Homework #6

Statistical Methods with R, Spring 2019

Due in class on Wednesday, March 13, 2019. *Instructions for "theoretical" questions: Answer all of the following questions. The theoretical problems should be neatly numbered, written out, and solved. Please do not turn in messy work. Working in small groups is allowed, but it is important that you make an effort to master the material and hand in your own work;Identical solutions will be considered as a violation of the Student Honor Code.* **Note that you are also required to turn in the computational portion of this assignment. It is on Canvas, under the name:**

`A4570SP19HW06-computational.ipynb.`

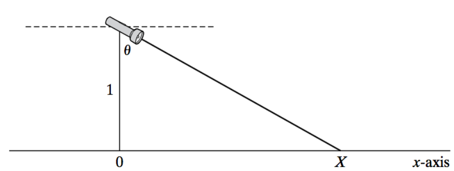
ON THE FRONT OF YOUR HOMEWORK CLEARLY PRINT THE FOLLOWING:

- Your full name.
- Your lecture number (either **APPM 4570** or **APPM 5570** or **STAT 4000** or **STAT 5000**).
- Homework number.
- **Points will be deducted if these instructions are not followed.**

Remember that writing style, clarity and completeness of explanations is always important. Justify your answers. (Be sure to place your homework in the correct pile, either "undergraduate" or "graduate".)

Theoretical Questions

1. Suppose a narrow-beam flashlight is spun around its center, which is located a unit distance from the x-axis. (See figure below.) Consider the point X at which the beam intersects the x-axis when the flashlight has stopped spinning. (If the beam is not pointing toward the x-axis, repeat the experiment.)



It can be shown that the point at which the beam intersects the x-axis is $X = \tan(\theta)$ and if we assume that θ is uniformly distributed between $-\pi/2$ and $pi/2$ then the pdf of X is given by

$$f(x) = \frac{1}{\pi(1+x^2)} \text{ for } -\infty < x < \infty$$

and X is said to have a *Cauchy distribution*. Show that $E[X]$ does not exist..

2. (a) Use the shortcut formula for the covariance to show $\text{Cov}(aX, bY) = ab\text{Cov}(X, Y)$.
(b) If $\text{Cov}(X, Y) = 0.3$, what is $\text{Cov}(100X, Y)$?
(c) If $\text{Corr}(X, Y) = 0.1$, what is $\text{Corr}(100X, Y)$

3. Let X be a normally distributed random variable with mean 3 and variance 4.
(a) Let $Y = 5X + 2$, what is the distribution of Y? What are it's mean and variance? Show all work.
(b) Find $P(Y < 10)$. You may leave your answer in terms of Φ, the cdf of the normal r.v.
(c) What is the 99^{th} percentile of the distribution of X?

♚PROBLEM #4 & #5 on the other side.♛

4. A rock specimen is randomly selected and weightd two different times. Let w denote the true weight (a number) of the rock, and let X_1 ad X_2 be the two measured weights. Then, $X_1 = w + E_1$ and $X_2 = w + E_2$, where E_1 and E_2 are the two measurement errors. Suppose that E_1 and E_2 are independent and distributed normally with mean 0 and variance 0.1 (*i.e.*, $E_1,\ E_2 \sim \mathrm{N}(0, 0.1)$).

   (a) Find the mean and variance of X_1.

   (b) Find $\mathrm{Cov}(X_1, X_2)$.

5. (**APPM 5570/STAT 5000 Students Only**) Let X be a nonnegative random variable with mean $E(X) = \mu_X$ and $\mathrm{Var}(X) = \sigma_X^2$, $k > 0$.

   (a) Prove that $P(X \geq k) \leq \dfrac{\mu_X}{k}$.

   (b) Prove that $P(|Y - \mu_Y| \geq k\sigma_Y) \leq \dfrac{1}{k^2}$. (*Hint:* Use the result of part (a) to first show $P(|Y - \mu_Y| \geq m) \leq \dfrac{\sigma_Y^2}{m^2}$ for $m > 0$ then substitute the appropriate value for m to finish the proof.)