Juan Menduiña

Trabajo Práctico N° 2: Límites.

Ejercicio 1.

Calcular los límites de las siguientes funciones, si existen.

(a) *f (x)=* $\begin{cases} x^2, si\ x > 2 \\ 1, si\ x \leq 2 \end{cases}$. *Calcular* $\lim_{x \to 2^+} f(x)$ *y* $\lim_{x \to 2^-} f(x)$.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 1 = 1$.

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = 2^2 = 4$.

Por lo tanto, ya que $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$, $\nexists \lim_{x \to 2} f(x)$.

(b) *Sea g la función valor absoluto, g (x)=* $|x| = \begin{cases} x, si\ x \geq 0 \\ -x, si\ x < 0 \end{cases}$. *Calcular* $\lim_{x \to 0^+} g(x)$ *y* $\lim_{x \to 0^-} g(x)$. *¿Qué se puede decir respecto al* $\lim_{x \to 0} g(x)$*?*

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} -x = 0$.

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} x = 0$.

Por lo tanto, ya que $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x)$, $\exists \lim_{x \to 0} g(x)$.

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Ejercicio 2.

Dada la función f (x)= $\begin{cases} \frac{x-1}{x^2-1}, si\ x \neq 1 \\ 3, si\ x = 1 \end{cases}$*, calcular* $\lim\limits_{x \to 1^+} f\ (x)$ *y* $\lim\limits_{x \to 1^-} f\ (x)$.

$\lim\limits_{x \to 1^-} f\ (x) = \lim\limits_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = (\frac{0}{0})$.

$\lim\limits_{x \to 1^-} f\ (x) = \lim\limits_{x \to 1^+} \frac{x-1}{(x+1)(x-1)}$

$\lim\limits_{x \to 1^-} f\ (x) = \lim\limits_{x \to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$.

$\lim\limits_{x \to 1^+} f\ (x) = \lim\limits_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = (\frac{0}{0})$.

$\lim\limits_{x \to 1^+} f\ (x) = \lim\limits_{x \to 1^+} \frac{x-1}{(x+1)(x-1)}$

$\lim\limits_{x \to 1^+} f\ (x) = \lim\limits_{x \to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$.

Ejercicio 3.

Calcular, si existen, los siguientes límites:

(a) $\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$.

$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{\sqrt{0^2+9}-3}{0^2} = \frac{\sqrt{0+9}-3}{0} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = (\frac{0}{0}).$$
$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$$
$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{x^2+9+3\sqrt{x^2+9}-3\sqrt{x^2+9}-9}{x^2(\sqrt{x^2+9}+3)}$$
$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$$
$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+9}+3} = \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{0+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}.$$

(b) $\lim\limits_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2}$.

$$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \frac{1^2-2*1+1}{1^2-3*1+2} = \frac{1-2+1}{1-3+2} = (\frac{0}{0}).$$
$$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \to 1} \frac{(x-1)^2}{(x-2)(x-1)} \qquad \text{(*) y (**)}$$
$$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \to 1} \frac{x-1}{x-2} = \frac{1-1}{1-2} = \frac{0}{-1} = 0.$$

(*) $x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2-4*1*1}}{2*1}$

$x_1, x_2 = \frac{2 \pm \sqrt{4-4}}{2}$

$x_1, x_2 = \frac{2 \pm \sqrt{0}}{2}$

$x_1, x_2 = \frac{2 \pm 0}{2}$

$x_1 = \frac{2+0}{2} = \frac{2}{2} = 1.$

$x_2 = \frac{2-0}{2} = \frac{2}{2} = 1.$

(**) $x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2-4*1*2}}{2*1}$

$x_1, x_2 = \frac{3 \pm \sqrt{9-8}}{2}$

$x_1, x_2 = \frac{3 \pm \sqrt{1}}{2}$

$x_1, x_2 = \frac{3 \pm 1}{2}$

$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2.$

$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1.$

Ejercicio 4.

Calcular los siguientes límites:

(a) $\lim_{x \to 3} \frac{x+2}{x+3}$.

$\lim_{x \to 3} \frac{x+2}{x+3} = \frac{3+2}{3+3} = \frac{5}{6}$.

(b) $\lim_{x \to 1} x^3 + 5x^2 + 10$.

$\lim_{x \to 1} x^3 + 5x^2 + 10 = 1^3 + 5 * 1^2 + 10 = 1 + 5 * 1 + 10 = 1 + 5 + 10 = 16$.

(c) $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x}$.

$\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x} = \frac{(-2)^3+2(-2)^2-1}{5-3(-2)} = \frac{-8+2*4-1}{5+6} = \frac{-8+8-1}{11} = \frac{-1}{11}$.

(d) $\lim_{x \to \pi} sen\ (x - \pi)$.

$\lim_{x \to \pi} sen\ (x - \pi) = \text{sen}\ (\pi - \pi) = \text{sen}\ 0 = 0$.

(e) $\lim_{x \to 2} \frac{\sqrt[3]{x^2+2}}{x}$.

$\lim_{x \to 2} \frac{\sqrt[3]{x^2+2}}{x} = \frac{\sqrt[3]{2^2+2}}{2} = \frac{\sqrt[3]{4+2}}{2} = \frac{\sqrt[3]{6}}{2}$.

(f) $\lim_{x \to 1} ln \frac{2}{x^3+1}$.

$\lim_{x \to 1} \ln \frac{2}{x^3+1} = \ln \frac{2}{1^3+1} = \ln \frac{2}{1+1} = \ln \frac{2}{2} = \ln 1 = 0$.

(g) $\lim_{x \to 0} \frac{ln(x+1)}{e^x}$.

$\lim\limits_{x \to 0} \frac{\ln(x+1)}{e^x} = \frac{\ln(0+1)}{e^0} = \frac{\ln 1}{1} = \frac{0}{1} = 0.$

(h) $\lim\limits_{x \to 0} 3 \cos x^2 \, (1+x)^4.$

$\lim\limits_{x \to 0} 3 \cos x^2 \, (1+x)^4 = 3 \cos 0^2 \, (1+0)^4 = 3 \cos 0 * 1^4 = 3 * 1 * 1 = 3.$

(i) $\lim\limits_{x \to 3} |x-3|.$

$\lim\limits_{x \to 3} |x-3| = |3-3| = |0| = 0.$

(j) $\lim\limits_{x \to 3} \frac{|x-3|}{x}.$

$\lim\limits_{x \to 3} \frac{|x-3|}{x} = \frac{|3-3|}{3} = \frac{|0|}{3} = \frac{0}{3} = 0.$

(k) $\lim\limits_{x \to 0} \frac{x}{x^2+x}.$

$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \frac{0}{0^2+0} = \frac{0}{0+0} = (\frac{0}{0}).$
$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \lim\limits_{x \to 0} \frac{x}{x(x+1)}$
$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \lim\limits_{x \to 0} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1.$

(l) $\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4}.$

$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \frac{2^2-2-2}{2^2-4} = \frac{4-2-2}{4-4} = (\frac{0}{0}).$
$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim\limits_{x \to 2} \frac{(x-2)(x+1)}{(x+2)(x-2)}$ (*)
$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim\limits_{x \to 2} \frac{x+1}{x+2} = \frac{2+1}{2+2} = \frac{3}{4}.$

(*) $x_1, x_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4*1(-2)}}{2*1}$
$x_1, x_2 = \frac{1 \pm \sqrt{1+8}}{2}$
$x_1, x_2 = \frac{1 \pm \sqrt{9}}{2}$

$x_1, x_2= \frac{1\pm3}{2}$

$x_1= \frac{1+3}{2}= \frac{4}{2}= 2.$

$x_2= \frac{1-3}{2}= \frac{-2}{2}= -1.$

(m) $\lim\limits_{x\to-1} \frac{x^2-1}{x^2+3x+2}.$

$\lim\limits_{x\to-1} \frac{x^2-1}{x^2+3x+2}= \frac{(-1)^2-1}{(-1)^2+3(-1)+2}= \frac{1-1}{1-3+2}= (\frac{0}{0}).$

$\lim\limits_{x\to-1} \frac{x^2-1}{x^2+3x+2}= \lim\limits_{x\to-1} \frac{(x+1)(x-1)}{(x-2)(x-1)}$ (*)

$\lim\limits_{x\to-1} \frac{x^2-1}{x^2+3x+2}= \lim\limits_{x\to-1} \frac{x+1}{x-2}= \frac{-1+1}{-1-2}= \frac{0}{-3}= 0.$

(*) $x_1, x_2= \frac{-(-3)\pm\sqrt{(-3)^2-4*1*2}}{2*1}$

$x_1, x_2= \frac{3\pm\sqrt{9-8}}{2}$

$x_1, x_2= \frac{3\pm\sqrt{1}}{2}$

$x_1, x_2= \frac{3\pm1}{2}$

$x_1= \frac{3+1}{2}= \frac{4}{2}= 2.$

$x_2= \frac{3-1}{2}= \frac{2}{2}= 1.$

(n) $\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}.$

$\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}= \frac{1-1}{\sqrt{1}-1}= \frac{0}{1-1}= (\frac{0}{0}).$

$\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}= \lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}\frac{\sqrt{x}+1}{\sqrt{x}+1}$

$\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}= \lim\limits_{x\to1} \frac{(x-1)(\sqrt{x}+1)}{x+\sqrt{x}-\sqrt{x}-1}$

$\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}= \lim\limits_{x\to1} \frac{(x-1)(\sqrt{x}+1)}{x-1}$

$\lim\limits_{x\to1} \frac{x-1}{\sqrt{x}-1}= \lim\limits_{x\to1} \sqrt{x}+1= \sqrt{1}+1= 1+1= 2.$

(ñ) $\lim\limits_{x\to0} \frac{x^3}{x^2+x}.$

$\lim\limits_{x\to0} \frac{x^3}{x^2+x}= \frac{0^3}{0^2+0}= \frac{0}{0+0}= (\frac{0}{0}).$

$\lim\limits_{x\to0} \frac{x^3}{x^2+x}= \lim\limits_{x\to0} \frac{x^3}{x(x+1)}$

$\lim\limits_{x\to0} \frac{x^3}{x^2+x}= \lim\limits_{x\to0} \frac{x^2}{x+1}= \frac{0^2}{0+1}= \frac{0}{1}= 0.$

(o) $\lim_{x \longrightarrow 2} \frac{x+2}{2-x}$.

$\lim_{x \longrightarrow 2} \frac{x+1}{2-x} = \frac{2+1}{2-2} = \frac{3}{0} = +\infty$.

(p) $\lim_{x \longrightarrow 3} \frac{x}{x-3}$.

$\lim_{x \longrightarrow 3} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0} = +\infty$.

(q) $\lim_{x \longrightarrow -2} \frac{x}{|x+2|}$.

$\lim_{x \longrightarrow -2} \frac{x}{|x+2|} = \frac{-2}{|-2+2|} = \frac{-2}{|0|} = \frac{-2}{0} = -\infty$.

Ejercicio 5.

Calcular los siguientes límites al infinito:

(a) $\lim\limits_{x \to -\infty} x - 3x^4$.

$\lim\limits_{x \to -\infty} x - 3x^4$= -∞.

(b) $\lim\limits_{x \to -\infty} \frac{5}{x} - \frac{3}{x}$.

$\lim\limits_{x \to -\infty} \frac{5}{x} - \frac{3}{x}$= 0 - 0= 0.

(c) $\lim\limits_{x \to +\infty} \frac{1}{x^2+3x-1}$.

$\lim\limits_{x \to +\infty} \frac{1}{x^2+3x-1}$= 0.

(d) $\lim\limits_{x \to +\infty} \frac{3x^2+2x-16}{x^2-x-2}$.

$\lim\limits_{x \to +\infty} \frac{3x^2+2x-16}{x^2-x-2}$= $(\frac{\infty}{\infty})$.

$\lim\limits_{x \to +\infty} \frac{3x^2+2x-16}{x^2-x-2}$= $\lim\limits_{x \to +\infty} \frac{x^2(3+\frac{2}{x}-\frac{16}{x^2})}{x^2(1-\frac{1}{x}-\frac{2}{x^2})}$

$\lim\limits_{x \to +\infty} \frac{3x^2+2x-16}{x^2-x-2}$= $\lim\limits_{x \to +\infty} \frac{3+\frac{2}{x}-\frac{16}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}$= $\frac{3+0-0}{1-0-0}$= $\frac{3}{1}$= 3.

(e) $\lim\limits_{x \to -\infty} \frac{x^2-2x-3}{x^3-x^2-6x-2}$.

$\lim\limits_{x \to -\infty} \frac{x^2-2x-3}{x^3-x^2-6x-2}$= 0.

(f) $\lim\limits_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}}$.

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4(1-\frac{6}{x^3})}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4}\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{x^2\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^2(1-\frac{2}{x^2})}{\sqrt{1-\frac{6}{x^3}}} = +\infty.$$

Ejercicio 6.

Calcular los siguientes límites, si es que existen:

(a) *Dada h (x)=* $\begin{cases} \sqrt{x-4}, si\ x > 4 \\ 8-2x, si\ x < 4 \end{cases}$. *Calcular* $\lim_{x \to 4} h\ (x)$.

$\lim_{x \to 4^-} h\ (x)= \lim_{x \to 4^-} 8-2*4$= 8 - 8= 0.

$\lim_{x \to 4^+} h\ (x)= \lim_{x \to 4^+} \sqrt{x-4}= \sqrt{4-4}= \sqrt{0}$= 0.

Por lo tanto, ya que $\lim_{x \to 4^-} h\ (x)= \lim_{x \to 4^+} h\ (x)$= 0, entonces, $\lim_{x \to 4} h\ (x)$= 0.

(b) *Sea f (x)=* $\begin{cases} \frac{x}{e^x}, si\ x \leq 0 \\ x^2, si\ x > 0 \end{cases}$. *Calcular* $\lim_{x \to 0} f\ (x)$ *y* $\lim_{x \to -1} f\ (x)$.

$\lim_{x \to 0^-} f\ (x)= \lim_{x \to 0^-} \frac{x}{e^x}= \frac{0}{e^0}= \frac{0}{1}$= 0.

$\lim_{x \to 0^+} f\ (x)= \lim_{x \to 0^+} x^2= 0^2$= 0.

Por lo tanto, ya que $\lim_{x \to 0^-} f\ (x)= \lim_{x \to 0^+} f\ (x)$= 0, entonces, $\lim_{x \to 0} f\ (x)$= 0.

$\lim_{x \to -1} f\ (x)= \lim_{x \to -1} x^2= (-1)^2$= 1.

Por lo tanto, $\lim_{x \to -1} f\ (x)$= 1.

(c) $\lim_{x \to +\infty} \frac{(ln\, x)^2}{x}$.

$\lim_{x \to +\infty} \frac{(\ln x)^2}{x}$= 0.

(d) $\lim_{x \to +\infty} \frac{3x^2+2x+1}{e^x}$.

$\lim_{x \to +\infty} \frac{3x^2+2x+1}{e^x}$= 0.

(e) $\lim_{x \to 0^+} x\, ln\, x$.

$$\lim_{x \longrightarrow 0^+} x \ln x = \lim_{y \longrightarrow +\infty} \frac{1}{y} \ln \frac{1}{y}$$

$$\lim_{x \longrightarrow 0^+} x \ln x = \lim_{y \longrightarrow +\infty} \frac{\ln \frac{1}{y}}{y}$$

$$\lim_{x \longrightarrow 0^+} x \ln x = \lim_{y \longrightarrow +\infty} \frac{\ln 1 - \ln y}{y}$$

$$\lim_{x \longrightarrow 0^+} x \ln x = \lim_{y \longrightarrow +\infty} \frac{0 - \ln y}{y}$$

$$\lim_{x \longrightarrow 0^+} x \ln x = \lim_{y \longrightarrow +\infty} \frac{-\ln y}{y} = 0.$$

Ejercicio 7.

Calcular las asíntotas verticales y horizontales, si existen, de las funciones dadas a continuación:

(a) $f(x)=\frac{x^2}{x+1}$.

$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-} \frac{x^2}{x+1}=\frac{(-1)^2}{-1+1}=\frac{1}{0}=-\infty$.

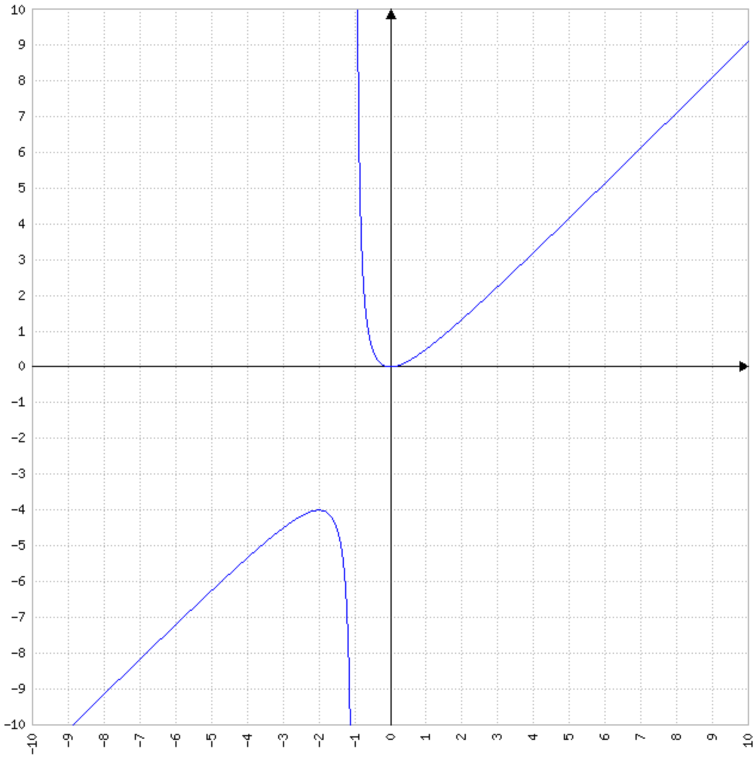
$\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+} \frac{x^2}{x+1}=\frac{(-1)^2}{-1+1}=\frac{1}{0}=+\infty$.

Por lo tanto, f (x) tiene un asíntota vertical en x= -1.

$\lim_{x\to -\infty} f(x)=\lim_{x\to -\infty} \frac{x^2}{x+1}=-\infty$.

$\lim_{x\to +\infty} f(x)=\lim_{x\to +\infty} \frac{x^2}{x+1}=+\infty$.

Por lo tanto, f (x) no tiene asíntotas horizontales.



(b) $g(x)=\frac{4x^2+2x-2}{3x-1}$.

$$\lim_{x\to\frac{1}{3}^-} g(x)= \lim_{x\to\frac{1}{3}^-} \frac{4x^2+2x-2}{3x-1}= \frac{4(\frac{1}{3})^2+2\frac{1}{3}-2}{3(\frac{1}{3})-1}= \frac{4\frac{1}{9}+\frac{2}{3}-2}{1-1}= \frac{\frac{4}{9}+\frac{2}{3}-2}{0}= \frac{\frac{-8}{9}}{0}= +\infty.$$

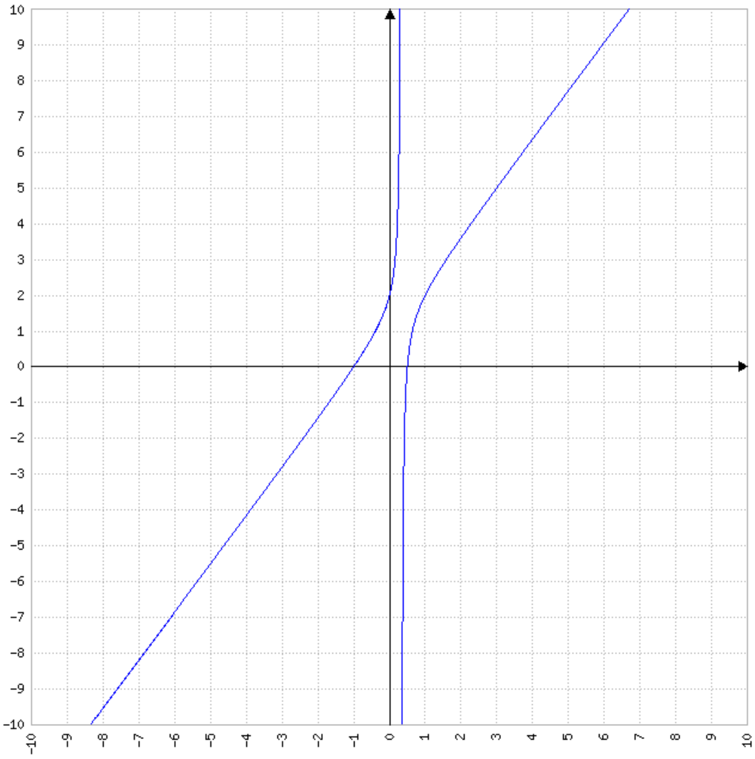
$$\lim_{x\to\frac{1}{3}^+} g(x)= \lim_{x\to\frac{1}{3}^+} \frac{4x^2+2x-2}{3x-1}= \frac{4(\frac{1}{3})^2+2\frac{1}{3}-2}{3(\frac{1}{3})-1}= \frac{4\frac{1}{9}+\frac{2}{3}-2}{1-1}= \frac{\frac{4}{9}+\frac{2}{3}-2}{0}= \frac{\frac{-8}{9}}{0}= -\infty.$$

Por lo tanto, g (x) tiene un asíntota vertical en x= $\frac{1}{3}$.

$$\lim_{x\to-\infty} g(x)= \lim_{x\to-\infty} \frac{4x^2+2x-2}{3x-1}= -\infty.$$

$$\lim_{x\to+\infty} g(x)= \lim_{x\to+\infty} \frac{4x^2+2x-2}{3x-1}= +\infty.$$

Por lo tanto, g (x) no tiene asíntotas horizontales.



(c) $h(x)= \frac{x-2}{x^2-4x+4}$.

$$x_1, x_2= \frac{-(-4)\pm\sqrt{(-4)^2-4*1*4}}{2*1}$$

$$x_1, x_2= \frac{4\pm\sqrt{16-16}}{2}$$

$$x_1, x_2= \frac{4\pm\sqrt{0}}{2}$$

$$x_1, x_2= \frac{4\pm 0}{2}$$

$x_1 = \frac{4+0}{2} = \frac{4}{2} = 2.$
$x_2 = \frac{4-0}{2} = \frac{4}{2} = 2.$

h (x)= $\frac{x-2}{(x-2)^2}$
h (x)= $\frac{1}{x-2}$

$\lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} = -\infty.$

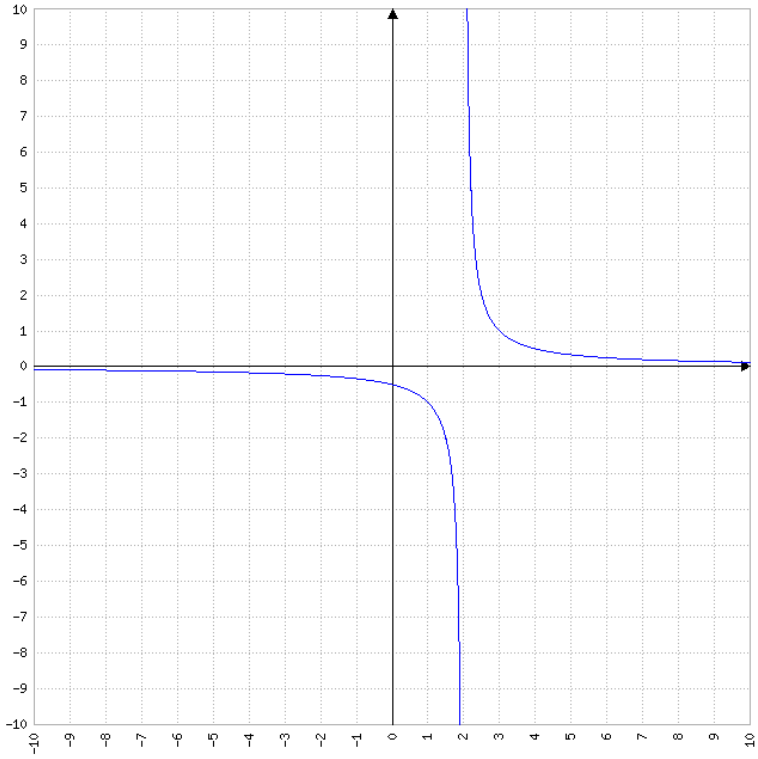
$\lim_{x \to 2^+} h(x) = \lim_{x \to 2^+} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} = +\infty.$

Por lo tanto, h (x) tiene un asíntota vertical en x= 2.

$\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{1}{x-2} = 0.$

$\lim_{x \to +\infty} h(x) = \lim_{x \to +\infty} \frac{1}{x-2} = 0.$

Por lo tanto, h (x) tiene un asíntota horizontal en y= 0.



(d) *k (x)*= $\begin{cases} \frac{x}{x^2-1}, si\ x < 2 \\ 3x^3 - 2x, si\ x \geq 2 \end{cases}.$

$\lim\limits_{x \to -1^-} k\ (x) = \lim\limits_{x \to -1^-} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$
$\lim\limits_{x \to -1^+} k\ (x) = \lim\limits_{x \to -1^+} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$

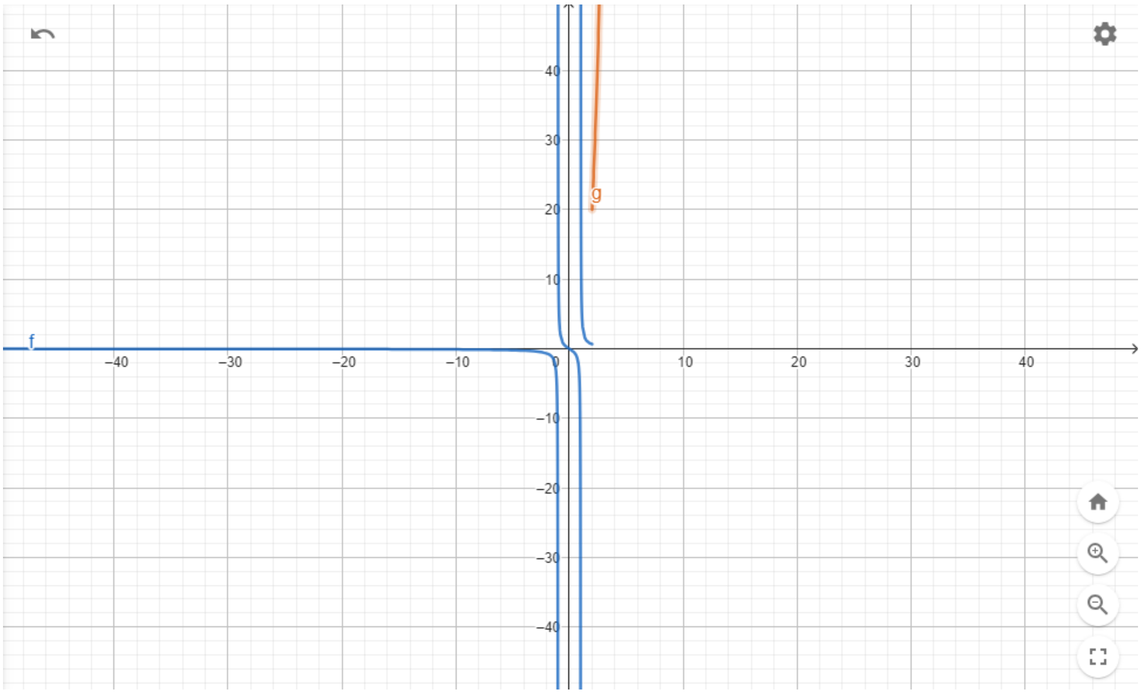
$\lim\limits_{x \to 1^-} k\ (x) = \lim\limits_{x \to 1^-} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$
$\lim\limits_{x \to 1^+} k\ (x) = \lim\limits_{x \to 1^+} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$

Por lo tanto, k (x) tiene asíntotas verticales en x= -1 y x= 1.

$\lim\limits_{x \to -\infty} k\ (x) = \lim\limits_{x \to -\infty} \frac{x}{x^2-1} = 0.$

$\lim\limits_{x \to +\infty} k\ (x) = \lim\limits_{x \to +\infty} 3x^3 - 2x = +\infty.$

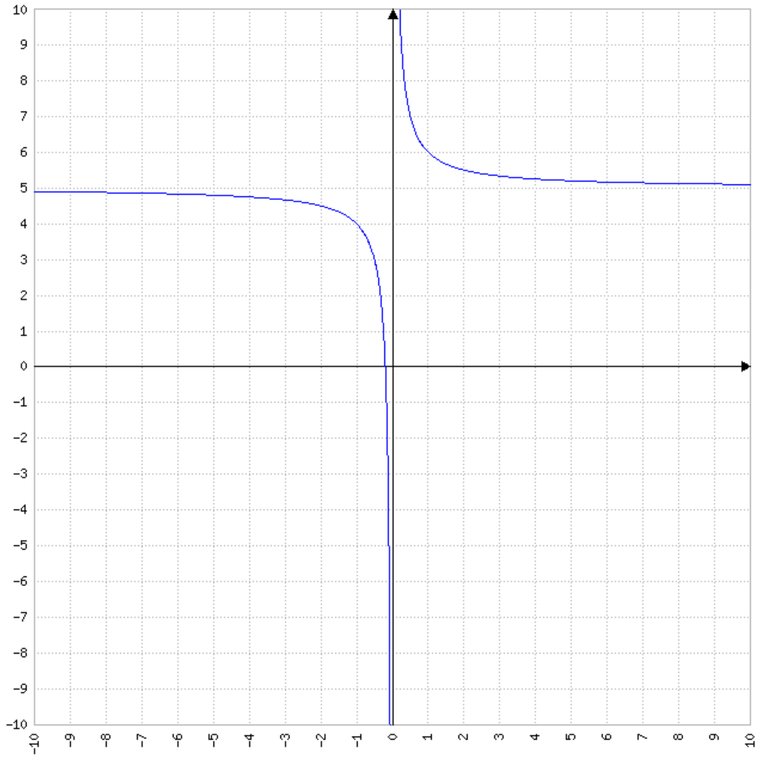
Por lo tanto, k (x) tiene un asíntota horizontal en y= 0.



Ejercicio 8.

Utilizar GeoGebra para verificar, gráficamente, el comportamiento de las funciones de los Ejemplos (11) y (15).

Ejemplo 11:



Ejemplo 15:

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