MA 293

Homework 8

Due Mar 28 at 4 pm

Recall the homework guidelines for this course. Do not forget to justify your answers.

1. Recall that "bit" is short for "binary digit," so a bit string is a string of binary digits. The binary digits are simply the numbers 0 and 1. The number of bits (0's or 1's) in the string is the **length** of the string. The number of 1's in a bit string is the **weight** of the string. An n-bit is a bit of length n.

   Let n and k be integers with $0 \leq k \leq n$. Find a bijective function from the set of n-bits of weight k to the set $B_{n,k}$ of subsets of $\{1, \ldots, n\}$ of cardinality k. Conclude that the number of n-bits of weight k is the entry in row n and column k of the Pascal triangle.

2. Count the following quantities using the sum and product principles.

   (a) The number of 11-bit strings of weight 4 start with either 11 or 00.

   (b) The number of 11-bit strings of weight 4 have at least one 1 in their first three bits.

3. Recall that the number of lattice paths from $(0, 0)$ to (x, y) is $\binom{x+y}{x}$. Count the following quantities using counting principles.

   (a) The number of lattice paths from $(0, 0)$ to $(10, 10)$ that pass through $(4, 7)$.

   (b) The number of lattice paths from $(0, 0)$ to $(10, 10)$ that pass through $(2, 2)$ or $(6, 6)$.

4. For how many three-digit numbers (10 to 99) is the sum of the digits even? (For example, 35 has an even sum of digits: $3 + 5 = 8$ which is even.) Find the answer and explain why it is correct in at least two different ways.

5. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, \ldots, 10\}$. Count the following quantities using counting principles or facts about functions.

   (a) The number of functions from A to B.

   (b) The number of injective functions from A to B.

   (c) The number of surjective functions from A to B.

6. How many different pizzas can you make if you can choose from 10 toppings and you can have any number of toppings (0-10) on your pizza?