

Homework 2

MA 293

Due Jan 31 at 4 pm

Recall the homework guidelines for this course.

1. Determine whether each statement below is true or false, or whether it is impossible to determine. Assume you do not know what my favorite number is (but you do know which numbers are prime).
 - (a) 7 is prime and 16 is not prime
 - (b) If 7 is not prime, then 7 is my favorite number.
 - (c) If 16 is my favorite number, then $16 + 1$ is my favorite number
 - (d) 16 is prime or 7 is prime
 - (e) 17 is my favorite number and 16 is not prime.
 - (f) If 16 is not prime, then 16 is my favorite number.
2. Consider the statement “*If Oscar eats Chinese food, then he drinks milk.*”
 - (a) Write the converse of the statement.
 - (b) Write the contrapositive of the statement.
 - (c) Is it possible for the contrapositive to be false? If it was, what would that tell you?
 - (d) Suppose the original statement is true, and that Oscar drinks milk. Can you conclude anything (about his eating Chinese food)? Explain.
 - (e) Suppose the original statement is true, and that Oscar does not drink milk. Can you conclude anything (about his eating Chinese food)? Explain.
3. Give explicit examples of statements P and Q that make the following false. Be creative, no repeats from class are allowed and you should pick different statements for each part.
 - (a) $(P \rightarrow Q) \wedge \neg(Q \rightarrow P)$.
 - (b) $\neg(P \wedge Q)$.
 - (c) $P \vee \neg Q$.
4. Let P and Q be mathematical statements. Show that $P \rightarrow Q$ is logically equivalent to $\neg P \vee Q$.
5. Show De Morgan’s laws: If P and Q are mathematical statements, then the following hold:
 - (a) $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$;
 - (b) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$.
6. Are the statements $(P \vee Q) \rightarrow R$ and $(P \rightarrow R) \vee (Q \rightarrow R)$ logically equivalent? If so, please prove that they are. Otherwise, give specific statements that make one true and the other one false.