Homework 4

Due Feb 14 at 4 pm

Recall the homework guidelines for this course.

1. Let $A = \{n^3 : n \in \mathbb{N}, n > 1\}$ and $B = \{k : k \in \mathbb{N}, k > 5\}$. Show that $A \subsetneq B$.
2. Let A and B be sets. Show that the following hold.
   (a) $\emptyset \in \mathscr{P}(A)$ and $A \in \mathscr{P}(A)$.
   (b) $\mathscr{P}(A) \neq \emptyset$.
   (c) $A \subseteq B \leftrightarrow \mathscr{P}(A) \subseteq \mathscr{P}(B)$.
   (d) $A = B \leftrightarrow \mathscr{P}(A) = \mathscr{P}(B)$.
3. Let A, B, and C be sets. Determine whether each of the following statements is true or false. If a statement is true, provide a proof. If it is false, give a counterexample.
   (a) If $A \cup B = A \cup C$, then $B = C$.
   (b) If $A \cap B = A \cap C$, then $B = C$.
   (c) If $A \cup B = A \cup C$, and $A \cap B = A \cap C$, then $B = C$.
   (d) A and $\emptyset$ are disjoint.
   (e) If A and B are disjoint and B and C are disjoint, then A and C are disjoint.
   (f) If A has 5 elements and B has 6 elements, then $A \cup B$ has 11 elements.
4. Let A, B, and C be subsets of a set U. Prove directly that the following set properties hold.
   (a) $A \cup A = A$ and $A \cap A = A$.
   (b) $A \cup B = B \cup A$ and $A \cap B = B \cap A$.
   (c) $A \subseteq A \cup B$ and $A \cap B \subseteq B$.
   (d) $A \cup \emptyset = A$ and $A \cap U = A$.
   (e) $A \cap \emptyset = \emptyset$ and $A \cup U = U$.
   (f) $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$.
5. Show De Morgan's laws for numbered sets: Let $A = \{A_i : i \in I\}$ be a set such that, for all $i \in I$, A_i is a set. Then
   (a) $\overline{\left(\bigcap_{i \in I} A_i\right)} = \bigcup_{i \in I} \overline{A_i}$
   (b) $\overline{\left(\bigcup_{i \in I} A_i\right)} = \bigcap_{i \in I} \overline{A_i}$

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6. Let $A, B \subseteq U$ and let $x, y \in U$. Match each statement on the left with an equivalent statement on the right. **You do not need to justify.**

(a) $\{x, y\} - \{x\} = \emptyset$	(1) $x \notin A$
(b) $\overline{A} = \emptyset$	(2) $A = B = \emptyset$
(c) $A \subseteq B$	(3) $A \subseteq \{x\}$
(d) $A - \{x\} = \emptyset$	(4) $\mathscr{P}(A) \subseteq \mathscr{P}(B)$
(e) $\{x\} \neq \{y\}$	(5) $x = y$
(f) $A \cup \{x\} = A$	(6) $\overline{A} \subseteq \overline{B}$
(g) $x \in \emptyset$	(7) $A = \{\emptyset\}$
(h) $A \cup B = \emptyset$	(8) $\mathscr{P}(A) = \{\emptyset\}$
(i) $A - \{x\} = A$	(9) $\{x\} \cap \{y\} = \emptyset$
(j) $A \cap B = \emptyset$	(10) $\mathscr{P}(\emptyset) = \{\}$
(k) $\overline{A} = U - \{\emptyset\}$	(11) $x \in A$
(l) $A = \emptyset$	(12) $A \subseteq \overline{B}$
(m) $A \cap B = B$	(13) $A = U$