appendix B.4 shows that we observe similar empirical patterns for properties listed for rent. The average sale price per square foot is $162 (expressed in constant 2017 dollars) and the average duration on the market is 10.5 months. Properties are relatively old, with an average age of around 26 years. Each listing is, on average, viewed 800 times per month and receives 45 clicks and 3 emails per month.

4.2 Cross-sectional Empirical Facts

We now use our data to provide a set of facts about the cross-section of listed capital units associated with the model's micro-level predictions.

Measuring predicted and residual prices Following the model's identification strategy discussed in Section 3.2, we begin by measuring the component of a listed price that can be predicted based on the property's characteristics included in the listing. We do so by estimating the following hedonic pricing regression:

$$\log(q_{it}) = \nu_{l(i)t} + \gamma X_i + \varepsilon_{it}, \tag{18}$$

where q_{it} is the real price per square foot of capital unit i in location $l(i)$, listed in month t; $\nu_{l(i)t}$ are location-by-time fixed effects; X_i is a set of observable characteristics included in the listing (see Table 1); and ε_{it} is a random error term.[13] Similar to Section 3.2, using

the histograms of duration for two groups of listings: those that closed the listing because the property was rented out or sold and those that do not provide an explanation. Those histograms are virtually identical. It is worth noting that Idealista is a paid service, so it is costly for the seller to keep a dormant listing after the property has been sold.

[13] Location fixed effects are defined, for each unit, at the finest geographic level possible in the platform: neighborhood level in the case of big cities like Madrid or Barcelona and city level in smaller cities. Results are similar if we focus only on cities that have available neighborhood information. In model (18), we focus on the average listed price during the lifetime of the listing. Table B1 in Appendix B.3 shows that when we estimate a version of model (18) using the entire panel dataset and including a listing fixed effect, less than 6% of the variation in prices can be accounted for by properties that change their price during the lifetime of the listing. To understand this result, Table B2 in Appendix B.3 shows that between 5% and 7% of listings change price in a given month.

Table 1: Descriptive Statistics

	Mean	Std. dev.
Price	162.27	131.71
Duration	10.47	11.21
Construction Date	1987.63	19.50
Area	3008.89	4619.22
New	0.05	0.22
Needs Restoration	0.14	0.35
Good Condition	0.81	0.40
Rooms	2.31	2.99
Restrooms	1.21	1.54
Heating	0.27	0.45
AC	0.64	0.48
E-Mails	2.75	2.12
Views	799.91	1273.95
Clicks	44.28	59.08
Number of Obs.	4.4e+05	4.4e+05

Note: Price is the price per square foot in constant 2017 dollars. Duration is the number of months a property was listed in the database. Construction date is the year the property was built. Property area is measured in square feet. "New" is a categorical variable that takes the value of 1 if the property is new. "Needs Restoration" is a categorical value that takes the value of 1 if the owner declares the property needs reparations. "Good Condition" takes the value of 1 if the property does not need reparations. "Rooms" is the number of separate rooms the property has, and similar for "Restrooms." Heating and AC are categorical variables that take the value of 1 when the property has some heating and air conditioning technologies. Emails is the number of times per month a property received an email from a potential customer. Views is the number of times a property appeared on the screen of a potential customer per month. Clicks is the number of times per month a potential customer clicked on the property listing to see its details. For those variables that change over time, we first take the average of the variable for each listing and report the average of that variable across listings.

the estimated coefficients $\{\hat{\nu}_{lt}, \hat{\gamma}\}$, we refer to $\hat{q}_{it} \equiv \hat{\nu}_{lt} + \hat{\gamma} X_i$ as "(log) predicted prices" and $\hat{\varepsilon}_{it} \equiv \log(q_{it}) - \hat{q}_{it}$ as "(log) residual prices."

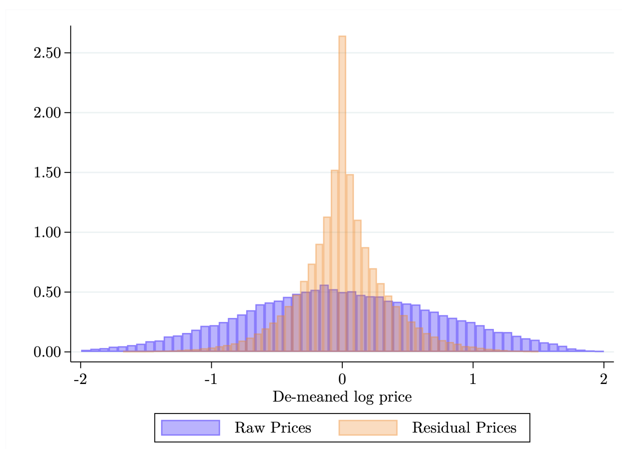
Using the estimated model (18), Table 2 shows that more than 70% of the variation in listed prices can be accounted for by characteristics included in the listing. The geographic dimension plays a salient role and explains almost 50% of the differences in listed prices. To illustrate this, Appendix Figure B8 shows large differences in sale prices across regions at different levels of aggregation. These maps demonstrate that locations vary significantly in their capital prices. Table 2 also shows that the time dimension explains 12% of the variation in listed prices, which is substantially smaller than the geographic dimension, despite the large fluctuations in capital prices Spain experienced during the Euro crisis (illustrated in Appendix Figure B9). Finally, Table 2 shows that the standard deviation of residual prices, which we obtain after including all available controls, is 37%, which is approximately 50% of the variation observed in the raw data. Figure 5 shows the distribution of price residuals, which illustrates the relevance of the dispersion in prices not accounted for by the characteristics in the listings.

TABLE 2: Price Variation Accounted for by Listed Characteristics

	St. Dev.	R^2
Raw data	0.75	0.00
Year	0.71	0.12
Location	0.54	0.48
Year $\times$ Location	0.49	0.57
... $\times$ Type	0.48	0.59
... $\times$ Area	0.38	0.74
... $\times$ Age	0.37	0.75
Benchmark	0.36	0.77

Note: This table reports the R^2 and standard deviation of residuals from estimating equation (18). The row labeled Raw data presents statistics for the demeaned raw log prices. The following rows include the fixed effects in the regression. Year and location denote fixed effects. Type (office and retail space, or warehouse), area, and age are sets of fixed effects for each of these characteristics. The last row includes additional controls for the variables listed in Table 1.

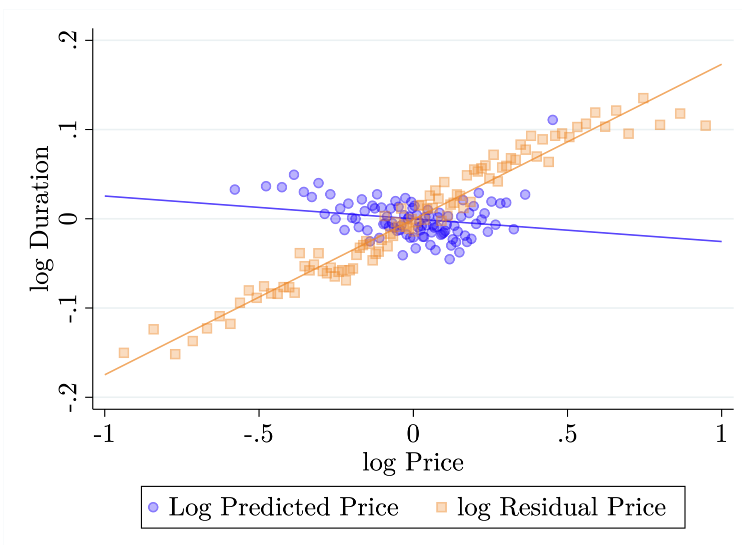
FIGURE 5: Distribution of Price Residuals



Note: This figure shows the distribution of log prices per square foot relative to its mean for the raw data and price residuals after including the fixed effects in Table (2).

Relationship between prices and duration Guided by our model predictions, we now analyze the relationship between units' predicted and residual prices and their duration on the market. Figure 6 shows that units with higher predicted prices tend to have a shorter duration on the market, while units with higher residual prices tend to have a higher duration on the market. Table 3 presents the same results in a regression framework. In column (1), we regress (log) duration on (log) prices and obtain a negative and statistically significant relation. In the second column, we split the (log) price into two components—predicted and residual prices—and run the same regression. While we obtain a positive and statistically significant relationship between duration and predicted prices, we obtain a positive and statistically significant relationship between duration and residual prices. In the last two columns, we estimate similar regressions but include location-time-property-type fixed effects and obtain similar results.[14]

FIGURE 6: Relationship between log Duration and log Prices



Note: This figure shows the relationship between log prices and duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (18)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table B3 and Figure B11 in Appendix B.3 reproduce the same analysis by replacing duration with the average monthly clicks received by a listing (as a proxy for search intensity). Results are consistent with those found for duration. Properties with high predicted prices receive more clicks on average, which is consistent with a shorter duration, and properties with high residual prices receive fewer clicks on average, which is consistent with a longer duration. This last set of results is important, because it shows that listed prices do play an important role in attracting or repelling potential buyers by affecting their search behavior.

Through the lens of the model, the different relations that residual and predicted prices

[14]The reason for the inclusion of time-location fixed effects in the regression is to allow for the process of duration on the market to differ over time and location (e.g., the match efficiency could be market-specific). However, the theory predicts that if a better observable location contributes positively to the quality of the property, it should also have a positive effect on the trading probability. Therefore, the inclusion of fixed effects is absorbing part of this effect as well.

Table 3: Prices and Duration

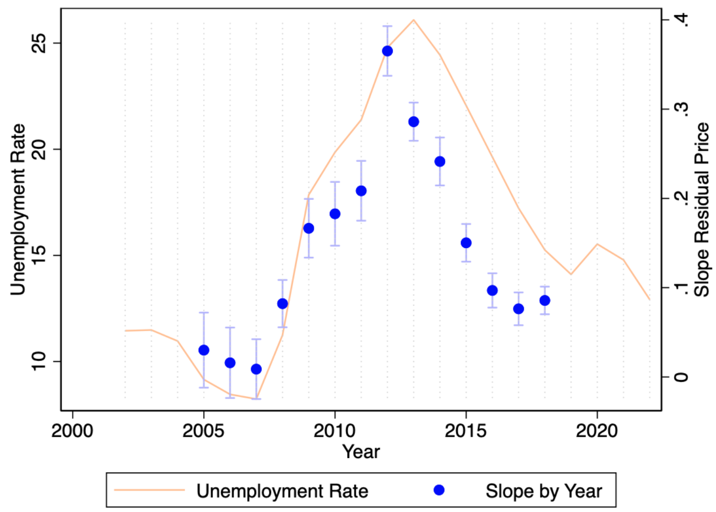
	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	0.013***		0.128***	
	(0.004)		(0.004)	
log Predicted Price		-0.071***		-0.025**
		(0.005)		(0.011)
log Residual Price		0.148***		0.148***
		(0.004)		(0.004)
Constant	1.961***	2.361***	1.407***	2.141***
	(0.018)	(0.026)	(0.019)	(0.053)
Observations	456351	439680	439680	439680
R^2	0.000	0.009	0.226	0.228
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The left-hand-side variable is the log duration of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of duration on prices. Column 2 regresses duration on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

have with duration suggest an important role of information asymmetries. When higher prices stem from listed characteristics, such as the location of the unit—which can be perfectly observed by buyers—they tend to be associated with shorter time to sell. When high prices cannot be linked easily to observable characteristics, they are associated with longer time to sell. Under the null hypothesis of full information, according to our model, residual prices reflect characteristics of properties not observed by the econometrician. Thus, we should expect a negative relation with duration, as is the case with predicted prices. The fact that we estimate a positive relation provides evidence that the extent of asymmetric information cannot be zero. This conclusion is more formally supported in the estimation exercise of the model, which allows us to provide a quantitative magnitude of the deviation from full information.

Business-cycle fluctuations Economic downturns are often characterized as times when information about the quality of assets deteriorates. To study this in our data, we estimate the relationship between a unit's duration on the market and its residual price separately for each year in our sample. Figure 7 shows that the slope between duration and residual prices exhibit substantial business-cycle variation, showing a strong comovement with the fluctuations of labor unemployment and more than doubling during the Euro crisis that started in 2008. Through the lens of our model, these patterns are consistent with the degree of asymmetric information decreasing during downturns. Motivated by this evidence, in the next section, we examine the macroeconomic effects of changes in the accuracy of information technologies ψ_t.

FIGURE 7: Cyclical Fluctuations in the Slope between Duration and Residual Prices



Note: This figure shows the relationship between log prices and duration over the business cycle. The blue points represent the regression coefficient of log residual prices when estimating the specification (4) in Table 3 for each year in the sample separately (vertical bars denote 95% confidence intervals). The solid line depicts the national unemployment rate in Spain during the sample period (source: Statistical Agency of Spain INE).

4.3 Discussion of alternative interpretations

So far, we have interpreted cross-sectional facts through the lens of our model with decentralized capital markets and asymmetric information. In Online Appendix B.5, we provide additional evidence to indicate that these empirical patterns would be hard to account for by alternative explanations that do not involve asymmetric information.

To briefly summarize, we first explore the possibility that the positive relationship between residual price and duration on the market that relies on sellers' indifference across these variables (an explanation akin to that in Burdett and Mortensen, 1998, for labor- and product-markets). To study whether the trade-off between residual prices and duration can account for their positive relationship in the data, we compute the expected net present discounted revenue for properties with different residual prices under alternative preferences. The results show that the expected net present discounted revenue monotonically increases in the listed price, indicating that sellers' indifference cannot explain the observed relationship between residual prices and duration. We then analyze the possibility that sellers have varying expenses associated with holding onto their property (e.g., maintenance costs, taxes, and debt service costs) that they must pay every period until their property is sold. If some sellers have higher costs than others, they might have to sell their property quickly and at a lower price. We used our data to determine the minimum cost that would make it reasonable for a seller to choose a lower residual price and found that these are implausibly large (e.g., the cost of holding 1 square foot of a property for 1 additional month would have to be larger than the price at which the owner can sell that unit). Finally, we consider the possibility that differences in buyers' liquidity for different units could explain our facts. However, the positive relationship between residual prices and duration holds for both high- and low-priced units (in terms of total and predicted prices), indicating that buyer liquidity is not the main factor driving the results.

5 The Macro Effects of Asymmetric Information

This section combines the model and empirical measurement to study the macroeconomic effects of asymmetric information. Section 5.1 discusses the model parameterization. Section 5.2 studies the impact of asymmetric information on steady-state macroeconomic variables, and Section 5.3 its impact on economic fluctuations.

5.1 Calibration

We calibrate the model in two steps. First, we fix a subset of parameters. Second, we calibrate the remaining parameters—which govern the degree of trading frictions—to match key data moments discussed in Section 4.

Fixed parameters The parameters we fix in the calibration are detailed in Table 4. The model is calibrated at a monthly frequency. A subset of these parameters is shared with the neoclassical stochastic-growth model and is set to standard values from the literature. For preferences, we set the discount factor to $\beta = 0.996$, which is associated with a 4% annual rate of time preference; the Frisch elasticity of labor supply, $1/\xi$, to one; and the disutility of labor, ϖ, to target steady-state hours worked, $\overline{h} = 1/3$. Regarding the firm's technology, we set the share of capital to $\alpha = 0.35$ (consistent with Fernald, 2014). We set the depreciation

Table 4: Fixed Parameters

Parameter	Description	Value
β	Discount factor	0.9966
α	Share of capital	0.35
δ	Depreciation rate	0.0074
γ	Technology growth	1.004
γ_n	Population growth	1.0027
φ	Firms' exit rate	0.0027
η	Curvature matching technology	0.8
ϕ	Bargaining power of seller	0.5

Note: This table shows the parameters we fix in the calibration.

rate to $\delta = 0.0074$, which corresponds to an annual rate for nonresidential capital of 8.5% (source: BEA, Fixed Asset tables); the growth rate of technical progress to $\gamma = 1.004$, which is associated with an annual technology growth rate of 1.6%—the growth rate per worker the U.S. economy experienced from 1980 to 2015 (data source: BEA)—and the population growth to $\gamma_n = 1.0027$, which is associated with an annual growth rate of the working-age population in the period of analysis of 1% (population aged 15–64, data source: Federal Reserve Bank of St. Louis and OECD). For the exit rate of firms, which governs separation flows, we set $\varphi = 0.008$, which corresponds to the 3.2% average exit rate of U.S. establishments, obtained as a weighted average of exit rates for establishments of different sizes reported by the U.S. Census Bureau. For search-and-matching frictions, we set the curvature of the matching technology to $\eta = 0.8$ (as estimated by Ottonello, 2017) and the bargaining parameter to $\phi = 0.5$ as a benchmark (used in the context of labor markets, for example, by Shimer, 2010), and analyze how the results vary with alternative parameter values.

Fitted parameters We calibrate the remaining parameters, $\{\psi, \sigma_\omega, \sigma_a, \overline{m}\}$, following the identification strategy proposed in Section 3.2 and targeting four key data moments measured in Section 4 and reported in Table 5. These moments are the average selling probability, which is mostly governed by the matching efficiency $\bar{m}$; the standard deviation of predicted and residual prices, which are mostly governed by the standard deviation of capital qualities, σ_ω and σ_a; and the slope in the regression of duration on residual prices, which is informative of the quality of information technology ψ. The calibration strategy of these parameters proceeds as follows. For a given set of parameters, we compute the equilibrium choices of prices and transaction probabilities for each type of capital. Then, we simulate the evolution of multiple units of capital, generate a sample of listed units (similar to that of listed properties in our dataset), and perform the same measurement analysis to obtain these moments in the model-simulated data as we performed with the data in Section 4. Finally, we use a minimum-distance estimator to choose parameter values that match the moments in the data. Table 5 shows that our parameterized model matches fairly well the moments targeted in our calibration. Table 6 also reports the results from running in model-simulated data the regressions between duration and predicted and residual prices considered in the empirical analysis of Section 4, which indicates that the model is aligned with the (untargeted) relationship between duration and predicted prices.

Table 5 also reports the parameters obtained from the calibration. The calibrated parameter for the accuracy of information technology is $\psi = 0.98$, which indicates that the probability that a lemon goes unnoticed is 2%. Therefore, the economy features moderate levels of information asymmetry in the steady state. To use as a benchmark in our quantitative exercises, we also estimate the accuracy of the information technology that would correspond to an economy with the larger degree of asymmetric information measured during the Euro crisis. As reported in Figure 7, in this episode the slope between duration and residual prices reached a level of 0.38, which would correspond to a level of $\psi = 0.96$ (keeping the rest of the model parameters constant). Section 5.3 considers an experiment in which the degree of asymmetric information measured by the slope between duration and residual prices increases only temporarily, as observed in the crisis episode.

Table 5: Fitted Parameters and Targeted Moments

Parameter	**Description**	**Value**	**Target**	**Model**	**Data**
$\bar{m}$	Matching efficiency	0.2777	Mean Duration	11.4	11.4
σ_ω	SD observed quality	0.34	SD of log predicted prices	0.30	0.34
σ_a	SD unobserved quality	0.64	SD of log residual prices	0.56	0.54
ψ	Accuracy information technology	0.9839	Regression coefficient	0.154	0.154

Note: This table shows the parameters that we calibrate by minimizing the distance between four moments in the data and in our simulated model.

TABLE 6: Relationship between Duration and Prices: Data and Model

	Data log Duration	Model log Duration
log Predicted Price	-0.02	-0.08
log Residual Price	0.154	0.154
Constant	2.11	1.99

Note: This table reproduces the regression coefficients in the data and the model. The left-hand-side variable is log duration and we regress it on a constant and our measures of predicted and residual prices. Refer to the empirical section for further details.

5.2 Steady-state Analysis

Aggregate channels To decompose the channels through which asymmetric information affects the macroeconomy, we can express aggregate output, $Y_t \equiv \int y_{jt}\,\mathrm{d}j$, as:

$$\begin{aligned} Y_t &\equiv \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha} \qquad (19) \\ &= \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega\in\omega} \sum_{a\in\mathcal{A}} K_t(\omega,a) \right] \left[\mathbb{E}\left(\omega a\right)\left(1-\mathbb{E}\left(u_t(\omega,a)\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega,a)\right)\right] \right)^{\alpha}, \end{aligned}$$

where $L_t \equiv h_t(\mathbf{k})\gamma_n^t - \int\int \sum_{\omega}\sum_{\hat{a}} v_{jt}(\omega,\hat{a},q)\,\mathrm{d}q\,\mathrm{d}j$ denotes labor used in production; $\mathcal{K}_t \equiv \int \mathcal{K}_{jt}\,\mathrm{d}j$ denotes aggregate capital input used in production; and $u_t(\omega,a)$ and $K_t(\omega,a)$ denote the aggregate unemployment rate and the aggregate stock of capital of type (ω,a), respectively. Equation (19) indicates that there are four channels through which asymmetric information can affect aggregate output. First, by affecting prices and trading probabilities, asymmetric information distorts the return to capital investment, which reduces the aggregate units of capital in the economy. Second, asymmetric information affects the aggregate utilization rate of capital in two ways, since it distorts both the average unemployment rate of capital and the composition of the pool of unemployed capital, captured by the covariance between the type-specific unemployment rate and the efficiency units of capital $\mathbb{C}ov\left(\omega a, u_t(\omega,a)\right)$. These three channels affect aggregate capital input used in production, which is high when the number of units of capital is high, the average efficiency units is high, and the average unemployment rate is low. Finally, by distorting the terms of trade, asymmetric information affects search efforts and the demand for labor used in production.

Effects of asymmetric information Figure 8 shows how the degree of asymmetric information affects aggregate economic activity through the channels discussed above. Panel

(a) shows that a lower accuracy of the information technology, ψ, is associated with a lower capital stock. This is because higher information asymmetries are associated with lower revenue for sellers of high-quality capital, which decreases the returns to producing capital goods. The effects are quantitatively large: Even though the economy features moderate levels of asymmetric information, the steady-state capital stock is 4.5% lower relative to its full-information level.[15]

Panel (b) shows that higher information asymmetries lead to a higher unemployment rate of capital. As information asymmetries increase, so do the listed prices of high-quality capital sellers, which decreases selling probabilities and increases the duration in unemployment of listed units up to 1 p.p. relative to a full-information unemployment rate of 5%. This effect is compounded by the fact that information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities (see Panel (c)), although this channel has the smallest independent effect on output. By combining these three effects, Panel (d) shows that a higher degree of asymmetric information is associated with a lower quality of employed capital of approximately 5% relative to the full information benchmark. A lower equilibrium level of capital input reduces the demand for labor, resulting in a 1% lower labor input and 2.3% lower output.

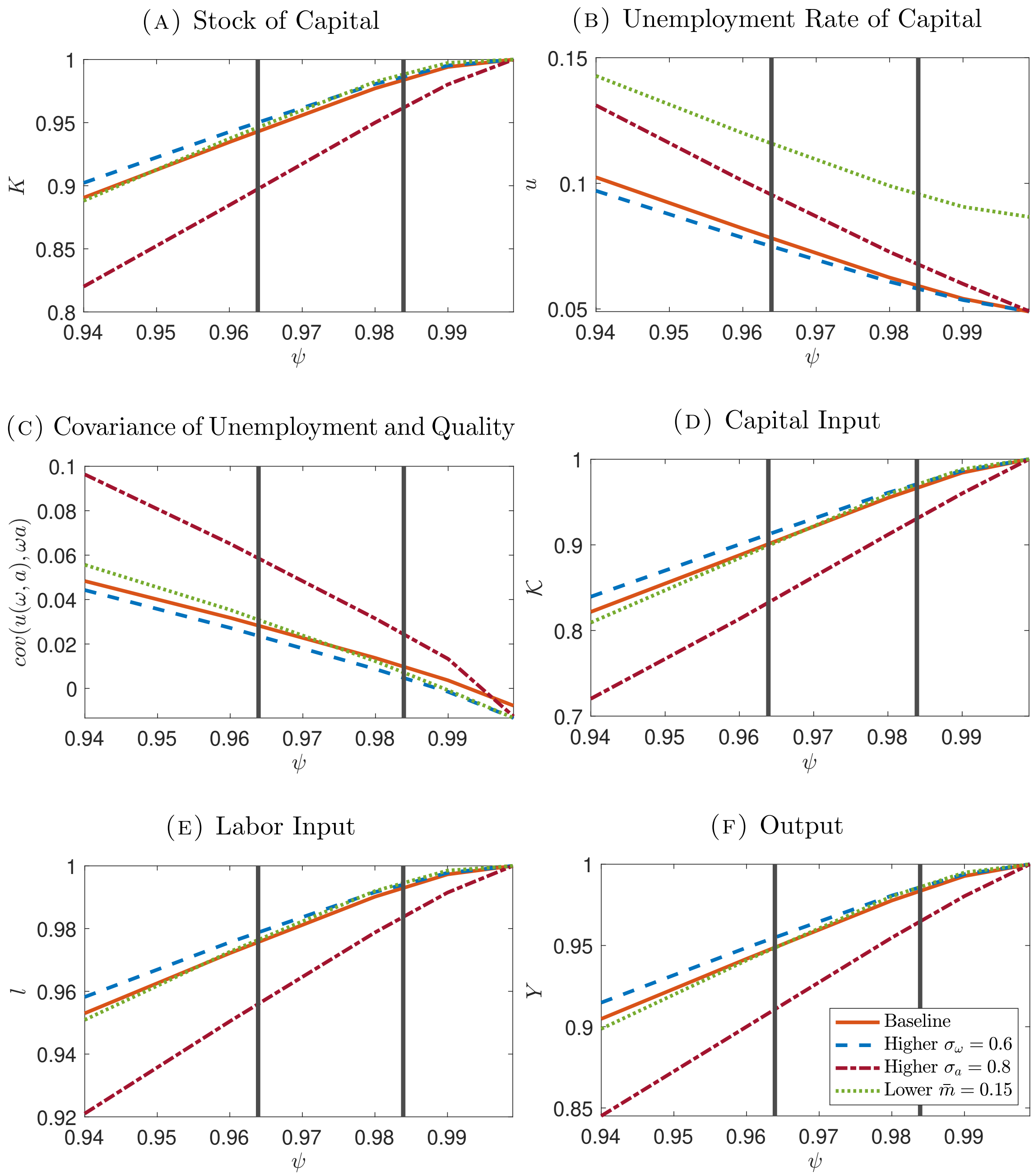
5.3 Economic Fluctuations

We now study the macroeconomic effects of changes in information technologies, ψ_t. Our experiment is motivated by the dynamics of capital markets observed during the Euro crisis documented in Figure 7, in which the slope between duration and residual prices sharply increased during the economic downturn. Through the lens of our model, such dynamics can be accounted for by a decline in the accuracy of information technologies, which increases the degree of asymmetric information in the economy.

Figure 9 shows that increases in the degree of asymmetric information driven by changes

[15] To understand the large aggregate effect for moderate levels of asymmetric information, we note that the relationship between expected duration on the market and residual prices is informative of the expected gains from mimicking of low-quality sellers. High gains from mimicking can arise either because the information technology is bad (i.e., ψ is low) or because the seller with the highest incentive to mimic is of very low quality. In our calibrated model, the constraint that mostly binds in the sequence of problems $\mathcal{P}_j(\omega)$ is that of the lowest quality. Thus, the expected gains from mimicking informed by the data can be rationalized with a relatively good information technology.

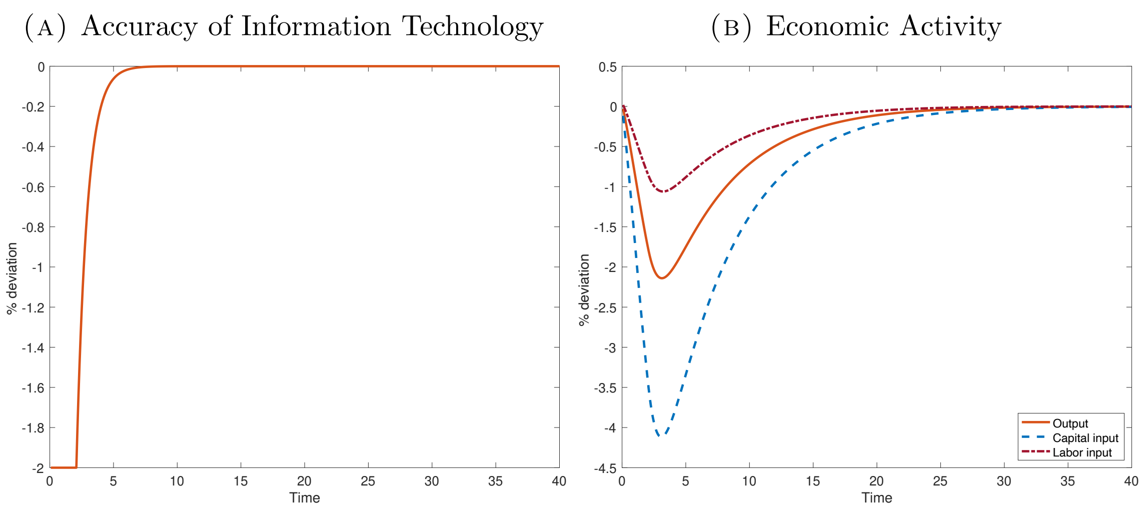
FIGURE 8: Degree of Asymmetric Information and Macroeconomic Variables



Note: This figure shows the decomposition of the macroeconomic effects of changes in the quality of the information technology for four parameterizations of the model. Our baseline calibration is represented by the solid red line. We also report a calibration with a higher variance of observed quality in dashed blue; a calibration with a higher variance of unobserved quality in dash-dot crimson; and a calibration with a lower efficiency of the matching technology in dotted green. All numbers are reported as a percentage of the value of output under full information, except for the capital unemployment rate, which is shown in rates.

in information technologies lead to large contractions in economic activity. Panel (A) depicts the dynamics of ψ_t considered in our experiment, aimed at generating an increase in the slope between duration and residual prices or the magnitude and duration observed during the Euro crisis reported in Figure 7. In particular, we assume that in $t = 0$ the economy experiences a 2 p.p. unexpected decline in the accuracy of information technology to $\psi_0 = 0.96$, which lasts for 3 years, and reverts to its steady-state value following a first-order autoregressive process.[16] Panel (B) shows that this decline in the accuracy of information technologies is associated with a more than 2% decline in output in the three years following the shock, followed by a slow recovery, which takes more than 5 years to recover half of the output decline. This decline in economic activity is mostly driven by a decline in capital input, explained both by a decline in the production of new capital goods and an increase in capital unemployment.

FIGURE 9: Macroeconomic Reponses to Changes in Information Technologies



Note: This figure shows the impulse responses of output, capital input, and labor input to an unexpected decline in the accuracy of the information technology ψ_t. Panel (A) depicts the assumed path for ψ_t considered in the exercise. Panel (B) shows the response of aggregate output Y_t, capital input $\mathcal{K}_t$, and labor input L_t. The horizontal axis displays years after the shock. Impulse responses are expressed in percentage deviations from the detrended steady state.

6 Conclusion

In this paper, we show that information asymmetries in capital markets have important macroeconomic implications, affecting an economy's investment, capital allocation, and economic

[16]More specifically, we assume that, during the recovery, the accuracy of the information technology follows the process $\psi_t = \rho_\psi \psi_{t-1}$. We parameterize this process to match the half-life of the slope between duration and residual prices observed in Figure 7.

activity. This conclusion emerges from conducting a micro-to-macro approach, which combines microlevel data on capital units listed for trade with a quantitative capital-accumulation model with illiquid capital markets and asymmetric information. The results of our paper suggest the importance of studying capital-market policies designed to address potential inefficiencies that arise from information asymmetries. For example, one can use our quantitative framework to investigate the welfare benefits of implementing a pooling equilibrium (e.g., by setting taxes on capital prices that prevent signaling). In addition, our results suggest the importance of further studying agents' incentives for developing information technologies that mitigate information frictions, in a version of the model in which the accuracy of information technologies is endogenous. We leave this analysis for future research.

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A Theory Appendix

In this section, we provide all the proofs of our theory. Instead of focusing on a specific post-inspection trading protocol, as we did in the main text, here we provide a set of general sufficient conditions that the inspection-adjusted price function must satisfy and generalize the proof to any protocol that satisfies the following assumption.

***Assumption* 1.** *The inspection-adjusted price function $q_t^P(\omega, a, \hat{a}, q) : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to \mathbb{R}_+$ has the following properties:*

(i) it is non-decreasing in the true quality:

$$\forall (a, a') \in \mathcal{A}^2 \text{ such that } a' > a,\ \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ : q_t^P(\omega, a', \hat{a}, q) \geq q_t^P(\omega, a, \hat{a}, q),$$

(ii) it is non-increasing in the announced quality:

$$\forall (\hat{a}, \hat{\hat{a}}) \in \mathcal{A}^2 \text{ such that } \hat{a} > \hat{\hat{a}},\ \forall (\omega, a, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ : q_t^P(\omega, a, \hat{a}, q) \leq q_t^P(\omega, a, \hat{\hat{a}}, q),$$

(iii) it is weakly lower (resp. higher) than the buyer's (resp. seller's) value for the unit:

$$q_t^P(\omega, a, \hat{a}, q) \in [\min(q, \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a), \min(q, \nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}))]$$

$\forall \omega \in \Omega, \hat{a}, a \in \mathcal{A}, q \in \mathbb{R}+, \mathbf{K}_{Ht} \in \mathbb{R}+,$

(iv) it is such that buyers obtain at least a fraction $1 - \eta$ of the surplus:

$$q_t^P(\omega, a, \hat{a}, q) \leq \eta \nu_t^{\mathrm{b}}(\omega, a, \mathbf{K}_{Ht}) + (1 - \eta)\left(\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right),$$

(v) it does not decrease "too fast" as the announced quality increases, i.e.:

$$\frac{\eta(\nu_t^{\mathrm{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i)}{q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i} \geq \frac{q^B(\omega, a_j, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}{q^B(\omega, a_j, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}$$

$\forall a_j < a_i < a_k \in \mathcal{A}, \omega \in \Omega.$

The first assumption requires that for a given announced quality, sellers obtain a weakly higher post-inspection price the higher their true revealed quality is. Relatedly, the second assumption states that after the inspection reveals the true quality, sellers of higher announced quality are weakly worse off. This protocol captures a variety of contractual arrangements that punish sellers for lying about the true quality of their units. For example, these assumptions allow for a post-inspection bargaining price that sanctions sellers more severely when the difference between true and announced quality increases. They also allow for sanction-less bargaining, which we consider in our quantitative analysis. The third assumption states that the inspection-adjusted price is bounded by the buyer's valuation of the unit and the seller's outside option, which corresponds to its continuation value if the transaction does not happen. This implies that a transaction occurs as long as the gains from trade are positive. Notice that this assumption also incorporates the seller's commitment to sell at the initially posted price q. The fourth assumption requires the inspection-adjusted price to be weakly lower than the price the seller would obtain under full information, which we derive below. Intuitively, after units are inspected and their true qualities are revealed, sellers should not be able to transact at a higher price than they would have received if all information were publicly available. The post-inspection price we propose can capture situations in which sellers who lie about the quality of their units are "punished" with a lower transacted price (e.g., by increasing the buyer's bargaining power). The final assumption limits how large this "punishment" can be. This is a sufficient condition that ensures that the separating equilibrium derived below has sellers truthfully reporting their quality and rules out pathological equilibria.

In the quantitative analysis, we use a standard Nash bargaining protocol to determine post-inspection price $q^P(\omega, a, \hat{a}, q)$. At the end of this section, Lemma 2 shows that such Nash solution satisfies the above assumptions if the seller's bargaining power satisfies $\phi \leq \eta$.

A.1 Proofs

Proof of Proposition 1

Here we prove a more general version of Proposition 1, in which we endogenize all selling and buying decisions of firms and households. Let us denote by $b^{FI}(\omega, \hat{a}, q, a) \in \{0, 1\}$ the decision of buyers to purchase the unit on submarket $(\omega, \hat{a}, q)$ conditional on learning from the inspection that it is of quality a. Similarly, let $b(\omega, \hat{a}, q) \in \{0, 1\}$ be the decision of buyers to purchase the unit conditional on visiting the submarket $(\omega, \hat{a}, q)$ and not learning the true quality from the inspection. Let $s(\omega, a)$ be the seller's decision to post the unit of quality

(ω, a) for sale. In what follows, we drop the subscript j of an individual firm.

Household's problem. The recursive optimization of the household can be written as

$$V_{Ht}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a),c,h,i>0\}}} u(c,h)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

subject to the per-period budget constraint

$$c\gamma_n^t + i + \delta \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \omega a \left(k'(\omega,a) - ig(\omega,a)\right) = w_t h \gamma_n^t + x^{\mathrm{s}} - x^{\mathrm{b}} + Div_{Ft},$$

the law of motion of capital of quality (ω, a)

$$k'(\omega,a) = k^{\mathrm{b}}(\omega,a) - k^{\mathrm{s}}(\omega,a) + k(\omega,a) + ig(\omega,a) + \varphi K_{Ft}(\omega,a)$$

and the nonnegativity constraints $v(\omega,\hat{a},q) \geq 0 \; \forall(\omega,\hat{a},q) \in \Omega\times\mathcal{A}\times\mathbb{R}_+$, where total purchases of quality (ω, a) are given by

$$k^{\mathrm{b}}(\omega,a) = \sum_{\hat{a}\in\mathcal{A}} \int_{q\in\mathbb{R}_+} \iota_t(a|\omega,\hat{a},q)[\psi b^{FI}(\omega,\hat{a},q,a) + (1-\psi)b(\omega,\hat{a},q)]\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) v(\omega,\hat{a},q)\,\mathrm{d}q,$$

total sales are given by

$$k^{\mathrm{s}}(\omega,a) = \left(\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) + (1-\psi)b_t(\omega,\hat{a}(\omega,a),q(\omega,a))\right) p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)))s(\omega,a)k(\omega,a)$$

total costs of buying capital are given by

$$\begin{aligned} x^{\mathrm{b}} = \sum_{\hat{a}\in\mathcal{A}}\sum_{\omega\in\Omega}\int_{q\in\mathbb{R}_+} \Big[&(\psi \sum_{a\in\mathcal{A}} \iota_t(a|\omega,\hat{a},q) q_t^P(\omega,a,\hat{a},q) b^{FI}(\omega,\hat{a},q,a) \\ &+ (1-\psi) q b(\omega,\hat{a},q))\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t \Big] v(\omega,\hat{a},q)\,\mathrm{d}q, \end{aligned}$$

and total revenues from selling capital are given by

$$\begin{aligned} x^{\mathrm{s}} = \sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}} \Big[&\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) q^P(\omega,a,\hat{a}(\omega,a),q(\omega,a)) \\ &+ (1-\psi) b_t(\omega,\hat{a}(\omega,a),q(\omega,a)) q(\omega,a) \Big] p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))) s(\omega,a) k(\omega,a). \end{aligned}$$

The optimal level of investment, provided that $i > 0$, is given by the first-order condition

$$1 = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} g(\omega, a) \lambda_t(\mathbf{k}) \nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta \gamma_n \frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$, $\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy function for capital accumulation associated with problem (2.2), and $\nu^{\mathrm{s}}_t(\omega, a, \mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega,a)} \frac{1}{u_{ct}(\mathbf{k})\gamma^t}$ is the marginal value of capital of type (ω, a) measured in final goods, which satisfies the recursive problem (its notation anticipates the result whereby households only sell capital, which is derived below):

$$\begin{aligned}
&\nu^{\mathrm{s}}_t(\omega, a, \mathbf{k}) = \max_{\{\hat{a}(\omega,a), q(\omega,a)\}} \\
&s(\omega, a) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))) \left[\psi b^{FI}_t(\omega, \hat{a}(\omega, a), q(\omega, a), a) q^P_t(\omega, a, \hat{a}(\omega, a), q(\omega, a)) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a)\right] \\
&+ \Big(1 - (\psi b^{FI}_t(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))) s(\omega, a) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big) \left[\lambda_t(\mathbf{k}) \nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{k}_{Ht+1}(\mathbf{k})) - \delta \omega a\right].
\end{aligned}$$

Firm's problem. The recursive optimization problem faced by firms can be written as

$$V_F(\mathbf{k}) = \max_{\substack{\{l, v(\omega,\hat{a},q), q(\omega,a), b(\omega,\hat{a},q), \\ b^{FI}(\omega,\hat{a},q,a), s(\omega,a), \hat{a}(\omega,a), k'(\omega,a)\}}} \mathbb{E}_a[div + \Lambda' \left((1-\varphi) V'_F(\mathbf{k}') + \varphi V^{\mathrm{exit}}_{t+1}(\mathbf{k}')\right)],$$

subject to the nonnegativity constraints $v(\omega, \hat{a}, q) \geq 0 \; \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+$, the definition of per-period dividends

$$div = \left(\sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a)\right)^{\alpha} (\gamma^t l)^{1-\alpha} - wl - \delta \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a) - x^b + x^s,$$

and the law of motion of capital of quality (ω, a)

$$k'(\omega, a) = k^{\mathrm{b}}(\omega, a) - k^{\mathrm{s}}(\omega, a) + k(\omega, a), \tag{A.1}$$

where total purchases of quality (ω, a) are given by

$$k^{\mathrm{b}}(\omega, a) = \sum_{\hat{a} \in \mathcal{A}} \int_{q \in \mathbb{R}_+} \iota_t(a|\omega, \hat{a}, q)[\psi b^{FI}(\omega, \hat{a}, q, a) + (1-\psi) b(\omega, \hat{a}, q)] \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v(\omega, \hat{a}, q) \, \mathrm{d}q, \tag{A.2}$$

total sales are given by

$$k^{\mathrm{s}}(\omega, a) = \left(\psi b^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b(\omega, \hat{a}(\omega, a), q(\omega, a))\right) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))) s(\omega, a) k(\omega, a), \tag{A.3}$$

total costs of buying capital are given by

$$\begin{aligned} x^{\mathrm{b}} = & \sum_{\hat{a}\in\mathcal{A}} \sum_{\omega\in\Omega} \int_{q\in\mathbb{R}_+} \Big[(\psi \sum_{a\in\mathcal{A}} \iota_t(a|\omega, \hat{a}, q) q^P(\omega, a, \hat{a}, q) b^{FI}(\omega, \hat{a}, q, a) \\ & + (1-\psi) q b(\omega, \hat{a}, q)) \mu\left(\theta\left(\omega, \hat{a}, q\right)\right) + w_t \Big] v(\omega, \hat{a}, q) \, \mathrm{d}q, \end{aligned} \tag{A.4}$$

and total revenues from selling capital are given by

$$\begin{aligned} x^{\mathrm{s}} = & \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}} \Big[\psi b^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q^P(\omega, a, \hat{a}(\omega, a), q(\omega, a)) \\ & + (1-\psi) b(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a) \Big] p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))) s(\omega, a) k(\omega, a). \end{aligned} \tag{A.5}$$

The recursive problem of the firm features a static choice of labor demand and only depends on the number of efficiency units of capital $\mathcal{K}' = \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}} \omega a k'(\omega, a)$. The first-order condition with respect to l is given by

$$\mathcal{K}'^{\alpha} \gamma^{t(1-\alpha)} (1-\alpha) l^{-\alpha} = w_t,$$

which can be rewritten as

$$l = \mathcal{K}' \left(\frac{(1-\alpha)\gamma^{t(1-\alpha)}}{w_t} \right)^{\frac{1}{\alpha}}.$$

Hence, labor demand is linear in $\mathcal{K}'$, which proves the last part of Proposition 1. We can express the revenue from production as

$$\Phi_t(\mathbf{k}') = \mathcal{K}'^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l.$$

Replacing our expression for the optimal labor demand, we obtain that $\Phi_t(\mathbf{k}') = Z_t \mathcal{K}'$, where

$$Z_t \equiv \alpha \left(\frac{\gamma^t (1-\alpha)}{w_t} \right)^{\frac{1-\alpha}{\alpha}}.$$

Given this result, we can now re-express the problem of the firm as

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a)\}}} \mathbb{E}_a\Big[\,(Z_t-\delta)\,\mathcal{K}'-x^b+x^s\Big]+\Lambda_{t,t+1}\left((1-\varphi)V_{Ft+1}(\mathbf{k}')+\varphi V_{t+1}^{\text{exit}}(\mathbf{k}')\right),$$

subject to (A.1), (A.2), (A.3), (A.4), (A.5), and the nonnegativity constraint $v(\omega,\hat{a},q)\geq 0\ \forall(\omega,\hat{a},q)\in\Omega\times\mathcal{A}\times\mathbb{R}_+$.

Next, we conjecture that $V_{Ft}(\mathbf{k})=\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\nu_t^{\text{b}}(\omega,a)k(\omega,a)$. Let us denote by $\xi_t(\omega,\hat{a},q)$ the Lagrange multiplier associated with the nonnegativity constraint for vacancies in all submarkets $(\omega,\hat{a},q)\in\Omega\times\mathcal{A}\times\mathbb{R}+$. The first-order condition with respect to $v(\omega,\hat{a},q)$ is

$$\mathbb{E}_a[\psi b^{FI}(\omega,\hat{a},q,a)+(1-\psi)b(\omega,\hat{a},q)]\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)\left((Z_t-\delta)\omega a+\Lambda_{t,t+1}((1-\varphi)\nu_{t+1}^{\text{b}}(\omega,a)+\varphi\nu_{t+1}^{\text{s}}(\omega,a,\mathbf{K}_{Ht+1}))\right)=$$
$$[(\psi\mathbb{E}_a q_t^P(\omega,a,\hat{a},q)b^{FI}(\omega,\hat{a},q,a)+(1-\psi)qb(\omega,\hat{a},q))\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)+w_t]+\xi_t(\omega,\hat{a},q),$$

together with the complementary slackness condition $\xi_t(\omega,\hat{a},q)v(\omega,\hat{a},q)=0$. These conditions do not depend on the firm's individual capital holdings and state that the purchased units of capital are bought at a cost equal to their marginal value. We multiply the first-order condition above by $v(\omega,\hat{a},q)$ and replace it in the objective of the firm, which then becomes

$$\begin{aligned}
V_{Ft}(\mathbf{k}) = &\max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),k'(\omega,a)\}}} \mathbb{E}_a\Bigg\{\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\\
&\Bigg[(Z_t-\delta)\omega a\left(1-[\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)+(1-\psi)b(\omega,\hat{a}(\omega,a),q(\omega,a))]p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right)s(\omega,a)\right)k(\omega,a)\\
&+[\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)q_t^P(\omega,a,\hat{a}(\omega,a),q)+(1-\psi)b(\omega,\hat{a}(\omega,a),q(\omega,a))q(\omega,a)]p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right)s(\omega,a)k(\omega,a)\\
&+\Lambda_{t,t+1}((1-\varphi)\nu_{t+1}^{\text{b}}(\omega,a)+\varphi\nu_{t+1}^{\text{s}}(\omega,a,\mathbf{K}_{Ht+1}))\\
&\times\left(1-[\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a)+(1-\psi)b(\omega,\hat{a}(\omega,a),q(\omega,a))]p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right)s(\omega,a)\right)k(\omega,a)\Bigg]\Bigg\},
\end{aligned}$$

where we have used the fact that the total cost of the newly purchased units equals their marginal value, so the terms x^b and k^b cancel each other. This shows the linearity of the firm's value function with respect to $\mathbf{k}$. In what follows, we show that households are the sellers and firms are the buyers in the capital market. We also show that under certain assumptions about $q_t^P(\omega,a,\hat{a}(\omega,a),q)$, buyers always choose to buy the capital unit after matching with a seller.

Firm's selling decision and household's buying decision. Here, we show that firms buy capital but do not sell it, and that households sell capital but do not buy it (although they invest to produce capital). For this, recall that the value of a capital unit is symmetric among all firms and households because it does not depend on individual capital holdings.

Notice that the problem of households is a particular case of the firms' problem with productivity Z_t set to zero. Hence, the marginal value of a capital unit for firms is larger than the marginal value of a capital good for the household as long as $Z_t > 0$. As a consequence, if firms want to sell (the value from operating the unit of capital is lower than the value from selling it), then households also prefer to sell, as they cannot obtain a higher value from this unit than an operating firm would. Hence, there is no market for the unit considered. Similarly, if households do not want to sell (they obtain a higher value by keeping the unit), firms also will not want to sell. Again, there will be no market for the unit considered, since no one wants to sell. This implies that we can simplify the problem: Households never buy capital (otherwise there are no sellers), and firms never sell capital as long as $Z_t > 0$ (otherwise there are no buyers). Thus, the optimal firm's policy is $s(\omega, a) = 0$, which simplifies the firm's marginal value of capital of quality $k(\omega, a)$ to

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right],$$

which proves the second result of Proposition 1. This result also simplifies the household's marginal value of a capital unit to

$$\begin{aligned}
\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = &\max_{\{q_t(\omega,a), \hat{a}_t(\omega,a)\}} p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big[\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q_t^P(\omega, a, \hat{a}(\omega, a), q) \\
&+ (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a)\Big] \\
&+ \Big(1 - (\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big) \\
&\times \left[\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}_{Ht+1}(\mathbf{k})) - \delta\omega a\right].
\end{aligned}$$

Optimal purchase decision. Here, we characterize the buyer's optimal purchase decision. There are two cases to consider: Either the inspection is unsuccessful and only $(\omega, \hat{a})$ is known, or the inspection is successful and the true type (ω, a) is revealed. We handle both cases successively.

In the first case, the firm's first-order condition with respect to $b(\omega, \hat{a}, q)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b(\omega, \hat{a}, q)} = (1-\psi)\mu_t(\theta(\omega, \hat{a}, q))[\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right) - q].$$

Hence, the optimal purchase policy when the inspection is not informative is given by

$$b(\omega, \hat{a}, q) = \begin{cases} 1 \text{ if } \mathbb{E}_a \left(\nu_t^{\mathrm{b}}(\omega, a) | \omega, \hat{a}, q\right) \geq q \\ 0 \text{ otherwise} \end{cases} .$$

In the second case, the firm's first-order condition with respect to $b^{FI}(\omega, \hat{a}, q, a)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b^{FI}(\omega, \hat{a}, q, a)} = \psi \mu_t(\theta(\omega, \hat{a}, q))[\nu_t^{\mathrm{b}}(\omega, a) - q_t^P(\omega, a, \hat{a}, q)].$$

Hence, the optimal purchase policy when the inspection is informative is given by

$$b^{FI}(\omega, \hat{a}, q, a) = \begin{cases} 1 \text{ if } \nu_t^{\mathrm{b}}(\omega, a) \geq q_t^P(\omega, a, \hat{a}, q) \\ 0 \text{ otherwise} \end{cases} .$$

Given the partial derivative with respect to $b^{FI}(\omega, \hat{a}, q, a)$ derived above, Assumption 1 ensures that a transaction always takes place after an informative inspection.

Proof of Proposition 2

The proof is split into five steps that we list below. It makes use of the particular case of the full-information case from Proposition 3, for which a separate proof is presented below. The proof proceeds as follows:

Step 1: We describe the link between prices and market tightness under a fully revealing separating equilibrium.

Step 2: We construct the unique, fully revealing separating equilibrium during the transition path recursively. We proceed in two substeps:

(a) First, we show that under certain conditions, the full-information allocation can be sustained when $\psi < 1$.

(b) Second, we prove the existence and uniqueness of a separating equilibrium when the full-information optimum is not part of the possible strategies under asymmetric information.

Step 3: We apply the result from the previous step to a balanced-growth path.

Step 4: We show that there cannot be a pooling equilibrium if $a \to \phi q^P(\omega, a, \hat{a}, q) - \nu^{\mathrm{b}}(\omega, a)$ is monotonous in a during the transition path.

Step 5: We show that there is no pooling equilibrium on the balanced-growth path by applying the previous step.

Step 1: Prices and market tightness in a fully revealing separating equilibrium. The following Lemma characterizes the equilibrium prices and market tightness in a fully revealing separating equilibrium.

Lemma* 1. *Equilibrium market tightness in a fully revealing separating equilibrium

In a fully revealing separating allocation, sellers never misreport their true unobserved quality—i.e., $\hat{a}(\omega, a) = a \ \forall(\omega, a) \in \Omega \times \mathcal{A}$. *Then, market tightness is given by*

$$\theta(\omega, \hat{a}, q) = \mu_t^{-1}\left(\frac{w_t}{\nu^{\mathrm{b}}(\omega, \hat{a}) - (1-\psi)q - \psi q^P(\omega, \hat{a}, \hat{a}, q)}\right). \tag{A.6}$$

Proof. In a fully revealing separating allocation, the vector $(\omega, \hat{a}, q)$ reveals the unobserved quality a by definition. Hence, using the indifference condition of buyers, we obtain

$$\theta(\omega, \hat{a}, q) = \mu_t^{-1}\left(\frac{w_t}{\mathbb{E}_a(\nu^{\mathrm{b}}(\omega, \hat{a}) - (1-\psi)q - \psi q^P(\omega, a, \hat{a}, q)|\omega, \hat{a}, q)}\right).$$

Since the allocation is fully revealing, $\hat{a}(\omega, a) = a$. Therefore, we can drop the expectation term. We then obtain the result of Lemma 1. ∎

Step 2: Recursive construction of the unique fully revealing separating equilibrium. Let us consider the case with $\psi < 1$. The case in which $\psi = 1$, corresponding to full information, is solved in the proof of Proposition 3 below. We first define the notation used within this step of the proof. In what follows, we fix $\omega \in \Omega$, a time t and omit all references to ω and t (there will be no ambiguity: All variables depend on t except for the continuation values taken at $t+1$). We note $\bar{v}(a) = \Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a) - \delta\omega a$ the continuation value of a seller of unobserved quality a and observed quality ω. We also note $V(a) = p(\theta(\omega, q(\omega, a)))\left[\nu^{\mathrm{b}}(\omega, a) - \bar{v}(a)\right] - \chi\theta(\omega, q(\omega, a))$, where $\chi \equiv \frac{w_t}{\gamma^t}$. The set of qualities $\mathcal{A}$ is ordered and we note $\mathcal{A}_k = \{a \in \mathcal{A} | a \leq a_k\}$ the subset of its k lowest elements. We denote $q^P(\omega, a_i, a_j, q) = min(q, q^B(\omega, a_i, a_j))$ so that the post-inspection price is equal to some $q^B(\omega, a_i, a_j)$ unless the seller committed to a lower price pre-inspection. Finally, we use Lemma 1 to define $q(a, \theta) = \nu^{\mathrm{b}}(\omega, a) - \frac{w_t}{\mu_t(\theta)}$ as the price corresponding to quality a and tightness θ. We will show the following assertion by induction on $k \in [1, N_a]$.

***Assertion* 1. *Assertion at rank* $\mathbf{k} \in [\mathbf{1}, \mathbf{N_a}]$**

The unique separating fully revealing equilibrium allocation $\Theta_k = \{\theta(a_1), \ldots, \theta(a_k)\}$ *on* $\mathcal{A}_k$ *that satisfies the D1 criterion is constructed recursively:*

(i) The seller of the lowest unobserved quality a_1 *chooses the full-information strategy* $\hat{a}(\omega, a_1) = a_1$, $q(\omega, a_1) = q^{FI}(\omega, a_1)$ *and* $\theta(\omega, \hat{a}(\omega, a_1), q(\omega, a_1)) = \theta^{FI}(\omega, a_1)$, *which is characterized by*

$$q^{FI}(\omega, a_1) = \nu^{\mathrm{b}}(\omega, a_1) - \frac{w}{\mu(\theta^{FI}(\omega, a_1))}$$

and

$$p'(\theta^{FI}(\omega, a_1)) \left(\nu^{\mathrm{b}}(\omega, a_1) - \bar{v}(\omega, a_1)\right) = \chi.$$

(ii) The seller of any unobserved quality $a_i > a_1$ *signals his true quality—i.e.,* $\hat{a}(\omega, a_i) = a_i$. *Regarding the terms of trade, there are two cases to consider:*

(a) If for all $l < i$, *the constraint* (11) *evaluated at* $q^{FI}(\omega, a_i)$ *and* $\theta^{FI}(\omega, a_i)$ *is slack, then the seller of quality* a_i *chooses the full-information terms of trade—i.e.,* $q(\omega, a_i) = q^{FI}(\omega, a_i)$ *and* $\theta(\omega, \hat{a}(\omega, a_i), q(\omega, a_i)) = \theta^{FI}(\omega, a_i)$.

(b) If at least one of the constraints (11) *binds for* $l < i$, *then let* $\underline{\theta}_l^i$ *denote the lowest solution* θ *to*

$$\nu^{\mathrm{s}}(\omega, a_l) = p\,(\theta) \left((1-\psi)\nu^{\mathrm{b}}(\omega, a_i) + \psi q^P(\omega, a_l, \hat{a}(\omega, a_i), q)\right) + (1 - p(\theta))\, \bar{v}(\omega, a_l) - \chi\theta.$$

The seller of quality a_i *chooses*

$$\theta(\omega, a_i) = min\left\{\underline{\theta}_j^i,\ j \in \{1, \ldots, i-1\}\right\},$$

and the corresponding price, as long as $\bar{v}(\omega, a_i) \geq \bar{v}(\omega, a_l)$ *for all* $l < i$. *In this case, the optimal market tightness is lower than under the full-information terms of trade—i.e.,* $\theta(\omega, a_i) < \theta^{FI}(\omega, a_i)$.

Initialization: $\mathcal{A}_1 = \{a_1\}$**.** We begin the construction by noting that since in the set $\mathcal{A}_1$ there is no lower type that needs to be disincentivized from mimicking for type a_1 and type a_1 does not want to mimic any higher type because there is none, we have that in any separating equilibrium in $\mathcal{A}_1$:

$$\begin{cases} \hat{a}(a_1) = a_1 \\ q(a_1) = q^{FI}(a_1) \end{cases}, \tag{A.7}$$

which proves the assertion for $k = 1$.

Recursion. Let us fix $k \in [2, N_a]$ and suppose that the assertion is true for $k - 1$.

Step 2(a): The full-information optimum can be sustained under asymmetric information. We first study the case in which the full-information terms of trade can be sustained for quality a_k. Given the sequence Θ_{k-1}, the full-information strategy of the seller of quality a_k can be part of a fully revealing separating equilibrium if and only if no seller of a lower quality wants to deviate from its current strategy to mimic her. Formally, the incentive compatibility constraint must be just binding or slack for every quality $a_i \leq a_k$ when type a_k implements its full-information allocation:

$$\begin{aligned} &V(a_i) \equiv p(\theta(a_i)) \left[\nu^{\mathrm{b}}(a_i) - \bar{v}(a_i)\right] - \chi\theta(a_i) \geq \\ &p(\theta^{FI}(a_k)) \left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_i, a_k, q^{FI}(a_k)) - \bar{v}(a_i)\right] - (1-\psi)\chi\theta^{FI}(a_k). \end{aligned} \tag{A.8}$$

The seller of quality a_k is then allowed to implement its full-information strategy, which maximizes its unconstrained objective conditional on $\hat{a}(a_k) = a_k$. Since a_k is the highest quality on $\mathcal{A}_k$, the seller would not be able to obtain a higher value by mimicking a lower quality. Then, since no seller has an incentive to deviate to mimic quality a_k, the previous allocation Θ_{k-1} remains.

The last step is to discuss off-equilibrium beliefs conditional on being on other submarkets in which a_k is the announced quality. We can first rule out that quality a_k is expected by buyers in these other submarkets. Indeed, the seller of quality a_k is not better off deviating to any other tightness θ as it is achieving its unconstrained optimum. If the seller of any lower quality is better off deviating to a submarket $(\hat{a} = a_k, \theta)$ conditional on quality a_k being expected, then this seller would be better off under a larger set of beliefs than the seller of quality a_k. The D1 criterion would then impose that its quality is the one expected instead of a_k, which rules out the deviation.

Finally, suppose that a seller of some quality $a_i < a_k$ has a profitable deviation by choosing $(\hat{a}(a_i) = a_k, \theta)$ and that the expected quality on this submarket is $a_j < a_k$. Then, since $q^B(a_i, a_k) \leq q^B(a_i, a_j)$ from Assumption 1, the seller of quality a_i would have a profitable deviation absent quality a_k as well. Using the recursion at rank $k-1$, we know that this is not the case since Θ_{k-1} is an equilibrium on $\mathcal{A}_{k-1}$. As a consequence, no seller of lower quality has a profitable deviation to either on-path or off-path submarkets in which a_k is announced, and quality a_k is never expected by buyers on any off-equilibrium submarket.

Thus, using the Assertion for $k-1$, we obtain a unique separating equilibrium in which the allocation is $\Theta_k = \Theta_{k-1} \cup \{\theta^{FI}(a_k)\}$ and all sellers announce their true quality.

Step 2(b): The full-information optimum cannot be sustained under asymmetric information. Let A_{k-1} denote the set of qualities that want to mimic sellers of quality a_k when they play their full-information market tightness. We now have that for all $a_j \in A_{k-1}$:

$$V(a_j) < p(\theta^{FI}(a_k))\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q^{FI}(a_k)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta^{FI}(a_k). \quad \text{(A.9)}$$

Let $R_j^k(\theta) = p(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(a_k, \theta)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta$. Then, $R_j^k(\theta) + \bar{v}(a_j)$ represents the revenue the seller of quality a_j receives when mimicking the seller of quality a_k when the latter plays θ.

Single-peaked shape of $R_j^k(\theta)$. We next analyze the properties of the expected value from mimicking $R_j^k(\theta)$. We have two intervals to consider. Suppose first that $q(a_k, \theta) > q^B(a_j, a_k)$. Then, $q^P(a_j, a_k, q(a_k, \theta)) = q^B(a_j, a_k)$ does not depend on θ. The second-order derivative with respect to θ is then

$$p''(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^B(a_j, a_k) - \bar{v}(a_j)\right] < 0.$$

Hence, on this interval, the function is strictly concave.[17]

Now suppose that $q(a_k, \theta) \leq q^B(a_j, a_k)$. Then, $q^P(a_j, a_k, q(\theta, a_k)) = q(a_k, \theta)$ and the

[17] We have $\nu^{\mathrm{b}}(a_k) \geq \nu^{\mathrm{s}}(a_k) > \bar{v}(a_k) \geq \bar{v}(a_j)$, where the first inequality is due to discounting and the presence of search frictions, the second inequality is due to the definition of the value, and the last inequality comes from the assertion. In addition, $q^B(a_j, a_k) \geq \bar{v}(a_j)$ from Assumption 1.

second-order derivative writes

$$p''(\theta)\left[\nu^{\mathrm{b}}(a_k)-\bar{v}(a_j)\right]<0.$$

Hence, the revenue from mimicking is also strictly concave on that interval. The entire function is piece-wise concave. Thus, it could have either one or two peaks.

We now show that it is indeed single-peaked. Let us define θ^B as the tightness at which the two concave parts connect: $q^B(a_j,a_k)=\nu^{\mathrm{b}}(a_k)-\frac{w}{\mu(\theta^B)}$. For $\theta>\theta^B$, the revenue from mimicking is equal to

$$R_j^k(\theta)=p(\theta)\left[\nu^{\mathrm{b}}(a_k)-\bar{v}(a_j)\right]-\chi\theta$$

and its maximum θ^* is characterized by the first-order condition:

$$p'(\theta^*)\left[\nu^{\mathrm{b}}(a_k)-\bar{v}(a_j)\right]-\chi=0.$$

Substituting in the buyer's indifference condition, we obtain

$$q^*=\eta\nu^{\mathrm{b}}(a_k)+(1-\eta)\bar{v}(a_j).$$

Recall that the full information price of the seller of quality a_j satisfies

$$q^{FI}(a_j)=\eta\nu^{\mathrm{b}}(a_j)+(1-\eta)\bar{v}(a_j).$$

Since $\nu^{\mathrm{b}}(a_k)\geq\nu^{\mathrm{b}}(a_j)$ and by assumption $q^{FI}(a_j)\geq q^B(a_j,a_k)$, we obtain $q^*\geq q^{FI}(a_j)\geq q^B(a_j,a_k)$. Hence, the function $R_j^k(\theta)$ is strictly decreasing for $\theta>\theta^B$, which in turn implies that it is single-peaked.

Disincentivizing mimicking from lower types. Next, we derive the set of tightnesses (and corresponding prices) the seller of quality a_k can choose without having other types mimicking him.

Let $\tilde{\theta}_l$ denote the market tightness that causes the incentive-compatibility constraint between types a_k and $a_l<a_k$ to bind. Then, $\tilde{\theta}_l$ is characterized by the equation

$$p(\theta_l)(\nu^{\mathrm{b}}(a_l)-\bar{v}(a_l))-\chi\theta_l=R_l^k(\tilde{\theta}_l). \tag{A.10}$$

Since $R_l^k(\cdot)$ is single-peaked, there are at most two values $\tilde{\theta}_l$ that satisfy the above equation. In addition, $\lim_{\theta \to +\infty} R_l^k(\theta) = -\infty$ and $\lim_{\theta \to 0} R_l^k(\theta) = 0$ and $p(\theta_l)(\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)) - \chi\theta_l > 0$. Hence, there are either two or no solutions to the equation above. Let us denote $\underline{\theta}_l^k$ and $\bar{\theta}_l^k$ the two solutions to (A.10) provided that they exist. As $R_l^k(\cdot)$ is single-peaked, we have that for all $\theta \in [0, \underline{\theta}_l^k] \cup [\bar{\theta}_l^k, +\infty)$, the right-hand side of equation (A.10) is lower than its left-hand side. As a consequence, for any tightness on these two subintervals, the incentive-compatibility constraint is slack; i.e., type a_l does not want to mimic type a_k.

We now show that the sets $\left\{\underline{\theta}_l^k : \ l \in \{1, \dots, k-1\}\right\}$ and $\left\{\bar{\theta}_l^k : \ l \in \{1, \dots, k-1\}\right\}$ are non-empty. Because the seller of quality a_k cannot choose its full-information tightness, there exists at least one quality $a_j \in \mathcal{A}_{k-1}$ such that

$$R_j^k(0) = 0 < p(\theta_j)\left[\nu^{\mathrm{b}}(a_j) - \bar{v}(a_j)\right] - \chi\theta_j < R_j^k(\theta^{FI}(a_k)).$$

Using the inequality above and the fact that $R_j^k(\cdot)$ is continuous, we can find a $\tilde{\theta}_j < \theta^{FI}(a_k)$ such that equation (A.10) holds, which implies that $\underline{\theta}_j^k$ and $\bar{\theta}_j^k$ are well defined. As a consequence, the two sets are non-empty.

Let $\underline{\theta} = \min\left\{\underline{\theta}_l^k : \ l \in \{1, \dots, k-1\}\right\}$ and $\bar{\theta} = \max\left\{\bar{\theta}_l^k : \ l \in \{1, \dots, k-1\}\right\}$. It follows that if type a_k chooses any $\theta \geq \bar{\theta}$ or $\theta < \underline{\theta}$, then no seller of a quality $a_j \in \mathcal{A}_{k-1}$ wants to mimic the seller of quality a_k. Finally, note that since $\underline{\theta}_j^k < \theta^{FI}(a_k)$ and $\bar{\theta}_j^k > \theta^{FI}(a_k)$ we necessarily have that $\underline{\theta} < \theta^{FI}(a_k)$ and $\bar{\theta} > \theta^{FI}(a_k)$.

Any tightness $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ is a possible value for a separating equilibrium. We just proved that for any $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ played by the seller of quality a_k, all types in $\mathcal{A}_{k-1}$ do not want to mimic, which ensures that a separating equilibrium can be constructed using any of these values. Note that without a further refinement of off-equilibrium beliefs, we can always find a set of buyers' beliefs that can sustain any θ in that set. We can, for example, assume that buyers believe that any off-path terms of trade are only picked by the lowest type and make any θ in the set part of a separating equilibrium. Also, note that any $\theta < \underline{\theta}$ or $\theta > \bar{\theta}$ imposes a cost to the seller without any further benefit, since the seller is already signaling its true type. Next, we follow the signaling literature and impose the D1 criterion, which is an equilibrium refinement that isolates the "most relevant" equilibrium.

The only values consistent with the D1 criterion are $\underline{\theta}$ or $\bar{\theta}$. Choose any $\theta_k \notin [\underline{\theta}, \bar{\theta}]$. The fact that $\theta_k > \bar{\theta}$ or $\theta_k < \underline{\theta}$ implies that the constraint "quality a_j does not want to mimic type a_k" is not binding; i.e.,

$$V(a_j) > p(\theta_k)\left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(a_k, \theta_k)) - \bar{v}(a_j)\right) - (1-\psi)\chi\theta_k.$$

Suppose first that $\theta_k < \underline{\theta}$. We now need to determine which seller is most likely to deviate and choose $(\hat{a} = a_k, \underline{\theta})$, so that we can set beliefs in accordance with the D1 criterion. We know that the seller of quality a_k is strictly better off if the price is $q(a_k, \underline{\theta})$, as the expected revenue of the seller of quality a_k is strictly increasing in market tightness for $\theta_k < \underline{\theta}$. At the same time, any seller of a quality lower than a_k would not, by construction, be better off by deviating to the submarket with tightness $\underline{\theta}$ and price $q(a_k, \underline{\theta})$. This implies that the seller of quality a_k is better off under a larger set of beliefs than other sellers, as he is better off for a larger set of prices (in the sense of inclusion). The D1 criterion then requires that type a_k is the one expected on any submarket with $\theta \in (0, \underline{\theta}]$. A symmetric reasoning implies that quality a_k is also expected on submarkets with $\theta \in [\bar{\theta}, +\infty)$.

We then invoke the fact that the unconstrained objective of sellers of quality a_k is strictly increasing on $(0, \underline{\theta}]$ because $\underline{\theta} < \theta^{FI}(a_k)$ and strictly decreasing on $[\bar{\theta}, +\infty)$ because $\bar{\theta} > \theta^{FI}(a_k)$. As a consequence, the seller of quality a_k has a profitable deviation for any $\theta_k < \underline{\theta}$ and $\theta_k > \bar{\theta}$, which leaves only $\underline{\theta}$ and $\bar{\theta}$ as possible equilibrium values after applying the D1 criterion.

The seller of quality a_k (weakly) prefers $\underline{\theta}$ to $\bar{\theta}$ if continuation values are (weakly) increasing in a. We proceed in two steps. We first show that the seller of quality a_k has a higher value at $\underline{\theta}$ if the incentive-compatibility constraint is binding with the same quality for $\underline{\theta}$ and $\bar{\theta}$. We then show that the same is true if the incentive-compatibility constraint binds with different qualities for $\underline{\theta}$ and $\bar{\theta}$.

Let $\nu^{\mathrm{s}}(a_k; \theta)$ denote the value of the seller of quality a_k when the market tightness is θ and the posted price is $q(a_k, \theta)$.

(i) Case 1: The same quality binds at $\underline{\theta}$ and $\bar{\theta}$. Let this quality be a_l.

We start with the sub-case with $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) = q^B(a_l, a_k)$, which implies that $q(a_k, \bar{\theta}_l^k) \geq q^B(a_l, a_k)$. Since $\underline{\theta}_l^k < \theta^{FI}(a_k)$, we always have $q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) = q^B(a_l, a_k)$.

The binding incentive-compatibility constraint with quality a_l at tightness $\underline{\theta}_l^k$ can be written as

$$V(a_l) = p(\underline{\theta}_l^k)\left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - \bar{v}(a_l)\right) - (1-\gamma)\chi\underline{\theta}_l^k.$$

After adding and subtracting $(1-\psi)(1-p(\underline{\theta}_l^k))\bar{v}(a_k)$ on the right-hand side, we obtain

$$V(a_l) = (1-\psi)\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - (1-\psi)\bar{v}(a_k) + p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right].$$

A similar expression applies when evaluating the incentive-compatibility constraint at $\bar{\theta}_l^k$. Subtracting the expression above from its counterpart at $\bar{\theta}_l^k$, we obtain

$$\begin{aligned}(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)\right] = &\left(p(\bar{\theta}_l^k) - p(\underline{\theta}_l^k)\right)\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right] \\ &+ \psi p(\underline{\theta}_l^k)\left(q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) - q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k))\right).\end{aligned} \tag{A.11}$$

Given the sub-case we started from, we have that $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k) = q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) = q^B(a_l, a_k)$; therefore the second term on the left-hand side in (A.11) is zero. This yields

$$(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)\right] = \left(p(\bar{\theta}_l^k) - p(\underline{\theta}_l^k)\right)\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right].$$

Because $p(\bar{\theta}_l^k) > p(\underline{\theta}_l^k)$ and $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) \geq \bar{v}(a_l)$ from Assumption 1, the right-hand side is (weakly) positive if continuation values are (weakly) increasing in unobserved quality. Hence, the left-hand side is (weakly) positive; i.e. $\underline{\theta}_l^k$ gives a (weakly) higher utility to the seller of quality a_k.

Let us now handle the remaining sub-case with $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) = q(a_k, \bar{\theta}_l^k)$. The binding incentive-compatibility constraint at $\bar{\theta}_l^k$ is given by

$$V(a_l) = p(\bar{\theta}_l^k)\left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) - \bar{v}(a_l)\right) - (1-\gamma)\chi\bar{\theta}_l^k.$$

Substituting in $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) = q(a_k, \bar{\theta}_l^k) = \nu^{\mathrm{b}}(a_k) - \frac{w}{\mu(\bar{\theta}_l^k)}$, we obtain

$$V(a_l) = p(\bar{\theta}_l^k)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_l)\right) - \chi\bar{\theta}_l^k$$

Next, we add and subtract $(1-p(\bar{\theta}_l^k))\bar{v}(a_k)$:

$$V(a_l)=\nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)-\bar{v}(a_k)+p(\bar{\theta}_l^k)\left[\bar{v}(a_k)-\bar{v}(a_l)\right].$$

As in the previous sub-case, the binding incentive-compatibility constraint at $\underline{\theta}_l^k$ is given by

$$V(a_l)=\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\bar{v}(a_k)+p(\underline{\theta}_l^k)\left[\psi(q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))-q(a_k,\underline{\theta}_l^k))+\bar{v}(a_k)-\bar{v}(a_l)\right].$$

Subtracting these two last expressions, we finally obtain

$$\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k)-\nu^{\mathrm{s}}(a_k;\bar{\theta}_l^k)=\left(p(\bar{\theta}_l^k)-p(\underline{\theta}_l^k)\right)\left[\bar{v}(a_k)-\bar{v}(a_l)\right]+p(\underline{\theta}_l^k)\psi\left[q(a_k,\underline{\theta}_l^k)-q^P(a_l,a_k,q(a_k,\underline{\theta}_l^k))\right].$$

The first term on the right-hand side is (weakly) positive because continuation values are (weakly) non-decreasing in a by assumption. The second term is weakly positive by the commitment assumption on $q^P(\cdot)$. Hence, $\underline{\theta}_l^k$ is (weakly) preferred when the incentive-compatibility constraint binds with the same quality on both sides.
(ii) Case 2: Different qualities bind at $\underline{\theta}$ and $\bar{\theta}$. Let us now take $l,m<k$ with $m\neq l$ such that $\bar{\theta}=\bar{\theta}_m^k$ and $\underline{\theta}=\underline{\theta}_l^k$.

By definition of the bounds, we have $\bar{\theta}_l^k\leq\bar{\theta}_m^k$ and $\underline{\theta}_l^k\leq\underline{\theta}_m^k$. Since $\bar{\theta}_l^k$ is on the decreasing part of the revenue function $R_l^k(\theta)$ and $\bar{\theta}_l^k\leq\bar{\theta}_m^k$, we have that $R_l^k(\bar{\theta}_l^k)\geq R_l^k(\bar{\theta}_m^k)$. By definition of $\bar{\theta}_l^k$, the last inequality can be written as

$$V(a_l)>R_l^k(\bar{\theta}_m^k).$$

We subtract $V(a_m)$ from both sides of the last inequality and use the definition of $\bar{\theta}_m^k$ to obtain

$$V(a_l)-V(a_m)>p(\bar{\theta}_m^k)\left[\psi(q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))-q^P(a_m,a_k,q(a_k,\bar{\theta}_m^k))+\bar{v}(a_m)-\bar{v}(a_l))\right]. \tag{A.12}$$

Let us now compare the value obtained by the seller of quality a_k at the two market tightnesses $\bar{\theta}_m^k=\bar{\theta}$ and $\underline{\theta}_l^k=\underline{\theta}$. We start by analyzing the sub-case $q^P(a_l,a_k,q(a_k,\bar{\theta}_m^k))<q(a_k,\bar{\theta}_m^k)$. As before, the binding incentive-compatibility constraint with respect to quality a_l

can be written as

$$V(a_l) = (1-\psi)\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - (1-\psi)\bar{v}(a_k) + p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right].$$

We subtract the above expression from its counterpart evaluated at $\bar{\theta}_m^k$ and obtain

$$\begin{aligned}(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)\right] = {} & V(a_l) - V(a_m) \\ & + p(\bar{\theta}_m^k)\left[\psi q^P(a_m, a_k, q(a_k, \bar{\theta}_m^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_m)\right] \\ & - p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right].\end{aligned}$$

Using inequality (A.12), we obtain

$$\begin{aligned}(1-\psi)\left[\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)\right] \geq {} & (p(\bar{\theta}_m^k) - p(\underline{\theta}_l^k))\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(a_l)\right] \\ & + \psi p(\bar{\theta}_m^k)\left[q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) - q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k))\right].\end{aligned}$$

In this sub-case, the second term on the right-hand side is zero. The first term on the right-hand side is positive, since $q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) \geq \bar{v}(a_l)$ and continuation values are increasing in a. Hence, $\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) \geq \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k)$; therefore, $\underline{\theta}$ is preferred by the seller of quality a_k.

We now address the second sub-case, namely $q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) = q(a_k, \bar{\theta}_m^k)$. In this case, the binding incentive-compatibility constraint at $\bar{\theta}_m^k$ can be written as

$$V(a_m) = \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k) - \bar{v}(a_k) + p(\bar{\theta}_m^k)\left[\bar{v}(a_k) - \bar{v}(a_m)\right],$$

and the binding incentive-compatibility constraint at $\underline{\theta}_l^k$ can be written as

$$V(a_l) = \nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \bar{v}(a_k) + p(\underline{\theta}_l^k)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - q(a_k, \underline{\theta}_l^k)) + \bar{v}(a_k) - \bar{v}(a_l)\right].$$

Subtracting one from the other, we obtain

$$\begin{aligned}\nu^{\mathrm{s}}(a_k;\underline{\theta}_l^k) - \nu^{\mathrm{s}}(a_k;\bar{\theta}_m^k) = {} & V(a_l) - V(a_m) + p(\bar{\theta}_m^k)\left[\bar{v}(a_k) - \bar{v}(a_m)\right] \\ & - p(\underline{\theta}_l^k)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - q(a_k, \underline{\theta}_l^k)) + \bar{v}(a_k) - \bar{v}(a_l)\right].\end{aligned}$$

Using inequality (A.12) to replace the difference $V(a_l) - V(a_m)$, substituting in the sub-case expression $q^P(a_l, a_k, q(a_k, \bar{\theta}^k_m)) = q(a_k, \bar{\theta}^k_m)$ and rearranging terms, we obtain the following inequality:

$$\begin{aligned}\nu^s(a_k; \underline{\theta}^k_l) - \nu^s(a_k; \bar{\theta}^k_m) \geq &\left(p(\bar{\theta}^k_m) - p(\underline{\theta}^k_l)\right)[\bar{v}(a_k) - \bar{v}(a_l)] \\ &+ p(\bar{\theta}^k_m)\psi\left[q(a_k, \bar{\theta}^k_m) - q^P(a_m, a_k, q(a_k, \bar{\theta}^k_m))\right] \\ &+ p(\underline{\theta}^k_l)\psi\left[q(a_k, \underline{\theta}^k_l) - q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l))\right].\end{aligned}$$

All the terms on the right-hand side are positive. Hence, the left-hand side is positive and $\nu^{\mathrm{s}}(a_k; \underline{\theta}^k_l) \geq \nu^{\mathrm{s}}(a_k; \bar{\theta}^k_m)$; therefore, $\underline{\theta}$ is preferred by the seller of quality a_k.

In conclusion, the only possible value for a separating equilibrium is $\underline{\theta}$ as long as continuation values are increasing in a. We also show that under beliefs set in accordance with the D1 criterion, there is no profitable deviation on $(0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$. This guarantees that no seller has an incentive to deviate to these submarkets.

Downward incentive-compatibility constraints. By construction, $\Theta_{k-1} \cup \{\underline{\theta}\}$ satisfies the "upward" incentive compatibility constraints, since no seller of a lower quality has an incentive to mimic a_k at $\underline{\theta}$. We now need to show that for this allocation, sellers have no incentives to mimic the strategy of qualities lower than themselves. The recursion at rank $k-1$ implies that all qualities lower than a_k satisfy the downward incentive-compatibility constraints, so that we only need to show the property for sellers of quality a_k. Let us fix $a_i < a_k$. We prove that type a_k does not want to mimic the strategy of type a_i. Let $\theta_k = \underline{\theta}$ and θ_i be the market tightness chosen by the seller of quality a_i in equilibrium.

(i) Case 1: $\theta_i < \theta_k$. By construction, $\theta_k \leq \theta^{FI}(a_k)$. We know that the unconstrained objective of sellers of quality a_k is strictly concave, with a maximum reached at $\theta^{FI}(a_k)$. This implies that $p(\theta)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta$ is strictly increasing in θ for $\theta \in [\theta_i, \theta_k]$, which yields

$$p(\theta_i)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_i < p(\theta_k)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_k.$$

We then use $\nu^{\mathrm{b}}(a_k) > \nu^{\mathrm{b}}(a_i)$, which implies:

$$p(\theta_i)\left(\nu^{\mathrm{b}}(a_i) - \bar{v}(a_k)\right) - \chi\theta_i < p(\theta_k)\left(\nu^{\mathrm{b}}(a_k) - \bar{v}(a_k)\right) - \chi\theta_k.$$

Intuitively, if $\theta_k > \theta_i$, then the seller of quality a_k benefits from both a higher selling probability and a higher price. As a consequence, the seller of quality a_k does not want to mimic the strategy of the seller of quality a_i.

(ii) Case 2: $\theta_i \geq \theta_k$. For this case, we need to introduce quality $a_j < a_k$ such that the "upward" incentive-compatibility constraint between a_k and a_j is binding. We proceed with three sub-cases:

(i) *Case 2.1:* $a_i = a_j$. The "upward" incentive-compatibility constraint between a_i and a_k is binding, i.e.:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_i,a_k)-\bar{v}(a_i)\right]=p(\theta_i)\left[q(a_i)-\bar{v}(a_i)\right],$$

which can be rewritten as

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right]=p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+(p(\theta_i)-p(\theta_k))(\bar{v}(a_k)-\bar{v}(a_i))+\psi p(\theta_k)\left[q(a_k)-q^P(a_i,a_k)\right].$$

Since $\theta_i \geq \theta_k$, $q(a_k) \geq q^{FI}(a_k) > q^P(a_i,a_k)$ and continuation values are increasing, we have unambiguously:

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]$$

Hence the seller of quality a_k does not want to mimic type a_i's strategy.

(ii) *Case 2.2:* $a_j > a_i$. We start from the binding incentive-compatibility constraint between a_j and a_k:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right]=p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right]$$

Using the recursion, sellers of quality a_j satisfy the downward incentive-compatibility constraint with a_i:

$$p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_j)\right]$$

Injecting this inequality in the previous equation:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_j)\right]$$

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+\psi p(\theta_k)\left[q(a_k)-q^P(a_j,a_k)\right]+(p(\theta_i)-p(\theta_k))(\bar{v}(a_k)-\bar{v}(a_j)).$$

Since $\theta_i \geq \theta_k$, $q(a_k) \geq q^P(a_j,a_k)$ and continuation values are increasing, we obtain

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right].$$

Hence, the seller of quality a_k does not want to mimic type a_i's strategy.

(iii) *Case 2.3:* $a_j < a_i$. We start again from the binding incentive-compatibility constraint between a_j and a_k:

$$p(\theta_k)\left[q(a_k)(1-\psi)+\psi q^P(a_j,a_k)-\bar{v}(a_j)\right] = p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right].$$

This time we use the upward incentive-compatibility constraint between a_i and a_j:

$$p(\theta_j)\left[q(a_j)-\bar{v}(a_j)\right] \geq p(\theta_i)\left[q(a_i)(1-\psi)+\psi q^P(a_j,a_i)-\bar{v}(a_j)\right].$$

Injecting this inequality in the previous expression:

$$\begin{aligned}(1-\psi)p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq (1-\psi)p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right] + (p(\theta_i)-p(\theta_k))((1-\psi)\bar{v}(a_k)-\bar{v}(a_j))\\ -\psi p(\theta_k)q^P(a_j,a_k)+\psi p(\theta_i)q^P(a_j,a_i).\end{aligned}$$

From Assumption 1 $q^P(a_j,a_i) \geq q^P(a_j,a_k)$, which yields

$$\begin{aligned}(1-\psi)p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq (1-\psi)p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right]+\\ (p(\theta_i)-p(\theta_k))(\psi q^P(a_j,a_i)+(1-\psi)\bar{v}(a_k)-\bar{v}(a_j)).\end{aligned}$$

Since $\theta_i \geq \theta_k$, continuation values are increasing and $q^P(a_j,a_i) \geq \bar{v}(a_j)$ (from Assumption 1), the second term on the right-hand side is positive. As a consequence:

$$p(\theta_k)\left[q(a_k)-\bar{v}(a_k)\right] \geq p(\theta_i)\left[q(a_i)-\bar{v}(a_k)\right].$$

Thus, the downward incentive-compatibility constraint is satisfied.

As a consequence, the allocation $\Theta_{k-1} \cup \{\underline{\theta}\}$ satisfies all upward and downward incentive-compatibility constraints and forms a separating equilibrium. In order to conclude, we still need to analyze off-equilibrium beliefs and show the absence of strictly profitable deviations to off-equilibrium submarkets.

Off-equilibrium beliefs. Next, we show that no seller strictly improves its payoff by deviating to an off-equilibrium submarket. Beliefs on off-equilibrium submarkets are set in accordance with the D1 criterion, so that the expected quality on a given submarket is the one that is better off deviating under the largest set of beliefs, i.e., prices. For the equilibrium to survive the D1 criterion, it is sufficient to show that the seller of the expected quality on any off-equilibrium submarket does not have a strictly profitable deviation to that submarket. Since we focus on the seller that is better off under the largest set of beliefs, no other seller will obtain a strictly positive payoff from deviating to this submarket.

To prove this, fix an unobserved quality $a \in \mathcal{A}_k$. Conditional on being expected on a given off-equilibrium submarket $(\hat{a}, \theta')$, the seller of quality a faces the same objective function as in equilibrium, but evaluated at market tightness θ'. As a consequence, if in equilibrium the seller of unobserved quality a is implementing its full-information strategy, it cannot have a strictly profitable deviation to this submarket because its equilibrium strategy already maximizes its unconstrained objective. Hence, without loss, we can restrict our attention to the case in which the seller has a binding incentive-compatibility constraint with a lower unobserved quality on equilibrium.

Given a binding constraint, we know that the seller of quality a deviates from its full-information strategy in order to disincentivize mimicking from lower types by picking a lower tightness than it would have under full information. Using the concavity of the unconstrained objective of sellers with respect to market tightness, this implies that there is a range of tightnesses $[\underline{\theta}_a, \bar{\theta}_a]$, where the seller of quality a would be better off deviating conditional on being the only quality expected. Note that $\underline{\theta}_a$ is the market tightness picked by the seller of quality a on equilibrium. We need to therefore ensure that the seller of quality a is never expected on off-equilibrium submarkets $(\hat{a}, \theta')$ with $\theta' \in [\underline{\theta}_a, \bar{\theta}_a]$—i.e.,that there is always a seller of a lower quality that is better off on this submarket under a larger set of beliefs. We show this result in three steps corresponding to the three possible cases.

(i) Case 1: $\hat{a} = a$. Choose any $a \in \mathcal{A}_k$ and let $a_l < a$ be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. Let $q(a, \underline{\theta}_a)$ denote the price obtained by the seller of quality a in the separating equilibrium. Choose any $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$

and let q' be the corresponding price on the off-equilibrium submarket (a, θ'). The revenue of the seller of quality a at θ' would be $p(\theta')\left[q' - \bar{v}(a)\right] + \bar{v}(a)$. Hence, the net gain that the seller of quality a would receive from deviating to θ' is

$$\Delta = p(\theta')[q' - \bar{v}(a)] - p(\underline{\theta}_a)\left[q(a, \underline{\theta}_a) - \bar{v}(a)\right].$$

Similarly, the net gain the seller of quality a_l would receive from deviating to the same submarket, conditional on these beliefs, is

$$\Delta_l = p(\theta')[(1-\psi)q' + \psi q^P(a_l, a, q') - \bar{v}(a_l)] - p(\theta(a_l))\left[q(a_l, \theta(a_l)) - \bar{v}(a_l)\right]. \tag{A.13}$$

Let us now use the binding incentive-compatibility constraint between qualities a_l and a, which is given by

$$p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)] = p(\theta(a_l))\left[q(a_l, \theta(a_l)) - \bar{v}(a_l)\right].$$

Combining this equality with equation (A.13), we obtain

$$\Delta_l = p(\theta')[(1-\psi)q' + \psi q^P(a_l, a, q') - \bar{v}(a_l)] - p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)]. \tag{A.14}$$

The break-even price of the seller of quality a at (a, θ') that makes $\Delta = 0$, which we denote by $\tilde{q}$, is characterized by

$$p(\theta')\tilde{q} = (p(\theta') - p(\underline{\theta}_a))\bar{v}(a) + p(\underline{\theta}_a)q(a, \underline{\theta}_a). \tag{A.15}$$

Next, we show that $\Delta_l > 0$ when evaluated at this price. Suppose first that $\tilde{q} \leq q^B(a_l, a)$. Then, $q^P(a_l, a, \tilde{q}) = \tilde{q}$ and equation (A.14) becomes

$$\Delta_l = p(\theta')[\tilde{q} - \bar{v}(a_l)] - p(\underline{\theta}_a)[(1-\psi)q(a, \underline{\theta}_a) + \psi q^P(a_l, a, q(a, \underline{\theta}_a)) - \bar{v}(a_l)].$$

Replacing in the expression for $\tilde{q}$ from (A.15), we obtain

$$\Delta_l = (p(\theta') - p(\underline{\theta}_a))[\bar{v}(a) - \bar{v}(a_l)] + p(\underline{\theta}_a)\psi\left(q(a, \underline{\theta}_a) - q^P(a_l, a, q(a, \underline{\theta}_a))\right).$$

Since $\theta' > \underline{\theta}_a$ and continuation values are increasing, the first term is positive. In addition, by Assumption 1, $q(a, \underline{\theta}_a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$ implies that the second term is also weakly positive. Hence, $\Delta_l > 0$; i.e., there exists a price $\tilde{q}$ for which there is a strictly profitable deviation for sellers of quality a_l but not for sellers of quality a.

Now suppose that $\tilde{q} > q^B(a_l, a)$. Then, $q^P(a_l, a, \tilde{q}) = q^B(a_l, a)$. Following similar steps, we evaluate (A.14) at price $\tilde{q}$ and replace the expression for $p(\theta')\tilde{q}$ using (A.15) to obtain

$$\Delta_l = (p(\theta') - p(\underline{\theta}_a))[\psi q^B(a_l, a) + (1-\psi)\bar{v}(a) - \bar{v}(a_l)] + p(\underline{\theta}_a)\psi \left(q^B(a_l, a) - q^P(a_l, a, q(a, \underline{\theta}_a))\right).$$

Since $\theta' > \underline{\theta}_a$, continuation values are increasing, $q^B(a_l, a) \geq \bar{v}(a_l)$, and $q^B(a_l, a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$ by Assumption 1, we can conclude again that $\Delta_l > 0$.

To summarize, for any submarket (a, θ') with $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$, sellers of quality a_l still find a strictly profitable deviation at the break-even price of sellers of quality a. This implies that sellers of quality a_l are better off under a larger set of beliefs (those that justify the price $\tilde{q}$) than sellers of quality a. The D1 criterion then requires that buyers do not expect to find units of quality a in these submarkets, which proves our claim in Case 1.

(ii) Case 2: $\hat{a} > a$. Let $a_j < a$ be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. Choose any $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$. Let q' denote the price on submarket $(\hat{a}, \theta')$. By deviating to this submarket, the seller of quality a obtains a change in expected revenues of

$$\Delta = p(\theta') \left[(1-\psi)q' + \psi q^P(a, \hat{a}, q') - \bar{v}(a)\right] - V(a). \tag{A.16}$$

Similarly, the change in revenue from this deviation for the seller of quality a_j is given by

$$\Delta_j = p(\theta') \left[(1-\psi)q' + \psi q^P(a_j, \hat{a}, q') - \bar{v}(a_j)\right] - V(a_j).$$

Hence, we have

$$\Delta_j - \Delta = V(a) - V(a_j) + p(\theta') \left[\psi(q^P(a_j, \hat{a}, q') - q^P(a, \hat{a}, q')) + \bar{v}(a) - \bar{v}(a_j)\right].$$

Let us use the binding incentive-compatibility constraint between a and a_j, which can be rewritten as

$$V(a_j) - V(a) = p(\underline{\theta}_a)\left[\psi(q^P(a_j, a, q(a)) - q(a)) + \bar{v}(a) - \bar{v}(a_j)\right].$$

Since $q(a) \geq q^{FI}(a) \geq q^B(a_j, a)$, $q^P(a_j, a, q(a)) = q^B(a_j, a)$. Substituting in the previous equation, we obtain

$$\Delta_j - \Delta \geq (p(\theta') - p(\underline{\theta}_a))(\bar{v}(a) - \bar{v}(a_j)) + \psi p(\theta')(q^P(a_j, \hat{a}, q') - q^P(a, \hat{a}, q')) + p(\underline{\theta}_a)\psi(q(a) - q^B(a_j, a)). \tag{A.17}$$

Let us now set q' such that $\Delta = 0$. First assume that $q' \leq q^B(a_j, \hat{a})$. Since $a_j < a$, $q^P(a_j, \hat{a}, q') \leq q^P(a, \hat{a}, q')$ and $q^P(a_j, \hat{a}, q') = q^P(a, \hat{a}, q') = q'$. Hence, the second term in (A.17) is zero. The other terms are positive as continuation values are increasing and $\theta' > \underline{\theta}_a$. As a consequence $\Delta_j \geq 0$: The seller of quality a_j is better off deviating for a larger set of prices than the seller of quality a.

We now consider the case where $q^B(a, \hat{a}) \geq q' \geq q^B(a_j, \hat{a})$. The break-even price for the seller of quality a is such that

$$p(\theta')(1 - \psi)q' = p(\underline{\theta}_a)q(a) + (p(\theta') - p(\underline{\theta}_a))\bar{v}(a) - \psi p(\theta')q^P(a, \hat{a}, q').$$

Provided that $q' \leq q^B(a, \hat{a})$, this equation becomes

$$p(\theta')q' = p(\underline{\theta}_a)q(a) + (p(\theta') - p(\underline{\theta}_a))\bar{v}(a). \tag{A.18}$$

Combining the expression for Δ_j, with the binding incentive-compatibility constraint between qualities a and a_j and the expression above characterizing the break-even price q', we obtain

$$\Delta_j = p(\theta')\left(\psi q^B(a_j, \hat{a}) + (1 - \psi)\bar{v}(a) - \bar{v}(a_j)\right) - p(\underline{\theta}_a)\left(\psi q^B(a_j, a) + (1 - \psi)\bar{v}(a) - \bar{v}(a_j)\right).$$

We now combine equation (A.18) together with the condition $q' \leq q^B(a, \hat{a})$ to obtain

$$p(\theta') \geq p(\underline{\theta}_a)\frac{q(a) - \bar{v}(a)}{q^B(a, \hat{a}) - \bar{v}(a)}.$$

Finally, we substitute this inequality into the previous equation to obtain

$$\Delta_j \geq p(\underline{\theta}_a)\left[\frac{q(a)-\bar{v}(a)}{q^B(a,\hat{a})-\bar{v}(a)}\left(\psi q^B(a_j,\hat{a})+(1-\psi)\bar{v}(a)-\bar{v}(a_j)\right)-\left(\psi q^B(a_j,a)+(1-\psi)\bar{v}(a)-\bar{v}(a_j)\right)\right].$$

The term between brackets is positive if

$$\frac{q(a)-\bar{v}(a)}{q^B(a,\hat{a})-\bar{v}(a)} \geq \frac{\psi q^B(a_j,a)+(1-\psi)\bar{v}(a)-\bar{v}(a_j)}{\psi q^B(a_j,\hat{a})+(1-\psi)\bar{v}(a)-\bar{v}(a_j)}$$

Since $q^B(a_j,a) \geq q^B(a_j,\hat{a})$, the right-hand side is an increasing function of ψ. We also have that $q(a) \geq q^{FI}(a)$. Therefore, a sufficient condition for the above inequality to hold is

$$\frac{q^{FI}(a)-\bar{v}(a)}{q^B(a,\hat{a})-\bar{v}(a)} \geq \frac{q^B(a_j,a)-\bar{v}(a_j)}{q^B(a_j,\hat{a})-\bar{v}(a_j)},$$

which is true by Assumption 1. As a consequence, the seller of quality a_j is better off deviating to submarket $(\hat{a},\theta')$ for a larger set of beliefs than the seller of quality a, which therefore cannot be expected on that submarket.

Next let us discuss the remaining case in which $q' > q^B(a,\hat{a})$. Note that (A.16) implies that the q' that makes $\Delta = 0$ is a continuous, decreasing function of θ'. Hence, if $q' \geq q^B(a,\hat{a})$ for a given tightness θ, we have $q' \geq q^B(a,\hat{a})$ for all $\theta' \leq \theta$. This implies that the right-hand side of inequality (A.17) is a linear function of $p(\theta')$ on an interval of the form $(\underline{\theta}_a, \theta^B)$, where θ^B is defined as the market tightness in submarket $(\hat{a}, q')$ when $q' = q^B(a,\hat{a})$. The right-hand side of inequality (A.17) is positive at $\underline{\theta}_a$. We just showed that (A.17) is positive when $q' = q^B(a,\hat{a})$. Hence, it is positive in the entire range $(\underline{\theta}_a, \theta^B)$, or equivalently for any $q' \geq q^B(a,\hat{a})$.

Thus, for any submarket (a,θ') with $\theta' \in (\underline{\theta}_a, \bar{\theta}_a)$, sellers of quality a_j still find a strictly profitable deviation at the break-even price of sellers of quality a. So, the D1 criterion then requires that buyers do not expect to find units of quality a in these submarkets.

(iii) Case 3: $\hat{a} < a$. Let a_l be the quality whose "upward" incentive-compatibility constraint with a is binding in equilibrium. There are two cases to consider.

(i) *Case 3.1:* If $\hat{a} > a_l$, then the proof is identical to Case 1. Indeed, the only required change is to replace the announced quality in the expression of q^P. This change does not alter any argument made in that step of the proof.

(ii) *Case 3.2:* Suppose now that $\hat{a} \leq a_l$ and set q' the price on submarket $(\hat{a}, \theta')$. The net gain from deviating for sellers of quality a is

$$\Delta = p(\theta')[q' - \bar{v}(a)] - V(a),$$

where we have used the fact that the quality a is higher than the announced quality $\hat{a}$, so sellers of a still sell at price q' and not at the inspection-adjusted price $q^P(\cdot)$. Similarly, the net gain from deviating for sellers of quality a_l is

$$\Delta_l = p(\theta')(q' - \bar{v}(a_l)) - V(a_l).$$

The incentive-compatibility constraint between qualities a_l and a can be rewritten as

$$V(a_l) = V(a) + p(\underline{\theta}_a) \left[\psi(q^P(a_l, a, q(a, \underline{\theta}_a)) - q(a, \underline{\theta}_a)) + \bar{v}(a) - \bar{v}(a_l)\right]$$

Combining these three expressions we obtain:

$$\Delta_l - \Delta = (p(\theta') - p(\underline{\theta}_a))(\bar{v}(a) - \bar{v}(a_l)) + p(\theta(a))\psi(q(a, \underline{\theta}_a) - q^P(a_l, a, q(a, \underline{\theta}_a))).$$

Continuation values are increasing, $\theta' > \underline{\theta}_a$ and $q(a, \underline{\theta}_a) \geq q^P(a_l, a, q(a, \underline{\theta}_a))$. Hence, $\Delta_l > \Delta$—i.e., sellers of quality a_l are better off under a larger set of prices (and the corresponding beliefs) than sellers of quality a. Hence, quality a cannot be expected on such off-equilibrium submarkets.

In conclusion, if a quality is expected on a given off-equilibrium submarket, the seller of this quality does not have a strictly profitable deviation to that submarket. As a consequence, the recursive construction of $\Theta_{k-1} \cup \{\theta_k\}$ as the only fully revealing separating equilibrium that survives the D1 criterion is justified.

Conclusion of the recursion. We proved that there exists a unique fully revealing equilibrium on $\mathcal{A}_k$ that satisfies the D1 criterion and that the construction is done using the procedure described in the assertion. Hence, the assertion is true at rank k, which concludes the induction.

A non-revealing separating equilibrium is always weakly dominated by a fully revealing separating equilibrium. For a given quality a_k, it is possible to construct bounds $\underline{\theta}$ and $\bar{\theta}$ on any submarket with announced quality $\hat{a} < a_k$ and similarly construct a separating, but not fully revealing, equilibrium. Because the inspection-adjusted price $q^P(\cdot)$ is weakly decreasing in the announced quality, the revenue from mimicking of sellers of all lower qualities will be weakly *larger* when the announced quality is $\hat{a}$ than a_k. Thus, in order to disincentivize mimicking, the bound $\underline{\theta}$, conditional on an announced quality $\hat{a} < a_k$, will be lower than for an announced quality $\hat{a} = a_k$, and the bound $\bar{\theta}$ will be larger. At the same time, the transacted price obtained by the seller of quality a_k remains the same as the posted price (since there is no penalty for announcing a quality lower than its own). This implies that by announcing $\hat{a} < a_k$, the seller of quality a_k will have to choose a market tightness further from his full-information tightness than he would have had he signaled quality a_k, yielding a lower payoff. Therefore, non-revealing separating equilibria are at least weakly dominated by the fully revealing separating equilibrium. Since we assume $q^P(\cdot)$ to be weakly decreasing in the announced quality, we cannot rule out non-revealing separating equilibria as strictly dominated. If instead $q^P(\cdot)$ was strictly decreasing in the announced quality, then the fully revealing equilibrium would be also the unique separating equilibrium.

Step 3: Balanced-growth path under asymmetric information. The proof of Proposition 2 is then a simple application of the recursion above on $\mathcal{A} = \mathcal{A}_{N_a}$.

Step 4: There is no pooling equilibrium in transitional dynamics if $\psi q^P(\omega, a, \hat{a}, q) - \bar{v}(\omega, a)$ is monotonous in a. Assume that for all $(\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$, $\psi q^P(\omega, a, \hat{a}, q) - \bar{v}(\omega, a)$ is monotonous in a. Suppose that there exists an equilibrium such that the subset of types $A \subset \mathcal{A}$ are pooled together in submarket $(\omega, \hat{a}, q)$ with some strictly positive probability. Let $\bar{a} = \max\{a : a \in A\}$. As in previous steps, we suppose that they all have the same observed quality ω and omit it for convenience.

We next show that under the D1 criterion, there exists a strictly profitable deviation for sellers of quality $\bar{a}$, ruling out any equilibria in which pooling occurs with strictly positive probability. Let θ be the tightness on submarket $(\hat{a}, q)$ where pooling occurs. We can assume without loss of generality that buyers visit this submarket.[18] We then proceed by setting

[18]Otherwise, sellers of any quality could deviate to any submarket in which buyers would purchase for a strictly positive price, which would be a strictly profitable deviation.

beliefs consistent with the D1 criterion on submarkets where $\hat{a}$ is the announced quality and θ' is the market tightness.

Consider a deviation of a seller of quality $a \in A$, with $a < \bar{a}$, to another submarket with the same announced quality $\hat{a}$ and market tightness θ'. The seller would be better off for any price weakly greater than $\tilde{q}$, which is defined by the indifference condition:

$$p(\theta')((1-\psi)\tilde{q}+\psi q^P(a,\hat{a},\tilde{q}))+(1-p(\theta'))\bar{v}(a) = p(\theta)((1-\psi)q+\psi q^P(a,\hat{a},q))+(1-p(\theta))\bar{v}(a). \tag{A.19}$$

Let us now set beliefs. Let $\Delta_{\bar{a}}$ denote the net gain for the seller of quality $\bar{a}$ from deviating to a submarket with tightness θ' and price $\tilde{q}$:

$$\Delta_{\bar{a}} = p(\theta')((1-\psi)\tilde{q}+\psi q^P(\bar{a},\hat{a},\tilde{q}))+(1-p(\theta'))\bar{v}(\bar{a})-p(\theta)((1-\psi)q+\psi q^P(\bar{a},\hat{a},q))-(1-p(\theta))\bar{v}(\bar{a}).$$

Consider first the case with $\theta' < \theta$. Then, from equation (A.19) we have $\tilde{q} > q$. It can be verified that $q^P(\bar{a},\hat{a},q) - q^P(a,\hat{a},q)$ is weakly increasing in q. Therefore, $q^P(\bar{a},\hat{a},\tilde{q}) - q^P(a,\hat{a},\tilde{q}) \geq q^P(\bar{a},\hat{a},q) - q^P(a,\hat{a},q)$, which implies

$$\Delta_{\bar{a}} \geq (p(\theta)-p(\theta'))\left[\psi(q^P(a,\hat{a},q) - q^P(\bar{a},\hat{a},q)) + \bar{v}(\bar{a}) - \bar{v}(a)\right].$$

We now use the assumption that $\psi q^P(a,\hat{a},q) - \bar{v}(a)$ is monotonous in a. More specifically, assume the function is decreasing in a. This implies that the second term on the right-hand side is positive, as is the first term on the right-hand side. Hence, for all $a \in A$ and $\theta' < \theta$, the seller of quality $\bar{a}$ always has a larger benefit from deviating than the seller of inferior quality—by definition, at price $\tilde{q}$, the net benefit from deviating for sellers of quality a is zero. This ensures that the seller of quality $\bar{a}$ will be better off under the worst consistent beliefs. The D1 criterion then requires that quality $\bar{a}$ is expected in submarkets with tightness $\theta' < \theta$. We can now construct a profitable deviation for the seller of quality $\bar{a}$. Using the indifference condition of buyers, we have that $(1-\psi)q = \mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) - \frac{\chi}{\mu(\theta)}$. Given the monotonicity assumption, $\mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) = \nu^{\mathrm{b}}(\bar{a}) - \psi q^P(\bar{a},\hat{a},q) - \epsilon$ for some $\epsilon > 0$. The net gain from deviating to tightness θ' for the seller of quality $\bar{a}$ is then

$$\Delta_{\bar{a}} = p(\theta')[\nu^{\mathrm{b}}(\bar{a})-\bar{v}(\bar{a})]-p(\theta)\left[\mathbb{E}_a\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) + \psi q^P(\bar{a},\hat{a},q) - \bar{v}(\bar{a})\right]+\chi(\theta-\theta'),$$

or

$$\Delta_{\bar{a}} = (p(\theta') - p(\theta))[\nu^{\mathrm{b}}(\bar{a}) - \bar{v}(\bar{a})] + \chi(\theta - \theta') + p(\theta)\epsilon.$$

Hence, we can find a θ' sufficiently close to θ such that the deviation yields a strictly positive $\Delta_{\bar{a}}$, which in turn implies a strictly profitable deviation for the seller of quality $\bar{a}$. Therefore, the pooling equilibrium cannot be sustained. If the function $\psi q^P(a, \hat{a}, q) - \bar{v}(a)$ is instead increasing in a, we can make the exact symmetric reasoning with $\theta' > \theta$.

Step 5: There are no pooling equilibria on the balanced-growth path. Let us now apply the result from Step 4 to a balanced-growth-path equilibrium. Suppose that some qualities are pooled together at some announced quality $\hat{a}$ and market tightness θ. Let quality a be one of them and let q be the associated price.

The value of the seller of type a on the balanced-growth path is:

$$\nu^{\mathrm{s}}(a) = p(\theta)((1-\psi)q + \psi q^P(a, \hat{a}, q)) + (1 - p(\theta))(\Lambda \nu^{\mathrm{s}}(a) - \delta\omega a).$$

We multiply by Λ, subtract $\delta\omega a$, and reorganize the terms to obtain

$$\bar{v}(a)(1 - \Lambda(1 - p(\theta))) = -\delta\omega a + \Lambda p(\theta)((1-\psi)q + \psi q^P(a, \hat{a}, q)).$$

Solving for $\bar{v}(a)$ and subtracting $\psi q^P(a, \hat{a}, q)$ we obtain

$$\bar{v}(a) - \psi q^P(a, \hat{a}, q) = \frac{1}{(1 - \Lambda(1 - p(\theta)))}\left[-\delta\omega a + \Lambda(1-\psi)p(\theta)q - \psi q^P(a, \hat{a}, q)(1 - \Lambda)\right].$$

Hence, $a \to \bar{v}(a) - \psi q^P(a, \hat{a}, q)$ is monotonous in a for all of the unobserved qualities $a \in A$ that are pooled in submarket $(\hat{a}, \theta)$. We can then apply the previous step, which rules out any pooling equilibrium on a balanced-growth path. ■

A.1.1 Proof of Proposition 3 and Corollary 1

We first characterize the equilibrium in transitional dynamics.

Transitional dynamics in the full-information case. We now characterize the equilibrium under full information—namely the equilibrium when the signal is always informative of the unobserved quality ($\psi = 1$).

We fix the continuation value of sellers $\nu^{\mathrm{s}}_{t+1}(\omega, a)$ for all ω and a and describe the equilibrium in period t conditional on agents' continuation values. From Proposition 1, we have that the values of sellers and buyers are given by equations (3) and (7).

Under full information, the first-order condition with respect to vacancies posted is given by

$$\mu_t(\theta(\omega, a, q))(\nu^{\mathrm{b}}_t(\omega, a) - q^{FI}_t(\omega, a)) = w_t,$$

which relates the expected benefit from searching in a given submarket to the expected cost, and provides an indifference condition between sale prices and trading probabilities. Given this condition, the seller's maximization problem is then given by

$$\max_{\theta} p(\theta)\left(\nu^{\mathrm{b}}_t(\omega, a) - \frac{w_t\theta}{\gamma^t p(\theta)}\right) + (1 - p(\theta))(\Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}) - \delta\omega a),$$

which gives the following first-order condition with respect to θ:

$$p'(\theta)(\nu^{\mathrm{b}}_t(\omega, a) - \Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a) = \frac{w_t}{\gamma^t}.$$

We then replace the right-hand side using the indifference condition of buyers to obtain

$$(1 - \eta)(\nu^{\mathrm{b}}_t(\omega, a) - \Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a) = \nu^{\mathrm{b}}_t(\omega, a) - q^{FI}_t(\omega, a),$$

from which we can solve for the equilibrium full-information price

$$q^{FI}_t(\omega, a) = \eta\nu^{\mathrm{b}}_t(\omega, a) + (1 - \eta)(\Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}) - \delta\omega a).$$

To find the associated optimal market tightness, we replace this price in the seller's first-order condition to obtain

$$\theta^{FI}_t(\omega, a) = \left(\frac{\bar{m}\gamma^t}{w_t}(1 - \eta)(\nu^{\mathrm{b}}_t(\omega, a) - \Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a, \mathbf{K}_{Ht+1}) + \delta\omega a)\right)^{1/\eta}.$$

Balanced-growth path under full-information. We next use these results to obtain closed-form solutions for values and terms of trade on the balanced growth path equilibrium.

Let $\chi = \frac{w_t}{\gamma^t}$ denote the detrended wage in the balanced-growth path. From our expression above we have

$$q^{FI}(\omega,a)=\eta\nu^{\mathrm{b}}(\omega,a)+(1-\eta)(\Lambda\nu^{\mathrm{s}}(\omega,a)-\delta\omega a)$$

and

$$\theta^{FI}(\omega,a)=\left(\frac{\bar{m}}{\chi}(1-\eta)(\nu^{\mathrm{b}}(\omega,a)-\Lambda\nu^{\mathrm{s}}(\omega,a)+\delta\omega a)\right)^{1/\eta}.$$

From Proposition 1, the seller's and buyer's values in the balanced-growth path under full information are given by

$$\begin{aligned}
\nu^{\mathrm{b}}(\omega,a)&=(Z-\delta)\omega a+\Lambda\left[(1-\varphi)\nu^{\mathrm{b}}(\omega,a)+\varphi\nu^{\mathrm{s}}(\omega,a)\right],\\
\nu^{\mathrm{s}}(\omega,a)&=q^{FI}(\omega,a)p(\theta^{FI}(\omega,a))+\left(1-p(\theta^{FI}(\omega,a))\right)(\Lambda\nu^{\mathrm{s}}(\omega,a)-\delta\omega a).
\end{aligned}$$

Replacing these values in the optimal market tightness $\theta^{FI}(\omega,a)$, we obtain

$$p(\theta^{FI}(\omega,a))=\bar{m}\left(\frac{Z\omega a\bar{m}(1-\eta)}{\chi\left(1-\Lambda(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a)))\right)}\right)^{\frac{1-\eta}{\eta}},$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dlogp(\theta^{FI}(\omega,a))}{dlog(\omega)}=\frac{1-\eta}{\eta}\frac{\left(1-\Lambda\left(1-\varphi\right)\left(1-\eta p(\theta^{FI}(\omega,a))\right)\right)}{\left(1-\Lambda\left(1-\varphi\right)\left(1-p(\theta^{FI}(\omega,a))\right)\right)}>0.$$

Similarly, we replace the buyer's and seller's values in the optimal price $q^{FI}(\omega,a)$ to obtain

$$q^{FI}(\omega,a)=\frac{\omega a}{1-\Lambda}\left[\eta Z\frac{1-\Lambda(1-p(\theta^{FI}(\omega,a)))}{1-\Lambda(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a)))}-\delta\right]\equiv\frac{\omega a}{1-\Lambda}F(p(\theta^{FI}(\omega,a))).$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dq^{FI}(\omega,a)}{d\omega}=\frac{a}{1-\Lambda}\left[F(p(\theta^{FI}(\omega,a)))+\omega F'(p(\theta^{FI}(\omega,a)))\frac{dp(\theta^{FI}(\omega,a))}{d\omega}\right],$$

where

$$F'(p(\theta^{FI}(\omega,a)))=\eta Z\Lambda\frac{1-(1-\varphi)[\eta+\Lambda(1-\eta)]}{(1-\Lambda(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a))))^2}>0.$$

Thus, when $q^{FI}(\omega,a)\geq 0$, we obtain $\frac{dq^{FI}(\omega,a)}{d\omega}>0$. Since qualities ω and a have similar effects on optimal terms of trade, the same comparative statics apply to changes in a under full information. ■

A.1.2 Proof of Proposition 4

In the case with $\mathcal{A} = \{a_L, a_H\}$ with $a_L < a_H$, the seller of quality a_L chooses the full-information price and market tightness. The strategy of the seller of quality a_H is then determined by the binding incentive-compatibility constraint between him and sellers of quality a_L, which is given by

$$\begin{aligned} p(\theta^{FI}(a_L))(q^{FI}(a_L) - &\Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) = \\ &p(\theta(a_H))[(1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L]. \end{aligned}$$

There exists a threshold ψ^* that triggers the non-full-information solution. For a small degree of asymmetry of information, the constraint above might not be binding. Instead, the constraint becomes binding for a threshold value ψ^* defined by

$$\begin{aligned} p(\theta^{FI}(a_L))(q^{FI}(a_L) - &\Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) = \\ &p(\theta^{FI}(a_H))[(1-\psi^*)q^{FI}(a_H) + \psi^* q^P(a_L, a_H, q^{FI}(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L]. \end{aligned}$$

By Assumption 1, f$q^P(a_L, a_H, q^{FI}(a_H)) < q^{FI}(a_L)$; thus, we can rewrite the constraint and solve for ψ^* and obtain

$$\psi^* = \frac{\left(\nu^{S,FI}(a_H) - \nu^{S,FI}(a_L)\right)\left[1 - \Lambda(1 - p(\theta^{FI}(a_H)))\right] + \delta\omega\left(a_H - a_L\right)\left(1 - p(\theta^{FI}(a_H))\right)}{p(\theta^{FI}(a_H))(q^{FI}(a_H) - q^P(a_L, a_H, q^{FI}(a_H)))}. \tag{A.20}$$

Thus, for ψ below ψ^*, the incentive-compatibility constraint evaluated at the full-information terms of trade is not satisfied. Then, the optimal market tightness for sellers of quality a_H is determined by

$$\begin{aligned} p(\theta^{FI}(a_L))(q^{FI}(a_L) - &\Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) \\ &= p(\theta(a_H))\left((1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\right). \end{aligned}$$

Replacing the price $q(a_H)$ from the buyer's indifference condition, this condition can be rewritten as

$$p(\theta^{FI}(a_L))(q^{FI}(a_L) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L)$$
$$= p(\theta(a_H))\left((1-\psi)\nu^b(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\right) - (1-\psi)\theta(a_H)\chi. \tag{A.21}$$

Comparative statics. Let us differentiate the constraint with respect to ψ in the region in which the incentive-compatibility constraint binds (i.e., $\psi \leq \psi^*$). Differentiating (A.21) with respect to ψ, we obtain

$$\frac{d\log(\theta(a_H))}{d\psi} = \frac{q(a_H) - q^P(a_L, a_H, q(a_H))}{(1-\eta)((1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) - \eta(1-\psi)\frac{\chi}{\mu(\theta(a_H))}}. \tag{A.22}$$

To sign this expression, we need to make sure that the denominator is positive since $q(a_H) \geq q^{FI}(a_H) > q^{FI}(a_L) \geq q^P(a_L, a_H, q(a_H))$. Under full information, we know that

$$q^{FI}(a_H) = \nu^{\mathrm{b}}(a_H) - \frac{w}{\mu(\theta^{FI}(a_H))} = \eta\nu^{\mathrm{b}}(a_H) + (1-\eta)(\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H),$$

hence

$$\frac{\chi}{\mu(\theta^{FI}(a_H))} = (1-\eta)(\nu^{\mathrm{b}}(a_H) - \Lambda\nu^{\mathrm{s}}(a_H) + \delta\omega a_H).$$

We also know that the binding incentive-compatibility constraint (A.21) has two solutions for $\theta(a_H)$ and that as long as $\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H \geq \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$, the lowest one will be chosen by the seller of quality a_H. In particular, this implies $\theta(a_H) \leq \theta^{FI}(a_H)$. Since $\mu(\theta)$ is a decreasing function of θ, we obtain

$$\frac{\chi}{\mu(\theta(a_H))} \leq (1-\eta)(\nu^{\mathrm{b}}(a_H) - \Lambda\nu^{\mathrm{s}}(a_H) + \delta\omega a_H).$$

The denominator in (A.22) can be rewritten as:

$$(1-\eta)((1-\psi)q(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L) - \eta(1-\psi)\frac{\chi}{\mu(\theta(a_H))} =$$
$$(1-\eta)\left((1-\psi)\nu^{\mathrm{b}}(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\right) - (1-\psi)\frac{\chi}{\mu(\theta(a_H))}.$$

Using the previous inequality and the fact that $q^P(a_L, a_H, q(a_H)) > \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$:

$$(1-\eta)\Big((1-\psi)\nu^{\mathrm{b}}(a_H) + \psi q^P(a_L, a_H, q(a_H)) - \Lambda\nu^{\mathrm{s}}(a_L) + \delta\omega a_L\Big) - (1-\psi)\frac{\chi}{\mu(\theta(a_H))} >$$
$$(1-\eta)(1-\psi)\left[\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H - (\Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L)\right].$$

Since continuation values are increasing, $\Lambda\nu^{\mathrm{s}}(a_H) - \delta\omega a_H \geq \Lambda\nu^{\mathrm{s}}(a_L) - \delta\omega a_L$. This implies that the denominator in (A.22) is positive, which in turn implies

$$\frac{dlog(\theta(a_H))}{d\psi} > 0.$$

Thus, the optimal market tightness for sellers of quality a_H is increasing in the informativeness of the inspection. From the buyer's indifference condition, we can also conclude that their optimal posted price is decreasing in the informativeness of the inspection.

There exists a threshold $\underline{\psi}$ such that sellers' continuation values are increasing for $\psi \geq \underline{\psi}$. The last item we need to verify is that $\bar{v}(a_H) > \bar{v}(a_L)$, with $\bar{v}(a) \equiv \Lambda\nu^{\mathrm{s}}(a) - \delta\omega a$, in the case in which the incentive-compatibility constraint is binding so that we can guarantee the existence of the equilibrium.

Rewriting the incentive-compatibility constraint, we have

$$\nu^{\mathrm{s}}(a_L) - \bar{v}(a_L) = \nu^{\mathrm{s}}(a_H) - \bar{v}(a_H) + p(\theta(a_H))[\psi(q^P(a_L, a_H, q(a_H)) - q(a_H)) + \bar{v}(a_H) - \bar{v}(a_L)].$$

Using the fact that $q(a_H) > q^{FI}(a_H)$ and $q^P(a_L, a_H, q(a_H)) = q^B(a_L, a_H)$, this constraint can be written as

$$(1 - \Lambda(1 - p(\theta(a_H))))(\bar{v}(a_H) - \bar{v}(a_L)) = \Lambda\psi p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] - \delta\omega(a_H - a_L).$$

Hence, $\bar{v}(a_H) \geq \bar{v}(a_L) \geq 0 \iff \Lambda\psi p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$.

For δ low enough, there exists a ψ such that $\psi\Lambda p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$. As ψ decreases, $\theta(a_H)$ decreases and since the function on the left-hand side is concave in $\theta(a_H)$, the left-hand side becomes increasing in ψ at optimal choices of $\theta(a_H)$. This implies that there exists a threshold $\underline{\psi}$ such that

$$\Lambda\underline{\psi}p(\theta(a_H))[q(a_H) - q^B(a_L, a_H)] = \delta\omega(a_H - a_L).$$

Thus, assuming that δ is low enough to have $\Lambda\psi^* p(\theta^{FI}(a_H))[q^{FI}(a_H) - q^B(a_L, a_H)] \geq \delta\omega(a_H - a_L)$, then $\underline{\psi} \leq \psi^*$ and we obtain that the inequality is satisfied on the interval $[\underline{\psi}, 1]$, which in turn ensures that continuation values are increasing on that interval. Finally, notice that $\lim\limits_{\delta \to 0} \underline{\psi} = 0$. That is, as the depreciation rate becomes negligible, it is always true that sellers' continuation values are increasing in the quality a.

There is no pooling equilibrium in the two-quality case. Using our earlier proof that rules out pooling equilibria, we need to verify the condition that for all $(\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$, $\psi q^P(\omega, a, \hat{a}, q) - \nu^{\mathrm{s}}(\omega, a) + \delta\omega a$ is monotonous in a. Here, this condition is trivially satisfied. Indeed, the function is always monotonous over the set of unobserved qualities since there are only two (either one is greater than the other or the reverse). ■

A.1.3 Proof of Proposition 5

In a fully revealing separating equilibrium, the buyer's indifference condition is given by

$$q_t(\omega, a) = \nu_t^{\mathrm{b}}(\omega, a) - \frac{w_t}{\mu_t(\theta(\omega, a))},$$

where

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left((1-\varphi)\,\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right).$$

As $\varphi \to 0$, this value can be expressed as

$$\begin{aligned} \nu_t^{\mathrm{b}}(\omega, a) &\approx \omega a\left((Z_t - \delta) + \sum_{j=1}^{\infty}\left(\prod_{i=0}^{j}\Lambda_{t,t+i}\right)(Z_{t+j} - \delta)\right) \\ &\approx \omega a f(Z_t), \end{aligned}$$

where we have used the fact that Z_t follows a Markov process to summarize the effect of future productivity in terms of the function of current productivity $f(Z_t)$. Then, taking logs, this value can be expressed as

$$\log \nu_t^{\mathrm{b}}(\omega, a) \approx \log \omega + \log a + \iota_t,$$

where ι_t denote time fixed effects. Finally, making the assumption that search costs represent a small fraction of the buyer's value of a unit of capital—i.e., $\frac{\frac{w_t}{\mu_t(\theta(\omega,a))}}{\nu_t^{\mathrm{b}}(\omega,a)} \to 0$—we obtain the following expression for the price of a unit of capital:

$$\begin{aligned}\log q_t(\omega, a) &\approx \log \omega + \iota_t + \log a \\ &= \tau X + \iota_t + \log a,\end{aligned}$$

where the second line imposes the mapping between observed characteristics and observed efficiency units $\log \omega = \tau X$. Thus, using microdata we can regress

$$\log q_{it} = \iota_\omega X_i + \iota_t + \varepsilon_{it}^q,$$

resulting in a consistent estimator for τ (and thus ω) and the unobserved quality a from ι_ω and the residuals ε_{it}^q, respectively.

In the second step, we need to estimate the following regression:

$$\log(Duration_{it}) = \upsilon_\omega \log(\omega_{it}) + \upsilon_q \log(q_{it}) + \iota_t + \varepsilon_{it}^d.$$

Given our assumptions that yield $\log q_{it} \approx \log \omega_{it} + \iota_t + \log a_{it}$ and the independence between ω_{it} and a_{it}, we can recover $\upsilon_q = \frac{cov(\log Duration_{it}, \log a_{it})}{var(\log a_{it})}$.

To map this result to our model, note that $Duration_{it} \equiv Duration(\omega_{it}, a_{it}) \sim Geometric(1/p(\omega_{it}, a_{it}))$. Thus,

$$Duration(\omega_{it}, a_{it}) = \frac{1}{p(\omega_{it}, a_{it})} + \eta_{it},$$

with $\mathbb{E}(\eta_{it}|\omega_{it}, a_{it}) = 0$. Making a first-order approximation of $\log Duration_{it}$ around $\eta_{it} = 0$, we obtain

$$\log Duration(\omega_{it}, a_{it}) \approx \log\left(\frac{1}{p(\omega_{it}, a_{it})}\right) + p(\omega_{it}, a_{it})\eta_{it}.$$

Using the law of total variance and the approximation, the estimated covariance simplifies to

$$cov(\log Duration(\omega_{it}, a_{it}), \log a_{it}) \approx -cov\left(\log\left(p(\omega_{it}, a_{it})\right), \log a_{it}\right).$$

Taking a first-order approximation of the equilibrium price around the mean qualities $(\bar{\omega}, \bar{a})$, and of $\log a$ around $\bar{a}$,

$$\log\left(p(\omega_{it}, a_{it})\right) \approx \log\left(p(\bar{\omega}, \bar{a})\right) + \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial \omega_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\left(\omega_{it} - \bar{\omega}\right) + \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\left(a_{it} - \bar{a}\right)$$

and

$$\log a_{it} \approx \log \bar{a} + \frac{1}{\bar{a}}\left(a_{it} - \bar{a}\right).$$

Replacing this expression in the covariance,

$$\begin{aligned}
cov\left(\log\left(p(\omega_{it}, a_{it})\right), \log a_{it}\right) &\approx cov\left(\frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial \omega_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\left(\omega_{it} - \bar{\omega}\right) + \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\left(a_{it} - \bar{a}\right), \frac{1}{\bar{a}}\left(a_{it} - \bar{a}\right)\right) \\
&= \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\bar{a} var\left(\frac{a_{it}}{\bar{a}} - 1\right) \\
&\approx \frac{\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}}{p(\bar{\omega}, \bar{a})}\bar{a} var(\log a_{it}),
\end{aligned}$$

where the second step follows from the independence of ω_{it} and a_{it}. Thus,

$$\begin{aligned}
\upsilon_q &= \frac{cov(\log Duration_{it}, \log a_{it})}{var(\log a_{it})} \\
&\approx -\frac{cov\left(\log\left(p(\omega_{it}, a_{it})\right), \log a_{it}\right)}{var(\log a_{it})} \\
&\approx -\frac{\partial p(\omega_{it}, a_{it})}{\partial a_{it}}|_{\bar{\omega},\bar{a}}\frac{\bar{a}}{p(\bar{\omega}, \bar{a})} \\
&\approx -\frac{\partial \log p(\omega_{it}, a_{it})}{\partial \log a_{it}}|_{\bar{\omega},\bar{a}}.
\end{aligned}$$

As shown in 4, our model predicts that $\frac{\partial \log p(\omega_{it}, a_{it})}{\partial \log a_{it}}$ is a strictly monotonic function of ψ, which means ψ can be recovered by inverting the function $\upsilon_q(\psi)$.

A.2 Additional results

A.2.1 Bargaining as a special case of inspection-adjusted price function

***Lemma* 2.** *Suppose that $q^P(\omega, a, \hat{a}, q)$ is determined by Nash bargaining. Let us denote ϕ the bargaining power of sellers so that*

$$q_t^P(\omega, a, \hat{a}, q) = min(q, \phi\nu_t^{\mathrm{b}}(\omega, a) + (1-\phi)\left[\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right]). \tag{A.23}$$

Then, q_t^P satisfies Assumption 1 if and only if $\phi \leq \eta$.

Proof. Let us assume that the post-inspection price function $q^P(\cdot)$ is determined by a Nash bargaining protocol and the bargaining power of the seller is ϕ with $\phi < \eta$:

$$q_t^P(\omega, a, \hat{a}, q) = \min\left(\phi \nu_t^{\mathrm{b}}(\omega, a) + (1-\phi)\left[\Lambda_t \nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right], q\right). \tag{A.24}$$

Let us note $q_t^B(\omega, a, \hat{a}) = \phi \nu_t^{\mathrm{b}}(\omega, a) + (1-\phi)\left[\Lambda_t \nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a\right]$ so that $q_t^P(\omega, a, \hat{a}, q) = min(q_t^B(\omega, \hat{a}, a), q)$. We now verify that this function satisfies all of the conditions in Assumption 1. To do so, we make use of the result whereby both $\nu^{\mathrm{b}}(\omega, a)$ and $\nu^{\mathrm{s}}(\omega, a) - \delta\omega a$ are increasing in the unobserved quality a, as we showed in the proof of Proposition 2.

(i) $q^P(\omega, a, \hat{a}, q)$ *is non-decreasing in the true quality:* For any two qualities $a' > a$,

$$\begin{aligned} &q_t^P(\omega, a', \hat{a}, q) - q_t^P(\omega, a, \hat{a}, q) \\ &\qquad = \phi(\nu_t^{\mathrm{b}}(\omega, a') - \nu_t^{\mathrm{b}}(\omega, a)) + (1-\phi)\left[\Lambda_t(\nu_{t+1}^{\mathrm{s}}(\omega, a') - \nu_{t+1}^{\mathrm{s}}(\omega, a)) - \delta\omega(a'-a)\right] \geq 0, \end{aligned}$$

which proves the first condition.

(ii) $q^P(\omega, a, \hat{a}, q)$ *is non-increasing in the announced quality:* The second condition is trivially satisfied, since $q^P(\omega, a, \hat{a}, q)$ in (A.24) does not depend on the announced quality.

(iii) $q^P(\omega, a', \hat{a}, q)$ *is weakly lower (resp. higher) that the buyer's (resp. seller's) value for the unit:* This condition is also trivially satisfied as $q^P(\omega, a', \hat{a}, q)$ in (A.24) is the minimum of q and a convex combination of $\nu^{\mathrm{b}}(\omega, a)$ and $\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a) - \delta\omega a$.

(iv) *Buyers obtain at least a fraction $1-\eta$ of the surplus:* The Nash bargaining solution implies that buyers get a fraction $1-\phi$ of the surplus. Hence, as long as $\phi \leq \eta$ this condition is satisfied.

(v) q_t^P *does not decrease too fast as the announced quality increases:* We need to show that

$$\frac{\eta(\nu_t^{\mathrm{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i)}{q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i} \geq \frac{q_t^B(\omega, a_j, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}{q_t^B(\omega, a_j, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_j) + \delta\omega a_j}.$$

Since $q^B(\cdot)$ does not depend on the announced quality (i.e., $q^B(\omega, a_j, a_k) = q^B(\omega, a_j)$), the right-hand side of the inequality is equal to one. Hence, the inequality simplifies to

$$\eta(\nu_t^{\mathrm{b}}(\omega, a_i) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i) \geq q^P(\omega, a_i, a_k) - \Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) + \delta\omega a_i,$$

or

$$\eta\nu_t^{\mathrm{b}}(\omega, a_i) + (1-\eta)(\Lambda_{t+1}\nu_{t+1}^{\mathrm{s}}(\omega, a_i) - \delta\omega a_i) \geq q^P(\omega, a_i, a_k),$$

which is satisfied since it corresponds to the fourth condition of Assumption 1.

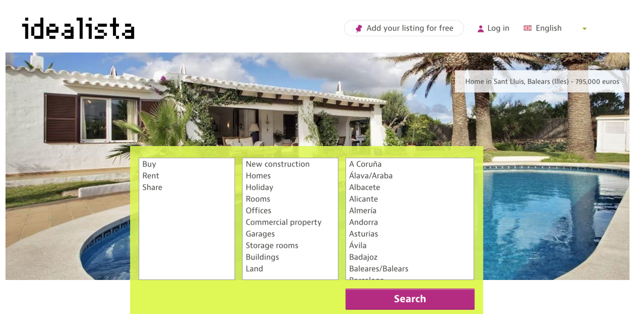
■

B Appendix Empirical Appendix

B.1 The online platform

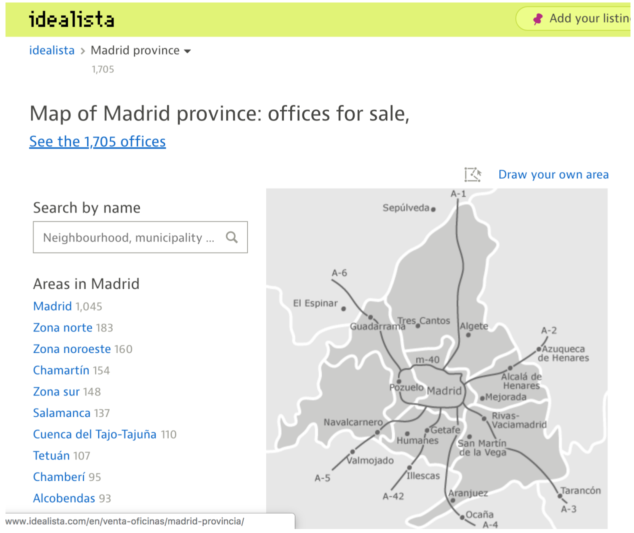
This subsection describes how the platform works. When entering the website, the buyer encounters the screen shown in Figure B1. The platform asks the client to choose a type of transaction (buy, rent, or find a shared space), the type of property (retail store, office, etc.), and the location.

FIGURE B1: Main Website



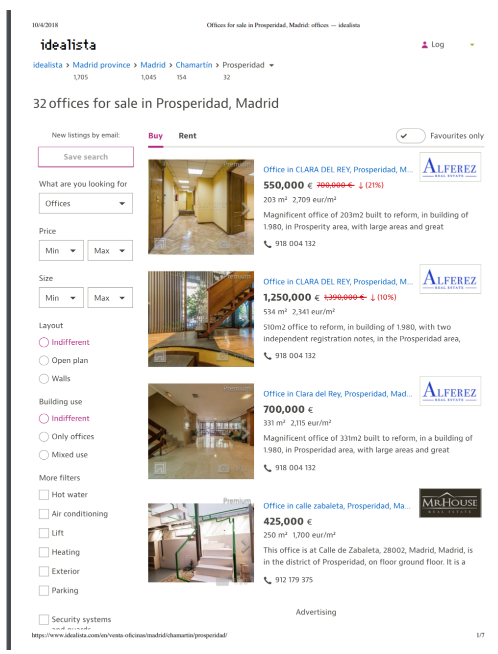
Once those options are selected (suppose the client wants to find a unit in Madrid—see Figure B2), then the website shows the number of properties available for sale by area in the city.

FIGURE B2: Options in Madrid



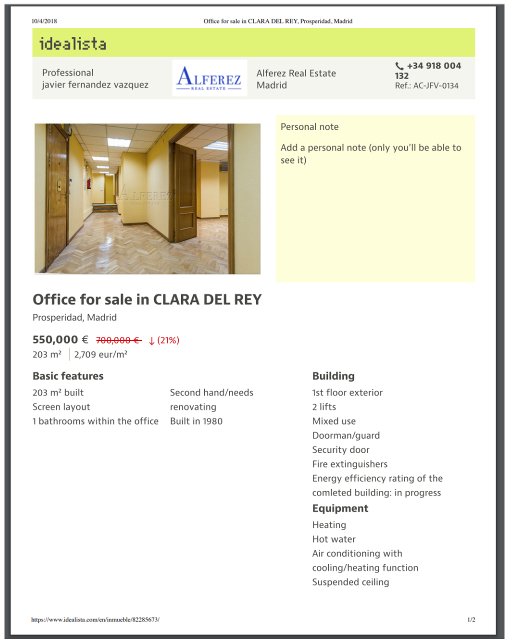
After choosing a narrower location within the city (not shown here), the client finds a scrolling list of the available units that meet her requirements, as shown in Figure (B3). There, the user can include more filters depending on her requirements for layout and amenities.

FIGURE B3: Available Listings in a Narrow Location in Madrid



When the user finds a unit that may be to her taste and clicks on it, a window pops up with the details shown in Figure B4 and additional text details not shown here. The information the listing contains is the unit description with pictures, price, change in price, area, construction date, and other amenities and equipment.

FIGURE B4: A Listing on the Website



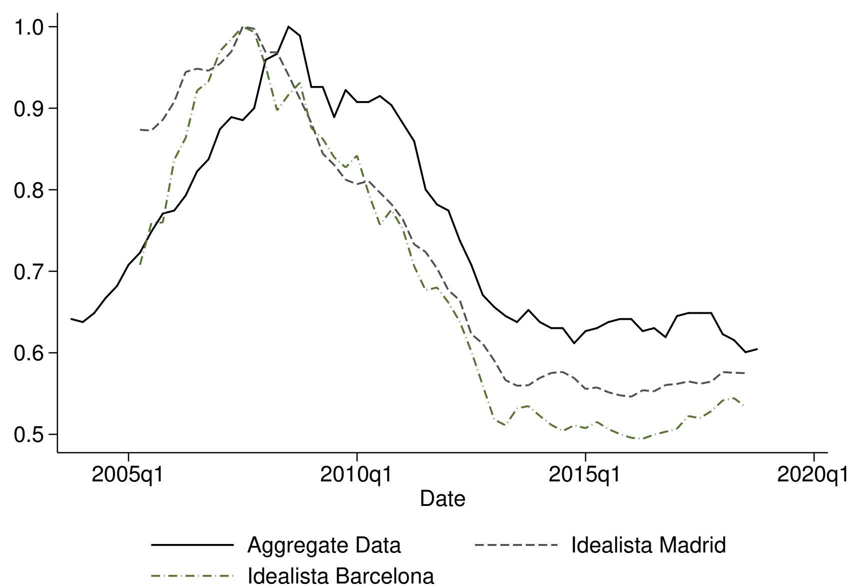
B.2 Representativeness of the dataset

In this subsection, we analyze the representativeness of the dataset and show that our data is consistent with aggregate patterns observed in Spain over this period. We provide two pieces of evidence regarding our data. First, we show that the price index exhibits the patterns of aggregate data. Second, we show that the patterns of sales follow those of aggregate sales of structures in Spain.

Figure B5 shows the index of listed prices for properties for sale in our sample and the index of transacted prices of retail space in Spain (obtained from official transaction records). Both indexes are normalized to 1 at their respective peak. We highlight the fact that the fall in prices we observe is consistent and very similar in size to that observed for retail space in Spain during the recent financial crisis. Moreover, our index leads the aggregate index, which is expected since our index consists of listed prices and it will take properties some months to exit the database, be registered as sales, and be recorded in national statistics. These patterns are consistent with the evidence presented in Guren (2018) who shows that the modal property sells at its listed price and that the average property sells within 1.6% of its listed price.

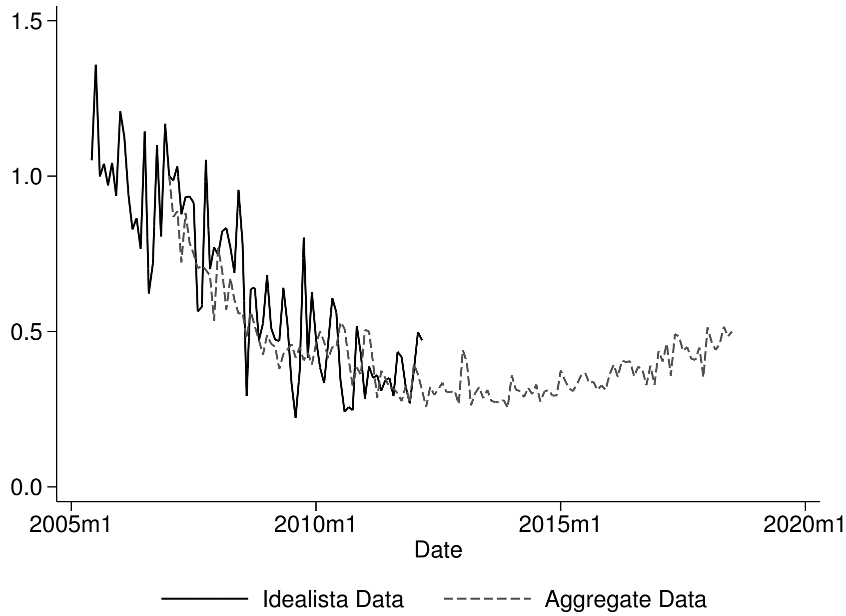
Figure B6 shows the index of sales for properties for sale in our sample and the aggregate sales index of real estate in Spain. Both indexes are normalized to take the value of 1 in the first month of 2007. The index for our data is constructed by computing the share of units that exit the database with respect to the number of active posts in that month. In the case of the aggregate number, we normalize the number of sale transactions recorded by the Statistical Agency of Spain (INE). In doing this, we assume that the total stock of units during this period is fairly constant (we do not have information on the size of the stock). Although our index is noisier than the national estimates, the patterns of the two series are close to each other.

Figure B5: Price Index: Idealista Data versus Aggregate Data



Note: The solid line shows the price index for properties for sale in Barcelona and Madrid in our dataset. The dashed line shows the aggregate retail space price index gathered from the National Registry of Property (*Registradores de España*). All indices are normalized to their respective peak.

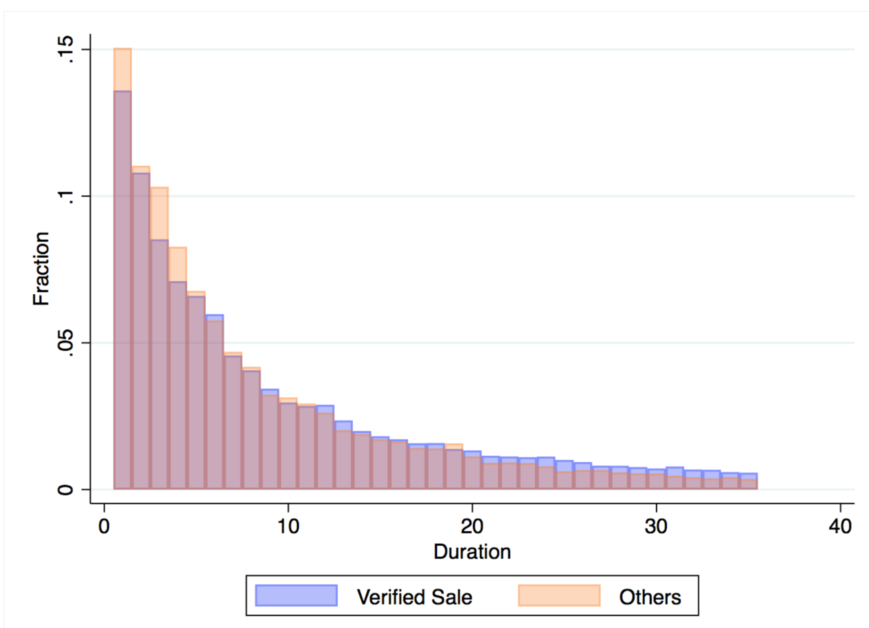
Figure B6: Sales Rate: Idealista Data versus Aggregate Data



Note: The solid line shows the sales rate for properties in our dataset. The dashed line shows the aggregate sales index of real estate gathered from the Statistical Agency of Spain (INE). Both indices take the value of one in January 2007.

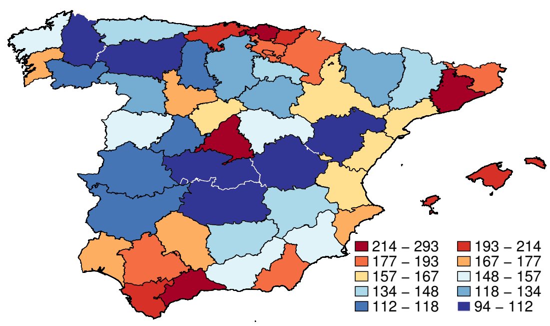
B.3 Additional Figures and Tables

Figure B7: Distribution of Duration: Confirmed Sales

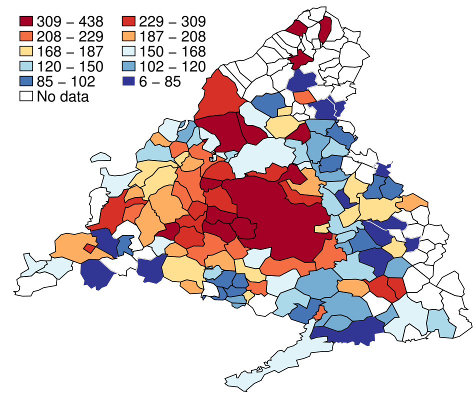


Note: This figure compares the histogram of duration for two subgroups of listings: those that, after removing the listing from the platform, explained that they did so because the property was rented out or sold, and those that did not provide an explanation.

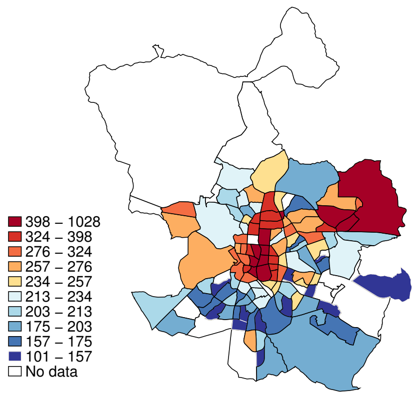
Figure B8: Capital Prices Across Locations



(A) Spain



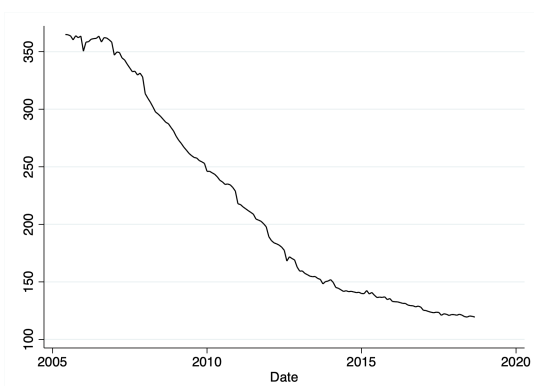
(B) Province of Madrid



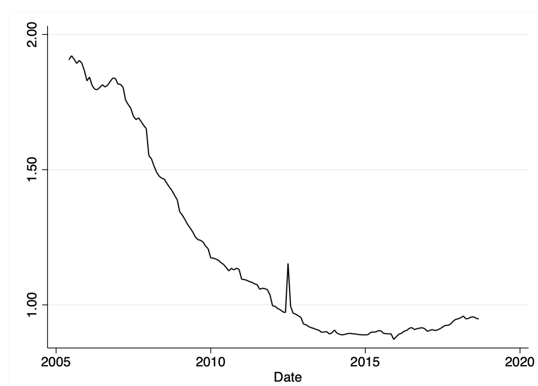
(C) City of Madrid

Note: Each map shows average prices by location expressed in constant 2017 dollars per square foot. Panel (A) shows average prices across provinces in Spain. Panel (B) zooms in on the province of Madrid to show substantial heterogeneity across municipalities within this province. Panel (C) shows that, after zooming in on the municipality of Madrid, there is still significant geographical dispersion of prices across neighborhoods.

Figure B9: Evolution of Prices of Capital Units



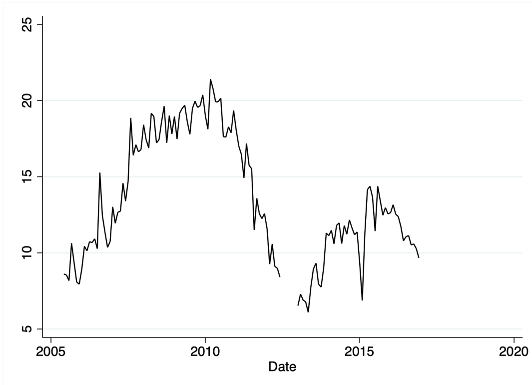
(A) Capital for Sale



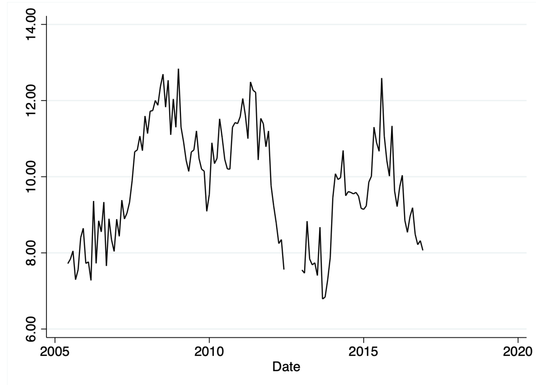
(B) Capital for Rent

Note: The left panel shows the evolution of mean prices at the daily frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Prices are denominated in constant 2017 dollars per square foot. To compute these price indices, we averaged the prices of all active listings in a given day.

Figure B10: Evolution of Average Duration



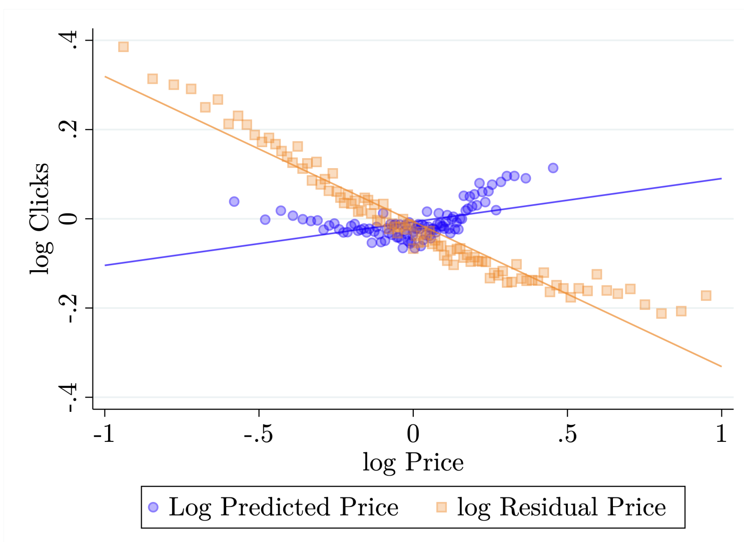
(a) Capital for Sale



(b) Capital for Rent

Note: The left panel shows the evolution of mean time to sell (in months) at monthly frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Time to sell is measured as the time difference between the entry and exit dates of each listing. Each observation contains the average time to sell for listings that entered the online platform in a given month.

Figure B11: Relationship between log Clicks and log Prices



Note: This figure shows the relationship between log prices and log average monthly clicks. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (18)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table B1: Price Variation Accounted for by Listed Characteristics in New Entrants

	Sale		Rent	
Statistic	IQR	R^2	IQR	R^2
Raw Data	0.666	0.000	0.802	0.000
Benchmark	0.284	0.776	0.198	0.845
Property Fixed Effect	0.119	0.946	0.116	0.937

Note: This table extends Table 2 by including a property fixed effect, which gathers inference from properties that change their prices while they are active in the dataset. We find that after including property fixed effects, nonparametrically absorbing all of the property's time-invariant price determinants, the IQR is roughly 11% and the R^2 is roughly 0.94.

Table B2: Frequency of Price Changes for Capital

	Rent Office	Sale Office	Rent Warehouse	Sale Warehouse
Frequency of Price Changes	0.07	0.07	0.05	0.07
Frequency of Price Increases	0.02	0.02	0.02	0.02
Frequency of Price Decreases	0.05	0.05	0.04	0.05
Absolute Size of Price Changes	0.15	0.12	0.16	0.15
Absolute Size of Price Increases	0.19	0.15	0.19	0.18
Absolute Size of Price Decreases	0.14	0.11	0.15	0.14

Note: This table presents price adjustment statistics by property type and operation. In order to compute the table, we first compute statistics on price changes within each property and then take averages across properties in a given time period. Finally, we compute the average over time. The first row shows the frequency of price changes, which is the average share of properties that exhibit a price change in a given month. The following two rows show the share of listings with price increases and decreases. The absolute size of price changes is computed as the absolute value of the log difference in prices over consecutive months (ignoring the zeros).

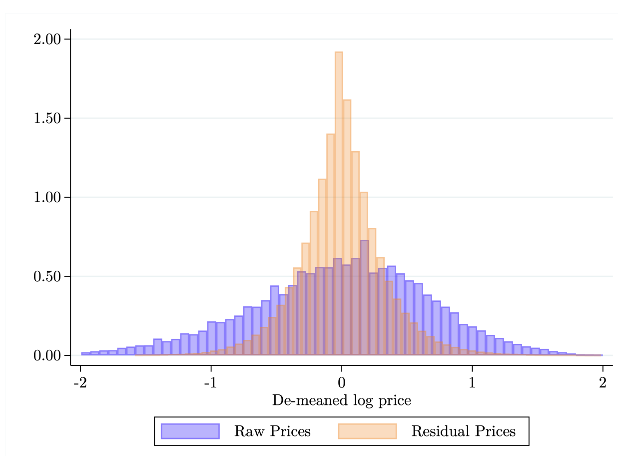
Table B3: Prices and Clicks

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.027***		-0.226***	
	(0.008)		(0.006)	
log Predicted Price		0.248***		0.116***
		(0.012)		(0.011)
log Residual Price		-0.270***		-0.272***
		(0.007)		(0.007)
Constant	3.299***	2.265***	4.509***	2.889***
	(0.035)	(0.056)	(0.029)	(0.051)
Observations	398260	387213	386163	386163
R^2	0.000	0.035	0.421	0.425
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log average monthly clicks on the two components of prices, residual and predicted prices. The left-hand-side variable is the log average monthly clicks of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of log clicks on prices. Column 2 regresses log clicks on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

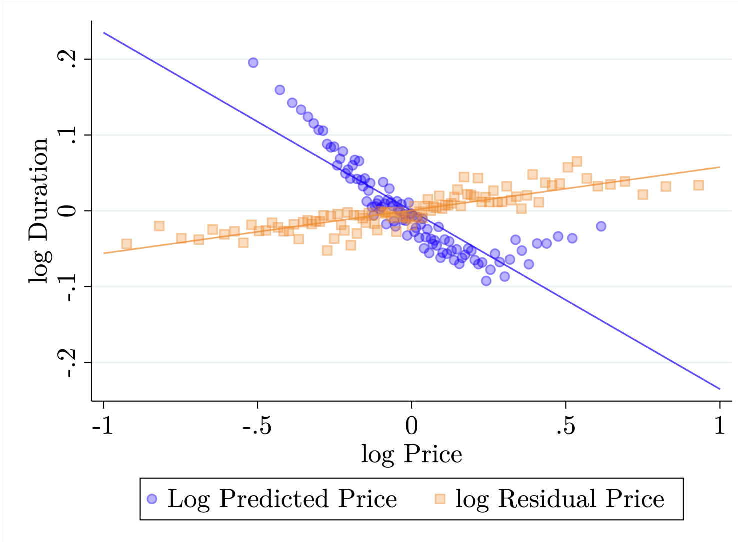
B.4 Results for properties listed for rent

Figure B12: Distribution of Price Residuals



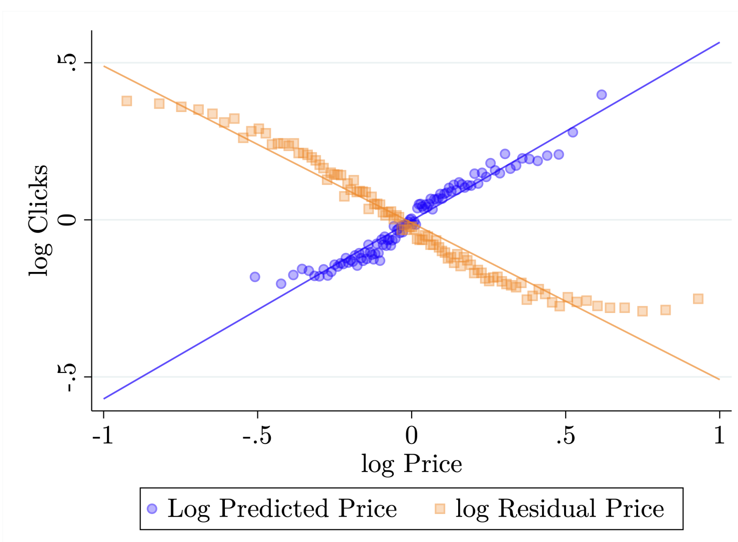
Note: This figure shows the distribution of log prices per square foot relative to its mean for the raw data and price residuals after including the fixed effects in Table (2).

Figure B13: Relationship between log Duration and log Prices



Note: This figure shows the relationship between log prices and duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (18)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Figure B14: Relationship between log Clicks and log Prices



Note: This figure shows the relationship between log prices and log average monthly clicks. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics (see equation (18)). Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table B4: Price Variation Accounted for by Listed Characteristics

	St. Dev.	R^2
Raw data	0.69	0.00
Year	0.68	0.04
Location	0.51	0.45
Year $\times$ Location	0.48	0.51
... $\times$ Type	0.47	0.54
... $\times$ Area	0.37	0.71
... $\times$ Age	0.36	0.73
Benchmark	0.35	0.73

Note: This table reports the R^2 and standard deviation of residuals from estimating equation (18). The row labeled Raw data presents statistics for the demeaned raw log prices. The following rows include the fixed effects in the regression. Year and location denote fixed effects. Type (office and retail space, or warehouse), area, and age are sets of fixed effects for each of these characteristics. The last row includes additional controls for the variables listed in Table 1.

Table B5: Prices and Duration

	(1) log Duration	(2) log Duration	(3) log Duration	(4) log Duration
log Price	-0.092***		-0.012***	
	(0.004)		(0.003)	
log Predicted Price		-0.175***		-0.228***
		(0.006)		(0.007)
log Residual Price		0.032***		0.032***
		(0.004)		(0.004)
Constant	1.848***	1.838***	1.857***	1.832***
	(0.004)	(0.004)	(0.000)	(0.001)
Observations	696874	680553	680553	680553
R^2	0.007	0.014	0.182	0.186
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The left-hand-side variable is the log duration of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of duration on prices. Column 2 regresses duration on predicted prices and residual prices. Columns 3 and 4 include location$\times$time$\times$type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

B.5 Alternative Explanations for the Price-Duration Relationship

Sellers' indifference We begin by considering potential explanations for the positive relationship between residual price and duration on the market that rely on sellers' indifference across these variables: Although all else equal, sellers prefer higher residual prices; this is also associated with longer duration on the market. This type of explanation is akin to that of

Table B6: Prices and Clicks

	(1) log Clicks	(2) log Clicks	(3) log Clicks	(4) log Clicks
log Price	0.176***		-0.195***	
	(0.011)		(0.007)	
log Predicted Price		0.528***		0.565***
		(0.017)		(0.010)
log Residual Price		-0.351***		-0.353***
		(0.009)		(0.009)
Constant	3.883***	3.950***	3.829***	3.957***
	(0.013)	(0.013)	(0.001)	(0.002)
Observations	578653	567847	566704	566704
R^2	0.013	0.088	0.437	0.460
Fixed Effects	No	No	Yes	Yes

Note: This table presents the results of a regression of log average monthly clicks on the two components of prices, residual and predicted prices. The left-hand-side variable is the log average monthly clicks of a listing and the right-hand-side variable is the mean price over the lifetime of the listing. The first column shows a regression of log clicks on prices. Column 2 regresses log clicks on predicted prices and residual prices. Columns 3 and 4 include location×time×type fixed effects. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

labor- and product-market models such as those of Burdett and Judd (1983) and Burdett and Mortensen (1998). To study whether this trade-off can explain the positive relationship observed between residual prices and duration in the data, we compute the expected net present discounted revenue for properties with different residual prices, given their observed trading probabilities implied by the relation in Figure 6. We begin by assuming homogeneous risk-neutral sellers; in this case, the expected net present revenue from choosing residual price ε_{it} is given by

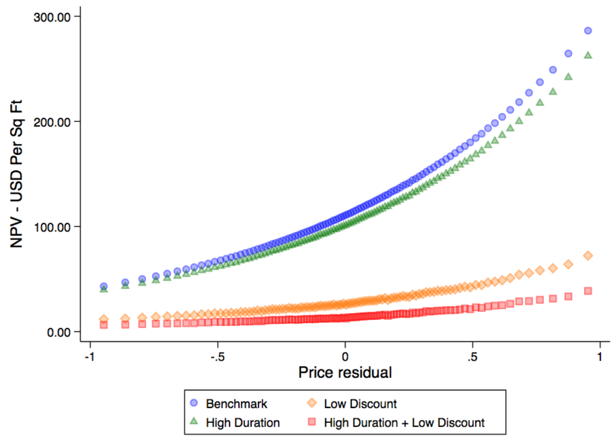
$$\mathcal{R}(\varepsilon_{it}, \beta) \equiv \sum_{t=0}^{\infty} \beta^t (1 - p(\varepsilon_{it}))^t p(\varepsilon_{it})\varepsilon_{it} = \frac{p(\varepsilon_{it})\varepsilon_{it}}{(1 - \beta(1 - p(\varepsilon_{it})))}, \tag{B.25}$$

where $p(\varepsilon_{it})$ is the associated selling probability implied by the empirical relationship between residual prices and duration, depicted in Figure 6, and $\beta \in [0, 1)$ is the discount factor used in the exercise.[19]

Figure B15 shows the results of this exercise for a wide range of discount factors ($\beta = 0.99$ and $\beta = 0$), which indicate that the expected net present discounted revenue is monotonically

[19]Note that we abstract from price changes in this calculation and use the mean price instead, since the frequency of price changes is small. The trading probability in a given month is computed from the duration of each property.

Figure B15: Net Present Value of Price-Duration Trade-off



Note: This figure reports the net present value estimates from (B.25). The blue line ("Benchmark") shows the net present value for a discount factor of $\beta = 0.99$ given the empirical relationship between residual prices and average duration in the data. The orange line ("Low Discount") shows a similar net present value calculation for a discount factor of $\beta = 0$. The green line replaces the average duration by the "pessimistic" measure of duration and recomputes the net present value. The red lide ("High Duration + Low Discount") reproduces this analysis for a discount factor of $\beta = 0$.

increasing in the listed price. Lower discount factors disproportionally affect properties that have lower trading probabilities and high prices, which flattens the net present value profile. However, even in the extreme scenario of $\beta = 0$, the relation between prices and duration in the data is such that we still find that the net present value is monotonically increasing in the listed price. These results indicate that sellers' indifference cannot explain the observed relationship between residual prices and duration: Any seller facing such a price-duration trade-off will maximize expected revenue by choosing the highest residual price we see in the data.

To study how the conclusions from this exercise are affected by sellers' risk aversion, we also compute the expected net present value in (B.25) under "pessimistic" selling probabilities associated with each residual price, $\pi(\varepsilon_{it})$. For this, we create quantiles of the price residual, and within each quantile we compute the standard deviation of duration across listings. Then, to compute the selling probability of each property, we use the realized duration plus 2 standard deviations of the duration within the quantile to which each property belongs. The green triangles and red squares in Figure B15 show the results from this exercise for different discount factors. Introducing pessimistic probabilities flattens the net present revenue schedule. This is because the distribution of durations is more dispersed (riskier) for higher prices, which decreases the net present revenue associated with listing a higher residual price. However, the expected net present revenue profile is still upward-sloping, which indicates that even with extreme risk-averse sellers, the positive relationship between residual prices and duration cannot be the result of the indifference of sellers, who would be better off listing units at larger residual prices.

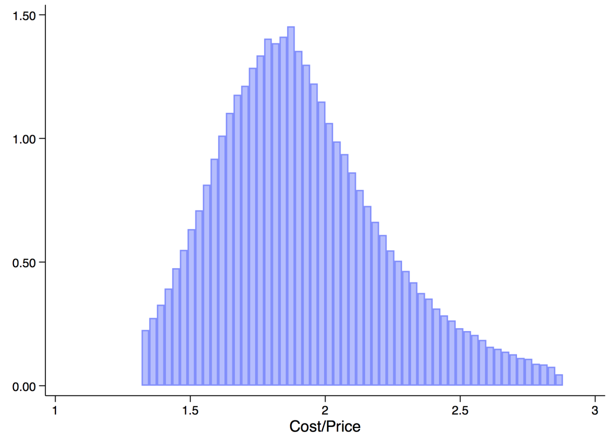
Sellers' heterogeneity We now consider whether the positive relationship between residual prices and duration can be explained by heterogeneity across sellers. First, the results presented in Figure B15 indicate that the positive relationship between residual prices and duration cannot be explained by heterogeneity in sellers' preferences (i.e., discount factor or risk aversion). To see this, note that, as shown in Figure B15, the expected net present revenue is increasing in both the computation with a high discount factor ($\beta = 0.99$) and with a low discount factor ($\beta = 0$). Therefore, if, under the preferences of the most impatient seller, a higher residual price with lower selling probability is preferred, the higher residual price would also be preferred under any other possible discount factor. A similar argument applies to the comparison between risk-neutral and (extreme) risk-averse agents: All sellers would maximize their expected net present value by choosing the highest residual price observed in the data.

Second, we consider the possibility that sellers have heterogeneous holding costs. For this, assume that sellers must pay a fixed cost each period until the property is sold (e.g., maintenance costs, taxes, debt service costs, etc). If sellers face different costs, then some sellers might be forced to list properties at low prices in order to sell their property faster, as would occur in a fire sale. Using our data, we ask how large must the cost be in order to rationalize a seller's choice of a lower residual price. Thus, for each residual price ε_{it}, we compute the (unobserved) cost, $\xi(\varepsilon_{it}, \beta)$, that would make risk-neutral sellers with discount factor β indifferent between choosing that residual price and the highest observed residual price ($\overline{\varepsilon}_{it}$) by solving the following condition:

$$\frac{p(\varepsilon_{it})\varepsilon_{it} - \xi(\varepsilon_{it}, \beta)(1 - \varepsilon_{it})}{1 - \beta(1 - \varepsilon_{it})} = \frac{p(\overline{\varepsilon}_{it})\overline{\varepsilon}_{it} - \xi(\varepsilon_{it}, \beta)(1 - p(\overline{\varepsilon}_{it}))}{1 - \beta(1 - p(\overline{\varepsilon}_{it}))}. \tag{B.26}$$

Figure B16 presents the results, which indicate that in order for differential holding costs to explain the differences in returns in the data, they must be extremely large. To illustrate, the cost of holding 1 square foot of a property for 1 additional month would have to be larger than the price at which the owner can sell that unit. We conclude that it is unlikely that the bulk of the positive relation between residual prices and duration is explained by the presence of heterogeneous holding costs.

Figure B16: Required Holding Costs



Note: This figure reports the distribution of holding costs obtained from (B.26) as a fraction of the property's price.