MA 293

Homework 9

Due Apr 4 at 4 pm

Recall the homework guidelines for this course. Do not forget to justify your answers.

1. Suppose you roll five regular 6-sided dice.
   (a) How many different outcomes are possible from a single roll? The order of the dice does not matter.
   (b) For how many of those outcomes is the sum of the results an even number?
2. How many non-negative integer solutions does the equation $x + y + z + w = 25$ have? Justify.
3. How many length-6 lists can be made from the symbols $\{A, B, C, D, E, F, G\}$, if repetition is allowed and the list is in alphabetical order? (Examples: $BBCEGG$, but not $BBBAGG$.)
4. Consider functions $f\colon \{1, 2, 3, 4, 5, 6, 7\} \to \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We say that a function is *non-decreasing* if $a < b$, then $f(a) \leq f(b)$.
   (a) Write examples of two distinct non-decreasing functions.
   (b) Find an example of a function that is not non-decreasing.
   (c) Use the sticks and stones method to count the number of the functions that are non-decreasing. Explain.
5. Let $0 \leq k \leq n$. Use combinatorial proof to show
$$\binom{n}{k} = \binom{n}{n-k}.$$
6. Let $n \geq 1$. Use combinatorial proof to show
$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$
Hint: Consider selecting two elements from a set of $2n$ items partitioned into two equal groups of size n.