MA 293

Homework 3

Due Feb 7 at 4 pm

Recall the homework guidelines for this course.

1. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Compute the following sets by listing all their elements.
   (a) $\{(x, y) : x \in A, y \in B\}$.
   (b) $\{\{x, y\} : x \in A, y \in B\}$.
   (c) $\{\{x, y\} : x \subseteq A, y \subseteq B\}$.

2. Let $P(x)$ and $Q(x)$ be predicates for $x \in \mathbb{N}$.
   (a) Let A and B be the sets:
   $$A = \{x \in \mathbb{N} : \neg P(x)\};$$
   $$B = \{x \in \mathbb{N} : P(x) \to Q(x)\}.$$
   Explain why $A \subseteq B$.
   (b) Let A and B be the sets:
   $$A = \{x \in \mathbb{N} : \neg(P(x) \wedge Q(x))\};$$
   $$B = \{x \in \mathbb{N} : P(x) \to \neg Q(x)\}.$$
   Explain why $A = B$.

3. Let A be a set. The *power set* of A, denoted $\mathscr{P}(A)$, is the set of all subsets of A. For example, $\mathscr{P}(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. Compute the following sets by listing all their elements.
   (a) $\mathscr{P}(\emptyset)$.
   (b) $\mathscr{P}(\{a\})$.
   (c) $\mathscr{P}(\{\emptyset\})$.
   (d) $\mathscr{P}(\mathscr{P}(\{\emptyset\}))$.

4. Determine whether each statement below is true or false. Please explain.
   (a) For $A = \{2n : n \in \mathbb{Z}\}$, $\exists x \in A(x \text{ is prime})$.
   (b) $1 \subseteq \{1, \{1\}\}$.
   (c) $\mathbb{R}^2 \subseteq \mathbb{R}^3$.
   (d) For any set A, $A \subseteq \mathscr{P}(A)$.

5. If A is a finite set of cardinality n, how many elements do you think $\mathscr{P}(A)$ has? Please explain.[1]

[1] I do not care about you getting the right answer; I am looking for you to show that you thought deeply about the problem.

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6. Determine whether each statement below is true or false. Please explain.

   (a) $\forall A$ set$(\emptyset \subseteq A)$.

   (b) $\forall A$ set$(\emptyset \in A)$.

   (c) $\forall A$ set$(\{\emptyset, A\} \subseteq \mathscr{P}(A))$.

   (d) $\{A : A \text{ is a set and } A \neq A\} = \emptyset$.