Magma scavenger hunt

This is a short scavenger hunt to get you started as a new Magma user. Some resources are: first steps in Magma, general examples, and handbook. I have also compiled a list of random Magma tricks I like.

$$19 - 25 - 4 - 14 - 5 - 25$$

1. Start with $A := 55489564$.
2. Let B be the largest prime factor of A.
3. Define C as the discriminant of the polynomial $x^3 + x + B \in \mathbb{Q}[x]$.
4. The number D is the class number of the quadratic field $\mathbb{Q}(\sqrt{C})$.
5. Construct the elliptic curve $E\colon y^2 + xy + y = x^3 - x^2 - 96x + D$ over $\mathbb{Q}$. *Hint: you can define an elliptic curve in Magma using* `EllipticCurve([a,b,c,d,e]);`. *Find out what the appropriate values of the elements in the list are.*
6. Let F be the rank of E.
7. Define G as the conductor of E.
8. By adding one digit of G at a time from right to left, how many of the intermediate numbers you form are a prime numbers? Let H be this quantity. For example, 103 gives 3 prime numbers: 3, 03, and 103.
9. Define $I := \mathbb{Q}(\zeta_H)$ as the cyclotomic field where ζ_H is a primitive H-th root of unity.
10. Find the trace of $\zeta_H + 2 \in I$ and call it J.
11. Let K be the number of elements $\zeta_H + x \in I$ have norm at most 700 for $x \in [1, \ldots, 100]$.
12. Find L, the list of prime numbers up to 100 (ordered in increasing order) that split in the field $I = \mathbb{Q}(\zeta_H)$.
13. The number M is the third element of L.
14. Now change the base field of the elliptic curve E from $\mathbb{Q}$ to $I = \mathbb{Q}(\zeta_H)$. Let N denote the number of points on E up to naive height bound of 20 whose coordinates lie in $\mathbb{Q}(\zeta_H)$ but not in $\mathbb{Q}$.

Where does Magma live?

$$\text{M - K - F - N - J - K}$$